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ADVANCED CAPITAL BUDGETING

<u>Learning objectives</u>

* Sensitivity analysis
* Inflation in capital budgeting
* Abandonment decision
* Replacement decision
* Base case NPV and adjusted NPV
* Probability analysis
* Certainity equivalent approach.
* Risk adjusted discount rate.
* Simulation technique
* Decision tree approach.
* Miscellaneous.

Before you start with advanced capital budgeting, it is highly recommended to have fantastic clarity on basics and fundamentals of capital budgeting which covers all concepts like ——

* Pay back
* Discounted pay back
* Annual rate of return
* NPV
* IRR
* PI
* modified NPV & modified IRR.
* Capital rationing
* Life & timing disparity analysis.

The advanced CB chapter covers concepts which stems out of the above said basics and therefore I have annexed CA Inter CB chapter notes before Adv. CB. more over, in terms of video classes also 50 hrs of CB from CA Inter is available in google drive. watch the videos and consider the notes (Both wrt CA Inter) before you start Adv. CB orelse this chapter may not be dearer to you.

1. ## BASICS

In this chapter we are going to learn the following:-

- Inflation in CB
- Techniques in CB
  - I Statistical
    - Probability
    - Variance/sd
    - Coefficient of variation
  - II Conventional
    - Risk-adjusted discount rate (RADR)
    - Certainity equivalent (CE)
  - III Miscellaneous
    - Sensitivity analysis
    - Scenario analysis
    - Simulation analysis
      - Basic Simulation
      - Monte-carlo simulation
    - Decision tree approach.
- Replacement and abandonment decisions

SEQUENCE OF STUDY

I. Replacement and abandonment decision.
II. Inflation in CB.
III. Special techniques.

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1\. Replacement decision :-

Life disparity – Refer CA Inter notes also

* NPV sees the quantum of return, while IRR sees the speed (or) rate of return.
* Longer the project, more the quantum, more the NPV.
* Faster the cash flows, more the IRR.
* If project A has longer life and project B has faster CFs, NPV selects project A and IRR selects project B. Hence, conflict arises.
* It is wrong to compare NPV of project with another project with inequal life. We should not compare a project having say, 3 years life with another project with say 2 years life.
* The right way is to make the projects into equal life and then compare. This can be done 2 ways

↓

Repetition method ↓ Perform 3 years project 2 times and 2 years project 3 times make it into 6 yrs CF and then compare NPV.

EAB / EAC ↓ calculate NPV on p.a. basis and then we compare.

* NPV p.a is what we refer as EAB (Equated annuity benefit).

$$EAB = \frac{NPV}{\text{annuity factor}}$$

* If it is outflow dominated question, we will go for EAC.

The above theory, can be practically taken as under ———

	Project – A	Project – B
Outflow	(₹ 50)	(₹ 50)
Inflow		
Year-1	₹ 30	₹ 90
Year-2	₹ 90	–

PV@10%

In this example, one cannot compare the project A & B directly because, project A is a 2 years project and project B is a 1 year project. So, the projects are having unequal lives. Therefore, it resulted in timing disparity.

Before resolving this conflict / disparity, we need to first understand existing NPV & IRR.

	Proj. A	Proj. B
NPV	[(₹30×0.909)+(₹90×0.826)]−₹50 = ₹101.61 − ₹50 = ₹ 51.61	[(₹90×0.909)]−₹50 = ₹81.81 − ₹50 = ₹31.81
IRR	Low (Due to lower recovery)	High (Due to higher recovery)

In this case NPV favours project A and IRR favours project 1. This is due to unequal lives and can be resolved using 2 approaches

Approach – 1 Repetition method

Under this method, both are compared by bringing CFs to equivalent stage. In the given case, project X is 2 years life and project Y is of 1 year life, to compare on evenly basis, project X should be carried out 1 time (2 yrs) and project Y should be carried out 2 times (1 yr × 2 = 2 yrs). The result is as follows ——

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Project X (₹)

Yr	CF	PV@10%	PVCF
0	(50)	1	(50)
1	30	0.909	27.27
2	90	0.826	74.34
		NPV =	₹51.61

Project Y (₹)

Yr	CF	PV@10%	PVCF
	Cycle - 1		
0	(50)	1	(50)
1	90	0.909	81.81
	Cycle - 2		
1	(50)	0.909	(45.45)
2	90	0.826	74.34
		NPV =	₹60.70

∴ It is clear that project Y is recommended.

