# CH-6
# Sequence & Series

## Introduction & Definition

→ It is an arrangement of numbers in specific order and following definite rule.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29 Order / Rule

1, 4, 7, 10, 13, 16 Order / Rule

Summation of ~~sq~~ sequence is called as series.

# 'माई - बाप' of the sequence ($T_n$ and $S_n$)

$T_n \rightarrow n^{th}$ term of the sequence. (Rule)

$\rightarrow$ $T_n = 3n - 2$      1, 4, 7, ......

$n = 1$    $3 \times 1 - 2 = 1$

$n = 2$    $3 \times 2 - 2 = 4$

$n = 3$    $3 \times 3 - 2 = 7$

$n = 20$    $3 \times 20 - 2 = 58$

$T_n = n^2 - 2n + 7$

$T_{12} = 12^2 - 2 \times 12 + 7$

$= 144 - 24 + 7$

$= 127$

$S_n \rightarrow$ Sum of n terms of the sequence

| | | | |
|---|---|---|---|
| $T_1 = 1$ | $T_2 = 4$ | $T_3 = 7$ | $T_4 = 10$ |
| $S_1 = 1$ | $S_2 = 5$ | $S_3 = 12$ | $S_4 = 22$ |
| $S_1 = T_1$ | $S_2 = T_1 + T_2$ | $S_3 = T_1 + T_2 + T_3$ | $S_4 = T_1 + T_2 + T_3 + T_4$ |

$T_3 = S_3 - S_2$

$T_4 = S_4 - S_3$

$\boxed{S_1 = T_1}$

$\boxed{T_n = S_n - S_{n-1}}$

(i) $T_n = 5n^2 - 6$ , $T_{10} = ?$

$T_{10} = 5 \times 10^2 - 6$

$= 494$

(ii) $S_n = 3n^2 + 5n + 2$ , $T_{15} = ?$

$S_{15} = 3 \times 225 + 5 \times 15 + 2$

$= 752$

(iii) $S_n = 2n^2 - 3$ , $T_8 = ?$

$T_8 = S_8 - S_7$ $\quad$ $S_8 = 2(8)^2 - 3 = 125$

$= 125 - 95$ $\quad$ $S_7 = 2(7^2) - 3 = 95$

$= 30$

(iv) $S_n = n^2 - 3n + 6$ , $T_{16} = ?$

$T_{16} = S_{16} - S_{15}$ $\quad$ $S_{16} = 16^2 - 3 \times 16 + 6 = 214$

$= 214 - 186$ $\quad$ $S_{15} = 15^2 - 3 \times 15 + 6 = 186$

$= 28$

** (v) $S_n = 3n^2 + n$ , $T_n = ?$ $\quad$ [Half sol. Half knock out]

$S_1 = T_1$

$S_1 = 3 \times 1^2 + 1$

$= 4$

# Arithmetic Progression (AP)

$\overset{a}{1}$, 4, 7, 10, 13, 16, 19, ... (AP)

+3 +3 +3 +3 +3 +3

d

9, 7, 5, 3, 1, ... (AP)

−2 −2 −2 −2

a → First Term

d → Common Difference $d = 2^{nd} - 1^{st}$

$d = 4 - 1 = 3$

$d = 7 - 9 = -2$

-35, -33, -31, -29, ...

$a = -35$

$d = -33 - (-35) = -33 + 35 = 2$

$T_n = a + (n-1)d$

| | |
|---|---|
| $a = 1,\ d = 3$ | |
| $T_1 = 1$ | $T_{15} = a + 14d$ |
| $T_2 = 1 + 3$ | $T_{11} = a + 10d$ |
| $T_3 = 1 + 2 \times 3$ | $T_{20} = a + 19d = 1 + 19 \times 3 = 58$ |
| $T_4 = 1 + 3 \times 3$ | |
| $T_n = a + (n-1)d$ | |

By Cal. →

$T_n = a + d = \ldots\ldots (n+1)$ counter

$T_{20} = 1 + 3 = \ldots\ldots\ldots 21$

□ $S_n = \frac{n}{2}(a + l)$ a → 1st Term
l → Last Term

□ $S_n = \frac{n}{2}[2a + (n-1)d]$ a → 1st Term
d = Comm. diff.

