CA FOUNDATION JANUARY'25

QUANTITATIVE APTITUDE

👉🏻CHAPTER NAME : PERMUTATIONS & COMBINATIONS

👉🏻CONCEPTS : FACTORIAL NOTATION , PERMUTATIONS , CIRCULAR, COMBINATIONS & SHORTCUTS

🔥KEY FORMULAS & CONCEPTS REVISION IN 20 MINUTE

Permutations & Combinations

1) FACTORIAL :

$n!$ or $\lfloor n$

$n! = n(n-1)(n-2)\ldots 3 \times 2 \times 1$

$\boxed{\text{OR}}$

$n! = n(n-1)!$ or $n(n-1)(n-2)!$

REMEMBER,

$0! = 1$	$4! = 24$	$8! = 40320$
$1! = 1$	$5! = 120$	$9! = 362880$
$2! = 2$	$6! = 720$	$10! = 3628800$
$3! = 6$	$7! = 5040$	

2) RULES OF COUNTING : 1) AND/Multiplication Rule

↓

Total ways = $m \times n$ ways

2) OR/ Addition Rule

↓

Total ways = $m + n$ ways

3) PERMUTATIONS : 1) Arrangements

2) $^nP_r = \dfrac{n!}{(n-r)!}$, $n \geq r$ [Repetition not Allowed]

3) n^r ways (Repetition Allowed)

4) No. of factors in ${}^nP_r = r$

5) $(n+1)! - n! = n \cdot n!$

6) ${}^nP_n = {}^nP_{n-1}$

4) Problem on summation of all possible number

No repetition.

$$\boxed{(n-1)! \times (\text{sum of digits}) \times \frac{10^n - 1}{9}}$$

repetition

$$\boxed{n^{n-1} \times \text{sum of digits} \times \frac{10^n - 1}{9}}$$

5) WORD FORMATION

Always Together

1) FIRST ARRange objects that should be always together
(Jiske upar condition hai)

2) Now, consider these as "1" single object and "Add" to Remaining objects and Now ARRange THEM

3) Total ways = step 1 x step 2

Never Together

$$\boxed{\text{Total ways} - \text{Always Together}}$$

6) Permutations when letters are alike

$$\boxed{\text{No. of ways} = \frac{n!}{n_1!\, n_2!\, n_3!}}$$

7) Circular permutations :

1) To be used when we arrange objects along a "circle"

2) No. of ways $= (n-1)!$ ways

3) For garlands, necklace

↓

$$\boxed{\frac{1}{2}(n-1)! \text{ ways}}$$

6) Some key points to be remembered:

1) ${}^{n}P_{r} = {}^{n-1}P_{r} + r \cdot {}^{n-1}P_{r-1}$ eg: ${}^{6}P_{4} + 4 \cdot {}^{6}P_{3} = {}^{7}P_{4}$

2) Total number of arrangements of 2 particular things never occur together out of n things in $(n-2)(n-1)!$ Ways

3) $\sum_{r=1}^{n} r \cdot r! = (n+1)! - 1$

4) No. of Rearrangements $= (n! - 1)$ ways

5) $2n! = 2^{n}[1.3.5 \ldots (2n-1)] \cdot n!$

6) Number of Permutations of 'n' distinct objects taken 'r' at a time when a particular object is not taken in any arrangement is ${}^{n-1}P_{r}$ ways (NEVER included)

7) Always Included : $r \cdot {}^{n-1}P_{r-1}$ ways.

7) COMBINATION : 1) Selection

2) Order of selection → Not Imp

3) $\boxed{{}^nC_r = \dfrac{n!}{r!\,(n-r)!}}$

4) ${}^nC_0 = 1$, ${}^nC_n = 1$, ${}^nC_1 = n$

5) ${}^nC_r = \dfrac{{}^nP_r}{r!}$ or ${}^nP_r = {}^nC_r \times r!$

6) ${}^nC_r = {}^nC_{n-r}$ $({}^7C_5 = {}^7C_2)$

7) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

[eg: ${}^7C_5 + {}^7C_4 = {}^8C_5$]

8) If ${}^nC_x = {}^nC_y$, then

$x = y$ or $x + y = n$

9) No. of Handshakes = ${}^nC_2$

10) Combination of 'n' different things taken one or more out of 'n' things taken at a time : $2^n - 1$

11) If a task is to be done 'n' times with 'r' choices for every task, then total ways of doing task = n^r

12) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

13) ${}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1$

14) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = {}^nC_1 + {}^nC_3 + \dots = 2^{n-1}$

