





What is Regression?

Let us take the example of Investment and return

Y is dependendent on X

Test Score and CA foundation Clear

Y is dependendent on x Mathematical sulation - aquation

A mathematical equation that allows us to predict value of one variable from known values of one or more variables is called a regression equation.

Yno trundendeboi X er Y no X



WHAT IS REGRESSION?

7	=2+	32
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X	Y	
2	2	
5	8	
8	12	
(0	14	

dimple Regnession only two variable Jonzier Egyntim Jonzier = atlax xmy: x= c+dy





The variable whose value is to be predicted is called the dependent variable or explained variable.





The variables which are used to predict the values of a dependent variable are called independent variables or explanatory variables

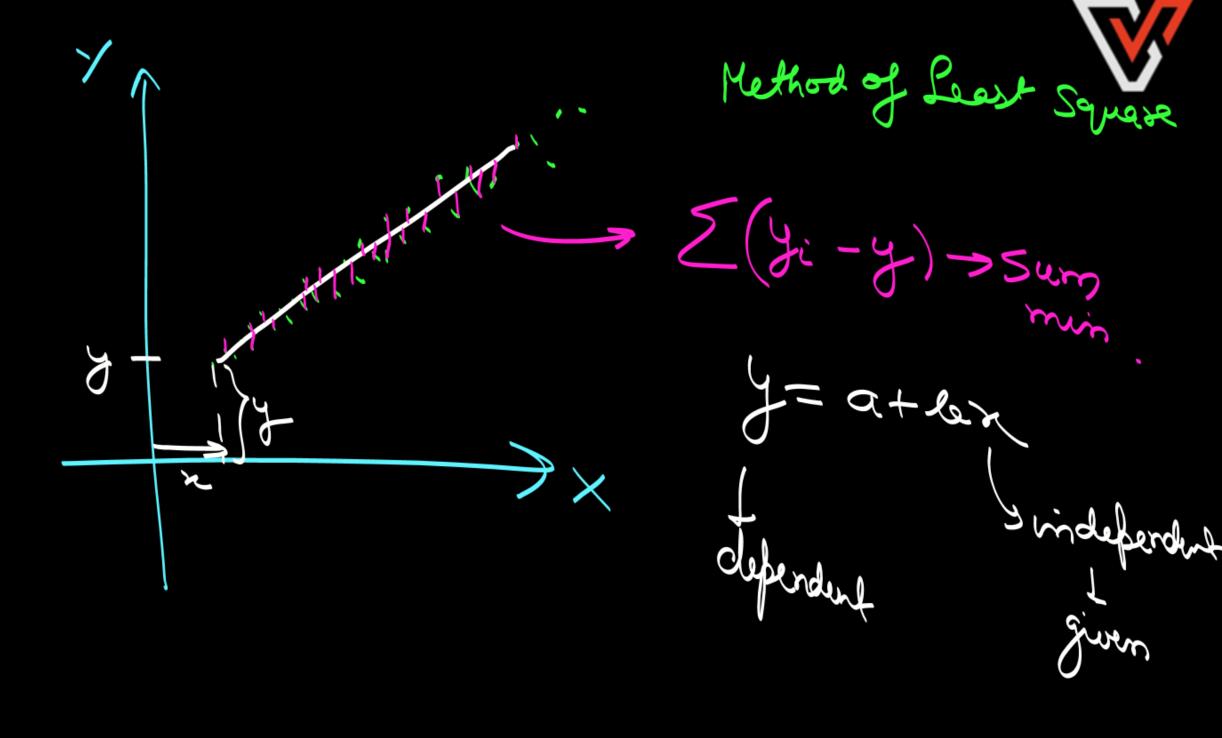


SIMPLE REGRESSION ANALYSIS AND SIMPLE LINEAR ANALYSIS

The regression analysis confined to the study of only two variables, a dependent variable and an independent variable, is called simple regression analysis.

When the relationship between the dependent variable and the independent variable is linear, the technique for prediction is called simple linear regression.