Approach - 2 **Equate annuity benefit model**

This model is just opposite to the previous model where any project of any life is then expressed in per annum terms.

↓

$$\frac{NPV}{PVAF}$$

Projects

↓

X | Y

X

NPV = ₹51.61

PVAF = $\frac{1}{1.736}$

EAB = ₹29.73

Y

NPV = ₹31.81

↓

This is already in 1 year terms.

Since EAB of project Y is higher, it is to be accepted.

Notes

Though it can be done 2 ways namely the repetition model and EAB model, EAB model is most recommended since repetition model is not practical. This can be understood as under ———

	Project X	Project Y	Action to be taken
Life	2 years	1 year	* Carry X 1 time
			* Carry Y 2 times
Life	4 years	3 years	* Carry X 3 times
			* Carry Y 4 times
Life	9 years	11 years	* Carry X 11 times
			* Carry Y 9 times

When life of projects are very big like in the 3rd case in above example, the project should be repeated for 99 times and hence one need to analyse 99 years cash flows which is eventually impracticable. Hence, EAB model is best fit for projects with unequal lives.

AN EXAMPLE

1. Discount rate : 10%
2. CF details

Year	0	1	2	3
Outlay	−1,000	—	—	—
Revenue	—	+900	+800	+700
Costs	—	−400	−350	−350
Scrap	—	+650	+400	+150

* We have 3 options in this case.
* In this case, the question is not about whether a machine is required. It is all about when to replace the machine.
* We require a machine continuously, which means machine is used perpetually. But the problem is machine cant have a perpetual life. So, we need to replace the machine at an appropriate time. What is the appropriate time is subject matter of discussion.
* In the given case, we can replace the machine as follows ———
  1. once in every year.

2. once in 2 years
3. once in 3 years
and not further since life is 3years.

* Analysis on numbers

Holding period	Cash flow analysis
1 year	Investment = (₹1,000) sales = ₹900 costs = (₹400) scrap = ₹650
2 years	Investment = (₹1,000) sales (1-yr) = ₹900 sales (2-yr) = ₹800 costs (1-yr) = (₹400) costs (2-yr) = (₹350) scrap (2-yr) = ₹400
3 years	Investment = (₹1,000) sales (1-yr) = ₹900 sales (2-yr) = ₹800 sales (3-yr) = ₹700 costs (1-yr) = (₹400) costs (2-yr) = (₹350) costs (3-yr) = (₹350) scrap (2-yr) = ₹400

Notes:

* As the life exhausts, scrap value reduces.
* Logic is, which ever option gives us highest NPV, that should be selected.
* But, the problem is, we cant compare all 3 options as it is because, option 1 is having 1 year CF whereas 2 has 2 years CF and option 3 with 3years of CF. Hence, there exists a life

disparity.

* we need to use EAB/ EAC due unequal lives.

Solution

year	PVF @10%	option 1 CF	option 1 DCF	option 2 CF	option 2 DCF	option 3 CF	option 3 DCF
0	1	(1,000)	(1,000)	(1,000)	(1,000)	(1,000)	(1,000)
1	0·9091	1,150 (900 − 400 + 650)	1,045	500	455	500	455
2	0·8264	–	–	850	702	450	372
3	0·7513	–	–	–	–	500	376
NPV			45		157		203

Further analysis:

It is wrong to compare opt 1, 2, 3 NPV's directly because, higher the holding period higher will be the NPV. That means, if we compare opt-1 with opt-3 (say), option 1 can be carried out 3 times and NPV will be ₹ 45 X 3. So, we use EAB. (Refer CA Inter notes for indepth application).

