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Part	Chapters	Weightage
Part A: Business Maths (40 Marks)	1. Ratio and Proportion, Indices and Logarithms	20%-30%
	2. Equations and Matrices	20%-30%
	3. Linear Inequalities with Objective Functions and Optimization	20%-30%
	4. Time value of Money	30%-40%
	5. Permutations and Combinations	30%-40%
	6. Sequence and Series	30%-40%
	7. Sets, Relations, and Functions	30%-50%
	8. Basic applications of Differential and Integral calculus	30%-50%
Part B: Logical Reasoning (20 Marks)	1. Number series, Coding, and Decoding and odd man out	60%-70%
	2. Direction Tests	60%-70%
	3. Seating Arrangements	60%-70%
	4. Blood Relations	30%-40%
	5. Syllogism	30%-40%
Part C: Statistics (40 Marks)	1. Statistical description of Data	45%-50%
	2. Measures of Central tendency and Dispersion	45%-50%
	3. Probability	25%-30%
	4. Theoretical Distributions	25%-30%
	5. Correlation and Regression	10%-15%
	6. Index Numbers and Time Series	10%-15%

RATIO, PROPORTION, INDICES & LOGARITHM

RATIO (written as x : y)

Comparison of two or more things of same kind

$\frac{x}{y}$

x ⟵ First term or **antecedent**

y ⟵ Second term or **Consequent**

- Ratio is expressed in lowest terms. E.g. 10:6 ⇒ 5:3
- Order of Ratio is Maintained. E.g. 4:3 ≠ 3:4
- Ratio exists with quantities having same unit (& kind).
- To Compare Ratios, convert to equivalent Fractions
  - $\underbrace{2\frac{1}{3} : 3 \ \& \ 4 : 5}_{\text{Compare}} \rightarrow \underbrace{\frac{7}{3} : 3 \ \& \ 4 : 5}$ Convert into improper fraction

Step 1 : $\frac{7}{3} : 3$, Multiply by 3 both sides, we get, **7:9**

Step 2 : For, 7:9 & 4:5, Take LCM and compare

$$\frac{7}{9} : \frac{4}{5} \Rightarrow \frac{5 \times 7}{5 \times 9} : \frac{4 \times 9}{5 \times 9} \Rightarrow \frac{35}{45} : \frac{36}{45}$$

4:5 is larger ratio compared to 7/3 : 3

- If a quantity increases or decreases in ratio a:b
  - New quantity = b/a of original quantity a
  - **b/a** is factor multiplying ratio
  - Put b as the Higher term if increasing and lower term if decreasing. (refer to question)

Q : If Sachin used to eat 10 pizzas in a month but has now **reduced** in the ratio 3:5, How many pizzas does he eat in a month now?

- $10 \times \frac{3}{5} \Rightarrow$ **6 Pizza's** (3 is written above as pizzas were reduced)

- **Inverse Ratio** - a:b & b:a are inverse as product is 1.
- **Duplicate ratio** of a : b is $a^2 : b^2$
- **Triplicate ratio** of a : b is $a^3 : b^3$
- **Sub-Duplicate ratio** of a : b is $\sqrt{a} : \sqrt{b}$
- **Sub-Triplicate ratio** of a : b is $3\sqrt{a} : 3\sqrt{b}$
- **Commensurable** : If the terms of the ratio are integers, E.g. 3 :2
- **Incommensurable** : If the terms of the ratio are not integers, E.g. $\sqrt{3} : \sqrt{2}$
- **Compound Ratio** of a:b & c:d is **ac : bd.**
- **Continued Ratio** of three similar kinds is a:b:c.
- **Greater Inequality** of a : b if a > b (E.g. 7 : 5)
- **Less Inequality** of a : b if a < b (E.g. 3 : 4)

PROPORTION (written as a:b :: c:d)

- An equality of two ratios, Here, a & d are **extremes** whereas c & d are **mean terms (or middle terms)**
- **d** is also called fourth proportional
- **Cross product rules** says if a/b = c/d, then ad = bc
  - Product of extremes = Product of means

- **Continuous proportion** of a, b, c (same kind or unit)
  - a : b = b : c ⇒ $\mathbf{b^2 = ac}$
  - Here, b is mean proportional between a & c
- E.g. Mean proportional of 16, 25 is
  - $b^2 = 16 \times 25 \Rightarrow b^2 = 400 \Rightarrow b = 20$

Properties of Proportion, if, a:b = c:d, then

- b:a = d:c (**Invertendo**)
- a:c = b:d (**Alternendo**)
- a +b:b = c + d:d (**Componendo**)
- a–b:b = c–d:d (**Dividendo**)
- a+b:a–b = c+d:c–d (**Componendo and Dividendo**)

- If a:b = c:d = e:f =…., then each of these ratios (**Addendo**) is equal to (a + c + e +…):(b + d + f+––)

Q. if 2/3 = 4/6, then if 2/4 = 3/6 is **Alternendo**

Q. if a/4 = b/5 = c/9, then (a+b+c)/3 is given by
- Suppose, a/4 = b/5 = c/9 = k

⇒(4k + 5k + 9k)/9 = 2

Indices (written as a^n)

- a is the **base** and n is the **power** or **index**
- Note : if a is Real – {0} and n = 0, then, $a^n = 1$

- **Law 1 :** $a^m \times a^n = a^{m+n}$
- **Law 2** : $a^m / a^n = a^{m-n}$
- **Law 3 :** $(a^m)^n = a^{mn}$
- **Law 4 :** $(ab)^n = a^n b^n$

- **Negative Power :** $a^{-n} = 1/a^n$
- **Equal Power** : If, $a^x = a^y$, then, $x = y$

Roots (or Fractional index)

$\sqrt{a} = (a)^{1/2}$	$\sqrt[3]{a} = (a)^{1/3}$	$\sqrt[4]{a} = (a)^{1/4}$	$\sqrt[m]{a} = (a)^{1/m}$

$$\left\{\frac{(x+y)^{2/3} \times (x-y)^{3/2}}{\sqrt{x+y} \times \sqrt{(x+y)^3}}\right\}^6$$

Solve the question

$$= \left\{\frac{(x+y)^{2/3} \times \cancel{(x-y)^{3/2}}}{(x+y)^{1/2} \times \cancel{(x-y)^{3/2}}}\right\}^6$$

Roots are written as power of 1/2 and red mark according to law 2

$$= \left\{\frac{(x+y)^4}{(x+y)^3}\right\} = (x+y)^{4-3} = (x+y)$$

Power 6 goes inside bracket and gets multiplied according to Law 3

Question for Reference

LOGARITHM

- Helps to find power or index to which base must be raised to produce the number on other side.
- E.g. if, $2^x = 10$; x can be written as $\mathbf{x = \log_2 10}$.
- If $a^x = n$, where $a,n > 0$ and $a \neq 1$, then $\mathbf{x = \log_a n}$
- Note : $a^0 = 1 \Rightarrow \log_a 1 = 0$ & $a^1 = 1 \Rightarrow \log_a a = 1$

$\log_a mn = \log_a m + \log_a n$	$\log_a(m/n) = \log_a m - \log_a n$
$\log_a(m)^n = n\log_a m$	$\log_p m + \log_m p = 1$

Change of Base can be done with following formula

$$\log_a m = \log_p m \times \log_a p \Rightarrow \log_p m = (\log_a m/\log_a p)$$

Trick to Find Log of any number (n)

1. Type number (n) and press root (√) for 15 times
2. Subtract 1 from above result
3. Divide the end result with number 0.000070274

Basics of Antilogarithm

- If $\mathbf{x = \log_a n}$, then $\mathbf{n = antilog_a x}$.

Trick to Find AntiLog of any number (n)

1. Multiply the number (n) with 0.000070274
2. Add 1 into the result above
3. Press "Multiply & =" for 15 times.

EQUATIONS

Mathematical statement of Equality (=)

- If Equality true for certain value of variable, **Conditional** Equation
  - $2x + 3y = 8$ (is true only if $x = 1$ & $y = 2$)
- If Equality true for all value of variable, **Identity**
  - $(x+2) + (x+3) = 2x + 5$

Solution of variable satisfying equation is **ROOT.**

Linear Equation (Simple Equation)

- An Equation of the form $ax + b = 0$, where a,b are constants and $a \neq 0$

Q. A student had to divide half of no. by 6 and the other half by 4, and add, but mistakenly divides the whole by 5. If answer is 4 short from correct, what's the no.?

Way of Approaching question Let the number be x

Correct way	Incorrect way	Difference
$\frac{\frac{1}{2}x}{6} + \frac{\frac{1}{2}x}{4}$ $\frac{x}{12} + \frac{x}{8}$	$\frac{x}{5}$	$\frac{x}{5} - \frac{x}{12} + \frac{x}{8} = 4$ Take LCM and solve for x

Linear Equation in two variable

- General Form ax + by + c = 0

Solving simultaneous Equations

- **Elimination Method**
  - Reduce two variables into one by operating over equations (addition/subtraction)

① $3x - y = 2$ ② $x + 2y = 10$	$6x - \cancel{2y} = 4$ $x + \cancel{2y} = 10$	$7x = 14$ $x = 2$
Multiply ① by 2	Add it to ② to eliminate y	Solving, we get x

  - Put x = 2 in either ① or ② to get y

- **Cross Multiplication Method**
  - Given, $ax_1 + by_1 + c_1 = 0$ & $ax_2 + by_2 + c_2 = 0$
  - Result

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

  - Solution

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \ \& \ y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Quadratic Equation

- An Equation of the form $ax^2 + bx + c = 0$, where a,b,c are constants and $a \neq 0$
  - $b = 0 \Rightarrow$ Pure Quadratic Equation
  - $b \neq 0 \Rightarrow$ Affected Quadratic Equation

Important Concepts/Formulas of Quadratic Equation

if roots of quadratic equation are α, β

$\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$	$\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

- Sum of Roots, $\alpha + \beta = -b/a$
- Product of Roots, $\alpha.\beta = c/a$

- Constructing a quadratic equation
  - x^2 - (Sum of Roots)x + (Product of roots) = 0

Nature of Roots, $b^2 - 4ac$ is discriminant, D	
$b^2 - 4ac = 0$	Real & Equal Roots
$b^2 - 4ac > 0$	Real but Distinct Roots
$b^2 - 4ac < 0$	Roots Are Imaginary
D = Perfect square	Real, rational and unequal roots
D ≠ Perfect square	Real, irrational and unequal roots

Cubic Equation

- An Equation of 3rd Degree, $ax^3 + bx^2 + cx + d = 0$

Method to Find Root of Quadratic Equation

- if roots of Cubic equation are α, β, γ

- Sum of Roots, $\alpha + \beta + \gamma = \mathbf{-b/a}$
- Product of Roots, $\alpha.\beta.\gamma = \mathbf{-d/a}$
- $\alpha.\beta + \beta.\gamma + \alpha.\gamma = \mathbf{c/a}$

TIME VALUE FOR MONEY

Why is Interest Paid?

- **Time Value of money** : Sum of Money received in future will have less value than at present.
- **Opportunity Cost** : Lending incurs opportunity cost due to the possibilities of alternative use.
- **Inflation** : Given amount of money buys fewer goods in future than it will now.
- **Liquidity Preference** : People prefer to have resources that are cash convertible.
- **Risk Factor** : Borrower may go bankrupt, Thus it is determinable factor for Rate of Interest.

Important Definitions.

- **Interest :** Cost of borrowing or the return on investment, expressed as a percentage.
- **Principal** : Initial amount of money borrowed or invested before interest or returns.
- **Rate of Interest** : It is the percentage at which money grows or the cost of borrowing.
- **Accumulated Amount** : The total sum of principal and interest after a specified period.

SIMPLE INTEREST (S.I)

- It is a fixed percentage of the principal amount, paid or earned over time without compounding.
- Directly Proportional to Principal Amount (P), Rate of Interest (i or R/100) and Time (T).

$SI = P\left(\frac{R}{100}\right)T$	$A. = P\left(1 + \frac{R}{100}\right)T$

Q. *Certain sum amounts to Rs. 15748 in 3 years at simple interest at r% p.a. The same sum amount to Rs. 16,510 at (r + 2)% pa. simple interest in the same time. What is r?*

Way of Approaching question   Let the Principal be P

1st Case : P for 3 Years at r% with A = 15748

$15748 = P(1 + \frac{r}{100}).3$

2nd Case : P for 3 Years at (r+2)% with A = 16510

$16510 = P(1 + \frac{r+2}{100}).3$

Divide both the cases

$$\frac{15748 = P(1 + \frac{r}{100}).3}{16510 = P(1 + \frac{r+2}{100}).3} = \frac{15748 = (100 + r)}{16510 = (100 + r+2)}$$

Solve for r

COMPOUND INTEREST VS SIMPLE INTEREST

- Let Principal = 1000, Interest = 5% (or 0.05) & T = 2.
- Simple Interest Calculated
  - First year Interest = 1000 x 0.05 = 50
  - Second year Interest = 1000 x 0.05 = 50
- Compound Interest Calculated
  - First year Interest = 1000 x 0.05 = 50
    - Principal now becomes 1050
  - Second year Interest = 1050 x 0.05 = 52.5
- Total **Simple Interest = 100** whereas total **Compound Interest = 102.5.**

HOW TO CALCULATE COMPOUND INTEREST

The accumulated money is	$A = P(1+i)^n$

$$i = \frac{\text{Annual Interest Rate}}{\text{Conversions Per Year (m)}} = \frac{R}{100.\,m}$$

Conversion Per year

$$\text{Interest} = A_n - P = P\left[(1+i)^n - 1\right]$$

n is total conversions, i.e. **T x conversions per year**

Conversion period	Conversion Per year (m)
1 Day	365 (Compounded daily)
1 Month	12 (Compounded Monthly)
3 Months	4 (Compounded Quarterly)
6 Months	2 (Compound Semi Annually)
1 Year	1 (Compounded Annually)

Q. *In what time will ₹4,000 amount to ₹4,410 at 10% per annum interest compounded half-yearly.*

Way of Approaching question Let the Time Period be T

We know, n = T x conversions per year = 1 x 2

$$\underbrace{4410}_{\text{Accumulated money}} = \underbrace{4000}_{\text{Principal}} \left(1 + \underbrace{\frac{10}{100}}_{\text{Interest}} \times \frac{1}{\underbrace{2}_{\text{Conversions per year}}}\right)^{T \times 2}$$

EFFECTIVE RATE OF INTEREST (E)

- If interest is compounded more than once a year, then effective rate of interest exceeds per annum interest rate.

