

* Derivatives

⇒ Diff. w.r.t "x"

$$i) \frac{d}{dx} (\text{constant}) = 0$$

$$ii) \frac{d}{dx} (x) = 1$$

$$iii) \frac{d}{dx} (x)^n = n \cdot x^{n-1}$$

$$iv) \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$v) \frac{d}{dx} a^x = a^x \cdot \log a$$

$$vi) \frac{d}{dx} e^x = e^x$$

* chain rule

⇒ Agar "x" ki jagah koi aur value aa raha hai toh waha chain rule apply karege.

eg i) $\frac{d}{dx} (x+2)^2 = 2(x+2) \cdot \frac{d}{dx} (x+2)$

ii) $\frac{d}{dx} \sqrt{x+1} = \frac{1}{2\sqrt{x+1}} \cdot \frac{d}{dx} (x+1)$

iii) $\frac{d}{dx} a^{2x} = a^{2x} \cdot \log a \cdot \frac{d}{dx} (2x)$

* Functions $\longrightarrow$ Limits $\longrightarrow$ Derivatives $\longrightarrow$ Integration.

* Sum based on L'Hospital Rule:-

- We use Method of L'Hospital Rule whenever the question is of Indeterminate form with N^x and D^x both becoming zero after putting the value of x .
- In L'Hospital rule, we find derivate of N^x and D^x both equal number of times till D^x is $\neq 0$.

* Sums Sequence.

(A) Algebraic type

(1) , (2) , (3) , (4) , (9)
(8) , (10) , (11) , (12) , (13)
(16) , (18)

(B) Exponential type

(24) , (26) , (27) , (39) , (23)
(25)

(C) Determinant type

(5) , (6) , (7) , (17)

(D) Infinite limits

(14) , (32) , (34) , (40) , (29) ,
(28) , (30) , (33)

(E) (Variable)^{variable} type - Finite limits

(35) , (36)

(F) (Variable)^{variable} type - Infinite limits

(21)

(G) Miscellaneous

(31)

① Algebraic type

$$1) \lim_{x \rightarrow 4} \frac{x^2 - 3x - 4}{x^2 - 2x - 8}$$

Method I:- Original Method

Solⁿ:- let $L = \lim_{x \rightarrow 4} \frac{x^2 - 3x - 4}{x^2 - 2x - 8}$

$$= \lim_{x \rightarrow 4} \frac{x^2 - 4x + 1x - 4}{x^2 - 4x + 2x - 8}$$
$$= \lim_{x \rightarrow 4} \frac{\cancel{(x-4)}(x+1)}{\cancel{(x-4)}(x+2)}$$
$$= \lim_{x \rightarrow 4} \frac{x+1}{x+2}$$
$$= \frac{4+1}{4+2} = \boxed{\frac{5}{6}} \quad (b)$$

Method II:- L'Hospital Rule.

Solⁿ:- let $L = \lim_{x \rightarrow 4}$

$$\frac{x^2 - 3x - 4}{x^2 - 2x - 8}$$

Diagram showing the differentiation of the numerator and denominator:
- The term x^2 in the numerator is circled with an arrow pointing to x^n .
- The term $3x$ in the numerator is circled with an arrow pointing to x .
- The constant term 4 in the numerator is circled with an arrow pointing to "constant".
- The term x^2 in the denominator is circled with an arrow pointing to x^n .
- The term $2x$ in the denominator is circled with an arrow pointing to x .
- The constant term 8 in the denominator is circled with an arrow pointing to "constant".

$$= \lim_{x \rightarrow 4} \left[\frac{2x^{2-1} - 3(1) - 0}{2x^{2-1} - 2(1) - 0} \right] \frac{d}{dx}(x^2) \text{ \& } \frac{d}{dx}(0^x)$$

$$= \lim_{x \rightarrow 4} \frac{2x - 3}{2x - 2}$$

$$= \frac{2(4) - 3}{2(4) - 2} = \boxed{\frac{5}{6}} \quad (b)$$

$$(2) \lim_{x \rightarrow -\frac{1}{3}} \frac{3x^2 + 7x + 2}{9x^2 - 1}$$

→ L'Hospital Rule.

Solⁿ:- Let $L = \lim_{x \rightarrow -\frac{1}{3}}$

$$\frac{3x^2 + 7x + 2}{9x^2 - 1}$$

Diagram showing the terms of the numerator and denominator with arrows indicating their power: x^2 for $3x^2$, x for $7x$, and constant for 2 in the numerator, and x^2 for $9x^2$ and constant for -1 in the denominator.

$$= \lim_{x \rightarrow -\frac{1}{3}} \left[\frac{3 \times 2x^{2-1} + 7(1) + 0}{9 \times 2x^{2-1} - 0} \right] \frac{d}{dx} (N^2) \& \frac{d}{dx} (D^2)$$

$$= \lim_{x \rightarrow -\frac{1}{3}} \frac{6x + 7}{18x}$$

$$= \frac{6x - \frac{1}{3} + 7}{18x - \frac{1}{3}}$$

$$= \frac{-2 + 7}{-6}$$

$$= \boxed{\frac{-5}{6}} \quad (b)$$

$$(3) \quad \lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 + 8x + 12}$$

Solⁿ: Let $L = \lim_{x \rightarrow -2}$

$$\frac{\overbrace{x^3 + 8}^{\text{constant}}}{\underbrace{x^2 + 8x + 12}_{\text{constant}}}$$

$\nearrow x^n$ $\nearrow \text{constant}$
 $\nwarrow x^n$ $\nwarrow x$ $\nwarrow \text{constant}$

$$= \lim_{x \rightarrow -2} \frac{3x^{3-1} + 0}{2x^{2-1} + 8(1) + 0} \left[\frac{d}{dx} (x^3) \text{ \& } \frac{d}{dx} (8) \right]$$

$$= \lim_{x \rightarrow -2} \frac{3x^2}{2x + 8}$$

$$= \frac{3(-2)^2}{2(-2) + 8}$$

$$= \frac{12}{4} = \boxed{3} \quad (c)$$

$$(4) \quad \lim_{x \rightarrow 2} \frac{x^3 - 2x - 4}{x^3 - x^2 - x - 2}$$

Solⁿ:- Let $L = \lim_{x \rightarrow 2} \frac{x^3 - 2x - 4}{x^3 - x^2 - x - 2}$

$\nearrow x^n$
 $\circledast$ $\circledast$
 $\nwarrow x^n$

$$= \lim_{x \rightarrow 2} \frac{3x^2 - 2(1) - 0}{3x^2 - 2x - 1 - 0} \left[\frac{d}{dx} (x^3) \text{ \& } \frac{d}{dx} (0^2) \right]$$

$$= \lim_{x \rightarrow 2} \frac{3x^2 - 2}{3x^2 - 2x - 1}$$

$$= \frac{3(2)^2 - 2}{3(2)^2 - 2(2) - 1}$$

$$= \boxed{\frac{10}{7}} \quad (d)$$

$$(9) \quad \lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{x-3}$$

Solⁿ: Let $L = \lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{x-3}$

$\xrightarrow{\text{constant}}$
 $\xrightarrow{\text{chain rule}}$

$$= \lim_{x \rightarrow 3} \frac{\frac{1}{2\sqrt{x+2}} \cdot \frac{d}{dx}(x+2) - 0}{1 - 0} \left[\frac{d}{dx}(x^2) \neq \frac{d}{dx}(x^2) \right]$$

