BUSINESS MATHEMATICS CHART BY MAYANK MAHESHWARI

INDICES

- a x a x a x ---- upto n terms = a^n
  where a = Base
  where n = the index of power
- $a^{-m} = 1/a^m$ and $1/a^{-m} = a^m$
- $(a^m)^n = a^{mn}$
- $(a.b)^m = a^m.b^m$
- $(a/b)^n = a^n/b^n$
- $\sqrt[n]{a} = a^{1/n}$
- $a^m \times a^n = a^{m+n}$ (base must be same)
- $a^m / a^n = a^{m-n}$ (base must be same)
- $a^0 = 1$
- $a^x = a^y \rightarrow x = y$ (base must be same)
- $a^x = b^x \rightarrow a = b$ (power must be same)
- $a^x = b^x$ & $a \neq b \rightarrow$ when x = 0
- $a^x = y \rightarrow a = y^{1/x}$

PERMUTATION

- Number of Permutations when r objects are chosen out of n different objects. Denoted by- $^nP_r = \frac{n!}{(n-r)!}$
  Or
  ^nP_r = n (n-1) (n-2)……….(n-r+1),
  where the product has exactly r factors.
- **1x1! + 2x2! + 3x3! + ………. + nxn! = (n+1)! – 1 or $\sum_{r=1}^{n} r.\,^rP_r = \,^{n+1}P_{n+1} - 1$**
- **(n-1)! = n!/n**
- $^nP_r = \,^nC_r \,.\, r!$ where, $n \geq r$
- $^nP_r = \,^{n-1}P_r + r.\,^{n-1}P_{r-1}$
- The no. of arrangements when things can be repeated is n^r

Linear permutations of n articles having some articles of same nature

- Arrangements = $\frac{n!}{Repetition!}$

Sum of all possible arrangements of given digits

1111.. (no. of digits) x sum of digits x (no. of digits-1)!

Sum of digits containing 0.

[1111.. (no. of digits) x sum of digits x (no. of digits-1)!] - [111.. (no. of digits -1) x sum of digits x (no. of digits-2)!]

Sum of digits containing repetitive digits

1111.. (no. of digits) x sum of digits x (no. of digits-1)! / Repetitions!

- The number of circular permutations of n different things chosen at a time is (n – 1)!
- The number of ways of arranging n persons along a round table so that no person has the same two neighbours is = $\frac{1}{2}$(n-1)!
- Number of necklaces formed with n beads of different colours = 1/2 (n-1)!

COMBINATIONS

- Number of combinations of n different things taken r at a time.

Denoted by- $^nC_r = \frac{n!}{r!(n-r)!}$ & $0 \leq r \leq n$
Or
^nC_r = [n (n-1) (n-2)……….(n-r+1)]/r!

- $^nC_0 = 1$
- $^nC_n = 1$
- $^nC_r = \,^nC_{n-r}$ Where, **$0 \leq n - r \leq n$**
- $^{n+1}C_r = \,^nC_r + \,^nC_{r-1}$
- $^nC_r + \,^nC_{r+1} = \,^{n+1}C_{r+1}$
- $^{n-1}C_r + \,^{n-1}C_{r-1} = \,^nC_r$
- $^nP_r = \,^nC_r \,.\, r!$
- $^nC_1 + \,^nC_2 + \,^nC_3 + \,^nC_4 + \ldots\ldots + \,^nC_n$ equals to $(2^n - 1)$

Some Important Tricks –

- How to count no. parallelograms using n1 & n2 parallel lines intersecting each other = $^{n1}C_2 \times \,^{n2}C_2$
- How to count no. of lines that can be made using n points (no 3 or more points are collinear) Or How to find no. of chords in a circle having n points = $^nC_2$
- How to count no. of lines that can be made using n points out of which m points lie on the same line (collinear) = $^nC_2 - \,^mC_2 + 1$
- How to count diagonals in a polygon with n sides = $^nC_2$ - n
- How to count Triangles out of n Points
  - No 3 are collinear = $^nC_3$
  - 3 or more are collinear = $^nC_3 - \,^mC_3$ where, m = points lie on the same line

LOGARITHMS

- $\log_a 1 = 0$ (where a ≠ 0)
- $\log_a a = 1$
- $\log_a a^x = x$
- $\log a^x = x \log a$
- $\log_a y = \log y / \log a = 1 / \log_y a$
- $\log_a\left(\frac{1}{m}\right) = -\log_a m$
- $\log_a b = \log_c b / \log_c a$
- $\log_a b = \log_c b \times \log_a c$
- $\log_a y = \log y / \log a = m \log y / m \log a = \log y^m / \log a^m = \text{Log}_{a^m} y^m$
- $\log_{a^n} y^m = \frac{m}{n} \log_a y$
- log a + log b = log a.b
- log a – log b = log $\frac{a}{b}$
- log a + log b – log c = log$\frac{a \times b}{c}$
- $\log_a b \times \log_b a = 1$
- $\log_c b \times \log_b a = \log_c a$
- If $\log_a x = \log_a y$, then x = y
- $a^{\log b} = b^{\log a}$
- $\text{Log}_a n = x$, then $a^x = n$
- $e^{\log a} = a$

TIME VALUE OF MONEY

SIMPLE INTEREST

SI = $\frac{PRT}{100}$, A = $P\left[1 + \frac{RT}{100}\right]$, A = P + SI

COMPOUND INTEREST

A = $P\left(1 + \frac{R}{100 * m}\right)^{T*m}$

CI = $P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$

Where, P=Principal; R=Rate; T=Time
SI =Simple Interest
CI=Compound Interest
m=No. of conversion period

Conversion Period	m
Compounded daily	365
Compounded monthly	12
Compounded quarterly	4
Compounded bi-monthly	6
Compounded semi annually	2
Compounded annually	1

EFFECTIVE RATE OF INTEREST

Effective Rate = $\left(1 + \frac{R}{100 * m}\right)^m - 1$

FUTURE VALUE (FV)

FV = $PV\left(1 + \frac{R}{100 * m}\right)^{T*m}$

PRESENT VALUE (PV)

PV = FV / $\left(1 + \frac{R}{100 * m}\right)^{T*m}$

ANNUITY

1. **FV of Annuity**
   - ✓ Annuity Regular (1st Payment at the end of 1st period)
   - ✓ Annuity Due (1st Payment at the beginning of 1st period)
2. **PV of Annuity**
   - ✓ Annuity Regular (1st Payment at the end of 1st period)
   - ✓ Annuity Due (1st Payment at the beginning of 1st period)

