Chapter 7

4 marks

SETS , RELATIONS AND FUNCTIONS

TOPIC :

- SETS
- RELATIONS
- FUNCTIONS
- LIMITS AND CONTINUITY

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QUANTITATIVE APTITUDE ROMANOVA

- 7 years of experience in teaching Mathematics
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- Was #1 Maths Faculty in Magnet Brains
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- Taught 30000+ students with highest score as 100

COLLECTION OF 3 BEST BOLLYWOOD ACTORS

COLLECTION OF ALL THE DAYS OF A WEEK

SETS

A well-defined collection of objects is called a set.

Well defined?

Collection which does not change from person to person.

$$A = \{a, e, i, o, u\}$$

- We denote sets by capital letters A, B, C, X, Y, Z, etc.
- The objects in a set are called its members or elements.
- Elements are usually denoted by small letters.
- If a is an element of a set A, we write, $a \in A$, which means that a belongs to A or that

  a is an element of A.

POINT TO REMEMBER

i. The elements of a set may be listed in any order

Thus, $\{1, 2, 3\} = \{2, 1, 3\} = \{3, 2, 1\}$.

ii. The repetition of elements in a set has no meaning.

Thus, $\{1, 2, 3\} = \{1, 1, 2, 3, 2\} = \{1, 1, 2, 2, 3, 3, 3\}$, etc.

CARDINALITY / CARDINAL NUMBER

The number of distinct elements in a set is called as Cardinal number of it .
For a set A , we represent cardinality with n(A) .

Example:

Let A = {1, 3, 5}

Then,

n(A) = 3

PRESENTATION OF A SET

Descriptive Form

D = The set of odd digits between 1 and 9 both inclusive.

Roster/Tabular Form/ Braces Form

$D = \{1, 3, 5, 7, 9\}$

Set Builder Form/Algebraic Form/Rule Method/ Property Method

$D = \{ x : x = 2n - 1, \text{ where } n \in N \text{ and } 1 \leq n \leq 5\}$

Roster/Tabular Form

- In the roster form, we list all the members of the set within braces { } and separate them by commas.

Set builder Form

- In the set-builder form, we list the property or properties satisfied by all the elements of the set.
- We write,

  $\{x : x \text{ satisfies properties } P\}$, which is read as 'the set of all those x such that each x has properties P'.

TYPES OF SETS

Empty Set:

- A set containing no element at all is called the empty set or the null set or the void set, denoted by ϕ. or $\{\}$.

Example:

i. $\{x : x \in N \text{ and } 2 < x < 3\} = \phi$.

TYPES OF SETS

Singleton Set:

- A set containing exactly one element is called a singleton set.

Example:

i. $\{x : x \in Z \text{ and } x + 4 = 0\} = \{-4\}$, which is a singleton set.

TYPES OF SETS

Finite sets

- An empty set or a non-empty set in which the process of counting of elements surely comes to an end is called a finite set.
- The number of distinct elements contained in a finite set A is denoted by n(A).

Example:

i. Let A = { 2, 4, 6, 8, 10, 12 }.

Then, A is clearly a finite set and n(A) = 6.

TYPES OF SETS

Infinite Sets:

- A set which is not finite is called an infinite set .

EXAMPLE

ii. N : the set of all natural numbers .

iii. Z : the set of all integers.

TYPES OF SETS

Equal Set:

- Two non-empty sets A and B are said to be equal, if they have exactly the same elements and we write, A = B.

EXAMPLE :

Let A = Set of letters in the word ‘follow’

B = Set of letters in the word ‘wolf ‘

Here ,

A= B

TYPES OF SETS

Equivalent Set:

- Two finite sets A and B are said to be equivalent, if n(A) = n(B).
- Equal sets are always equivalent. But, equivalent sets need not be equal.

Example:

i. Let A = {1, 3, 5} and B = {2, 4, 6}.

Then, n(A) = n(B) = 3.

So, A and B are equivalent.

SUBSETS

A set A is called a subset of a set B if every element of A is also element of B

SUPERSET

If ‘A’ is subset of ‘B’ then ‘B’ is called a Superset of ‘A’

PROPER SUBSET

When A is a subset of B but A is not equal to B , then A is a proper subset of B .

$$A \subset B$$

IMPROPER SUBSET

If A is a subset of B and also B is a subset of A , then both are improper subsets of each other , this is possible only when both the sets are equal.

$$A \subseteq B$$

FORMULAE

- No. of possible subsets of set containing n elements

$$2^n$$

- No. of proper subsets of set containing n elements

$$2^n - 1$$

FORMULAE

- Every set is a subset of itself.

- The empty set is a subset of every set .

POWER SET

- Let A be a set . Then the collection of all subsets of A is called the power set of A and is denoted by P(A)

- A is a finite set having n elements , then P (A) has 2^n elements

UNIVERSAL SET

Example:

- Let A = {1, 2, 3}, B = {2, 3, 4, 5} and C = {6, 7}.

  If we consider the set U = {1, 2, 3, 4, 5, 6, 7}, then clearly, U is a superset of each of the given sets.

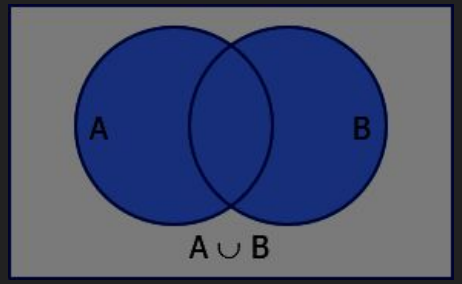
  Hence, U is the universal set.

If there are some sets under consideration, then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set for those sets. We shall denote a universal set by U.

OPERATIONS ON SETS

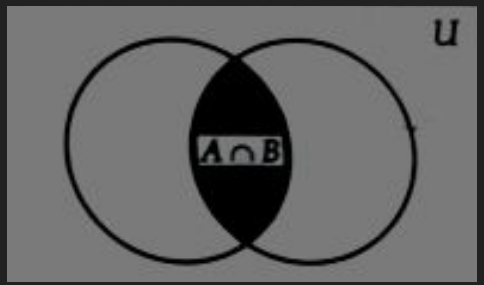
UNION

$A \cup B = \{x : x \in A \text{ or } x \in B\}$.



