CA FOUNDATION - PAPER 3 - BUSINESS MATHEMATICS, LOGICAL REASONING AND STATISTICS

At the Foundation level the concept of Probability is used in accounting and finance to understand the likelihood of occurrence or non-occurrence of a variable. It helps in developing financial forecasting in which you need to develop expertise at an advanced stage of chartered accountancy course. Here in this capsule an attempt is made for solving and understanding the concepts of probability.

Chapter 16 : Probability

The terms 'Probably' 'in all likelihood', 'chance', 'odds in favour', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics, known as Probability. In recent time, probability has developed itself into a full-fledged subject and become an integral part of statistics.

Random Experiment: An experiment is defined to be random if the results of the experiment depend on chance only. For example if a coin is tossed, then we get two outcomes—Head (H) and Tail (T). It is impossible to say in advance whether a Head or a Tail would turn up when we toss the coin once. Thus, tossing a coin is an example of a random experiment. Similarly, rolling a dice (or any number of dice), drawing items from a box containing both defective and non—defective items, drawing cards from a pack of well shuffled fifty—two cards etc. are all random experiments.

Events: The results or outcomes of a random experiment are known as events. Sometimes events may be combination of outcomes. The events are of two types:

(i) Simple or Elementary,

(ii) Composite or Compound.

An event is known to be simple if it cannot be decomposed into further events. Tossing a coin once provides us two simple events namely Head and Tail. On the other hand, a composite event is one that can be decomposed into two or more events. Getting a head when a coin is tossed twice is an example of composite event as it can be split into the events HT and TH which are both elementary events.

Mutually Exclusive Events or Incompatible Events: A set of events A_1, A_2, A_3, ... is known to be mutually exclusive if not more than one of them can occur simultaneously. Thus, occurrence of one such event implies the non-occurrence of the other events of the set. Once a coin is tossed, we get two mutually exclusive events Head and Tail.

Exhaustive Events: The events A_1, A_2, A_3, ... are known to form an exhaustive set if one of these events must necessarily occur. As an example, the two events Head and Tail, when a coin is tossed once, are exhaustive as no other event except these two can occur.

Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events: The events of a random experiment are known to be equally likely when all necessary evidence are taken into account, no event is expected to occur more frequently as compared to the other events of the set of events. The two events Head and Tail when a coin is tossed is an example of a pair of equally likely events because there is no reason to assume that Head (or Tail) would occur more frequently as compared to Tail (or Head).

CLASSICAL DEFINITION OF PROBABILITY OR A PRIORI DEFINITION

Let us consider a random experiment that result in n finite elementary events, which are assumed to be equally likely. We next assume that out of these n events, $n_A(\leq n)$ events are favourable to an event A. Then the probability of occurrence of the event A is defined as the ratio of the number of events favourable to A to the total number of events. Denoting this by P(A), we have

$$P(A) = \frac{n_A}{n} = \frac{\text{Number of equally likely events favourable to A}}{\text{Total Number of equally likely events}}$$

However, if instead of considering all elementary events, we focus our attention to only those composite events, which are mutually exclusive, exhaustive and equally likely and if $m(\leq n)$ denotes such events and is furthermore $m_A(\leq n_A)$ denotes the no. of mutually exclusive, exhaustive and equally likely events favourable to A, then we have

$$P(A) = \frac{m_A}{m} = \frac{\text{"Number of mutually exclusive,exhaustive and equally likely events favourable to A"}}{\text{"Total Number of mutually exclusive,exhaustive and equally likely events"}}$$

PROBABILITY AND EXPECTED VALUE BY MATHEMATICAL EXPECTATION

For this definition of probability, we are indebted to Bernoulli and Laplace. This definition is also termed as a priori definition because probability of the event A is defined based on prior knowledge.

This classical definition of probability has the following demerits or limitations:

(i) It is applicable only when the total no. of events is finite.

(ii) It can be used only when the events are equally likely or equi-probable. This assumption is made well before the experiment is performed.

(iii) This definition has only a limited field of application like coin tossing, dice throwing, drawing cards etc. where the possible events are known well in advance. In the field of uncertainty or where no prior knowledge is provided, this definition is inapplicable.

