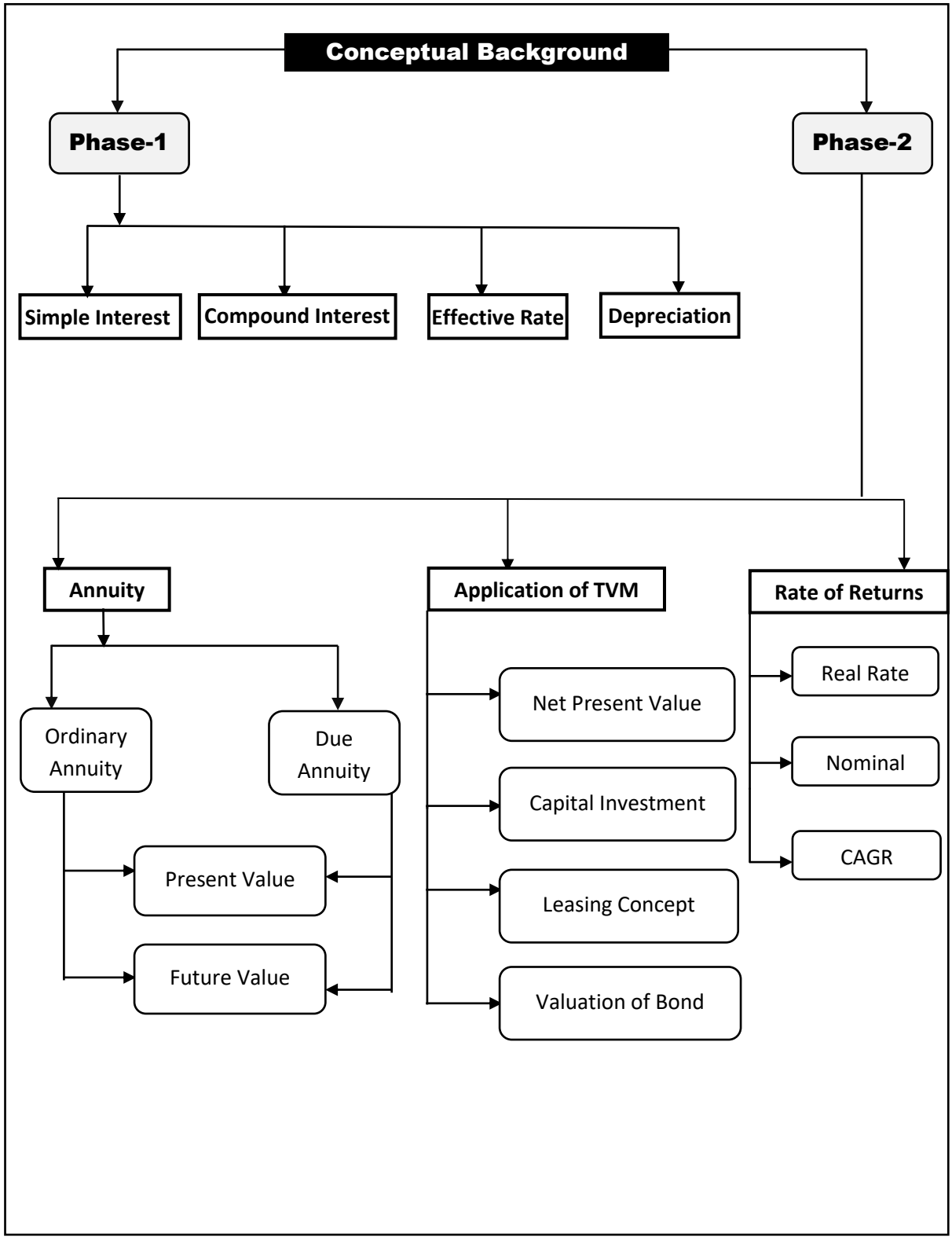
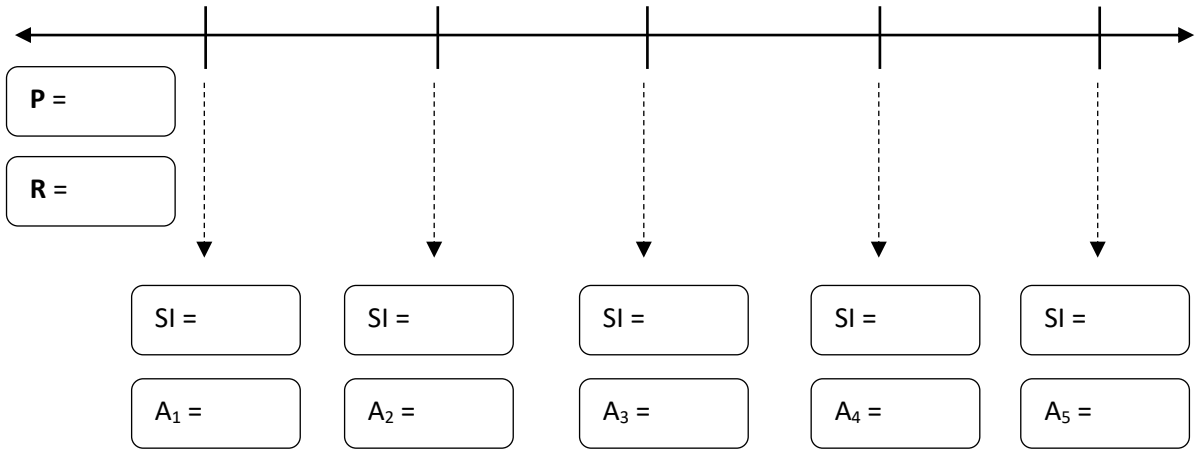
Mathematics of Finance