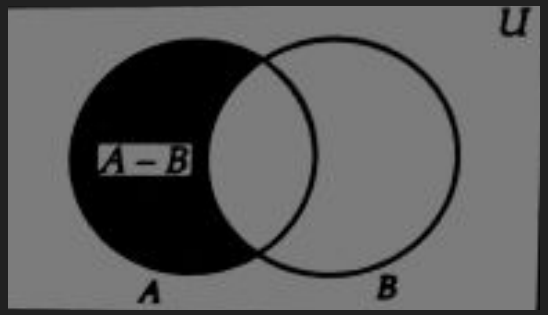
INTERSECTION

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

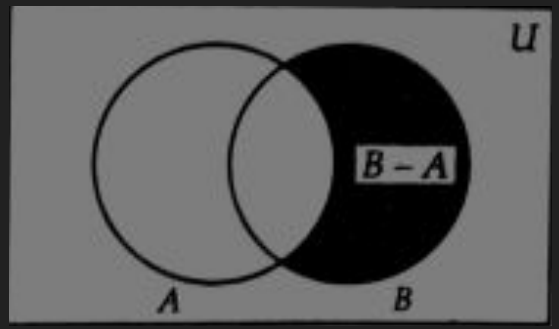


DIFFERENCE OF SETS

$A - B = \{x : x \in A \text{ and } x \notin B\}$

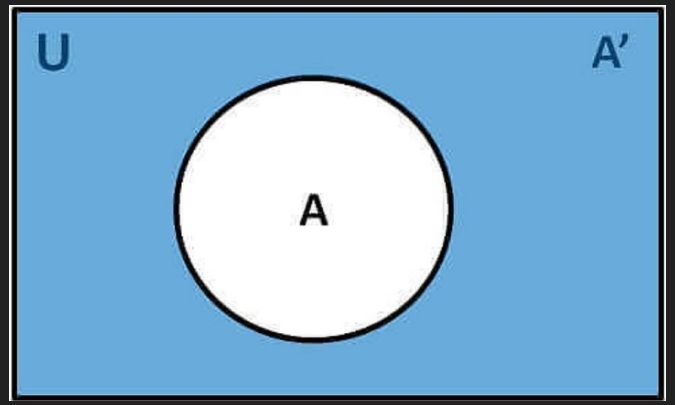


$B - A = \{x \in B \text{ and } x \notin A\}$



COMPLEMENT OF A SET

- Let U be the universal set and let A be a set such that $A \subset U$. Then, the complement of A with respect to U is denoted by A' or A^c or $U - A$ and is defined the set of all those elements of U which are not in A



DE MORGAN'S LAW

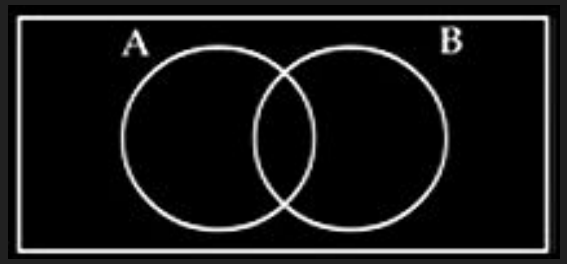
For any two sets A and B,

(i) $(A \cup B)' = (A' \cap B')$

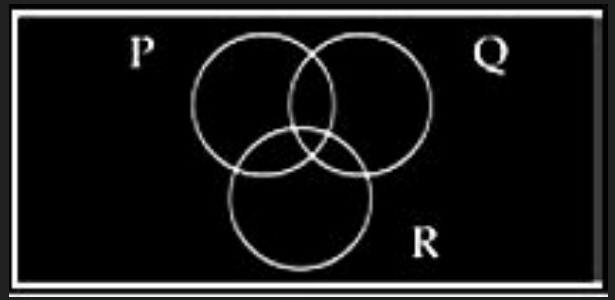
(ii) $(A \cap B)' = (A' \cup B')$

FORMULAE

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



- $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$



EXERCISE 7(A)

Que1. The number of subsets of the set {2, 3, 5} is

a. 3

b. 8

c. 6

d. None of these

Que 2. The number of subsets of a set containing n elements is

a. 2^n

b. 2^{-n}

c. n

d. None of these

Que 3. The null set is represented by.

a. {ϕ}

b. {0}

c. ϕ

d. None of these

Que 4. $A = \{2, 3, 5, 7\}$, $B = \{4, 6, 8, 10\}$ then $A \cap B$ can be written as

a. $\{\}$

b. $\{\phi\}$

c. $(A \cup B)'$

d. None of these

Que 5. The set $\{x \mid 0 < x < 5\}$ represents the set when x may take integral values only

a. $\{0, 1, 2, 3, 4, 5\}$

b. $\{1, 2, 3, 4\}$

c. $\{1, 2, 3, 4, 5\}$

d. None of these

Que 6. The set {0, 2, 4, 6, 8, 10} can be written as

a. $\{2x \mid 0 < x < 5\}$

b. $\{x : 0 < x < 5\}$

c. $\{2x : 0 \leq x \leq 5\}$

d. None of these

Que 7. The data to be used If P = {1, 2, 3, 5, 7}, Q = {1, 3, 6, 10, 15}, Universal Set S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

The cardinal number of P ∩ Q is

a. 3

b. 2

c. 0

d. None of these

Que 8. The data to be used If P = {1, 2, 3, 5, 7}, Q = {1, 3, 6, 10, 15}, Universal Set S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

The cardinal number of P ∪ Q is

a. 10

b. 9

c. 8

d. None of these

Que 9. The data to be used If P = {1, 2, 3, 5, 7}, Q = {1, 3, 6, 10, 15}, Universal Set S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

n (P') is

a. 10

b. 5

c. 6

d. None of these

Que 10. The data to be used If P = {1, 2, 3, 5, 7}, Q = {1, 3, 6, 10, 15}, Universal Set S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

n (Q') is

a. 4

b. 10

c. 4

d. None of these

Que 11. The set of cubes of the natural number is

a. A finite set

b. An infinite set

c. A null set

d. None of these

Que 12. The set $\{ 2^x \mid x \text{ is any positive rational number} \}$ is

a. An infinite set

b. A null set

c. A finite set

d. None of these

Que 13. $\{1 - (-1)^x\}$ for all integer x is the set

a. $\{0\}$

b. $\{2\}$

c. $\{0, 2\}$

d. None of these

Que14. E is a set of positive even numbers and O is a set of positive odd numbers, then E ∪ O is a

a. Set of whole numbers

b. N

c. A set of rational number

d. None of these

Que 15. If R is the set of positive rational numbers and E is the set of real numbers then

a. $R \subseteq E$

b. $R \subset E$

c. $E \subset R$

d. None of these

Que.16 If N is the set of natural numbers and I is the set of positive integers, then

a. $N \subset I$

b. $N \subset I$

c. $N \subseteq I$

d. None of these

Que.17 If I is the set of isosceles triangles and E is the set of equilateral triangles, then

a. $I \subset E$

b. $E \subset I$

c. $E = I$

d. None of these

Que 18. If R is the set of isosceles right angled triangles and I is set of isosceles triangles, then

a. $R = I$

b. $R \supset I$

c. $R \subset I$

d. None of these

Que19. { n(n + 1)/2 : n is a positive integer) is

a. A finite set

b. An infinite set

c. Is an empty set

d. None of these

Que20. If $A = \{1, 2, 3, 5, 7\}$, and $B = \{x^2 : x \in A\}$

a. $n(b) = n(A)$

b. $n(B) > n(A)$

c. $n(A) = n(B)$

d. $n(A) < n(B)$

Que21. A ∪ A is equal to

a. A

b. E

c. ϕ

d. None of these

Que22. $A \cap A$ is equal to

a. ϕ

b. A

c. E

d. None of these

Que23. $(A \cup B)'$ is equal to

a. $(A \cap B)'$

b. $A \cup B'$

c. $A' \cap B'$

d. None of these

Que24. $(A \cap B)'$ is equal to

a. $(A' \cup B)'$

b. $A' \cup B'$

c. $A' \cap B'$

d. None of these

Que25. A ∪ E is equal to (E is a superset of A)