⊕ chain rule

$$= \lim_{x \rightarrow 3} \frac{1}{2\sqrt{x+2}} \cdot (1 + 0) - 0$$

$$= \lim_{x \rightarrow 3} \frac{1}{2\sqrt{x+2}}$$

$$= \frac{1}{2\sqrt{3+2}} = \boxed{\frac{1}{2\sqrt{5}}} \quad (b)$$

(8)

$$\lim_{x \rightarrow 0}$$

$$\frac{1 - \sqrt{1-x}}{x}$$

Solⁿ:-

Let $L = \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x}$

$$= \lim_{x \rightarrow 0} \frac{0 - \frac{1}{2\sqrt{1-x}} \cdot \frac{d}{dx}(1-x)}{1} \left[\frac{d}{dx} (1^2) \& \frac{d}{dx} (0^2) \right]$$

Chain rule

$$= \lim_{x \rightarrow 0} \frac{-1}{2\sqrt{1-x}} (0-1)$$

$$= \lim_{x \rightarrow 0} \frac{-1 \times -1}{2\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{1-0}} = \boxed{\frac{1}{2}} \quad (d)$$

$$(10) \quad \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$$

Solⁿ:- Let $L = \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{2\sqrt{1+x}} \cdot \frac{d}{dx}(1+x) - \frac{1}{2\sqrt{1-x}} \cdot \frac{d}{dx}(1-x)}$$

chain rule chain rule

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{2\sqrt{1+x}} \times (0+1) - \frac{1}{2\sqrt{1-x}} \times (0-1)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{2\sqrt{1+x}} - \frac{1 \cdot (-1)}{2\sqrt{1-x}}}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{1-x}}}$$

$$= \frac{1}{\frac{1}{2\sqrt{1+0}} + \frac{1}{2\sqrt{1-0}}}$$

$$= \frac{1}{\frac{1}{2} + \frac{1}{2}} = \boxed{1} \quad (a)$$

$$(13) \quad \lim_{x \rightarrow 2} \frac{\sqrt{x+7} - 3}{\sqrt{x+2} - 2}$$

Solⁿ:- let $h = \lim_{x \rightarrow 2} \frac{\sqrt{x+7} - 3}{\sqrt{x+2} - 2}$

$$= \lim_{x \rightarrow 2} \frac{\frac{1}{2\sqrt{x+7}} \cdot \frac{d}{dx}(x+7) - 0}{\frac{1}{2\sqrt{x+2}} \cdot \frac{d}{dx}(x+2) - 0}$$

chain rule

chain rule

$$= \lim_{x \rightarrow 2} \frac{\frac{1}{2\sqrt{x+7}} \times (1+0)}{\frac{1}{2\sqrt{x+2}} \times (1+0)}$$

$$= \lim_{x \rightarrow 2} \left[\frac{\frac{1}{2\sqrt{x+7}} \quad a}{\frac{1}{2\sqrt{x+2}} \quad c} \right] \quad \left[\frac{a}{b} \right] \quad \left[\frac{c}{d} \right]$$

$$= \lim_{x \rightarrow 2} \frac{2\sqrt{x+2}}{2\sqrt{x+7}}$$

$$= \frac{\sqrt{2+2}}{\sqrt{2+7}} = \boxed{\frac{2}{3}} \quad (b)$$

$$(16) \quad \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$$

Sol:[∴] Let $L = \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$

$$= \lim_{x \rightarrow 2} \frac{2x - 0}{\frac{1}{2\sqrt{x+2}} \cdot \frac{d}{dx}(x+2) - \frac{1}{2\sqrt{3x-2}} \cdot \frac{d}{dx}(3x-2)} \quad \left. \vphantom{\lim_{x \rightarrow 2}} \right] \text{chain rule}$$

$$= \lim_{x \rightarrow 2} \frac{2x}{\frac{1}{2\sqrt{x+2}} \times (1+0) - \frac{1}{2\sqrt{3x-2}} \times (3-0)}$$

$$= \lim_{x \rightarrow 2} \frac{2x}{\frac{1}{2\sqrt{x+2}} - \frac{3}{2\sqrt{3x-2}}}$$

$$= \frac{2(2)}{\frac{1}{2\sqrt{2+2}} - \frac{3}{2\sqrt{3(2)-2}}}$$

$$= \frac{4}{\frac{1}{2 \times 2} - \frac{3}{2 \times 2}}$$

$$= \frac{4}{\frac{1}{4} - \frac{3}{4}}$$

$$= \frac{4}{-\frac{2}{4}} \left] \frac{a}{b} \div \frac{c}{d} = \frac{a}{c} \cdot \frac{d}{b} \right]$$

$$= \frac{4 \times 4}{-2} = \boxed{-8} \quad (d)$$

$$(18) \quad \lim_{x \rightarrow 1} \frac{x^{5/2} - 1}{x^{3/2} - 1}$$

Sol:-

Let $L = \lim_{x \rightarrow 1}$

$$\frac{x^{5/2} - 1}{x^{3/2} - 1} \rightarrow x^n$$

$$= \lim_{x \rightarrow 1} \frac{\frac{5}{2} \times x^{\frac{5}{2}-1} - 0}{\frac{3}{2} \times x^{\frac{3}{2}-1} - 0}$$

$$= \lim_{x \rightarrow 1} \frac{\frac{5}{2} \times x^{\frac{5-2}{2}}}{\frac{3}{2} \times x^{\frac{3-2}{2}}}$$

$$= \lim_{x \rightarrow 1} \frac{5x^{3/2}}{3x^{1/2}}$$

$$= \frac{5(1)^{3/2}}{3(1)^{1/2}} = \boxed{\frac{5}{3}} \quad (C)$$

$$(12) \quad \lim_{x \rightarrow 2} \frac{x^3 - 8}{\sqrt{x} - \sqrt{2}}$$

Solⁿ:- Let $L = \lim_{x \rightarrow 2} \frac{\overset{\rightarrow x^r}{\cancel{x^3}} - 8}{\sqrt{x} - \sqrt{2}}$

$$= \lim_{x \rightarrow 2} \frac{3x^2 - 0}{\frac{1}{2\sqrt{x}} - 0}$$

$$= \lim_{x \rightarrow 2} \left[\frac{3x^2}{\frac{1}{2\sqrt{x}}} \right] \left[\frac{a}{b \cdot \frac{c}{d}} \right]$$

$$= \lim_{x \rightarrow 2} 3x^2 \times 2\sqrt{x}$$

$$= 3(2)^2 \times 2\sqrt{2}$$

$$= \boxed{24\sqrt{2}} \quad (c)$$

$$(11) \quad \lim_{x \rightarrow 1} \frac{\sqrt{x-1}}{\sqrt{x^2-1} + \sqrt{x^3-1}}$$