Chapter 4

Background of Chapter



Simple Interest



Crux: From above we can conclude that in Simple interest we get the interest on same amount **(i.e., Principle)** every year

Formula

Simple Interest = Principle × $\frac{Rate\ \%}{100}$ × Time

Amount = Principle + Interest

Note: if you are taking Rate in Percentage then make sure to divide it by 100

Note

Note: If we are taking the rate in **decimals** then **no need** to divide it by 100

Note: While applying the formula of S.I always take Time in **"Years"** if **time** is given in months, then convert them in Years

E.g. 1.5 Years =

E.g. 1.5 Years =

E.g. 1 Years & 3 Months =

E.g. 1 Years & 5 Months =

E.g. 2 Years & 8 Months =

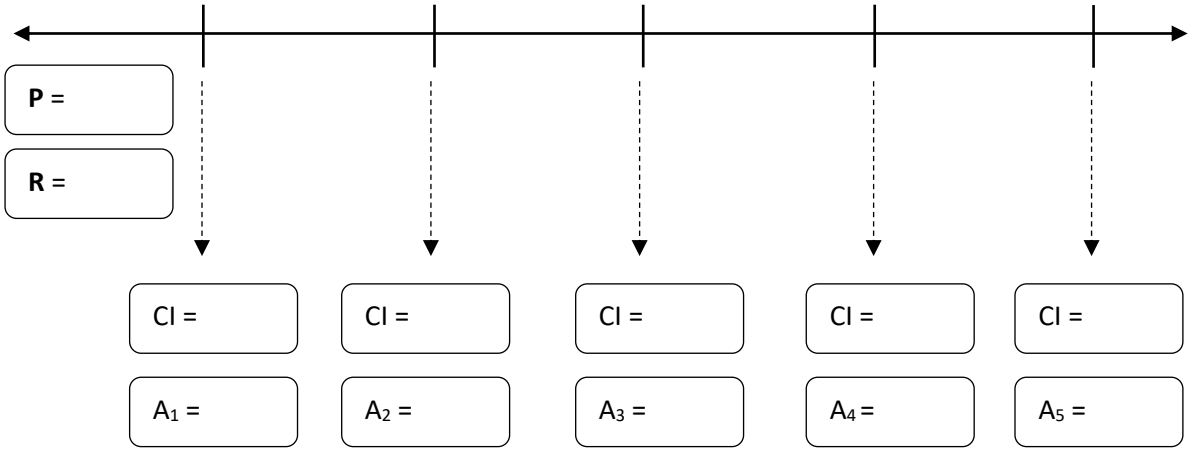
Advance Shortcuts

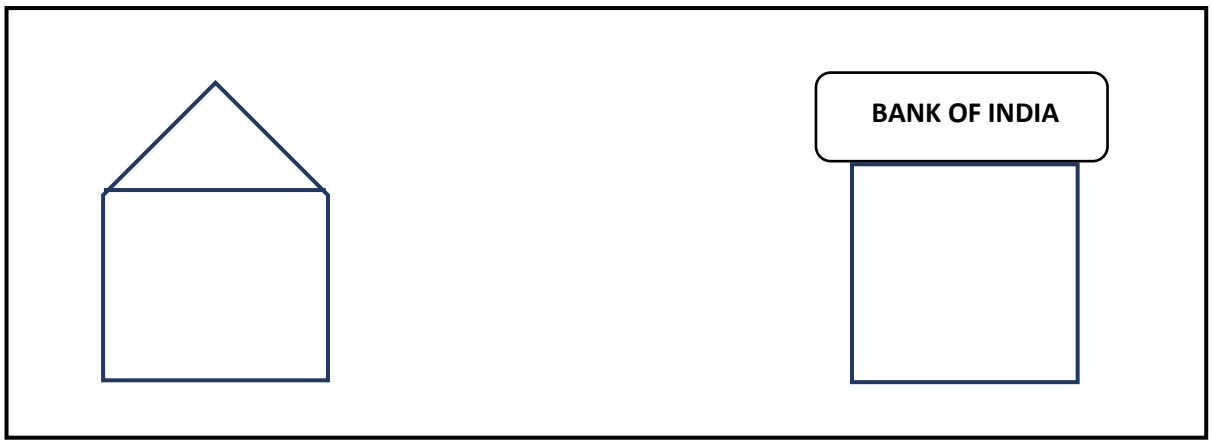
How to find Time or Rate to multiple a sum at S.I.