Final solution

Particulars	opt-1	opt-2	opt-3
NPV	45	157	203
Annuity factor	0·9091	1·7355	2·4868
EAB	49	90	82

$EAB = \frac{NPV}{PVAF}$

Replace once in 2 years since EAB is highest in option-2.

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2. Inflation in Capital Budgeting:-

DISCUSSION AND ANALYSIS

* Capital budgeting is an exercise company's will undertake to select (or) reject projects.
* If a project is estimated to give a positive NPV, it shall be accepted and negative means rejected. + NPV maximises the wealth and negative NPV erodes the wealth.
* NPV shall be calculated as follows —

Year	FCF	PV@RR	PVCF
1	XXX	XXX	XXX
2	XXX	XXX	XXX
3	XXX	XXX	XXX
		ΣDISCI =	XXX
		(-) ΣDISCO =	(XXX)
		NPV =	XXX

S1: Estimate future cash flow

S2: Estimate an appropriate discount rate

S3: Calc sum of all discounted cash flows.

S4: Deduct the Initial cost from the cumulative discounted CFs.

S5: Resultant is called as NPV

- +ve NPV → Accept.
- -ve NPV → Reject.

* while estimating the cash flows we need to estimate two things namely —
  - Volume estimate
  - cost estimate

* **Volume estimate.**

We predict to sell say, 10,000 units in 1st yr, 15,000 units in 2nd yr and 8000 units in 3rd yr. These are only mere estimates and not being guaranteed. Therefore, the sales volume estimated has a risk. This risk is called as risk of the project undertaken. If project is for essential goods/services, the risk is low but otherwise risk is very high.

* **Cost/price estimate**

For the volume estimated, we do the following-

- **Sales value** → Product of estimated sales volume and estimated sale price. → NOU x SP
- **Material cost** → Product of estimated sales volume, no. of units of raw material for 1 unit and cost per kg of raw material. → NOU x No. of kgs x cost per kg.
- **Labour and other OH cost** → Product of labour hrs for each unit and no. of units estimated and also we estimate all other variable & fixed OHs. → NOU x No. of hrs x cost per hour. etc.

* When we estimate the future cash flows there are 2 options available namely ——

- Estimate as per the current purchasing power.
- (or)
- Estimate as per the respective year's estimated purchasing power.

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costs, sales and everything is estimated at today's costs.	costs, sales and everything is estimated at inflation applied values
1. sales units x sales price	1. sales units x yr-1 SP @ I_1, yr-2 SP @ I_2, yr-3 SP @ I_3
All estimates for future at today's inflation only. That means, inflation is ignored.	All estimates are made not at today's PP, but at the respective year's PP.
2. sales volume x cost per unit	2. sales volume x cost per unit
material, labour, OHs	material, labour, OHs
Estimated at current prices for all future years.	Estimated at the inflated prices for all future years.

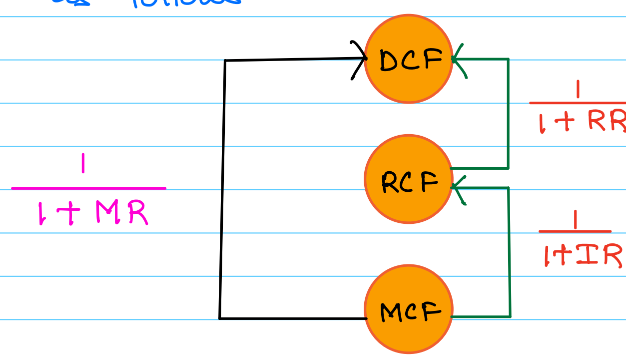
* There are 3-types of rates

- money rate/ Nominal rate → compensates for inflation and risk.
- Real rate → compensates for risk of the project.
- Inflation rate → compensates the effect of the inflation.