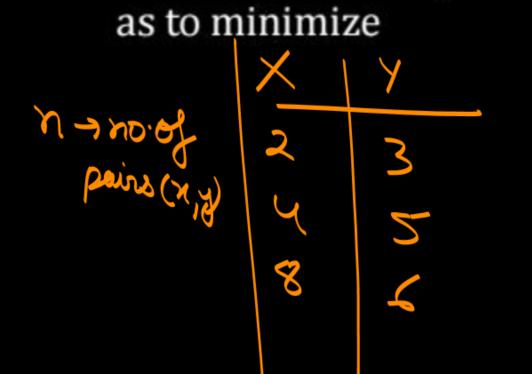
If let say y depends on x, then equation will be :-

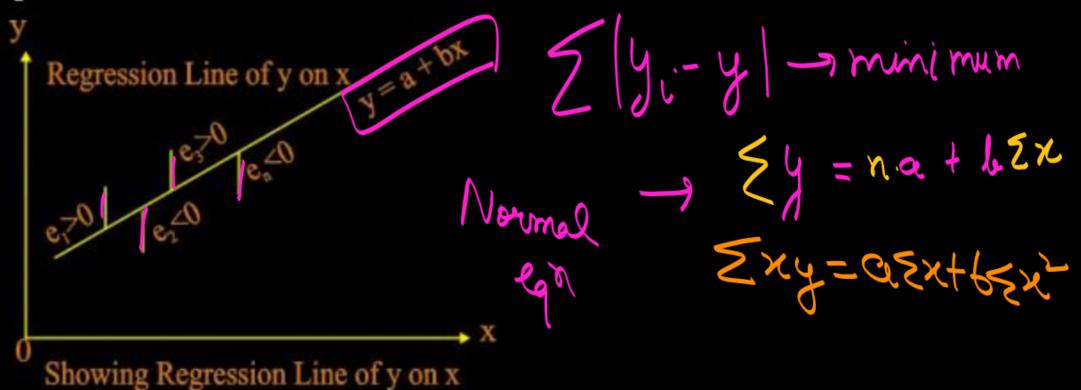






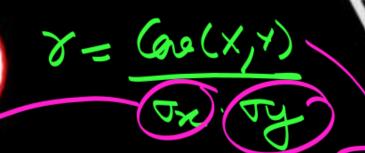
If a line of best fit approximating the given data has the equation then the method of least squares requires that we must determine constants a and b so





These equations known as the normal equation for estimating a and b, are given by





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Solving equation simultaneously for a and b we obtain

$$\frac{1}{2} = \frac{1}{2} \frac{$$

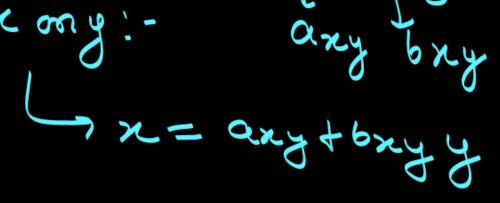
* youx = y = ayx + byxx - D

* coeff of youx

$$\Rightarrow \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{2} \int$$

3 ayx = 3 - byx x -0

* x ony:- 2 axy bxy



 $\frac{1}{2} = \frac{1}{2} = \frac{1}$



Calculate the regression coefficients from the following information:

$$\sum X = 50$$
, $\sum Y = 30$, $\sum XY = 1000$, $\sum X^2 = 3000$, $\sum Y^2 = 1800$, n=10

$$y = \frac{2y}{x} = \frac{30}{10} = \frac{50}{10} = 5$$

$$byx = \frac{72xy - 5x2y}{72x^2 - (2x)^2} = \frac{10(1000) - 50x30}{10(3000) - (50)^2} = 0.309$$

(or
$$y = 3 - y = 64x(x - x)$$
)



xony

Calculate the regression coefficients from the following information:

$$\sum X = 50$$
, $\sum Y = 30$, $\sum XY = 1000$, $\sum X^2 = 3000$, $\sum Y^2 = 1800$, $n = 10$
 $X = 5$, $y = 3$
 X