Particular	1.5 Times	Double	Trebled	4 Times
Time Req.	T = $\frac{0.5}{R}$ yrs.	T = $\frac{1}{R}$ yrs.	T = $\frac{2}{R}$ yrs.	T = $\frac{3}{R}$ yrs.

Example: A sum of money doubles itself in 10 years. The number of years it would triple itself is

Compound Interest





Conversion

Year	Half Year	Quatre	Months
1			
2			
1.5			
2.5			
6 Months			
3 Months			

Calculator Trick to Compute Compound Interest

Payment Period	Calculator Trick
Payable Yearly	
Payable Half Yearly	
Payable Quarterly	
Payable Monthly	

Example A person Invested 8000 at 7% Calculate Amount and compound interest in following situation

A. Interest is payable Yearly			**B.** Interest is payable half yearly		
Years	**Amount**	**Interest**	**Years**	**Amount**	**Interest**
1 Years			**1 Years**		
1.5 Years			**2 Years**		
2 Years			**1.5 Years**		
2.5 Years			**6 Months**		
C. Interest is payable Quarterly			**D. Interest is payable Monthly**		
Years	**Amount**	**Interest**	**Years**	**Amount**	**Interest**
1 Years			**1 Years**		
1.5 Years			**1.5 Years**		
6 Months			**6 Months**		

Reverse Approach

Whenever question Demands Anything Backwards of equals to Button (i.e., P, R, T)

⬇

We Will Use Reverse Approach

Note

When the Question Does not give you the exact Data what to do?

In that case assume the **Principle =** 100 and construct the question according to the information given in the question.

Example: The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

(a) 7 years **(b)** 10 years **(c)** 17 years **(d)** none

Note

When Years are in Decimal?

Example: The Amount on Rs. 16000 for 11.6 years at 10% p.a. Compounded annually?

Advance Shortcuts of Compound Interest

1

Amounts in C.I Forms GP

Forward Term of GP:

Backward Term of GP:

2

Difference Between C.I & S.I

3

If Rate Keeps on Changing every year?

Differential Rate of Interest:

4

If a sum of money amounts to **"m times"** in **"n years"** then **"q years"** it will amount to:

Effective Rate of Interest

Effective rate of interest can be defined as the equivalent annual rate of interest compounded annually if interest is compounded more than once in a year

Calculator Trick to Compute Effective Rate

Payment Period	Trick
Payable Yearly	
Payable Half Yearly	
Payable Quarterly	
Payable Monthly	

Note:

Depreciation

Calculator Trick to Compute Depreciation

Reverse Approach -

When Years are in Decimal?

Example: Machine Value 10,000, Rate of Depreciation 10 % Calculate Scrape at the end of 5.8 years.

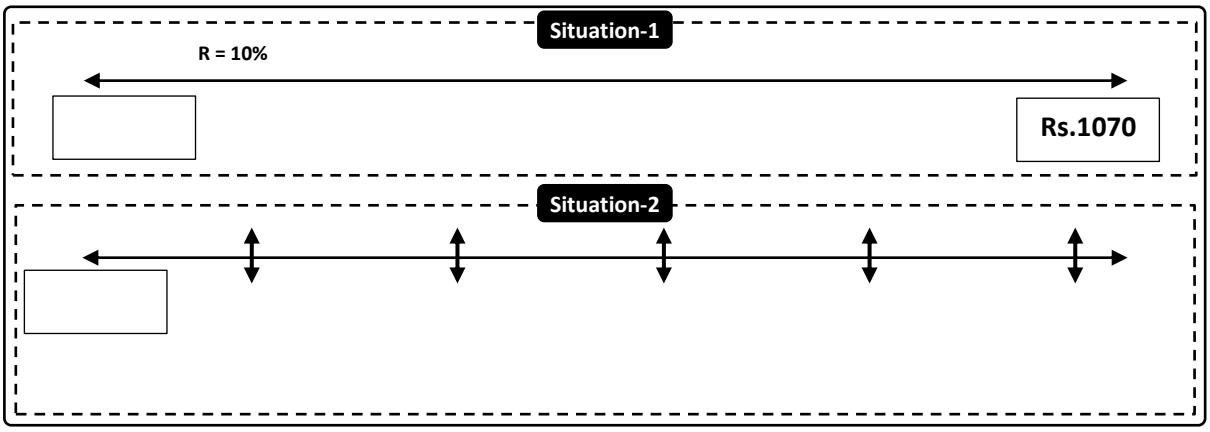
Sol:

