

CHEATSHEET (फ़ररी)

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Ratio
Proportion
Logarithm
Indices

#1] Ratios

Comparison of 2 or more
(quantities)

Same kind same units

$\frac{a}{b}$ is written as $a:b$

a = Numerator (First term)
(Antecedent)

b = Denominator (Second term)
(Consequent)

If more than two quantities of same kind are given in Ratio then this is called continued Ratio

continued Ratio of a, b & c
is denoted by $a:b:c$

{ If two quantities are in $a:b$
then first quantity = aK
& second quantity = bK

eg The internal angles of a triangle are in $1:2:3$.
find three angles.

Sol: let three angles are $K, 2K$ & $3K$

$$\text{so } (K) + (2K) + (3K) = 180^\circ$$

$$6K = 180^\circ$$

$$\{K = 30^\circ\}$$

so angles are $30^\circ, 60^\circ$ & 90°

{ Terms of a Ratio can be
multiplied or Divided by any
non zero number }

$$\begin{array}{l|l} \text{e.g.} & 2 : 3 \\ & \text{multiply by 5} \\ & 10 : 15 \end{array} \quad \begin{array}{l} \text{e.g.} \quad \frac{2}{3} : 5 \\ \text{multiply by 3} \\ \frac{6}{3} : 15 \\ 2 : 15 \end{array}$$

$$\begin{array}{l} \text{e.g.} \quad \frac{1}{2} : \frac{1}{3} : 5 \\ \text{multiply by 6 (LCM of 2 \& 3)} \\ 3 : 2 : 30 \end{array}$$

$a : b$ or $c : d$
which is greater?

use calculator to find $\frac{a}{b}$ & $\frac{c}{d}$

Q $4:5$ & $7:9$

which is greater?

Sol: $\frac{4}{5} = 0.80$

$$\frac{7}{9} = 0.77$$

So $\frac{4}{5} > \frac{7}{9}$

$a:b = 2:3$

$$b:c = 5:4$$

Find $a:b:c = ?$

Sol:

$a : \left\{ \begin{matrix} b \\ b \end{matrix} \right\} : c$

$2 : \left(\begin{matrix} 3 \\ 5 \end{matrix} \right) : 4$

$$10 : 15 : 12$$

Inverse of $a:b = b:a$

Ratio compounded of

$$a:b \text{ \& } c:d = ac:bd$$

i.e. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

Duplicate Ratio of $a:b = a^2:b^2$

Triplicate Ratio of $a:b = a^3:b^3$

Sub Duplicate Ratio of $a:b = (a)^{\frac{1}{2}}:(b)^{\frac{1}{2}}$

Sub Triplicate Ratio of $a:b = (a)^{\frac{1}{3}}:(b)^{\frac{1}{3}}$

Commensurable Quantities



If Ratio of two quantities
is a rational number

Q 5 & 2 are commensurable
as $\frac{5}{2}$ is a Rational no.

Q $\sqrt{5}$ & 2 are incommensurable
as $\frac{\sqrt{5}}{2}$ is not a Rational no.