a. A

b. E

c. ϕ

d. None of these

Que26. A ∩ E is equal to (E is a superset of A)

a. A

b. E

c. ϕ

d. None of these

Que 27. E ∪ E is equal to E is a superset of A

a. E

b. ϕ

c. 2E

d. None of these

Que 28. A∩ E’ is equal to E is a superset of A

a. E

b. ϕ

c. A

d. None of these

Que 29. $A \cap \phi$ is equal to E is a superset of A

a. A

b. E

c. ϕ

d. None of these

Que 30. A ∪ A’ is equal to E is a superset of A

a. E

b. ϕ

c. A

d. None of these

Que 31. If E = {1, 2, 3, 4, 5, 6, 7, 8, 9}, the subset of E satisfying 5 + x > 10 is

a. {5, 6, 7, 8, 9}

b. {6, 7, 8, 9}

c. { 7, 8, 9}

d. None of these

Que 32. If $A \Delta B = (A-B) \cup (B-A)$ and $A = \{1, 2, 3, 4\}$, $B = \{3, 5, 7\}$ than $A \Delta B$ is

a. $\{1, 2, 4, 5, 7\}$

b. $\{3\}$

c. $\{1, 2, 3, 4, 5, 7\}$

d. None of these

Jan 2021

The set of cubes of natural number is

(a) Null set

(b) A finite set

(c) An infinite set

(d) Singleton Set

ANS : C

JUNE 2012 , MAY 2018

The numbers of proper subsets of the set { 3, 4, 5, 6, 7} is:

(a) 32

(b) 31

(c) 30

(d) 25

ANS : b

JUNE 2022 , DEC 2020

Two finite sets have m and n elements. The total number of sub - sets of the first set is 56 more than the total number of sub -sets of the second set. The values of m and n are

(a) 6 , 3

(b) 7 , 6

(c) 5 , 1

(d) 8 , 7

ANS : a

JUNE 2019

If A = {1, 2, 3, 4, 5, 6, 7, 8, 9} ; B = {1, 3, 4, 5, 7, 8} ; C = {2, 6, 8}

Then find (A−B) ∪ C

(a) {2, 6}

(b) {2, 6, 8}

(c) {2, 6, 8, 9}

(d) None

ANS : c

JUNE 2019

If A = {1, 2, 3, 4, 5, 6, 7} and B={2, 4, 6, 8}. Cardinal number of A - B is:

(a) 4

(b) 3

(c) 9

(d) 7

Ans : a

May 2018

Let U be the universal set, A and B are the subsets of U. If n(U)=650, n(A) = 310, n(A ∩ B) = 95 and n(B) = 190. then $n(\bar{A} \cap \bar{B})$ is equal to

(a) 400

(b) 200

(c) 300

(d) 245

Ans : d

Ordered Pair

- Two numbers a and b listed in a specific order and enclosed in parentheses form an ordered pair (a, b).

$$(a, b) \neq (b, a)$$

Cartesian Product of Two Sets

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}.$$

RELATION

- Let A and B be two sets. Then a relation R from set A to set B is a subset of A × B.
- Thus, R is a relation from A to B ⇔ R ⊆ A × B

- If A and B are finite sets consisting of m and n elements respectively then A × B has mn ,

total number of relations from A to B is 2^{mn}.