Relation of Nominal & Effective interest	$I = P.E.T$
Other way of computing m = conversion per year	$E = (1 + i)^m - 1$

Q. *The effective rate of return for 24% per annum convertible monthly is given as:*

$$E = \left(1 + \frac{24}{100} \times \frac{1}{\underbrace{12}_{\text{Conversions per year}}}\right)^{\overbrace{12}^{\text{Conversions per year}}} - 1$$

ANNUITY

- When we pay (or receive) a fixed amount of money periodically over a specified period of time.
- When Payment takes place forever, it's **Perpetuity.**

- For a recurring payment to be Annuity,
  - Amount must be constant
  - Time interval of two payments must be same.

Annuity

Annuity Regular	Annuity due or intermediate
↓	↓
First payment at end of the period	First payment in the first period

FUTURE VALUE

- Cash value of an investment in future

Future value & Single cashflow are related as	$\mathrm{F.V.} = \mathrm{C.F.}\,(1+\mathrm{i})^{\mathrm{T}}$

T = Time Period ;C.F. = Cashflow ;F.V. = Future value

Q. You invest \$10,000 at 5% for 2 years than what will be the Future value of money invested after 2 year.

$$\text{F.V.} = \underbrace{10000}_{\text{Cash Flow}}\left(1 + \underbrace{\frac{5}{100}}_{\text{Interest}}\right)^{\overset{\text{Time Period}}{2}}$$

Way of Approaching question

FUTURE VALUE OF ANNUITY REGULAR

If A be the periodic payments, the future value A(n, i) of the annuity is given by

$$A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

*i should be used in decimals

n = no. of payments

***Q.** \$500 is invested at the end of each month in an account paying interest 12% per year compounded monthly. What is the future value after 9th payment?*

$$i = \frac{12}{100} \times \frac{1}{12} = 0.01 \quad t = \frac{9}{12}\text{ yrs} \quad n = \frac{9}{12} \times 12 = 9$$

$$A(9, 0.01) = 500\left(\frac{(1+0.01)^9 - 1}{0.01}\right)$$

FUTURE VALUE OF ANNUITY DUE OR INTERMEDIATE

Future value of annuity due or intermediate = Future value of annuity regular **x (i+1)**

Step-1 Calculate the future value as though it is an ordinary annuity.

Step-2 Multiply the result by (1+ i)

$$A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right] \times (i+1)$$

- *To distinguish between annuity regular and annuity due/intermediate, search the question for keywords.*
    - **Starting** of the year/month : Annuity **regular**
    - **End** of the year/month : Annuity **Intermediate**

PRESENT VALUE

- Value of future money in the present.

Present value Formula A_n = Amount due at end of n period at rate i.	$P.V. = \dfrac{A_n}{(1+i)^n}$

***Q.** Find the present value of 5000 to be required after 4 years if the interest was 7%?*

$$P.V. = \frac{5000}{(1+0.07)^4}$$

PRESENT VALUE OF ANNUITY REGULAR

$$V = \frac{A(n,i)}{(i+1)^n} = A\left[\frac{(1+i)^n - 1}{i.(i+1)^n}\right] = A.P(n,i)$$

Q. The present value of an annuity of ₹80 a year for 20 years at 5% p.a is

Way of Approaching question A = ₹80; i = 0.05; n = 20

$$V = 80\left[\frac{(1+0.05)^{20} - 1}{0.05.(0.05+1)^{20}}\right]$$

PRESENT VALUE OF ANNUITY INTERMEDIATE/DUE

Step-1 Calculate the Present value of **n-1 period** of annuity regular

Step-2 Add the initial cash payment (A) to the step 1

Q. Your Papa gives you ₹10000 every year starting from today for next 5 years as a gift. So, you invest it at the interest rate of 10% in mutual funds today morning. What should be the present value of this annuity?

Way of Approaching question A = 10000; i = 0.15; n = 5

Step 1 : Present value for 4 Years i.e n-1

V = 10000 x P (4, 0.10) = 31698.70

Step 2 : Add one cash payment to above

V = 10000 + 31698.70 = 41698.70

Sinking Fund : Fund credited for a specific purpose

Sinking fund deposited is **A = P.A(n,i)**

Here, P = Periodic Payment and A = amount to be saved

Application : Leasing

- Financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time(lease period) for a consideration given period of time.

Easy Example : *A company has a machine worth 5 Lacs, it can lease out at 2 Lacs P.A. for 4 years, If company invests the rent at 14% P.A., is leasing favourable?*

Application : Capital Expenditure

- Capital expenditure means purchasing an asset today in result of benefits of tomorrow which would flow across the life of the investment.

Easy Example : *You buy a factory worth 10 Crore by borrowing money at 10% interest, if you generate a return of 3 Crore every year,* ***Will you be able to recover the cost in 4 years?***

To Check whether Buying an asset/Leasing is favourable or not, Check for the present value of the amount with given interest and periodic cash for fixed period of time

Application : Valuation of a bond

- A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issue

To Calculate present value of bond,

$$\frac{I}{(1+i)}+\frac{I}{(1+i)^2}+\frac{I}{(1+i)^3}+\cdots\frac{I}{(1+i)^n}+\frac{B.I.}{(1+i)^n}$$

- Here, $\mathbf{I}$ = Interest amount provided by issuer,
- B.I. = Bond value
- i = Interest Percentage that investor requires

Perpetuity

- When you get annuity for unlimited amount of time

$$PVA_{\infty}=\frac{A}{i}$$

A=Particular amount received

PVA = Amount to be Paid to get A

Growing perpetuity means the periodic installment is increasing with fixed interest rate

$$PVA_{\infty}=\frac{A}{i-g}$$

$$g=\frac{G}{m\times 100}$$

Rate of Return

- Net Present Value (NPV)= Present value of cash inflow - Present value of cash outflow
- RULE : To make decision
  - If NPV > 0 Accept the Proposal
  - If NPV < 0 Reject the Proposal

- Easy Example : You invest 1,00,000/- in a machine and expect a return of 30000 in 1st year, 80000 in 2nd, 50000 in 3rd, **is it worth investing?**

Check for the present value of the all the returns with given interest and add them all to get NPV.

Compound Annual Growth Rate (CAGR)

To find the annualized gain of an investment over a given time period

$$\mathrm{CAGR} = i = \left(\frac{\mathrm{F.V.}}{\mathrm{P.V.}}\right)^{\frac{1}{n}} - 1$$

In Other Words, $$\mathrm{CAGR}(t_0, t_n) = \left(\frac{V_{t_n}}{V_{t_0}}\right)^{\frac{1}{t_n - t_0}} - 1$$

V_{t_n} = End Period	V_{t_0} = Beginning Period

PERMUTATION & COMBINATION

Fundamental Principle of multiplication

- If one task can be completed in m ways and another task (independent of first task) can be completed in n ways (after first task has ended)
  - Total ways in which both tasks can be performed simultaneously is **m x n**
- Easy example : if there are 12 gates to enter hall, & exiting is allowed from a different gate.
  - Since remaining gates to exit are 11
  - Total no. of ways : $12 \times 11 = 132$

Factorial Notation : n!

- Product of several consecutive integers
  - $n! = n.(n-1).(n-2).(n-3)....3.2.1$
  - $n! = n.(n-1)! = n.(n-1).(n-2)!$
- Example : $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1.$
- Remember that $0! = 1$ and $1! = 1$

Permutations (Order Matters)

- Arrangement of objects in a specific order

$$^{n}P_{r} = \frac{n!}{(n-r)!} = n(n-1)(n-2)\ ....\ (n-r+1)$$

Total objects : **n** ; Objects taken : **r** $(0 \leq r \leq n)$

- Remember, ${}^nP_n = n!$ and ${}^nP_1 = n$
- Easy Example : 8 People participate in competition : What are the ways to give gold, silver and bronze.
  - Since the order matters and only 3 people can be selected out of 8
  - ${}^8P_3 = 336$ ways to give medals
- Easy Example : 10 Students are arranged in such a way that the Tallest and Shortest never come together.
  - a = Total arrangements without condition = 10!
  - b = Arrangements to keep Tallest and Shortest together = ${}^9P_9 \times {}^2P_2 = 9! \times 2!$
  - Conditional permutation = a – b = 10! – (9! x 2!)

Circular Permutation

- Permutations in Circular Arrangement
- Arrangement of n things in n places.

Linear Permutation
${}^nP_n = n!$

Circular Permutation
$(n-1)!$

- The number of ways arranging n persons in round table (no person has two same neighbours)

$= \frac{1}{2}(n-1)!$

- The number of necklaces formed with n beads of different colours)

$= \frac{1}{2}(n-1)!$

Permutation with restriction

1. No. of Permutations of n objects taken r at a time when a particular object is not taken in any : $^{n-1}P_r$

- Easy Example : Ways to serve 6 Plates of Pasta to 10 people, if one doesn't eat maida.
  - Total ways $^{10-1}P_6 = {}^9P_6$

2. No. of Permutations of n objects taken r at a time when a particular object is taken in all : $^{n-1}P_{r-1}$

- Easy Example : Ways to arrange 10 students when tallest and shortest are never together.
  - Total ways $(^{10-1}P_{10-1}).{}^2P_2$

Combinations (Order Doesn't Matter)

- Selection of objects, without arrangement.

$$C(n, r) \text{ or } {}^nC_r = \frac{n!}{r!(n-r)!} \quad \text{where, } (0 \le r \le n)$$

Remember : $^nC_0 = {}^nC_n = 1$ & $^nC_r = {}^nC_{n-r}$

- Easy Example : Ways to hire 3 Employees out of 10 applicants. ⇒ Total ways to select : $^{10}C_3$

Note the following results

$^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$	$^nP_r = {}^{n-1}P_r + r.{}^{n-1}P_{r-1}$

Permutation with REPETITION involved

- n different objects taken r at a time when **each object** may be repeated any no. of times is $\mathbf{n^r}$.

e.g. 7 letters and 5 letter-boxes, Total ways to put letters

Every letter has 5 boxes to choose : $\mathbf{5^7}$

Permutation of ALIKE Objects

- n different objects all at once p objects are of 1st kind, q objects are of 2nd kind, then ways : **n!/(p!q!)**

e.g. Ways to arrange 'SWALLOW' = 7!/(2!2!) = 1260

Combination Standard Results

- n different Objects taken all or some at once = $\mathbf{2^n-1}$
- n Objects taken all or some at once, n_1 are alike, n_2 are alike & n_3 are alike = $\mathbf{(n_1+1) \times (n_2+1) \times (n_3+1)-1}$
- If we have to select the combination such that r things to be selected from n, and t things to be selected from m. Then, total selections are $\mathbf{{}^nC_r \times {}^mC_t}$

SETS

- **Sets :** A well-defined and well distinguished collection of objects. Notation being,
  - **Set** → Capital alphabets
  - **Elements** (inside set) → Small alphabets

Important Signs

$\exists$	there exists	:	such that
$\vert$	such that	,	and
$\forall$	for every/ for all	$\in$	belongs to
$\notin$	doesn't belongs to	$\Rightarrow$	implies
$\subset$	Subset	iff	if and only if

Representation of sets

Roster form

- Elements separated by comma & enclosed in braces.
- Elements not repeated
- Order of writing a set doesn't matter

E.g. $A = \{a, e, i, o, u\}$ or $\{e, i, a, u, o\}$

Set Builder form

- Elements of a set possess a common property. **e.g.**
  - A = Set of vowels in the alphabet series
  - $B = \{x: x \text{ is a natural number and } 5 < x < 10\}$

Few Important sets

- **N** : Set of All Natural Numbers
- **Z** : Set of All Integers
- **Q** : Set of all Rational Numbers
- **R** : Set of Real Numbers
- **Z+** : Set of All Positive Integers
- **Q+** : Set of All Positive Rational Numbers
- **R+** : Set of All Positive Real Numbers

IMPORTANT

Special Sets

- **Empty/Null Set** : Set with no elements. Represented by { }, ϕ.
- **Singelton Set** : Set with only one element.
- **Finite Set** : Set with Finite no. of Elements.
- **Infinite Set** : Set with Infinite no. of Elements. e.g. **N, Z, Q, R.**
- **Universal Set (U) :** Containing all possible elements.
- **Equal Sets** : A=B, if every element of A is in B
- **Equivalent Sets** : finite sets, if n(A) = n(B)
- **Disjoint Sets** : When no elements of A occur in B

Subset, Superset and Properties

- **Superset** : $A \subset B$: B is superset of A
- **Subset** : $A \subset B$: if & only if all elements of A are in B.
    - If $A \subset B$ & $B \subset A \Leftrightarrow A = B$
    - Every Set is a subset of itself. $A \subseteq A$
    - Null set ϕ is a subset to all sets.