In connection with classical definition of probability, we may note the following points:

(a) The probability of an event lies between 0 and 1, both inclusive.

i.e. $0 \leq P(A) \leq 1$

When P(A) = 0, A is known to be an impossible event and when P(A) = 1, A is known to be a sure event.

(b) Non-occurrence of event A is denoted by A′ or A^C or and it is known as complimentary event of A. The event A along with its complimentary A′ forms a set of mutually exclusive and exhaustive events.

i.e. $P(A)+P(A')=1$, $P(A')=1-P(A) = 1 - \frac{m_A}{m} = \frac{m-m_A}{m}$

STATISTICS

Statistical definition of Probability: Owing to the limitations of the classical definition of probability, there are cases when we consider the statistical definition of probability based on the concept of relative frequency. This definition of probability was first developed by the British mathematicians in connection with the survival probability of a group of people.

Let us consider a random experiment repeated a very good number of times, say n, under an identical set of conditions. We next assume that an event A occurs F_A times. Then the limiting value of the ratio of F_A to n as n tends to infinity is defined as the probability of A.

i.e. $P(A) = \lim_{n\to\infty} \frac{F_A}{n}$

This statistical definition is applicable if the above limit exists and tends to a finite value.

Two events A and B are mutually exclusive if P (A ∩ B) = 0 or more precisely

$P(A\cup B) = P(A) + P(B)$

Similarly, three events A, B and C are mutually exclusive if.

$P(A\cup B\cup C) = P(A) + P(B) + P(C)$

Two events A and B are exhaustive if.

$P(A\cup B) = 1$

Similarly, three events A, B and C are exhaustive if.

$P(A\cup B\cup C) = 1$

Three events A, B and C are equally likely if

$P(A) = P(B) = P(C)$

Axiomatic or modern definition of probability: Then a real valued function p defined on s is known as a probability measure and p(a) is defined as the probability of A if P satisfies the following axioms:

(i) $P(A) \le 0$ for every $A \subseteq S$ (subset)

(ii) $P(S) = 1$

(iii) For any sequence of mutually exclusive events A_1, A_2, A_3,..

$P(A_1 U A_2 U A_3 U\ldots) = P(A_1) + P(A_2) + P(A_3)$

Addition theorems or theorems on total probability: For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.i.e. P (A∪B) or P(A + B) = P(A) + P(B) or P(A or B) whenever A and B are mutually exclusive

For any three events A, B and C, the probability that at least one of the events occurs is given by

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C)+ P(A\cap B\cap C)$

(d) For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

i. e. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

For any three events A, B and C, the probability that at least one of the events occurs is given by

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C)+ P(A\cap B\cap C)$

(e) Two events A and B are mutually exclusive if

$P(A\cup B) = P(A) + P(B)$

Similarly, three events A, B and C are mutually exclusive if $(A\cup B\cup C) = P(A) + P(B) + P(C)$

(f) $P(A-B) = P(A\cap B') = P(A) - P(A\cap B)$

And $P(B-A) = P(B\cap A') = P(B) - P(A\cap B)$

Some important Results

1.If A and B are two independent events, then the probability of occurrence of both is given by P (A∩B) = P(A). P(B)

2.If A, B and C are three events, then. P(A∩B∩C) = P(A). P(B/A). P(C/A∩B)

3.If A and B are two mutually exclusive events of a random experiment, then.

A∩B = ϕ , P(AUB) = P(A) +P(B)

4. If A and B are associated with a random experiment, then.

P(AUB) = P(A) +P(B) -P(A∩B)

5. If A, B and C are three events connected with random experiment, then

P(AUBUC) = P(A) +P(B) +P(C) -P(A∩B) -P(B∩C)-P(C∩A) +P(A∩B∩C)

(g) Compound Probability or Joint Probability

$P(B/A) = \frac{P(B\cap A)}{P(A)} = \frac{P(A\cap B)}{P(A)}$

(h) For any three events A, B and C, the probability that they occur jointly is given by

$P(A\cap B\cap C) = P(A)\ \ P(B/A)\ \ P(C/(A\cap B))$ Provided $P(A\cap B) > 0$

(i) $P(A'/B) = \frac{P(A'\cap B)}{P(B)} = \frac{P(B) - P(A\cap B)}{P(B)}$

(j) $P(A/B') = \frac{P(A\cap B')}{P(B')} = \frac{P(A) - P(A\cap B)}{1 - P(B)}$

(k) $P(A'/B') = \frac{P(A'\cap B')}{P(B')}$

$= \frac{P(A\cup B)'}{P(B')}$ [by De-Morgan's Law $A'\cap B' = (A\cup B)'$]