USE MY CODE : SS12

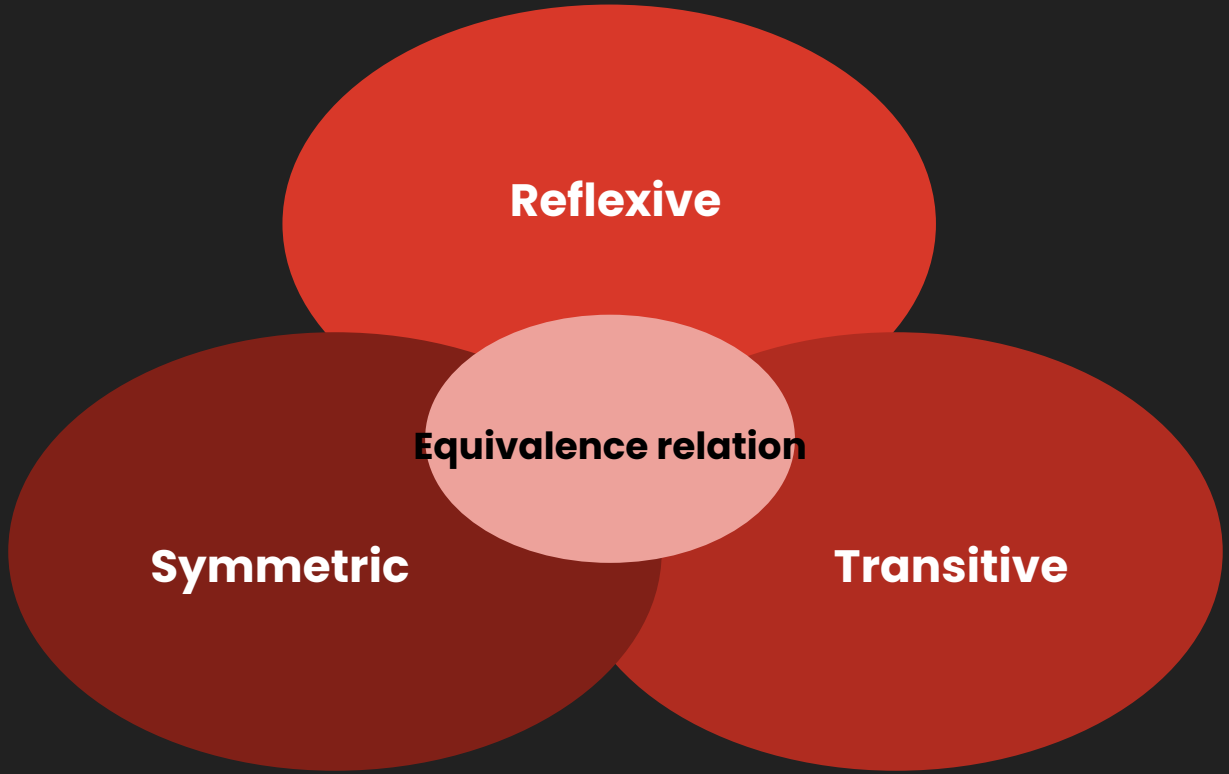
DOMAIN ,RANGE , CODOMAIN OF A RELATION

➢ If A = {1, 3, 5, 7}

B = {2, 4, 6, 8, 10} and R is relation from A to B

R = {(1, 8), (3, 6), (5, 2), (1, 4)}

USE MY CODE : SS12


Reflexive
Equivalence relation
Symmetric
Transitive

REFLEXIVE RELATION

Example: A = {1, 2, 3} and R_1, R_2, R_3 be the relations given as

$R_1 = \{(1, 1), (2, 2), (3, 3)\}$ — Reflexive relation

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3)\}$ — Reflexive relation

$R_3 = \{(2, 2), (2, 3), (3, 2), (1, 1)\}$ — Not Reflexive relation

SYMMETRIC RELATION

Example: Let A = {1, 2, 3} and R_1 , R_2 , R_3 be the relations on A

$R_1 = \{(1,2), (2, 1)\}$,

Symmetric relation

$R_2 = \{(1, 2), (2, 1), (1,3), (3, 1)\}$

Symmetric relation

$R_3 = \{(1, 3), (3, 1), (2, 3)\}$

Not Symmetric relation

TRANSITIVE RELATION

Example: Let A = {1, 2, 3} and R_1 and R_2 in A be defined as

$R_1 = \{(1, 2), (2, 3), (1, 3), (3, 2)\}$

Not Transitive relation

$R_2 = \{(1, 3), (3, 2), (1, 2)\}$

Transitive relation

IDENTITY RELATION

If A = {1, 2, 3}

Let R_1 , R_2, R_3 be relation on A

$R_1 = \{(1, 1), (2, 2), (3, 3)\}$ — Identity relation

$R_2 = \{(1, 1), (2, 2)\}$ — Not Identity relation

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$ — Not Identity relation

INVERSE RELATION

$R^{-1} = \{ (b, a) : (a, b) \in R \}$

EXAMPLE

Let $A = \{ 1, 2, 3 \}$,

$B = \{ a, b, c, d \}$

be two sets and

let $R = \{ (1, a), (1, c), (2, d), (2, c) \}$ be a relation from A to B .

Then , $R^{-1} = \{ (a, 1), (c, 1), (d, 2), (c, 2) \}$ is a relation from B to A .

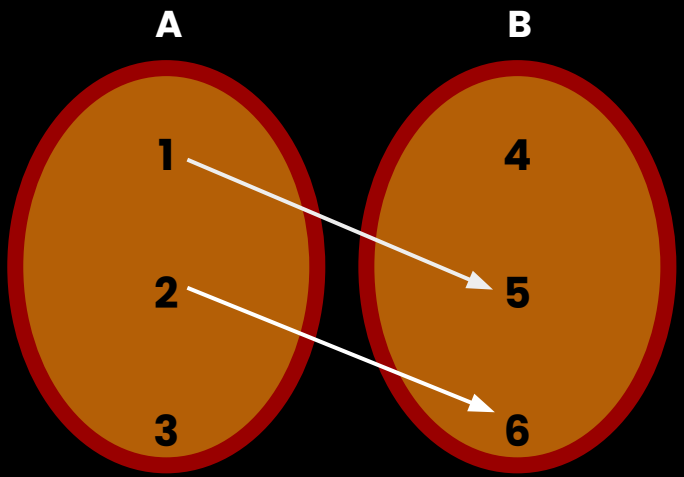
FUNCTION

- Let A and B be two non-empty sets. Then a function 'f' from set A to set B is a rule or method or correspondence which associates elements of set A to elements of set B such that:

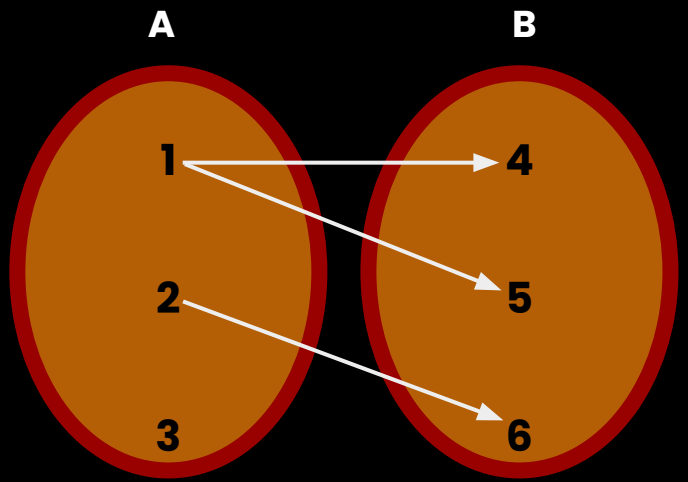
i. All elements of set A are associated to elements in set B.

ii. An element of set A is associated to a unique element in set B.

Que. Identity which of them is a function from A to B ?



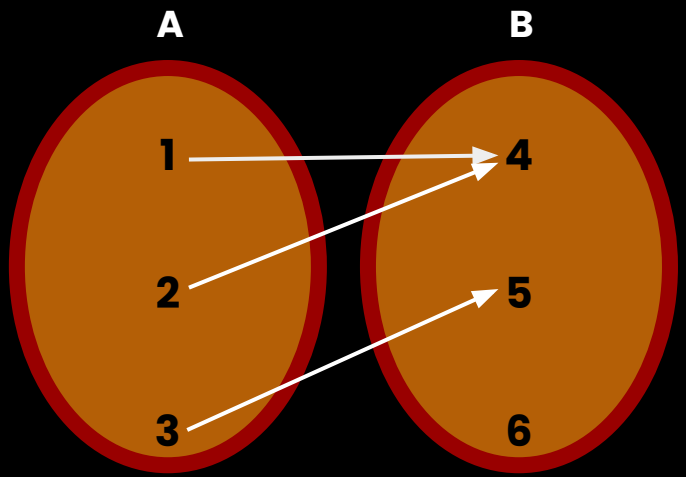
Not a Function



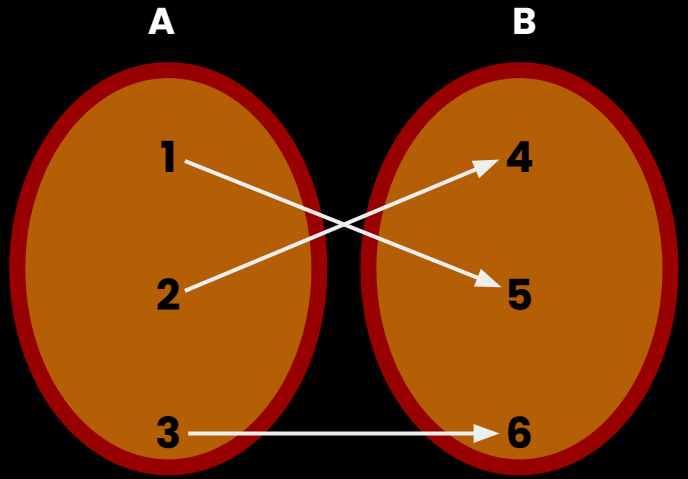
Not a Function

USE MY CODE : SS12

Que. Identity which of them is a function from A to B ?