FV of Annuity (Regular) = $C\left[\frac{(1+r)^n - 1}{r}\right]$

FV of Annuity (Due) = $C\left[\frac{(1+r)^n - 1}{r}\right](1+r)$

PV of Annuity (Regular) = $C\,\frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r}$

PV of Annuity (Due) = $C\,\frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r}(1+r)$

where, C = Cash flows per period
r=Rate/100*m
n = T*m

PERPETUITY

PV of perpetuity = C/R
PV of growing perpetuity = C/(R-G)
where, C = Cash flows per period
R=Rate per period
G=Growth rate

NET PRESENT VALUE (NPV)

NPV = PV of cash inflow – PV of cash outflow

Decision Rule:

If NPV > 0 Accept the Proposal
If NPV < 0 Reject the Proposal
If NPV = 0 Accept the Proposal

DEPRECIATION

WDV/Scrap value = Cost$\left(1 - \frac{R}{100}\right)^T$

NOTES:

- **In Loan Ques use PV of Annuity (Regular) Formula**

Loan Amount = Installment $\frac{\left[1 - \frac{1}{(1+r)^n}\right]}{r}$

- **In Sinking Fund ques use FV of Annuity Formula**
- **In valuation of bond ques use PV & PV of annuity(regular) formula**

RATIO

- Ratio = $\frac{a}{b}$ or a : b where b ≠ 0

Where, a = First term or Antecedent
b = Second term or Consequent

- Both terms of ratio can be multiplied or divided by the same (non-zero) number
- If a quantity increases or decreases in the ratio a : b then
  new quantity = $\frac{b}{a}$ x Original Qty.
- The reciprocal of a given ratio is called **Inverse ratio**
- The ratio **compounded** of the two ratios a : b & c : d is ac : bd
- The **duplicate ratio** of a : b is $a^2 : b^2$
- The **triplicate ratio** of a : b is $a^3 : b^3$
- The **sub-duplicate ratio** of a : b is √a : √b or $a^{\frac{1}{2}} : b^{\frac{1}{2}}$
- The **sub-triplicate ratio** of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$ or $a^{\frac{1}{3}} : b^{\frac{1}{3}}$

SEQUENCE & SERIES

ARITHMETIC PROGRESSION:

- A sequence $a_1, a_2, a_3, \ldots, a_n$ is called an arithmetic progression when $a_2 - a_1 = a_3 - a_2$.
- t_n = a + (n-1) d
  Where, a = first term
  n = number of terms
  d = common difference
  t_n = last term/ n^{th} term
- S = $\frac{n}{2}$[2a + (n-1) d] or $\frac{n}{2}$[a + t_n]
  Where, S = Sum of n terms
  a = first term
  n = number of terms
  d = common difference
  t_n = last term/ n^{th} term
- Sum S_n of the first n natural numbers = n(n+1)/2
- Sum S_n of first n odd numbers = n^2
- Sum of the Squares of the first n natural numbers = S = n(n + 1)(2n + 1) / 6
- Sum of the cubes of first n natural numbers = $[n(n+1)/2]^2$

GEOMETRIC PROGRESSION:

- A sequence a, ar, ar^2, ar^3……, ar^n is called Geometric Progression.
- n^{th} term of GP: $t_n = ar^{n-1}$
  Where, a = first term
  n = number of terms
  r = common ratio
  t_n = last term/ n^{th} term
- Common ratio = $\frac{Any\ Term}{Preceding\ Term} = \frac{ar}{a}$
  = $\frac{ar^2}{ar}$ = r
- If a, b, c are in GP we get $\frac{b}{a} = \frac{c}{b}$ which gives b^2 = a c, (b = $\sqrt{ac}$), b is called the geometric mean between a & c.
- S_n = a ($1 - r^n$) / (1 – r) when r < 1
  [Sum of GP of n terms]
  S_n = a (r^n - 1) / (r – 1) when r > 1
  [Sum of GP of n terms]
  where, a = first term
  n = number of terms
  r = common ratio
  a_n = last term/ n^{th} term
  S_n = Sum of n terms

$S_\infty = \frac{a}{1-r}$, for r < 1.
[Sum of infinite terms]

DIFFERENTIATION & APPLICATION

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(a^x) = a^x \log_e a$
- $\frac{d}{dx}(constant) = 0$
- $\frac{d}{dx}(e^{ax}) = ae^{ax}$
- $\frac{d}{dx}(logx) = 1/x$
- $\frac{d}{dx}f(x) = f'(x)$
- *Product Rule:* $\frac{d}{dx}f(uv) = u'v + uv'$
- *Quotient Rule:* $\frac{d}{dx}f\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

APPLICATION

Cost Function =C(x)
Average cost (AC) = TC/Output = C(x)/x
Marginal cost = C'(x)
Revenue Function R(x) = px
Marginal Revenue = R'(x)
Profit Function P(x) = R(x) – C(x)
Marginal profit = P'(x)

PROPORTION

- Equality of two ratios is called proportion.
  If a, b, c, d are said to be in proportion then a : b = c : d
  Here, a and d are **Extremes**; b and c are **Means**
  $\frac{a}{b} = \frac{c}{d}$ → ad = bc
- Product of extremes = Product of means *(Cross product rule)*
- If a, b, c are in continuous proportion then a : b = b : c
- b^2 = a c *(by cross product rule)*
- a:b=c:d → b:a=d:c **(Invertendo)**
- a:b=c:d → a:c=b:d **(Alternendo)**
- a:b=c:d → (a+b):b=(c+d):d **(Componendo)**
- a:b=c:d → (a-b):b=(c-d):d **(Dividendo)**
- a:b=c:d → (a+b):(a-b)=(c+d):(c-d) **(Componendo & Dividendo)**
- a:b=c:d → (a+c):(b+d) **(Addendo)**
- a:b=c:d → (a-c):(b-d) **(Subtrahendo)**
- **Formula for inverse variable**
  If y is inversely proportional to x
  i.e. y ∝ 1/x, then, y = (k ⁄ x)
  Here, K is the constant of proportionality

LINEAR INEQUALITY

- The Inequality is not affected by adding/subtracting any number.
- The Inequality is not affected by multiplying/dividing by a non-zero, positive number.
- When is inequality is multiplied/ divided by a negative number the inequality symbol is reversed.