$= \frac{1 - P(A\cup B)}{1 - P(B)}$

CA FOUNDATION - PAPER 3 - BUSINESS MATHEMATICS, LOGICAL REASONING AND STATISTICS

This capsule is in continuation to the previous edition featured in March 2021. It presents the concepts of Random Variable, Expected value, Variance and Standard Deviation of a random variable. These concepts are extensively applied and widely used in areas such as Finance, Risk Management and Costing. Here an attempt is made to enable the students to understand these concepts of probability calculation with the help of examples and help them attempt diverse questions based on these concepts.

Chapter 16 : Probability - II

(7) **A random variable or stochastic variable** is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.

(8) **Expected value or Mathematical Expectation or Expectation** of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

When x is a discrete random variable with probability mass function f(x), then its expected value is given by

$$E(x) = \mu = \sum_x x f(x)$$

and its variance is

$$V(x) = \sigma^2 = E(x^2) - \mu^2$$

$$\text{Where } E(x^2) = \sum_x (x)^2 f(x)$$

For a continuous random variable x defined in [–∞, ∞], its expected value (i.e. mean) and variance are given by

$$E(x) = \int x f(x) dx$$

$$\text{and } \sigma^2 = E(x^2) - \mu^2$$

$$\text{where } E(x^2) = \int x^2 f(x) dx$$

Properties of Expected Values

1. Expectation of a constant is k
   i.e. E(k) = k for any constant k
2. Expectation of sum of two random variables is the sum of their expectations.
   i.e. E(x + y) = E(x) + E(y) for any two random variables x and y.
3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.
   i.e. E(k x) = k.E(x) for any constant k
4. Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.
   i.e. E(xy) = E(x).E(y)
   Where x and y are independent.

IMPORTANT EXAMPLES:

1. A speaks truth in 60% and B in 75% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?
   Solution:
   The Probability that A speaks the truth and B a lie =
   $$\frac{60}{100} \times \frac{(100-75)}{100} = \frac{60}{100} \times \frac{25}{100} = \frac{3}{20}$$
   The Probability that B speaks the truth and A a lie =
   $$\frac{75}{100} \times \frac{(100-60)}{100} = \frac{75}{100} \times \frac{40}{100} = \frac{3}{10}$$
   $$\therefore \text{ Total Probability} = \frac{3}{20} + \frac{3}{10} = \frac{9}{20}$$
   Hence, the percentage of cases in which they contradict each other = (9/20) × 100 or 45%

2. A Committee of 4 persons is to be appointed from 7 men and 3 women. The probability that the committee contains (i) exactly two women, and (ii) at least one woman is
   Solution:
   Total number of persons = 7+3 = 10. Since 4 out of them can be formed in 10 C_4 ways, where the exhaustive number of cases is 10 C_4 or 210 ways.
   (i) P (exactly 2 women in a committee) of four = 7 C_2X 3C_2/ 210 = 63/210 = 3/10.
   (ii) P (at least one women in committee)
   = 1−p (no women) =1-(7C_4/10C_4)=1-(35/210)=1-1/6=5/6

3. If A and B are two events, such that P(A) = 1/4, P(B) = 1/3 and P(A ∪ B) = 1/2; then P(B/A) is equal to
   Solution: $P(A \cup B)$ = P(A) + P (B) - P(A ∩ B)
   1/2 = 1/4 + 1/3 - P(A ∩ B)
   Or P(A ∩ B) = 1/4+1/3-1/2= 1/12
   $$\text{Hence, P (B/A)} = \frac{P(A \cap B)}{P(A)} = \frac{1/12}{1/4} = \frac{1}{3}$$

4. A person applies for a job in two firms, say X and Y. the probability of his being selected in firm X is 0.7 and being rejected in firm Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected in one of the two firms?