Solⁿ: Let $L = \lim_{x \rightarrow 1} \frac{\sqrt{x-1}}{\sqrt{x^2-1} + \sqrt{x^3-1}}$

Solving by
original
method

$$\rightarrow \underline{A^2 - B^2} = (A - B)(A + B)$$

$$\rightarrow \underline{A^3 - B^3} = (A - B)(A^2 + AB + B^2)$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x-1}}{\sqrt{(x-1)(x+1)} + \sqrt{(x-1)(x^2+x+1)}}$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x-1}}{\sqrt{x-1} \times \sqrt{x+1} + \sqrt{x-1} \sqrt{x^2+x+1}}$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x-1}}{\sqrt{x-1} [\sqrt{x+1} + \sqrt{x^2+x+1}]}$$

$$= \lim_{x \rightarrow 1} \frac{1}{\sqrt{x+1} + \sqrt{x^2+x+1}}$$

$$= \frac{1}{\sqrt{1+1} + \sqrt{1^2+1+1}}$$

$$= \boxed{\frac{1}{\sqrt{2} + \sqrt{3}}} \quad (a)$$

(B) Exponential type :-

$$(24) \quad \lim_{x \rightarrow 0} \frac{3^{2x} + 2^{3x} - 2}{x}$$

Solⁿ : Let $L = \lim_{x \rightarrow 0} \frac{3^{2x} + 2^{3x} - 2}{x}$

$$= \lim_{x \rightarrow 0} \frac{3^{2x} \cdot \log 3 \cdot \frac{d}{dx}(2x) + 2^{3x} \cdot \log 2 \cdot \frac{d}{dx}(3x) - 0}{1}$$

chain rule chain rule

$$= \lim_{x \rightarrow 0} 3^{2x} \cdot \log 3 \times 2(1) + 2^{3x} \cdot \log 2 \times 3(1)$$
$$= \lim_{x \rightarrow 0} 3^{2x} \cdot 2 \times \log 3 + 2^{3x} \times 3 \times \log 2$$
$$= 3^{2(0)} \times 2 \times \log 3 + 2^{3(0)} \times 3 \times \log 2$$
$$= 2 \log 3 + 3 \log 2$$
$$= \log 3^2 + \log 2^3$$
$$= \log 9 + \log 8 \quad \textcircled{A}$$
$$= \log (9 \times 8)$$
$$= \log 72 \quad \textcircled{B}$$

$[n \cdot \log A = \log A^n]$
 $[\log A + \log B = \log (A \times B)]$

option (c)

$$(26) \quad \lim_{x \rightarrow 0} \frac{a^{5x} + a^{2x} - 2}{x}$$

Solⁿ: Let $L = \lim_{x \rightarrow 0} \frac{a^{5x} + a^{2x} - 2}{x}$

$$= \lim_{x \rightarrow 0} \frac{a^{5x} \cdot \log a \cdot \frac{d}{dx}(5x) + a^{2x} \cdot \log a \cdot \frac{d}{dx}(2x) - 0}{1}$$

chain rule *chain rule*

$$= \lim_{x \rightarrow 0} a^{5x} \cdot \log a \times 5 + a^{2x} \cdot \log a \times 2$$
$$= \lim_{x \rightarrow 0} a^{5x} \times 5 \times \log a + a^{2x} \times 2 \times \log a$$
$$= a^{5(0)} \times 5 \times \log a + a^{2(0)} \times 2 \times \log a$$
$$= 5 \log a + 2 \log a$$
$$= \log a (5+2)$$
$$= 7 \log a \quad (d)$$

$$(27) \quad \lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x}$$

Solⁿ:- Let $L = \lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x}$

$$= \lim_{x \rightarrow 0} \frac{e^{3x} \cdot \frac{d}{dx}(3x) - e^{2x} \cdot \frac{d}{dx}(2x)}{1}$$

chain rule *chain rule*

$$= \lim_{x \rightarrow 0} e^{3x} \times (3) - e^{2x} \times (2)$$
$$= e^{3(0)} \times 3 - e^{2(0)} \times 2$$
$$= 3 - 2$$
$$= \boxed{1} \quad (b)$$

$$(39) \quad \lim_{x \rightarrow 0} \frac{9^x - 3^x}{4^x - 2^x}$$

Solⁿ:- Let $L = \lim_{x \rightarrow 0} \frac{9^x - 3^x}{4^x - 2^x}$

$$= \lim_{x \rightarrow 0} \frac{9^x \cdot \log 9 - 3^x \cdot \log 3}{4^x \cdot \log 4 - 2^x \cdot \log 2}$$

$$= \frac{9^{(0)} \cdot \log 9 - 3^0 \cdot \log 3}{4^{(0)} \cdot \log 4 - 2^0 \cdot \log 2}$$

$$= \frac{\log 9 - \log 3}{\log 4 - \log 2}$$

$$= \frac{\log \left(\frac{9}{3} \right)}{\log \left(\frac{4}{2} \right)}$$

$$= \frac{\log 3}{\log 2}$$

$$= \log_2 3$$

option (c)

$$[\log A - \log B = \log \left(\frac{A}{B} \right)]$$

(A)