SETS, RELATIONS & FUNCTIONS

- **Sub Sets**: A subset of a main set is a set which is formed by choice of any number of elements from the main set. Number of possible subsets = 2^n where n = no. of elements.
  Also, in all possible sets, one is improper subset and remaining are proper Subsets.
  Therefore, Proper subsets = 2^n – 1 and improper subset = 1
- **Power Set**: The collection of all possible subsets of a given set A is called the power set of A, to be denoted by P(A).
  No of elements in power set = n[P(A)] = 2^n
  No. of elements in Power set of a power set n[P(P(A))] = 2^{2^n}
- n(AXB) = n(A) x n(B)

FORMULAS -

1. n(AUBUC) = n(A) + n(B) + n(C) – n(A∩B) – n(B∩C) – n(C∩A) + n(A∩B∩C) [Not disjoint sets]
2. n(AUBUC) = n(A) + n(B) + n(C) [If A and B are disjoint sets]
3. n(AUB) = n(A) + n(B) – n(A∩B) [If A and B are not disjoint sets]
4. n(AUB) = n(A) + n(B) [If A and B are disjoint sets]
5. n(A – B) = n(A) – n(A∩B)
6. n(A'UB') = n[(A∩B)'] = n(S) – n(A∩B)
7. n(A'∩B') = n[(AUB)'] = n(S) – n(AUB)
8. (P U Q)' = P' ∩ Q'
9. (P ∩ Q)' = P' U Q'

FUNCTIONS

- **One-One Function** (Injective): Let f : A → B. If different elements in A have different images in B, then f is said to be a one-one or an injective function or mapping
- **Into function**: If in A → B, there exist even a single element in B having no pre-image in A, then f is said to be an into function.
- **Onto function** (Surjective): A function f defined from the set X to set Y (i.e. f : X → Y) is said to be an onto function if every element in the co-domain is mapped to by some element in its domain.
- **Bijection (One-One onto)**: A mapping which is both injective and surjective is called a bijection.

EQUATIONS

- An equation of degree 1 is called linear equation
- An equation of degree 2 is called quadratic equation.
  e.g. $ax^2 + bx + c = 0$, Where, a, b and c are constants and a ≠ 0
  - If b = 0, then the equation is called pure quadratic equation
  - If b ≠ 0, then the equation is called a mixed or affected quadratic equation
- A quadratic equation has two roots (i.e. x has two values)
- Roots of a quadratic equation $ax^2 + bx + c = 0$, where a ≠ 0
  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Sum of roots ($x_1 + x_2$) = $\frac{-b}{a}$
- Product of roots ($x_1.x_2$) = $\frac{c}{a}$
- Discriminant (D) = $b^2 - 4ac$
- If 2 roots of a quadratic equation are given, then quadratic equation is
  X^2 – (Sum of roots) x + Product of roots = 0

Nature of Roots:

- If D > 0 but not a perfect square then the roots are real, irrational and unequal
- If D < 0 then the roots are imaginary or not real
- If D = 0 then roots are real and equal
- If D > 0 and perfect square then the roots are real, rational and unequal

INTEGRATION

Integration is the reverse process of differentiation.

f(x) → Differentiate → f'(x)
f'(x) → Integrate → f(x)

Integration Formulas:

1. ∫ 1 dx = x + C
2. ∫ a dx = ax + C
3. ∫ x^n dx = ((x^{n+1})/(n+1)) + C
4. ∫ (1/x) dx = log x + C
5. ∫ e^x dx = e^x+ C
6. ∫ e^{ax} dx = e^{ax} / a + C
7. ∫ a^x dx = (a^x/log a) + C; a>0, a≠1
8. ∫ c f(x) dx can be written as c∫f(x) dx
9. ∫ f(x) dx ± g(x) dx can be written as ∫ f(x) dx ± ∫ g(x) dx

STANDARD FORMULA

a) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$
b) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$
c) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$
d) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$
e) $\int e^x [f(x) + f'(x)]dx = e^x f(x) + c$
f) $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left(x + \sqrt{x^2 + a^2}\right) + c$
g) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left(x + \sqrt{x^2 - a^2}\right) + c$
h) $\int \frac{f'(x)}{f(x)}dx = \log f(x) + c$

INTEGRATION BY PARTS

$\int uv\,dx = u\int v\,dx - \int\left[\frac{d(u)}{dx}\int v\,dx\right]dx$

where u and v are two different functions of x

APPLICATION

- If Marginal cost = C'(x) then Total cost C(x) = $\int C'(x)$
- If Marginal Revenue = R'(x) then Total revenue R(x) = $\int R'(x)$
- If Marginal profit = P'(x) then Total Profit P(x) = $\int P'(x)$

"Don't settle for average. Bring your best to the moment. Then, whether it fails or succeeds, at least you know you gave all you had."

ALL THE BEST!!!

Chart prepared by Mayank Maheshwari

STATISTICS CHART BY MAYANK MAHESHWARI

STATISTICAL DESCRIPTION OF DATA

I. BASIC

- **Meaning**
  The Word "Statistics" has different meanings when used in "Singular" and "Plural" Senses.
  - In **Plural sense** Statistics refers to the **data**, qualitative as well as quantitative.
  - In **Singular sense** Statistics refers to the **scientific method**
- **Applications of Statistics**
  - Economics
  - Business Management
  - Commerce and Industry
- **Characteristics (Attributes)**
  - Aggregate of facts
  - Affected to marked extent by large number of causes
  - Expressed Numerically
  - Reasonable percent of assurance
  - Systematic manner
  - Pre-defined purpose
- **Limitations of Statistics**
  - It ignores the quality aspect
  - No importance to an individual data
  - Does not reveal real story
  - Data should uniform and homogeneous

II. DATA

- **Types of data - Primary and Secondary Data**
  - Data which is collected & used for the first time is known as Primary Data
  - Data as being already collected, is used by a different person or agency is secondary data
- **Methods of collecting data**

  Primary Sources
  - Interview Method
    - ✓ Personal Interview – quick, accurate
    - ✓ Indirect Interview – problem in reaching
    - ✓ Telephone Interview – less consistent, wide coverage, non – responses are high
  - Mailed Questionnaire – wide coverage, maximum non-responses
  - Observation Method – best accuracy, time consuming, laborious, best method
  - Questionnaires - used for larger enquiries

  Secondary Sources
  - International sources
  - Government sources
  - Private and quasi-government sources
  - Unpublished sources
- **Classification of data**
  - **Chronological or Temporal or Time Series Data** - data are classified in respect of successive time points or intervals
  - **Geographical or Spatial Series data** - Data arranged region wise
  - **Qualitative or Ordinal Data** - Data classified in respect of an attribute
  - **Quantitative or Cardinal Data** - When the data is classified in respect of a variable
  - **Frequency and Non-Frequency group**
    - ✓ Frequency – Qualitative & Quantitative
    - ✓ Non-frequency – Chronological & Geographical

