

CH-6

Sequence & Series

Introduction & Definition

→ It is an arrangement of numbers in specific order and following definite rule.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Order / Rule

1, 4, 7, 10, 13, 16

Order / Rule

Summation of sequence is called as series.

'माई-काप' of the sequence (T_n and S_n)

$T_n \rightarrow n^{\text{th}}$ term of the sequence. (Rule)

$\rightarrow T_n = 3n - 2$ 1, 4, 7, ...

$n=1$ $3 \times 1 - 2 = 1$

$n=2$ $3 \times 2 - 2 = 4$

$n=3$ $3 \times 3 - 2 = 7$

$n=20$ $3 \times 20 - 2 = 58$

$T_n = n^2 - 2n + 7$

$T_{12} = 12^2 - 2 \times 12 + 7$

$= 144 - 24 + 7$

$= 127$

$S_n \rightarrow$ Sum of n terms of the sequence

$T_1 = 1$

$T_2 = 4$

$T_3 = 7$

$T_4 = 10$

$S_1 = 1$

$S_2 = 5$

$S_3 = 12$

$S_4 = 22$

$S_1 = T_1$

$S_2 = T_1 + T_2$

$S_3 = T_1 + T_2 + T_3$

$S_4 = T_1 + T_2 + T_3 + T_4$

$T_3 = S_3 - S_2$

$T_4 = S_4 - S_3$

$S_1 = T_1$

$T_n = S_n - S_{n-1}$

$$(i) \quad T_n = 5n^2 - 6, \quad T_{70} = ?$$

$$T_{10} = 5 \times 10^2 - 6$$

$$= 494$$

$$(ii) \quad S_n = 3n^2 + 5n + 2, \quad T_{15} = ?$$

$$S_{15} = 3 \times 225 + 5 \times 15 + 2$$

$$= 752$$

$$(iii) \quad S_n = 2n^2 - 3, \quad T_8 = ?$$

$$T_8 = S_8 - S_7$$

$$= 125 - 95$$

$$= 30$$

$$S_8 = 2(8)^2 - 3 = 125$$

$$S_7 = 2(7)^2 - 3 = 95$$

$$(iv) \quad S_n = n^2 - 3n + 6, \quad T_{16} = ?$$

$$T_{16} = S_{16} - S_{15}$$

$$= 214 - 186$$

$$= 28$$

$$S_{16} = 16^2 - 3 \times 16 + 6 = 214$$

$$S_{15} = 15^2 - 3 \times 15 + 6 = 186$$

$$** (v) \quad S_n = 3n^2 + n, \quad T_n = ?$$

[Half sol.
Half knock out]

$$S_1 = T_1$$

$$S_1 = 3 \times 1^2 + 1$$

$$= 4$$

Arithmetic Progression (AP)

$\nearrow a$

$$1, 4, 7, 10, 13, 16, 19, \dots \quad (AP)$$

$+3 \quad +3 \quad +3 \quad +3 \quad +3 \quad +3$

d

$$9, 7, 5, 3, 1, \dots \quad (AP)$$

$-2 \quad -2 \quad -2 \quad -2$

$a \rightarrow$ First Term

$d \rightarrow$ Common Difference

$$d = 2^{\text{nd}} - 1^{\text{st}}$$

$$d = 4 - 1 = 3$$

$$d = 7 - 9 = -2$$

o $-35, -33, -31, -29, \dots$

$$a = -35$$

$$d = -33 - (-35) = -33 + 35 = 2$$

$$T_n = a + (n-1)d$$

$$a = 1, d = 3$$

$$T_1 = 1$$

$$T_2 = 1 + 3$$

$$T_3 = 1 + 2 \times 3$$

$$T_4 = 1 + 3 \times 3$$

$$T_n = a + (n-1)d$$

$$T_{15} = a + 14d$$

$$T_{11} = a + 10d$$

$$T_{20} = a + 19d = 1 + 19 \times 3 = 58$$

By Cal. $\rightarrow$

$$T_n = a + d = \dots (n+1) \text{ counter}$$

$$T_{20} = 1 + 3 = \dots 21$$

$$\square S_n = \frac{n}{2} (a + L)$$

$a \rightarrow 1^{\text{st}} \text{ Term}$

$L \rightarrow \text{Last Term}$

$$\square S_n = \frac{n}{2} [2a + (n-1)d]$$

$a \rightarrow 1^{\text{st}} \text{ Term}$

$d = \text{Comm. diff.}$

By Cal. $\rightarrow$

$$a + d = \dots (n+1) \text{ Counter} \quad GT + a$$

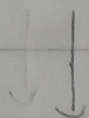
$$T_{15} = 1 + 3 = \dots (16) \quad GT + 1 =$$