(B)

$$\left[\frac{\log A}{\log B} = \log_B A \right]$$

(23)

$$\lim_{x \rightarrow 0} \frac{a^{3x} - a^{2x} - a^x + 1}{x^2}$$

Sol:- Let $L = \lim_{x \rightarrow 0} \frac{a^{3x} - a^{2x} - a^x + 1}{x^2}$

$$\begin{aligned} * \quad a^{3x} - a^{2x} - a^x + 1 &= a^{x+2x} - a^{2x} - a^x + 1 \\ &= \frac{a^x \times a^{2x} - a^{2x} - a^x + 1}{1} \\ &= a^{2x} [a^x - 1] - [a^x - 1] \\ &= [a^x - 1] [a^{2x} - 1] \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{[a^x - 1] \times [a^{2x} - 1]}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{[a^x - 1]}{x} \times \frac{[a^{2x} - 1]}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^x - 1}{x} \times \lim_{x \rightarrow 0} \frac{a^{2x} - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^x \cdot \log a - 0}{1} \times \lim_{x \rightarrow 0} \frac{a^{2x} \cdot \log a \cdot \frac{d}{dx}(2x) - 0}{1}$$

$$= \lim_{x \rightarrow 0} a^x \cdot \log a \times \lim_{x \rightarrow 0} a^{2x} \cdot \log a \times 2$$

$$= a^0 \cdot \log a \times a^{2(0)} \cdot \log a \times 2$$

$$= \log a \times \log a \times 2$$

$$= 2 (\log a)^2 \quad (d)$$

$$(25) \quad \lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x^2}$$

Solⁿ: Let $L = \lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x^2}^*$

$$\begin{aligned} * \quad 10^x - 2^x - 5^x + 1 &= (2 \times 5)^x - 2^x - 5^x + 1 \\ &= \frac{2^x \times 5^x - 2^x - 5^x + 1}{1} \\ &= 2^x [5^x - 1] - 1 [5^x - 1] \\ &= [5^x - 1] [2^x - 1] \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{[5^x - 1] [2^x - 1]}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{[5^x - 1]}{x} \times \frac{[2^x - 1]}{x}$$

$$= \lim_{x \rightarrow 0} \frac{(5^x - 1)}{x} \times \lim_{x \rightarrow 0} \frac{(2^x - 1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x \cdot \log 5 - 0}{1} \times \lim_{x \rightarrow 0} \frac{2^x \cdot \log 2 - 0}{1}$$

$$= \lim_{x \rightarrow 0} 5^x \cdot \log 5 \times \lim_{x \rightarrow 0} 2^x \cdot \log 2$$

$$= 5^0 \cdot \log 5 \times 2^0 \cdot \log 2$$

$$= \log 5 \times \log 2 \quad \textcircled{C}$$

© DETERMINANT TYPE

$$(5) \quad \lim_{x \rightarrow 0} \left[2 + \frac{3}{4 + \frac{5}{x}} \right]$$

Solⁿ:- Let $L = \lim_{x \rightarrow 0} \left[2 + \frac{3}{4 + \frac{5}{x}} \right]$

$$= \lim_{x \rightarrow 0} \left[2 + \frac{3 \frac{a}{c}}{\frac{4x+5}{x} \frac{d}{e}} \right] \left(\frac{\frac{a}{c}}{\frac{b}{d} \frac{e}{f}} \right)$$

$$= \lim_{x \rightarrow 0} \left[2 + \frac{3x}{4x+5} \right]$$

$$= 2 + \frac{3(0)}{4(0)+5}$$

$$= 2 + \frac{0}{5}$$

$$= 2 \quad (b)$$

$$(6) \quad \lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{3}{x^2-3x} \right]$$

Solⁿ:- Let $L = \lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{3}{x^2-3x} \right]$

$$= \lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{3}{x(x-3)} \right]$$
$$= \lim_{x \rightarrow 3} \left[\frac{x}{x(x-3)} - \frac{3}{x(x-3)} \right]$$
$$= \lim_{x \rightarrow 3} \left[\frac{\cancel{x} - 3}{x(\cancel{x-3})} \right]$$
$$= \lim_{x \rightarrow 3} \left[\frac{1}{x} \right]$$
$$= \frac{1}{3} \quad \textcircled{d}$$

$$(7) \quad \lim_{x \rightarrow 0} \left[\frac{1}{x} \left(\frac{2x+9}{x+3} - 3 \right) \right]$$

Solⁿ: Let $L = \lim_{x \rightarrow 0} \left[\frac{1}{x} \left(\frac{2x+9}{x+3} - 3 \right) \right]$

$$= \lim_{x \rightarrow 0} \left[\frac{1}{x} \left(\frac{2x+9-3(x+3)}{x+3} \right) \right]$$
$$= \lim_{x \rightarrow 0} \left[\frac{1}{x} \left(\frac{2x + \cancel{9} - 3x - \cancel{9}}{x+3} \right) \right]$$
$$= \lim_{x \rightarrow 0} \left[\frac{1}{x} \left(\frac{-x}{x+3} \right) \right]$$
$$= \lim_{x \rightarrow 0} \left[\frac{-1}{x+3} \right]$$
$$= \frac{-1}{0+3}$$
$$= \frac{-1}{3} \quad (a)$$

$$(17) \quad \lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{1}{x^2-3x+2} \right]$$

Solⁿ: Let $L = \lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{1}{x^2-3x+2} \right]$

$$= \lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{1}{x^2-2x-1x+2} \right]$$

$$= \lim_{x \rightarrow 2} \left[\frac{1}{(x-2)} - \frac{1}{(x-2)(x-1)} \right]$$

$$= \lim_{x \rightarrow 2} \left[\frac{(x-1)}{(x-1)(x-2)} - \frac{1}{(x-2)(x-1)} \right]$$

$$= \lim_{x \rightarrow 2} \left[\frac{x-1-1}{(x-1)(x-2)} \right]$$

$$= \lim_{x \rightarrow 2} \left[\frac{\cancel{(x-2)}}{(x-1)\cancel{(x-2)}} \right]$$

$$= \lim_{x \rightarrow 2} \left[\frac{1}{x-1} \right]$$

$$= \frac{1}{2-1}$$

$$= 1 \quad \textcircled{a}$$

* FORMULA'S

$$a) \quad 1 + 2 + 3 + 4 + \dots \quad n = \boxed{\frac{n(n+1)}{2}}$$

$$\text{i.e.} \quad \sum_{r=1}^n r = \frac{n(n+1)}{2}$$

$$b) \quad 1^2 + 2^2 + 3^2 + 4^2 + \dots \quad n^2 = \boxed{\frac{n(n+1)(2n+1)}{6}}$$

$$\text{i.e.} \quad \sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$c) \quad 1^3 + 2^3 + 3^3 + 4^3 + \dots \quad n^3 = \boxed{\frac{n^2(n+1)^2}{4}}$$

$$\text{i.e.} \quad \sum_{r=1}^n r^3 = \frac{n^2(n+1)^2}{4}$$

$$d) \quad 1 + 1 + 1 + 1 \dots + 1 \quad \text{n times} = \boxed{n}$$

① SUMS BASED ON INFINITE LIMITS

(14) $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{(2n+3)(n-4)}$ } $\frac{\text{Polynomial}}{\text{Polynomial}}$ type.