* There are 3-types of Cash flows:

- **Money CF** → Loaded with risk and the inflation
- **Real CF** → Loaded only with risk.
- **Discounted CF** → No risk, No inflation.

* This concept of rates and Cash flows can be merged as follows ———



$$\therefore \frac{1}{1+MR} = \frac{1}{1+RR} \times \frac{1}{1+IR}$$

$$1+MR = (1+RR) \times (1+IR)$$

$$MR = [(1+RR)(1+IR)] - 1$$

↓

$$RR = \frac{1+MR}{1+IR} - 1$$

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Example

a. Investment proposal = ₹ 8,00,000
b. Real cash inflows p.a = ₹ 2,80,000 (todays P.P)
c. monetary cost of capital = 9%
d. Inflation = 3.2% p.a

Compute

(i) Real COC
(ii) PV of cash inflows in real terms.
(iii) compute nominal cash inflow from real cash inflows and also calculate PV on basis of nominal cash inflows.

Solution

Step 1: Calculation of real rate

$$RR = \frac{1+MR}{1+IR} - 1$$

$$= \frac{1+0.09}{1+0.032} - 1$$

$$= \frac{1.09}{1.032} - 1$$

$$= 5.62\%$$

Step 2: Calculation of NPV

Year	Cash inflow	PV@5.62%	PVCF
1 – 4	₹ 2,80,000	3.496	9,78,824
		(–) DISCO	(8,00,000)
		NPV	+₹1,78,824

Step 3: Calculation of NPV using money CFs

Year	Cash inflow (₹)	PV@9%	PVCF
1	2,80,000 × 1.032 = 288960	0.917	2,65,092
2	2,88,960 × 1.032 = 2,98,207	0.842	2,51,000
3	2,98,207 × 1.032 = 3,07,749	0.772	2,37,643
4	3,07,749 × 1.032 = 3,17,597	0.708	2,24,985
		DISCI	9,78,720
		– DISCO	(8,00,000)
		NPV	+ ₹1,78,720.

3. Special techniques

Sequence of learning

a. Decision tree
b. Probability in Capital Budgeting
c. Variance analysis
d. coefficient of variation
e. sensitivity analysis
f. Simulation
g. RADR
h. CE approach.
i. Scenario analysis

Decision tree analysis in capital Budgeting

Decision tree approach (DTA) is widely used to analyse a project with multiple distributed probable Cash flow. The aim of DTA is to calculate probable NPV from various scenarios.

Example

1. Investment = ₹ 40,000
2. Life = 2 years
3. salvage value = 0
4. Probability & Cashflows

		prob		
CF in year1	₹ 25,000	0·40	₹ 30,000	0·60
CF in year2	₹ 12,000	0·20	₹ 20,000	0·40
	₹ 16,000	0·30	₹ 25,000	0·50
	₹ 22,000	0·50	₹ 30,000	0·10

Discount rate @ 10%.

5. Required

* construct a Decision tree.
* NPV if worst outcome is realised. What is the probability of occurence of this NPV?
* What is the best outcome and its prob?
* will the project be accepted?

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Solution

* We want to start a project with 2 years life.
* Today we need to invest ₹ 40,000.
* 1st year the project may give ₹25,000 (or) ₹30,000. (with prob of 40% & 60%)
* 2nd year CF depends on what happened in 1st year. That means, if 1st year CF is ₹25,000 then, 2nd year may give ₹ 12,000 (or) ₹16,000 (or) ₹ 22,000 with probabilities 0·20, 0·30, 0·50 and if 1st year CF is ₹30,000, then 2nd year may give ₹20,000, ₹25,000 (or) ₹ 30,000 respectively.

Part I: Calculation of possible NPVs.

Step 1:

Year	CF (₹)	PV @ 10%	PVCF (₹)
1	25,000	0·9091	22,728
2	12,000	0·8264	9,917
		DISCI	32,645
		(–) DISCO	(40,000)
		NPV	– 7,355.

