CA Foundation
Business Mathematics
FORMULA SHEET

General Algebraic Rules

Squares:

$(a \pm b)^2 = a^2 \pm 2ab + b^2$
$(a - b)^2 = (a + b)^2 - 4ab$
$a^2 - b^2 = (a - b)(a + b)$
$(a^2 + b^2) = \frac{1}{2}[(a + b)^2 + (a - b)^2]$

Cubes:

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Ratios

If ratio is $a : b \Rightarrow \frac{a}{b}$ then,

- Duplicate ratio $= a^2 : b^2$
- Sub duplicate ratio $= \sqrt{a} : \sqrt{b}$
- Triplicate ratio $= a^3 : b^3$
- Sub triplicate ratio $= \sqrt[3]{a} : \sqrt[3]{b}$
- Compound ratio of a: b and c: d $= ac : bd$

Proportion

a : b = c : d then a, b, c and d are 1st, 2nd, 3rd and 4th proportional.

Continued Proportion

- $a : b = b : c \Rightarrow \frac{a}{b} = \frac{b}{c}$, where a, c are 1st and 3rd proportions; b is mean proportion.
- 1st proportion (a) = $\frac{(Mean)^2}{3rd\ proportion}$ $= \frac{b^2}{c}$
- 3rd proportion (c) = $\frac{(Mean)^2}{1st\ proportion}$ $= \frac{b^2}{a}$
- Mean proportion (b) = $\sqrt{1st\ proportion \times 3rd\ proportion}$ $= \sqrt{ac}$

Square of 3 numbers = $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Laws of Indices

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m \times b^m$
- $a^0 = 1$
- $a^{-m} = \frac{1}{a^m}$
- $(\frac{a}{b})^m = \frac{a^m}{b^m}$
- $a^m = b^m \rightarrow a = b$
- $a^m = a^n \rightarrow m = n$
- $\sqrt[m]{a} = a^{\frac{1}{m}}$
- If net sum of powers = 0 , then result of equation = 1 i.e.,

  $a^{x-y}.a^{y-x}.a^{z-x} = 1$

Logarithm

- If $\log_b N = p$ then, $b^p = N$. Normally base (b) = 10 (Common) for calculus or b = e (where e = constant)

- **Properties (fixed base) :**

i) $\log a + \log b = \log(ab)$

ii) $\log a - \log b = \log(\frac{a}{b})$

iii) $\log a^m = m \log a$

iv) $\log_b b = 1$ (i.e. log 10 = 1, commonly in algebra and log e = 1, in calculus)

v) $\log 5 = 1 - \log 2$ (w.r.t base = 10)

vi) $\log 1 = 0$

vii) $b^{\log_b N} = N$, $b^{m \log_b N} = N^m$

viii) $\log_{b^n} a^n = \frac{n}{m} \log_b a$, like $\log_{b^2} a^3 = \frac{3}{2} \log_b a$.

- **Change of base :**

ix) $log_y x = \frac{\log x}{\log y}$

x) $\frac{1}{\log_y x} = \log_x y$

xi) $\log_b a \times \log_a c = \log b$

- **Note :**

(a) $\log 3 + \log 2 = \log 6 \ \ (\neq \log 5)$

(b) $\frac{\log 3}{\log 2} = \log_2 3 \ \ (\neq \log 3 - \log 2)$

(c) $(\log x)^2 = (\log x)(\log x) \ \ (\neq 2 \log x)$

(d) $\log x = \log y \Rightarrow x = y$ (if base is same)

Equations

Simple Equation ;

- $ax + b = 0$ where a, b are known constants and $a \neq 0$

Method of solving simultaneous linear equation with 2 variables:

- Elimination Method [Equations are reduced to one unknown by eliminating the other unknown]
- Cross Multiplication Method

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Conditions of Two Equations :

Two equations $a_1 x + b_1 y + c_1 = 0$, $a_2 x + b_2 y + c_2 = 0$ have,

- Unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
- Infinite solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- No solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ i.e., if $a_1b_2 = a_2b_1$

Quadratic Equation :

- $ax^2 + bx + c = 0$ has two solutions of x (i.e. 2 roots of x)
- $x = \frac{-b \pm \sqrt{D}}{2a} (= \alpha, \beta)$ where, $D = b^2 - 4ac$
- If $b = 0$, then two roots α, β are equal but opposite in sign.
- If $c = a$, then α, β are reciprocal i.e. $\alpha = \frac{1}{\beta}$
- If one root is $\alpha = m + \sqrt{n}$, then other root $\beta = m - \sqrt{n}$.
- Sum of roots $= \frac{-b}{a} = \frac{\text{coefficient of x}}{\text{coefficient of } x^2}$
- Product of roots $= \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$
- If two roots are known then quadratic equation is, $x^2 - (Sum\ of\ roots)x + (Product\ of\ roots) = 0$

Nature of Roots :

- If $D > 0$
  - D is a perfect square (α, β are unequal, rational)
  - D is not a perfect square (α, β are unequal, irrational)
- If $D = 0$
  - α, β are real and equal i.e., $\boldsymbol{b^2 = 4ac}$
- If $D < 0$
  - α, β are non-real and unequal.

Cubic Equation : $ax3 + bx2 + cx + d = 0$

Linear Inequality

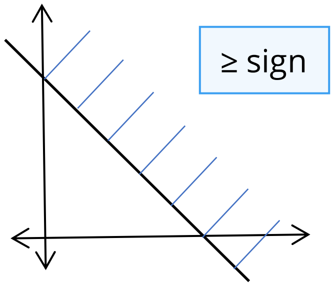
(i) $a > b$ then $-a < -b$ & $\frac{1}{a} < \frac{1}{b}$

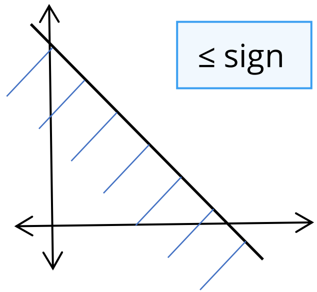
(ii) $|x| > b \Rightarrow x > b$ or $x < -b$

(iii) $|x| < b \Rightarrow -b < x < b$

(iv) For graph, shade the region whose any point satisfies the inequality ≤ means usually towards origin i.e. (0, 0) should satisfy it and ≥ sign means away from the origin.

Graph of linear inequality:





Simple interest & Compound interest

Simple Interest (SI):

- Simple Interest = $\frac{Prt}{100}$
- Amount = $P\left(1 + \frac{rt}{100}\right)$

[A=Future Amount, P=Principal, r=rate of interest per annum and t=time in years]

Compound Interest (CI):

- Compound Interest $= A - P$

$$= P\left\{\left(1 + \frac{r}{100}\right)^n - 1\right\}$$

(compounded annually)

Compound Interest:

- If annually compounded,

$$A = P\left(1 + \frac{r}{100}\right)^n$$

- If half-yearly compounded,

$$A = P\left(1 + \frac{r}{200}\right)^{2n}$$

- If quarterly compounded,

$$A = P\left(1 + \frac{r}{400}\right)^{4n}$$

- If monthly compounded,

$$A = P\left(1 + \frac{r}{1200}\right)^{12n}$$

Effective Interest Rate (E) :

$$E = \left(1 + \frac{r}{100}\right)^n - 1$$

Compound Annual Growth Rate (CAGR) :

$$CAGR = r\% = \left(\frac{End\ Value}{Beginning\ Value}\right)^{\frac{1}{t_n - t_0}} - 1$$

Depreciation :

$$Scrap\ Value = P\left(1 - \frac{r}{100}\right)^n$$

(where r = annual rate of depreciation and P = original price)

- **Nominal Rate of Return** = Real Rate of Return + Inflation
- **Net Present Value** = Present value of cash inflow – Present value of cash outflow

Perpetuity:

- PV of multi period perpetuity $= \frac{R}{i}$

  (where R = payment or receipt each period & i = interest rate per payment or receipt period)
- PV of growing perpetuity $= \frac{R}{i-g}$

  (where R = cash flow, i = interest rate & g = growth rate in interest)

Annuity

- **Annuity** – a sequence of periodic payments regularly over a specified period.