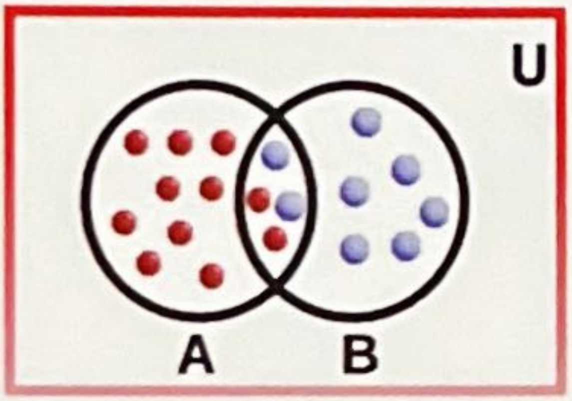
Cardinality/Cardinal Number/Order of a set

- Number of distinct elements in a set.
- Represented as n(A), O(A), |A| for a set A.
    - Cardinality of A = Set of Vowels is **5**

Venn Diagram

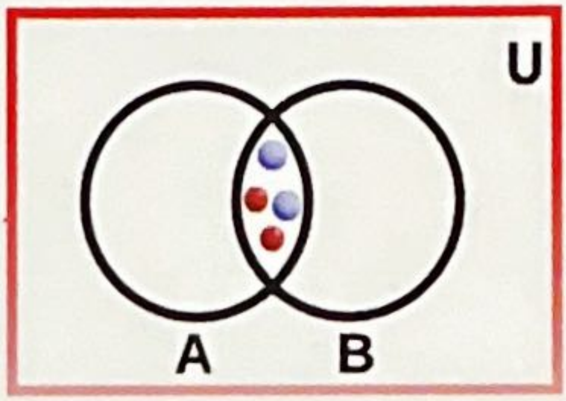
UNION

$A \cup B = \{x : x \in A \text{ or } x \in B\}$



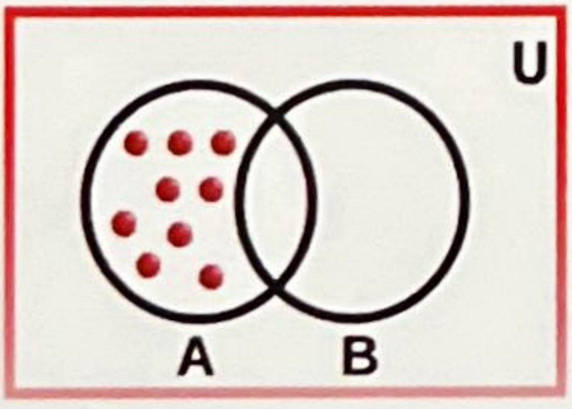
INTERSECTION

$A \cap B = \{x : x \in A \text{ and } x \in B\}$



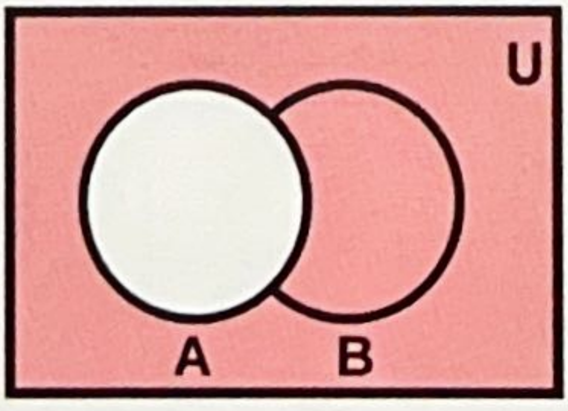
DIFFERENCE

$A - B = \{x : x \in A \text{ and } x \notin B\}$



COMPLIMENT

$A^c = U - A$



Sets with no common element are called **Disjoint**

Laws of Sets

- **$n(A \cup B)$** $= n(A) + n(B) - n(A \cap B)$
  - If $A \cap B = \phi$, then, $n(A \cup B) = n(A) + n(B)$
- **$n(A \cup B \cup C)$** $= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
  - If A,B,C are disjoint: **$n(A \cup B \cup C)$**$=n(A)+n(B)+n(C)$
- **$n(A - B)$** $=$ **$n(A \cap B^c)$** $= n(A) - n(A \cap B)$

Cartesian Product of Sets

- Ordered Pair : Two elements a & b listed in specific order, written as (a,b)
- **Cartesian Product of sets** : Set of ordered pairs (a,b) such that a $\in$ A & b $\in$ B is cartesian product (A x B) .
  - $A \times B = \{ (a,b) : a \in A \ \& \ b \in B \}$

1	2	3
(1,x)	(2,x)	(3,x)
(1,y)	(2,y)	(3,y)

A×B

Set A

Set B

RELATIONS & FUNCTIONS

- **Relations :** A subset of $A \times B$ defined as $(R : A \to B)$

  Total Relations from $A \to B$: $2^{n(A \times B)} = 2^{n(A)} 2^{n(B)}$

Classification of Relations, $R : A \to A = \{1,2,3\}$

- **Identity Relation :** $I = \{ (a,a) \, , \, a \in A \}$
  - **e.g.** $R_1 = \{ (1,1) \, ; (2,2) \, ; (3,3) \}$
- **Reflexive Relation** : $(a,a) \in R$
  - e.g. $R_1' = \{ (1,1) \, ; (2,2) \, ; (3,3) \, ; (1,2) \, ; (3,2) \}$
- **Symmetric Relation** : $(a,b) \in R_1 \Rightarrow (b,a) \in R_1 \; a,b \in A$
  - e.g. $R_2 : \{ (1,2) \, ; (2,1) \, ; (1,1) \}$
- **Transitive Relation** : $(a,b) \in R_2 \; \& (b,c) \in R_2 \Rightarrow (a,c) \in R_2$
  - e.g. $R_3 : \{ (1,2) \, ; (2,3) \, ; (1,3) \, ; (2,2) \}$
- **NOTE** : Every Identity relation is a reflexive relation but every reflexive relation need not be an Identity.
- **Equivalence Relation :** If a relation is Reflexive, Symmetric and Transitive.

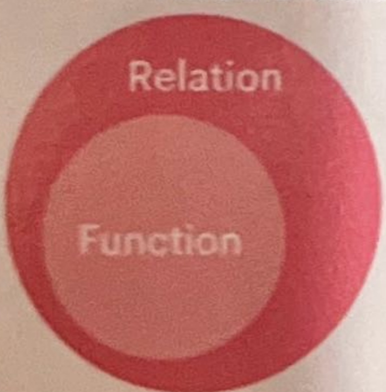
Inverse Relation

- If Relation R is defined on A, then there exists R^{-1} such that
  - $R^{-1} = \{(b,a) : (a,b) \in R\}$
  - Here, $Dom(R^{-1}) = Range\,(R)$
  - $Range\,(R^{-1}) = Dom\,(R)$

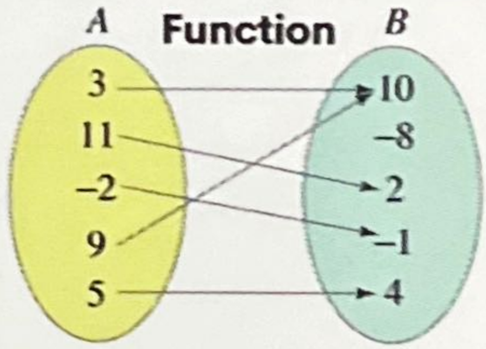
Domain and Range of a Relation

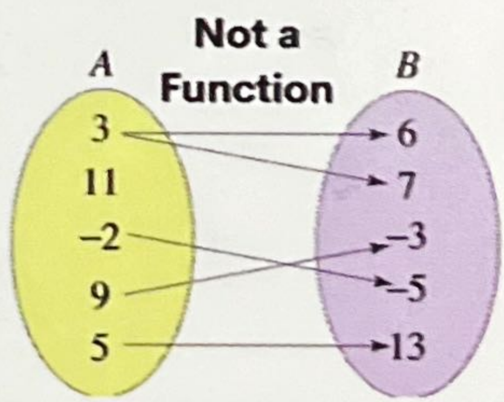
- If A = {1,2,3,4} & B = {4,5,6} and we define,
  - R_1 = {(1,4), (1,5), (2,4), (3,5)} **(Relation)**
  - Domain = {1, 2, 3} and Range = {4, 5}
- In General,
  - **Dom (R) = {a : (a,b) ∈ R}**
  - **Range (R) = {b : (a,b) ∈ R}**

- **Functions :** A relation *f*:A→B is considered function if
  - Every element of A is associated with some B
  - Association is unique



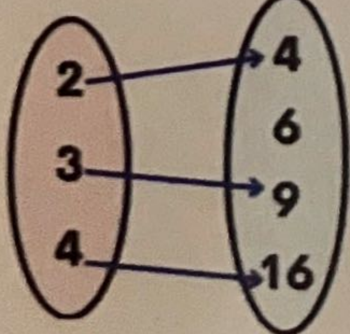
UNIQUE





Domain and Range of a Function

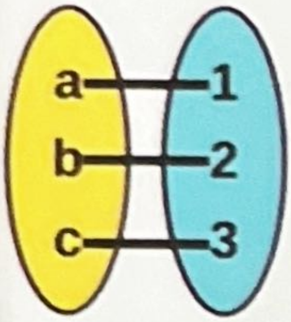
- If A = {2, 3, 4} & B = {4, 6, 9, 16} and f(x) = x^2



f = { (2,4), (3,9), (4,16) }

- Domain = { 2, 3, 4 }
- Range = { 4, 9, 16}
- Co-Domain = { 4, 6, 9, 16}

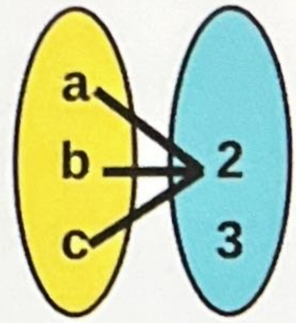
Classification of Functions



one one
(Injective)

if $f(x_1) = f(x_2)$
$\Rightarrow x_1 = x_2$

Followed



Many one
(Surjective)

Codomain = Range : **Onto**
Codomain ≠ Range : **Into**

One-One + Onto Function is Bijective

Other Functions

- **Identity Function** = I : A → A : I (x) = x for all x ∈ A
  - One-One Function with Domain A & Range A
- **Constant Function** : Let f : A → B such that all elements in A have same image in B.
- **Equal Functions** : if two functions have same domain & they satisfy condition f(x) = g(x), for all x.
- **Inverse Function** : if f : A→B is one-one & onto , then there exists f^{-1} : A→B, for which ,
  - $f(x) = y \Rightarrow f^{-1}(y) = x$
  - Inverse function is also one-one onto

Composite Function

- Let f: A → B and g: B → C be two functions. Then the function gof: A → C defined by
  - $(gof)(x) = g(f(x))\ \forall\ x \in A$
- $f(x) = 2x^2$; $g(x) = 3x \Rightarrow fog = f(g(x)) = 2(3x)^2 = 18x^2$

LIMITS AND CONTINUITY

Concept of RHL and LHL

$\lim_{x \to a^-} f(x) = \lim_{x \to h} f(a - h)$ **Left Hand Limit**

$\lim_{x \to a^+} f(x) = \lim_{x \to h} f(a + h)$ **Right Hand Limit**

if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = l$ (finite)

then, $\lim_{x \to a} f(x) = l$ ⇒ Limit Exists

Algebra of Limits

(a) $\lim_{x\to a}(f+g)(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(b) $\lim_{x\to a}(f-g)(x) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

(c) $\lim_{x\to a}(c\cdot f)(x) = c\lim_{x\to a} f(x)$ [c is a constant]

(d) $\lim_{x\to a}(fg)(x) = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$

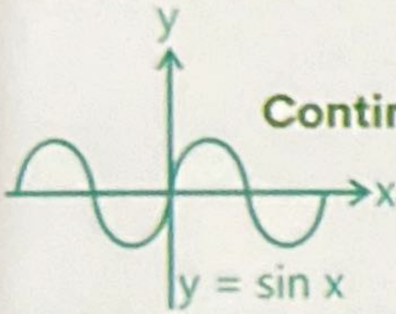
(e) $\lim_{x\to a}\left(\frac{f}{g}\right)(x) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

(f) $\lim_{x\to a}\left(f(x)\right)^n = \left(\lim_{x\to a} f(x)\right)^n$

Some Important Limits

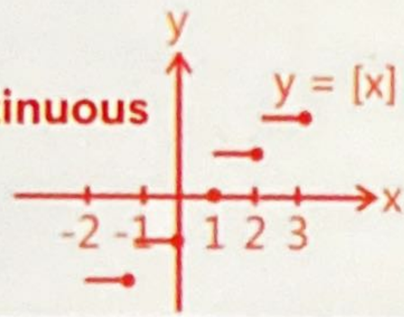
$\lim_{x\to 0}\frac{e^x-1}{x}=1$	$\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$
$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$	$\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

CONTINUITY



Continuous

Discontinuous



Continuity at a point

- A function is said to be continuous at a point if,
- If f(a) exists and $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

- Sum, Difference & Product of two continuous functions is a continuous function
- The quotient of two continuous functions is continuous provided denominator is not equal to zero.

SEQUENCE & SERIES

Some Terms

- $a_1, a_2, a_3 \ldots a_n$ are in sequence when numbers follow a particular order or law.

Some Important Sequences

Natural Numbers	1, 2, 3, 4, 5, 6....
n odd natural numbers	1, 3, 5, 7, 9, 11....
n even natural numbers	2, 4, 6, 8, 10....
Squares of natural no.	$1^2, 2^2, 3^2, 4^2, 5^2 \ldots.$
Sequence of (1/n)	1, 1/2, 1/3, 1/4....
Sequence of 1/(n+2)	1/3, 1/4, 1/5, 1/6 .....

Series

- Adding elements of sequence forms a sequence,
  - $a_1 + a_2 + a_3 \ldots + a_n$ are in series.
- **Finite Series** : Number of terms are finite
  - e.g. First 100 natural numbers
- **Infinite Series** : Number of terms are infinite
  - e.g. All Integers
- If S_n is the sum of n terms of a series then, it is equal to $\sum_{r=1}^{n} a_n$ or $\sum a_n$

Arithmetic Progression (A.P.)	
Series : $a, a + d, a + 2d \ldots\ldots \underbrace{a + (n-1)d}_{\text{nth term}}$	
Common Difference	$d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 \ldots$
nth term of series (t_n)	$[a + (n-1)d]$
Sum of series (ℓ = last term)	• $S_n = (n/2)\,[2a + (n-1)d]$ • $S_n = (n/2)\,[a + \ell]$

Arithmetic Mean (A.M.)

- Arithmetic mean of two numbers a & b is $(a+b)/2$
- A.M. of n numbers : $(a_1 + a_2 + a_3 \ldots + a_n)/n$

Geometric Progression (G.P.)	
Series : $a, ar, ar^2, ar^3 \ldots\ldots \underbrace{ar^{n-1}}_{\text{nth term}}$ r is common ratio	
nth term (T_n)	ar^{n-1}
Common Ratio (r)	r = (Any Term / Preceding Term)
Sum of series	• $S_n = a(1-r^n)/(1-r)$, if $r < 1$ • $S_n = a(r^n-1)/(r-1)$, if $r > 1$ • $S_n = na$, if $r = 1$
Provided $\|r\| > 1$ & $n \to \infty$	$S_\infty = \dfrac{a}{1-r}$

Geometric Mean (G.M.)

- If a, b, c are in G.P. then, $b/a = c/b \Rightarrow b^2 = ac$
- Here, b is called Geometric mean of a and c

Some Important Results

Sum of Natural Numbers	$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$
Sum of the first n odd natural numbers	$\sum_{r=1}^{n} (2r-1) = n^2$
Sum of the first n even natural numbers	$\sum_{r=1}^{n} 2r = n(n+1)$
Sum of the squares of the first n natural numbers	$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$
Sum of the cubes of first n natural numbers	$\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

DIFFERENTIAL CALCULUS

Differential coefficient	
• The instantaneous rate of change of a function with respect to the dependent variable.	
• if $y = f(x)$ and h be small increment in x, then derivative of f(x)	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
• The above is denoted as $f'(x)$, dy/dx or $df(x)/dx$ and known as differential coefficient	
• A function is said to be differentiable at $x = c$, if derivative $f'(c)$ exists.	