Function



Function

USE MY CODE : SS12

June 2019

If A= {a, b, c, d}; B = {p, q, r, s} which of the following relation is a function from A to B

(a) R_1 = {(a, p), (b, q), (c, s)}

(b) R_2 = {(p, a), (b, r), (d, s)}

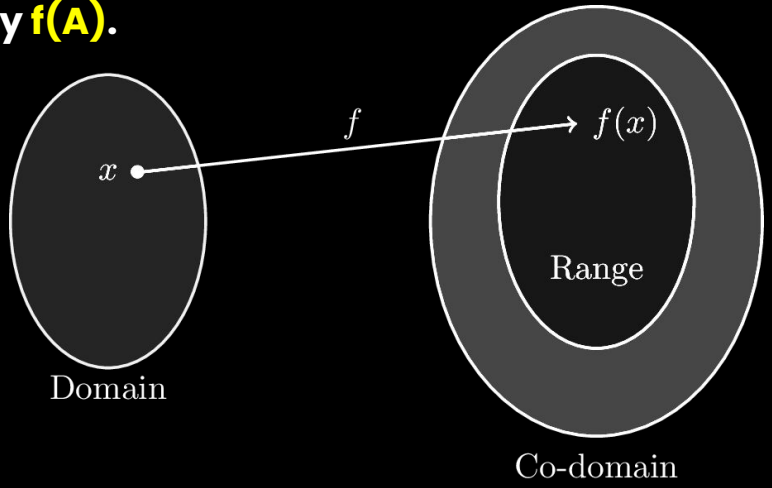
(c) R_3 = {(b, p), (c, s), (b, r)}

(d) R_4 = {(a, p), (b, r), (c, q), (d, s)}

Ans : d

Domain, Codomain and Range of Function

- If f : A ⟶ B, the set A is known as the **domain** of f and the set B is known as the **co-domain** of f.
- The set of all f-images of elements of A is known as **the range of f or image set of A under f** and is denoted by **f(A)**.



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Nov 2018

A is {1, 2, 3, 4} and B is { 1, 4, 9, 16, 25} if a function f is defined from set A to B where $f(x) = x^2$ then the range of f is:

(a) {1, 2, 3, 4}

(b) {1, 4, 9, 16}

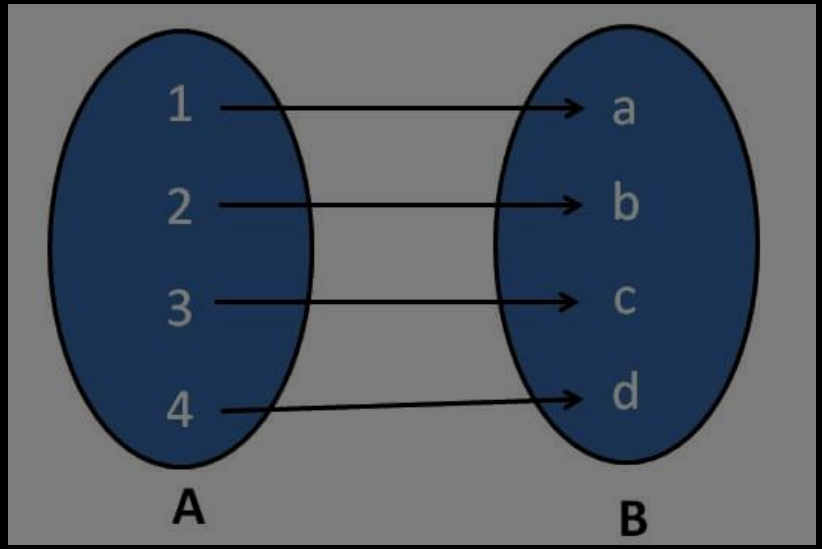
(c) {1, 4, 9, 16, 25}

(d) None of these

Ans : b

TYPES OF FUNCTION

ONE – ONE FUNCTION



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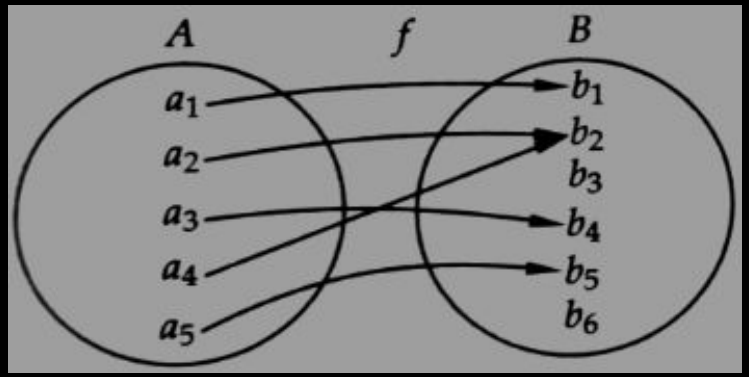
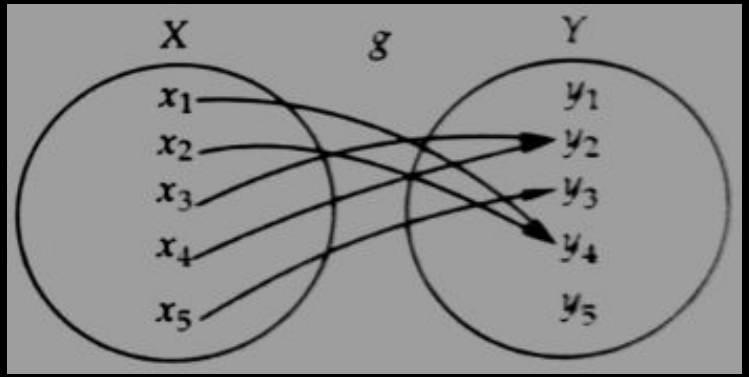
TYPES OF FUNCTION

MANY ONE FUNCTION





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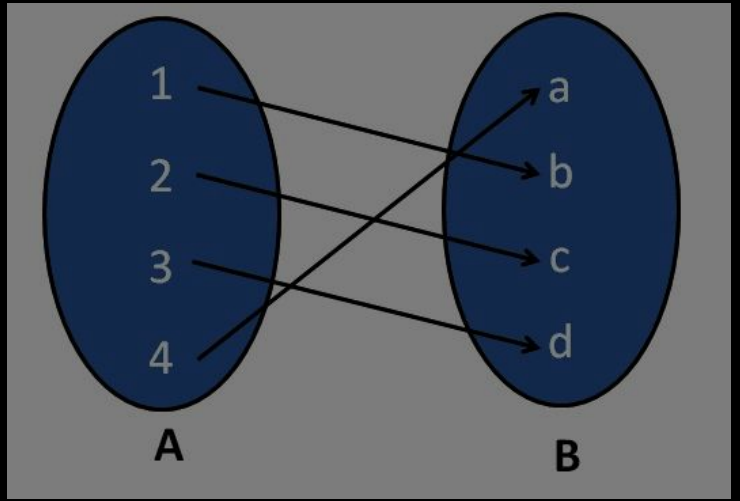
TYPES OF FUNCTION

ONTO FUNCTION



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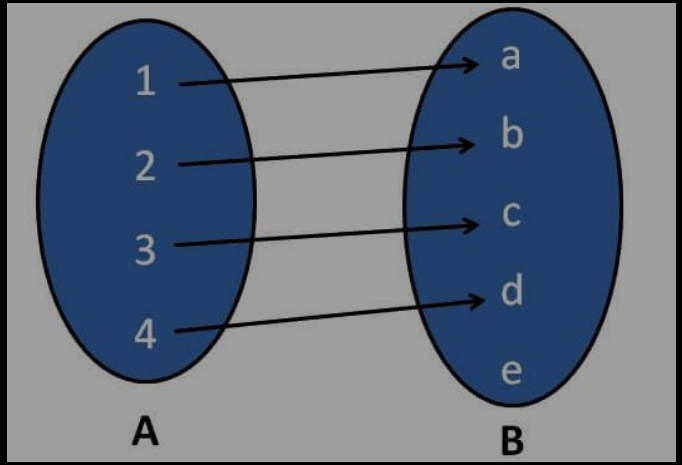
TYPES OF FUNCTION