Following table gives the age of cars of a certain make and annual maintenance costs. Obtain the regression equation for costs related to age:

Age of cars in years: X

2

8

Maintenance cost (in hundred) 10

20 25

30

A-A= PAN (X-2)

PAN = MEXX3-(SX)

NEXX3-(SX)

NEXX3-(SX)

NEXX3-(SX)

NEXX3-(SX)

2X10= =44 =0 EXP as 25 6125= 30 8x30=15T8x=

64x=4x490-20x85 4x120-(20)2-64x=1960-1700-3.25

9-91-52=3-92(x-2)





- The constant b is called the regression coefficient of Y on X is denoted by b_{vx}.
- It measures the change in Y corresponding to a unit change in X.
- Thus, b_{yx} represent the slope of the line of regression of Y on X

y-y-byx(x-x) or y=ayx+

The equation of the line of regression of Y on X can also be written as





On the other hand, if we wish to estimate a value of X for a given value of Y, we have to obtain regression line of X on Y:

 $y = \frac{x^2 + 5x^2}{5x^2 - 5x^2}$ $y = \frac{x^2 + 5x^2}{5x^2 - 5x^2}$ $y = \frac{x^2 + 5x^2}{5x^2 - 5x^2}$ $y = \frac{x^2 + 5x^2}{5x^2 - 5x^2}$

The two normal equation for estimating c and d are given by Solving these normal equations simultaneously for c and d, we obtain

| (x-x) = y-y = - (x-x)+y = -





The constant d is called the regression coefficient of X on Y and is denoted by b_{xv}.it measure the change in X corresponding to a unit change in Y.

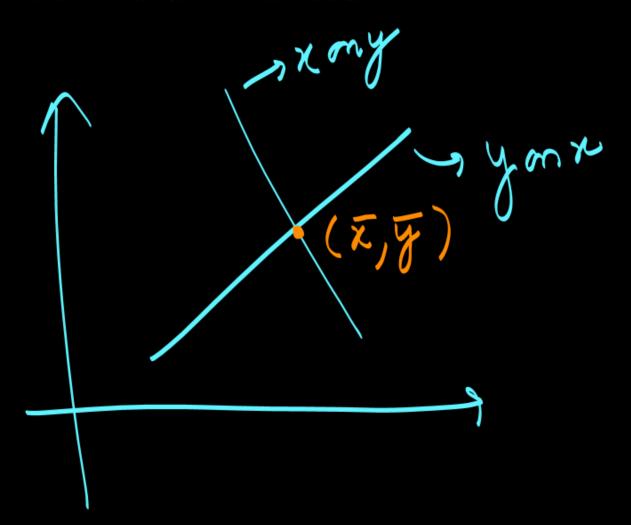
 \diamond Clearly ,1/b_{xy} represent the slope of the regression line of X on Y.

Slope (= f m x x = axy + bxy y \Rightarrow 6xyy=x-axySlope





Line of regression of X and Y passes the point (X, Y) and hence the equation of the line of regression of X on Y can be written as





In the estimation of regression equation of two variables X and Y, the following results were obtained:

$$\sum X = 900$$
, $\sum Y = 700$, $\sum X^2 = 6360$, $\sum Y^2 = 2860$ $\sum XY = 3900$, n=10
Obtain two regression equations

$$\frac{b_{4}x = 10(3900) - 9000000}{10(6360) - (900)^{2}} = 0.791$$

$$\sum XY = 3900, n=10$$

$$x = \frac{100}{7} = \frac{100}{10} = \frac{100}$$



Given $\bar{x} = 50$, $\bar{y} = 20$, $\sigma_x = 20$, $\sigma_y = 30$ and cov (X, Y) = -100 find:

- 1. Correlation coefficient
- 2. Both the regression coefficients

$$\frac{1}{8} = \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

2)
$$byx = (ax(x,y) = -100) = -100 = -1$$

$$\frac{3}{3} = \frac{3}{3} = \frac{3}$$

y = y = byx(x - x) $\frac{\partial}{\partial x} = \frac{\partial}{\partial x} (x - x)$ xon y - x - x - bxy(y-y) $x - x = 8 \times (9 - 9)$





(15 min) Ques + Botak

The following data relate to the mean and SD of the prices of two shares in a stock Exchange:

Share: Mean (in Rs.)