Step 2:

NPV of Step 1	=	(₹ 7,355)
(+) PV of increased inflow in 2nd year (₹16,000 – 12,000) × 0·8264	=	₹ 3,306
NPV	=	(₹ 4,049)

Step 3:

NPV of Step 2	=	(₹4,049)
(+) PV of increased inflow in year 2 (₹ 22,000 – ₹16,000) × 0·8264	=	₹ 4,958
NPV	=	₹ 909

Step 4:

Year	CF (₹)	PV@10%	PVCF (₹)
1	30,000	0·9091	27,273
2	20,000	0·8264	16,528
		DISCI	43,801
		(–) DISCO	(40,000)
		NPV	+ 3,801

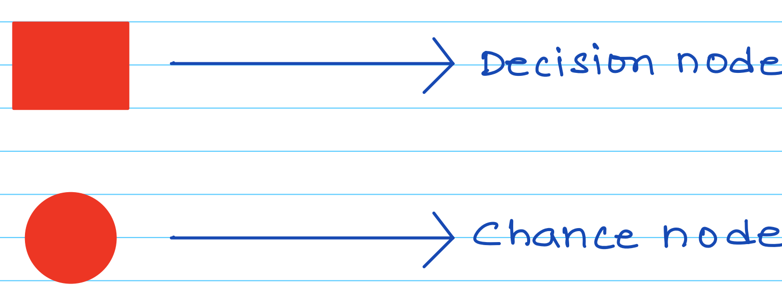
Step 5:

NPV of step 4 = ₹3,801
(+) PV of increased inflow
in 2nd year = ₹ 4,132
(25,000 − 20,000) × 0·8264
NPV = ₹7,933

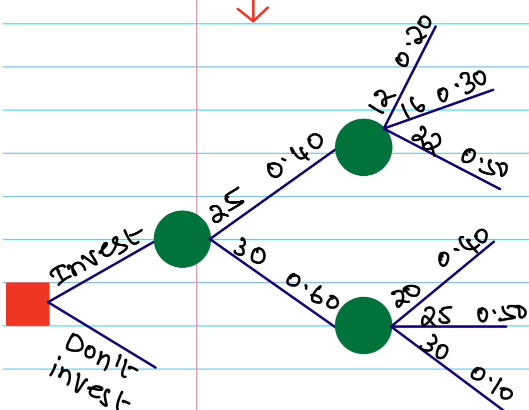
Step 6:

NPV of step 5 = ₹ 7,933
(+) PV of increased inflow
in 2nd year = ₹ 4,132
(30,000 − 25,000) × 0·8264
NPV = ₹ 12,065

Decision Tree



All amounts in 000's



NPV	Joint prob	Expected NPV
(7,355)	0.08	(588)
(4,049)	0.12	(486)
909	0.20	182
3,801	0.24	912
7,933	0.30	2,380
12,065	0.06	724
	1.00	3,124

1. NPV for worst out come = 7,355 with prob of 0.08 (8%)
2. NPV for best out come = 12,065 with prob of 0.06 (6%)
3. Project will be accepted since NPV is positive.

Extra Example (Advanced) — Foreign author (CMA US)