INTO FUNCTION



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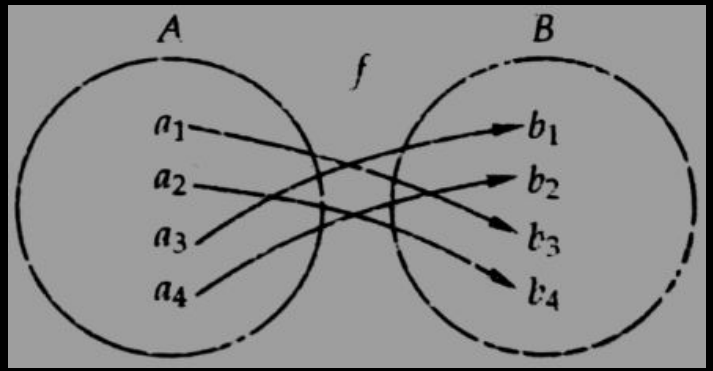
TYPES OF FUNCTION

BIJECTIVE FUNCTION (ONE ONE ONTO)



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Dec 2014

Let N be the set of all Natural number; E be the set of all even natural numbers then the function

F : N → E defined as $f(x) = 2x$; $\forall X \in N$ is:

(a) One-one into

(b) One-one onto

(c) Many-one into

(d) Many-one onto

Ans : b

IDENTITY FUNCTION

- The function $f : R \rightarrow R$:

$$f(x) = x$$

- Dom (f) = R and Range (f) = R

CONSTANT FUNCTION

$f : R \rightarrow R :$

$f(x) = k$

- Dom (f) = R and
- Range (f) is the singleton set { k}

Dec 2014

The No. of elements in range of constant function is

(a) One

(b) Zero

(c) Infinite

(d) None

Ans : a

COMPOSITION OF FUNCTION

- $fog(x) = f(g(X))$
- $gof(x) = g(f(x))$

June 2019

If $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(a) go f(3) = 3

(b) go f(−3) = 9

(c) go f(9) = 3

(d) go f(−9) = 3

Ans : a

July 2021

If $f(x) = x^2 - 1$ and $g(x) = |2x + 3|$, then $fog(3) - g\,of(-3) =$

(a) 71

(b) 61

(c) 41

(d) 51

Ans : b

INVERSE OF FUNCTION

- Let $f : A \longrightarrow B$ be one -one and onto function , then there exist a unique function $g : B \longrightarrow A$,

  such that $f(x) = y \Leftrightarrow g(y) = x \ \forall \ x \in A , y \in B$.

. Then g is said to be inverse of f . Thus $g = f^{-1}$

INVERSE OF FUNCTION

- **ALGORITHM**

Let $f : A \rightarrow B$ be a bijection . To find the inverse of f we follow the following steps :

STEP 1 : Put $f(x) = y$

STEP 2: Solve $f(x) = y$ to obtain x in terms of y

STEP 3: In the relation obtained in Step 2 replace x by $f^{-1}(y)$ to obtain the required inverse

Dec 2021 , June 2022

If $u(x) = 1/(1-x)$, then $u^{-1}(x)$ is

(a) $1/(x-1)$

(b) $1-x$

(c) $1-(1/x)$

(d) $(1/x)-1$

Ans : C

EQUAL FUNCTION

- Two function f and g are said to be equal , written as f=g

  if they have the same domain and they satisfy the

  condition f(x) = g(x) , for all x

Venn diagram

1. Out of a group of 20 teachers in a school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry. 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics? How many teach only Physics ?

Venn diagram

2. A survey shows that 74% of the Indians like grapes, whereas 68% like bananas. What percentage of the Indians like both grapes and bananas?

Venn diagram

3. In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Find the number of students who like

   (i) Maths only (ii) Science only (iii) either Maths or Science

   (iv) neither Maths nor Science

Exercise 7(B)

Que 1. If A = {x, y, z}, B = {p, q, r, s} which of the relations on A to B are functions.

a. {(x, p), (x, q), (y, r), (z, s)}

b. {(x, s), (y, s), (z, s)}

c. {(y, p), (y, q), (y, r),(z, s)}

d. {(x, p), (y, r), (z, s)}

Que.2 $\{(x, y) \mid x + y = 5\}$ where $x, y \in R$ is a

a. Not a function

b. A composite function

c. One-one mapping

d. None of these

Que 3. $\{(x, y) \mid x = 4\}$ where $x, y \in R$ is a

a. Not a function

b. Function

c. One-one mapping

d. None of these

Que.4 $\{(x, y), y = x^2\}$ where $x, y \in R$ is

a. Not a function

b. A function

c. Inverse mapping

d. None of these

Que 5. $\{(x, y) \mid x < y\}$ where $x, y \in R$ is

a. Not a function

b. A function

c. One-one mapping

d. None of these

Que 6. The domain of {(1, 7), (2, 6)} is

a. {1, 6}

b. {7, 6}

c. {1, 2}

d. {6, 7}

Que 7. The range of {(3, 0), (2, 0), (1, 0), (0, 0)} is

a. {0, 0}

b. {0}

c. {0, 0, 0, 0}

d. None of these

Que 8. The domain and range of $\{(x, y) : y = x^2\}$ where $x, y \in R$ is

a. (Reals, Natural Numbers)

b. (Reals, Reals)

c. (Reals, Non-negative reals)

d. None of these

Que 9. Let the domain of x be the set {1}. Which of the following functions gives values equal to 1

a. $f(x) = x^2, g(x) = x$

b. $f(x) = x, g(x) = 1-x$

c. $f(x) = x^2 + x + 2, g(x) = (x+1)^2$

d. None of these

Que.10 If $f(x) = 1/1 - x$, $f(-1)$ is

a. 0

b. 1/2

c. 0

d. None of these

Que.11 If $g(x) = (x - 1)/x$, $g(-½)$ is

a. 1

b. 2

c. 3/2

d. 3

Que 12. If $f(x) = 1/1 - x$ and $g(x) = (x - 1)/x$, then $fog(x)$ is

a. x

b. $1/x$

c. $-x$

d. None of these

Que 13. If $f(x) = 1/1 - x$ and $g(x) = (x - 1)/x$, then $gof(x)$ is

a. $x-1$

b. x

c. $1/x$

d. None of these

Que 14. The function $f(x) = 2^x$ is

a. One-one mapping

b. Many-one

c. One-many

d. None of these

Que 15. The range of the function $f(x) = \log_{10}(1 + x)$ for the domain of real values of x when $0 \leq x \leq 9$ is

a. $[0, 1]$

b. $[0, 1, 2]$

c. $\{0, 1\}$

d. None of these

Que 16. The Inverse function f^{-1} of $f(x) = 2x$ is

a. $1/2x$

b. $x/2$

c. $1/x$

d. None of these

Que 17. If $f(x) = x + 3$, $g(x) = x^2$, then $fog(x)$ is

a. $x^2 + 3$

b. $x^2 + x + 3$

c. $(x + 3)^2$

d. None of these

Que 18. If $f(x) = x + 3$, $g(x) = x^2$, then $f(x).g(x)$ is

a. $(x + 3)^2$

b. $x^2 + 3$

c. $x^3 + 3x^2$

d. None of these

Que 19. The Inverse h1 when $h(x) = \log_{10} x$ is

a. $\log_{10} x$

b. 10^x

c. $\log_{10}(1/x)$

d. None of these

Que 20. For the function $h(x) = 10^{1+x}$ the domain of real values of x where $0 \leq x \leq 9$, the range is

a. $10 \leq h(x) \leq 10^{10}$

b. $0 \leq h(x) \leq 10^{10}$

c. $0 < h(x) < 10$

d. None of these

EXERCISE 7(c)