Annuity

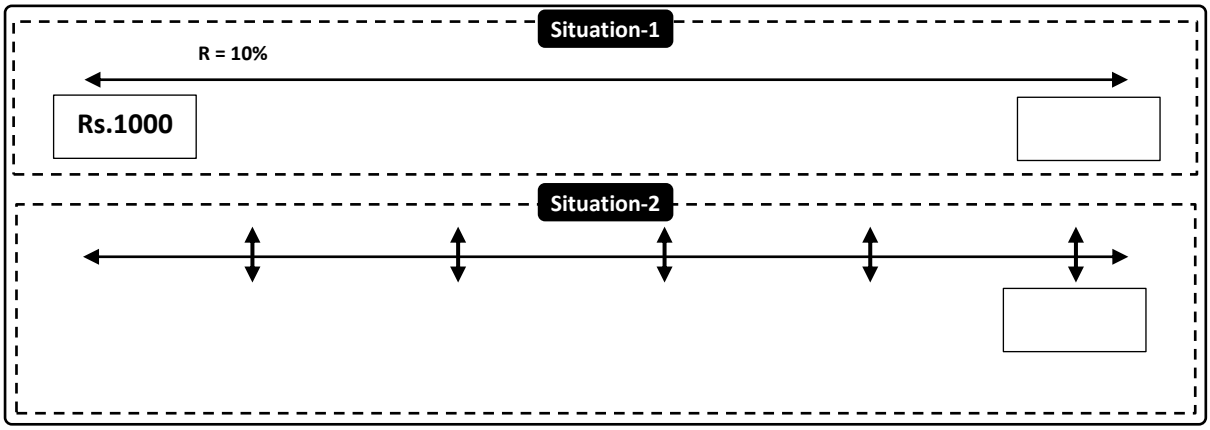
Some Important Definition

Annuity: A Sequence of payment (Regular payment) made at the end or beginning of periodical period is termed as Annuity e.g.**: Rent, Premium of LIC etc.**

Present Value: Present value is **Today's Value** of **Tomorrows money** Discounted at the Interest Rate.



Future Value: Future value is **Tomorrow's money** of **Today's Value** Discounted at the Interest Rate.



Types of Annuities

Ordinary Annuity

When the Annuity is Paid or Received at the [ ] of Periodical term it is termed as Ordinary Annuity

Annuity Due

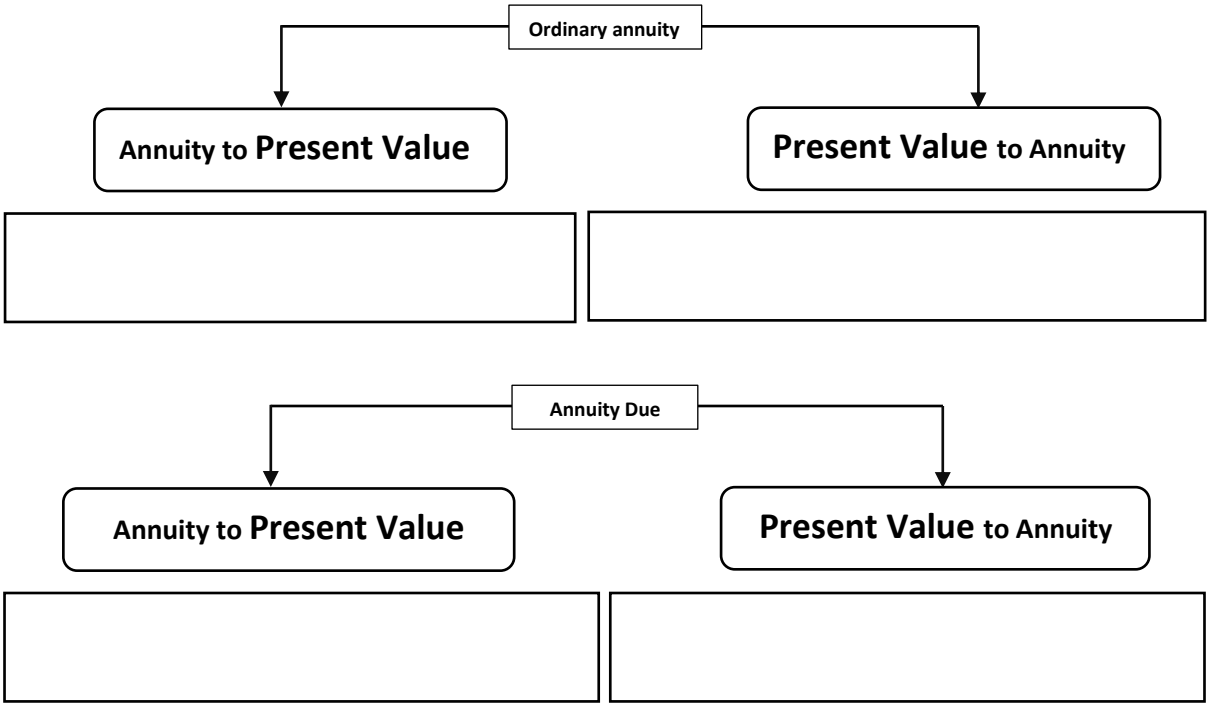
When the Annuity is Paid or Received at the of P[ ] is termed as Annuity Due

Note: If the question is **silent** then we will **Assume** it to be ordinary annuity