By Cal. →

$a + d = \ldots\ldots\ldots\ldots (n+1)$ GT + a
Counter

$S_{15} = 1 + 3 = \ldots\ldots\ldots$ 16 GT + 1 =

Q. If 5th AP → 19
11th AP → 37
20th AP → ?

6d
9d

6d → 18
d = 3
9×3

27+37 = 64

$a + 4d = 19$
$a + 10d = 37$ } X

Q. $T_4 : T_8 = 5:8$ , $T_5 : T_9 = ?$

$$\frac{a+3d}{a+7d} = \frac{5}{8}$$

$$8a + 24d = 5a + 35d$$

$$8a - 5a = 35d - 24d$$

$$3a = 11d$$

$$a = \frac{11d}{3}$$

$$\frac{a}{d} = \frac{11}{3}$$

$$\frac{a+4d}{a+8d} = \frac{11 + 4\times 3}{11 + 8\times 3}$$

$$= \frac{11+12}{11+24}$$

$$= \frac{23}{35}$$

—4 $\left(\frac{T_4}{T_8} = \frac{5}{8}\right)$

$$4d = 3$$

$$d = \frac{3}{4}$$

$$\frac{T_5}{T_9} = \frac{5 + 3/4}{8 + 3/4} = \frac{23/\cancel{4}}{35/\cancel{4}}$$

$$= \frac{23}{35}$$

Q. $T_5 : T_9 = 7:12$ , $T_4 : T_{10}$

$\left(\frac{T_5}{T_9} = \frac{7}{12}\right)$

$$4d = 5$$

$$d = \frac{5}{4}$$

$$\frac{T_4}{T_{10}} = \frac{7 - 5/4}{12 + 5/4}$$

$$= \frac{23/4}{}$$

$$= \frac{23}{53}$$

# Points to Remember

(i) If three number a, b and c are in A.P., then the necessary and sufficient condition is

$$\boxed{2b = a + c}$$

Average Type =) $T_{13}$ $\quad$ $T_{27}$

$\uparrow$
$T_{20}$

(ii) If we have to assume numbers in AP as a, b, c, d, e, ... then we take them as 1, 2, 3, 4, 5, ....

→ $\underset{x+7}{a}$, $\underset{2x+5}{b}$, $\underset{x+9}{c}$ are in AP then find $x$?

$$2(2x+5) = x+7+x+9$$
$$4x+10 = 2x+16$$
$$4x-2x = 16-10$$
$$2x = 6$$

(iii) If a, b and c are in AP and $a+b+c = S$, then $b = \frac{S}{3}$ OR

a, b, c, d and e are in AP and $a+b+c+d+e = S$, then $c = \frac{S}{5}$ and so on.

★ Only applicable on odd terms of A.P.

$a + b + c = 12$

$\left\{\begin{matrix} 1 & 4 & 7 \\ 2 & 4 & 6 \\ 2.5 & 4 & 5.5 \end{matrix}\right\}$

$b = \frac{12}{3} = 4$

It value will be fixed

(iv) Numbers of terms $(n) = \frac{\text{last term} - \text{first term}}{\text{Common Difference}} + 1$

$= \frac{l - a}{d} + 1$

□ 2, 6, 10, ...... 134, n = ?

$n = \frac{134 - 2}{4} + 1$

$n = 34$

□ 20 → 200 divisible by 3 then how many no. are there?

$\frac{20}{3} = 6.66......$ age 7 × 3 = 21

Peeche , $\frac{200}{3} = 66.66....$

$= 66 \times 3$

$= 198$

198 − 21 ÷ 3 + 1

(v) For inserting AMs between two numbers 'a' and 'b'

last term

$$\text{Common Difference } (d) = \frac{b - a}{n - 1}$$

- 8 14 20 26 32

$$\frac{32 - 8}{4} = 6$$

- If 5 AMs are inserted between 8 and 35, find 4th AM

8 12.5 17 21.5 $\boxed{26}$ 30.5 35

$a = 8$
$b = 35$
$n = 7$

$$d = \frac{35 - 8}{7 - 1} = \frac{27}{6}$$

$= 4.5$

# Geometric Progression (G.P.)

AP

a = 2 5 8 11 14 . . . . . .