Some Standard Derivatives	
$\frac{d}{dx}(\text{constant}) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(\log_e x) = \frac{1}{x}$	$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$	$\frac{d}{dx}(a^x) = a^x \log_e a$

Some Useful Rules	
Product Rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
Division Rule $v \neq 0$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$
Chain Rule	$\frac{d}{dx}(f(g(x))) = f'(g(x)) . g'(x)$
Addition/ Subtraction	$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$
Constant Rule	$\frac{d}{dx}(k\, f(x)) = k\frac{d}{dx}(f(x))$

Derivative of a function of function

- if, $y = f[h(x)]$, then $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = f'(t) \times h'(x)$
  - here $t = h(x)$

- Example : $y = \log(ax^2 + b) = \log t$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{1}{t} \times (2ax + 0)$$

- The above method is known as chain rule.

DIFFERENTIATION OF IMPLICIT FUNCTION

- When in a function the dependent variable is not explicitly isolated on either side of the equation

Working Method

- Every term of f(x, y) = 0 should be differentiated with respect to x
- The value of dy/dx should be obtained by rearranging the terms.

EXAMPLE

$$x^4 + y^3 - 3x^2y = 0$$

$$4x^3 + 3y^2\frac{dy}{dx} - 3\left(2xy + x^2.\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{4x^3 - 6xy}{3x^2 - 3y^2}$$

DIFFERENTIATION OF PARAMETRIC FORM

if x = f(t) and y = g(t), then $\frac{dy}{dx} = \frac{dy\,/\,dt}{dx\,/\,dt}$

Example, x = a(θ + sinθ) and y = a(1 – cosθ)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan\frac{\theta}{2}$$

DIFFERENTIATION OF LOGARITHMIC FUNCTION

- if $y = x^x$ (or any function with power in terms of x), then take log both sides to form
  - $\log y = x.\log x$
  - Now, differentiate wrt x

Gradient of a curve

- To find Gradient of Curve, Differential f(x) w.r.t x
- Now put the coordinates of (x,y) in the equation

Applications of Differential Calculus ①

- **Cost Function :** Consists of two parts
  - Variable Cost V(x) & Fixed Cost F(x)

$$\text{Average Cost } (\text{AC or } \overline{\text{C}}) = \frac{\text{Total Cost}}{\text{Output}} = \frac{\text{C(X)}}{\text{X}}$$

$$\text{Average Variable Cost (AVC)} = \frac{\text{Variable Cost}}{\text{Output}} = \frac{\text{V(X)}}{\text{X}}$$

$$\text{Average Fixed Cost (AFC)} = \frac{\text{Fixed Cost}}{\text{Output}} = \frac{\text{F(X)}}{\text{X}}$$

- **Marginal Cost** : If C(x) is total cost of producing x units, then increase in cost of making one more unit is marginal cost, given by dC/dx.

$$\text{Marginal Cost (MC)} = \frac{dC}{dx}$$

Finding Maxima or Minima (for Applications)

1. Differentiate the function $f(x)$ to find $f'(x)$
2. Set the Derivative to 0 and solve for x to find critical Points (values of x for which slope of curve is zero)
3. Find $f''(x)$ by differentiating $f'(x)$
4. Put critical point values in $f''(x)$
   a. $f''(x) < 0$; then function has local maximum
   b. $f''(x) > 0$; then function has local minima
   c. $f''(x) = 0$, further investigation needed
5. To find maximum or minimum values, Put value of critical point in $f(x)$ and compare.

Applications of Differential Calculus ②

- **Revenue Function :** $R(x)$ represents total turnover after selling x units at a price P $\Rightarrow$ $\mathbf{R(x) = P.x}$
- **Marginal Revenue :** Rate of change of revenue with a unit change in output. $\Rightarrow$ **M.R. = $\mathbf{dR/dx}$**
- **Profit Function :** $P(x)$ represents the difference between Revenue & Total cost $\Rightarrow$ $\mathbf{P(x) = R(x) - C(x)}$
- **Marginal Profit :** Rate of change of profit with a unit change in output. $\Rightarrow$ **M.P. = $\mathbf{dP/dx}$**

INTEGRAL CALCULUS

Basic Integration	
$\int 0.dx = c$	$\int 1.dx = x + c$
$\int k.dx = kx + c$	$\int e^x dx = e^x + c$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c$	$\int \frac{1}{x} dx = \log_e x + c$
$\int a^x dx = \frac{a^x}{\log_e a} + c = a^x \log_a e + c$	
Basic Theorems	
$\int k\, f(x)dx = k\int f(x)dx$	$\frac{d}{dx}\left(\int f(x)dx\right) = f(x)$
$\int \left[f(x) \pm g(x)\right]dx = \int f(x)dx \pm \int g(x)dx$	
Integration by Parts	
$\int (u.v)dx = u\left(\int v\, dx\right) - \int \left(\frac{du}{dx}\right)\left(\int v\, dx\right)dx$	
• Order ILATE (Inverse circular, logarithmic, Algebraic, Trigonometric, Exponential)	

Method of Substitution with Example

- Put z = f(x) and also adjust dz = f′(x).dx

$y = \int (3x+4)^4 . dx$, Let $3x + 4 = z$

Differentiate, $3. dx = dz \Rightarrow dx = dz/3$

$$\int (3x+4)^4 . dx = \frac{1}{3}\int z^4 . dz = \frac{z^5}{15} = \frac{(3x+4)^5}{15}$$

Two Classic integrals

- $\int e^x (f(x) + f'(x))\, dx = e^x . f(x) + c$
- $\int (f(x) + x.f'(x))\, dx = x.f(x) + c$

Standard Substitutions

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$$

$$\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2 + a^2}\right| + c$$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + c$$

$$\int \frac{f'(x)}{f(x)} . dx = \log . f(x) + c$$

Method Of Partial Fractions

In R(x)/g(x), factorize g(x) and then write partial fractions

1. Non-repeated linear factor in the denominator

$$\frac{1}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$$

2. Every repeated linear factor in the denominator.

$$\frac{1}{(x-a)^3(x-b)}=\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)^3}+\frac{D}{(x-b)}$$

3. Non-repeated quadratic factor in the denominator

$$\frac{1}{(ax^2+bx+c)(x-d)}=\frac{Ax+B}{ax^2+bx+c}+\frac{C}{x-d}$$

Properties of Definite Integration

$\int_a^b f(x).dx=\int_a^b f(t).dt$	$\int_a^b f(x).dx=-\int_b^a f(x).dx$

$$\int_a^b f(x).dx=\int_a^c f(x).dx+\int_c^b f(x).dx$$

$$\int_a^b f(x).dx=\int_a^b f(a+b-x).dx$$

Application of Integral Calculus

- **Cost Function :** C(x) = ∫(Marginal cost).dx
- **Revenue Function :** R(x) = ∫(Marginal Revenue).dx

NUMBER SERIES

NUMBER SERIES	
• Identify the missing number from the series using a pattern or code. Types Discussed Below	
• **4, 16, 36, 64, _, 144, 196** Squares of numbers with gap of 2 ◦ 2^2, 4^2, 6^2, 8^2, $\underline{10^2}$, 12^2, 14^2	**Perfect Square Series**
• **1, _, 27, 64** Cubes of number starting from 1 ◦ 1^3, $\underline{2^3}$, 3^3, 4^3	**Perfect Cube Series**
• $1 \xrightarrow{+5} 6 \xrightarrow{+5} 11 \xrightarrow{+5} 16 \xrightarrow{+5} \underline{21}$, 26	**Arithmetic Series**
• $5 \xrightarrow{\times 5} \underline{25} \xrightarrow{\times 5} 125 \xrightarrow{\times 5} 625$, 3125	**Geometric Series**
• **1, 4, 10, 22, _** (governed by Previous no. x 2 + 2) ◦ $1 \xrightarrow{\times 2 + 2} 4 \xrightarrow{\times 2 + 2} 10 \xrightarrow{\times 2 + 2} 22 \xrightarrow{\times 2 + 2} \underline{46}$	**Mixed Series**

- **3, 5, 8, 12, 17, _, 30**

  +2 +3 +4 +5 +6

  - 3, 5, 8, 12, 17, **23**, 30

Alternating Series

- **2, 5, 17, 72, _**

  x2 +1 x3 +2 x4 +3 x5 +4

  - 2, 5, 17, 72, **365**

Combination Series

- **Can you Solve These Series?**

  a. **2, 3, 3, 5, 10, 13, 39, ?, 172, 177**

  b. **10, 12, 22, 34, 56, 90, ?**

  c. **7, 15, 29, 59, 117, ?**

  d. **8, 13, 21, 32, 46, 63, 83, ?**

ODD MAN OUT

- When there are few words, numbers or letters are given to you, such that all have a relation except one.

- **Select the Odd one out**
  - 10, 14, 16, 23, 28, 30
    - 23 as it is odd no. in series of even.
  - 16, 25, 36, 62, 144, 196, 225
    - 62 as it is not a perfect square

ALPHABET NUMBERING

Alphabet	Number	Alphabet	Number
A	1	N	14
B	2	O	15
C	3	P	16
D	4	Q	17
E	5	R	18
F	6	S	19
G	7	T	20
H	8	U	21
I	9	V	22
J	10	W	23
K	11	X	24
L	12	Y	25
M	13	Z	26

ALPHABET & LETTER SERIES

ALPHABET SERIES

- Series of particular set of alphabets forming pattern.
- Series : **P M T, O O S, N Q R, ?**
  - First Alphabets : **P→O→N** (Decreasing by 1)
  - Second Alphabets : **M→O→Q** (Increasing by 2)
  - Third Alphabets : **T→S→R** (Decreasing by 1)
  - By this : Last set of alphabets is **M S Q**

LETTER SERIES

- Series of small alphabets forming a pattern.
- Series : **a _ c a a _ b c c _ a a b b b _ c c**
  - First blank is **b**, since it will create series of individual abc.
  - Second blank is also **b**, since it creates double pattern of aa bb cc.
  - Third Blank is **a**, since it creates triple pattern of aaa bbb.
  - Fourth blank is **c**, to complete the triple pattern
- Answer is : **a b c a a b b c c a a a b b b c c c**

CODING DECODING

Letter Coding

- If **MONKEY** is **KMLICW,** then **ORANGE** is **?.**
  - Code for MONKEY is 13 15 14 11 5 25 decreased by 2
  - Code for KMLICW is 11 13 12 9 3 23
  - Code for ORANGE is 15 18 1 14 7 5 decreased by 2
  - Code for new word is 13 16 25 5 3
  - New word is **MPYLEC**

Number Coding

Type 1 : When numerical values are assigned to words

- If 'GLOSSORY' is coded as '97533562' & 'GEOGRAPHY' as 915968402' then **'GEOLOGY'** ?
  - From above we elucidate,
  - G→9, E→1, O→5, L→7, Y→2
  - Code for GEOLOGY is **9157592**

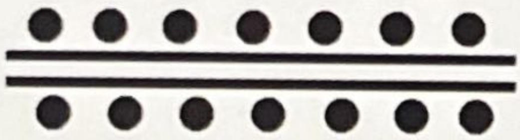
Type 2 : Number to letter coding

- In a certain code , a number 18462 is written as BETKO and 7935 is written as RAHU. How is **43857** written in that code ?
  - From above we elucidate,
  - 4→T, 3→H, 8→E, 5→U, 7→R
  - Word for 43857 is **THEUR**

SEATING ARRANGEMENT

Types of Arrangements

- **Linear Arrangement**

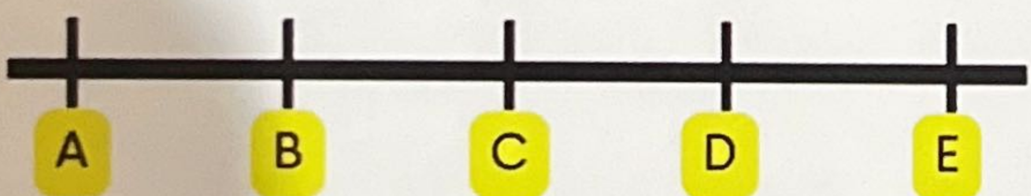


- **Two Row Sequence**

- **Circular Arrangement**

- **Polygon Arrangement**

Linear Arrangements



- **Immediate Left** : Just left side of Reference pt.
    - A is immediate left of B
- **Immediate Right** : Just Right side of Reference pt.
    - C is Immediate Right of B
- **To the Left** : Moving towards left from Reference pt.
    - A, B, C, D are to the left of E
- **To the Right** : Moving towards Right from Reference.
    - B, C, D, E are to the right of A

- **In Between :** Object or Positions located in Between.
  - B is in between A and C.

Type 1 : When Direction of facing is not known

- There are five houses P, Q, R, S, T. P is immediate right of Q and T is immediate left of R and immediate right of P. Q is on right of S. Elucidate the pattern.
  ① Q P (immediate right of Q)
  ② P T R (T imm. right of P and imm. left of R)
  ③ S Q P (Since Q is right of S)
  - Final Elucidation : **S Q P T R**

Type 2 : When Direction of facing is known

- **Five boys are standing in a row facing East. Pavan is left of Tavan. Vipin and Chavan to the left of Nakul. Chavan is between Tavan and Vipin. Vipin is fourth from the left. How Far is Tavan to the Right?**

① Pavan is left to Tavan

→ Pavan
→ Tavan

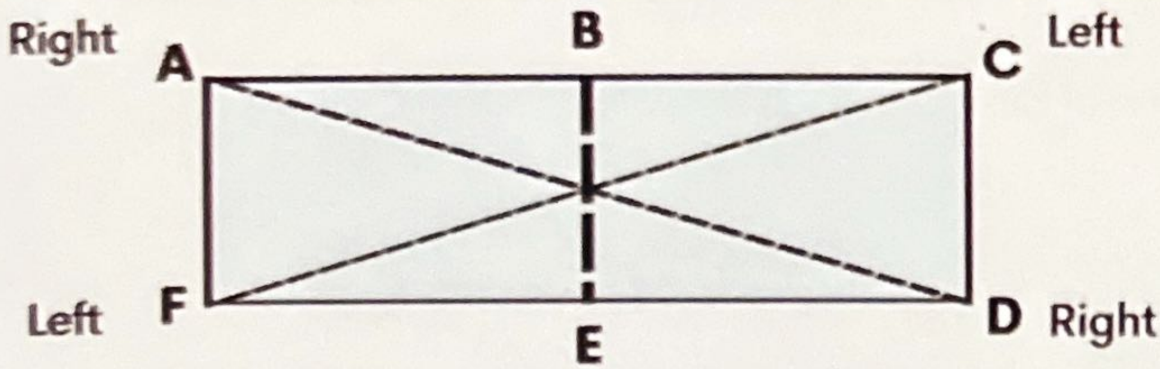
② Chavan b/w Tavan & Vipin & Vipin is fourth left to Pavan