Solution:
Event A; Person is selected in firm X, and
Event B : person is selected in Firm Y .
Then, $P(A) = 0.7$, $P(B^c) = 0.5$ and $P(A^c \cup B^c) = 0.6$
Therefore $P(B) = 1-0.5 = 0.5$
$P(A^c \cup B^c) = P[(A \cap B)^C] = 1-P(A \cap B)$
This implies that $P(A \cap B) = 1- P(A^c \cup B^c) = 1-0.6 = 0.4$
Hence, $P(A \cup B) = 0.7+0.5-0.4 = 0.8$

5. A person is known to hit a target in 5 out of 8 shots, whereas another person is known to hit in 3 out of 5 shots. Find the probability that the target is hit at all when they both try.

Solution: Event A = First person hits the target and
Event B = Another person hits the target.
P (A) = 5/8 and P(B) = 3/5
$P(A^C) = 1-5/8 = 3/8$ and $P(B^C) = 1-3/5 = 2/5$
Event X = target is hit when they both try i.e.,
When at least one of them hit the target.
$P(X^C)$ = P (the target is not hit at all)
$= P(A^C \cap B^C) = P(A^C) \times P(B^C) = 3/20$
Hence $P(X) = 1-P(X^C) = 1-3/20 = 17/20$

6. The probability that a man will be alive in 25 years is 3/5, and the probability that his wife will be alive in 25 years in 2/3. Find the probability that :
(i) Both will be alive (ii) at least one of the will be alive

Solution:
P (M) = 3/5 and P(W) = 2/3
$P(M^C) = 1 - 3/5$ and $P(W^C) = 1-2/3 = 1/3$.
The probability that both will be alive
$= P(M) \times P(W) = 3/5 \times 2/3 = 2/5$.
Probability that at least one of them will be alive is given by
$P(M \cup W) = P(M) + P(W) - P(M \cap W)$
$= 3/5+2/3-6/15 = 13/15$.

7. Given the data in Previous Problem find the probability that (i) only wife will be alive, (ii) only man will be alive.

Solution.
(i) Probability that only wife will be alive.
= Probability that wife will be alive but not man
$= P(W) \times P(M^C) = 2/3 \times 2/5 = 4/15$
(ii) Probability that only man will be alive
= Probability that man will be alive but not wife
$= P(M) \times P(W^C) = 3/5 \times 1/3 = 1/5$.

8. A random variable X has the following probability distribution:

Value of X	0	1	2	3
P[X = x]	1/3	1/2	0	1/6

Find $E(\{X - E(X)\}^2)$

Solution :
$E(X) = 0\times1/3+1\times1/2+2\times0+3\times1/6 = 1$
$E(X^2) = 0\times1/3+1\times1/2+4\times0+9\times1/6 = 2$
$E[X - E(X)]^2 = E(X^2) - [E(X)]^2 = 2-1 = 1$

9. Given the data in previous Problem, Find Var (Y), where Y = 2X – 1.

Solution:
$E(Y) = E(2X-1) = 2E(X) - 1 = 1$
$E(Y^2) = E(2x-1)^2 = 2E(X^2) - 4E(X)+1 = 1$
$\text{Var.}(Y) = E(Y^2) - [E(Y)]^2 = 1-1 = 0$

10 Daily demand for pen drive is having the following probability distribution. Determine the expected demand and variance of the demand:

Demand	1	2	3	4	5	6
Probability	0.10	0.15	0.20	0.25	0.18	0.12

Solution:
$E(X)=1\times0.10+2\times0.15+3\times0.20+4\times0.25+5\times0.18+6\times0.12 = 3.62$
Variance of the demand.
$E(X^2) = 1\times0.10+4\times0.15+9\times0.20+16\times0.25+25\times0.18+36\times0.12= 15.32$
$\text{Var.}(X) = 15.32 - (3.62)^2 = 2.22$

11. An investment consultant predicts the odds against the price of a certain stock going up are 2:1 and odds in favor of the price remaining the same are 1:3. What is the price of stock will go down?
Solution:

$$\text{P(the prices will go up)} = \frac{1}{2+1} = \frac{1}{3}$$

$$\text{P (the prices will remain the same)} = \frac{1}{1+3} = \frac{1}{4}$$

Therefore P (the prices will go down)
= P (the price will neither go up nor remain same)
=1-P (the price will go up or will remain the same)

$$= 1 - \left(\frac{1}{3} + \frac{1}{4}\right) = 1 - \frac{7}{12} = \frac{5}{12}$$

