Mathematics of finance

Simple Interest

Concept	1. $SI = PiT$ 2. $A = P + SI$ 3. $A = P(1+iT)$ 4. $A_n = P + n \times SI_1$
Raju bn gaya gentleman	5 yrs $\xrightarrow{\times 2}$ = 10 yrs. $P \longrightarrow 2P$ $\quad$ $P \longrightarrow 3P$ $SI = 1P$ $\xrightarrow{\times 2}$ $SI = 2P$

now, today, Commences now or at end...

single cash flow or Annuity...

Compound Interest

Compounded monthly, quarterly, semi-annually..

Concept	1. $CI = P\left\{\left(1+\frac{i}{k}\right)^{nk} - 1\right\}$ 2. $A = P + CI$ 3. $A = P\left(1+\frac{i}{k}\right)^{nk}$ # When compounded ⇒ annually, $k = 1$ ⇒ semi-annually, $k = 2$ ⇒ quarterly, $k = 4$ ⇒ monthly, $k = 12$ ⇒ daily, $k = 365$
Raju bn gaya gentleman	P —— $2P$ (×3) —— 10 yrs P —— $8P$ (×3) —— 30 yrs
Amount & CI in case of diff. rates	Suppose rates are 10%, 8% & 12% for 1st, 2nd & 3rd yr resp. $P = ₹\ 10{,}000$ $A = 10{,}000 + 10\% + 8\% + 12\%$ or $A = 10{,}000 \times 1.1 \times 1.08 \times 1.12$ $CI = A - P$
Depreciation	Using reducing balance metho Scrap Value (S.V.) = Cost (C) $(1-i)^n$ # i = rate of depreciation n = useful life of asset
How to find log value # used for finding n	1. Enter the no. 2. Press $\sqrt{\ }$ key 19 times 3. Subtract 1 4. Multiply 227695

Calculator steps

$i \div k + 1 \times = ---$ $(nk-1)$ times
M+ will give I.F. then
$-1 \div \frac{i}{k}$ will give F.F. then
$\div$ MRC = will give P.F.

Don't forget to note i/k

Annuity

Concept	Fixed amount paid / received in fixed intervals of time
Terminology	Interest factor = I.F. = $\left(1+\frac{i}{k}\right)^{nk}$ Future value factor = F.F. = $\frac{\left(1+\frac{i}{k}\right)^{nk}-1}{i/k} = \frac{I.F.-1}{i/k}$ Present value factor = P.F. = $\frac{\left(1+\frac{i}{k}\right)^{nk}-1}{i/k \times \left(1+\frac{i}{k}\right)^{nk}} = \frac{F.F.}{I.F.}$
Phir Phir Hera Pheri Scheme	F.V. $\xrightarrow{\times I.F.}$ P.V. ; F.V. $\xrightarrow{\times P.F.}$ Annuity ; P.V. $\xrightarrow{\times P.F.}$ Annuity
Ordinary Annuity or Annuity Regular	1st payment / receipt done at the end of the period. P.V. $\xrightarrow{\times I.F.}$ F.V. Annuity regular $\xrightarrow{\times F.F.}$ F.V. Annuity regular $\xrightarrow{\times P.F.}$ P.V.
Annuity immediate or Annuity due	1st payment / receipt done at the start or beginning of the period Annuity due $\xrightarrow{\times F.F. \times (1+i)}$ F.V. Annuity due $\xrightarrow{\times P.F. \times (1+i)}$ P.V. or P.V. = Annuity due + P.V. for (n-1) yrs using ordinary annuity method.
Perpetuity	Infinite time tak chalne wali Annuity P.V. of perpetuity [fixed] = $\frac{\text{Annual Cash flow}}{\text{Int. Rate}}$ P.V. of perpetuity [Growing] = $\frac{\text{Annual Cash flow}}{\text{Int. Rate} - \text{Growth Rate}}$