→ Pavan
→ Tavan
→ Chavan
→ Vipin

③ Vipin & Chavan to the left of Nakul

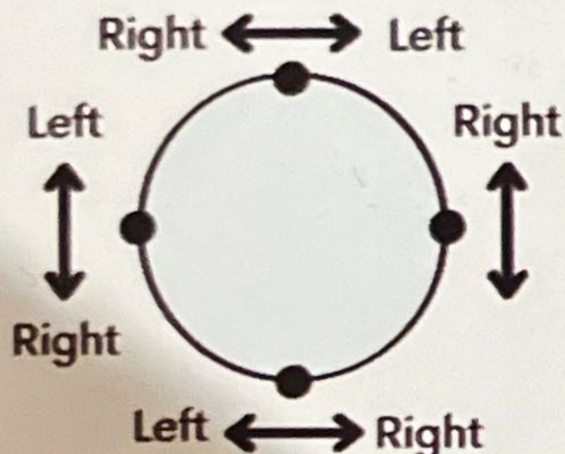
→ **Pavan**
→ **Tavan**
→ **Chavan**
→ **Vipin**
→ **Nakul**

Double Row Arrangements



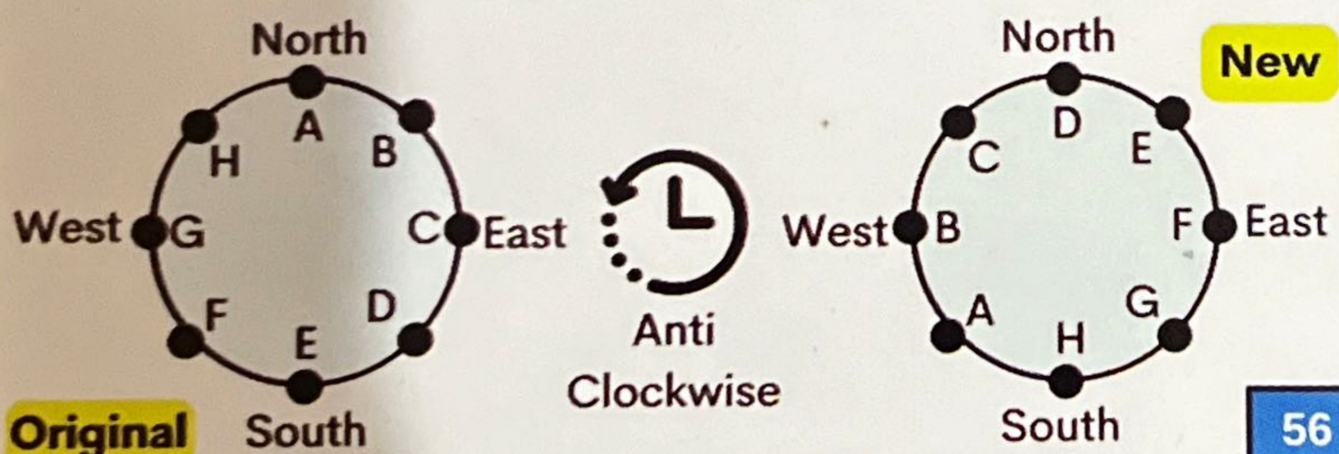
- A is sitting Opposite of F
- D & C are sitting diagonally opposite.

Circular Arrangements

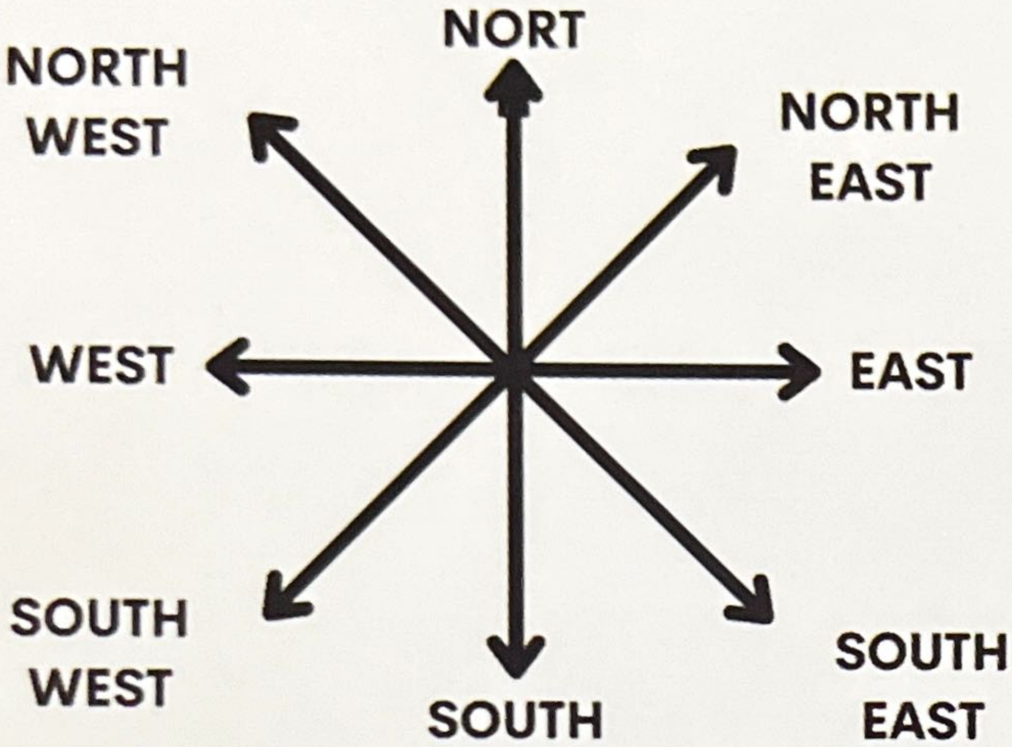


- Left movement is clockwise rotation
- Right moment is Anti-Clockwise Rotation

- Eight Persons A, B, C, D, E, F, G & H are sitting around circle, facing the direction opposite to center, If they move three places anticlockwise, Elucidate.



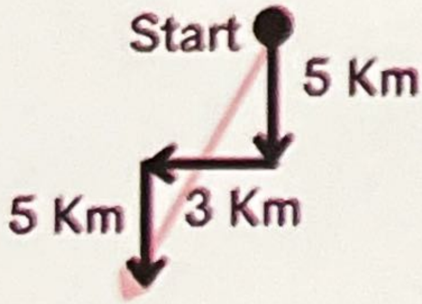
DIRECTION SENSE



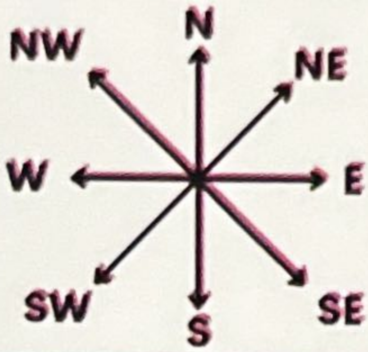
DIRECTION GIVEN	DIRECTION INDICATED
• Left + Left	• Down
• Left + Right	• Up
• Right + Left	• Up
• Right + Right	• Down
• Up + Left	• Left
• Up + Right	• Right
• Down + Left	• Right
• Down + Right	• Left

Some Questions on Direction Sense

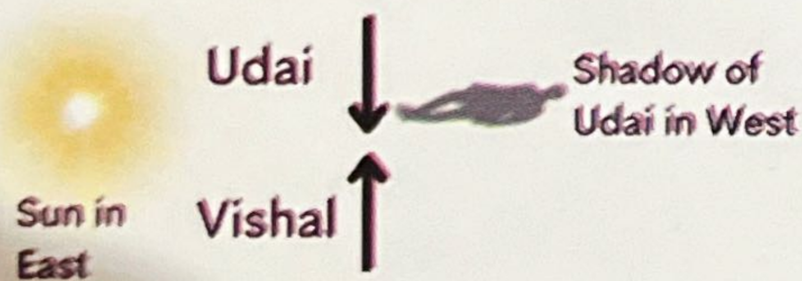
- A man walks 5 km toward south and then turns to the right. After walking 3 km he turns to the left and walks 5 km. Which direction is he from the starting place?



SW from the start

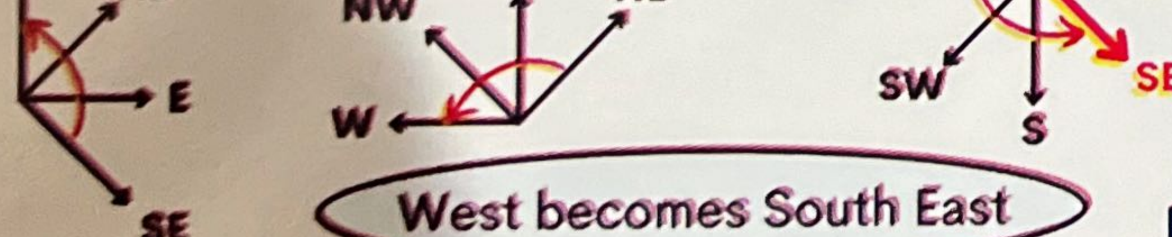


- One morning Udai and Vishal were talking to each other face to face. If Vishal's shadow was exactly to the left of Udai, which direction was Udai facing?
    - Since Sun rises from the East, Shadow will be formed in **West.**



Udai Facing South

- If South-East becomes North, North-East becomes West and so on. What will West become?



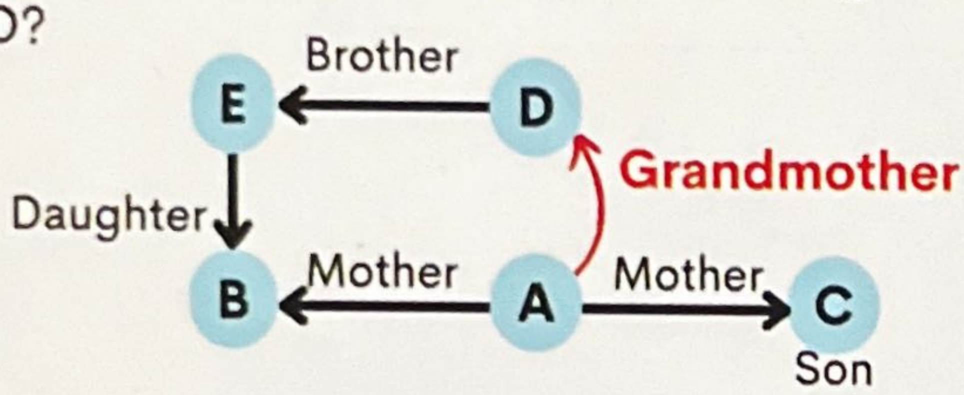
West becomes South East

BLOOD RELATIONS

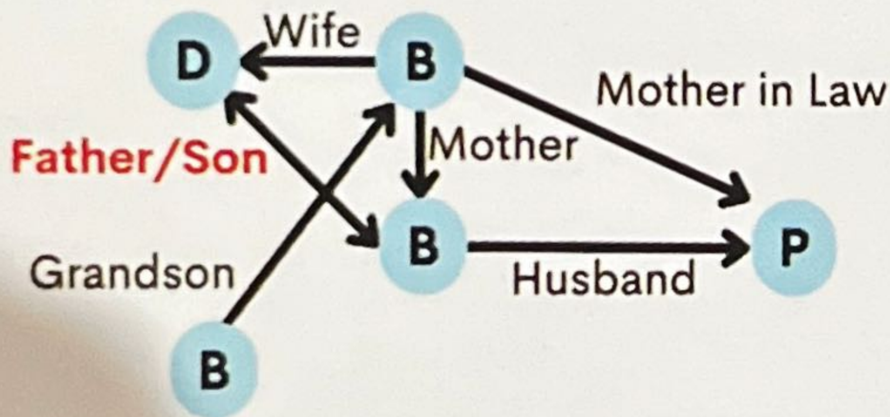
Expanded Relationship	Direct Relationship
Mother's or Father's son	Brother
Mother's or Father's Daughter	Sister
Mother's or Father's Father	Grandfather
Mother's or Father's Mother	Grandmother
Son's Wife	Daughter in Law
Daughter's Husband	Son in Law
Husband's or Wife's Sister	Sister in Law
Husband's or Wife's Brother	Brother in Law
Brother's or Sister's Son	Nephew
Brother's or Sister's Daughter	Niece
Uncle's or Aunt's Daughter/Son	Cousin
Brother's Wife	Sister in Law
Sister's Husband	Brother in Law
Grandson's or Grand daughter's Son/Daughter	Great Grand Son/ Daughter

Some Questions on Blood Relations

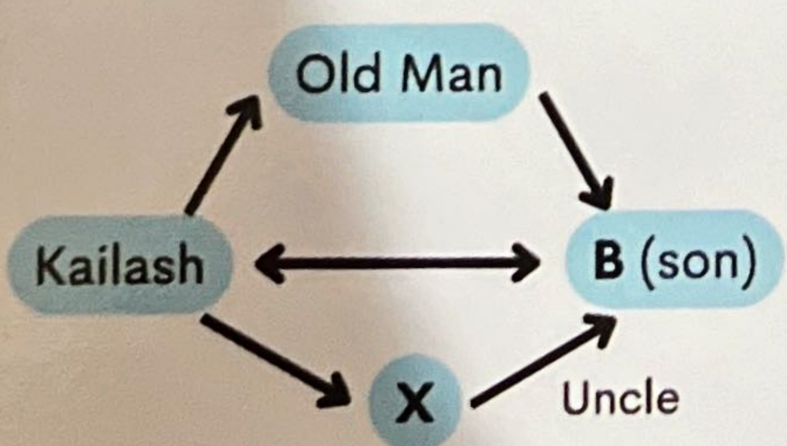
- A is the mother of B, C is the son of A, D is the brother of E, E is the daughter of B, Who is the grandmother of D?



- B is the husband of P. Q is the only Grandson of E, who is the wife of D and mother in Law of P. How is B related to D?



- Pointing to the old man, Kailash said "his son is my son's uncle". How is Kailash Related to the old man?



Kailash is either son or son-in-law.

STATISTICAL DESCRIPTION

Definition of Statistics

- Statistics, in plural, encompasses qualitative and quantitative data for analysis. In singular, it is the scientific method for collecting, analyzing, and presenting data, drawing statistical inferences—a 'science of counting' or 'averages.'