12. A pair of dice is rolled. If the sum of the two dice is 9, find the probability that one of the dice shows 3
Solution:
Let A: Sum of on the two dice is 9. B: one of the dice showed 3.
Total outcomes when two dice are thrown = 6×6 = 36

$$P(A) = P\{(6,3), (5,4), (4,5), (3,6)\} = \frac{4}{36}$$

$$P(A \cap B) = P\{(6,3), (3,6)\} = \frac{2}{36}$$

Therefore, required probability = P(B/A) =

$$= \frac{P(A \cap B)}{P(A)} = \frac{2/36}{4/36} = \frac{2}{4} = \frac{1}{2}$$

13. The overall percentage of failures in a certain examination was 30. What is the probability that out of a group of 6 candidates at least four passed the examination?

Solution:

Let passing the examination be a success.

Take n= 6, P= P(a student passes) = $1-\frac{30}{100}=\frac{70}{100}=\frac{7}{10}$

q = P (a student fails) = $\frac{30}{100}=\frac{3}{10}$

Therefore P (at least 4 students pass) = P(4 or 5 or 6)

$$= 6_{C_4}\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right)^2+6_{C_5}\left(\frac{7}{10}\right)^5\left(\frac{3}{10}\right)+6_{C_6}\left(\frac{7}{10}\right)^6\left(\frac{3}{10}\right)^0$$

$$=15\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right)^2+6\left(\frac{7}{10}\right)^5\left(\frac{3}{10}\right)+\left(\frac{7}{10}\right)^6$$

$$=\left(\frac{7}{10}\right)^4\left[\frac{135}{100}+\frac{126}{100}+\frac{49}{100}\right]$$

$$=\frac{31}{10}\left(\frac{7}{10}\right)^4=0.74431.$$

14. What is the probability that a leap year selected at random would contain 53 Saturdays?

Solution:

A normal year has 52 Mondays, 52 Tuesdays, 52 Wednesdays, 52 Thursdays, 52 Fridays, 52 Saturdays and 52 Sundays

52 Saturdays, 52 + 1 day that could be anything depending upon the year under consideration.

- In addition to this, a leap year has an extra day which might be a Monday or Tuesday or Wednesday or Sunday.

Our sample space is S: {Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday-Friday, Friday-Saturday, Saturday-Sunday, Sunday-Monday} =Number of elements in S = n(S) = 7

set A (say) that comprises of the elements Friday-Saturday and Saturday-Sunday i.e. A : {Friday-Saturday, Saturday-Sunday}

Number of elements in set A = n(A) = 2,

By definition, probability of occurrence of A = n(A)/n(S) = 2/7

Therefore, probability that a leap year has 53 Saturdays is =2/7

15. If two unbiased coin is tossed three times, what is the probability of getting more than one head.

Solution:

One toss can give two (2) possible outcomes - head and tail. So, three tosses can give (2×2×2) = 8 possible outcomes.

2 heads and 1 tail out of 3 tosses can occur in $(3C_2)\times(1C_1)$ = 3 ways.

So, the probability = (3/8).

16. If two unbiased are rolled, what is the probability of getting points neither 6 nor 9?

Solution:

Two dice can make 6 in 5 ways: {1,5}, {2,4}, {3,3}, {4,2} and {5,1}.

Two dice can make 9 in 4 ways: {3,6}, {4,5}, {5,4} and {6,3}.

There are 36 possible ways the two dice can fall. Therefore, the probability of 6 or 9 is (5+4)/36 = 1/4.

The probability of not (6 or 9) is therefore 1–1/4 = 3/4.

17. What is probability that 4 children selected at random would have different birthdays

Solution:

There are 365 out of 365 ways to select the birthday of first person. Therefore, the number of ways that we can choose a birthday for second person is 364 out of 365.

The probability that the second child has a different birthdate than the first is 364/365.

The probability that the third child has a different birthday than the first two is 363/365.

The probability that the fourth child has a different birthday than the first three is 362/365.

Since all three of these situations must occur, multiply the three probabilities.

364/365 × 363/365 × 362/365 = 98.364%

18. A box contains 5 white and 7 black balls. Two successive drawn of 3 balls are made (i) with replacement (ii) without replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively.