- **Types of annuity :**
  - **Annuity regular** – First payment/receipt at the end of the period
  - **Annuity due or immediate** – First payment/receipt in the beginning of the period

Future Value (FV) :

- Used for investments.
- FV of annuity regular $= \frac{A}{i}[(1+i)^n - 1]$
- FV of annuity due / annuity immediate = FV of annuity regular $\times (1+i)$

(where i = Adjusted rate of interest like $\frac{r}{200}$ for half yearly etc.; n = No. of instalments)]\'

Present Value (PV) :

- Used for repayments.
- PV of annuity regular $= \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$
- PV of annuity due / annuity immediate = PV of annuity regular for (n-1) years + initial receipt or payment in beginning of the period.

Permutations and Combinations

Permutation :

- ${}^{n}P_{r} = \frac{n!}{(n-r)!} = n(n-1)(n-2)\cdots\cdots\cdots(n-r+1)$
- ${}^{n}P_{n} = n!$
- $0! = 1$
- $n! = m!$ if n = 1, m = 0 or m = 1, n = 0
- AND → Multiply & OR → Add
- No. of Rearrangements = No. of Arrangements – 1.
- If some objects are always together, take them as one object & multiply their arrangement with total arrangements.
- TAN Rule: Total − Always together = Never Together
- If *n* objects to be arranged in which m particular objects are always together $= {}^{n-m+1}P_{n-m-1} \times m!$
- No. of arrangements of *n* things with *p*, *q*, *r* alike objects $= \frac{n!}{p!q!r!}$

Circular Permutation:

- Used for ring , round table, etc.
- Number of arrangements $= (n-1)!$
- The number of necklaces formed with n beads of different colours $= \frac{1}{2}(n-1)!$

Combination :

- ${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$
- ${}^{n}C_{x} = {}^{n}C_{y}$ then x = y or x + y = n
- ${}^{n+1}C_{r} = {}^{n}C_{r} + {}^{n}C_{r-1}$
- ${}^{n}C_{r} = {}^{n}C_{n-r}$
- ${}^{n}C_{0} = 1$
- ${}^{n}C_{n} = 1$

- If n different objects are to be selected any number at a time, then number of selections $= 2^n - 1$
- Among n persons, number of handshakes = ${}^{n}C_{2}$
- For n sided polygon, no. of diagonals = ${}^{n}C_{2} - n$
- If m different parallel straight lines intersected with n different parallel straight lines, then number of different parallelograms formed = ${}^{m}C_{2} \times {}^{n}C_{2}$
- At least ⇒ Minimum
- At most ⇒ Maximum
- If atleast one object to be selected, then TNA rule is applicable.
- Number of selections of n different objects taken r at a time such that
  - m objects are always present = ${}^{n-m}C_{r-m}$
  - m objects are never present = ${}^{n-m}C_{r}$

Arithmetic Progression (AP) & Geometric Progression (GP)

AP :

- Common difference (d) = $T_2 - T_1 = T_3 - T_2$ etc.
- Terms : a, a+d, a+2d, ...
- If a, b, c are in A.P., then $b - a = c - b$ or $a + c = 2b$
- nth term (T_n) = $a + (n - 1)d$

$$\Rightarrow n = \left(\frac{T_n - a}{d}\right) + 1$$

$$\Rightarrow d = \frac{T_n - a}{n - 1} = \frac{T_m - T_n}{m - n}$$

- Sum of n terms (S_n) = $\frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}(a + l)$$

- Arithmetic Mean between a & $b = \frac{1}{2}(a + b)$
- $T_n = S_n - S_{n-1}$
- $T_1 = S_1$, $T_2 = S_2 - S_1$

GP :

- Common ratio (r) = $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ etc.
- Terms : a, ar, ar^2, ar^3, ...
- nth term= $a(r)^{n-1}$
- $r^{m-n} = \frac{T_m}{T_n}$
- Sum of n terms (S_n) = $\frac{a(1-r^n)}{1-r}$ when $r < 1$

$$= \frac{a(r^n-1)}{r-1} \text{ when } r > 1$$

- G.M. between a and b = $\sqrt{ab}$
- If a, b, c are in GP, then $b^2 = ac$ (b is called the geometric mean between a and c)

HP : A sequence of numbers is said to be a **harmonic progression** if the reciprocal of the terms are in arithmetic progression.

Important Summation :

(a) Sum of n natural numbers, 1 + 2 + 3 +.... + n $= \frac{1}{2}n(n+1)$

(b) Sum of squares of n natural numbers, $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}n(n+1)(2n+1)$

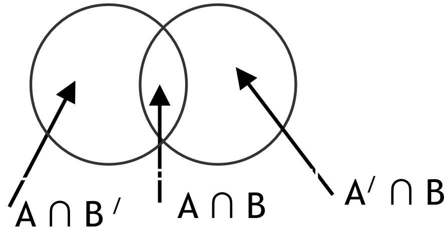
(c) Sum of first n odd numbers, 1+ 3 + 5 +........+ (2n – 1) = n^2

(d) Sum of cubes of n natural numbers, $1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{1}{2}n(n+1)\right]^2$

(e) Sum of infinite GP = a + ar + ar^2 + ... $= \frac{a}{1-r}$ where $-1 \le r < 1$.

Set Theory

- Cardinal number of a set = Size of the set A = n(A);
  For Empty Set or null set { } = ∅
- Number of subsets of a set whose cardinal number is n = 2^n and number of proper subsets = $2^n - 1$
- Power Set: Set of all subsets of a set i.e., 2^n subsets in power set
- Singleton Set: A set containing one element
- Equal Set: Two sets A & B are said to be equal, written as A = B if every element of A is in B and every element of B is in A.



Product Set : $A \times B = \{(a, b)\ where\ a \in A\ and\ b \in B\}$

[Note : $B \times A \neq A \times B$ but $n(A \times B) = n(B \times A)$]

Relation (R) :

$R \subseteq (A \times B)$ i.e. R is sub- set of some ordered pairs of the set (A × B). R be may one to one, many to one or one to many, but mapping is one to one or many to one.

Types of Relations :

(1) Reflexive:

$(a, a) \in R$ for all $a \in A$ (E.g. parallel, congruent, etc.)

(2) Symmetric:

$(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$ (e.g., reciprocal, perpendicular or | a – b | = K)

(3) Transitive:

$(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ for all $a, b, c \in A$ (e.g. greater than relation, smaller than relation etc.)