Note 1 : No of straight lines with the given n points : $^nC_2$

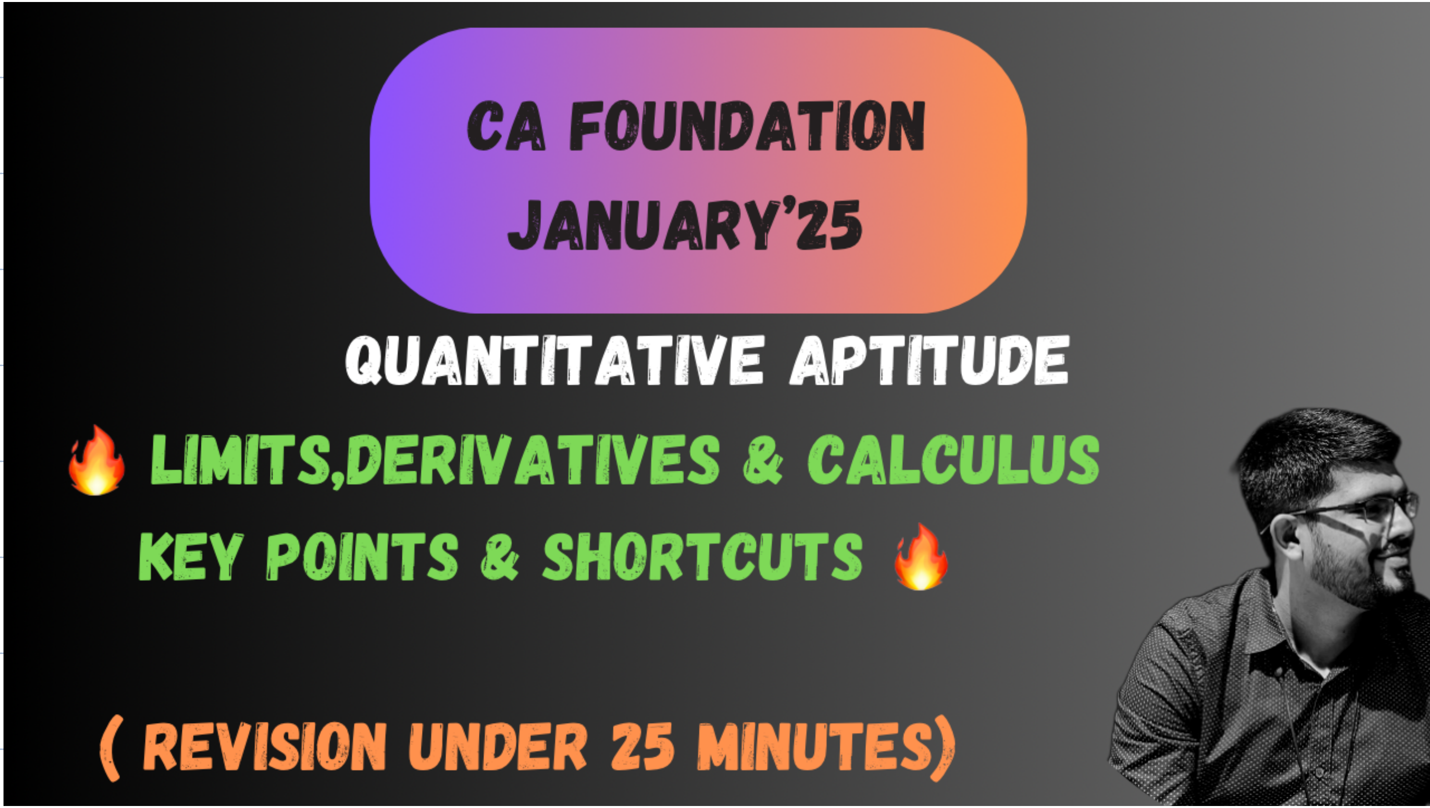
Note 2 : No of Triangle with the given n points : $^nC_3$

Note 3 : No of straight lines with the given n points where m points are collinear: $^nC_2 - {^mC_2} + 1$

Note 4 : number of triangles with the given n points where m points are collinear: $^nC_3 - {^mC_3}$

Note 5 :No of parallelogram with the given one set of m parallel lines And another set of n parallel lines : $^nC_2 \times {^mC_2}$

Note 6 : No of diagnols with n sides : $^nC_2 - n$ or $\frac{n(n-3)}{2}$


CA FOUNDATION
JANUARY'25
QUANTITATIVE APTITUDE
🔥 LIMITS,DERIVATIVES & CALCULUS
KEY POINTS & SHORTCUTS 🔥
(REVISION UNDER 25 MINUTES)

Limits, Derivatives & integral calculus

1) **LIMITS**: a) limit exist only If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

b) $\lim_{x \to a^-}$ ⇒ left hand limit $(x < a)$

c) $\lim_{x \to a^+}$ ⇒ Right hand limit $(x > a)$

d) If $\lim_{x \to a} \frac{f(x)}{g(x)}$ → a) Orally substitute limit value

b) If $\frac{0}{0}$, then do not apply limit

INSTEAD

- factorization
- Rationalize
- Standard formula

FORMULA :

1) $$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot (a)^{n-1}$$

2) $$\lim_{x \to 0} \frac{a^x - 1}{x} = \log a$$

3) $$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

4) $$\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$$

5) $$\lim_{x \to 0} (1+x)^{1/x} = e$$

6) $$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{1}{x^n} = 0$$

SHORTCUTS :

1) $$\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m} = \frac{n}{m} \cdot (a)^{n-m}$$

2) $$\lim_{x \to 0} (1 + kx)^{1/x} = e^k$$

3) $$\lim_{x \to \infty} (1 + k \cdot 1/x)^x = e^k$$

4) $$a^x + a^{-x} - 2 = \frac{(a^x - 1)^2}{a^x}$$

5) $$\lim_{x \to a} \frac{f(x)}{g(x)} \Rightarrow \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

$\downarrow$

Directly Differentiate N and D and

Substitute limit-

6) $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)}$

Degree of N = Degree of D

$\downarrow$

$\dfrac{\text{Coeff of Highest Power of N}}{\text{Coeff of Highest Power of D}}$

Degree of N > Degree of D

$\downarrow$

N.D

Degree of N < Degree of D

$\downarrow$

0

Differential Calculus

1. $\frac{d}{dx}(x^n) = n \cdot (x)^{n-1}$

2. $\frac{d}{dx}(e^x) = e^x$

3. $\frac{d}{dx}(a^x) = a^x \cdot \log a$

4. $\frac{d}{dx}(k) = 0$

5. $\frac{d}{dx}(\log x) = \frac{1}{x}$

6. $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

7. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\frac{d}{\delta x}(x) = 1$