III. PRESENTATION OF DATA

- **Textual Presentation** - This method comprises presenting data with the help of a paragraph or a number of paragraphs. This type of presentation can be taken as the first step towards the other methods of presentation. It is dull, monotonous and comparison between different observations is not possible
- **Tabular Presentation** - There are two types of table – Simple & Complex.
  The Table under consideration should be divided into Caption, Box-head, Stub and Body. Caption is the upper part of the table, describing the columns and sub-columns, if any. The Box-head is the entire upper part of the table which includes columns and sub-column numbers, unit(s) of measurement along with caption. Stub is the left part of the table providing the description of the rows. The body is the main part of the table that contains numerical figures.
  - It facilitates comparison between rows and columns.
  - Complicated data can also be represented using tabulation.
  - It is a must for diagrammatic representation.
  - Without tabulation, Statistical Analysis of data is not possible
- **Diagrammatic Presentation** - An attractive representation of statistical data is provided by Charts, Diagrams and Pictures.
  Unlike the first two methods of representation of data, diagrammatic representation can be used for both the educated section and uneducated section of the society. Furthermore, any hidden trend present in the given data can be noticed only in this mode of representation. Diagrams can be (B.P.L) - Bar Diagram, Pie Chart and Line Diagram
  - **Bar Diagram**: Rectangle of equal width & usually of varying length. Bar Diagrams may be –
    (a) Horizontal Bar Diagram (used for qualitative data or data varying over space), or
    (b) Vertical Bar Diagram (used for quantitative data or time series data).
  - **Pie diagram**: This type of diagram shows the components of a variate as parts of a Circle.
  - **Line Diagram:** When the data vary over time, we take recourse to line diagram.
    - ✓ **Logarithmic and Ratio Charts**: When the time series exhibit a wide of fluctuations.
    - ✓ **Multiple line and Multiples Axis charts**: **Multiple line** charts are used for representing two or more related time series data expressed in the **same unit**, and **multiple - axis chart** in somewhat similar situations if the variables are expressed in **different units**.
- **Graphical Presentation** - The various types of graphical representation of a Frequency Distribution are as follows -
  - **Histogram or Area Diagram** - It is the most convenient way to represent a Frequency Distribution. With a Histogram, an idea of the Frequency Curve of the Variable under study can be obtained. A comparison among the frequencies for different Class Intervals is possible with Histograms
  - **Frequency Polygon** - A Frequency Curve can be regarded as a limiting form of Frequency Polygon & Histogram.
    Area of Histogram = Area of Polygon
  - **Ogives or Cumulative Frequency Graphs** – There are two types of Ogives –
    - ✓ **Less than type Ogives**: Plotting less than Cumulative Frequency
    - ✓ **More than type Ogives**: Plotting more than Cumulative Frequency

    Ogives may be considered for obtaining median, quartiles, deciles & percentiles graphically. Ogives are used for making short term projections.

Particulars	Histogram	Bar diagram
Space	No	Yes
Mode	Yes	No
Variable	Continuous series	Discrete & continuous series
Width	Important	Not Important

IV. FREQUENCY DISTRIBUTION

- **Meaning**: Frequency Distribution is a Tabular Representation of Statistical Data that distributes the total frequency to a number of classes.
- **Width or Size or length of a Class Interval**: The width of a Class Interval is the difference between the UCB and the LCB of that class. [Class Interval = UCB – LCB]
- **Class Limit** – inclusive & exclusive series
- **Class Boundary** – exclusive series only
- **Mid-Point or Class Mark**
  Mid-Point = $\frac{UCL+LCL}{2}$ or $\frac{UCB+LCB}{2}$
- **Frequency Density** = Frequency of Given Class / Class width
- **Relative Frequency** = Class Frequency / Total Frequency
- **Percentage Frequency** = Relative Frequency x 100

THEORETICAL DISTRIBUTIONS

Discrete Probability Distributions – Binomial, Poisson distributions
Continuous Probability Distribution – Normal distribution

BINOMIAL DISTRIBUTION

$P(x) = {}^nC_x\, p^x q^{n-x}$ where, n = no. of trials (n = 0, 1, 2, …., n)
x = Success required (x = 0,1,2,3,…n)
p = Probability of success of single event
q = Probability of Failure of single event

Properties:
- Binomial distribution is bi-parametric. 2 parameters are (n and p)
- Mean = μ = np; Variance = σ^2 = npq; SD = $\sigma = \sqrt{npq}$
- Variance is always less than Mean
- Variance will be highest when p = q (i.e. p = q = 1/2) = n/4
- Mode = (n+1)p; if (n+1)p is non integer then mode = highest integer value. (i.e. Uni-modal); if (n+1)p is integer then Mode = (n+1)p & (n+1)p – 1 (i.e. Bi-modal)

POISSON DISTRIBUTION

$P(x) = \frac{e^{-m}\, m^x}{x!}$; where, e = exponential function (e = 2.71828)
m = Average or mean = np = μ
x = no. of success required (0,1,2,3,….∞)

Properties:
- It is Uni-parametric. 1 parameter is m
- Mean = μ = m; Variance = σ^2 = m; SD = $\sigma = \sqrt{m}$
- Mode = m, if m integer, then mode = m, m-1 (bi-modal); if m non-integer, then mode = m (uni-modal)

NORMAL DISTRIBUTIONS

$F(x) = \frac{1}{\sigma\sqrt{2\Pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for -∞ < x < ∞, where, σ = S.D., μ = mean

Or

A function f(x) is Probability Density Function (PDF) if –
i. F(x) ≥ 0, -∞ < x < +∞
ii. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

- The normal distribution curve is a bell-shaped curve.
- At the center of the curve lies Mean, Median & Mode (i.e. μ = Mean, Median & Mode)
- Normal distribution curve is Uni-modal
- The curve never touches the x-axis
- The total area under the curve = 1 or 100%
- The point of inflection are $\mu + \sigma$ & $\mu - \sigma$
- For a standard normal variate, value of Mean = 0, SD = 1
- The skewness of the normal distribution curve is zero

Properties:
- The normal distribution has 2 parameters i.e. μ & σ
- Q1 = $\mu - 0.675\,\sigma$; Q3 = $\mu + 0.675\,\sigma$
- QD : MD : SD = 10 : 12 : 15; MD = 0.8 σ; QD = 0.675 σ
- If X and Y are 2 independent normal variables with mean as a & b and SD as x & y, then normal distribution (X+Y) is distributed with Mean = a+b & SD = $\sqrt{x^2+y^2}$

MEASURES OF CENTRAL TENDENCY

- Types of Series
  - Individual Series
  - Discrete Series
  - Continuous Series
- Types of Continuous Series
  - Inclusive Series
  - Exclusive Series

Central tendency is an **average** which represent the characteristics of the entire data and help us to compare the given data with another data. This average has a tendency to be somewhere at the centre and hence called Measure of Central Tendency.