Q. If $\left(\begin{array}{l} 5^{\text{th}} \text{ AP} \rightarrow 19 \\ 11^{\text{th}} \text{ AP} \rightarrow 37 \end{array} \right)$ $\begin{array}{l} 6d \rightarrow 18 \\ d = 3 \\ 9 \times 3 \end{array}$

$$\left. \begin{array}{l} a + 4d = 19 \\ a + 10d = 37 \end{array} \right\} \times$$

9d $\left(\begin{array}{l} 20^{\text{th}} \text{ AP} \rightarrow ? \end{array} \right)$
 $27 + 37 = 64$

Q. $T_4 : T_8 = 5 : 8$, $T_5 : T_9 = ?$



$$\frac{a+3d}{a+7d} = \frac{5}{8}$$

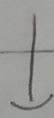
$$8a + 24d = 5a + 35d$$

$$8a - 5a = 35d - 24d$$

$$3a = 11d$$

$$a = \frac{11d}{3}$$

$$\frac{a}{d} = \frac{11}{3}$$



$$\frac{a+4d}{a+8d} = \frac{11 + 4 \times 3}{11 + 8 \times 3}$$

$$= \frac{11 + 12}{11 + 24}$$

$$= \frac{23}{35}$$

→

$$\left(\frac{T_4}{T_8} = \frac{5}{8} \right)$$

$$4d = 3$$

$$d = \frac{3}{4}$$

$$\frac{T_5}{T_9} = \frac{5 + \frac{3}{4}}{8 + \frac{3}{4}} = \frac{23/4}{35/4}$$

$$= \frac{23}{35}$$

Q. $T_5 : T_9 = 7 : 12$, $T_4 : T_{10} = ?$

$$\left(\frac{T_5}{T_9} = \frac{7}{12} \right)$$

$$4d = 5$$

$$d = \frac{5}{4}$$

$$\frac{T_4}{T_{10}} = \frac{7 - \frac{5}{4}}{12 + \frac{5}{4}} = \frac{23/4}{53/4}$$

$$= \frac{23}{53}$$

Points to Remember

- (i) If three numbers a, b and c are in A.P., then the necessary and sufficient condition is

$$\boxed{2b = a + c}$$

Average Type $\Rightarrow$ T_{10} $\uparrow$ T_{27}
 T_{20}

- (ii) If we have to assume numbers in AP as $a, b, c, d, \dots$, then we take them as $1, 2, 3, 4, 5, \dots$

$\rightarrow$ a, b, c are in AP then find x ?
 $x+7, 2x+5, x+9$

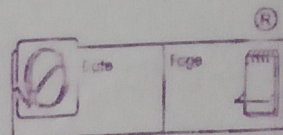
$$2(2x+5) = x+7 + x+9$$

$$4x+10 = 2x+16$$

$$4x-2x = 16-10$$

$$\underline{2x = 6}$$

$$a-b+c$$



(iii) If a, b and c are in AP and $a+b+c=S$, then

$$b = \frac{S}{3} \quad \text{OR}$$

a, b, c, d and e are in AP and $a+b+c+d+e=S$, then

$$c = \frac{S}{5} \quad \text{and so on.}$$

* Only applicable on odd terms of A.P.

$$a+b+c=12$$

$$\left\{ \begin{array}{ccc} 1 & 4 & 7 \\ 2 & 4 & 6 \\ 2.5 & 4 & 5.5 \end{array} \right\}$$

$$b = \frac{12}{3} = 4$$

It value will be fixed

(iv) Numbers of terms $(n) = \frac{\text{last term} - \text{first term} + 1}{\text{Common Difference}}$

$$= \frac{l - a + 1}{d}$$

□ $2, 6, 10, \dots, 134, n = ?$

$$n = \frac{134 - 2}{4} + 1$$

$$n = 34$$

□ $20 \rightarrow 200$ divisible by 3 then how many no. are there?

