

Drawback \rightarrow Effected by extreme values

$$\bar{x} = \frac{\sum x}{n}$$

Measures of Central Tendency

AM $\rightarrow \bar{x}$ both x & f given

$$\bar{x} = \frac{\sum fx}{\sum f(n)}$$

\rightarrow Best, Accurate, reliable method

Note \rightarrow If class lengths are equal or unequal the formula is same.

Find the AM of 5, 10, 15, 20, ... 75?

$$\bar{x} = \frac{\text{First term} + \text{last term}}{2}$$

Note - If the given data is in AP

AP is difference is constant

Find the AP of 1, 2, 3, ... n?

$$\bar{x} = \frac{\sum x}{n} = \frac{1+2+3+\dots+n}{n}$$

\therefore Sum of natural numbers

$$= \frac{n+1}{2}$$

Properties of AM \rightarrow

1) If data values are constant the AM values constant
for ex - 20, 20, ... ans = 20 //

2) AM shows linear relationship. for ex - $y = 5x + 3$
find \bar{y} , $\bar{x} = 10$ $y = a - x + b$

3) All averages ^{mean} (\bar{x}), median (M), mode (Z), GM, HM
shows linear relation property.

for ex - $5x + 3y = 10$ $\bar{x} = 5$ find y ?
Ans = -5 //

4) AM can be pooled (combined) $\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$
in case $n_1 = n_2$, $\bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}$

5) Sum of deviation from \bar{x} is always 0
($\sum x - \bar{x} = 0$)

6) If frequency of variable increases or decrease
by same variable of AM will be remain
constant.

7)

find \rightarrow
 $\Rightarrow \sum fx$
 \downarrow
If f is high then opposite class frequency in that distribution there will be ans

GM[√]

Application -

1. It is less affected by extreme values
2. Extensively used in the construction of index nos
3. GM cannot be calculated of any variable assumes 0 or negative value.

4. Less affected by sampling fluctuating compare to Z & M.

5. GM is most different avg to calⁿ & underst-anding (It involve log)

6. GM can be used in accounts to calculate depⁿ in WDV method.

7. It is used to calculate growth rate

Prop -

8. If data values are constant 'k' then $GM = k$

9. GM shows linear relation

10. GM can be combined

$$G_{12} = (G_1^{n_1} \times G_2^{n_2})^{\frac{1}{n_1+n_2}}$$

$GM = 0$, when data values are zero.

GM is only for +ve data values.

$$GM = \sqrt[n]{\text{Product of values}} = \sqrt[n]{x_1 \times x_2 \times \dots \times x_n}$$

When there is 0 or -ve we should not calculate GM.

GM

Ex - $n_1 = 3$ $n_2 = 5$ $GM = (4^3 \times 8^5)$

$G_1 = 4$ $G_2 = 3$

$$= \sqrt[8]{64 + 243}$$

$$= \sqrt[8]{15552} \rightarrow \text{Pinto}$$

$$= 1.82$$

* $GM = 0$, when data values are zero.

* GM is only for +ve data values.

When there is 0 or -ve we should not calculate GM .

$$GM = \sqrt{\text{Product of values}}$$

$$GM = \sqrt{x_1^f \times x_2^f \times x_3^f}$$

HM

$$\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$$

If in case

HM cannot be calculated when if any variable assumes 0

HM can be calculated for +ve or -ve data

If all observations are same and is same

$$HM = \frac{\sum f}{\sum \frac{f}{x}} \rightarrow \text{Grouped frequency}$$

Application of HM

HM can be combined (pooled HM)

$$H_{12} = \frac{n_1 + n_2}{\frac{n_1}{h_1} + \frac{n_2}{h_2}}$$

for equal data value

$$n_1 = n_2$$

$$H_{12} = \frac{2H_1 H_2}{H_1 + H_2}$$

Relation

$$AM \geq GM \geq HM$$

$AM = GM = HM \rightarrow$ when data is constant

$AM > GM > HM \rightarrow$ When data is distinct
relation

$$\text{Mathematical}^m = GM = \sqrt{AM \times HM}$$

Median \rightarrow Median is second best average.

\rightarrow It can be calculated for open end as well as close end frequency.

- Quartile
 - Decile
 - Percentile
- } Fractile family.

Median is dependent of scale ~~of~~ origin

Ungrouped \rightarrow step 1 \rightarrow Make ascending order
odd \rightarrow middle value is median
even \rightarrow middle two values
 $\neq 0$ divide by 2

$$\text{Grouped } X \left\{ \begin{array}{l} l + \left(\frac{N - f}{2} \right) \cdot \frac{1}{fm} \\ 1 + \left(\frac{n}{2} - f \right) \cdot \frac{1}{fm} \end{array} \right.$$

Total freq $\leftarrow M = \frac{N}{2}$

or
=

$$m = l_1 + \left(\frac{m - cf}{f} \right) (l_2 - l_1)$$

Mode → It is most repeated value.

↳ It is least used method.

↳ It is ill defined.

↳ It can be calculated for open end freq.

↳ It is dependent on change of origin & scale. It is not affected by extreme values.

↳ It is affected by sampling fluctuation.

X → 5 4 8 30 18 5

f → 5 10 20 90 18 90

Ans → 30, 5

Mode can be located graphically using histogram
or area diagram or freq diagram.