Special topics

Difference between CI and SI	for 2 years $D = Pi^2$ or $D = Pi \times i$ ↓ SI for 1 yr $\times i$ for 3 years $D = Pi^2(i+3)$
Effective Rate of Interest (ERI)	actual rate of interest when compounding is done more than once in a year. $ERI = 100\left\{\left(1+\frac{i}{k}\right)^{nk} - 1\right\}$
Compound Annual Growth Rate (CAGR) [leaps]	$\left\{\left(\frac{A}{P}\right)^{1/n} - 1\right\} \times 100$ Calculate of Rate from Amount formula
Lease Rent (nature of Annuity.)	Present value of Lease Rent is greater than Cost of Asset, then purchase the Asset.
Net Present Value	NPV = P.V. of cash inflows less: P.V. of cash outflows 1. NPV > 0, accept the proposal 2. NPV < 0, reject the proposal 3. NPV = 0, consider other factors.
Valuation of Bond	Purchase Price = P.V. of all inflows ↓ Returns + maturity value.
Real Rate of Return	Nominal Rate − Inflation Rate ERI > Nominal Rate > Real Rate

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Measures of Central Tendency

Arithmetic Mean

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Computation

Method / Series	Direct Method	Shortcut Method	Step Deviation Method
Individual Series	$\frac{\Sigma x}{N}$	$A + \frac{\Sigma d}{N}$ where $d = x - A$, A = Assumed Mean	$A + \frac{\Sigma d'}{N} \times C$ where A = Assumed Mean, C = Common factor, $d' = \frac{x-A}{C}$
Discrete Series	$\frac{\Sigma fx}{\Sigma f}$	$A + \frac{\Sigma fd}{\Sigma f}$ where A = Assumed Mean, $d = x - A$	$A + \frac{\Sigma fd'}{\Sigma f} \times C$ where A = Assumed Mean, C = Common factor, $d' = \frac{x-A}{C}$
Continuous Series	$\frac{\Sigma fm}{\Sigma f}$ where m = mid value $= \frac{UCB+LCB}{2}$ or $\frac{UCL+LCL}{2}$	$A + \frac{\Sigma fd}{\Sigma f}$ where A = Assumed Mean, $d = m - A$	$A + \frac{\Sigma fd'}{\Sigma f} \times C$ where A = Assumed Mean, C = Common factor, $d' = \frac{m-A}{C}$

Properties

1. A.M. of first n natural numbers $= \frac{n+1}{2}$ $\left\{ \frac{\frac{n(n+1)}{2}}{n} \right\}$
2. Sum of deviations from mean is zero. i.e. $\Sigma(x-\bar{x}) = 0$
3. Sum of squared deviations from mean is minimum i.e. mean se nikale gaye squared deviations ka sum hota h sabse kam $\Rightarrow \Sigma(x-\bar{x})^2 \le (x - \text{any no.})^2$
4. A.M. cannot be found graphically
5. Weighted Arithmetic Mean $= \bar{x}_w = \frac{\Sigma wx}{\Sigma w}$
6. Combined Arithmetic mean $= \bar{x}_{12} = \frac{\bar{x}_1 \times N_1 + \bar{x}_2 \times N_2}{N_1 + N_2}$
   where, $\bar{x}_1$ = mean of series 1, $\bar{x}_2$ = mean of series 2, n_1 = no. of items in series 1, n_2 = no. of items in series 2

Mode

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Computation

Series	formula
Individual Series	an observation with highest frequency.
Discrete Series	an observation with highest frequency.
Continuous series	Modal class with class – highest freq. $L_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$ where L_1 = lower limit of modal class, f_1 = freq. of modal class, f_0 = freq. of class just before modal class, f_2 = freq. of class just after modal class, i = class size

Properties

1. It is an obsⁿ with highest frequency not the highest freq.
2. It can be located graphically using histogram.
3. Not affected by extreme obsⁿ

Note: In case of inclusive series, only modal class will be converted into exclusive series

Median & Partition values

Computation

Particulars / Series	Individual series	Discrete Series	Continuous Series
Median	$\left(\frac{N+1}{2}\right)^{th}$ item	$\left(\frac{N+1}{2}\right)^{th}$ item in c.f.	Median class $= \frac{N}{2}^{th}$ item in c.f. $L_1 + \frac{N/2 - c.f.}{f} \times i$
Quartiles	$k\left(\frac{N+1}{4}\right)^{th}$ item, $k = 1, 2, 3$	$k\left(\frac{N+1}{4}\right)^{th}$ item in c.f., $k = 1, 2, 3$	Quartile class $= k\left(\frac{N}{4}\right)^{th}$ item in c.f. $L_1 + \frac{k\left(\frac{N}{4}\right) - c.f.}{f} \times i$
Deciles	$k\left(\frac{N+1}{10}\right)^{th}$ item, $k = 1, 2, \ldots 9$	$k\left(\frac{N+1}{10}\right)^{th}$ item in c.f., $k = 1, 2, \ldots 9$	Decile class $= k\left(\frac{N}{10}\right)^{th}$ item in c.f. $L_1 + \frac{k\left(\frac{N}{10}\right) - c.f.}{f} \times i$
Percentiles	$k\left(\frac{N+1}{100}\right)^{th}$ item, $k = 1, 2, \ldots 99$	$k\left(\frac{N+1}{100}\right)^{th}$ item in c.f., $k = 1, 2, \ldots 99$	Percentile class $= k\left(\frac{N}{100}\right)^{th}$ item in c.f. $L_1 + \frac{k\left(\frac{N}{100}\right) - c.f.}{f} \times i$