Note: Starting From **Today**

Present Value

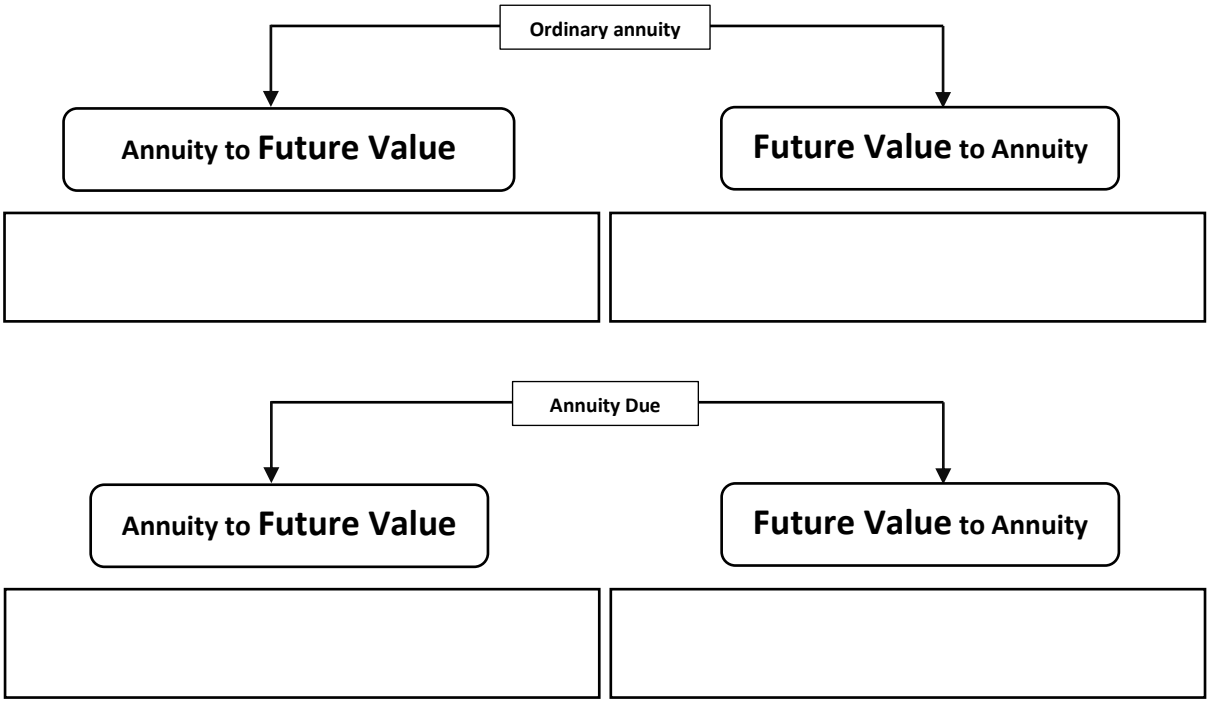
Calculator Trick to Compute Present Value



Note: Use 100 for annual ,200 for Semiannual ,400 for Qtrly, & 1200 for Monthly Compounding

Future Value

Calculator Trick to Compute Future Value



Present Value

Example: Given R = 8 % & Time = 3 Years
Compute Following

Ordinary Annuity			
Given A = 500 Find PV?	**Yearly**	**Half Yrly**	**Qtrly**
Given PV = 10,000 Find A?			
Annuity Due			
	Yearly	**Half Yrly**	**Qtrly**
Given A = 500 Find PV?			
Given PV = 10,000 Find A?			

Future Value

Example: Given R = 8 % & Time = 3 Years
Compute Following

Ordinary Annuity			
Given A = 500 Find FV?	**Yearly**	**Half Yrly**	**Qtrly**
Given FV = 10,000 Find A?			
Annuity Due			
	Yearly	**Half Yrly**	**Qtrly**
Given A = 500 Find FV?			
Given FV = 10,000 Find A?			

Perpetual Annuity

Formula to Compute Perpetual Annuity

Particulars	Ordinary annuity	Due Annuity
Without Growth		
With Growth		

Note

Annually	Half Yearly	Quarterly	Monthly
r %			

Note: As there is No Future date, we **Cannot** calculate **Future value** of perpetuity **only Present value** of Perpetuity can be Computed Known as **Perpetual Annuity**

Example: If money is worth 6 % per annum, find the present value of a perpetuity of Rs.3300 Payable annually.
Sol:

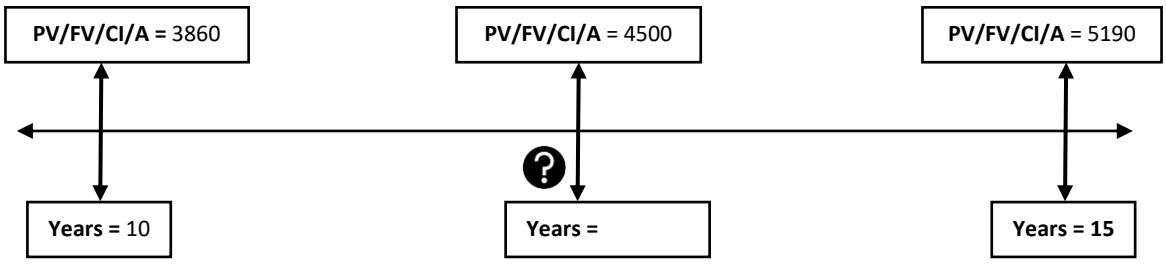
Example: Assuming that the discount rate is 7% p.a how much would you pay to receive Rs 50 Growing at 5% annually forever?
Sol:

What if Years are in Decimals (INTERPOLATION TECHNIQUE)

When the **Years are in decimal** then we will apply **Interpolation** techniques, basically interpolation method is used for finding any value between any two known values.