d = +3 +3 +3 +3 +3

GP

a = 2 6 18 54 . . . . . .

r = ×3 ×3 ×3 ×3

↓

Common Ratio

$$\boxed{T_n = a r^{n-1}}$$

$T_5 = 2 \times (3)^{5-1}$

$= 2 \times 3^4$

$= 162$

Cl. -4

$r \times a = \ldots\ldots\ldots (n+1)$

$3 \times 2 = \ldots\ldots\ldots 6$

$$S_n = \frac{a(r^n - 1)}{r-1}, \quad r > 1$$

$$S_n = \frac{a(1 - r^n)}{1-r}, \quad r < 1$$

Ex-

$r^3 \left( T_2 \to 6 \right.$

$r^5 \left( T_5 \to 48 \right.$

$T_{10} \to ?\ 2^5$

$r^3 = 48/6 = 8$

$r = 2$

$T_{10} = 48 \times 32$
$= 1536$

Ex-

$r^2 \left( T_7 \to 4 \right.$

$r^3 \left( T_9 \to 36 \right.$

$T_{12} \to 3^3$

$r^2 = \frac{36}{4} = 9$

$r = 3$

$T_{12} = 36 \times 27$
$= 972$

$\boxed{\text{Cal.} \longrightarrow r \times a = \text{-----} (n+1)\ GT + a}$

$$S_\infty = \frac{a}{1-r} \quad (r<1)$$

$r = \frac{T_2}{T_1}$

Ex- $2, \frac{2}{3}, \frac{2}{9}$ ------ $\infty$

$S_\infty = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = 2 \times \frac{3}{2}$

$= 3$

Ex- 10, 5, 2.5, .... $a = 10$, $r = \frac{5}{10} = 0.5$

$$S_\infty = \frac{10}{1-0.5} = 20$$

Ex- 10, 15, 22.5, .... $T_6$ & $S_{10}$ $a = 10$, $r = \frac{15}{10} = 1.5$

$T_6 \rightarrow 1.5 \times 10 = \ldots$ 7

$S_{10} = 1.5 \times 10 = \ldots$ 11 GT + 10 = 1133.30

Ex-

# Points to Remember

(i) If three number a, b & c are in GP, then the necessary and sufficient condition is,

$$\boxed{b^2 = ac}$$

Ex- Let $x+5$, $x+8$ & $x+4$ are in GP then find $x$?

$$(x+8)^2 = (x+5)(x+4)$$
$$\cancel{x^2}+64+16x = \cancel{x^2} + 9x + 20$$
$$7x = -44$$
$$x = -6.28$$

(ii) If we have to assume numbers in GP as a, b, c, d, e, ... then we take them as 1, 2, 4, 8, 16

(iii) If a, b and c are in GP and $a \times b \times c = P$, then $b = (P)^{1/3}$ or a, b, c, d and e are in GP and $a \times b \times c \times d \times e = P$, then $c = (P)^{1/5}$ and so on...

Ex-

| a | b | c | | |
|---|---|---|---|---|
| 2 | 4 | 8 | = 64 | $b = (64)^{1/3}$ |
| 1 | 4 | 16 | = 64 | $b = (64)^{1/3}$ |

(iv) for inserting GMs between two numbers 'a' and 'b'.

$$\text{Common Ratio } (\overset{r}{d}) = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$$

Ex- 2 __6__ __18__ __54__ 162 Insert 3 G.M.s

$$r = \left(\frac{162}{2}\right)^{\frac{1}{5-1}} = 81^{1/4} = 3$$

☆☆ Example.3 Module 6.12

Put $n = 2$

$$\frac{1}{27}(10^{n+1} - 9n - 10)$$ [∵ given in option]

$$\frac{1}{27}(10^3 - 9\times 2 - 10)$$

36

## Harmonic Progression

**Condition**

$a, b, c \rightarrow HP$

$$b = \frac{2ac}{a+c}$$