SD (in Rs.)

Company A × 44

5x 5.6

y Company B 58

Coefficient of correlation between the share prices = 0.48 = 8

Find the most likely price of share A corresponding to a price of Rs.60 of share B and also the most likely price of share B for a price of Rs. 50 of share A.

$$= \frac{1}{3} \times -\frac{1}{3} \times -\frac{1}{3}$$



PROPERTIES OF REGRESSION COEFFICIENTS



Property 1. The coefficients of correlation and two regression coefficients has the same signs.

Property 2. The coefficients of correlation are the geometric mean between the

regression coefficients.

ogx xbxy = 82



If r = -0.6 and one regression coefficient is -0.8, find the other regression coefficient.

$$\Rightarrow \frac{1}{2}$$
 $\Rightarrow \frac{1}{2}$
 $\Rightarrow \frac{$



PROPERTIES OF REGRESSION COEFFICIENTS



Property 3. If one of the regression coefficients is greater than unity, the other

must be less than the unity.

byx bxy =1

 $=) by x \leq \frac{1}{bx^{9}}$



PROPERTIES OF REGRESSION COEFFICIENTS



Property 4. The two lines of regression intersect at the point $(\overline{X}, \overline{Y})$ where x and y are the variables under consideration.



Property 5. The regression coefficients are independent of change of origin but

not for scale.



The regression coefficients of regression equation of X on Y is 2.4 and the same for regression equation of Y on X is 0.8 are the regression coefficients consistent?

byx.bxy < 1 $\Rightarrow 2.4 \times 0.8 = 1.92 \neq 1$ as product of reg. coeff is not less than !. They are not anothers.



If the relationship between two variables x and u is u + 3x = 10 and between two other variables y and v is 2y + 5v = 25, and the regression coefficient of y on x is known as 0.80, what would be the regression coefficient of v on u?



For some bivariate data, the following results were obtained:

Mean value of variable X = 53.2 and Y = 39.5 regression coefficient of Y on X = -1.5 and of X on Y = -0.38 what should be the most likely value of X when Y = 50? Also find the coefficients of correlation between two variables.

Xony -> x-x= 6xy(4y)

-> x-53:2=-0:38(50-395)

-> x=53:2-3:99

-> x= 49:21



Following are the two normal equations obtained for deriving the regression line of y

and x:

$$5a + 10b = 40$$
 ymx $2y = xa + b2x$
 $10a + 25b = 95$ $2xy = a2x + b2x^2$

The regression line of y on x is given by

$$2x + 3y = 5$$

B
$$2y + 3x - 5$$

$$y = 2 + 3x$$

$$D y = 3 + 5x$$



If the regression line of y on x and of x on y are given by 2x + 3y = -1 and 5x + 6y = -1 then the arithmetic means of x and y are given by

Q17, Set (B), (ICAI)

- A (1, -1)
- B (-1, 1)
- C (-1, -1)
- **D** (2, 3)

त्र

$$\frac{5}{5} \frac{\text{multareous}}{\text{substitutes}}$$

$$\frac{5}{5} \times 2 \times + 3 = -| > | 0 \times + | 2 = -3$$

$$\frac{3}{5} = -3$$

$$\frac{3}{5} = -1$$





Given the regression equations as 3x + y = 13 and 2x + 5y = 20, which one is the regression equation of y on x? $\frac{1}{2} = \frac{2}{2} \frac{1}{2} \frac{1}{2$

- A 1st equation
- B 2nd equation
- Both (a) and (b
- D None of these

3x+y=13 2x+5y=25 3x+5y=25 3x+5y=25

 $\frac{1}{1}$ $\frac{1}{3}$ $\frac{1}$

Check byxxbxy<1 > -3x-5 & 1 => 15

3x4y=13 2x+5y=20

3 - 1 x 2 / 2 x 3 / 2 x

on demose rue paral si



Given the following equations: 2x-3y = 10 and 3x + 4y = 15, which one is the regression x on y?