Ques

1. "Is smaller than" over the set of eggs in a box is

(a) Transitive (T) (b) Symmetric (S) (c) Reflexive (R) (d) Equivalence (E)

Ques 2. "Is equal to" over the set of all rational numbers is

(a) (T) (b) (S) (c) (R) (d) E

Ques

3. "has the same father as" ..... over the set of children

(a) R (b) S (c) T (d) E

Ques

4. "is perpendicular to" over the set of straight lines in a given plane is

(a) R (b) S (c) T (d) E

Ques

5. "is the reciprocal of" ..... over the set of non-zero real numbers is

(a) S (b) R (c) T (d) none of these

Ques

6. $\{(x, y)/x \in x, y \in y, y = x\}$ is

(a) R (b) S (c) T (d) E

Ques 7. {(x,y) / x + y = 2x where x and y are positive integers}, is

(a) R (b) S (c) T (d) E

Ques 8. "Is the square of" over n set of real numbers is

(a) R (b) S (c) T (d) none of these

Ques

9. If A has 32 elements, B has 42 elements and $A \cup B$ has 62 elements, the number of elements in $A \cap B$ is

(a) 12 (b) 74 (c) 10 (d) none of these

Ques

10. In a group of 20 children, 8 drink tea but not coffee and 13 like tea. The number of children drinking coffee but not tea is

(a) 6 (b) 7 (c) 1 (d) none of these

Ques

11. The number of subsets of the sets {6, 8, 11} is

(a) 9 (b) 6 (c) 8 (d) none of these

Ques

12. The sets $V = \{x / x+2=0\}$, $R=\{x / x^2+2x=0\}$ and $S = \{x : x^2 + x - 2 = 0\}$ are equal to one another if x is equal to

(a) –2 (b) 2 (c) ½ (d) none of these

Ques 13. If the universal set $E = \{x \mid x \text{ is a positive integer } < 25\}$, $A = \{2, 6, 8, 14, 22\}$, $B = \{4, 8, 10, 14\}$ then

(a) $(A \cap B)' = A' \cup B'$ (b) $(A \cap B)' = A' \cap B'$ (c) $(A' \cap B)' = \phi$ (d) none of these

Ques

14. If the set P has 3 elements, Q four and R two then the set $P \times Q \times R$ contains

(a) 9 elements (b) 20 elements (c) 24 elements (d) none of these

Ques

15. Given $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$ then $A \times (B \cap C)$ is

(a) $\{(2, 5), (3, 5)\}$ (b) $\{(5, 2), (5, 3)\}$ (c) $\{(2, 3), (5, 5)\}$ (d) none of these

Ques

16. A town has a total population of 50,000. Out of it 28,000 read the newspaper X and 23,000 read Y while 4,000 read both the papers. The number of persons not reading X and Y both is

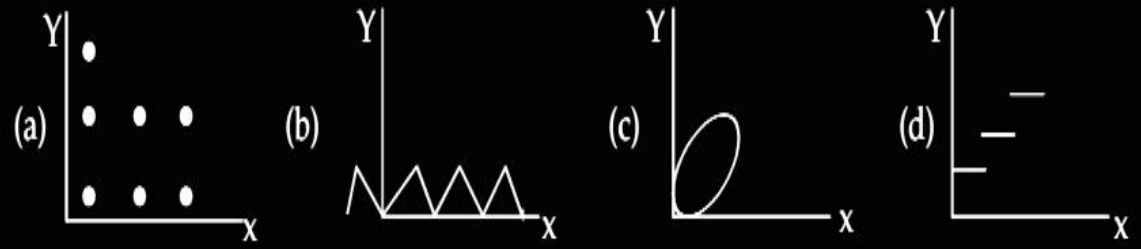
(a) 2,000 (b) 3,000 (c) 2,500 (d) none of these

Ques 17. If A = { 1, 2, 3, 5, 7} and B = {1, 3, 6, 10, 15}. Cardinal number of A-B is

(a) 3 (b) 4 (c) 6 (d) none of these

Ques

18. Which of the diagram is graph of a function



Ques

19. At a certain conference of 100 people there are 29 Indian women and 23 Indian men. Out of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreign doctors. The number of women doctors attending the conference is

(a) 2 (b) 4 (c) 1 (d) none of these

Ques 20. Let A = {a, b}. Set of subsets of A is called power set of A denoted by P(A). Now n(P(A) is

(a) 2 (b) 4 (c) 3 (d) none of these

Ques 21. Out of 2000 employees in an office 48% preferred Coffee (c), 54% liked (T), 64% used to smoke (S). Out of the total 28% used C and T, 32% used T and S and 30% preferred C and S, only 6% did none of these. The number having all the three is

(a) 360 (b) 300 (c) 380 (d) none of these

Ques 22. Referred to the data of Q. 21 the number of employees having T and S but not C is

(a) 200 (b) 280 (c) 300 (d) none of these

Ques

23. Referred to the data of Q. 21 the number of employees preferring only coffee is

(a) 100 (b) 260 (c) 160 (d) none of these

Ques

24. If $f(x) = x + 3$, $g(x) = x^2$, then g of (x) is

(a) $(x + 3)^2$ (b) $x^2 + 3$ (c) $x^2 (x + 3)$, (d) none of these

Ques 25. If $f(x) = 1/1-x$, then $f^{-1}(x)$ is

(a) $1-x$ (b) $(x-1)/x$ (c) $x/(x-1)$ (d) none of these

Dec 2014

If a relation S = { (1,1), (2,2), (1,2), (2,1) } on S = { 1,2,3 } is symmetric and

(a) Reflexive but not transitive

(b) Reflexive as well as transitive

(c) Transitive but not reflexive

(d) Neither transitive nor reflexive

Ans : c

Jan 2021

In the set of all straight lines on a plane which of the following is Not 'TRUE'?

(a) Parallel to an equivalence relation

(b) Perpendicular to is a symmetric relation

(c) Perpendicular to is an equivalence relation

(d) Parallel to a reflexive relation

Ans : c

June 2019

If A= {a, b, c, d}; B = {p, q, r, s} which of the following relation is a function from A to B

(a) R_1 = {(a, p), (b, q), (c, s)}

(b) R_2 = {(p, a), (b, r), (d, s)}

(c) R_3 = {(b, p), (c, s), (b, r)}

(d) R_4 = {(a, p), (b, r), (c, q), (d, s)}

Ans : d

Nov 2018

A is {1, 2, 3, 4} and B is { 1, 4, 9, 16, 25} if a function f is defined from set A to B where $f(x) = x^2$ then the range of f is:

(a) {1, 2, 3, 4}

(b) {1, 4, 9, 16}

(c) {1, 4, 9, 16, 25}

(d) None of these

Ans : b

Concept Of Limit

We say that $\lim_{x \to a} f(x) = l$ if whenever $x \to a$, $f(x) \to l$.