Here,
L_1 = lower limit of the concerned class
c.f. = c.f. of the class preceding concerned class
f = frequency of concerned class
i = class size

Important Points

1. Data has to be arranged in ascending order first.
2. In case of inclusive series, only the concerned class will be converted into exclusive series & then formula will be applied.

Properties

1. Median $= Q_2 = D_5 = P_{50} =$ average of 2 middle most items $= \frac{Q_1 + Q_3}{2}$
2. Median is not affected by extreme observations.
3. Median and Quartiles can be located graphically using Ogive Curve.
4. Sum of the absolute deviations taken from median is minimum i.e. median se nikale gaye absolute deviations ka sum hota h sabse kam ($\Sigma|x - Me|$ is minimum)

⇒ arrangement in ascending order

⇒ inclusive to exclusive series in case of mode & median

Geometric Mean

1. Computed as n^{th} root of the product of n numbers $= \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdots x_n} = (x_1 \times x_2 \times x_3 \cdots x_n)^{1/n}$
2. G.M. is the most accurate average.
3. G.M. is used for calculation of average rates.
4. If the series is in G.P., then G.M. can be calculated as $\sqrt{1^{st} \text{ term} \times \text{last term}}$
5. G.M cannot be calculated if any observation is zero or negative.
6. G.M. of xy = G.M. of x × G.M. of y
7. G.M. of $x/y = \frac{\text{G.M. of } x}{\text{G.M. of } y}$
8. log of G.M. for a set of obserⁿ is the A.M. of the log of the observations i.e. $\log G.M. = \frac{1}{3}(\log A + \log B + \log C)$
   [obsⁿ k G.M. ki log value – obsⁿ k individual log values k A.M. k barabar h.]

Harmonic Mean

1. H.M. is the reciprocal of the A.M. of the reciprocal of observations i.e. $H.M. = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} \cdots + \frac{1}{x_n}}$

[obserⁿ k reciprocal ki mean ka reciprocal.)

2. Combined H.M. $= \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$
3. Weighted H.M. $= \frac{\Sigma w}{\Sigma\left(\frac{w}{x}\right)}$

Application of Harmonic Mean

Average Speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

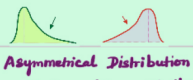
I. $A \xrightarrow{s_1} B \xrightarrow{s_2} C$, d_1, d_2 $= \frac{d_1 + d_2}{\frac{d_1}{s_1} + \frac{d_2}{s_2}}$

II. $A \rightleftarrows B$ $= \frac{2}{\frac{1}{s_1} + \frac{1}{s_2}} = \frac{2 \times s_1 \cdot s_2}{s_1 + s_2}$

Relationship

Between mean, median & mode

Symmetrical Distribution
$\bar{x} = Me = z$
Mean = Median = Mode



Asymmetrical Distribution
Mean – Mode = 3 (Mean – Median)
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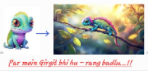
Mode = 3 Median – 2 Mean

Between AM, GM & HM

1. A.M. ≥ G.M. ≥ H.M.
   # A.M = G.M. = H.M., when all the obserⁿ are equal or constant
2. for 2 positive numbers $G.M. = \sqrt{A.M. \times H.M} \Rightarrow G^2 = AH$

Common Properties



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Property	Mean	Median	Mode	G.M.	H.M.
1. If all the obsⁿ are constant (k), then ____ will be equal to k.	✓	✓	✓	✓	✓
2. Affected by change of origin & scale in same proportion. $y = a + bx$, $\bar{y} = a + b\bar{x}$	✓	✓	✓	–	–