DIFFERENT MEASURES OF CENTRAL TENDENCY

- **Arithmetic Mean (AM)**
  Individual Series: $\bar{x} = \frac{\Sigma x}{n}$
  Discrete or Continuous Series: $\bar{x} = \frac{\Sigma fx}{n}$

Properties of AM
- If all the observations are same, say 'k', then the AM is also 'k'
- The algebraic **sum of deviations** of the given set of observations taken from the AM is always **ZERO.** i.e. $\sum f(x-\bar{x}) = 0$
- **(Change of Origin)** If each observation of a data is **increased or decreased** by a constant 'k', then the AM of new data also gets increased or decreased by 'k'
- **(Change of Scale)** If each observation of a data is **multiplied or divided** by a constant 'k', then the AM of new data also gets by multiplied or divided by 'k'
- **(Change of Origin & Scale)** AM is affected due to change of origin and/or scale which implies that if the original variable 'x' is changed to another variable 'y' by affecting a change of origin, say a, and change of scale, say b, of x, i.e. y = a + bx, then AM of y is given by $\bar{y} = a + b\bar{x}$
- The **sum** of **Square** of deviations of given set of observations is minimum when taken from **AM**. i.e. $\sum(x-\bar{x})^2$ is minimum
- **Correcting incorrect mean**
  Step 1: Calculate wrong total ($\bar{x}$ x n)
  Step 2: Calculate correct total = Wrong total – wrong observations + correct observations
  Step 3: Correct mean = $\frac{correct\ total}{no.\ of\ observations}$
- If there are two groups containing n_1 and n_2 observations and $\bar{x}_1$ and $\bar{x}_2$ as the respective arithmetic means, then the **combined AM** is given by
  $$\bar{x} = \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$$
- **Weighted AM** = $\bar{x}_w = \frac{w_1x_1+w_2x_2+\cdots+w_nx_n}{w_1+w_2+\cdots+w_n}$ or $\bar{x}_w = \frac{\Sigma wx}{\Sigma w}$

- **Median (Positional Average)**

Median in case of Individual Series
- In case of odd observations, Median = Middle Value or (n+1)/2 observation
- In case of even observations, Median = Average of Middle two Values or Average of n/2 and n/2+1 observation

Median in case of Discrete Series
Step 1: Prepare 'less than' c.f. distribution
Step 2: Find (n+1)/2, where n = no. of observations
Step 3: See the c.f. just greater than equal to (n+1)/2[th] observation.
Step 4: The variable corresponding to the c.f. is the median.

Median in case of Continuous Series
Step 1: Prepare 'less than' c.f. distribution
Step 2: Find n/2, where n = no. of observations
Step 3: See the c.f. just greater than equal to n/2[th] observation.
Step 4: Find the class corresponding to the c.f. obtained in Step 3. This class is called median class.
Step5: Apply the following formula

$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h$$

Where, l = lower limit of median class
c = c.f. of the class preceding the median class
f = frequency of the median class
h = size or width of the median class

Properties of Median
- The **sum of absolute deviations is minimum** when the deviations are taken from the **median.** i.e. $\sum|x-A|$ is minimum, where A = median
- **(Change of Origin & Scale)** If x and y are two variables, to be related by y = a + bx for any two constants a and b, then the median of y is given by $y_{me} = a + bx_{me}$

- **Mode**

Mode is the value which occurs **maximum number of times**. Therefore, it is also called as **fashionable average**

Individual Series
An observation repeated maximum number of times.

Discrete Series
Observation having Highest frequency.

Continuous Series
Step 1: Find Modal Class (i.e. Class with highest frequency)
Step 2: Apply following formula:

$$\textbf{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where, l = lower limit of modal class.
f_1 = frequency of modal class
f_0 = preceding frequency
f_2 = Succeeding frequency,
h = height of modal class

Properties of Mode
- **Change of Origin & Scale**: If x and y are 2 variables related as **y = a + bx, then $Y_{(mo)} = a + b.X_{(mo)}$**
- **Mode = 3 Median – 2 Mean**

- **Geometric Mean (GM)**

Geometric Mean is the n[th] root of n terms. It is the best measure of central tendency for ascertaining rate of change **over a period of time**

Individual Series
$$GM = (X_1 \times X_2 \times X_3 \times \ldots\ldots X_n)^{1/n}$$

Discrete or Continuous Series
$$GM = (X_1^{f_1} . X_2^{f_2} . X_3^{f_3} \ldots\ldots X_n^{f_n})^{1/n}$$

Properties of GM
- If **any observation is zero** (0) then **GM is not defined**
- If **all the observations are same**, say a, then **GM is also same**. i.e. a
- GM of the product of 2 variables is the product of their GM. i.e. if z = xy, then GM of Z = GM of x . GM of y
- GM of the ratio of 2 variables is the ratio of the GM's of 2 variables i.e. if z = x/y then GM of z = GM of x/ GM of y
- GM<AM
- It is the best measure of central tendency for ascertaining the average rate of change over a period of time
- It is the most appropriate average to be used for construction of index numbers
- It is the most suitable average to be used when it is desired to give more weightage to smaller items

- **Harmonic Mean (HM)**

It is defined as the reciprocal of the AM of the reciprocals of a given set of observations.

Individual Series
$$HM = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\cdots+\frac{1}{x_n}} \quad \text{OR } HM = \frac{n}{\sum\frac{1}{x}}$$

Discrete & Continuous Series
$$HM = \frac{n}{\sum\frac{f}{x}}$$

Properties of HM
- **If all the observation** taken by a variable **are same**, say k, then the **harmonic mean of the observations is also same**, i.e. k
- If **any one observation is 0**, then **HM is 'not defined'**
- The harmonic mean has the least value when compared to the geometric mean and the arithmetic mean (i.e. AM > GM > HM)
- Combined HM = $\frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_2}}$
- Weighted HM = $\frac{\sum w}{\sum\frac{w}{x}}$
- It is used primarily in averaging speeds when 'EQUAL' distances are covered.
- It is also used in averaging cost of commodity/ securities when 'EQUAL' amount is invested

- **Other Partitional Values**

QUARTILES	DECILES	PERCENTILES
Quartiles divide the set of observations into 4 equal parts	Deciles divide the set of observations into 10 equal parts.	Percentiles divide the set of observations into 100 equal parts.
Q_1, Q_2, Q_3	$D_1, D_2, D_3, \ldots\ldots\ldots, D_9$	$P_1, P_2, P_3, \ldots\ldots\ldots, P_{99}$
There are 3 Quartiles	There are 9 Deciles	There are 99 Percentiles

Quartiles (Q_k) - (Q_1, Q_2, Q_3)

Computation:

Individual Series
Step 1: Arrange data in order
Step 2: Find the rank of $\frac{K(n+1)}{4}$
Step 3: Corresponding Variable is Quartile.