8. $$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v \cdot \frac{du}{dx}$$

9. $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

10. If $f(x, y) = 0$ (Implicit function)

$$\frac{dy}{dx} = -\frac{[\text{Derivative w.r.t 'x' keeping 'y' constant}]}{\text{Derivative w.r.t 'y' keeping 'x' constant}}$$

$x^2 + 2xy + y^2 = 0$, $\frac{dy}{dx} = \boxed{-\frac{[2x + y(2)]}{2x + 2y}}$

(11) If $x^m \cdot y^n = (x+y)^{m+n}$, then

$$\frac{dy}{dx} = \frac{y}{x}, \quad \frac{d^2y}{dx^2} = 0$$

(12) If $ax^2 + 2hxy + by^2 = 0$, then

$$\frac{dy}{dx} = \frac{y}{x}, \quad \frac{d^2y}{dx^2} = 0$$

(13) If function $\left[\frac{x^n - y^n}{x^n + y^n}\right] = K$, then $\frac{dy}{dx} = \frac{y}{x}$

14 Slope of tangent = dy/dx (gradient)

15 Slope of normal = $-1/(dy/dx)$

16 If $y = f(x)$, then $f'(x) > 0$, Increasing function
$f'(x) < 0$, Decreasing function

17 **For minima/maxima:**

1. find $f(x)$
2. $f'(x)$
3. $f''(x)$
4. let $f'(x) = 0 \begin{cases} x = \_\_ \\ x = \_\_ \end{cases}$
5. Substitute value of x in $f''(x)$
   If $f''(x) > 0$, minima
   $f'(x) < 0$, maxima

(18) $\dfrac{d}{dx}(x^x) = x^x(1+\log x)$

(19) $y = A \cdot e^{mx} + B \cdot e^{nx}$ then

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$$

(20) $e^x = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dots \infty$

$e^{-x} = 1 - \dfrac{x}{1!} + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \dots \infty$

21) $C \rightarrow$ Total cost

$C = VC + FC$

VC $\downarrow$ $f(x)$ $\downarrow$ x = no. of items manufactured

FC $\downarrow$ Constant term $\downarrow$ Independent of x

22) Avg. cost $= \frac{C}{x}$, marginal cost $= \frac{d}{dx}(C)$

23) Total Revenue $\Rightarrow R = P \times D$

Avg. Revenue $= \frac{R}{D} = \frac{P \times D}{D} = P$

Marginal Revenue $\Rightarrow \frac{dR}{dP}$ or $\frac{dR}{dD}$

24) Profit $= R - C$

BEP $\Rightarrow R = C$

Integral calculus

① $$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

② $$\int 1 \, dx = x + c$$

③ $$\int e^x \, dx = e^x + c$$

④ $$\int e^{ax} \, dx = \frac{e^{ax}}{a} + c$$

⑤ $$\int \frac{dx}{x} = \log|x| + c$$

⑥ $$\int a^x \, dx = \frac{a^x}{\log a} + c$$

⑦ $$\int \frac{1}{\sqrt{x}} \, dx = 2\sqrt{x} + c$$

$$\int_a^b f'(x) \, dx = [f(x)]_a^b$$

$$= f(b) - f(a)$$

1) $f(x) \underset{I}{\overset{D}{\rightleftarrows}} f'(x)$

2) $C \underset{I}{\overset{D}{\rightleftarrows}} M \cdot C$

Note : when $x = 0$, $C =$ fixed cost

3) $R \underset{I}{\overset{D}{\rightleftarrows}} MR$

Note : when $x = 0$, $R = 0$

4) $x^x \underset{I}{\overset{D}{\rightleftarrows}} x^x(1+\log x)$

$\int u \cdot v \, dx$

$= u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v\right] dx$

5) $\int \log x \, dx = x \log x - x + c$

6) $\int u \cdot v \, dx = u \int v \, dx - \boxed{} \; \boxed{} + \boxed{} \; \boxed{} \ldots$

x^n exponential

n = natural no form

eg: $\int x^2 \cdot e^{3x}\,dx = x^2 \cdot \frac{e^{3x}}{3} - 2x\,\frac{e^{3x}}{9} + 2\,\frac{e^{3x}}{27} + c$

$x^2 \to x^n$, $e^{3x} \to e^x$

D

7) $\int_a^b \frac{f(x)}{f(x)+g(x)}\,dx = \frac{b-a}{2}$

8) i) $\int \frac{f'(x)}{f(x)}\,dx = \log|\text{Denominator}| + c$

ii) $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\,(\text{Denominator}) + c$

iii) $\int [f(x)]^n \cdot f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$