Application of Statistics

Economics

- **Overlapping areas:** Time Series Analysis, Index Numbers, Demand Analysis
- **Econometrics:** Positive interaction between Economics and Statistics
- **Socio-economic** surveys and analysis use various statistical methods
- **Regression analysis** crucial for future projections in Economic planning

Business Management

- **Past**: Decisions based on hunches, intuition, and trials
- **Present**: Complex business environments demand quantitative techniques
- **Decision-making** combines statistical methods and operations research

- **Statistical inferences** from samples crucial for developing criteria
- **Statistical decision** theory analyses business strategies and their alternatives

Statistics in Commerce and Industry

- Modern industrialists and businessmen use statistical procedures for **expansion**
- **Data** on sales, raw materials, wages, and similar products are collected and analysed
- Expert consultation to **maximize profits**

Limitation of Statistics

LIMITED

- Study of Quantitative data only
- Study of Aggregates only
- Homogeneity of Data, an essential Requirement
- Results are True only on an Average
- Results may Prove to be Wrong
- Can be used only by experts

Collection of Data

- On the nature of data, we can define data as
  - **Quantitative** Data : Represents Numeric Values covering measurements, Counts & Ratings.
  - **Qualitative** Data : Represents Characteristics covering opinions and Descriptive information.
- To Analyse qualitative information, it needs to be converted into quantitative information (into the numerical form)

Variables

Discrete Variable
- Only specific values can be taken
- No. of cars in parking lot
- No. of Children in family

Continuous Variable
- Any values can be taken in
- Height
- Temperature
- Time
- Weight

Classification on basis of source of Data

1. **Primary Data**
- Data collected for own's purpose.
- Collected from source of origin
- E.g. Surveys, Interviews, Experiments.

2. **Secondary Data**
- Data collected already exists.
- Collected for for some other purpose
- Obtained from Published and unpublished reports.
- E.g. Government Reports, Academic Studies & Historical Records.

Collection of Primary Data in 4 ways

Interview Method
- → Direct Interview
- → Indirect Interview
- → Telephonic Interview

- Telephonic interview is the fastest but may have high number of non responses

Mailed Questionnaire Method

- It involves a well-drafted questionnaire sent to respondents
- Covers important aspects with pre-paid stamps and guidelines
- Wide coverage possible, high risk of non-responses

Observation Method

- Collects data through direct observation/instruments
- Considered best method but is time-consuming.
- Limited coverage, suitable for specific, focused data.

Questionnaires filled and sent by enumerators

- Questionnaire method used for larger surveys
- Enumerators collect information via direct interviews
- Questions explained to respondents for collection.

Sources of Secondary Data

- **International** sources: WHO, ILO, IMF, World Bank.
- **Government** sources: CSO's Statistical Abstract, Ministry of Food and Agriculture's Indian Agricultural Statistics, etc.
- **Private** and **quasi-government** sources: ISI, ICAR, NCERT, etc.
- **Unpublished** sources from research institutes and researchers.

Scrutiny of Data

- Statistical analyses rely on accurate & consistent data
- Scrutiny is essential, requiring intelligence, patience, & experience
- Errors may occur during data collection, needing careful observer scrutiny
- Internal check crucial for related series of figures

Presentation of Data

- Collected and verified data must be presented neatly, emphasizing essential features.
- Common methods of presentation includes tables, graphs, charts & visualisation

Objective of Classification

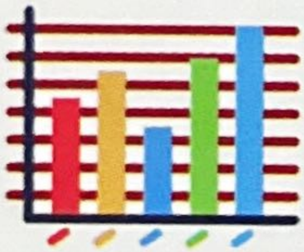
Simplification & Briefness	Comparibility
Statistical Analysis	Makes Data Understandable

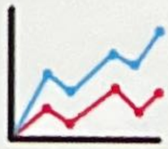
Classification of Data

- **Chronological** or Temporal or Time Series Data : Classifying monthly sales over past year into intervals of quarters or seasons.
- **Geographical** or Spatial Series Data : Classifying population data into North, South, East, West groups.
- **Qualitative** or Ordinal Data : Classifying survey responses into satisfied, Neutral, & Dissatisfied.
- **Quantitative** or Cardinal Data : Classifying test scores in grade ranges

Mode of Presentation

- **Textual Presentation**
  - Data presented using paragraphs in textual form
  - Common in official reports
  - Statisticians avoid it for being dull, monotonous, and lacking comparison
  - Not recommended for complex classifications.
- **Tabulation**
  - Tabulation defined as systematic presentation using statistical tables
  - Includes rows, columns, reference numbers, title, row and column descriptions, and footnotes.
- **Diagrammatic Representation of Data**
  - We will consider
    - Line diagram
    - Bar diagram
    - Pie chart.





Frequency Data

- Number of times the data value occurs
- E.g. : No. of times a word appears in a document

Non-Frequency Data

- Data where individual values and specific identity's are important.
- Recording the order details for an ecommerce brand in which customer details are unique.

Table Number: ....
Title: ....
(Head Note, if any)

<table>
<tr><td rowspan="3">Stub
(Row Heading)</td><td colspan="4">Caption (Column Heading)</td><td rowspan="3">Total (Rows)</td></tr>
<tr><td colspan="2">Sub-head</td><td colspan="2">Sub-head</td></tr>
<tr><td>Column-head</td><td>Column-head</td><td>Column-head</td><td>Column-head</td></tr>
<tr><td>Stub Entries
(Row Entries)
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. . .</td><td colspan="4">Body</td><td></td></tr>
<tr><td>Total Columns</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Source Note:
Footnote:

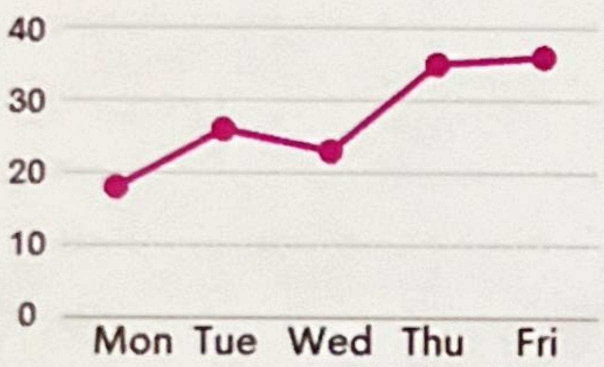
Tabulation

Line Diagram/ Histogram

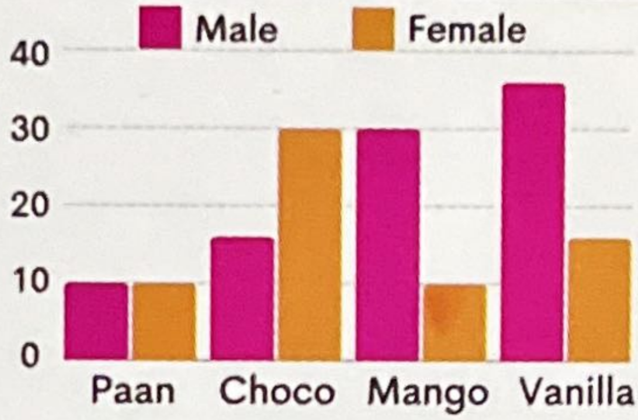
- Line diagram used when data vary over time
- Each pair of values (t, y_t) plotted on the t-y_t plane.

Bar Diagram

- Two bar diagram types: Horizontal for qualitative, Vertical for quantitative data.
- Multiple, Component, Divided Bars for comparisons



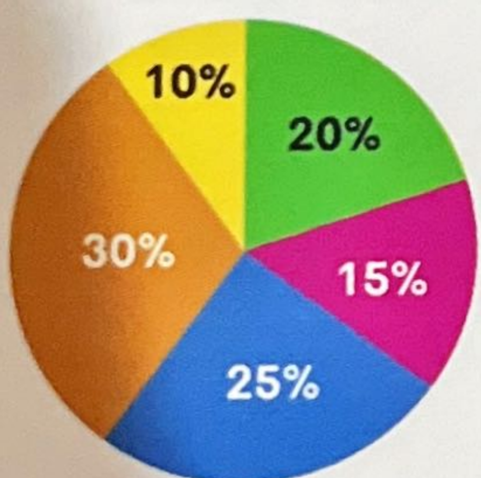
No. of Hot dogs sold per day



Favourite Ice Cream Flavour

Pie Diagram

- A pie chart is used to represent and compare components of a whole in a visually circular format.



To Find Central Angle, use

$$\frac{n}{\text{Total}} \times 360°$$

Pie charts show proportions intuitively; bars depict individual values distinctly.

Favourite sports among students

Badminton Cricket Basketball Vollyball Swim

Frequency Distribution

- Tabular representation of statistical data
- Total Frequency distributed among classes/intervals.
- Usually in ascending order

Classified by Category ⟵ 2 Types ⟶ Classified by Intervals

Ungrouped Frequency Distribution

Discrete variables	No. of Children	Frequency
Data set representing no. of children in family: 2, 4, 1, 3, 2, 4, 2, 1, 3, 4	1	2
	2	3
	3	2
	4	3

Grouped Frequency Distribution

Continuous variables	Heights	Frequency
Data set representing heights of group : 156, 168, 174, 160, 162, 170, 168, 172, 158, 164	150 - 160	2
	160 - 170	5
	170 - 180	3

Number of Classes in grouped frequency distribution = $\dfrac{\text{Range}}{\text{Class length}}$

Here, **Range** = Largest - Smallest Number

Important Terminologies

- **Class Limit** : Maximum and Minimum values of an interval, E.g. : in 100–200, 200 is Upper Class Limit **(UCL)** and 100 is Lower Class Limit **(LCL)**
- **Class Boundaries :** It is used to establish a common boundary between adjacent Classes.
  - In case of Overlapping class interval, Class boundaries and Limits are same.
- Lower Class Boundary **(LCB)** = LCL - D/2
- Upper Class Boundary **(UCB)** = UCL + D/2
  - where D = LCL of next class interval - UCL of given class interval.
- Class **Mid Point** = (LCL + UCL)/2 = (LCB + UCB)/2
- **Width or Size of Interval** : UCL – LCL = UCB – LCB

Cumulative Frequencies

Defined as running total of frequencies

Frequencies	Less than (Addition ↓)	More Than (Subtraction ↓)
12	12	50
9	21	38
14	35	29
10	45	15
5	50	5
Total = 50		

Graphical Representation of frequency distribution

- **Histogram or Area Diagram**
  - 2-D graphical representation of continuous frequency distribution.
  - Areas of Rectangles ∝ Frequencies
- **Frequency Polygon**
  - Used for single frequency distribution.
  - For Group distribution, width of the class interval should be same.
- **Ogives or Cumulative Frequency Graph**
  - By Plotting cumulative frequency against respective class boundary, we get ogives.
  - Two types : Less than type ogives & More than type ogives.

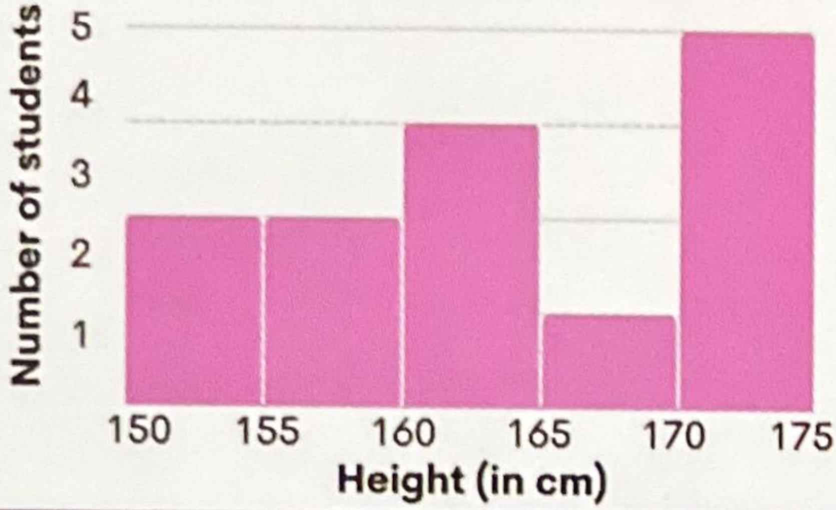
Grouped Frequency Distribution

Continuous variables	Heights	Frequency
Data set representing heights of group : 156, 168, 174, 160, 162, 170, 172, 158, 164, 151, 153, 157, 163, 174, 173	150 - 155	2
	155 - 160	2
	160 - 165	3
	165 - 170	1
	170-175	4

Graphical Representation of frequency distribution

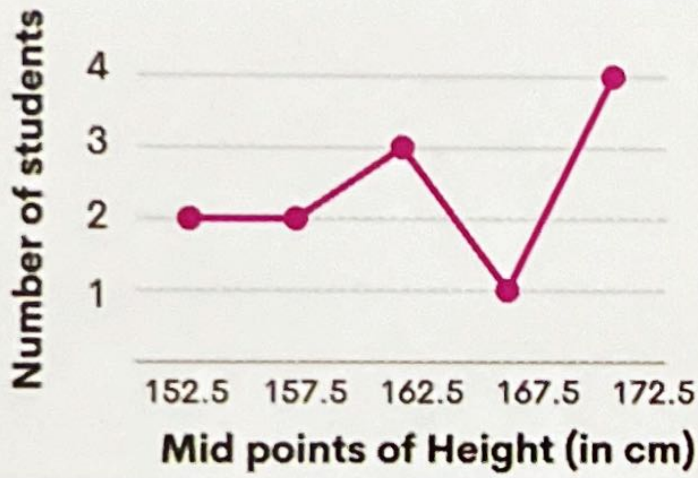
Histogram or Area Diagram

150 - 155	2
155 - 160	2
160 - 165	3
165 - 170	1
170-175	4



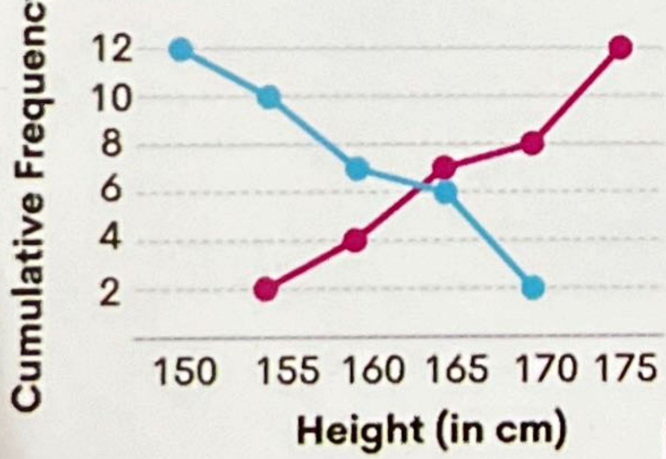
Frequency Polygon

152.5	2
157.5	2
162.5	3
167.5	1
172.5	4



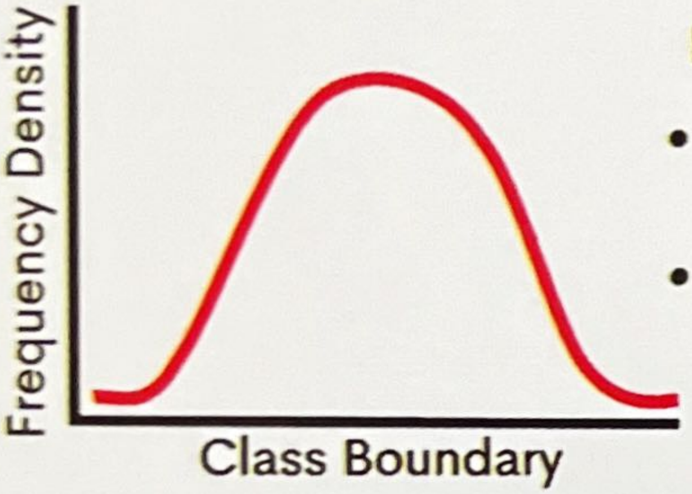
Ogives or Cumulative Frequency graph

Interval	Less	More
150 - 155	2	12
155 - 160	4	10
160 - 165	7	7
165 - 170	8	6
170-175	12	2



Histogram or Area Diagram

- Frequency curve: smooth, unit area curve, a limit of histogram/frequency polygon.
- Obtained by drawing through mid-points of histogram rectangles.