Discrete Series
Step 1: Arrange data in order
Step 2: Prepare c.f. distribution
Step 3: Find the rank of $\frac{K(n+1)}{4}$
Step 4: Then find the c.f. just greater than equal to $\frac{K(n+1)}{4}$
Step 5: Corresponding Variable is Quartile.

Continuous Series
Step 1: Prepare c.f. distribution
Step 2: Find $\frac{Kn}{4}$
Step 3: See the c.f. just greater than equal to $\frac{Kn}{4}$
Step 4: Find the Quartile class
Step 5: Apply the formula:

$$Q_k = l + \frac{\frac{kN}{4} - c}{f} \times h$$

Where, l = lower limit of Quartile class
c = c.f. of the class preceding the Quartile class
f = frequency of the Quartile class
h = size or width of the Quartile class

Note: For computation of Deciles, use same steps as used in Quartile calculation, just replace 4 with 10.
Note: For computation of Percentiles, use same steps as used in Quartile calculation, just replace 4 with 100.

- **Relationship between AM, GM & HM**
  - When observations are unequal, positive & greater than zero, AM > GM > HM always.
  - If all the observations are equal, AM = GM = HM
  - AM x HM = $(GM)^2$

MEASURES OF DISPERSION

The degree to which numerical data tend to spread about an average value is called the dispersion of data.

High variation/Dispersion - BAD

Low variation/ Dispersion - GOOD

Absolute Measure	Relative Measure
Absolute measures are dependent on the unit of the variable under consideration	Relative measure of dispersion are unit free.
Absolute measures are not considered for comparison.	For comparing 2 or more distributions, relative measures are considered.
Absolute measures are easy to compute and understand.	Relative measures are difficult to compute and understand

Types of Measures of Dispersion

Absolute Measure	Relative Measure
• Range • Quartile Deviation • Mean Deviation • Standard Deviation	• Coefficient of Range • Coefficient of Quartile Deviation • Coefficient of Mean Deviation • Coefficient of Variation

I. RANGE

Range is the simplest method of computing the dispersion.

Range = L – S

where, L = Largest value, S = Smallest value

Coefficient of Range = $\frac{L-S}{L+S}$ x 100

Properties of Range:

- Range is based on 2 extreme values of the observation & hence ill-defined.
- It is not possible to compute range in case of open-ended distribution

Merits of Range:

- It is easy to calculate and understand
- It requires minimum time to calculate

De-merits of Range:

- It is not based on all observations
- Range is a poor measure of dispersion

II. QUARTILE DEVIATION (SEMI INTER QUARTILE RANGE)

QUARTILES: Q_1, Q_2, Q_3

It is defined as half of the deviation between the upper Quartile & Lower Quartile of the distribution.

Q.D. = $\frac{Q_3-Q_1}{2}$

Coefficient of Q.D. = $\frac{Q_3-Q_1}{Q_3+Q_1} \times 100$

OR

Coefficient of Q.D. = $\frac{\frac{Q_3-Q_1}{2}}{Median/Q_2} \times 100$

Coefficient of Q.D. = $\frac{QD}{Median/Q_2} \times 100$

Inter Quartile range = $Q_3 - Q_1$

$Q_3 - Q_2 = Q_2 - Q_1$

Properties of Q.D.

- It is best suited measure of dispersion for an open-end distribution.
- It Is based on middle 50% of the values of the distribution
- First 25% & last 25% values are left out.

Merits of Q.D.

- It is simple to understand and calculate
- It is superior to Range
- It can be computed for distribution with Open-end classes
- Q.D. is not affected by extreme values

De-merits of Q.D.

- It is not based on all the observations
- It is not suitable for further mathematical treatment

III. MEAN DEVIATION (AVERAGE DEVIATION)

Mean Deviation is the A.M. of the absolute deviation of the observations from an appropriate measure of central tendency (i.e. Mean, Median or Mode)

M.D. = $\frac{\sum|x-A|}{n}$ = $\frac{\sum|D|}{n}$ (Individual Series)

M.D. = $\frac{\sum f|x-A|}{n}$ = $\frac{\sum f|D|}{n}$ (Discrete & Continuous Series)

Where, A = Mean, Median or Mode

D = X – A

Coefficient of M.D. = $\frac{MD}{A} \times 100$

Property of M.D.

- The M.D. is minimum when the deviations are taken from Median.

Merits of M.D.

- It is based on each and every observation
- It is rigidly defined
- It is easy to calculate and understand
- As compared with S.D., it is less affected by extreme observations

De-merit of M.D.

- Algebraic signs are ignored
- It is not suitable for further mathematical treatment
- It cannot be computed for distributions with open ended classes

All birds find shelter during the rain.
But eagle avoids the rain by flying above the clouds.
Be an Eagle
ALL THE BEST!!

CHART PREPARED BY MAYANK MAHESHWARI

IV. STANDARD DEVIATION (σ)

It is defined as the root mean square deviation when the deviations are taken from A.M.

Variance is Square of S.D. (i.e. Variance = σ^2)

Calculation:

Individual Series

S.D. or $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ OR $= \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$ OR $= \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

Discrete & Continuous series

S.D. or $\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{n}}$ OR $= \sqrt{\frac{\Sigma fx^2}{n} - \left(\frac{\Sigma fx}{n}\right)^2}$ OR $= \sqrt{\frac{\Sigma fx^2}{n} - (\bar{x})^2}$

Coefficient of Variation = $\frac{S.D.}{A.M.} \times 100$

Coefficient of S.D. = $\frac{S.D.}{A.M.}$

Properties of S.D.

- S.D. of first n natural numbers = $\sqrt{\frac{n^2-1}{12}}$
- Q.D. = $\frac{2}{3}\sigma$ M.D. = $\frac{4}{5}\sigma$ Q.D. = $\frac{5}{6}MD$
- S.D. of 2 numbers = $\frac{L-S}{2}$ OR = $\frac{|a-b|}{2}$
- Combined S.D. = $\sqrt{\frac{n_1s_1^2+n_2s_2^2+n_1d_1^2+n_2d_2^2}{n_1+n_2}}$

where, $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$,

S_1 = S.D. of 1st Group, S_2 = S.D. of 2nd Group;

n_1, n_2 = No. of observations in 1st and 2nd group respectively

$\bar{x}$ = Combined mean

Merits of S.D.

- It is the best measure of Dispersion
- It considers all observations
- It is rigidly defined
- It is useful for further mathematical treatment.

De-merits of S.D.