(4) Equivalence:

R is Reflexive, Symmetric and Transitive. (e.g., parallel to, equal to)

De Morgan's Law :

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

Difference property :

- $A - B = A \cap B'$
- $B - A = B \cap A'$

Total number of elements :

- For 2 sets :
  - $\mathbf{n(A \cup B)} = n(A) + n(B) - n(A \cap B)$
- For 3 sets :
  - $\mathbf{n(A \cup B \cup C)} = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
  - $\mathbf{n(only\ A)} = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
  - $\mathbf{n(only\ A, B\ but\ not\ C)} = n(A \cap B) - n(A \cap B \cap C)$

Mapping or Functions :

- It is denoted by f or f (x) ; $f \subseteq R$ (i.e. Subset of Relation is Mapping)
- $f: A \longrightarrow B$ such that $f(x) = y$, then A $\Rightarrow$ Domain set, B $\Rightarrow$ Co-domain set & $f(x) = y \Rightarrow$ Range set .
- f (x) = y is the set of outputs or results from given x.

Types of Functions :

(1) One-One Function :

Let $f: A \longrightarrow B$. If different elements in A have different images in B, then f is said to be a one-one or an **injective function** or mapping (only one x exists for each y).

(2) Onto Function :

Let : $A \longrightarrow B$. If every element in B has at least one pre-image in A, then f is said to be an onto or **surjective function** (i.e. all elements of B are in y i.e. f (x))

(3) Into Function :

Let Y $\subset$ B (i.e. some elements of B are not in y) then '**into** mapping'.

(4) Bijective Function ;

If 'f(x)' is onto and one to one then, it is **Bijective** Mapping.

(5) Inverse Function :

If f (x) = k , then x = f^{-1} (k). A function is **invertible** only if it is one-one and onto.

Note :

- $g \, o \, f(x) = g\{f(x)\}$ [put f (x) in place of x in g (x)]
- $f \, o \, g(x) = f\{g(x)\}$ [put g (x) in place of x in f (x)].

Limits and Continuity

- $\lim_{x\to a} f(x)$ is said to exist when both left-hand and right-hand limits exists and they are equal.
  i.e., $\lim_{x\to a-} f(x) = \lim_{x\to a+} f(x) = \lim_{x\to a} f(x)$
- If $\lim_{h\to 0} f(a+h) = \lim_{h\to 0} f(a-h)\,, (h>0)$ then $\lim_{x\to a} f(x)$ exists.

Useful theorems on limits:

Let $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$ then,

- $\lim_{x\to a}\{f(x)+g(x)\} = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = l+m$
- $\lim_{x\to a}\{f(x)-g(x)\} = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = l-m$
- $\lim_{x\to a}\{f(x).\,g(x)\} = \lim_{x\to a} f(x) \times \lim_{x\to a} g(x) = lm$
- $\lim_{x\to a}\{f(x)/g(x)\} = \left\{\lim_{x\to a} f(x)\right\} / \left\{\lim_{x\to a} g(x)\right\} = l/m$ if m ≠ 0
- $\lim_{x\to a} c = c$ where c is constant.
- $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$
- $\lim_{x\to a} F\{f(x)\} = F\left\{\lim_{x\to a} F(x)\right\} = F(l)$
- $\lim_{x\to 0+} \frac{1}{x} = \lim_{h\to 0} \frac{1}{h} \to +\infty$ (h > 0)
  $\lim_{x\to 0+} \frac{1}{x} = \lim_{h\to 0} \frac{1}{-h} \to -\infty$ (h > 0). Thus $\lim_{x\to 0+} \frac{1}{x}$ does not exist.

Important Limits:

1) $\lim_{x\to 0} \frac{(e^x-1)}{x} = 1$
2) $\lim_{x\to 0} \frac{(a^x-1)}{x} = \log_e a\,, (a>0)$
3) $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$
4) $\lim_{x\to x} \left(1+\frac{1}{x}\right)^x = e$
5) $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}}}{x} = e$
6) $\lim_{x\to 0} \frac{x^n-a^n}{x} = na^{n-1}$
7) $\lim_{x\to 0} \frac{(1+x)^n-1}{x} = n$

Continuity: A function f(x) is said to be continuous at x=a if and only if i) f(x) is defined at x = a , ii) $\lim_{x\to a-} f(x) = \lim_{x\to a+} f(x)$ and, iii) $\lim_{x\to a-} f(x) = f(a)$. Sum, difference, product & quotient of 2 continuous functions is a continuous function.

Derivative

1) $\frac{d}{dx}(x^n) = nx^{n-1}$ 4) $\frac{d}{dx}(x) = 1$ 7) $\frac{d}{dx}(\log x) = \frac{1}{x}$

2) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{-n}{x^{n+1}}$ 5) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$ 8) $\frac{d}{dx}(a^x) = a^x \log_e a$

3) $\frac{d}{dx}(k) = 0$ 6) $\frac{d}{dx}(e^x) = e^x$

Product Rule:- Derivative of $f(x).g(x) = gf' + fg'$

Division Rule:- Derivative of $\frac{f(x)}{g(x)} = \iint \frac{gf' - fg'}{g^2}$

Chain Rule:- $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (i.e.,finding the derivative of a composite function)

E.g derivative of $e^{f(x)} = e^{f(x)}.f'(x)$

Differential Calculus (application)

Cost Function:

Total Cost (T.C.) = $F.C. + V.C.$; A.C. (Average Cost) = $\frac{T.C.}{X}$

AVC (Average variable cost) = $\frac{V.C.}{x}$; Marginal Cost = $\frac{d}{dx}(T.C.)$

Revenue Function:

Selling Price (Per unit) = p = demand function

Then, Total Revenue $(R) = p.x$ where x is output

Marginal Revenue = $\frac{dR}{dx}$

Profit Function :

Profit (P) = $R - T.C.$

Marginal Profit = $\frac{dP}{dx}$

Break-even point: $T.C. = R$

Marginal Propensity to Consume (MPC) = $\frac{dC}{dy}$

Marginal Propensity to save (MPS) = $\frac{dS}{dy}$

Integration

Basic Formulae:

1) $\int x^n.dx = \frac{x^{n+1}}{n+1} + c$

2) $\int dx = x + c$

3) $\int e^x.dx = e^x + c$

4) $\int e^{ax}\,dx = \frac{e^{ax}}{a} + c$

5) $\int \frac{dx}{x} = \log x + c$

6) $\int a^x.dx = \frac{a^x}{log_e a} + c$

Special Cases:

1) $\int (x \pm k)^n dx = \frac{(x\pm k)^{n+1}}{n+1} + c$

2) $\int (x \pm k)^{-1} dx = \log(x \pm k) + c$

3) $\int (ax \pm b)^n dx = \frac{1}{a}\frac{(ax\pm b)^{n+1}}{n+1} + c$

4) $\int (ax \pm b)^{-1} dx = \frac{1}{a}\log(ax \pm b) + c$

Note:

1) $\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$

2) $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

3) $\int e^x[f(x) + f'(x)] = e^x f(x) + c$

[If degree of Numerator $\geq$ that of denominator, then divide and write as $\frac{N}{D} = Q + \frac{R}{D}$, then integrate.]

Substitution:

Let $I = \int f(x).dx$ and $x = g(t)$

Then, $I = \int f[g(t)].g'(t).dt$

Integration by parts:

$$\int uv\,dx = u\int v\,dx - \int \left[\frac{du}{dx}\int v\,dx\right] dx$$

Definite Integration :

$\int_a^b F(x)dx = f(b) - f(a)$

'b' is called the upper limit and 'a' the lower limit of integration.