Concept Of Limit

- We consider a function $f(x) = 2x$.
- If x is a number approaching to the number 2 then $f(x)$ is a number approaching to the value $2(2) = 4$
- The following table shows $f(x)$ for different values of x approaching 2

x	f(x)
1.90	3.8
1.99	3.98
1.999	3.998
1.9999	3.9998

- Here x approaches 2 from values of $x < 2$ and for x being very close to 2 ,f(x) is very close 4. This situation is defined as left-hand limit of f(x) as x approaches 2 and is written as $\lim f(x) = 4,\ x \longrightarrow 2^-$

x	f(x)
2.0001	4.0002
2.001	4.002
2.01	4.02

- Here x approaches 2 from values of x greater than 2 and for x being very close 2 ,f(x) is very close to 4. This situation is defined as right-hand limit of f(x) as x approaches 2 and is written as $\lim f(x) = 4$ as $x \longrightarrow_{2+}$

Existence of a Limit

- $\lim_{x \to a} f(x)$ is said to exist when both left-hand limits and right hand limits exists and they are equal

$$\lim_{x \mapsto a-} f(x) = \lim_{x \mapsto a+} f(x)$$

LHL and RHL Concept

Example: $fx = \begin{cases} -3x & \text{when } x < 0 \\ 2x & \text{when } x > 0 \end{cases}$ **s:**

Test the existence of $\lim_{x \to 0} f(x)$.

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Important Results

$$\lim_{x \to a} f\{f(x) + g(x)\} = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$\lim_{x \to a}\{f(x) - g(x)\} = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

$$\lim_{x \to a}\{f(x) \cdot g(x)\} = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

Important Results

$$\lim_{x \to a} \{ f(x) / g(x) \} = \left\{ \lim_{x \to a} f(x) \right\} / \left\{ \lim_{x \to a} g(x) \right\}$$

$$\lim_{x \to a} c = c \text{ where } c \text{ is constant}$$

$$\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$$

Methods Of Solving

- **DIRECT SUBSTITUTION**

Rule 1 : Put x = a in the given function. If f(a) is a definite value then

$$\lim_{x \to a} f(x) = f(a).$$

Example:

$$\lim_{x \to 1} (x^2 + 5x - 2) = (1^2 + 5 \times 1 - 2) = 4.$$

Example 1: Evaluate (i) $\lim_{x \to 2}(3x + 9);$

USE MY CODE : SS12

INDETERMINATE FORMS OF LIMITS

$$\frac{\infty}{\infty},\quad \frac{0}{0},\quad \infty-\infty,\quad 0^0,\quad 0\cdot\infty,\quad \infty^0,\quad 1^\infty$$

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- If f(a) is indeterminate, we adopt the rules given below.

Rule 2: If f(x) is a rational function then factorize the numerator and the denominator. Cancel out the common factors and then x = a.

Example: Evaluate (i) $\lim_{x \to 3}\left(\frac{x^2-9}{x-3}\right)$

USE MY CODE : SS12

- If f(a) is indeterminate, we adopt the rules given below.

Rule 3 : If the given function contains a surd then simplify it by using conjugate surds. After simplification, put x = a.

Example: Evaluate $\lim_{x \to 0} \left\{ \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \right\}$

USE MY CODE : SS12

Some Important Limits

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$$

$$\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to a} \left(\frac{x^n - a^n}{x - a}\right) = na^{n-1},$$

Where a > 0

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Some Important Limits

$$\lim_{x \to a}\left(\frac{x^n - a^n}{x - a}\right) = na^{n-1},$$ Where a > 0

Evaluate $\lim_{x \to a}\left\{\frac{x^{12} - a^{12}}{x - a}\right\}$.

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Some Important Limits

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

Evaluate $\lim_{x \to 0} \left(\frac{e^{3x} - 1}{x} \right)$.

$$\lim_{x \to a} f(x)^{g(x)} = e^{\lim_{x \to a} g(x)\{f(x)-1\}}$$

USE MY CODE : SS12

Example: Find $\lim_{x \to \infty} \left(1+\frac{9}{x}\right)^{x}$. (Form 1)

USE MY CODE : SS12

TRICK : Limits Of Rational Functions

$$\frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

USE MY CODE : SS12

TRICK : Limits Of Rational Functions

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \frac{a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots\ldots + a_mx^m}{b_0 + b_1x + b_2x^2 + \ldots\ldots\ldots\ldots + b_nx^n}$$

$m = n$	$m < n$	$m > n$	$m > n$
$\frac{a_m}{b_n}$	0	∞	$-\infty$
		$a_mb_n > 0$	$a_mb_n < 0$

USE MY CODE : SS12

Example: Find $\lim_{z\to\infty} \frac{2x+1}{x^3+1}$. $\left[\text{Form } \frac{\infty}{\infty}\right]$

USE MY CODE : SS12

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}, \quad \frac{0}{0},$$

Both f(x) and g(x) are continuous and differentiable at x = a

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Continue the process till you get a finite answer

USE MY CODE : SS12

- **By the term continuous we mean some thing which goes on without interruption and without abrupt changes.**

CONTINUITY

- **A function f(x) is said to be continuous at x = a if and only if**

(i) f(x) is defined x= a

(ii) $\lim_{x \mapsto a-} f(x) = \lim_{x \mapsto a+} f(x)$

(iii) $\lim_{x \mapsto a} f(x) = f(a)$

USE MY CODE : SS12

CONTINUITY

- **A function f(x) is said to be continuous at x = a if and only if**

$$\lim_{x \mapsto a-} f(x) = \lim_{x \mapsto a+} f(x) = f(a)$$

OR

$$\lim_{x \mapsto a} f(x) = f(a)$$

USE MY CODE : SS12

CONTINUITY

i. The sum, difference and product of two continuous functions is a continuous function.

ii. The quotient of two continuous functions is continuous function provided the denominator is not equal to zero.

Important results on continuous function

A constant function $f(x) = k$, is continuous everywhere.

Identity function $f(x) = x$, is continuous everywhere.

A polynomial function $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, $n \in N$, $x \in R$, is continuous everywhere.

Important results on continuous function

The modulus function $f(x) = |x|$ is continuous everywhere.

The logarithmic function is continuous in $(0, \infty)$.

The exponential function is continuous everywhere.

Example $f(x) = \left(\frac{1}{2} - x\right)$: $x<1/2$

$$= \frac{3}{2} - x \quad \text{when } \frac{1}{2}<x<1$$

$$= \frac{1}{2} \quad \text{when } x=\frac{1}{2}$$

Discuss the continuity of f(x) at $x= \frac{1}{2}$

USE MY CODE : SS12

Example Find points of discontinuity of the function $f(x) = \dfrac{x^2 + 2x + 5}{x^2 - 3x + 2}$