Big oil company is taking decision on oil drill as follows:-

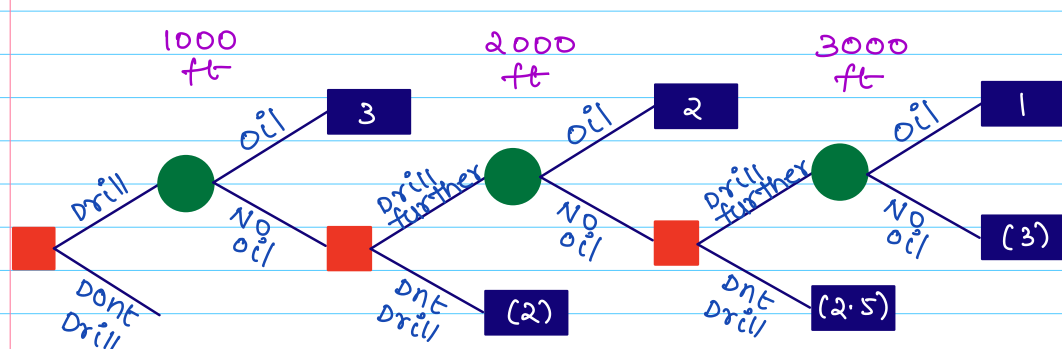
Depth of oil well (feet)	Cost (₹ in millions)	Cum. prop of finding oil	PV of oil well if found (₹ in millions)
1,000	2.00	0.50	5.00
2,000	2.50	0.60	4.50
3,000	3.00	0.70	4.00

Solution

Analysis on question

* The company can drill upto 1000 feet by spending ₹ 2 million. Finding the oil at 1000 feet is probable @ 0.50. In that case, the PV of oil value is ₹ 5 million. If we find the oil @ 1000 feet, it mean that well is deep and from 1000 feet everything found is oil and more extraction can happen & therefore more present value.
* If oil is not found @ 1000 feet, then we need to drill 1000 more feet till 2,000 feet. Then we need to spend ₹ 2.5 million. At 2,000 feet chance of finding oil is 0.60 (since more we drill, the chance of finding oil increases). It should be noted that ₹ 2.5 million is not additional amount to be spent rather it is total. That means, we need to spend additional 0.5 mil. Same way, probability of 0.60 is cumulative.
* If we dont find oil even at 2000 feet, we need to drill further more 1000 feet to reach 3000 feet and results are to be interpreted as before.

Basic Decision tree



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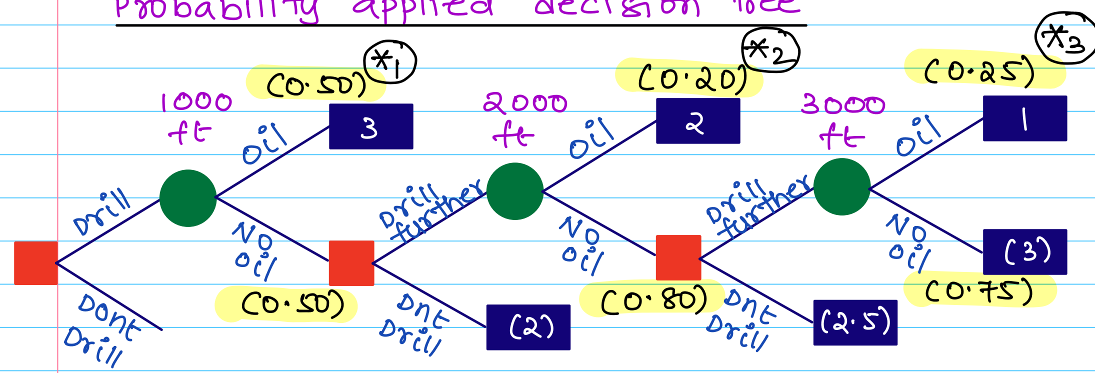
Probability applied decision tree



(*1) It is clearly mentioned in the question that probability of finding oil @ 1,000 ft is 0.50. Therefore, the probability of not finding oil @ 1000 ft will be 0.50 (1 − 0.50).

(*2) As mentioned in the question, the probability of finding oil by 2,000 feet has got 2 cumulative events.

(or)

Finding the oil at 1000 ft (0.50)

Finding the oil at 2,000 feet (0.10)

(0.60) (given)

(and)

Not finding the oil @ 1,000 feet (0.50)

Finding the oil @ 2000 feet (0.20) $\left[\frac{0.10}{0.50}\right]$

Notes:

* As mentioned in the question the cumulative prob of finding the oil by 2000 feet is 0.60. cumulative probability uses "OR". That means the probability of finding oil by 2000 feet will be more than probability of finding oil by 1000 feet. So, probability increased to 0.60.