Application of Integration:

- If marginal cost function is given, then total cost function can be found out using integration.
- If marginal revenue is given, then total revenue function can be found out using integration.

Important Standard Formulae :

1) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

2) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

3) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log(x + \sqrt{x^2 + a^2}) + c$

4) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log(x + \sqrt{x^2 - a^2}) + c$

5) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log(x + \sqrt{x^2 + a^2}) + c$

6) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log(x + \sqrt{x^2 - a^2}) + c$

Form of the rational function	Form of the partial fraction
1. $\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2. $\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3. $\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
4. $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

Properties of Definite Integral :

1) $\int_a^b f(x)dx = \int_a^b f(t)dt$

2) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, a < c < b$

3) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

4) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

5) When $\mathrm{f(x) = f(a + x)}$, then

$$\int_0^{na} f(x)dx = n\int_0^a f(x)dx$$

6) If $\mathrm{f(-x) = f(x)}$, then

$$\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$$

6) If $\mathrm{f(-x) = -f(x)}$, then

$$\int_{-a}^a f(x)dx = 0$$

CA Foundation – Logical Reasoning

KEY CONCEPTS & FORMULA SHEET

Number Series, Coding, De-coding & Odd Man Out

Number Series:

- Succeeding numbers are usually derived from preceding numbers. The pattern is usually based on the following:
  - Simple Addition of a number
  - Simple Subtraction of a number
  - Combination of addition & subtraction
  - Multiplication of a number
  - Division of a Number
  - Exponents
- First find out differences between succeeding numbers or difference of differences
- Try remembering up to 5th power of first 10 natural numbers.

If there are two large numbers one after the other – try dividing them and to get a relation between those two numbers, if that does not work out try exponents.

Coding & Decoding:

Coding is usually,

- Common difference of Succeeding alphabets
- Common difference of Preceding Alphabets
- Alphabet before A is Z
- Alphabet after Z is A
- Sometimes alphabets are allotted numbers as table below and the series may of alphabets may be based on these numbers or a summation of these numbers.
  E.g.: FAT may be represented as 6120 or 6 + 1 + 20 = 27

Odd Man Out:

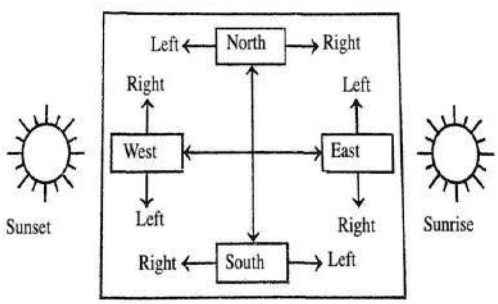
- Usually, combination of number series or alphabet series or other patterns.
- Be on lookout for prime numbers or factorials.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

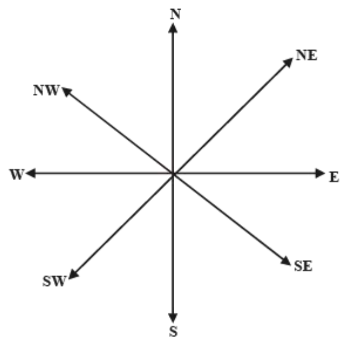
Number / Exponent	1	2	3	4	5
1	$1^1= 1$	$1^2= 1$	$1^3= 1$	$1^4= 1$	$1^5= 1$
2	$2^1=2$	$2^2=4$	$2^3=8$	$2^4=16$	$2^5=32$
3	$3^1=3$	$3^2=9$	$3^3=27$	$3^4=81$	$3^5=243$
4	$4^1=4$	$4^2=16$	$4^3=64$	$4^4=256$	$4^5=1024$
5	$5^1=5$	$5^2=25$	$5^3=125$	$5^4=625$	$5^5=3025$
6	$6^1=6$	$6^2=36$	$6^3=216$	$6^4=1296$	$6^5=7776$
7	$7^1=7$	$7^2=49$	$7^3=343$	$7^4=2401$	$7^5=16807$
8	$8^1=8$	$8^2=64$	$8^3=512$	$8^4=4096$	$8^5=32768$
9	$9^1=9$	$9^2=81$	$9^3=729$	$9^4=6561$	$9^5=59049$
10	$10^1=10$	$10^2=10$	$10^3=1000$	$10^4=10000$	$10^5=100000$

Direction Sense Test

- Always start the problem by imagining an India Map.
- Draw the map & start solving by assuming that you are in Nagpur, Centre of the country as starting point.
- When a question already provides the direction of the ending point relative to the starting point, it is easier to solve using that direction as a clue.



Main directions



- When a problem states that people are sitting in a circle determine based on the question if the students are facing inwards or outwards. If nothing is mentioned start with the assumption that they are facing inwards. E.g., If people are friends or a family, they are usually looking at each other and, hence facing inwards.
- When a person is upside down, east, and west directions become reverse.
  - E.g.: If a person is standing in the centre of the country & facing north, West will be to his left and East to his right. If the same person is upside down, and is facing north, east will be to his left and west will be to his right.

Left + left	Down
Left + right	Up
Right + left	Up
Right + right	Down
Up + left	Left
Up + right	Right
Down + left	Right
Down + right	Left

Seating Arrangements

Always use pen & paper to solve these problems.

Two types of seating arrangements & information provided in them,

- Circular seating
  - Clockwise seating - Left movement is called clockwise rotation.
  - Anti-Clockwise Seating - Right movement is called anti–clockwise rotation.
- Linear arrangements
  - Definite Information - When the place of any object or person is definitely mentioned then we say that it is a definite information, X is sitting on the right end of the bench.
  - Comparative Information - When the place of any object or person is not mentioned definitely but mentioned only in the comparison of another person or object, then we say that it is a comparative information.
  - Negative Information - A negative information does not tell us anything definitely but it gives an idea to eliminate a possibility.

Solve the problems by drawing lines and arranging positions based on information.

Blood Relations

- Solve Problems by placing people in layers one above / below the other based on generation – this helps in retaining clarity.
- Similarly, for people in same generation, place them side by side on the same layer.
- Use full terms like Boy / Girl or Mother / Father or Male / Female rather than using abbreviations like M / F as that can lead to confusion if M stands for Mother or Male
- If you (personally) have a large family imagine yourselves in the position of one of the characters and try and figure out relationships – this is risky as sometimes all relationships may not be there in your family.
- Mother's side – Maternal, Father's side - Paternal

Some important Relations:

Grandfather/Grandmother's son	Father or Uncle
Mother's or father's mother	Grandmother
Grandfather/Grandmother's only son	Father
Son's wife	Daughter-in-Law
Daughter's husband	Son-in-Law
Husband's or wife's sister	Sister-in-Law
Brother's son	Nephew
Brother's daughter	Niece
Uncle or aunt's son or daughter	Cousin
Sister's husband	Brother-in-Law
Brother's wife	Sister-in-Law
Granson's or granddaughter's daughter	Great grand Daughter

i) Relations of Paternal side:

- Father's father → Grandfather
- Father's mother → Grandmother
- Father's brother → Uncle
- Father's sister → Aunt
- Children of uncle → Cousin
- Wife of uncle → Aunt
- Children of aunt → Cousin
- Husband of aunt → Uncle

(ii) Relations of Maternal side:

- Mother's father → Maternal grandfather
- Mother's mother → Maternal grandmother
- Mother's brother → Maternal uncle
- Mother's sister → Aunt
- Wife of maternal uncle → Maternal aunty

CA Foundation - Statistics

KEY CONCEPTS & FORMULA SHEET

Statistical Description of Data

Origin of Statistics:

- Latin word - status
- Italian word - statista
- German word - statistik
- French word - statistique

Definition of Statistics:

- Plural sense - defined as data qualitative as well as quantitative.
- Singular sense - defined as the scientific method that is employed for collecting, analysing and presenting data.