iv) $\int e^x[f(x)+f'(x)]\,dx = e^x(f(x)) + c$

5) $R = P \cdot Q$

$= (10800 - 4x^2) \cdot x$

$R = 10800x - 4x^3$

$$\frac{dR}{dx} = 10800 - 12x^2$$

$-30 > x \qquad x > 30$

$x < -30$

$$\frac{dR}{dx} > 0$$

$10800 - 12x^2 > 0$

$10800 > 12x^2$

$900 > x^2$

$-30 > x > 30$

But, x never be negative

$\boxed{x > 30}$

6) $C = 47x + 300x^2 - x^4$

$$AC = \frac{C}{x}$$

$$= \frac{47x + 300x^2 - x^4}{x}$$

$$AC = 47 + 300x - x^3$$

$$\frac{d(AC)}{dx} = 300 - 3x^2$$

i) AC is increasing

$$\frac{d(AC)}{dx} > 0$$

$$300 - 3x^2 > 0$$

$$300 > 3x^2$$

$$100 > x^2$$

$$10 > x$$

i.e. $\boxed{x < 10}$

$$\frac{d(AC)}{dx} < 0$$

$$300 - 3x^2 < 0$$

$300 < 3x^2$

$100 < x^2$

$10 < x$ i.e. $\boxed{x > 10}$

12) $TC = 40 + 2x$

$P = 120 - x$

1) $R = P \cdot Q$

$= (120 - x) \cdot x$

$R = 120x - x^2$

$\dfrac{dR}{} = 120 - 2x$

2) $\pi = R - C$

$= (120x - x^2) - (40 + 2x)$

$= 120x - x^2 - 40 - 2x$

$\pi = 118x - x^2 - 40$

$\dfrac{d\pi}{dx} = 118 - 2x$

dx

$$\frac{dR}{dx} > 0$$

$$120 - 2x > 0$$

$$120 > 2x$$

$$60 > x$$

i.e. $\boxed{x < 60}$

$$\frac{d\pi}{dx} > 0$$

$$118 - 2x > 0$$

$$118 > 2x$$

$$59 > x$$

i.e. $\boxed{x < 59}$

(iii) $E_d = -\frac{P}{D} \times \frac{dD}{dP}$

$$= \frac{-80}{40+2x} \times 2$$

$$= \frac{-160}{40+160}$$

$$= \frac{-160}{200}$$

$$= -\frac{4}{5}$$

(Relatively Inelastic)

$$= -0.8$$

$$Ed = -\frac{P}{D} \times \frac{dD}{dP}$$

$$Rm = RA\left[1 - \frac{1}{Ed}\right]$$

$$R_m = R_A\left[1 - \frac{1}{E_d}\right]$$

$$-40 = 40\left(1 - \frac{1}{E_d}\right)$$

$$-1 = 1 - \frac{1}{E_d}$$

$$\frac{1}{E_d} = 2$$

$$E_d = \frac{1}{2}$$

$$R = 120x - x^2$$

$$R_m = 120 - 2x$$

$$= 120 - 160$$

$$R_m = -40$$

$$R_A = \frac{R}{x}$$

$$= \frac{120x - x^2}{x}$$

$$R_A = 120 - x$$

$\boxed{E_d = 0.5}$

$RA = 120 - 80$

$RA = 40$

$$\boxed{\begin{aligned} E_d' &= -\frac{P}{D} \times \frac{dD}{dP} \\ &= -\frac{80}{120 - x} \times -1 \\ &= \frac{80}{40} \\ &= 2 \end{aligned}}$$

$P = 120 - x$

$= -\frac{120 - x}{x} \times -1$

$= \frac{40}{80}$

$= 1/2$

$$= -\frac{p}{x} \times \frac{dp}{dp}$$

$$p = 120 - x$$

$$x = 120 - p$$

$$= -\frac{p}{x} \times \frac{dx}{dp}$$

$$= -\frac{(120 - x)}{x} \times -1$$

$$= \frac{120 - 40}{40} = \frac{80}{40} = 2$$

$$Ed = -\frac{P}{D} \times \frac{dD}{dP}$$

$P = 120 - x$

$x = 120 - P$

$x = 120 - 80$

$x = 40$

$$= -\frac{P}{x} \times \frac{dx}{dP}$$

$$= -\frac{80}{40} \times -1$$

$$= \frac{80}{40}$$

$Ed = 2$ (relatively elastic)

8) $D = \dfrac{P+6}{P-3}$

$$\frac{dD}{dP} = \frac{(P-3)\cdot 1 - (P+6)(1)}{(P-3)^2}$$

$$= \frac{\cancel{P} - 3 - \cancel{P} - 6}{(P-3)^2}$$

$$= \frac{-9}{(P-3)^2}$$

$$E_d = \frac{-P}{D} \times \frac{dD}{dP}$$

$$= \frac{-P}{\left(\dfrac{P+6}{\cancel{P-3}}\right)} \times \frac{-9}{(P-3)^{\cancel{2}}}$$

At $P = 4$

$$= \frac{-4 \times -9}{10 \times 1}$$

$$= \frac{36}{10}$$

$$E_d = 3.6$$

$$D = \frac{2P+3}{3P-1}$$

$$\frac{dD}{dP} = \frac{(3P-1) \cdot 2 - (2P+3) \cdot 3}{(3P-1)^2}$$

$$= \frac{\cancel{6P} - 2 - \cancel{6P} - 9}{(3P-1)^2}$$

$$\frac{dD}{dP} = \frac{-11}{(3P-1)^2}$$

$$Ed = -\frac{P}{D} \times \frac{dD}{dP}$$

$$\frac{\cancel{11}}{14} = \frac{-P}{\left(\frac{2P+3}{\cancel{3P-1}}\right)} \times \frac{-\cancel{11}}{(3P-1)^{\cancel{2}}}$$

