RATIO, PROPORTION, INDICES AND LOGARITHM

RATIO :-

- Ratio is comparison of two quantities of same kind and expressed in same unit. (Ratio as a whole has no units).

- If 'a' and 'b' are two quantities then their ratio is $a : b$ read as 'a is to b'.

- Ratio is a fraction i.e. $a : b = \frac{a}{b}$

- In the ratio $a : b$:-
  'a' is called antecedent (first term)
  'b' is called consequent (second term)

- The value of a ratio remains same if both the antecedent and the consequent are multiplied or divided by the same number.

- Usually a ratio is expressed in simplest form (reduced form) i.e. the antecedent and the consequent have no common factor.

Some Important Ratios :-

- Duplicate ratio of $a : b$ is $a^2 : b^2$ [Ratio of Squares]

- Triplicate ratio of $a : b$ is $a^3 : b^3$ [Ratio of Cubes]

- Sub - duplicate ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$ [Ratio of Square - root]
- Sub - triplicate ratio of $a : b$ is $\sqrt[3]{a} : \sqrt[3]{b}$ [Ratio of cube - root]
- Inverse ratio of $a : b$ is $b : a$
- Reciprocal ratio of $a : b$ is $\frac{1}{a} : \frac{1}{b}$
- Compounded ratio of $a : b$ and $c : d$ is $ac : bd$
  [(product of antecedents) : (product of consequents)]
- Continued Ratio of $a : b$ and $b : c$ is $a : b : c$
  [It is the ratio between three or more quantities of same kind]
- <u>Commensurable quantity</u> :- If the ratio can be expressed as a ratio of two integers then the quantities are commensurable
- <u>Incommensurable quantity</u> :- If the ratio cannot be expressed as a ratio of two integers then the quantities are incommensurable

PROPORTION :-

- Equality of two ratios is a proportion.
  i.e. $a : b = c : d$ this means a, b, c, d are in proportion.

- It is written as $a : b :: c : d$ and it is read as
  'a is to b is proportional to c is to d' where
  a is called the first proportional, b is called the second proportional,
  c is called the third proportional, d is called the forth proportional

- In $a : b :: c : d$:- a and d are extremes whereas b and c are means.

- $a : b :: c : d \Rightarrow \dfrac{a}{b} = \dfrac{c}{d} \Rightarrow$ $\boxed{\underset{\text{(product of extremes)}}{a \times d} = \underset{\text{(product of means)}}{b \times c}}$

- Continued proportion :-

Three quantities a, b and c are in continued proportion if :-

$$a : b = b : c$$

$$\therefore \frac{a}{b} = \frac{b}{c}$$

$$\therefore b^2 = ac \quad \text{or} \quad b = \sqrt{ac}$$

where, a is called the first proportional
b is called the mean proportional or G.M. of a and c
c is called the third proportional

$\boxed{\text{Important properties}}$:-

- <u>Invertendo</u> :- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$

- <u>Alternendo</u> :- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$

- <u>Componendo</u> :- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$

- <u>Dividendo</u> :- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$

- <u>Componendo & Dividendo</u> :- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

- <u>Addendo</u> :- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

- <u>Subtrahendo</u> :- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$

<u>NOTE</u> :- Only Addendo & Subtrahendo are equal to original ratio.

→ INDICES :-

- Consider $a^n = a \times a \times a \times \ldots\ldots n$ times.
  where a is the base and n is called the power | index | exponent

→ LAWS OF INDICES :-

1.	$a^m \times a^n = a^{m+n}$	2.	$\dfrac{a^m}{a^n} = a^{m-n}$
3.	$(a^m)^n = a^{m \times n}$	4.	$a^n = \dfrac{1}{a^{-n}}$ or $a^{-n} = \dfrac{1}{a^n}$
5.	$(ab)^n = a^n \times b^n$	6.	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
7.	$\sqrt[n]{a} = a^{1/n}$	8.	$\sqrt[n]{a^p} = a^{p/n}$
9.	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$	10.	$\left(\dfrac{a}{b}\right)^{\frac{m}{n}} = \left(\dfrac{b}{a}\right)^{-\frac{m}{n}}$
11.	$a^0 = 1$ $a^1 = a$	12.	$1^a = 1$ $0^a = 0$
13.	If $a^x = a^y$ then $x = y$	14.	If $a^x = b^x$ then $a = b$

Important Basic Formulae :-

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a^2 - b^2) = (a+b)(a-b)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- If $(a+b+c) = 0$ then $a^3 + b^3 + c^3 = 3abc$

Important Result :-

- If $x = a^{1/3} - a^{-1/3}$ then $x^3 + 3x = a - \frac{1}{a}$
- If $x = a^{1/3} + a^{-1/3}$ then $x^3 - 3x = a + \frac{1}{a}$

→ LOGARITHM :-

- Let a, b and m be any three numbers such that their Exponential form is $b^m = a$ then they can be written as :-
  Logarithmic form $m = \log_b a$ and is read as logarithm of a to the base b.
  Here, m is the exponent
  a is the argument, $a > 0$
  b is the base, $b > 0$ and $b \neq 1$

→ Types of Logarithm :-

A) Common Logarithm :- Logarithms with base 10. eg :- $\log_{10} a$

B) Natural Logarithm :- Logarithms with base e. eg :- $\log_e a$
'e' is a constant called Euler's number $e = 2.7183\ldots$

- Common logarithms are used for numerical calculations whereas Natural logarithms are used in calculus.
- When the base in not specified we take the base as 10.

→ Properties of Logarithm :-

- $\log_a (m \cdot n) = \log_a m + \log_a n$
- $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$
- $\log_{a^b} m^n = \frac{n}{b} \log_a m$
- $\log_a a = 1$

- $\log_a 1 = 0$
- $\log_b a = \dfrac{1}{\log_a b}$
- $\log_b a = \dfrac{\log_m a}{\log_m b} = \dfrac{\log a}{\log b}$
- $a^{\log_a m} = m$

→ Calculator trick to calculate logarithm of any number :-

A) With any base :-

$$\log_b a = \frac{a \sqrt{}\ \sqrt{}\ \sqrt{} \ldots\ 12 \text{ times} - 1}{b \sqrt{}\ \sqrt{}\ \sqrt{} \ldots\ 12 \text{ times} - 1}$$

A) With base 10 :-

$$\log_{10} a = (a \sqrt{}\ \sqrt{}\ \sqrt{} \ldots\ 12 \text{ times} - 1) \times 1778.37$$

→ Calculator trick to calculate n^{th} power of any number :-

- $x^n = x \sqrt{}\ \sqrt{}\ \sqrt{} \ldots\ 12 \text{ times} - 1 \times n + 1 \quad X =, X =, \ldots\ 12 \text{ times}$

→ Calculator trick to calculate n^{th} root of any number :-

- $x^{1/n} = x \sqrt{}\ \sqrt{}\ \sqrt{} \ldots\ 12 \text{ times} - 1 \times 1 \div n + 1 \quad X =, X =, \ldots\ 12 \text{ times}$

- for $(25)^{1/2} = \sqrt{25} = 25 \sqrt{}$

$(25)^{1/4} = \sqrt[4]{25} = 25 \sqrt{}\ \sqrt{}$

$(25)^{1/8} = \sqrt[8]{25} = 25 \sqrt{}\ \sqrt{}\ \sqrt{}$