Solⁿ: Let $L = \lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{(2n+3)(n-4)}$

$= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2} \quad \begin{matrix} a \\ b \end{matrix}}{(2n+3)(n-4) \quad c} \quad \left[\begin{matrix} a \\ b \\ c \\ d \end{matrix} \right]$

$= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2(2n+3)(n-4)}$

$= \lim_{n \rightarrow \infty} \frac{n(n+1)}{(4n+6)(n-4)}$

$= \lim_{n \rightarrow \infty} \frac{n(n+1)}{4n^2 - 16n + 6n - 24}$

$= \lim_{n \rightarrow \infty} \frac{n^2 + n}{4n^2 - 10n - 24} \quad \left[\begin{matrix} \rightarrow \text{Polynomial} \\ \rightarrow \text{Polynomial} \end{matrix} \right]$

Dividing n^2 and n^2 by n^2

$= \lim_{n \rightarrow \infty} \frac{\frac{n^2}{n^2} + \frac{n}{n^2}}{\frac{4n^2}{n^2} - \frac{10n}{n^2} - \frac{24}{n^2}}$

Compare Degree.
If $n^x = n^y$
Degree = Degree

then
Ans = Ratio of
co-efficient of
highest degree

$\frac{1}{4}$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \\ &= \lim_{\frac{1}{n} \rightarrow 0} \end{aligned}$$

$$\frac{1 + \frac{1}{n}}{4 - 10\left(\frac{1}{n}\right) - 24\left(\frac{1}{n}\right)^2}$$

$$\frac{1 + \left(\frac{1}{n}\right)}{4 - 10\left(\frac{1}{n}\right) - 24\left(\frac{1}{n}\right)^2}$$

$$= \frac{1 + 0}{4 - 10(0) - 24(0)}$$

$$= \frac{1}{4} \quad \textcircled{b}$$

$$(32) \quad \lim_{n \rightarrow \infty} \frac{\varepsilon n^2}{n^3 + 1}$$

Solⁿ:- Let $L = \lim_{n \rightarrow \infty} \frac{\varepsilon n^2}{n^3 + 1}$

$$= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6} \begin{matrix} a \\ b \\ c \end{matrix}}{n^3 + 1} \left[\begin{matrix} \frac{a}{b} \\ \frac{c}{d} \end{matrix} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6(n^3 + 1)}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^2 + n)(2n+1)}{6n^3 + 6}$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + n^2 + 2n^2 + n}{6n^3 + 6}$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^2 + n}{6n^3 + 6} \begin{matrix} \rightarrow \text{Polynomial} \\ \rightarrow \text{Polynomial} \end{matrix} \left. \begin{matrix} \text{Compare Degree} \\ \text{If } N^{\text{th}} \text{ Degree} = D^{\text{th}} \text{ Degree} \end{matrix} \right\}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3} \quad (b)$$

then
Ans = Ratio of
co-efficient of
highest degree.

$$\downarrow$$

$$\left(\frac{2}{6} \right)$$

$$(34) \quad \lim_{n \rightarrow \infty} \frac{\sum (n+1)}{2n^2 + 5n - 6}$$

Solⁿ:- Let $L = \lim_{n \rightarrow \infty} \frac{\sum (n+1)}{2n^2 + 5n - 6}$

$$= \lim_{n \rightarrow \infty} \frac{\sum n + \sum 1}{2n^2 + 5n - 6}$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2} + n}{2n^2 + 5n - 6}$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1) + 2n}{2}}{2n^2 + 5n - 6} \left. \begin{array}{l} a \\ b \\ c \end{array} \right\} \frac{a}{b} = \frac{c}{d}$$

$$= \lim_{n \rightarrow \infty} \frac{n(n+1) + 2n}{2(2n^2 + 5n - 6)}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2 + n + 2n}{4n^2 + 10n - 12} \rightarrow \frac{\text{Polynomial}}{\text{Polynomial}} \left. \begin{array}{l} \text{Compare Degree} \\ \text{If } n^{\text{th}} \text{ Degree} = n^{\text{th}} \text{ Degree} \end{array} \right\}$$

then

Ans = Ratio of co-efficient of highest degree.

$$= \frac{1}{4} \quad (C)$$

$$\frac{1}{4}$$

$$(40) \quad \lim_{n \rightarrow \infty} \frac{(n+2) + (n+4) + (n+6) + \dots + (n+2n)}{n^2}$$

$$\underline{\text{Sol}^n} \therefore \text{Let } L = \lim_{n \rightarrow \infty} \frac{(\overset{*}{n} + \overset{*}{2}) + (\overset{*}{n} + \overset{*}{4}) + (\overset{*}{n} + \overset{*}{6}) + \dots + (\overset{*}{n} + \overset{*}{2n})}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(\overset{*}{n} + \overset{*}{n} + \overset{*}{n} + \dots + \overset{*}{n}) + (\overset{*}{2} + \overset{*}{4} + \overset{*}{6} + \dots + \overset{*}{2n})}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{n(1+1+1+\dots+1) + 2(1+2+3+\dots+n)}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{n(n) + \cancel{2} \left[\frac{n(n+1)}{2} \right]}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2 + n^2 + n}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{2n^2 + n}{n^2} \rightarrow \text{Polynomial} \quad \left. \begin{array}{l} \text{Compare Degree.} \\ \text{If } n^r \text{ degree} = 0^r \text{ degree} \end{array} \right\}$$

$$= 2 \quad (b)$$

then

Ans = Ratio of
co-efficient of
highest degree.

$$\downarrow$$

$$\left(\frac{2}{1} \right)$$

$$(29) \quad \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n-6} - \sqrt{3n^2-6}}{2n+5}$$

Solⁿ: Let $L = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n-6} - \sqrt{3n^2-6}}{2n+5}$