$$\frac{1}{14} = \frac{P}{(2P+3)(3P-1)}$$

$$(2P+3)(3P-1)$$

$$(2P+3)(3P-1) = 14P$$

$$6P^2 - 2P + 9P - 3 = 14P$$

$$6P^2 - 7P - 3 = 0$$

$$6P^2 - 9P + 2P - 3 = 0$$

$$3P(2P-3) + 1(2P-3) = 0$$

$$3P+1 = 0 \qquad 2P-3 = 0$$

$$3P = -1 \qquad 2P = 3$$

$$P = -1/3 \qquad \boxed{P = 3/2}$$

X

13) $E.C = (0.0003) I^2 + (0.075 I)$

$$MPC = \frac{d(EC)}{dI}$$

$$= 0.0006 I + 0.075$$

At $I = 1000$

$$= 0.0006 \times 1000 + 0.075$$

$$= \frac{6}{10\cancel{000}} \times 1\cancel{000} + 0.075$$

$$= 0.600 + 0.075$$

$$MPC = 0.675$$

$$MPS = 1 - MPC$$

$$= 1 - 0.675$$

$$MPS = 0.325$$

$$APC = \frac{Ec}{I}$$

$$APC = 0.0003 I + 0.075$$

At $I = 1000$

$APC = 0.375$

$APS = 1 - APC$

$= 1 - 0.375$

$APS = 0.625$

$$\left[\frac{(6x+4)^{3/2}}{3/2}\right]_0^2$$

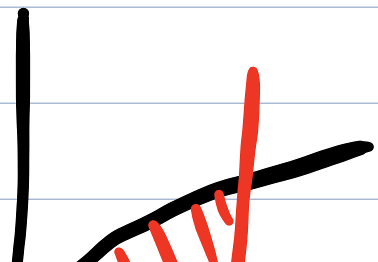
$$\frac{2}{3}\left[(16)^{3/2} - (4)^{3/2}\right]$$

$$\frac{2}{3}\left[(4)^{\cancel{2} \times 3/\cancel{2}} - (2)^{\cancel{2} \times 3/\cancel{2}} \right]$$

$$\frac{2}{3}\left(64 - 8 \right]$$

$$= \frac{2 \times 56}{3}$$

$$= \frac{112}{3} \text{ sq. units}$$



$$y^2 = 4x$$

$$y = \pm 2\sqrt{x}$$

$$\boxed{y = 2\sqrt{x}}$$

$$A = 2\int_0^3 y \, dx$$

$$= 2\int_0^3 2\sqrt{x} \, dx$$

$$y^2 = 4x$$

$$x^2 = 4y$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^3$$

$$= \frac{8}{3}\left((3)^{3/2}\right)$$

$$= \frac{8}{\cancel{3}} \times \cancel{3}\sqrt{3}$$

$$= 8\sqrt{3} \text{ sq. unit}$$

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

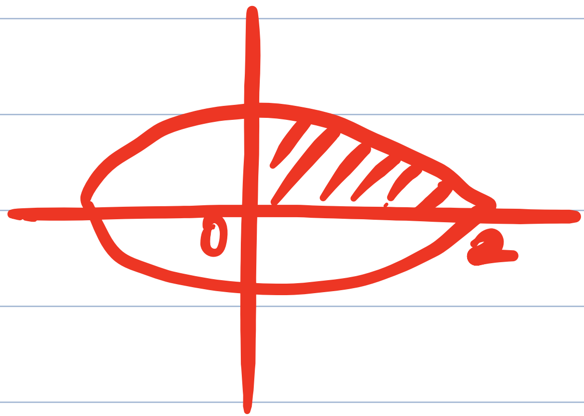
$$\frac{y^2}{25} = 1 - \frac{x^2}{4}$$

$a = 2 \qquad b = 5$

$$\frac{y^2}{25} = \frac{4 - x^2}{4}$$

$a < b$

$$y^2 = \frac{25}{4}(4 - x^2)$$



$$y = \pm \frac{5}{2}\sqrt{4 - x^2}$$

$$A = 4 \int^{2} \frac{5}{} \sqrt{4 - x^2}\, dx$$

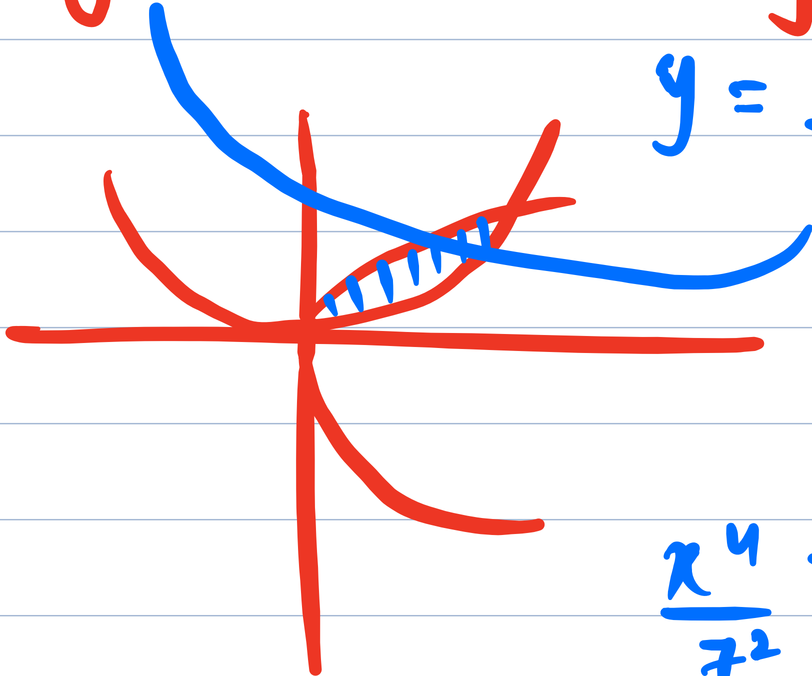
$$= 10\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= 10\left(2 \times \sin^{-1}(1)\right)$$