Let's understand it from situation below

Find the value of x?



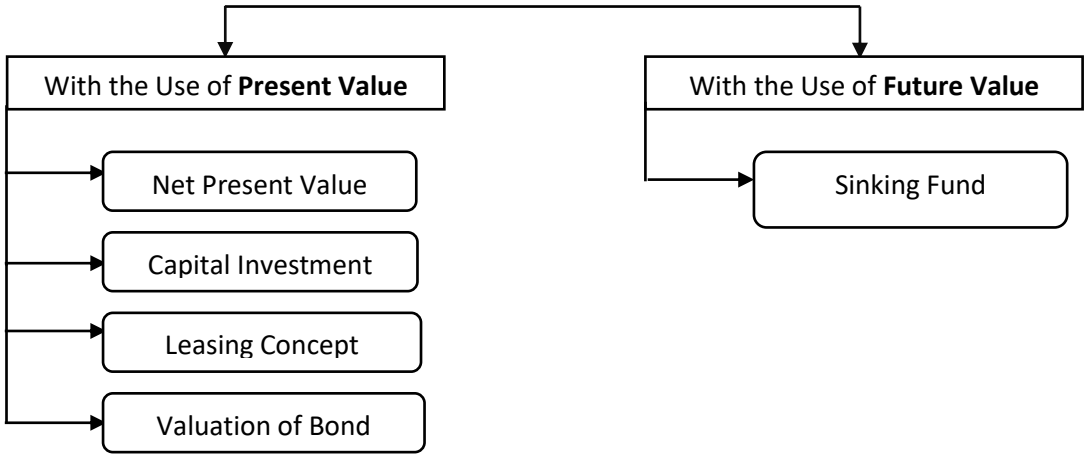
Example: A company borrows Rs. 10000 on condition to repay it with compound interest at 5% p.a. by annual installments of Rs. 1000 each. The number of years by which the debt will be clear is

(a) 14.2 yrs. **(b)** 10 yrs. **(c)** 12 yrs. **(d)** None of these

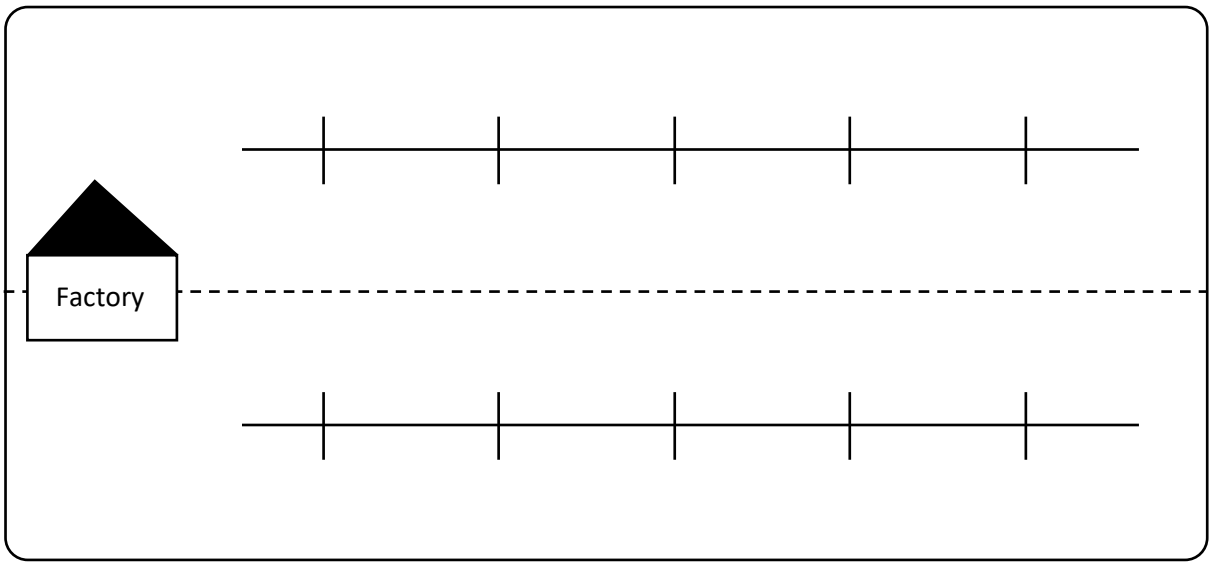
Example: Given annuity of Rs. 100 amounts to Rs. 5000 at 4.5% p.a C.I. The number of years will be