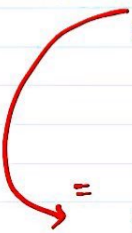
Dividing N^{∞} and D^{∞} by $\textcircled{n} \rightarrow$ highest power

$$= \lim_{n \rightarrow \infty} \frac{\frac{\sqrt{n^2+n-6}}{n} - \frac{\sqrt{3n^2-6}}{n}}{\frac{2n}{n} + \frac{5}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{n^2+n-6}{n^2}} - \sqrt{\frac{3n^2-6}{n^2}}}{2 + 5\left(\frac{1}{n}\right)}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{n^2}{n^2} + \frac{n}{n^2} - \frac{6}{n^2}} - \sqrt{\frac{3n^2}{n^2} - \frac{6}{n^2}}}{2 + 5\left(\frac{1}{n}\right)}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \left(\frac{1}{n}\right) - 6\left(\frac{1}{n}\right)^2} - \sqrt{3 - 6\left(\frac{1}{n}\right)^2}}{2 + 5\left(\frac{1}{n}\right)}$$


$$= \lim_{\frac{1}{n} \rightarrow 0} \frac{\sqrt{1 + \left(\frac{1}{n}\right) - 6\left(\frac{1}{n}\right)^2} - \sqrt{3 - 6\left(\frac{1}{n}\right)^2}}{2 + 5\left(\frac{1}{n}\right)}$$

$$= \frac{\sqrt{1 + 0 - 6(0)^2} - \sqrt{3 - 6(0)^2}}{2 + 5(0)}$$

$$= \frac{1 - \sqrt{3}}{2} \quad \textcircled{b}$$

$$(28) \quad \lim_{x \rightarrow \infty} \sqrt{x} (\sqrt{x+2} - \sqrt{x})$$

Sol:- Let $L = \lim_{x \rightarrow \infty} \sqrt{x} (\sqrt{x+2} - \sqrt{x})$

Rationalizing the N°

$$= \lim_{x \rightarrow \infty} \sqrt{x} \left(\sqrt{x+2} - \sqrt{x} \right) \times \frac{\left(\sqrt{x+2} + \sqrt{x} \right)}{\left(\sqrt{x+2} + \sqrt{x} \right)}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{x} \left[(\sqrt{x+2})^2 - (\sqrt{x})^2 \right]}{\sqrt{x+2} + \sqrt{x}} \quad [A-B][A+B] = A^2 - B^2$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{x} [x+2 - x]}{\sqrt{x+2} + \sqrt{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{2\sqrt{x}}{\sqrt{x+2} + \sqrt{x}}$$

Dividing N^x and O^x by $\sqrt{x} \rightarrow$ highest power

$$= \lim_{x \rightarrow \infty} \frac{\frac{2\sqrt{x}}{\sqrt{x}}}{\frac{\sqrt{x+2}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}}}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{\sqrt{\frac{x+2}{x}} + 1}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{\sqrt{\frac{x}{x} + \frac{2}{x}} + 1}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{\sqrt{1 + 2\left(\frac{1}{x}\right)} + 1}$$

$$\begin{aligned} & \searrow \\ &= \lim_{\frac{1}{x} \rightarrow 0} \frac{2}{\sqrt{1 + 2\left(\frac{1}{x}\right)} + 1} \end{aligned}$$

$$= \frac{2}{\sqrt{1 + 2(0)} + 1}$$

$$= \frac{2}{1 + 1}$$

$$= \frac{2}{2}$$

$$= 1 \quad \textcircled{C}$$

$$(3b) \quad \lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$$

Sol $\therefore$ Let $L = \lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$

Rationalizing the N°

$$= \lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x) \times \frac{(\sqrt{x^2 + 5x} + x)}{(\sqrt{x^2 + 5x} + x)}$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 5x})^2 - (x)^2}{\sqrt{x^2 + 5x} + x} \quad (A-B)(A+B) = A^2 - B^2$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + 5x - x^2}{\sqrt{x^2 + 5x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{5x}{\sqrt{x^2 + 5x} + x}$$

Dividing N^2 and D^2 by $(x) \rightarrow$ highest power

$$= \lim_{x \rightarrow \infty} \frac{\frac{5x}{x}}{\frac{\sqrt{x^2+5x}}{x} + \frac{x}{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{5}{\frac{\sqrt{x^2+5x}}{x^2} + 1}$$

$$= \lim_{x \rightarrow \infty} \frac{5}{\sqrt{\frac{x^2}{x^2} + \frac{5x}{x^2}} + 1}$$

$$= \lim_{x \rightarrow \infty} \frac{5}{\sqrt{1 + 5\left(\frac{1}{x}\right)} + 1}$$

$$\rightarrow = \lim_{\frac{1}{x} \rightarrow 0} \frac{5}{\sqrt{1 + 5\left(\frac{1}{x}\right)} + 1}$$

$$= \frac{5}{\sqrt{1 + 5(0)} + 1}$$

$$= \frac{5}{1+1}$$

$$= \frac{5}{2} \quad (d)$$

(33)

**

$$\lim_{n \rightarrow \infty}$$

$$\frac{\frac{\varepsilon n}{\sqrt{n^2-1}} \times \sqrt{n^2+3}}$$

Solⁿ:- Let $L = \lim_{n \rightarrow \infty} \frac{\varepsilon n}{\sqrt{n^2-1} \cdot \sqrt{n^2+3}}$

$$= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2} \begin{matrix} a \\ b \end{matrix}}{\sqrt{n^2-1} \cdot \sqrt{n^2+3} \begin{matrix} c \end{matrix}} \left] \begin{matrix} \frac{a}{b} \\ \frac{c}{d} \end{matrix} \right.$$

$$= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2(\sqrt{n^2-1}) \cdot (\sqrt{n^2+3})}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2+n}{2(\sqrt{n^2-1}) \cdot (\sqrt{n^2+3})}$$

Dividing N^x and D^x by $(n^2) \rightarrow$ highest power

$$= \lim_{n \rightarrow \infty} \frac{\frac{n^2}{n^2} + \frac{n}{n^2}}{\frac{2(\sqrt{n^2-1}) \cdot (\sqrt{n^2+3})}{n^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \frac{(\sqrt{n^2-1})}{n} \times \frac{\sqrt{n^2+3}}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \sqrt{\frac{n^2-1}{n^2}} \times \sqrt{\frac{n^2+3}{n^2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \sqrt{\frac{n^2}{n^2} - \frac{1}{n^2}} \times \sqrt{\frac{n^2}{n^2} + \frac{3}{n^2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \sqrt{1 - \left(\frac{1}{n}\right)^2} \times \sqrt{1 + 3\left(\frac{1}{n}\right)^2}}$$

$$\stackrel{=}{=} \lim_{\frac{1}{n} \rightarrow 0} \frac{1 + \left(\frac{1}{n}\right)}{2 \times \sqrt{1 - \left(\frac{1}{n}\right)^2} \times \sqrt{1 + 3\left(\frac{1}{n}\right)^2}}$$