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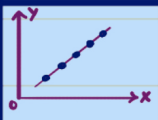
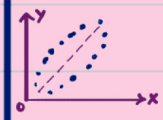
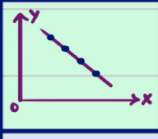
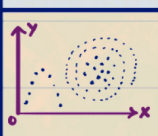
Correlation

$-1 \le r \le +1$

Scatter Diagram Method

Degree of Correlation	Coefficient of Correlation	Graphical Representation	Remarks
1. Perfect Positive Correlation 	$r = +1$ % change = same direction of change = same		positive or upward sloping straight line, points are scattered from lower left to upper right.
2. Positive Correlation	$0 < r < 1$ % change = diff. direction of change = same		points are scattered around positive sloping imaginary straight line.
3. Perfect negative Correlation 	$r = -1$ % change = same direction of change = opposite		negative or downward sloping straight line, points are scattered from upper left to lower right.
4. Negative Correlation	$-1 < r < 0$ % change = diff. direction of change = opposite		points are scattered around negative sloping imaginary straight line.
5. Zero or No Correlation 	$r = 0$		curvilinear or points are evenly distributed without following any specific pattern.

Concurrent Deviation Method

Formula

$$r_c = \pm\sqrt{\frac{\pm(2c-m)}{m}}$$

where c = no. of concurrent deviations or positive signs in product column

$m = n-1$

n = no. of pairs of x & y

Imp Note: On the basis of the sign of $(2c-m)$, the sign inside and outside the radical will be chosen

Steps:
1. Determine the sign of deviation of x & y series excluding the first pair.
2. Multiply the signs of deviation of x & y
3. Count the no. of positive signs in the product column, which is equal to c i.e. no. of concurrent deviations.
4. Determine the value of $(2c-m)$ & accordingly choose the sign inside & outside the radical.

Spearman Rank Correlation Method

When ranks are not repeated	When ranks are repeated
$r_k = 1 - \frac{6\Sigma d^2}{N(N^2-1)}$ where $d = R_x - R_y$, n = no. of pairs of x & y   	$r_k = 1 - \frac{6\left\{\Sigma d^2 + \frac{\Sigma m^3 - m}{12}\right\}}{N(N^2-1)}$ where $d = R_x - R_y$, N = No. of pairs of x & y, m = No. of times a particular rank repeats. Imp. Note: 1. (m^3-m) ki aahuti for every repeated rank. 2. purn-aahuti $= \frac{\Sigma(m^3-m)}{12}$

Imp. Points:
1. Sum of the deviations of rank will always be zero i.e. $\Sigma d = 0$
2. If the ranks of 2 series are exactly opposite of each other, then $r_k = -1$
3. In case of tied ranks, rank is assigned by taking average of the positions on which conflict is there.

Karl Pearson's or Product Moment Correlation Coefficient

Method	formula
Basic formula	$r_{xy} = \frac{\text{Covariance of } x \text{ \& } y}{SD_x \times SD_y}$
Actual Mean Method	$r_{xy} = \frac{\Sigma dx \cdot dy}{\sqrt{\Sigma dx^2 \times \Sigma dy^2}}$ where $dx = x - \bar{x}$, $dy = y - \bar{y}$
Direct Method	$r_{xy} = \frac{N \times \Sigma xy - \Sigma x \times \Sigma y}{\sqrt{N \times \Sigma x^2 - (\Sigma x)^2} \times \sqrt{N \times \Sigma y^2 - (\Sigma y)^2}}$ 
Shortcut or Assumed Mean Method	$r_{xy} = \frac{N \times \Sigma dx \cdot dy - \Sigma dx \cdot \Sigma dy}{\sqrt{N \times \Sigma dx^2 - (\Sigma dx)^2}\sqrt{N \times \Sigma dy^2 - (\Sigma dy)^2}}$ where $dx = x$ – Assumed mean of x series, $dy = y$ – Assumed mean of y series

Coefficient of determination	$r^2 = \frac{\text{Explained / accounted variance}}{\text{Total variance}}$ 
Coefficient of non determination	$1 - r^2 = \frac{\text{Unexplained / unaccounted variance}}{\text{Total variance}} = 1 - \frac{\text{Explained / accounted variance}}{\text{Total variance}}$

Coefficient of determination is calculated in order to determine the extent of change of one factor on the second factor, keeping other factors constant.