Q19, Set (B), (ICAI)

- A 1st equation
- B) 2nd equation
- both the equations
- none of these



If u = 2x + 5 and v = -3y-6 and regression coefficient of y on x is 2.4, what is the regression coefficient of y on u?

Q20, Set (B), (ICAI)

- **A** 3.6
- **B** -3.6
- **C** 2.4
- **D** –2.4





If 4y - 5x = 15 is the regression line of y on x and the coefficient of correlation between x and y is 0.75, what is the value of the regression coefficient of x on y?

- A 0.45
 - **B** 0.9375
- **c** 0.6
- D None of these

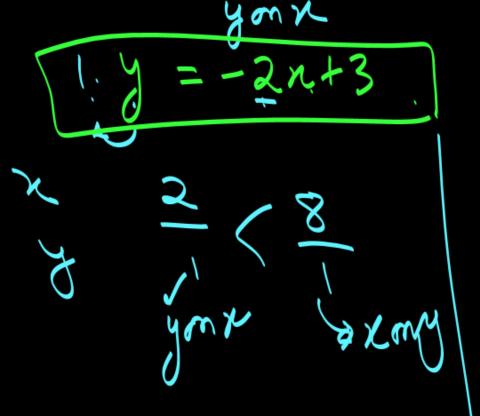
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Q21, Set (B), (ICAI)

y are given by y = -2x + 3 and 8x = y + 3

If the regression line of y on x and that of x on y are given by y = -2x + 3 and 8x = y + 3 respectively, what is the coefficient of correlation between x and y?

- **A** 0.5
- B $-1/\sqrt{2}$
- C -0.5
- None of these



between x and y?

$$8x = -y + 3$$

$$9x = -y +$$



If the regression coefficient of y on x, the coefficient of correlation between x and y and variance of y are -3/4, $\frac{\sqrt{3}}{2}$ and 4 respectively, what is the variance of x?

- **A** $2/\sqrt{3/2}$
- **B** 16/3
- C 4/3
- **D** 4

10 W 30 S

Q23, Set (B), (ICAI)

2 (3) X (3) = 15 3 (3) X (3) = 15



If y = 3x + 4 is the regression line of y on x and the arithmetic mean of x is -1, what is the arithmetic mean of y?

Q24, Set (B), (ICAI)

- A 1
- B) -i
- **G**
- D None of these

$$y = 3x + 4$$
 $y = 3x + 4$
 $y = 3x + 4$

/



If there are two variables x and y, then the number of regression equations could be

Q32, Set (A), (ICAI)

- (A) 1
- В
- C Any number
- **D** 3



Since Blood Pressure of a person depends on age, we need to consider

Q33, Set (A), (ICAI)

- A The regression equation of Blood Pressure on age
- B The regression equation of age on Blood Pressure
- Both (a) and (b)
- D Either (a) or (b).



The method applied for deriving the regression equations is known as

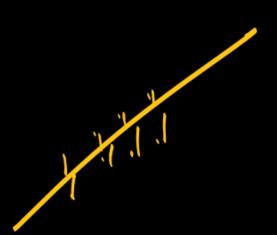
Q34, Set (A), (ICAI)

- A Least squares
- B Concurrent deviation
- C Product moment
- Normal equation



The difference between the observed value and the estimated value in regression analysis is known as Q35, Set (A), (ICAI)

- A Error
- B Residue
- **C** Deviation
- (A) or (B).





The errors in case of regression equations are

Q36, Set (A), (ICAI)

- A Positive
- B Negative
- C Zero
- D All these



The regression line of y on x is derived by

Q37, Set (A), (ICAI)

- A The minmisation of vertical distance in the catter diagram
- B) The minimisation of horizontal distances in the scatter diagram
- Both (A) and (B)
- (A) or (B).