(a) 25 yrs. (appx.) **(b)** 20 yrs. (appx.) **(c) 26.** **(d)** none

Application Of Time Value of Money



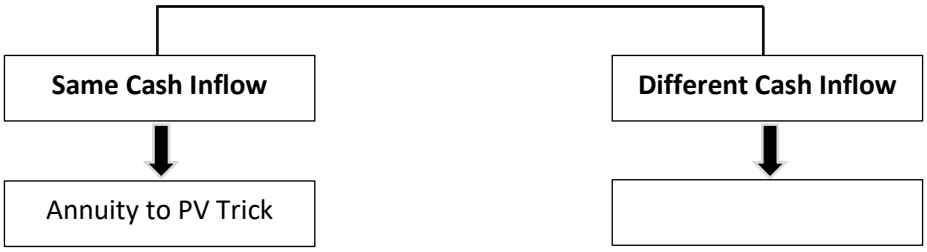
Net Present Value (NPV)



Conclusion: From above it is clearly evident that due to the time value of money, the value of money at present shall always have more value than the same amount of money in future. Due to Time value of Money, a person would prefer to receive the money at present rather than in future and would like to earn interest on the money held.

Formula to Compute Net Present Value

Note



Present Value of Future Money

Example: Find the present value of Rs.10,000 to be required after 5 years if the interest rate by 9%.

Example: Assume cash outflow are Rs. 1,20,000 followed by cash inflows of Rs. 25,000 per year for 8 years and cost of capital is 11% what is the net present value?

Sol:

Example: Assume that ABC Inc is considering projects namely Project X and wants to calculate the NPV for the project. Project X is four-year project and cash flows for four years are given below Suppose rate of interest is 10% per annum compounded annually on which money can be invested:

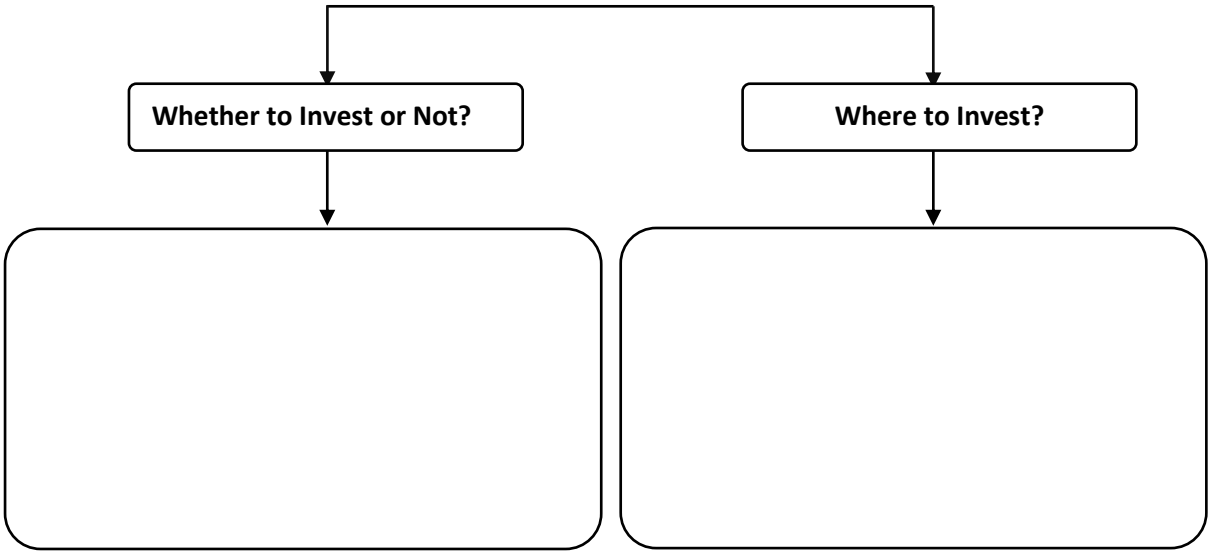
Year	0	1	2	3	4
Cash Flows	(Rs 10,000)	Rs. 1000	Rs. 3000	Rs. 4000	Rs. 6750

Sol:

Year	Cash Flows		
1			
2			
3			
4			

Capital Investment

Capital Investment means Purchasing an asset (Which results in Outflow) today in anticipation of benefits (cash inflow) Across the life of investment (asset).



Example: A Project will cost Rs. 1,50,000 followed by cash inflows of Rs. 45,000 per year for 5 years and cost of capital is 12% Should you accept the project?
Sol:

Example: A machine with useful life of 7 years costs Rs. 10,000 while another machine with useful life of 5 years costs Rs. 8,000. The first machine saves labor expenses of Rs. 1900 annually and the second one saves labor expenses by Rs. 2,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% per annum.
Sol:

Leasing Concept

Investor have two alternatives to Occupy particular asset one is **Purchasing option** and another is taking it on **lease**, He have to decide Which option is Preferable to him.