#2] Proportion

Equality of two Ratios
make a proportion

Denoted by $a:b :: c:d$

or $a:b = c:d$

or $\frac{a}{b} = \frac{c}{d}$

eg $\frac{3}{6} = \frac{8}{16}$

so $3:6 :: 8:16$

If a, b, c & d are in proportion

so $a:b :: c:d$

a, b, c & d are called terms

a & $d \Rightarrow$ Extremes

b & $c \Rightarrow$ means

Continuous proportion

four 3 terms a, b & c

if $\frac{a}{b} = \frac{b}{c} \Rightarrow \boxed{b^2 = ac}$

then a, b & c are called in

Continuous proportion

where a = First proportional
 c = Third proportional
 b = mean proportional

Continued proportion

for more than 3 terms

$a, b, c, d, e, f, \dots$

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \dots$$

eg 2, 4, 8 & 16 are in
continued proportion

as $\frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \dots$

Properties of Proportion

1) Cross multiplication Rule

$$\text{If } \frac{a}{b} = \frac{c}{d}$$

$$\text{then } \boxed{ad = bc}$$

2) Invertendo Rule

$$\text{If } \frac{a}{b} = \frac{c}{d}$$

$$\text{then } \frac{b}{a} = \frac{d}{c}$$

3) Alternendo Rule

$$\text{If } a:b = c:d$$

$$\text{then } a:c = b:d$$

4) Componendo Rule

$$\text{If } a:b = c:d$$

$$\text{i.e. } \frac{a}{b} = \frac{c}{d}$$

then

$$\frac{a+b}{b} = \frac{c+d}{d}$$

$$\text{eg } \frac{x}{2} = \frac{y}{5}$$

$$\text{then } \frac{x+2}{2} = \frac{y+5}{5}$$

5> Dividendo Rule

$$\text{If } \frac{a}{b} = \frac{c}{d}$$

$$\text{then } \frac{a-b}{b} = \frac{c-d}{d}$$

$$\text{eg. } \frac{x}{5} = \frac{y}{6}$$

$$\text{then } \frac{x-5}{5} = \frac{y-6}{6}$$

6) Componento & Dividendo Rule

$$\text{If } \frac{a}{b} = \frac{c}{d}$$

$$\text{then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\text{eg } \frac{x}{2} = \frac{y}{5}$$

$$\text{Then } \frac{x+2}{x-2} = \frac{y+5}{y-5}$$

7) Addendo Rule

$$\text{If } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$$

$$\text{then } \frac{a+c+e}{b+d+f} = k$$

$$Q \quad \frac{8}{4} = \frac{6}{3} = 2$$

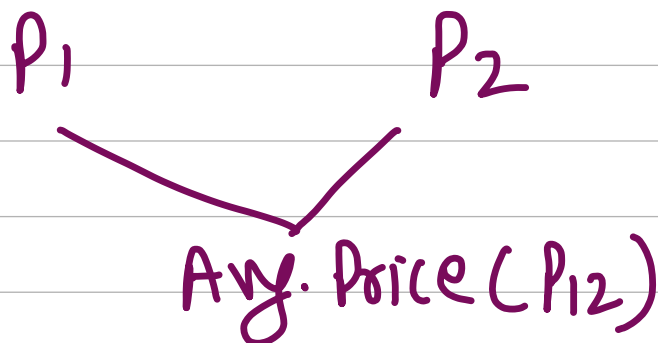
$$\text{then} \quad \frac{8+6}{4+3} = 2$$

8) Subtrahendo Rule

$$\text{If} \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K$$

$$\text{then} \quad \frac{a-c-e}{b-d-f} = K$$

Allegation Rule



$$Q_1 : Q_2 = |P_{12} - P_2| : |P_{12} - P_1|$$

g

£ 10/kg

£ 15/kg

Avg. Price = £ 12/kg

$$15 - 12 = 3$$

$$12 - 10 = 2$$

$$Q_1 : Q_2 = 3 : 2$$

#3 > Indices

If a number 'x' is multiplied 'n' times

$$x \cdot x \cdot x \cdot x \dots n \text{ times} = x^n$$

then $\underset{\text{Base}}{x}$ & $n \Rightarrow$ Power (Index) or Exponent

g $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

2 = Base

5 = Index

Properties

1) $a^0 = 1$ where $a \neq 0$

2) $a^{-1} = \frac{1}{a}$

calculator
 $\div =$

3) $a^{-n} = \frac{1}{a^n}$

Calculator Trick

$a^n = ?$

→ 'a' on the screen

→ Press 'x'