Effect of change of origin & scale on r_{xy}

1. Unaffected by change of origin but affected by sign of change of scale
2. Compare the signs of coefficients of x & y after isolating u & v (new variable)

If sign are same [++, --] then $r_{uv} = r_{xy}$	If signs are opposite (+-, -+) then $r_{uv} = -r_{xy}$

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Measures of Dispersion

Types

Absolute measures	Relative measures
1. Range	1. Coefficient of Range
2. Quartile Deviation	2. Coefficient of Quartile Deviation
3. Mean Deviation	3. Coefficient of Mean Deviation
4. Standard Deviation	4. Coefficient of Variation

absolute dev^n is to be taken in case of M.D.

Range & its Coefficient

Computation		Properties
MOD	formula	
Range 	largest obser^n − smallest obser^n [L−S]	1. Easiest to calculate.
		2. Affected by extreme obser^n
		3. Cannot be calculated for open ended class intervals.
Coefficient of Range	$\frac{L-S}{L+S} \times 100\%$	4. Inclusive series if given will be converted into exclusive series

Quartile Deviation & its Coefficient

Computation		Properties
MOD	formula	
Quartile Deviation	$\frac{Q_3 - Q_1}{2}$	1. Suitable for open ended class intervals.
		2. Not affected by extreme obser^n
Coefficient of Quartile Deviation 	$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100\%$ Or $\frac{\text{Quartile Deviation}}{\text{Median}} \times 100\%$	3. also known as semi-inter Quartile Range or Coefficient of skewness.
		4. Inter Quartile Range $= Q_3 - Q_1$
		5. Q.D. is half of IQR Or IQR is double of Q.D.

Mean Deviation & its Coefficient

Computation		Properties
MOD	formula	
Mean Deviation 	Mean of the absolute deviation taken from different averages Or $\frac{\text{Sum of absolute dev.}}{\text{No. of obser}^n}$ Or $\frac{\Sigma\lvert x - \text{any average}\rvert}{N}$	1. Based on all observations
		2. M.D. of two numbers $= \frac{\lvert a-b\rvert}{2}$ = Half of Range
		3. also known as average deviation
		4. disregarded because ignores negative sign of deviations.
Coefficient of Mean Deviation	$\frac{\text{Mean Deviation}}{\text{any average}} \times 100\%$	5. M.D. is minimum when calculated from median becoz absolute dev. from median is minimum

modulus function of k i.e. $|k|$ is multiplied to old MOD

new variable is to be isolated before finding k

Standard Deviation & its Coefficient

Computation		Properties
MOD	formula	
Standard Deviation 	square root of the mean of the squared deviation taken from mean $= \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}}$ $= \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$ $= \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$ $= \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$ where $d = x - A$	1. Variance $= (S.D.)^2$
		2. S.D. of 2 numbers $= \frac{\lvert a-b\rvert}{2}$ or Half of Range
		3. S.D. of 1st n natural numbers $= \sqrt{\frac{n^2-1}{12}}$
		4. Combined S.D. $= \sigma_{12}$ $\sqrt{\frac{N_1 \times (S.D_1)^2 + N_2 \times (S.D_2)^2 + N_1 \times d_1^2 + N_2 \times d_2^2}{N_1 + N_2}}$ Or $\sqrt{\frac{N_1 \times V_1 + N_2 \times V_2 + N_1 \times d_1^2 + N_2 \times d_2^2}{N_1 + N_2}}$ where $d_1 = \bar{x}_{12} - \bar{x}_1$, $d_2 = \bar{x}_{12} - \bar{x}_2$
Coefficient of Variation	$\frac{S.D.}{A.M.} \times 100\%$ # Compares the consistency of 2 or more series. # Higher the C.V., more will be the variation & lesser will be consistency	

Common Properties

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Properties	Range	Quartile Deviation	Mean Deviation	Standard Deviation
1. If all the observations are constant say 'k', then ____ will be equal to zero	✓	✓	✓	✓
2. Independent of change of origin but affected by change of scale. New Range/MD/QD/SD $= \lvert k\rvert$ old Range/MD/QD/SD; New variance $= k^2 \times$ old variance [agar sari obs^n 'k' se multiply hogi, Range/MD/QD/SD 'k' se nhi 'k' ke modulus function se multiply hogi or Variance k^2 se multiply hoga.]	✓	✓	✓	✓

Relationship

Between SD, MD & QD

1. 4 SD = 5 MD = 6 QD [Sadda Munda Qud Boliya see]
2. Ratio of SD : MD : QD = 15 : 12 : 10
3. SD > MD > QD