The two lines of regression become identical when

- $A \qquad r = 1$
- B r=-1
- \mathbf{c} $\mathbf{r} = \mathbf{0}$
- (A) or (B).

7=1

Q38, Set (A), (ICAI)



What are the limits of the two regression coefficients?

Q39, Set (A), (ICAI)

- A No limit
- B Must be positive
- One positive and the other negative
- Product of the regression coefficient must be numerically less than unity.

by x x bry 4



The regression coefficients remain unchanged due to a

Q40, Set (A), (ICAI)

- A Shift of origin
- B Shift of scale
- Both (a) and (b)
- (a) or (b).

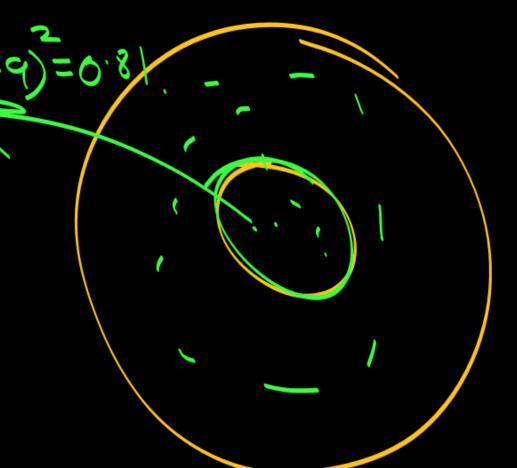


If the coefficient of correlation between two variables is -0.9, then the coefficient of determination is Q41, Set (A), (ICAI)

- **A** 0.9
- **B** 0.81
- C 0.1
- D 0.19.

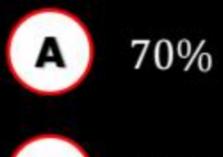
(segf of det. -82 = (-0.9)=0.8)

æff of nm-det. = 1-82





If the coefficient of correlation between two variables is 0.7 then the percentage of variation unaccounted for is Q42, Set (A), (ICAI)



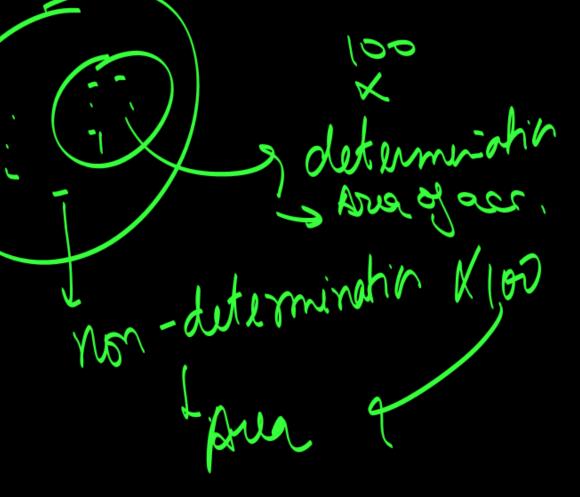
30%

Coeff of nondet = $(-(0.7)^2$ = (-0.49 = 0.5)



·/· ane a unacc.= 0.5/x1001.

49%





The following results relate to bivariate data on (x, y):

 $\sum xy = 414$, $\sum x = 120$, $\sum y = 90$, $\sum x^2 = 600$, $\sum y^2 = 300$ n = 30 Later or, it was known that two pairs of observations (12, 11) and (6, 8) were wrongly taken, the correct pairs of observations being (10, 9) and (8, 10). The corrected value of the correlation coefficient is

- A 0.752
- **B** 0.768
- **c** 0.846
- D 0.953

> Record as add

20min

Coefficient of correlation between x and y for 20 items is 0.4. The AM's and SD's of x and y are known to be 12 and 15 and 3 and 4 respectively. Later on, it was found that the pair (20, 15) was wrongly taken as (15, 20). Find the correct value of the correlation coefficient.

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Samean

20mi