→ Press '=' (n-1) times

2^{10}
→ '2' on screen
→ $\boxed{\times}$
→ $\boxed{=}$ 9 times

Calculator Trick

$$a^n = ?$$

→ 'a' on the screen

→ Press $\boxed{\div}$

→ Press $\boxed{=}$ n times

$$g \quad 2^{10} = ?$$

→ 2 on screen

→ $\boxed{\div}$

→ $\boxed{=}$ '10' times

$$u) \quad a^m \cdot a^n = a^{m+n}$$

(Same Base & multiplication
then power add)

$$g \quad x^2 x^{\frac{1}{3}} = (x)^{2+\frac{1}{3}} = (x)^{\frac{7}{3}}$$

$$5) \quad \frac{a^m}{a^n} = a^{m-n}$$

(Same Base & Division
then power subtract)

$$g \quad \frac{x^{10}}{x^3} = x^7$$

$$6) (a^m)^n = a^{mn}$$

$$g \quad (x^2)^5 = x^{10}$$

Note $a^m \neq a^{mn}$

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$$7) \sqrt[n]{x} = (x)^{1/n}$$

(n^{th} Root of a number)

$$g \quad \sqrt{x} = (x)^{1/2}$$

$$g \quad \sqrt[3]{x} = (x)^{1/3}$$

$$g \quad \sqrt[4]{x} = (x)^{1/4}$$

Calculator Trick

- $\sqrt{}$ 12 times
- -1
- $\div$ by n
- $+1$
- $\boxed{x=}$ 12 times

$\sqrt[5]{32} = (32)^{\frac{1}{5}}$ 32 on screen of calculator
 $= (2^5)^{\frac{1}{5}}$ $\rightarrow \sqrt{\quad}$ 12 times
 $= 2$ $\rightarrow \div$ by 5
 $\rightarrow \boxed{x=}$ 12 times

calculator's answer
 will not be exactly 2
 (It will be very close to 2)

8) If $a^m = a^n$
 then $m = n$

a) If $a^m = k$
 then $a = (k)^{\frac{1}{m}}$

g $x^3 = 10$
 then $x = (10)^{\frac{1}{3}}$

g $x^{\frac{1}{2}} = y^{\frac{1}{5}}$
 then $x^5 = y^2$

$$10> (a b c d)^m = a^m b^m c^m d^m$$

Class 10th identities

$$\Rightarrow (a+b)^2 = a^2 + b^2 + 2ab$$

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$\Rightarrow (a-b)^2 = a^2 + b^2 - 2ab$$

$$a^2 + b^2 = (a-b)^2 + 2ab$$

$$\Rightarrow (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$\Rightarrow (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$\Rightarrow a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$\Rightarrow a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$

$$\# \text{ If } x = a^{\frac{1}{3}} + a^{-\frac{1}{3}}$$

$$\text{then } x^3 - 3x = a + \frac{1}{a}$$

$$\& \text{ If } x = a^{\frac{1}{3}} - a^{-\frac{1}{3}}$$

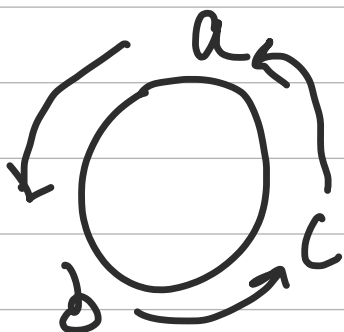
$$\text{then } x^3 + 3x = a - \frac{1}{a}$$

$$\text{eg } x = 5^{\frac{1}{3}} + 5^{-\frac{1}{3}}$$

$$\text{then } x^3 - 3x = ?$$

$$\underline{\text{sol:}} \quad x^3 - 3x = 5 + \frac{1}{5} = \frac{26}{5}$$

Cyclic order Triangles



$$\rightarrow (a-b) + (b-c) + (c-a) = 0$$

$$\rightarrow a(b-c) + b(c-a) + c(a-b) = 0$$

$$\rightarrow (a^2b - b^2c) + (b^2c - c^2a) + (c^2a - a^2b) = 0$$

$$\rightarrow (a^3b^3 - b^3c^3) + (b^3c^3 - c^3a^3) + (c^3a^3 - a^3b^3) = 0$$

$$\rightarrow (b-c)(b+c-a) + (c-a)(c+a-b) + (a-b)(a+b-c) = 0$$

$$\rightarrow \frac{1}{(a-b)(b-c)} + \frac{1}{(b-c)(c-a)} + \frac{1}{(c-a)(a-b)} = 0$$