Solution:

Two successive drawn of 3 balls are made: TOTAL = 12 Balls: (5White balls +7 Black balls)

1st draw white ball and second draw black ball with replacement = $\frac{5C_3}{12C_3}\times\frac{7C_3}{12C_3}=\frac{10}{220}\times\frac{35}{220}=\frac{7}{968}$

1st draw white ball and second draw black ball without replacement = $\frac{5C_3}{12C_3}\times\frac{7C_3}{9C_3}=\frac{10}{220}\times\frac{35}{84}=\frac{5}{264}$

P (both happening) = $\frac{7}{968}$ and $\frac{5}{264}$

19. There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of same colour?

Solution:

Either balls would be Red or white or blue

$=P(R_1\cap R_2\cap R_3) + P(W_1\cap W_2\cap W_3) + P(B_1\cap B_2\cap B_3)$

$=P(R_1)\times P(R_2)\times P(R_3)+P(W_1)\times P(W_2)\times P(W_3)+P(B_1)\times P(B_2)\times P(B_3)$

$=\frac{5}{18}\times\frac{4}{18}\times\frac{3}{9}$(Red Balls) $+\frac{7}{18}\times\frac{8}{18}\times\frac{4}{9}$ (White Balls) $+\frac{6}{18}\times\frac{6}{18}\times\frac{2}{9}$ (Black Balls)

$=\frac{89}{729}$

20. A number is selected at random from the first 1000 natural numbers. What is the probability that the number so selected would be a multiple of 7 or 11?

Solution:

First 1000 natural numbers belong to the following set {1,2, 3,...,1000} with cardinality = 1000

Multiples of 7 less than 1000 = Quotient of (1000/7) = 142

Multiples of 11 less than 1000 = Quotient of (1000/11) = 90

As 7 & 11 are both primes so multiples of 7*11 = 77 will be included in both multiples of 7 and multiples of 11

Multiples of 77 less than 1000 = Quotient of (1000/77) = 12
Hence, all-natural numbers below 1000 which are either multiples of 7 or of 11 = 142 + 90–12 = 220
So, Prob (this event) = 220/1000 = 0.22

21. A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw will produce 3 white balls and the second 3 red balls is

Solution:
There are total 13 balls out of which 8 are red and 5 are white.
Favourable case of first draw is to get 3 white balls out of 5 white balls.
Probability $P_1 = 5C_3/13C_3 = 5/143$
If this happens then remaining are - 2 white balls and 8 red balls.
Favourable case is to get 3 red balls out of 8 balls.
Probability $P_2 = 8C_3/10C_3 = 7/15$
Both the events are independent of each other, hence total probability is $P_1 * P_2 = (5C_3 \times 8C_3)/(13C_3 \times 10C_3) = 5/143 \times 7/15 = 7/429$

22. There are two boxes containing 5 white and 6 blue balls and 3 white and 7 blue balls respectively. If one of the the boxes is selected at random and a ball is drawn from it, then the probability that the ball is blue is

Solution:
First box: No. of white balls = 5, No. of blue balls = 6
Second box: No. of white balls = 3, No. of blue balls = 7
So, total no. of white balls = 8, Total no. of blue balls = 13
So, total no. of balls = 8+13=21
Now probability of getting blue ball: =13/21
Hence the probability of getting blue ball is =13/21

23. A problem in probability was given to three CA students A, B and C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

Solution:
Probability of A solving the problem = 1/3, Probability of A not solving the problem = 1-1/3 = 2/3
Probability of B solving the problem = 1/5, Probability of B not solving the problem = 1-1/5 = 4/5
Probability of C solving the problem = ½, Probability of C not solving the problem = 1-1/2 = ½
Probability of A, B and C not solving the problem = 2/3 x 4/5 x 1/2 = 4/15
Probability of A, B and C solving the problem = 1-4/15 = 11/15

24. There are three persons aged 60, 65 and 70 years old. The survival probabilities for these three persons for another 5 years are 0.7, 0.4 and 0.2 respectively. What is the probability that at least two of them would survive another five years?

Solution:
Probability (At least two alive) = P (two alive) +P (two alive)
=(0.7)(0.4)(1-0.2)+(0.7)(0.2)(1-0.4)+(0.4)(0.2)(1-0.7) +(0.7(0.4)(0.2)
$= 0.28 \times 0.8 + 0.14 \times 0.6 + 0.08 \times 0.3 + 0.056 = 0.224 + 0.084 + 0.056 = 0.388$

25. Tom speaks truth in 30 percent cases and Dick speaks truth in 25 percent cases. What is the probability that they would contradict each other?