Unimodal → one number repeated multiple times

bi → two numbers repeated multiple times

$$m_0 = l_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) (l_2 - l_1)$$

upper f_0

highest freq is f_1

Relation b/w mean, median & mode

Mean = Median = mode. \rightarrow Normal (symmetrical)

Mean \leq Median \leq mode \rightarrow left symmetry

mean \geq median \geq mode \rightarrow right symmetry

\hookrightarrow If it is not $=, \leq, \geq \rightarrow$ Asymmetrical.

$$\text{Mode} = 3 \text{ median} - 2 \text{ mean}$$

$$\text{mean} - \text{mode} = 3 (\text{mean} - \text{median})$$

\hookrightarrow logic \rightarrow Memo = 3 memo

$$Q_1 = 25\% \quad Q_2 = 50 \quad Q_3 = 75 \quad Q_4 = 100$$

\downarrow lower quartile $Q_2 \rightarrow$ median of data. upper quartile

Measures of Dispersion

Dispersion \rightarrow Deviation or Scatterness of values from their central values (mean & median)

- \rightarrow Variability in uniformity.
- \rightarrow If values are equal then MOD is always zero.
- \rightarrow All MOD are independent of change of origin but dependent on change of scale.

- Has 2 types

1) Absolute Dispersion

Relative Dispersion

same units

Expressed as ratios
or percentage &
unit free.

- Related to distinct
itself Range, QD,
MD, SD

- To compare variability
blw 2/more series.
- To check relative
accuracy of data.

Range - • Quickest method. Easy to calⁿ. less
time consuming.

- It does not depend in all observation
- Most unreliable.
- Unaffected by presence of frequency.
- Independence of change of origin, dependent on change of scale.

$$\text{Range} = HV - LV$$

$$\text{Co. of Range} = \frac{HV - LV}{HV + LV} \times 100$$

- It is not a accurate measure.
- It cannot be calⁿ for open end.

Quartile Deviation -

- Half of the Range b/w the quartiles.
- Based on upper & lower quartile & covers 50% of observation.
- Does not depend on all observation.
- QD is the best & only MOD for open end.
- QD is not accurate measure.

$$QD = \frac{Q_3 - Q_1}{2} \quad \text{Coeff. of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

For grouped freq dist

$$Q_1 = L_1 + \frac{m - (f \times (L_2 - L_1))}{f} \quad Q_2 = m = \frac{N}{4} \quad Q_3 = \frac{n - 3N}{4}$$

Mean Deviation -

- MD better than Range & QD as MOD as it depends on 100+ data.
- MD is not accurate MOD (becoz of module) - as it lesser absolute deviation.
- Cannot be used for open-end dist.

$$MD = \frac{\sum |x - \bar{x}|}{n} \quad \bar{x} = \frac{\sum x}{n}$$

$$\text{Coef} = \frac{MD}{\text{mean}} \times 100$$

Standard Deviation -

- Best, commonly used, Accurate, Reliable method
- Unduly effected by presence of extreme values
- It is also known as Root-Mean-Square Deviation (RMSD)

- It is denoted by σ .
- $SD^2 = \text{variance} = \sigma^2$
- If all observations are equal variance = SD = 0
- SD - Defined as the +ve square root of AM of the square deviations of the value from their AM

Simple series/w/o freq
ungrouped

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left[\frac{\sum x}{n}\right]^2}$$

$$\sigma^2 = \sqrt{\frac{\sum d^2}{n} - \left[\frac{\sum d}{n}\right]^2} \times i$$

where $d = \frac{x - A}{i}$, $x = \text{mid value}$

Simple / Grouped

Not imp
Just go through

$$\sigma = \sqrt{\frac{\sum f (x - \bar{x})^2}{\sum f}}$$

$$\sigma = \sqrt{\frac{\sum f x^2}{\sum f} - \left[\frac{\sum f x}{\sum f}\right]^2}$$

$$\sigma_x = \sqrt{\frac{\sum f d^2}{\sum f} - \left[\frac{\sum f d}{\sum f}\right]^2} \times i$$

$i = \text{class width}$

$A = \text{assumed mean}$

$$\text{coeff of variation} = \frac{SD}{\text{mean}} \times 100 = \frac{\sigma}{\bar{x}} \times 100$$

- Coeff of variation is the best measure of disp.
- It is used to compare variability or consistency b/w 2/more series.
- More CV implies more variability or reliability indicating less stability or consistency & vice versa.
- Regarding choice - always choose item which has less CV becoz item with lower CV is more stable.

$$SD \text{ of 2 no's } a, b = \frac{|a-b|}{2}$$

$$\text{variance of first 'n' natural no's} = \frac{n^2-1}{12}$$

$$\text{Sum of squares of observations } \sum x^2 = n(\sigma^2 + \bar{x}^2)$$

Linear R/s prop

$$R_y = |b| R_x$$

$$QD_y = |b| QD_x$$

$$MD_y = |b| MD_x$$

$$SD_y = |b| SD_x$$

Coefficients

$$\frac{R_y}{y} \times 100 = \frac{MD_y}{\bar{y}} \times 100$$

Relation b/w QD, MD & SD

$$SD \geq MD \geq QD$$

$$SD = MD = QD = 0$$

$$QD = \frac{2}{3} SD, MD = \frac{4}{3} SD$$