$$= 10 \times \cancel{2} \times \frac{\pi}{\cancel{2}}$$

$= 10\pi$ sq unit

$y^2 = 7x \qquad x^2 = 7y$



$y = \frac{x^2}{7}$

$\frac{x^4}{7^2} = 7x$

$x^4 = 7^3 x$

$x^4 - 7^3 x = 0$

$y^2 = 7x$

$y = \sqrt{7}\sqrt{x}$

$x(x^3 - 7^3) = 0$

$x = 0, \quad x = 7$

$$A = \int_0^7 \sqrt{7}\,\sqrt{x} = \sqrt{7}\left[\frac{x^{3/2}}{3/2}\right]_0^7$$

$$= \sqrt{7} \times \frac{2}{3} \times (7)^{3/2}$$

$$= \sqrt{7} \times \frac{2}{3} \times 7\sqrt{7}$$

$x^2 = 7y$

$y = \frac{x^2}{7}$

$$= \frac{98}{3}$$

$$A_2 = \int_0^7 \frac{x^2}{7}\, dx$$

$$= \frac{1}{7}\left[\frac{x^3}{3}\right]_0^7$$

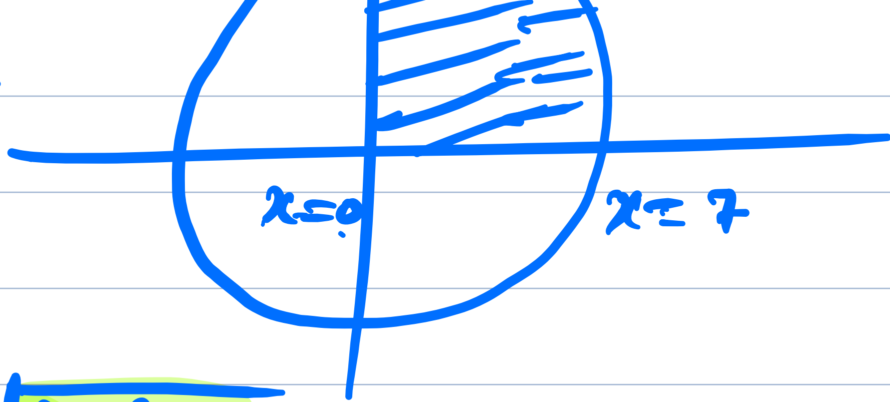
$$= \frac{(7)^3}{7 \times 3} = \frac{49}{3}$$

$$\frac{98}{3} - \frac{49}{3} = \frac{49}{3} \text{ sq un.}$$

$$x^2 + y^2 = 49$$

$$y^2 = 49 - x^2$$

$y = \sqrt{49 - x^2}$



$$A = 4\int_0^7 \sqrt{(7)^2 - x^2}\, dx$$

a^2 $\quad$ x^2

$$= 4\left[\frac{x}{2}\sqrt{49 - x^2} + \frac{49}{2} \times \sin^{-1}\left(\frac{x}{7}\right)\right]_0^7$$

$$= 4 \times \frac{49}{2} \times \sin^{-1}\left(\frac{7}{7}\right)$$

$$= 4 \times \frac{49}{2} \times \sin^{-1}(1)$$

$$= \cancel{4} \times \frac{49}{\cancel{2}} \times \frac{\pi}{\cancel{2}} = 49\pi$$