- Not that easy to calculate and understand
- It cannot be computed for distribution having open end class distributions

Common properties of measures of dispersion

- MOD are UNAFFECTED by CHANGE OF ORIGIN
- They CHANGE in the same ratio as CHANGE OF SCALE.
- If all the observations are same or zero than MOD is zero.
- If any 2 constants a, b and 2 variables are related by y = a + bx, then

Computation is as follows:

MOD	Value
Range	$R_y = |b|. R_x$
Quartile Deviation	$QD_y = |b|. QD_x$
Mean Deviation	$MD_y = |b|. MD_x$
Standard Deviation	$SD_y = |b|. SD_x$

PROBABILITY

Probability of n events refers to the chance of occurrence of such event in a Random Experiment.

P(A) = $\frac{\text{Occurrence of favourable event A}}{\text{Total outcomes}}$ OR = $\frac{n(A)}{n(S)}$

Property & Formulas –

- P(A) + P(A') = 1, or P(A') = 1 – P(A)
- P(AUB) = P(A) + P(B) [mutually exclusive events]
- P(AUB) = P(A) + P(B) – P(A∩B) [not mutually exclusive events]
- P(AUBUC) = P(A) + P(B) + P(C) - P(A∩B) - P(B∩C) - P(A∩C) + P(A∩B∩C) [not mutually exclusive events]
- P(AUBUC) = P(A) + P(B) + P(C) [mutually exclusive events]
- P(A-B) = P(A) – P(A∩B) [Probability of only A]
- P(B-A) = P(B) – P(A∩B) [Probability of only B]
- P(A∩B) = P(AB) = P(A and B) all are same
- P(AUB) = P(A or B) = P(A+B) all are same

Types of events

- **Independent Event –** If outcome of one event does not influence the occurrence of the other event.
  P(A∩B) = P(A) x P(B); P(A∩B') = P(A) x P(B'); P(A'∩B) = P(A') x P(B)
  P(A'∩B') = P(A') x P(B') ; P(A∩B∩C) = P(A) x P(B) x P(C)
- **Mutually exclusive events –** If occurrence of one event prevents the occurrence of the other events.
  Therefore, P(A∩B) = 0; P(A∩B∩C) = 0; P(AUB) = P(A) + P(B)
- **Mutually exhaustive events –** It means that the events together make up everything that can happen.
  P(AUB) = 1; P(AUBUC) = 1
- **Mutually exclusive & exhaustive events**
  P(AUBUC) = P(A) + P(B) + P(C) [when exclusive]
  P(AUBUC) = 1 [when exhaustive]
  P(A) + P(B) + P(C) = 1 [when exclusive & exhaustive]

Odd in Favour & Odd against

Odd in favour = Favourable outcomes : Unfavourable outcomes

Odd against = Unfavourable outcomes : Favourable outcomes

Total outcomes = Favourable + Unfavourable

Conditional Probability

P(B/A) = $\frac{P(A\cap B)}{P(A)}$; P(A/B) = $\frac{P(A\cap B)}{P(B)}$

Statistical Definition of Probability

Mean = Expected Value = μ = E(x) = ΣP(X).X or $\Sigma R_f.X_i$

Probability = P(X) = P(X_i) = R_f ; Variable = X = X_i

Expected value of x^2 in given by: E(X_i^2) = ΣP(X_i).X_i^2

Variance = σ^2 = E(x_i – μ)^2 = E(x_i^2) – μ^2 = ΣP(X_i).X_i^2 - μ^2

Properties

- E(x + y) = E(x) + E(y); E(x - y) = E(x) - E(y); E(xy) = E(x) x E(y)
- E(k.x) = k.E(x) [Change of scale]
- Variance of a constant k is V(k) = 0

CORRELATION & REGRESSION ANALYSIS

CORRELATION

Correlation analysis determines the relation between 2 variables. Also, it measures the extent of relationship between 2 variables by means of a single number called a correlation coefficient (r).

-1 ≤ r ≤ +1

Methods of finding correlation coefficient (r) -

- **Scatter Diagram** - It is a simple diagrammatic method to establish correlation between a pair of variables. It can be used to find linear & non-linear relation. It fails to measure the extent of relationship between the variables.
- **Karl Pearson's Coefficient of Correlation** – It is also known as Product Movement Correlation.

$$r = \frac{N\Sigma xy - \Sigma x\Sigma y}{\sqrt{N\Sigma x^2 - (\sum x)^2}\sqrt{N\Sigma y^2 - (\sum y)^2}}$$

OR

$$r = \frac{Cov(x,y)}{\sigma x\ \sigma y} \text{ OR } r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sigma_x\sigma_y}$$

- **Rank Correlation** - Rank correlation is applied to identify the correlation between the Qualitative Characteristics.

Rank correlation $(r) = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$

D = Difference of Ranks

n = no. of Observations

- **Coefficient of Concurrent Deviation** - This method does not take into account the magnitude of deviations of the 2 variables.

$$r_C = \pm\sqrt{\left|\frac{2c-n}{n}\right|}$$

Where, c = no. of pairs of concurrent deviations (i.e. no. of + sign)

n = no. of observations – 1

Property of Correlation

The correlation coefficient (r) is independent of change of origin and scale.

i.e. if u = a + bx & v = c + dy

then, $r_{uv} = \frac{b\times d}{|b|\times|d|} . r_{xy}$

Note: Coefficient of correlation between x & y and u & v will always remain equal. They would have opposite signs only when b & d differs in sign.

Note: r^2= coefficient of determination

1-r^2= coefficient of non-determination

Note: The coefficient of determination is such that 0 ≤ r^2 ≤ 1

REGRESSION

Regression is concerned with estimating the value of DEPENDENT Variable Corresponding to a known INDEPENDENT Variable.

In other words, known variable is independent variable and unknown variable is dependent variable.