$$\begin{aligned} \text{eg } & x^{a-b} x^{b-c} x^{c-a} \\ &= x^{a-b+b-c+c-a} \\ &= x^0 = 1 \end{aligned}$$

$$\begin{aligned} \text{eg } & \left(\frac{x^a}{x^b} \right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c} \right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a} \right)^{c^2+ac+a^2} \\ &= \binom{a-b}{(x)}^{(a^2+ab+b^2)} \binom{b-c}{(x)}^{(b^2+bc+c^2)} \binom{c-a}{(x)}^{(c^2+ac+a^2)} \\ &= \binom{a^3-b^3}{(x)} \binom{b^3-c^3}{(x)} \binom{c^3-a^3}{(x)} \\ &= x^0 = 1 \end{aligned}$$

#4] Logarithm

3/4 $\log_a(x) = y$ then $x = (a)^y$

Base Power where
 $x > 0$
 $a > 0$
 $a \neq 1$

value

a की Power क्या रखें जिससे
value x मिले

Q

$$\log_2(8) = ?$$

2 की Power में क्या रखें
जिससे answer 8 मिले ?

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

so $\log_2(8) = 3$

or

$$w \quad \log_2(8) = y$$

$$8 = (2)^y$$

$$(2)^3 = (2)^y$$

$$y = 3$$

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$$\log_a(x) = y$$

$$x = a^y$$

Common Log

$$y = \log_{10}(x)$$

calculator Trick

→ $\sqrt{\quad}$ 19 times

→ -1

→ $\times 227695$

There are many other tricks to find Log.

Natural Log

$$y = \log_e(x)$$

used in calculus

$$\log(200) = 2.3010$$

$$= \underbrace{1}_{\downarrow} + \underbrace{0.3010}_{\downarrow}$$

character mantissa

always positive
(Lies b/w 0 & 1)

$$\log(20) = 1.3010$$

$$= \underbrace{1}_{\downarrow} + \underbrace{0.3010}_{\downarrow}$$

character mantissa

$$\log(2) = 0.3010$$

$$= \underbrace{0}_{\downarrow} + \underbrace{0.3010}_{\downarrow}$$

character mantissa

$$\star \log(0.2) = -0.699$$

$$= \underbrace{0}_{\downarrow} - \underbrace{0.699}_{\downarrow}$$

subtract 1

Add 1

special case
↓

when log
is negative

$$= \underbrace{-1 + 0}_{\text{character}} - 0.699 + 1$$

$$= \underbrace{-1}_{\text{character}} + \underbrace{0.3010}_{\downarrow \text{mantissa}}$$

character

mantissa

negative
character

$$= \bar{1}.3010$$

$$\begin{aligned}
 \log(0.02) &= -1.699 \\
 &= \underbrace{-1}_{\text{Subtract 1}} - \underbrace{0.699}_{\downarrow \text{add 1}} \\
 &= \underbrace{-1-1}_{-2} - \underbrace{0.699+1}_{0.3010} \\
 &= \underbrace{-2}_{\downarrow \text{character}} + \underbrace{0.3010}_{\downarrow \text{mantissa}} \\
 &= \overline{2}.3010
 \end{aligned}$$

Properties of Log

$$1> \log_a(1) = 0$$

$$2> \log_a(a) = 1$$

$$3> \log(xy) = \log x + \log(y)$$

$$4> \log\left(\frac{x}{y}\right) = \log x - \log y$$

$$5> \log(x)^n = n \log x$$

नोट:

$$[\log(x)]^n \neq n \log x$$

↓
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$$6> \log_a(x) = \frac{\log_b(x)}{\log_b a}$$

$$7> \log_x(y) \times \log_y(x) = 1$$

$$8> \log_{a^b}(x) = \frac{1}{b} \log_a(x) = \log_a(x)^{1/b}$$

$$9> \log_{a^b}(x)^y = \frac{y}{b} \log_a(x)$$

$$10> a^{\log_a(x)} = x$$