2) $7x - 25 \le -4$

$7x \le -4 + 25$

$7x \le 21$

$\boxed{x \le 3}$

3) $0 < \dfrac{x-5}{4} < 3$

$0 < x - 5 < 12$

$\boxed{5 < x < 17}$

4) $|7x - 4| < 10$

$-10 < 7x - 4 < 10$

$-10 + 4 < 7x < 10 + 4$

$-6 < 7x < 14$

$-\dfrac{6}{7} < x < \dfrac{14}{7}$

$\boxed{-\dfrac{6}{} < x < 2}$

$\frac{}{7}$

4

1) $5x + 7 > 4 - 2x$

$7x > -3$

$\boxed{x > \frac{-3}{7}}$

2) $3x + 1 \geq 6x - 4$

$5 \geq 3x$

$\frac{5}{3} \geq x$

i.e. $\boxed{x \leq \frac{5}{}}$

3) $4 - 2x < 3(3 - x)$

$4 - 2x < 9 - 3x$

$\boxed{x < 5}$

4) $\frac{3}{4}x - \frac{6}{1} \leq x - 7$

$\frac{3x - 24}{4} \leq x - 7$

$3x - 24 \leq 4(x - 7)$

$3x - 24 \leq 4x - 28$

3

$$4 \le x$$

ie, $\boxed{x \ge 4}$

5) $-8 \le -(3x-5) < 13$

$$-8 \le -3x+5 < 13$$

$$-8-5 \le -3x < 13-5$$

$$-13 \le -3x < 8$$

$$\boxed{\frac{13}{3} \ge x > \frac{8}{-3}}$$

6) $-1 < \frac{3}{1} - \frac{x}{5} \le 1$

$$-1 < \frac{15-x}{5} \le 1$$

$$-5 < 15-x \le 5$$

$$-5-15 < -x \le 5-15$$

$$-20 < -x \le -10$$

$$\boxed{20 > x \ge 10}$$

7) $2|4-5x| \geq 9$

$|4-5x| \geq \frac{9}{2}$

$-\frac{9}{2} \geq 4-5x \geq \frac{9}{2}$

$-\frac{9}{2} - 4 \geq -5x \geq \frac{9}{2} - 4$

$-\frac{17}{2} \geq -5x \geq \frac{1}{2}$

$\frac{-17}{-10} \leq x \leq \frac{1}{-10}$

8) $|2x+7| \leq 25$

$-25 \leq 2x+7 \leq 25$

$-32 \leq 2x \leq 18$

$\frac{-32}{2} \leq x \leq \frac{18}{2}$

$\boxed{-16 \leq x \leq 9}$

$$\boxed{1.7 \le x \le -0.1}$$

ix) $2|x+3| > 1$

$|x+3| > \frac{1}{2}$

$-\frac{1}{2} > x+3 > \frac{1}{2}$

$-\frac{1}{2} - 3 > x > \frac{1}{2} - 3$

$-\frac{7}{2} > x > -\frac{5}{2}$

$$\boxed{-3.5 > x > -2.5}$$

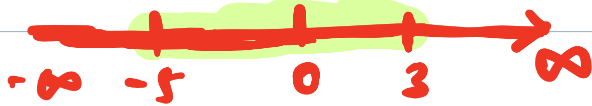
x) $\frac{x+5}{x-3} < 0$

$\frac{a}{b} < 0$, (i) $a>0,\ b<0$
(ii) $a<0,\ b>0$

CASE 1: $a>0,\ b<0$

$x+5>0$ OR $x-3<0$

$x>-5$ $\quad$ $x<3$

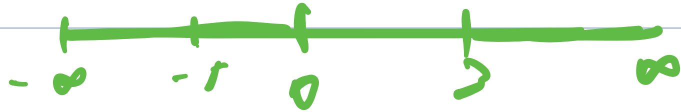


$$\boxed{-5 < x < 3}$$

CASE 2: $a<0,\ b>0$

$x+5<0$ or $x-3>0$

$x<-5$ $\quad$ $x>3$



Not possible.

xi) $\dfrac{x-2}{x+5}>0$

$\dfrac{a}{b}>0$ $\quad$ i) $a>0,\ b>0$

ii) $a<0,\ b<0$

1) $x-2>0$ or $x+5>0$

$x>2$ $\qquad$ $x>-5$

2) $x-2<0$ or $x+5<0$

$x<2$ $\qquad$ $x<-5$, $-5>x$

On combining,

$-5>x>2$

5) Average ≥ 60

$$\frac{x_1 + x_2 + x_3}{3} \geq 60$$

$$70 + 75 + x_3 \geq 180$$

$$145 + x_3 \geq 180$$

$$x_3 \geq 180 - 145$$

$$\boxed{x_3 \geq 35}$$

6) Average ≥ 90

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} \geq 90$$

$$87 + 92 + 94 + 95 + x_5 \geq 450$$

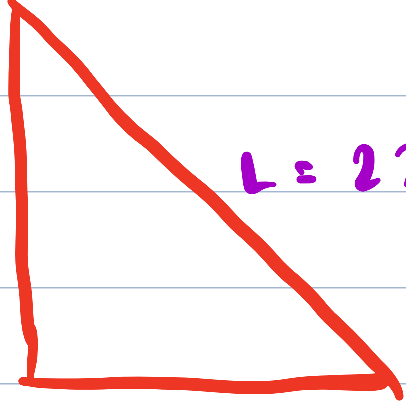
$$368 + x_5 \geq 450$$

$$x_5 \geq 450 - 368$$

$$\boxed{x_5 \geq 82}$$

9)