Limitations of Statistics:

- It deals with the aggregates.
- It is concerned with quantitative data.
- Future projections of are possible under a specific set of conditions. If any of these conditions is violated, projections are likely to be inaccurate.
- The theory of statistical inferences is built upon random sampling.

Types of Data:

- Qualitative – attribute [E.g. gender, nationality, the colour of a flower, etc.]
- Quantitative – variable
  - Discrete – finite or isolated value. [E.g. no. of petals in a flower, no. of road accidents in a locality]
  - Continuous – any value from a given interval. [E.g. height, weight, sale, profit, etc]

Method of Collection of data:

- Primary [first-hand data] - data collected for the first time by an investigator or agency
- Secondary [second hand data] - data already collected are used by a different person or agency.

Collection of Primary Data:

- Interview method
  - Personal Interview method, Indirect Interview method and Telephone Interview method.
- Mailed questionnaire method.
- Observation method
- Questionnaires filled and sent by enumerators.

Sources of Secondary Data:

a) International sources like WHO, ILO, IMF, World Bank etc.
b) Government sources
c) Private and quasi-government sources like ISI, ICAR, NCERT etc.
d) Unpublished sources of various research institutes, researchers.

Note:

- Personal Interview – used for natural calamity or an epidemic.
- Indirect Interview – used for rail accidents (info is collected from persons associated with the problems)
- Telephone Interview – quick, non-expensive but problem of non-responses
- Mailed questionnaire – wide area coverage, maximum no. of non-responses.
- Observation – best method, but time consuming, laborious and covers only small area.

Classification of Data:

a) Chronological or Temporal or Time Series Data
b) Geographical or Spatial Series Data
c) Qualitative or Ordinal Data
d) Quantitative or Cardinal Data

Mode of Presentation of Data:

- Textual presentation – simple, even a layman can present by this method, but is dull, monotonous, and not recommended for manifold classification.
- Tabular presentation or Tabulation – facilitates comparison, complicated data can be presented, must for diagrammatic representation.
- Diagrammatic representation – hidden trend can be noticed, less accurate than tabulation.

Statistical Table:

It has 4 parts namely,

a) Caption (upper part of table showing columns and sub – columns)
b) Box head (showing column, sub – columns, units etc)
c) Stub (left part showing row descriptions)
d) Body (main part showing numerical information.)

Types of diagrams:

a) Line diagram or Histogram - For Time Series data exhibiting wide range of fluctuations use Logarithmic or Ratio Chart. To compare two or more related time series data with same unit, Multiple Line Chart can be used.
b) Bar diagram - Horizontal bar diagram → qualitative data or data varying over space & Vertical bar diagram → quantitative data or time series data.
c) Pie chart - used to compare different components of variables. [Sub – divided or component bar diagram may also be used but Pie – chart is better]
Central Angle for Pie – Chart =

$$\frac{Individual\ Value}{Total} \times 360°$$

Types of Frequency Distribution:

- When tabulation is done in respect of a discrete random variable → **Discrete or Ungrouped** or simple Frequency Distribution.
- For a continuous variable → **Grouped** Frequency Distribution.

Types of graphical representation of frequency distribution:

- Histogram or Area diagram (Exclusive or Overlapping Classes are required and for unequal classes use frequency density)
- Frequency Polygon (used for single frequency distribution)
- Ogives or cumulative Frequency graphs [2 types - less than & more than, plotting c. f. on y axis & class boundaries on X axis gives ogives & intersection of two ogives give median]

Frequency Curve:

A smooth curve for which the total area is taken to be unity.

Types:

(a) Bell-shaped curve (most used)

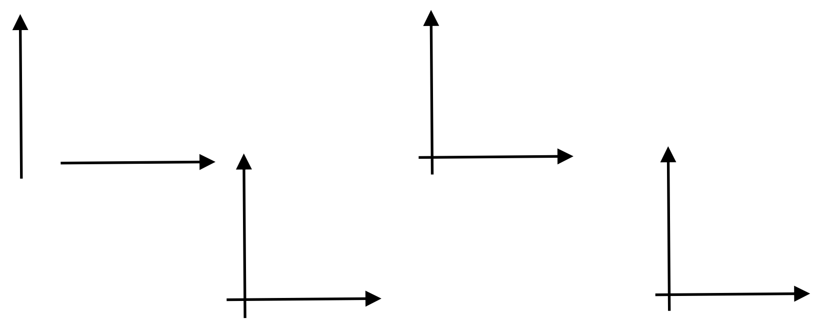
(b) U-shaped curve (used for no.of passengers)

(c) J-shaped curve (denotes less than Ogive)

(d) Mixed curve.

Some Important terms:

- No. of class interval × class lengths = Range. i.e., highest value – lowest value
- Class Limit (CL) - the minimum value and the maximum value the class interval may contain.
- Class Boundary (CB) - actual class limit of a class interval
    - For overlapping Class Intervals (upper end to be excluded), CB coincides with CL. (Used for Histogram)
    - For non – overlapping Class Intervals (both ends included), $CB = CL \pm \frac{D}{2}$
- Mid-point or Mid-value or class mark $= \frac{LCL+UCL}{2} = \frac{LCB+UCB}{2}$
- Width or size of a class interval $= UCB - LCB$
- Frequency Density $= \frac{Class\ frequency}{Class\ length}$
- Relative frequency $= \frac{Class\ frequency}{Total\ frequency}$
- Percentage frequency is relative frequency expressed in percentage.



Sampling

Population or Universe:
The aggregate of all the units under consideration. It may be finite or infinite; existent or hypothetical.

Sample:

- A part of a population selected with a view of representing the population in all its characteristics.
- If a sample contains n units, then n is known as **sample size**.
- The units forming the sample are known as "**Sampling Units**".

Parameter:
A characteristic of a population based on all the units of the population.

Population mean $(\mu) = \frac{\sum_{a=1}^{n} x_a}{N}$

Population proportion (P) $= \frac{X}{N}$

Population variance $(\sigma^2) = \frac{\sum (X_a - \mu)^2}{N}$

$\mathrm{SD}(\sigma) = \sqrt{\frac{\sum (X_a - \mu)^2}{N}}$

- The estimates of population mean, variance and population proportion are given by,

$$\hat{\mu} = \frac{\sum x_i}{n}$$

$$\widehat{\sigma^2} = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$\hat{p} = \frac{x}{n}$$

- The mean of the statistic is known as "Expectation".
- The standard deviation of the statistic is known as the "Standard Error (SE) ".
- For simple random sampling with replacement:

$$\mathrm{SE}\ \bar{x} = \frac{\sigma}{\sqrt{n}} \quad \& \quad \mathrm{SE}\ (p) = \frac{\sqrt{Pq}}{n}$$

- For simple random sampling without replacement:

$$\mathrm{SE}\ \bar{x} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$$

$$\mathrm{SE}\ (p) = \frac{\sqrt{Pq}}{n} \cdot \sqrt{\frac{N-n}{N-1}}$$

Note:

- **Simple random sampling** is effective when,

(i) the population is not very large
(ii) the sample size is not very small
(iii) the population is not heterogeneous.

