Theoretical Probability Distributions

A mathematical foundation for forecasting, population study, and statistical hypothesis testing.

STRATEGIC OVERVIEW

Nature of Random Variables

Discrete Distributions

Dealing with countable values or "counts". These distributions model scenarios where outcomes can be listed individually.

Example: Number of accidents per day.

Continuous Distributions

Dealing with infinite values within a range or "measurements". Variables are measured along a continuum.

Example: Weight of a student population.

Binomial Distribution Basics

Represents the number of successes in "n" independent Bernoulli trials.

- Fixed number of trials (n)
- Binary outcomes (Success/Failure)
- Constant probability of success (p)
- Trials are mutually independent

Mathematics of Binomial Distribution

The distribution is **bi-parametric**, uniquely defined by parameters *n* and *p*.

METRIC	MATHEMATICAL FORMULA	CONDITION / PROPERTY
Mean	np	Represents the average outcome
Variance	npq	Where q = 1 - p
Critical Property	$\text{Variance} < \text{Mean}$	Unique to Binomial Distribution

The Poisson Distribution

Designed for modeling **rare events** occurring over a large number of opportunities where *p* is extremely small.

- Uniparametric: Defined by 'm' (mean)
- Used when n is large and p is small
- Modeling phone calls or printing errors

$$P(x) = \frac{e^{-m} m^{x}}{x!}$$

Poisson Parameters and Identity

The Central Parameter

Equality of Moments

One of the most defining characteristics of the Poisson Distribution is that the **Mean** and **Variance** are perfectly equal.

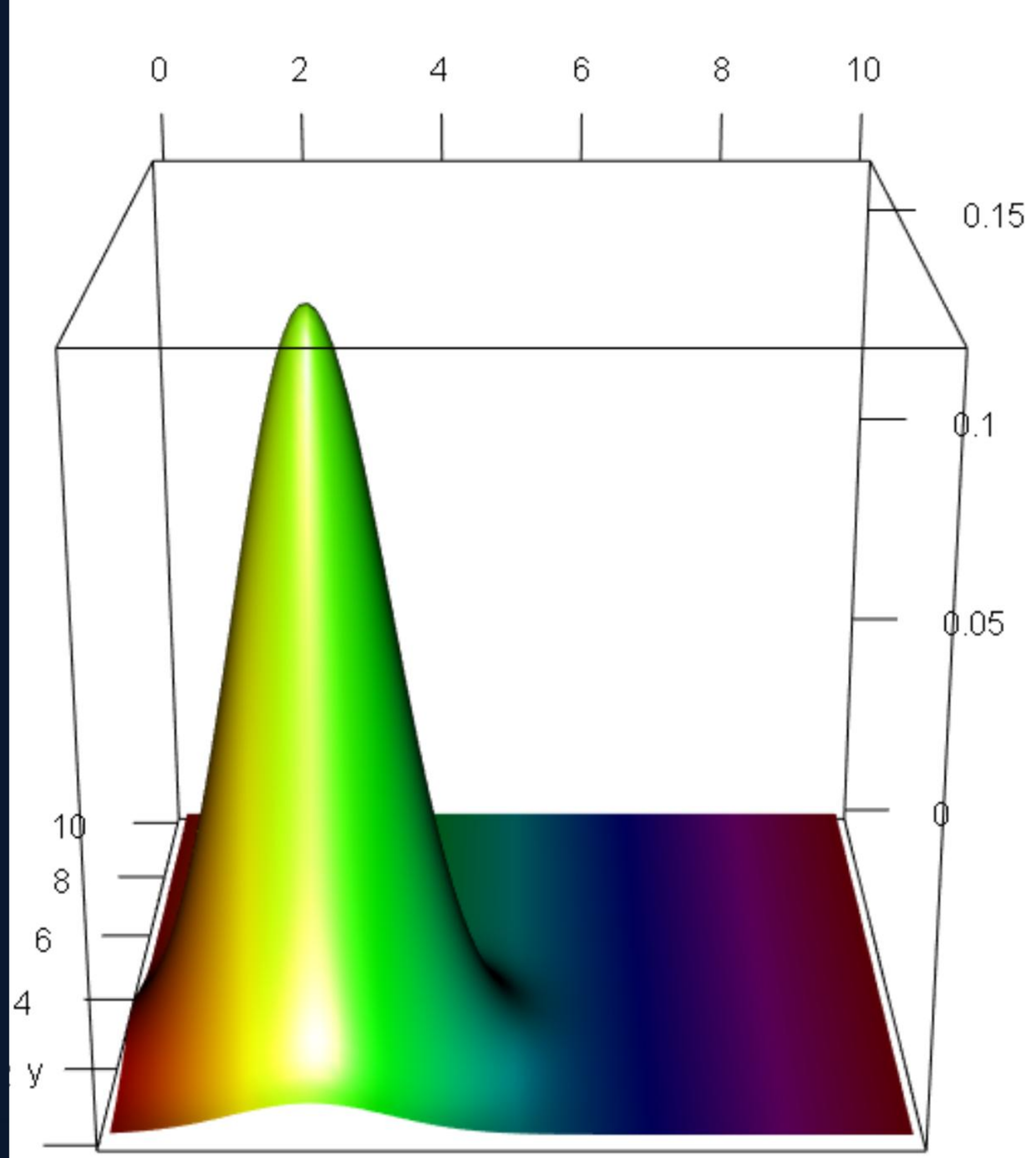
- Mean = Variance = m
- Standard Deviation = $\sqrt{m}$
- Applicable when $m = np$ (finite constant)