Regression Coefficient are b_{YX}, b_{XY}

b_{YX}	b_{XY}
$b_{YX} = \frac{n\Sigma XY - (\sum X)(\Sigma Y)}{n\sum X^2 - (\Sigma X)^2}$	$b_{XY} = \frac{n\Sigma XY - (\sum X)(\Sigma Y)}{n\sum Y^2 - (\Sigma Y)^2}$
$b_{YX} = r\frac{\sigma_y}{\sigma_x}$ here, r = Coefficient of correlation	$b_{XY} = r\frac{\sigma_x}{\sigma_y}$ here, r = Coefficient of correlation
$b_{YX} = \frac{Cov(x,y)}{(\sigma x)^2}$	$b_{XY} = \frac{Cov(x,y)}{(\sigma y)^2}$

Regression Equation

Y depends on X Y on X	X depends on Y X on Y
General Form: **Y = a + bX** **here b = b_{YX}**	**General Form:** **X = a + bY** **here b = b_{XY}**
Point Form: $y - \bar{y} = b_{yx}(x - \bar{x})$	**Point Form:** $x - \bar{x} = b_{xy}(y - \bar{y})$

Properties of Regression

- Coefficient of Regression remains UNCHANGED due to change of ORIGIN but CHANGES due to change of SCALE.
  Change of Origin ➔ No Change in Regression Coefficient
  Change of Scale ➔ Change in Regression Coefficient
  $b_{UV} = b_{XY} . \frac{M_x}{My}$
  $b_{VU} = b_{YX} . \frac{M_y}{Mx}$
- Relationship between r, b_{YX}, b_{XY} (Most Important)
  $r^2 = b_{YX} . b_{XY}$
- r, b_{YX}, b_{XY} all 3 bears the same sign.
- Both regression lines i.e. X on Y & Y on X intersect each other at their MEANS. i.e. on $\bar{x}$ & $\bar{y}$

CALCULATOR TRICKS:

Find a^n	Find 1/(a^n)	Find $a^{1/n}$
Steps - type a - Press × - Press = (n-1) times	Steps - type a - Press ÷ - Press = (n times)	Steps - type a - Press √ 12 times - Minus 1 = - ÷ n = - Add 1 = - Press ×= 12 times
Find a^n where n is non integer	**Find Scrap value in depreciation ques.**	**Find log**
Steps - type a - Press √ 12 times - Minus 1 = - × n = - Add 1 = - Press ×= 12 times	Steps - (1-Dep %) - Press × - Type cost of machine - Press = (n times)	Steps - Enter number - Press √ 13 times - Minus 1 - × 3558

AVJ ACADEMY

INDEX NUMBER

Index number shows movement of a variable

The base value of the index number is usually 100 and indicates either to price, date, a level of production, etc.

Expressed in Percentage, Measures of Net Changes, Measure change over a period of time

What are the types of Index Numbers?

- **Price Index Numbers** – Shows movement in price levels between 2 periods
- **Quantity Index Numbers** - Shows movement in quantity levels between 2 periods
- **Value Index Numbers** - Shows movement in Value levels between 2 periods

Some other points on Index Numbers

- P_{01} is the price index for time 1 on 0.
  Here, P_0 = Base year price, P_1 = Current year price
- P_{01} = Current year price / Base year price * 100 OR $\Sigma P_1/\Sigma P_0$*100
- P_{01} = Price Index, Q_{01} = Quantity Index, V_{01} = Value Index
- The ratio of the price of a **single** commodity in a given period to its price in other period is called the Price Relative.
  Price relative = P_1/P_0*100
- Index Numbers are constructed from the sample
- Weights play an important part in construction of Index Numbers
- The best average for construction of Index Number is GM. But in general practice AM is used.
- GM makes index number time reversable P_{01} → P_{10}
- Pure numbers are used in computing Price Relative
- Price index are used to measure economic strength
- Purchasing power of Money = 1/Price Index
- Cost of Living index is Price Index

Methods of constructing Index Numbers (Price Index P_{01})

- **Simple Method/ Unweighted Method**
  - **Simple Average Method**
    P_{01} = $\frac{\Sigma P_1}{\Sigma P_0} \times 100$
    here, $\sum P_1$ = Sum of all commodity prices in current year
    ΣP_0 = Sum of all commodity prices in Base year
  - **Simple Average of price/quantity relative**
    Using AM → P_{01} = $\frac{1}{n}\Sigma\left(\frac{P_1}{P_0} \times 100\right)$ OR = $\frac{1}{n}\Sigma P$
    Using GM → P_{01} = $AL\left[\frac{1}{n}\Sigma\ log\left(\frac{P_1}{P_0} \times 100\right)\right]$
- **Weighted Method**
  - **Weighted average method**
    General Form = $P_{01} = \frac{P_1w}{P_0w} \times 100$
    Where, w = weight
    ✓ Laspeyre's Price Index → P_{01} = $\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$
    ✓ Paasche's Price Index → P_{01} = $\frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100$
    ✓ Fisher's Ideal Price Index → P_{01} = $\sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}}$ x 100
    OR $P_{01} = \sqrt{L * P}$
    ✓ Dorbish & Bowley's Price Index
    P_{01} = $[\frac{\Sigma p_1q_0}{\Sigma p_0q_0} + \frac{\Sigma p_1q_1}{\Sigma p_0q_1}]/2$ * 100 OR P_{01} = $\frac{L+P}{2}$

Note:

- The result obtained by Marshall Edgeworth method is closest to Fisher's Index
- Fisher's Ideal Index is GM of Laspeyre's & Paasche's Index
  - **Weighted average of price/quantity relative**
    Using AM → P_{01} = $\frac{\Sigma WP}{\Sigma W}$ where P = $\frac{P_1}{P_0}$ × 100
    Using GM → P_{01} = $AL\left[\frac{\Sigma W\ logP}{\Sigma W}\right]$ where P = $\frac{P_1}{P_0}$ × 100

Methods of constructing Index Numbers (Quantity Index Q_{01})

All methods and formulas are same to determine Q_{01}

Just interchange p with q and q with p.

Value Index Numbers (V_{01})

Value Index numbers shows the movement in value levels between two periods.

Value = Price x Quantity

Note: It is used for computing growth rate in the economy.

Value Index → V_{01} = $\frac{\Sigma p_1q_1}{\Sigma p_0q_0} \times 100$

OR

V_{01} = $\frac{\Sigma V_1}{\Sigma V_0} \times 100$

Here, $V_1 = \Sigma p_1q_1$ & $V_0 = \Sigma p_0q_0$

Test of Adequacy

There are four tests of adequacy:

- **Unit Test** - Except for the simple average method all other formulae satisfy this test
- **Time reversal test** - P_{01} x P_{10} = 1 – Laspeyre's method and Paasche's method do not satisfy this test
- **Factor Reversal test** - P_{01} x Q_{01} = V_{01} - Only Fisher's Index satisfies Factor Reversal test
- Circular test - P_{01} x P_{12} x P_{20} = 1 - This test is not met by Laspeyres, or Paasche's or the Fisher's ideal index. The simple geometric mean of price relatives and the weighted average method with fixed weights meet this test. This test is extension of Time Reversal Test.

Other imp. Formulas-

CPI, CII, RPI = $\frac{\Sigma p_1q_0}{\Sigma p_0q_0}$ × 100,

Real Wages = $\frac{\text{Money wages}}{\text{CII}}$ x 100

Consumer Price Index (CPI),
Cost of Living Index (CII),
Real Price Index (RPI)