Solution:
P(Tom speaks truth)= $P(T_T)$ = 30/100= 3/10;
P(Dick speaks truth)= $P(D_T)$ = 25/100= ¼
$P(T_F)$ = 1-3/10= 7/10; $P(D_F)$ = 1-1/4= ¾
probability that they would contradict each other
= $P(T_T) \times P(D_F) + P(T_F) \times P(D_T)$
= (3/10×3/4) + (7/10×1/4) = 0.40

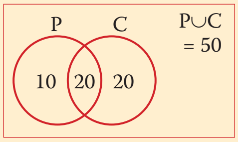
26. There are two urns. The first urn contains 3 red and 5 white balls whereas the second urn contains 4 red and 6 white balls. A ball is taken at random from the first urn and is transferred to the second urn. Now another ball is selected at random from the second urn. The probability that the second ball would be red is

Solution:
the first urn contains 3 red and 5 white balls => total = 8 Balls
second urn contains 4 red and 6 white balls. => Total = 10 Balls
There can be two cases : Ball taken from Urn is Red or White
Case 1 : Red is Taken from Urn:A
Probability of Red = (3/8)
then second urn contains 5 Red & 6 White => total = 11
Probability of Red from Urn:B = (3/8) (5/11) = 15/88
Probability of White from Urn:B = (3/8) (6/11) = 18/88
Case 1 : White is Taken from Urn:A
Probability of White = (5/8)
then second urn contains 4 Red & 7 White => total = 11
Probability of Red from Urn:B = (5/8) (4/11) = 20/88
Probability of White from Urn:B = (5/8) (7/11) = 35/88
probability of that the second ball would be Red = 15/88 + 20/88 = 35/88
probability of that the second ball would be White = 18/88 + 35/88 = 53/88

27. For a group of students, 30%, 40% and 50% failed in Physics, Chemistry and at least one of the two subjects, respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry?

Solution:
Let the total number of students = 100
Number of students failed in physics = 30% of 100 =30
Number of students failed in chemistry = 40% of 100 =40
Number of students failed at least one of the two subjects = 50% of 100 =50
We need to calculate.
P (He passed in Physics but Failed in Chemistry)/P (Failed Chemistry)
$n(P \cup C) = n(p) + n(c) - n(P \cap C) 50 = 30+40- n(P \cap C)$
$n(P \cap C) = 20$
P (He passed in Physics but Failed in Chemistry)/P (failed Chemistry)

$$= P(C\text{-}P)/(P(P) = \frac{\frac{20}{100}}{\frac{40}{100}} = \frac{20}{40} = \frac{1}{2}$$



28. A packet of 10 electronic components is known to include 2 defectives. If a sample of 4 components is selected at random from the packet, what is the probability that the sample does not contain more than 1 defective?

Solution:
P (No more than one defective) = P (No defective) + P (One defective)

$$= \frac{{}^{8}C_{4}}{{}^{10}C_{4}} + \frac{{}^{2}C_{1} \times {}^{8}C_{3}}{{}^{10}C_{4}}$$

$$= \frac{70}{210} + \frac{112}{210}$$

$$= \frac{91}{105} = \frac{13}{15}$$

29. 8 identical balls are placed at random in three bags. What is the probability that the first bag will contain 3 balls?

Solution:
The probability that a ball will be placed in the first bag is 1/3. The probability that exactly 3 of the 8 balls will end up in the first bag can be found by using the binomial distribution:

$${}^{8}C_{3}\left(\frac{1}{3}\right)^{3}\left(\frac{2}{3}\right)^{5} = \frac{{}^{8}C_{3} \times 2^{5}}{3^{8}} = \frac{56 \times 32}{6561} = 0.2731$$

30. X and Y stand in a line with 6 other people. What is the probability that there are 3 persons between them?

Solution:
There are altogether 8 people $p_1, p_2, \ldots, p_8$ including X & Y and these 8 people can be arranged in 8!= 40320 ways
Now, there should be 3 people between X and Y and these 3 people can be selected out of 6 in C(6,3)=20 =20ways. (6 people because X & Y are excluded from 8).
Now, take (X,*,*,*,Y)(X,*,*,*,Y)as one set of people and together with the remaining 3 people we can think of a total of 4 people which can be arranged in 4!=24 ways.
Again, the 3 people between X & Y can be arranged in 3!=6 ways.
Also, the position of X and Y can also be arranged in 2!=2ways.
So, total arrangements with 3 people between X & Y is 20×24×6× 2=5760
Hence, the required probability is =5760/40320=1/7.