Shortest side = x



$$x + 2x + x+2 > 166$$

$$4x + 2 > 166$$

$$4x > 164$$

$$x > \frac{164}{4}$$

$$\boxed{x > 41}$$

1, 3, 5, 7, 9, 11

7) let '2' consecutive odd number be

$2n-1, \; 2n+1$

$2n-1 < 10$ or $2n+1 < 10$

$2n < 11$ $\qquad$ $2n < 10-1$

$n < 11/2$ $\qquad$ $2n < 9$

$n < 5.5$ $\qquad$ $n < 9/2$

$n < 4.5 \rightarrow 2$

$2n - \cancel{1} + 2n + \cancel{1} > 11$

$4n > 11$

$n > 11/4$

$n > 2.75 \rightarrow$ 1

$2.75 < n < 4.5$

$n = 3, 4$

If $n = 3$

$2n-1 = 5$

$2n+1 = 7$

$(5, 7)$

If $n = 4$

$2n-1 = 7$

$2n+1 = 9$

$(7, 9)$

8) $2n, \quad 2n+2$

$2n > 5$

$n > 5/2$

$n > 2.5$

$2n+2 > 5$

$2n > 3$

$n > 3/2$

$n > 1.5$

$2n + 2n+2 < 23$

$4n < 21$

$n < 21/4$

$\boxed{n < 5.25}$

$2.5 < n < 5.25$

$n = 3, 4, 5$

$n = 3, \quad 2n = 6$

$2n+2 = 8$

$(6, 8)$

$n = 4, \quad 2n = 8$

$2n+2 = 10$

$(8, 10)$

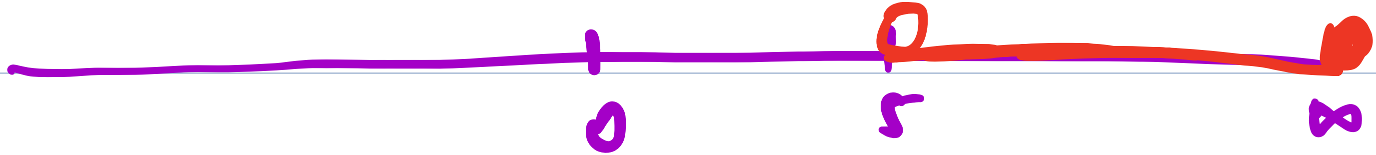
$n = 5, \quad 2n = 10$

$2n+2 = 12$

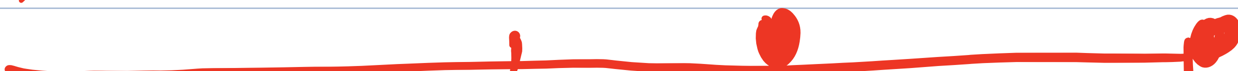
$(10, 12)$

3) 1) $x > 5$

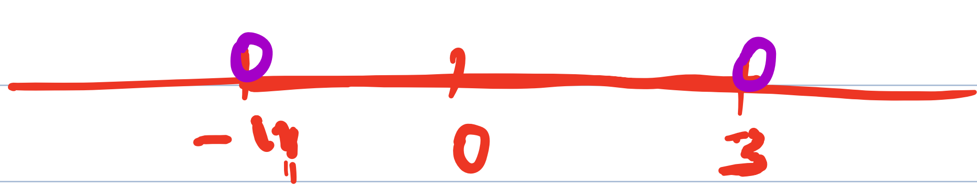
solution set: $(5, \infty]$



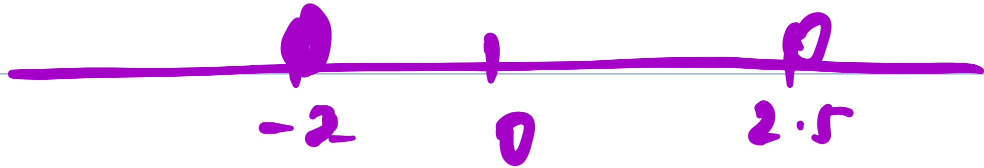
2) $x \geq 5$



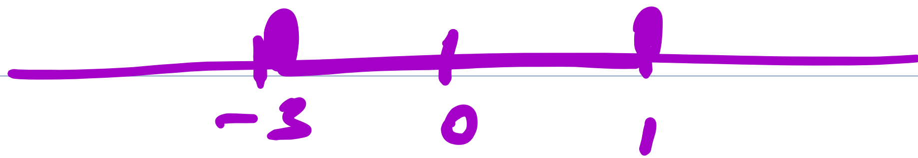
5)



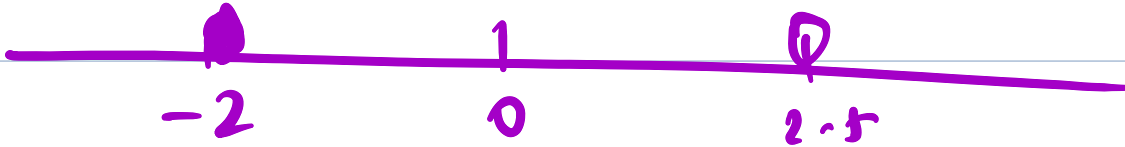
6)



7) $-3 \le x \le 1$

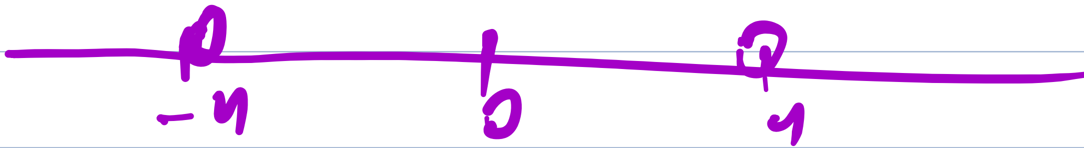


8) $-2 \leq x < 2.5$



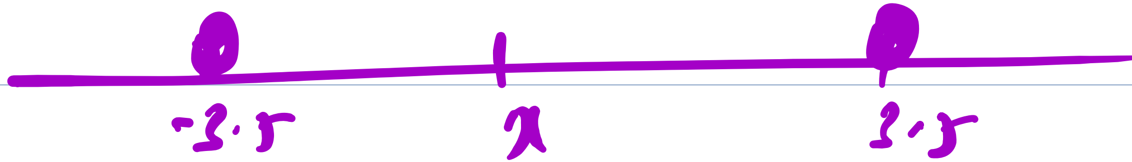
9) $|x| < 4$

$-4 < x < 4$



10) $|x| \geq 3.5$

$-3.5 \geq x \geq 3.5$



A B C D E F G

H I J K L M N

O P Q R S T U

V W X Y Z

1 7 11
2 8 12
3
4 9 13
5 10
6

A B C D E F G H

I J K L M N O P

Q R S T U V W X Y

Z

1 7 12 17 55 57

2 8 13 18 53 58

3 9 14 19 54 59

4

5 10 15 50 5 5 30

6 11 51 56 31