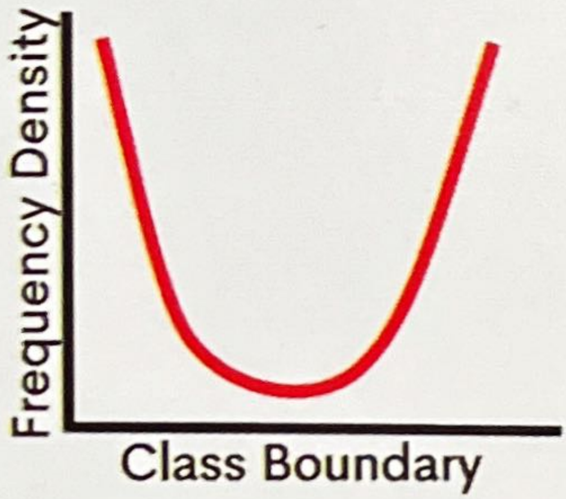


Bell Shaped Curve

- Represents distribution of height, weight, etc.
- Peaks in the center, gradually declines at extremes.

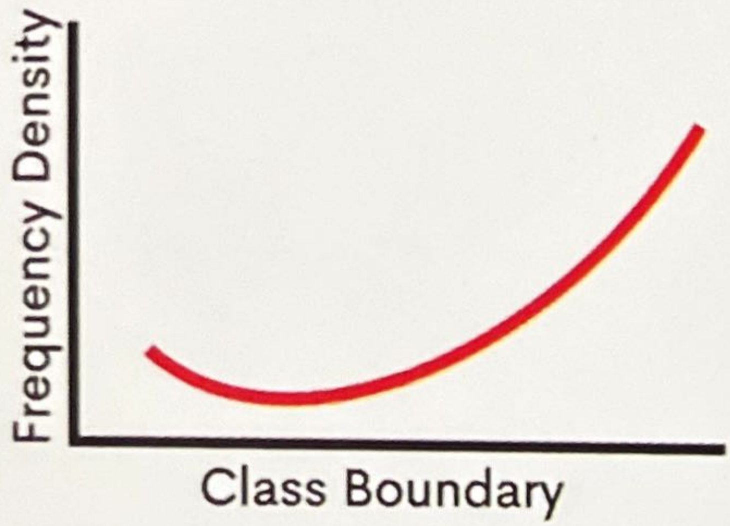
U - Shaped Curve

- U-shaped curve: minimal frequency at center, increases steadily towards extremities.
- Example: Kolkata-bound commuters during peak hours.



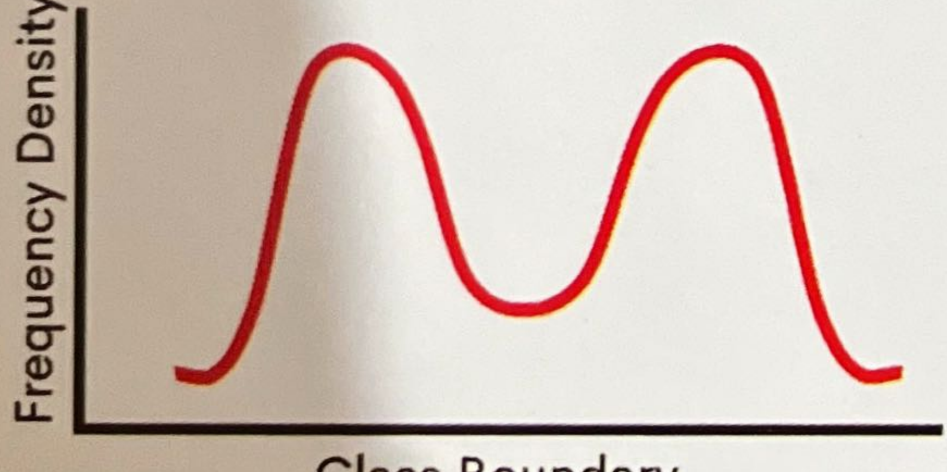
J Shaped Curve

- J-shaped curve: starts with minimal frequency, gradually peaks at the other extremity.
- Example: Kolkata commuters from early morning to peak morning.



Mixed Curve

We may have a combination of these frequency curves, known as mixed curve.



CENTRAL TENDENCY

Definition
• Statistical concept representing the midpoint or average of a dataset through **mean, median, mode.**
Arithmetic Mean
• Sum of all observations divided by no. of observations • A.M. of n numbers : $\overline{x} = (x_1 + x_2 + x_3 ... + x_n)/n$
$\overline{x} = \frac{\sum x_i}{n}$, where n = no. of observations $\overline{x} = \frac{\sum f_i x_i}{N}$, where f = Frequency & $N = \sum f_i$ Above formula is used in case of grouped frequency
The following formula can also be used $x = A + \frac{\sum f_i d_i}{N} \times C$ where $d_i = \frac{x_i - A}{C}$
Properties of Arithmetic Mean
• If all observations assumed by variable are constants, k then AM is also k. • If all observations are added, subtracted, multiplied by k, then AM is also done same way. • Algebraic sum of deviations from AM is zero.

- AM is affected due to a change of origin and/or scale, if x changes to a variable y with relation $y = a + bx$, then the AM of y is given by $\overline{y} = a + b\overline{x}$.
- Combined arithmetic mean of two observations n_1 & n_2 is given by $\overline{x} = \dfrac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2}$

Median - Partition Values

- It is positional average defined by middle most value when arranged in ascending order.

In case of grouped frequency distribution,	$M = l_1 + \left(\dfrac{\frac{N}{2} - N_l}{N_u - N_l}\right) \times C$

- l_1 : lower class boundary of median class
- N = Total Frequency; N_l and N_u are cumulative frequency corresponding to l_1 and l_2
- C = Length of median class = $l_2 - l_1$

Median of Individual Series

I : Arrange the data in ascending or descending order.

II Find Median

- If n is odd, then Median = value of the (1/2)(n + 1)th observation
- If n is even, then Median = mean of the (n/2)th and (n/2 + 1)th Observation

Median of Discrete Series

I : Arrange the data in ascending or descending order.

II : Prepare the cumulative Frequency table

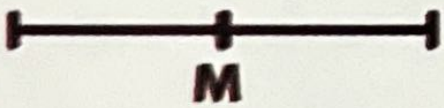
III : Median is the observation whose cumulative frequency is equal to or just greater than N/2, where N = sum of frequencies.

Properties of Median

- if x and y are related as y = ax + b, then median of y is $y_{me} = a + bx_{me}$
- Sum of absolute deviations is minimum when taken from median

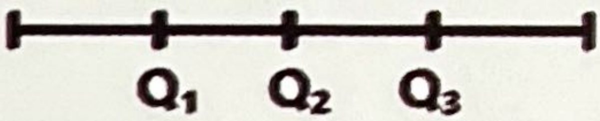
Quartiles or Partition Values

Median



Dividing in 2 Parts

Quartiles



Dividing in 4 equal Parts

- Similarly, Deciles divide in 10 equal parts
- Percentiles divide in 100 equal parts

- k^{th} quartile is k.(n+1)/4 term
- k^{th} decile is k.(n+1)/10 term
- k^{th} percentile is k.(n+1)/100 term

For ungrouped data of n observations

For Grouped data of **n** observations,	$M = l_1 + \left(\frac{N_p - N_l}{N_u - N_l}\right) \times C$

Measure of Dispersion

$\frac{1}{n}\sum \lvert x_i - A \rvert$ or $\frac{1}{n}\sum f_i \lvert x_i - A \rvert$	A can be mean or median or any no.

Mode

- Value that occurs maximum number of times

In case of grouped frequency dist.	$\text{mode} = l + \left(\frac{f_1 - f_o}{2f_1 - f_2 - f_o}\right) \times h$

- Where l = the lower limit of the modal class i.e. the class having maximum frequency;
- f_1 = frequency of the modal class;
- f_0 = frequency of the class preceding the modal class;
- f_2 = frequency of class succeeding the modal class
- h = width of the modal class

Mode of Individual Series

The number occurring the most frequently in the series

Mode of Discrete Series

By looking to that value of variable around which the items are most heavily concentrated.

DISPERSION

Definition
Central tendency measures the center of a dataset (mean, median, mode), while dispersion assesses its spread or variability(range, variance, standard deviation).
Absolute & Relative measure of dispersion
• **Absolute** measures are : Range, Standard Deviation, Mean Deviation & Quartile Deviation, while • **Relative** measures define Coefficient of above
NOTE
• Absolute measures are tied to variable units; relative measures are unit-free. • For distribution comparison, prefer relative over absolute measures. • Relative measures are complex to comprehend
RANGE, R = L - S
• Range = Largest - Smallest (values of set) • In case of **grouped** frequency, Range = Upper limit of highest class - Lower limit of smallest class.
Coefficient of Range

• **It is the relative measure of range,** given as	$\frac{L - S}{L + S} \times 100$

Properties of Range

- Origin shifts don't impact the range, but a scale change affects the range proportionally.
- if x and y are related as $y = a + bx$, then $R_y = |b| \times R_x$

MEAN DEVIATION

- Average absolute difference between each data point and the mean of the set.

$$\frac{1}{n}\sum |x_i - A| \text{ or } \frac{1}{n}\sum f_i|x_i - A|$$

A can be mean or median or any no.

Coefficient of Mean Deviation

$$\text{Coefficient of Mean Deviation} = \frac{\text{Mean Deviation of A}}{\text{A}} \times 100$$

A can be mean or median or any no.

Properties of Mean Deviation

- Mean deviation takes its minimum value when the deviations are taken from the median.
- It remains unchanged due to a change of origin but changes in the same ratio due to a change in scale
  - if $y = a + bx$, then, MD of $y = MD_y = |b| \times MD_x$

QUARTILE DEVIATION	
Interquartile Range	$Q_3 - Q_1$
Quartile Deviation	$(Q_3 - Q_1)/2$

Coefficient of Quartile Deviation

Coefficient of quartile deviation is a pure number independent of the units of measurement	$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

- Quartile Deviation gives the amount by which two quartiles differ from median
- In symmetrical Dist., $Q_2 = Q.D. + Q_1$ & $Q_3 = Q.D. + Q_2$ i.e. Q_1 & Q_3 are equidistant from median
- Median ± Q.D. covers exactly 50% of data

STANDARD DEVIATION

- Measure of data dispersion, indicating average distance of each point from the mean.
- Root mean square deviation from the arithmetic mean

In Discrete Values	In Continuous Values
$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$	$s = \sqrt{\frac{\sum f_i(x_i - \bar{x})^2}{N}}$

Variance : Square of Standard Deviation = s^2

Coefficient of Variation

Coefficient of Variation (C.V.) = (S.D/A.M) x 100

Properties of Standard Deviation

- If all the observations assumed by a variable are constant i.e. equal, then the SD is zero
- SD remains unaffected due to a change of origin but
  - if $y = a+bx$, then, $s_y = |b|\, s_x$

Properties of Coefficient of Variation

- When smaller, Distribution is said to be less variable or more consistent, more uniform.
- Coefficient of variation can be used to compare the variability of two or more sets of data even when expressed in different units of measurement.

Relationship between SD, MD, QD

4 S.D. = 5 M.D. = 6 Q.D.

Relationship between SD, MD, QD

for n Natural numbers	$SD = \sqrt{\dfrac{n^2-1}{12}}$

for two groups with observations n_1 and n_2 and SD's

$$s = \sqrt{\frac{n_1 s_1^{\,2} + n_2 s_2^{\,2} + n_1 d_1^{\,2} + n_2 d_2^{\,2}}{n_1 + n_2}}$$

CORRELATION - REGRESSION

Meaning of Correlation

- Correlation analysis checks if two things (like profit and investment) are linked, considering various correlation measures.
- In layman terms, To stay healthy, food if,
  - Junk : **Negative** Correlation (Opposite Direction)
  - Healthy : **Positive** Correlation (Same Direction)

Bivariate Data

- Two-variable information, like marks in stats and maths for a class of students.