31. Given that P (A) = 1/2, P (B) = 1/3, P (AB) = 1/4, what is P (A'/B')

Solution:

$$P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{P(AUB)'}{1-P(B)} = \frac{1-P(AUB)}{1-P(B)} = \frac{1-\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)}{1-\frac{1}{3}} = \frac{1-\left(\frac{6+4-3}{12}\right)}{\frac{3-1}{3}}$$

$$= \frac{1-\frac{7}{12}}{\frac{2}{3}} = \frac{\frac{5}{12}}{\frac{2}{3}} = \frac{5}{12} \times \frac{3}{2} = \frac{5}{8}$$

32. Four digits 1, 2, 4 and 6 are selected at random to form a four-digit number. What is the probability that the number so formed, would be divisible by 4?

Solution:
From four digits 1,2,4 and 6 last two digits (12,16,24,64) can be selected in (4 ways)
Total possible numbers are divisible by 4 are 4612, 6412, 2416, 4216, 1624, 6124, 1264, 2164 =8
Here are four ways of filling the last two digits. The remaining two places(100's, 1000's digits) can be filled in two ways. Thus there are total 4×2=8 ways
Total possible 4 digit numbers =4! =24

$$\text{Probability} = \frac{8}{4!} = \frac{8}{24} = \frac{1}{3}$$

33. A bag containing 6 white and 4 red balls. Rs 10 is received if he draws white ball and Rs. 20 for red ball. Find the expected amount when the person draws 2 balls.

Solution:
The probability of both being white ball would be $= \frac{{}^{6}C_{2}}{{}^{10}C_{2}} = \frac{15}{45}$

The probability of both being red ball would be $= \frac{{}^{4}C_{2}}{{}^{10}C_{2}} = \frac{6}{45}$

The probability of one being red ball and another being white ball would be $= \frac{{}^{6}C_{1} \times {}^{4}c_{1}}{{}^{10}C_{2}} = \frac{24}{45}$

Hence the expected amount when person draws two balls will be

$$= \frac{15}{45} \times (10+10) + \frac{6}{45}(20+20) + \frac{24}{45}(10+20)$$

= 6.7+5.33+16 = Rs.28

34. If two random variables x and y are related as Y =-3x +4 and Standard Deviation of Y is

Solution:
Given Y =-3x +4
Y = a + bx

$\sigma_y = \sigma(ax+b)$, $\sigma(b) = 0$

$\sigma_y = |a|.\sigma_x$

SD of y=$\sigma_y = 3 \times 2 = 6$

35. If 2x+3y+ 4 = 0 and v(x) = 6 then V(y) is

Solution:
Given that 2x+3y+ 4 = 0 and v(x) = 6 then SD of x = √6:V(y) = ?

$$SD_y = \left|-\frac{2}{3}\right|.SD_x = \frac{2}{3}\sqrt{6}$$

$$y^2 = \frac{4}{9}.6 = \frac{8}{3}$$

$$V(y) = \frac{8}{3}$$

36. A pocket of 10 electronic components is known to include 3 defectives. If 4 components are selected from the packet at random, what is the expected value of the number of defective?

Solution:
10 electronic components If 4 components are selected from the packet at random $= \frac{4}{10}$

Expected value of the number of defective $= 3 \times \frac{4}{10} = \frac{12}{10} = 1.2$

37. The Probability there is atleast one error in a account statement prepared by 3 persons A , B and C are 0.2 , 0.3 and 0.1 respectively . If A, B and C prepare 60, 70 and 90 such statements then expected number of correct statements.

Solution:
E(x)=A.(1-P(A))+B.(1-P(B))+C.(1-P(C))=(60×(1-0.2) + (70×(1-0.3) + (90×(1-0.1)
= (60×0.8) +(70×0.70) +(90×0.9) =178