∴ Probability of finding oil at 1000 feet (OR) 2000 feet is 0.60. The additional probability of finding oil @ 2000 feet is 0.10. This 0.10 is possible only if 2 events "jointly" occur. They are, ——

a) Not finding oil @ 1000 feet (and)
b) finding oil @ 2000 feet.

This all can be presented as under.

Prob of finding oil by 2000 feet = prob of finding oil @ 1000 feet (or) prob of finding oil @ 2000 feet

$0.60 = 0.50 + x$

$x = 0.10$

prob of finding oil @ 2000 feet = prob of not finding oil @ 1000 feet (AND) prob of finding only @ 2,000 feet

$0.10 = 0.50 \times y$
$(1-0.50)$

$y = 0.10/0.50 = 0.20$

Notations to be remembered

OR ⟶ +
AND ⟶ ×

(*3) Probability of finding oil on 3rd drill i.e. 3000 feet

Cumulative prob of finding oil in 3 drills $= 0.70$

(–) Prob of finding in 2 drills $= (0.60)$

Prob of finding in 3rd drill $= 0.10$

To find oil in 3rd drill, 3 events should have happened namely ——

* Not finding in 1st drill AND
* Not finding in 2nd drill AND
* Finding in 3rd drill.

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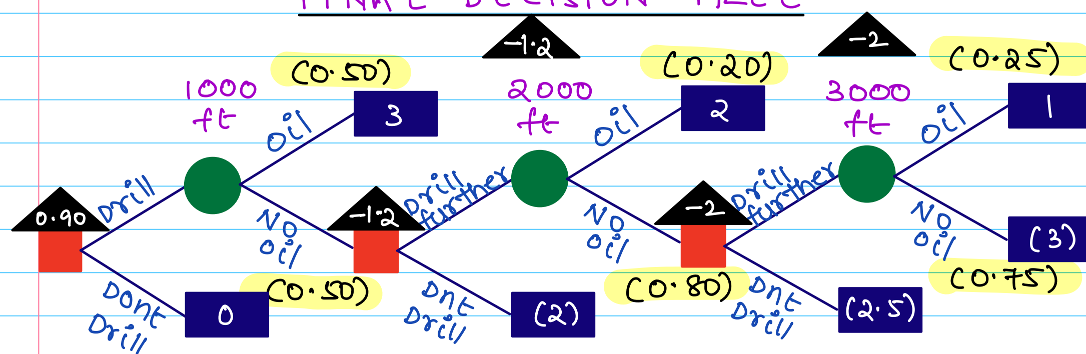
$0.10 = 0.50 \times 0.80 \times x$

$x = \frac{0.10}{0.40}$

$x = 0.25$

FINAL DECISION TREE



Notes:

In this decision tree, workings have to be done in reverse mode. (For NPV calculations)

Step 1 : Analysing Drill 3

At 3,000 feet, 2 situations can occur

- Drill Further
  - Oil: 1×0.25
  - No oil: -3×0.75
  - $= 0.25 - 2.25$
  - $= -2.00$
- Dont drill further: -2.50

To drill is better since abandonment results in more negative NPV.

Step 2 : Analysing Drill 2

At 2,000 feet, 2 situations can occur

- Drill Further
  - Oil → 2 × 0·20
  - No oil → −2 × 0·80
  - = 0·40 − 1·60
  - = −1·20
- Dont drill further → −2·00

To drill is better since abandonment results in more negative NPV.

Step 3 : Analysing Drill 1

At 1,000 feet, 2 situations can occur

- Drill Further
  - Oil → (0·50 × 3)
  - No oil → (0·5 × −1·20)
  - = 1·5 − 0·60
  - = 0·90
- Dont drill → 0

To drill is better since abandonment results in '0' NPV as against positive NPV of ₹0·90 millions.

Final Conclusion

The company have to drill till 3000 feet.