$$= \frac{1 + (0)}{2 \times \sqrt{1 - 0^2} \times \sqrt{1 + 3(0)^2}}$$

$$= \frac{1}{2 \times 1 \times 1}$$

$$= \frac{1}{2} \quad \textcircled{a}$$

(E) (VARIABLE)^{variable} TYPE - FINITE LIMITS

* FORMULA'S

$$1) \lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

$$2) \lim_{x \rightarrow 0} (1+mx)^{k/x} = e^{m \times k}$$

$$3) \lim_{x \rightarrow 0} \left(\frac{1+mx}{1+nx} \right)^{1/x} = e^{m-n}$$

$$(35) \quad \lim_{x \rightarrow 0} \left(\frac{3-x}{3+x} \right)^{1/x}$$

Solⁿ:- Let $L = \lim_{x \rightarrow 0} \left(\frac{3-x}{3+x} \right)^{1/x}$

Dividing N^x and D^x by 3

$$= \lim_{x \rightarrow 0} \left(\frac{\frac{3}{3} - \frac{x}{3}}{\frac{3}{3} + \frac{x}{3}} \right)^{1/x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 + \overset{m}{\left(-\frac{1}{3} \right) x}}{1 + \underset{n}{\left(\frac{1}{3} \right) x}} \right)^{1/x}$$

$$\rightarrow \lim_{x \rightarrow 0} \left(\frac{1 + mx}{1 + nx} \right)^{1/x} = e^{m-n}$$

$$= e^{-\frac{1}{3} - \frac{1}{3}}$$

$$= e^{-2/3} \quad (b)$$

$$(36) \quad \lim_{x \rightarrow 0} \left(\frac{3+2x}{3-x} \right)^{1/x}$$

Sol:- Let $L = \lim_{x \rightarrow 0} \left(\frac{3+2x}{3-x} \right)^{1/x}$

Dividing N^x and D^x by 3

$$= \lim_{x \rightarrow 0} \left(\frac{\frac{3}{3} + \frac{2x}{3}}{\frac{3}{3} - \frac{x}{3}} \right)^{1/x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 + \overset{m}{\left(\frac{2}{3} \right) x}}{1 + \underset{n}{\left(-\frac{1}{3} \right) x}} \right)^{1/x}$$

Now, $\lim_{x \rightarrow 0} \left(\frac{1+mx}{1+nx} \right)^{1/x} = e^{m-n}$

$$= e^{\frac{2}{3} - (-\frac{1}{3})}$$

$$= e^{2/3 + 1/3}$$

$$= e^{3/3}$$

$$= e \quad (C)$$

(F) (VARIABLE)^{variable} TYPE - INFINITE LIMITS

* FORMULA'S

$$1) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$2) \lim_{x \rightarrow \infty} \left(1 + \frac{m}{x}\right)^{k \cdot x} = e^{m \times k}$$

$$3) \lim_{x \rightarrow \infty} \left[\frac{1 + m \left(\frac{1}{x}\right)}{1 + n \left(\frac{1}{x}\right)} \right]^x = e^{m-n}$$

$$(21) \quad \lim_{n \rightarrow \infty} \left(\frac{n-2}{n+3} \right)^n$$

Solⁿ: $\therefore$ Let $L = \lim_{n \rightarrow \infty} \left(\frac{n-2}{n+3} \right)^n$

Dividing N^x and D^x by " n "

$$= \lim_{n \rightarrow \infty} \left[\frac{\frac{n}{n} - \frac{2}{n}}{\frac{n}{n} + \frac{3}{n}} \right]^n$$

$$= \lim_{n \rightarrow \infty} \left[\frac{1 + \overset{m}{(-2)} \left(\frac{1}{n} \right)}{1 + 3 \left(\frac{1}{n} \right)} \right]^n$$

$$\begin{aligned} \therefore \lim_{x \rightarrow \infty} \left[\frac{1 + m \left(\frac{1}{x} \right)}{1 + n \left(\frac{1}{x} \right)} \right]^x &= e^{m-n} \\ &= e^{-2-3} \\ &= e^{-5} \\ &= \frac{1}{e^5} \quad \textcircled{b} \end{aligned}$$

(4) MISCELLANEOUS

$$(31) \quad \lim_{x \rightarrow 0} \frac{7x + |x|}{2x - |x|}$$

Solⁿ:- Let $L = \lim_{x \rightarrow 0} \frac{7x + |x|}{2x - |x|}$

* Note:- $|x| = x$, for $x > 0$
 $|x| = -x$, for $x < 0$

Case I:- For $|x| = x$, when $x > 0$

$$L = \lim_{x \rightarrow 0} \frac{7x + |x|}{2x - |x|}$$

$$= \lim_{x \rightarrow 0} \frac{7x + x}{2x - x}$$

$$= \lim_{x \rightarrow 0} \frac{8x}{x}$$

$$= \boxed{8}$$

Case II:- For $|x| = -x$, When $x < 0$

$$L = \lim_{x \rightarrow 0} \frac{7x + |x|}{2x - |x|}$$

$$= \lim_{x \rightarrow 0} \frac{7x + (-x)}{2x - (-x)}$$

$$= \lim_{x \rightarrow 0} \frac{6x}{3x}$$

$$= \boxed{2}$$

Now $\lim_{x \rightarrow 0^+} f(x) = 8$ and $\lim_{x \rightarrow 0^-} f(x) = 2$

$$\text{As } \lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$$

$\therefore$ Limit does not exist. (d)

* CONTINUITY

→ Concept:- A given function $f(x)$ is said to be continuous if and only if its graph is continuous.

But since, It is difficult to draw the graph of each and every function hence for checking the continuity of a function we follow the following methods.

* METHODS:-

(A) Denominator = 0, that value of "x" is discontinuity

(B)
$$\left. \begin{array}{l} \lim_{\substack{x \rightarrow a \\ x \neq a}} f(x) = f(a) \\ \text{Limits} = \text{Functions} \end{array} \right\} \begin{array}{l} \Rightarrow \\ \text{L.H.S} = \text{R.H.S} \text{ then continuous} \\ \text{L.H.S} \neq \text{R.H.S} \text{ then discontinuous} \end{array}$$

(C)
$$\lim_{\substack{x \rightarrow a^- \\ x < a}} f(x) = \lim_{\substack{x \rightarrow a^+ \\ x > a}} f(x) = f(a)_{x=a}$$

(A) SUMS BASED ON 1st METHOD

i.e. check Denominator = 0

→ If Denominator = 0, $f(x) \rightarrow$ Discontinuous.

→ If Denominator $\neq 0$, $f(x) \rightarrow$ continuous.

(41) Given:- $f(x) = \frac{1}{x-2}$

To find:- Discontinuous at $x = ??$

Solⁿ:- $f(x) = \frac{1}{x-2}$

Let $x-2=0$

$x=2$

If $x=2$ then 0^x would become "0"

If $0^x = 0$ then $f(x)$ is discontinuous at $x=2$.

(C)

(42) (+) (43)

Given:-

$$f(x) = \frac{x^2 - 6x + 5}{x^2 - 3x + 2}$$

To find:- (42) Discontinuous at which value(s)
of x .

(43) Continuous at which value(s)
of x .