- **Stratified sampling** provides separate estimates for population means for different segments and also an overall estimate.
- Two types of allocation of sample size - Bowley's allocation & Neyman's allocation.
- **Multistage Sampling** adds flexibility into the sampling process.
- **Systematic sampling** is affected most if the sampling frame contains an undetected periodicity.
- **Purposive or Judgement sampling** is dependent solely on the discretion of the sampler.

Measures of Central Tendency

Different measures of central tendency:

(i) Mean

- Arithmetic Mean (AM)
- Geometric Mean (GM)
- Harmonic Mean (HM)

(ii) Median

(iii) Mode

Median (middle most value)

$$= l + \left(\frac{\frac{N}{2} - N_l}{N_u - N_l}\right) \times c$$

Mode (most frequent value)

$$= l + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1}\right) \times c$$

Weighted Averages:

- Weighted AM $= \frac{\sum w_i x_i}{\sum w_i}$
- Weighted GM $= \text{Antilog}\left(\frac{\sum w_i \log x_i}{\sum w_i}\right)$
- Weighted HM $= \frac{\sum w_i}{\sum\left(\frac{w_i}{x_i}\right)}$

[Weighted HM of first n natural numbers $= \frac{2n+1}{3}$]

AM $(\bar{x}) = \frac{\sum x}{n}$ or $\frac{\sum fx}{\sum f}$ or $A + \frac{\sum fd}{N} \times c$

Properties:

- If all the observations assumed by a variable are constants, say k, then the AM is also k.
- The algebraic sum of deviations of a set of observations from their AM is zero.
- If $y = a + bx$, then the AM of y is given by $\bar{y} = a + b\bar{x}$.
- If there are two groups containing n_1 and n_2 observations and $\bar{x}_1$ and $\bar{x}_2$ as the respective arithmetic means, then the combined AM is

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

- GM $= (x_1 x_2 \ldots \ldots x_n)^{\frac{1}{n}}$
- HM $= \frac{n}{\sum\left(\frac{1}{x}\right)}$
- Combined HM $= \frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_2}}$
- For a set of n distinct positive values, A.M. > G.M. > H.M.
- For a set of same values, A.M.= G.M.= H.M.
- Generally, A.M. ≥ G.M. ≥ H.M.
- For two values, A.M × H.M. = $(\text{G.M.})^2$
- If a set of data include any value equal to zero, only AM can be calculated.
- For two values x and y, AM= $\frac{x+y}{2}$, GM $= \sqrt{xy}$ and HM $= \frac{2xy}{x+y}$

Partition Values or Quartiles or Fractiles:

- $P_K = \frac{(n+1)K}{100}$th value of ascending data (K th Percentile)
- $D_K = \frac{(n+1)K}{10}$ th value of ascending data (K th Decile)
- Q_1 = Lower Quartile $= \frac{n+1}{4}$ th ascending value
- Q_3 = Upper Quartile $= \frac{3(n+1)}{4}$ th ascending value
- Q2 = Median $= \frac{n+1}{2}$ th ascending value
- $P_{10} = D_1$, $P_{20} = D_2$ …… , $P_{90} = D_9$, $P_{25} = Q_1$, $P_{75} = Q_3$
- $D_5 = P_{50} = Q_2$ = Median

Note:

- **Mean – Mode** = 3 (Mean – Median), **Mode** = 3Median – 2Mean, **Corrected Mean** $= \frac{n\bar{x} - wrong\ values + correct\ values}{n}$
- **Choice of averages:**
  - Usually, A.M. is applied.
  - For percentage change, like index number, population change, G.M. is used.
  - For rate per unit like speed, H.M. is used.

Measures of Dispersion

Dispersion - measure of scatteredness or variability or amount of deviation from the central tendency.

Measurement	Absolute formula	Relative formula (Coefficient)
1). Range	$L - S$	$\frac{L - S}{L + S} \times 100$
2). Mean Deviation	$\frac{1}{n}\sum \lvert x_i - M \rvert$ or $\frac{1}{n}\sum \lvert x_i - M \rvert f_i$, where M = Mean or Median or Mode.	$\frac{MD_M}{M} \times 100$
3). Standard Deviation	$\sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$ or $\sqrt{\frac{\sum f(x_i - \bar{x})^2}{n}}$	Coefficient of Variation (CV) $= \frac{SD}{AM} \times 100$
4). Quartile Deviation or Semi-inter quartile range	$\frac{Q_3 - Q_1}{2}$	$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

Note:

- Variance = $(SD)^2$
- If all values are constant (i.e., fixed) SD = 0
- For 1st n natural numbers, SD $= \frac{n^2 - 1}{12}$
- For two values a and b (a > b), SD = MD $= (a - b)/2$

- For change of origin, Range, QD, MD and SD **do not change** but due to change of scale Range, QD, MD and SD **always change** accordingly.
- If y = a + bx , then $R_y = |b| R_x$, $QD_y = |b| QD_x$, $MD_y = |b| MD_x$ and $SD_y = |b| SD_x$

Note:

- Combined SD $= \sqrt{\frac{n_1 s_1{}^2 + n_2 s_2{}^2 + n_1 d_1{}^2 + n_2 d_2{}^2}{n_1 + n_2}}$
- For Normal or Symmetric Distribution $Q.D. = Probable\ Error\ of\ S.D.$ & $2\,S.D. = 3\,Q.D$

Probability

- P(A) $= \frac{n_A}{n} = \frac{No.of\ equally\ likely\ events\ favourable\ to\ A}{Total\ no.of\ equally\ likely\ events}$
- $0 \leq$ P(A) ≤ 1
- P(A) $= 0 \Rightarrow$ A is an impossible event.
- P(A) $= 1 \Rightarrow$ A is a sure event.
- Non-occurrence of event A is denoted by A' or AC or $\bar{A}$ and it is known as complimentary event of A.
  P(A) + P(A') = 1
- Odds in favour of an event = happening : non – happening
  $= m : (n - m) = p : q$
- Odds against an event = non – happening : happening
  $= (n - m) : m = q : p$
- Probability for the above 2 events $= \frac{p}{p+q}$
- Probabilities of x heads (or x tails) from n tosses $= \frac{{}^nC_x}{2^n}$
- Probability of sum of two dice
  $= \frac{(\text{Sum}-1)}{36}$ when sum ≤ 7
  $= \frac{(13-\text{sum})}{36}$ when sum ≥ 8
- Probability of taking at least one = 1 – Probability of taking none.