Gaussian (Normal) Distribution

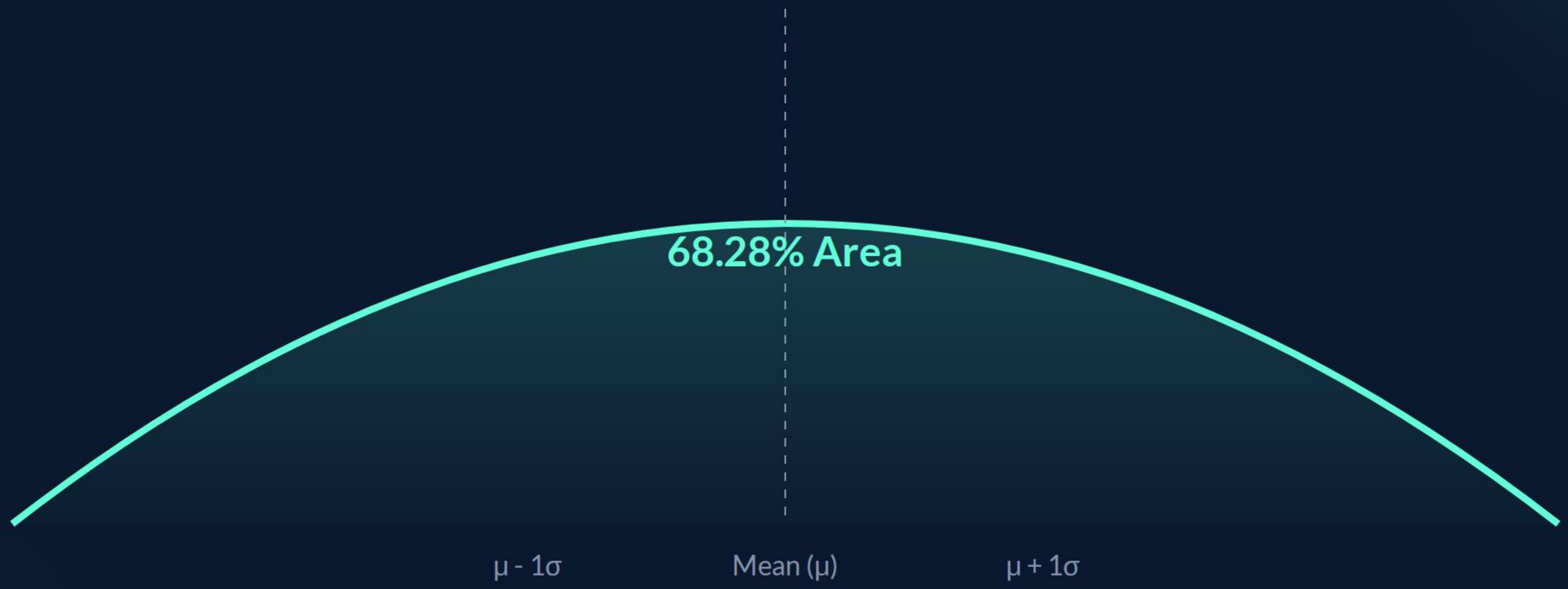
The cornerstone of continuous probability. It represents a perfectly symmetrical, bell-shaped curve where 99.7% of all values fall within three standard deviations.

Defined by two parameters:

- Mean (μ)
- SD (σ)



The Empirical Rule



95.46%

μ ± 2σ

99.73%

μ ± 3σ

Converting to the Z-Score

To standardize any Normal variable (X), we convert it into a Standard Normal Variable (Z), facilitating calculations using a single table.

- **Normalization Formula:** $$Z = \frac{X - \mu}{\sigma}$$
- **Standard Parameters:** Mean = 0, Standard Deviation = 1.
- **Interpretation:** A Z-score of +1.0 indicates a score is 1 Standard Deviation above the mean.

Strategic Selection Matrix

Binomial

Countable data with fixed number of trials and constant probability.

Coin Toss / Quality Control

Poisson

Rare events occurring in a continuous interval with no fixed trials.

Traffic / Call Logs

Normal

Continuous measurements where data clusters around the average.

Height / Weight / Exam Marks

Analogy for Mastery

> Think of these distributions as fishing nets: The Binomial net is for specific casts; the Poisson net is for rare, deep-sea fish; and the Normal net catches the average fish that represent the majority of the population.

Questions?

Thank you for your attention to Theoretical Probability Distributions.

ACADEMIC SESSION | STATISTICS FOUNDATION

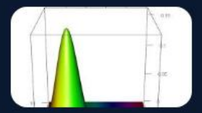
Image Sources

https://images.pond5.com/3d-golden-coin-flipping-upward-footage-275602824_iconl.jpeg

Source: www.pond5.com

http://controlroomsolution.com/wp-content/uploads/2025/08/High-Tech-Traffic-Police-Control-Room-with-Surveillance-Screens-1024x768.webp

Source: controlroomsolution.com



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Source: stackoverflow.com

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