$\downarrow$ age

$$\frac{20}{3} = 6.66 \dots \quad 7 \times 3 = 21$$

$$\frac{200}{3} = 66.66 \dots$$

$$= 66 \times 3$$

$$= 198$$

$$198 - 21 \div 3 + 1$$

(v) For inserting AMs between two numbers 'a' and 'b'

$$\text{Common Difference (d)} = \frac{b - a}{n - 1}$$

□ $8 \quad 14 \quad 20 \quad 26 \quad 32$

$$\frac{32 - 8}{4} = 6$$

□ If 5 AMs are inserted between 8 and 35, find 4th AM

$8 \quad 12.5 \quad 17 \quad 21.5 \quad \boxed{26} \quad 30.5 \quad 35$

$$a = 8$$

$$b = 35$$

$$n = 7$$

$$d = \frac{35 - 8}{7 - 1} = \frac{27}{6}$$

$$7 - 1$$

$$6$$

$$= 4.5$$

Geometric Progression (G.P.)

$$\begin{array}{ccccccc} a = & 2 & & 5 & & 8 & & 11 & & 14 & \dots & \dots & \dots & \text{AP} \\ d = & +3 & & +3 & & +3 & & +3 & & +3 & & & & \end{array}$$

$$\begin{array}{ccccccc} a = & 2 & & 6 & & 18 & & 54 & \dots & \dots & \dots & \text{GP} \\ r = & \times 3 & & \times 3 & & \times 3 & & \times 3 & & & & \end{array}$$



Common Ratio

$$T_n = ar^{n-1}$$

$$\begin{aligned} T_5 &= 2 \times (3)^{5-1} \\ &= 2 \times 3^4 \\ &= 162 \end{aligned}$$

Ex. - 1

$$\begin{aligned} r \times a &= \dots \dots \dots (n+1) \\ 3 \times 2 &= \dots \dots \dots 6 \end{aligned}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r > 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r}, \quad r < 1$$

Ex- $\begin{cases} T_2 \rightarrow 6 \\ T_5 \rightarrow 48 \\ T_{10} \rightarrow ? \end{cases}$ $r^3 = \frac{48}{6} = 8$
 $r = 2$

$$T_{10} = 48 \times 32$$

$$= 1536$$

Ex- $\begin{cases} T_2 \rightarrow 4 \\ T_9 \rightarrow 36 \\ T_{12} \rightarrow ? \end{cases}$ $r^2 = \frac{36}{4} = 9$
 $r = 3$

$$T_{12} = 36 \times 27$$

$$= 972$$

Cal. $\rightarrow r \times a = \dots \dots (n+1)GT + a$

$$S_{\infty} = \frac{a}{1-r} \quad (r < 1)$$

$$r = \frac{T_2}{T_1}$$

Ex- $2, \frac{2}{3}, \frac{2}{9}, \dots \infty$

$$S_{\infty} = \frac{2}{1 - \frac{1}{3}} = \frac{2}{\frac{2}{3}} = 2 \times \frac{3}{2}$$

$$= 3$$

Points to Remember

- (i) If three numbers a, b & c are in GP, then the necessary and sufficient condition is,

$$\boxed{b^2 = ac}$$

Ex- Let $x+5, x+8$ & $x+4$ are in GP then find x ?

$$(x+8)^2 = (x+5)(x+4)$$

$$\cancel{x^2} + 16x + 64 = \cancel{x^2} + 9x + 20$$

$$7x = -44$$

$$x = -6.28$$

- (ii) If we have to assume numbers in GP as $a, b, c, d, e, \dots$ then we take them as $1, 2, 4, 8, 16, \dots$

⁰⁰⁶
(iii) If a, b and c are in GP and $a \times b \times c = P$, then $b = (P)^{1/3}$ or a, b, c, d and e are in GP and $a \times b \times c \times d \times e = P$, then $c = (P)^{1/5}$ and so on...

Ex-

$a \quad b \quad c$

$2 \quad 4 \quad 8 \quad = 64$

$1 \quad 4 \quad 16 \quad = 64$

$$b = (64)^{1/3}$$

$$b = (64)^{1/3}$$

(iv) For inserting G.Ms between two numbers 'a' and 'b'.

$$\text{Common Ratio } (r) = \left(\frac{b}{a} \right)^{\frac{1}{n-1}}$$

Ex - 2 6 18 54 162 Insert 3 G.M.s

$$r = \left(\frac{162}{2} \right)^{\frac{1}{5-1}} = 81^{\frac{1}{4}} = 3$$

★★
Example 3
Module 6.12

Put $n = 2$

$$\frac{1}{27} (10^{n+1} - 9n - 10)$$

[∵ given in option]

$$\frac{1}{27} (10^3 - 9 \times 2 - 10)$$

36

Harmonic Progression

Condition

$$a, b, c \rightarrow \text{HP}$$

$$b = \frac{2ac}{a+c}$$