Set Concepts of Probability

- Notation $A \cup B = A + B$ and $A \cap B = AB$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cap B') = P(A) - P(A \cap B)$
- $P(B \cap A') = P(B) - P(A \cap B)$
- $P(A' \cup B') = 1 - P(A \cap B)$
- $P(A' \cap B') = 1 - P(A \cup B)$
- $P\left(\frac{A}{B}\right) = \frac{P(A \cup B)}{P(B)}$
- $P\left(\frac{B}{A}\right) = \frac{P(A \cup B)}{P(A)}$
- If two events A and B are independent then,
  $\boldsymbol{P(A \cup B) = P(A) \times P(B)}$ [i.e., Two events can happen together without affecting other's chance]
- Two events A and B are mutually exclusive (Incompatible Event) then,
  $\boldsymbol{P(A \cap B) = 0}$ [i.e., Two events cannot happen together]
- Two events A, B are exhaustive then
  $\boldsymbol{P(A \cup B) = 1}$ [i.e., one of the events must happen]
- Two events are equally likely then,
  $\boldsymbol{P(A) = P(B)}$

Note:

- For two events, probability of at least one event happening = 1− probability of none of the events happening.
- For two events, probability of any one (exactly one i.e. only one happening) = (Happening × Not happening) + (Not happening × happening)

Random Variable and Expectation:

- If x be a random variable depends on chance, then distribution has $\sum p = 1$.
- Mathematical Expectation of X, $E(x) = \sum x.p$.
- **Properties:**
  - Mean = Expectation = $E(x) = \sum x.p$
  - $E(x^2) = \sum x^2 .p$
  - $V(x) = E[x - E(x)]^2 = E(x^2) - [E(x)]^2$
  - $E(c) = 0$, if c = constant
  - $E(a + bx) = a + b\,E(x)$
  - $E(x \pm y) = E(x) \pm E(y)$
  - For Two independent variables x and y, $E(x.y) = E(x).E(y)$
  - Expected number of a die rolled once = 3. 5
  - For uniform distribution having n equally likely cases, Expected mean = $2\,n + 1$

Theorems:

- $P(A \cup B)$ or $P(A$ or $B) = P(A) + P(B)$, where A and B are mutually exclusive.
- $P(A \cap B)$ or $P(A$ and $B) = P(A) \times P(B)$
- $P(A \cup B) = P(A) - P(B) - P(A \cap B)$
- $P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$
- For any three events A, B and C, the probability that they occur jointly is given by, $P(A \cap B \cap C) = P(A) \times P\left(\frac{B}{A}\right) \times P\left(\frac{C}{A \cap B}\right)$

Theoretical Distribution

Theoretical Distributions is divided into,

- Discrete Probability Distributions (finite or isolated value)
- Continuous Probability Distributions (any value from a given interval)

Two important discrete probability distributions are,

- Binomial Distribution (Success & failures)
- Poisson distribution (rare accidental events)

Continuous probability distributions are of the following types,

- Bell Shaped – Normal & T distribution.
- Non bell shaped – Chi square & F distribution.

Binomial Distribution:

- It is denoted by $x \sim B(n, p)$ i.e., two parameters (n, p).
- Probability mass function: $P(X = x) = f(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$ where x= 0, 1, 2…n.
- **Properties:**
  - Mean $= E(x) = np$
  - Variance $= V(x) = npq = np(1 - p)$
  - Mean > Variance.
  - If p = q = $\frac{1}{2}$, distribution is symmetric when variance is maximum and the maximum value of variance = $\left(\frac{n}{4}\right)$
  - S.D. $= npq$
  - One mode - If $[(n + 1)p]$ has a decimal part, delete decimal part.
  - Two Modes – If $[(n + 1)p]$ has no decimal part, (n + 1)p and (n + 1)p – 1 are the modes.

Poisson Distribution:

- It is denoted by X~P (m) i.e., one parameter = m where m= np (finite) if n is very large and p is very small.
- Probability mass function: $P(X = x) = f(x) = \frac{e^{-m}.m^{x}}{x!}$ where x=0, 1, 2… ∞ .
- **Properties:**
  - It is a uni-parametric distribution as it is characterised by only one parameter m.
  - Mean = Variance = m
  - S.D. $= \sqrt{m}$
  - One Mode – If m has a decimal part, delete decimal part.
  - Two Modes – if m has no decimal parts, then m and (m – 1) are the modes.
  - Mean is generally greater than Mode (i.e., Positive Skewness)

Normal Distribution:

- It is denoted by X~N (μ, σ^2)i.e., two parameters μ and σ^2.
- Probability density function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ where $-\infty < x < \infty$.
- **Properties:**
  - Normal curve is symmetric and is always bell shaped.
  - Mean = Median = Mode = μ (Uni-modal)
  - Variance = σ^2 [2nd Central Moment]
  - S.D. = σ
  - Distribution is Symmetric about μ.
  - Two points of inflexion are $(\mu - \sigma)$, $(\mu + \sigma)$.
  - $P(\mu - \sigma < x < \mu + \sigma) = 0.6827$
  - $P(\mu - 3\sigma < x < \mu + 3\sigma) = 0.9973$
  - $P(-\infty < x < \infty) = 1$

- $Q_1 = \mu - 0.675\,\sigma$
- $Q_2 = \mu + 0.675\,\sigma$
- $Q.D. = 0.675\,\sigma$
- $M.D. = 0.8\,\sigma$

- $4\,S.D. = 5\,M.D. = 6\,Q.D.$
- $x \sim N(\mu_1, \sigma_1^2)$ and $y \sim N(\mu_2, \sigma_2^2)$ then, $(X + Y) \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

Standard Normal Distribution:

- It is denoted by z~N (0,1)
- $\mu = 0$; Var (z) = 1
- S.D (z) = 1

Correlation

- **Correlation** - Degree and nature of association between two or more variables
- **Scatter Diagram** - Graphical Analysis of Correlation Coefficient of correlation (Denoted by r)
- If x and y changes in same direction, there is positive correlation and if x and y changes in opposite direction, there will be negative correlation. [Like price and demand are in negative correlation while expenditure and income are positively correlated.]
- If x and y are lying in a straight line, they are perfectly correlated.
- If there is no significant relative change between two variables, they are uncorrelated. [Like size of shoe and income]
- Spurious correlation means no real association but are related due to any other factors.
- **Properties of Coefficient of Correlation (r):**
  - $-1 \leq r \leq 1$
  - r is independent of change of origin and change of scale, but sign of r depends on relative signs of two variables.
  - r is a pure number, and it has no unit.
- **Measures of correlation**:

  (a) Scatter diagram

  (b) Karl Pearson's Product moment correlation coefficient

  (Quantitative technique – generally applied)

  (c) Spearman's rank correlation co-efficient

  (Qualitative technique)

  (d) Co-efficient of concurrent deviations (change in signs)

Karl Person's Method:

- $r = \frac{Cov(x,y)}{\sigma_x \sigma_y}$

  $= \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 (y-\bar{y})^2}}$
- $Cov(x,y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{n}$ where n = no. of pairs of data.
- Sum of product of deviations of x, y from their means $= \sum (x-\bar{x})(y-\bar{y})$
- Sum of squares of deviations of x about $\bar{x} = \sum (x-\bar{x})^2$
- If $u = ax + b$ and $v = cy + d$ and a, c are scale change, then $r(u,v) = \pm r(x,y)$,if a, c have same sign or opposite sign.

Rank Correlation Coefficient:

- For different ranks,

$$r = 1 - \frac{6 \sum d^2}{n(n^2-1)}$$

- For tied ranks,

$$r = 1 - \frac{6\left[\sum d^2 + \sum \frac{(t^3-t)}{12}\right]}{n(n^2-1)}$$

Where t represents tie length.

Concurrent Deviation:

$$r = \pm\sqrt{\pm\left(\frac{2c-m}{m}\right)}$$

- $m = n - 1$
- c = No. of identical sign changes of x, y
- If $(2c – m) > 0$, then we take the positive sign both inside and outside the radical sign.
- If $(2c – m) < 0$, then we take negative sign both inside and outside the radical sign.