Solⁿ:-

$$f(x) = \frac{x^2 - 6x + 5}{x^2 - 3x + 2}$$

$$= \frac{x^2 - 5x - 1x + 5}{x^2 - 2x - 1x + 2}$$

$$= \frac{(x-5)(x-1)}{(x-2)(x-1)}$$

For (42)

$f(x)$ is discontinuous only when Denominator = 0

$$\therefore (x-2)(x-1) = 0$$

$$x-2 = 0 \quad \text{OR} \quad x-1 = 0$$

$$\therefore \boxed{x=2} \quad \text{OR} \quad \boxed{x=1}$$

$\therefore$ For $x=2$ OR $x=1$, $f(x)$ will be discontinuous.
(a)

For (43)

$f(x)$ is continuous only when Denominator $\neq 0$

$$\therefore (x-2)(x-1) \neq 0$$

$$x-2 \neq 0 \quad \text{OR} \quad x-1 \neq 0$$

$$\therefore \boxed{x \neq 2} \quad \text{OR} \quad \boxed{x \neq 1}$$

$\therefore f(x)$ is continuous for all values of "x"
except $x=2$ OR $x=1$
(c)

⑥ SUMS BASED ON 2nd METHOD

$$\Rightarrow \lim_{\substack{x \rightarrow a \\ x \neq a}} f(x) = f(x)_{x=a}$$

i.e. Limits = function then $f(x)$ is continuous.

(44) Given: i) $f(x) = \frac{\sqrt{x} - 2}{x - 4}$, $x \neq 4$ Limits

$f(x) = \frac{1}{k}$, $x = 4$ Function

ii) $f(x)$ is continuous at $x = 4$
 $\rightarrow$ Limits = function

To find:- $k = ??$

Solⁿ:- As the given function is continuous
at $x=4$

$$\rightarrow \text{Limits} = \text{function}$$

$$\therefore \lim_{\substack{x \rightarrow 4 \\ x \neq 4}} f(x) = f(x)_{x=4}$$

$$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} = f(4)$$

$$\lim_{x \rightarrow 4} \frac{\frac{1}{2\sqrt{x}} - 0}{1 - 0} = \frac{1}{k}$$

$$\lim_{x \rightarrow 4} \frac{1}{2\sqrt{x}} = \frac{1}{k}$$

$$\frac{1}{2\sqrt{4}} = \frac{1}{k}$$

$$\frac{1}{4} = \frac{1}{k}$$

$$\therefore \boxed{k=4}$$

(c)

(46) Given:- i) $f(x) = \frac{x^2 - 16}{x - 4}$, $x \neq 4$ Limits
 $f(x) = 2x^2$, $x = 4$ Function

ii) $f(x)$ is continuous at $x = 4$
 $\rightarrow$ Limits = Function

To find:- $k = ??$

Sol:- As the given function is continuous at $x = 4$

$\rightarrow$ Limits = Function

$$\therefore \lim_{\substack{x \rightarrow 4 \\ x \neq 4}} f(x) = f(4)_{x=4}$$

$$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4} = f(4)$$

$$\lim_{x \rightarrow 4} \frac{2x - 0}{1 - 0} = 2k^2$$

$$\lim_{x \rightarrow 4} 2x = 2k^2$$

$$\therefore 2(4) = 2k^2$$

$$\frac{8}{2} = k^2$$

$$k^2 = 4$$

$$\therefore k = \pm \sqrt{4}$$

$$\boxed{k = \pm 2}$$

(d)

© SUMS BASED ON 3rd METHOD

$$\lim_{\substack{x \rightarrow a^- \\ x < a}} f(x) = \lim_{\substack{x \rightarrow a^+ \\ x > a}} f(x) = f(x)_{x=a}$$

(47)
**
*

Given:- i) $f(x) = ax^2$

$$f(x) = x$$

$$f(x) = bx^3$$

$$\begin{aligned} 0 \leq x < 1 &\rightarrow x \rightarrow 1^- \\ 1 \leq x < 2 &\rightarrow x \rightarrow 1^+, x \rightarrow 2^- \\ 2 \leq x < 3 &\rightarrow x \rightarrow 2^+ \end{aligned}$$

ii) $f(x)$ is continuous at $x=1$

iii) $f(x)$ is continuous at $x=2$

$$\lim_{x \rightarrow a^-} = \lim_{x \rightarrow a^+}$$

To find:- $a = ??$, $b = ??$

Solⁿ:-

for $x=1$

$\therefore f(x)$ is continuous at $x=1$

$$\therefore \lim_{\substack{x \rightarrow 1^- \\ x < 1}} f(x) = \lim_{\substack{x \rightarrow 1^+ \\ x > 1}} f(x)$$

$$\lim_{x \rightarrow 1} ax^2 = \lim_{x \rightarrow 1} x$$

$$a(1)^2 = 1$$

$$\therefore \boxed{a=1}$$

for $x=2$

$\therefore f(x)$ is continuous at $x=2$

$$\therefore \lim_{\substack{x \rightarrow 2^- \\ x < 2}} f(x) = \lim_{\substack{x \rightarrow 2^+ \\ x > 2}} f(x)$$

$$\lim_{x \rightarrow 2} x = \lim_{x \rightarrow 2} bx^3$$

$$2 = b(2)^3$$

$$2 = 8b$$

$$b = \frac{2}{8}$$

$$\therefore \boxed{b = \frac{1}{4}}$$

$$\therefore a = 1 \quad \text{and} \quad b = \frac{1}{4} \quad \textcircled{a}$$

(45) Given:- $f(x) = a - \frac{x^3}{a^2}$, $x \leq a \rightarrow \lim_{x \rightarrow a^-}$
 $f(x) = a^2 - x^2$, $x > a \rightarrow \lim_{x \rightarrow a^+}$

To check:- $f(x)$ is continuous at $x=a$

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

Solⁿ:- i) $\lim_{\substack{x \rightarrow a^- \\ x < a}} f(x) = \lim_{x \rightarrow a^-} a - \frac{x^3}{a^2}$

$$= a - \frac{a^3}{a^2}$$

$$= a - a$$

$$= 0.$$

$$\therefore \boxed{\lim_{x \rightarrow a^-} f(x) = 0}$$

$$\begin{aligned}
 \text{ii) } \lim_{\substack{x \rightarrow a^+ \\ x > a}} f(x) &= \lim_{x \rightarrow a^+} a^2 - x^2 \\
 &= a^2 - a^2 \\
 &= 0
 \end{aligned}$$

$$\therefore \boxed{\lim_{x \rightarrow a^+} f(x) = 0}$$

$$\text{As } \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

$\therefore f(x)$ is continuous at $x=a$
 (a)