X → MARKS IN MATHS

Y ↓ MARKS IN STATS

	0-4	4-8	8-12	12-16	16-20	Total
0-4	1	1	2			4
4-8	1	4	5	1	1	12
8-12	1	2	4	6	1	14
12-16		1	3	2	5	11
16-20			1	5	3	9
Total	3	8	15	14	10	50

Supposing the above Bivariate Data, we can generate two types of Univariate distribution

Marginal Distribution

Marks	Students
0-4	4
4-8	12
8-12	14
12-16	11
16-20	9
Total	50

Marks in Statistics

Conditional Distribution

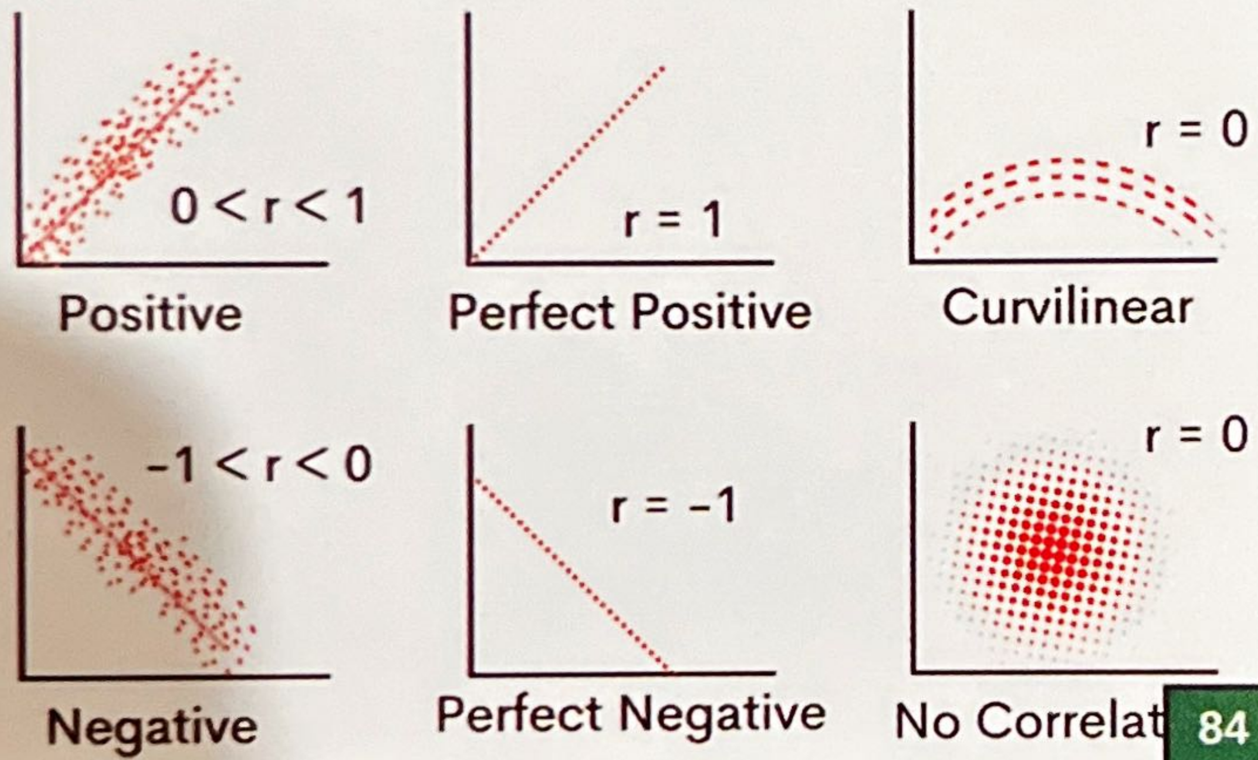
Marks	Students
0-4	2
4-8	5
8-12	4
12-16	3
16-20	1
Total	15

Marks in Statistics for students having marks in Maths 8-12

Condition Applied

Methods to Study Correlation

- **Scatter** diagram, Here r = Correlation Coefficient



Methods to Study Correlation

- **KARL PEARSON'S PRODUCT MOMENT CORRELATION COEFFICIENT**

For two observations, x and y,

$$r = r_{xy} = \frac{\text{Cov}(x,y)}{s_x \times s_y}$$

where, cov = covariance
& s = standard Deviation

- $$\text{cov}(x, y) = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n} = \frac{\Sigma x_i y_i}{n} - \bar{x}\bar{y}$$

- $$\text{Cov}(x,y) = \frac{\sum_{i,j} x_i y_i f_{ij}}{N} - \bar{x} \times \bar{y}$$

In case of frequency distribution

A Single Formula to compute the Coefficient

$$r = \frac{n\Sigma x_i y_i - \Sigma x_i \times \Sigma y_i}{\sqrt{n\Sigma x_i^2 - \left(\Sigma x_i\right)^2}\sqrt{n\Sigma y_i^2 - (\sum y_i)^2}}$$

Properties of Correlation Coefficient

- Unit–free measure.
- It remains invariant under a change of origin and/or scale of the variables under consideration depending on the sign of scale factors.
  - If x and y are related to u & v as $x = a + bu$; $y = c + dv$, then
    - $r_{xy} = (b.d.r_{uv})/|b||d|$
- Always lies between –1 and 1

Methods to Study Correlation

- **SPEARMAN'S RANK CORRELATION COEFFICIENT**
  - Beauty, intelligence etc. cannot be measured with numerically, thus we use spearman's rank
  - Given as $r_R = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$
    - $d_i = x_i - y_i$, difference in ranks of ith individual
    - n is the number of observations
    - r_r is the rank coefficient

$$r_R = 1 - \frac{6\left[\sum_i d_i^2 + \sum_j \frac{\left(t_j^3 - t_j\right)}{12}\right]}{n\left(n^2-1\right)}$$

In case of u individuals receive the same rank

- t_j represents the j tie length and summation of $t_j^3 - t_j$ extends over full length

Methods to Study Correlation

- **COEFFICIENT OF CONCURRENT DEVIATIONS**
  - Denoting the number of concurrent deviation by c and total number of deviations as m, the coefficient of concurrent deviation is given by

$$r_c = \pm\sqrt{\pm\frac{(2c-m)}{m}}$$

r also lies between -1 and 1

- $(2c-m) > 0$, positive sign both inside and outside
- $(2c-m) < 0$, negative sign both inside and outside

Meaning of Regression

- Regression analysis predicts one variable based on another, useful in business decisions.

- There can be Estimation of y when x is given,
  - Y : Dependent & X : Independent
  - It can be vice versa too

- When Correlation is imperfect, we create equation for y on x or x on y as
  - Regression of y on x as $y - \bar{y} = b_{yx} (x - \bar{x})$
  - Regression of x on y as $x - \bar{x} = b_{xy} (y - \bar{y})$

• Regression coefficient of y on x r = karl Pearson coeff	$b = \frac{Cov(x, y)}{S_x^{\ 2}} = \frac{r.S_y}{S_x}$

- b_{xy} can be calculated simultaneously

Regression coefficient b_{yx}

- Measures change in y corresponding: unit change in x
- It also denotes slope of the regression of y on x

Properties of Regression Lines

- Change of Origin : No Impact
- Change of Scale : If original Pair is x,y and modified pair is u,v. Then,

$$b_{uv} = b_{yx} = \frac{\text{Change of Scale of y}}{\text{Change of Scale of x}}$$

- Two regression lines will intersect at the point of their means. i.e. $(\bar{x}, \bar{y})$

Relation between Correlation and Regression

$$r = \pm\sqrt{b_{yx} \times b_{xy}}$$

b_{xy}, b_{yx} and r will always have same sign

- NOTE : b_{xy} & b_{yx} always have same sign.

Important Points

- Pearson's correlation is the best measure.
- Methods Limited to linear relationships.
- Correlation coefficient of zero doesn't imply independence; no linear relationship.

• **Coefficient of Determination**	r^2 = Experienced Variance/Total
• **Coefficient of non Determination**	$1 - r^2$

PROBABILITY THROUGH QUESTIONS AND TRICKS

Classic Probability

Talking about the Number of mutually exclusive, exhaustive and equally likely events

m_A = number of events favourable to A m = Total number of events	**$P(A) = m_A / m$**

- The probability lies between 0 & 1. i.e. $0 < P(A) < 1$
  - Sure Event : $P(A) = 1$; Impossible Event : $P(A) = 0$
- Non occurrence of event $P(A') = 1 - P(A) = (m-m_A)/m$
- Odds in favour of A = $m_A : m-m_A$

Everything about coins

- Total outcomes : $\mathbf{2^n}$, where n = no. of coins
  - 1 Coin : {H,T} : 2 Outcomes
  - 2 Coins : { (H,H); (H,T); (T,H); (T,T) } : 4 Outcomes

Q. Three unbiased coins are tossed. What is the probability of getting at most two heads?

- Events against A: (H,H,H) = 1 Outcome
- Total Events : $2^3 = 8$; Probability = 1/8
- Event in Favour of A = 1-(1/8) = **7/8**

Everything about Dice

- Total outcomes : **6^n** , where n = no. of dice rolls
  - 1 Dice : {1,2,3,4,5,6} = 6 Outcomes
  - 2 Dice = 36 outcomes etc.

Q. A dice is rolled twice. What is the probability of getting a difference of 2 points

- Events : { (6,4); (4,6); (1,3); (3,1); (2,4); (4,2); (3,5); (5,3) } = 8 Outcomes
- Total Outcomes : 36; **Probability = 8/36 = 2/9**

- Trick for probability of sum of number on dice roll

SUM	2	3	4	5	6	7	8	9	10	11	12
Events	1	2	3	4	5	6	5	4	3	2	1

Q. What is the probability of getting a sum 9 from two throws of a dice?

- Events from above table = 4
- Total Outcomes = 36; **Probability = 4/36 = 1/9**

Everything about Cards (Use Combination Formulas)

- Total Cards = 52; All Numbers are in Pairs of 4.
- J,K,Q are Face Cards
- Total Groups = 4 (Hearts, Diamond, Club, Spades)

Q. Three cards are drawn together from a pack of 52 cards. Probability to get One spade and Two hearts.

$$({}^{13}C_1 \times {}^{13}C_2) / {}^{52}C_3$$

Total Cards Selected

Cards in a group

Cards Selected

Total Cards

OR means Addition & AND means Multiplication

Miscellaneous question attempt

- A bag contains 6 white, 4 black & 3 blue balls, 2 balls are drawn at random. Probability that they are either blue or white.

$$({}^{6}C_2 + {}^{3}C_2) / {}^{13}C_2$$

Total Balls

Total Whites

Total Blues

Selecting 2

Important Terms to Note

- **Random experiment** : Unpredictable process with uncertain outcomes
- **Sample Space** : Total possible Outcomes
- **Event** : Total Probable Outcomes
  - Simple Event : Rolling a Dice
  - Composite Event : Drawing 2 cards simultaneously
- **Mutually Exclusive Events** : Not more than one outcome can appear at once, If coin is tossed only H or T can come.
- **Exhaustive Events** : All possible outcomes included
- **Equally Likely :** All outcomes equally likely

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) \\ - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$P(A^c) = 1 - P(A)$$

NOTE

- Two Events are mutually exclusive if $P(A \cap B) = 0$
  - $P(A \cup B) = P(A) + P(B)$
- For Three Mutually Exclusive Events : $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
- Two Events are exhaustive if $P(A \cup B) = 1$
- Three Events are exhaustive if $P(A \cup B \cup C) = 1$
- A, B and C are equally likely if $P(A) = P(B) = P(C)$.

Conditional Probability

- Probability of occurrence of A given B has already occurred represented as P(A/B) or P(A given B)

$$P\left(\frac{A}{B}\right) \text{ i.e. } P(A \text{ given } B) = \frac{n(A \cap B)}{n(B)} = \frac{P(A \cap B)}{P(B)}$$

- In General, $P(B \cap A) = P(A) \times P(B|A)$

$$P(A \cap B \cap C) = P(A) \times P\left(\frac{B}{A}\right) \times P\left(\frac{C}{A \cap B}\right)$$

Note : Two events are independent if,

$$P\left(\frac{A}{B}\right) = P(A) \Rightarrow P(A) \times P(B) = P(A \cap B)$$

Mean Value/expectation	$E(X) = \sum_{i=1}^{n} (x_i\, p_i)$
Variance	$V(x) = E(X^2) - [E(X)]^2$

Definition

- Index number helps simplify complex changes over time or across locations by condensing them into a single, easy-to-understand value
- Expressed in percentage and independent of unit.

Methods to Study

- Simple
  - Aggregative
  - Relative
- Weighted
  - Aggregative
  - Relative

For a Data of Stocks

Stock	1990	2000	2010	P_1/P_0
A	10	12	15	1.5
B	20	21	23	1.15
C	30	31	34	1.13
Total	60	64	72	3.78

Simple Aggregate Method

considering above example $\frac{\Sigma P_1}{\Sigma P_0} \times 100 \Rightarrow (72/60) \times 100 =$ **120%**

Simple Relative Method	
considering above example	$\frac{\Sigma(P_1/P_0)}{n} \times 100 \Rightarrow \frac{(3.78)}{3} \times 100 =$ **126%**

Weighted Aggregate Method	
• **Laspeyres' Index** : Base year quantities are used as weights	$\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$
• **Paasche's Index :** Current year are used as weights	$\frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$
• **Fisher's ideal Price :** G.M. of Laspeyres & Paasche's	$\sqrt{L \times P}$
• **Marshall-Edgeworth :** taking average of the base year and the current year	$\frac{\sum P_1 Q_0 + \sum P_1 Q_1}{\sum P_0 Q_0 + \sum P_0 Q_1} \times 100$

The Chain Index Number
$\frac{\text{Link Relatives (Current yr.)} \times \text{Chain Index (Previous yr.)}}{100}$
Link Relative of Current Year = $\frac{P_1}{P_0} \times 100$

Simple Relative Method	
considering above example	$\frac{\Sigma(P_1/P_0)}{n} \times 100 \Rightarrow \frac{(3.78)}{3} \times 100 = \mathbf{126\%}$

Weighted Aggregate Method	
• **Laspeyres' Index** : Base year quantities are used as weights	$\frac{\Sigma P_1Q_0}{\Sigma P_0Q_0} \times 100$
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• **Fisher's ideal Price :** G.M. of Laspeyres & Paasche's	$\sqrt{L \times P}$
• **Marshall-Edgeworth :** taking average of the base year and the current year	$\frac{\Sigma P_1Q_0 + \Sigma P_1Q_1}{\Sigma P_0Q_0 + \Sigma P_0Q_1} \times 100$

The Chain Index Number

$$\frac{\text{Link Relatives (Current yr.)} \times \text{Chain Index (Previous yr.)}}{100}$$

Link Relative of Current Year = $(P_1/P_0) \times 100$

Deflated Value

$$\frac{\text{Current Price}}{\text{Price Index of Current Year}} = \text{Current Price} \times P_0/P_1$$

SHIFTING AND SPLICING OF INDEX NUMBERS

Two indexes covering different bases may be combined

$$= \frac{\text{Original Price Index}}{\text{Price index of the year to which it has to be shifted}} \times 100$$

TEST OF ADEQUACY

- **Unit Test** : Formulas must be unit-independent; most meet this except simple aggregative.
- **Time Reversal Test** : Ensure formula works both ways current on base, base on current multiply to unity.
  - $P_{01} \times P_{10} = 1$ (Reciprocal of each other)
  - Laspeyres' method and Paasche's method do not satisfy this test, but Fisher's Ideal Formula does.
- **Factor Reversal Test** : Product of price and quantity indices equals corresponding value index
  - $P_{01} \times Q_{01} = V_{01}$
  - Fisher's Ideal Formula gives this test.
- Fisher's Ideal Formula is **IDEAL** index number
- **Circular Test :** This is an extension to Time Reversal test and no formula satisfied by this test.
  - Simple G.M. of price relatives and the weighted aggregative with fixed weights meet this test.