Note:

(a) If $Cov\ (x, y) = 0,\ r_{xy} = 0$

(b) If $Cov\ (x, y) > 0, r_{xy} > 0$ and if $Cov\ (x, y) < 0,\ r_{xy} < 0$

(c) Product of σ_x and σ_x is always greater than covariance (x, y)

(d) If $n = 2$,

$r = 1$ if x, y are changing in the same direction.

$r = -1$ if x, y are changing in the opposite sense and

$r = 0$ if y does not change.

(e) Sum of difference of ranks is always =0 i.e., $\sum d = 0$

(f) If ranks are in the opposite sense, value of $r = -1$ and if ranks are in the same sense, $r = 1$

Regression

Regression Equation:

- Y on X (y unknown, x known), $y - \bar{y} = b_{yx}(x - \bar{x})$
- X on Y (x unknown, y known), $x - \bar{x} = b_{xy}(y - \bar{y})$

where $\bar{x}, \bar{y}$ are the means of x, y respectively.

- Regression Coefficients are b_{yx} and b_{xy}

Regression Coefficients:

$$b_{yx} = \frac{Cov(x,y)}{Variance(x)} = r\,\frac{\sigma_y}{\sigma_x}$$

$$b_{xy} = \frac{Cov(x,y)}{Variance(y)} = r\,\frac{\sigma_x}{\sigma_y}$$

Properties of Regressions:

(1) Two regression lines intersect at mean of x and mean of y i.e. $(\bar{x}, \bar{y})$ is the point of intersection.

(2) $r = \pm\sqrt{b_{xy} \times b_{yx}}$ i.e., correlation coefficient is the G.M. of two regression coefficients.

(3) A.M. of b_{xy} and $b_{yx} \geq$ r.

(4) Sign of b_{xy} and b_{yx} be same as sign of r.

(5) Product of regression coefficients can not exceed 1. i.e., $b_{xy}.b_{yx} \leq 1$

(6) Slope of Y on X regression equation = byx.

(7) If r = ± 1, two regression lines coincide. (i.e., identical) & If r = 0, two regression lines are perpendicular or at right angle

(8) If only one regression equation exists between x and y, then r = 1, if slope is positive (upward line) and r = −1, if slope is negative (downward line)

- Regression coefficients depend on change of scale but are independent of change of origin, $b_{yx} = \frac{a}{c}\, b_{uv}$ and $b_{xy} = \frac{a}{c}\, b_{vu}$ where $u = ax + b$ and $v = cy + d$.
- **Identification**: Two regression equations be $a_1 x + b_1 y = c$ and $a_2 x + b_2 y = c$, then 1st equation is x on y and 2nd equation is y on x if $|a_1 b_2| > |a_2 b_1|$, otherwise choice is reverse.
- If $y = a + bx$ is the regression equation of y on x , then b = regression coefficient of y on x .
- If $x = a' + b'y$ is the regression equation of x on y, then b' = Regression Coefficient of x on y.

Coefficient of Determination:

- Coefficient of determination $= r^2 = \frac{Total\ Variance}{Explained\ Variance}$
- Coefficient of non – determination = $1 - r^2$ = ratio of unexplained variance to the total variance .
- Standard Error of $r = \frac{1-r^2}{\sqrt{n}}$
- Probable Error (P.E.) of $r = 0.6745\ \frac{1-r^2}{\sqrt{n}}$ where r= Correlation coefficient from n pairs of sample data.
- $P.E. = \frac{2}{3} S.E$
- The limit of the correlation coefficient of the population = $r \pm PE$

Index Numbers

- Average percentage change of price level of any country over a certain period is Index Number.
- Best average for Index Number is G.M. (But in practical problems A.M. is used)
- Base year is the year whose Index or Price level = 100.
- Index Number is denoted by I_{0n} or I_{01} = (100 ± x) % where x = percentage increase or decrease of price.
- Price Relative (P_i) = $\frac{p_1}{p_0} \times 100$ and Quantity relative (Q_i) = $\frac{q_1}{q_0} \times 100$ for each item.

Price Index :

- Simple Average Method = $\frac{\sum P_n}{n}$
- Weighted Average Method = $\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$
- Simple Aggregative Method = $\frac{\sum P_n}{\sum P_0} \times 100$
- Weighted Aggregative Method = $\frac{\sum P_n w}{\sum P_0 w} \times 100$
- Cost of Living Index (C.L.I.) or Consumer Price Index (C.P.I.) = $\frac{\sum I\, w}{\sum w} \times 100$ [If information given with price and quantity (p,q), CLI = $\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$
- Purchasing Power = $\frac{1}{CLI}$

Formula Name	Price Index	Quantity Index
Laspeyres' Index	$\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$	$\frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$
Paasche's Index	$\frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$	$\frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$
Fisher's Index	$\sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$	$\sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100$
Marshall-Edgeworth Index	$\frac{\sum p_1 (q_0 + q_1)}{\sum p_0 (q_0 + q_1)} \times 100$	$\frac{\sum q_1 (p_0 + p_1)}{\sum q_0 (p_0 + p_1)} \times 100$

Note:

- Laspeyres' Price Index is weighted by base year quantities & Paasche's Price Index is weighted by current year quantities.
- Fisher's Index is G.M. of Laspeyres' and Paasche's Index Number (Fisher's Index is Ideal Index)
- Laspeyres' Price Index $= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$ is also used for Cost of Living Index or, Consumer Price Index or whole sale Price Index.

Test of Adequacy or Test of Consistency:

(1) Unit Test: Index Number Formula should be independent of the unit.

(2) Factor Reversal Test: For any formula, Price Index × Quantity Index $= \frac{\sum p_1 q_1}{\sum p_0 q_0}$ = Value Relative.

(3) Time Reversal Test: For Price Index, $P_{01} \times P_{10} = 1$ (I_{01}= Index Number of Current year, I_{10} = Index Number of Base year and formula should be used without 100)

(4) Circular Test: (Generalization of Time Reversal Test for more than 2 years) It is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base.

Note:

i) Unit Test is satisfied by all formula except simple aggregative index.

ii) Time Reversal Test is satisfied by Fisher and Marshall – Edgeworth Index Number while Factor Reversal Test is satisfied by only Fisher Index Number.

iii) Circular Test is satisfied by none except simple G.M. of Price Relative and weighted aggregative (with fixed weights)

iv) Fisher's Index is Ideal as it can satisfy all tests except Circular Test.

- Expected salary calculated for current year

  $$= \frac{Previous\ Salary}{CLI\ of\ previous\ year} \times 100$$

- Dearness Allowance (D.A.)

  = *Current year calculated salary – base year salary.*

- Real wage (deflated wage)

  $$= \frac{Current\ year\ wage}{Current\ year\ CLI} \times 100$$

- Deflated value

  $$= \frac{Current\ value}{Price\ index\ of\ current\ year}$$

- Shifted Price Index

  $$= \frac{Original\ Price\ Index}{Price\ index\ of\ new\ base\ year} \times 100$$

- Splicing means joining two or more series of Index Numbers with different base years to a simple series having same base year.
- Chain Base Index Number =

  $$\frac{Current\ link\ relative \times Previous\ year\ chain\ index}{100}$$

  where link relative $= \frac{Current\ year\ price}{Previous\ year\ price} \times 100$

THANKS!

Smart CA!