Lessee Point of View
(For Lessee LR are Expense)

If

Lessor Point of View
(For Lessor LR are Income)

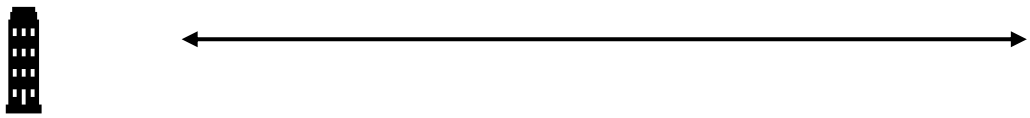
If

Example: A company is considering proposal of purchasing a machine either by making full payment of Rs. 4,000 or by leasing it for 4 years at an annual rent of Rs. 1,250. Which course of action is preferable, if the company can borrow money at 14% per annum? [Given: $(1.14)^4 = 1.68896$]
Sol:

Example: A person wants to lease out a machine costing Rs. 5,00,000 for a 10-year period. It has fixed a rental of Rs. 51,272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum compounded annually on which money can be invested. To whom this agreement is favorable?
Sol:

Valuation of Bond

A bond is a debt security in which the issuer owes the holder a debt & is obliged to repay principle (at **face value** of bond) & **Interest.**



Value of Bond =

Example: An investor intends purchasing a three-year Rs. 1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

Sinking Fund

Sinking fund is a fund created to accumulate the specified amount of sum by the way of regular periodic payment for some specific purpose

Rate of Returns

Nominal Rate of Return

A nominal rate of return is nothing but the total amount of money that is earned from a particular investing activity before taking, inflation, into the due consideration.

Real Rate of return

The real rate of return is the actual annual rate of return after taking into consideration the factors that affect the rate like inflation.

CAGR Model

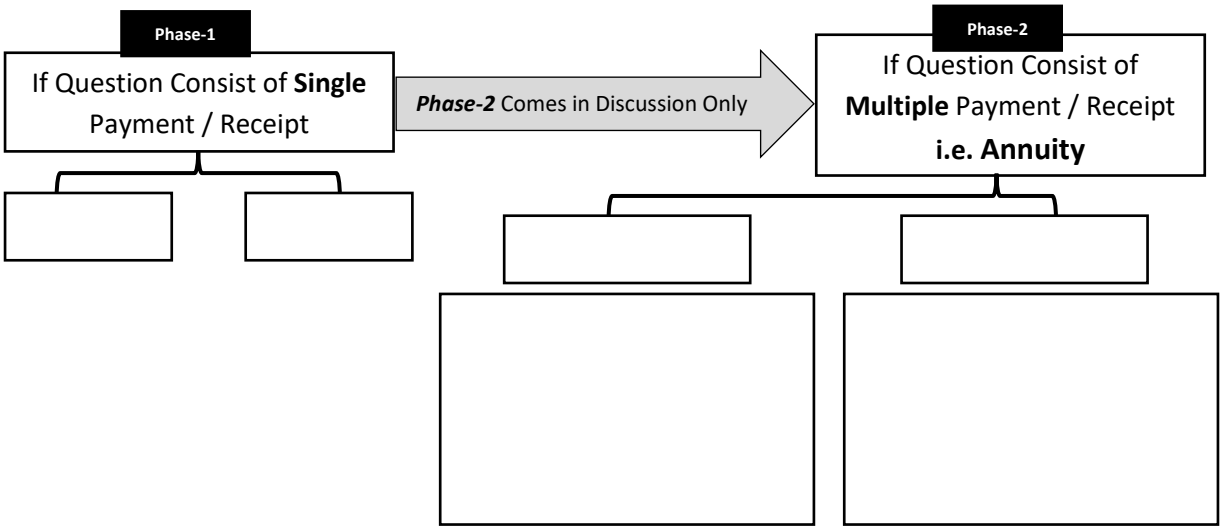
Compound annual growth rate, or CAGR, is the mean annual growth rate of an investment over a specified period of time longer than one year.

Formula

Calculator Trick to Compute any Power

Let's Remove Confusion whether Question is of
PRESENT VALUE | FUTURE VALUE | AMOUNT | PRINCIPLE

*1st of all look read the question very carefully and check whether question is of **Phase-1** or **Phase-2***



S.no	Question	Phase	What is Given	What is asked	Answer
1.	The amount of Rs. 150 after 12 years at 3.5% p.a. C.I is				
2.	The amount of an annuity certain of Rs. 150 for 12 years at 3.5% p.a. C.I is				
3.	You invest Rs 3000 in a two-year investment that pays you 12% per annum. Calculate the future value of investment				
4.	What is present value of Rs.1 to be received after two years compounded annually at 10% interest rate?				
5.	How much amount is required to be invested every year so as to accumulate Rs.300000 at the end of 10 years if the interest is compounded annually at 10%?				
6.	How much amount should I deposit today so as to accumulate Rs.300000 at the end of 10 years if the interest is compounded annually at 10%?				
7.	Raju buys a house for which he agrees to pay Rs. 5000 at the end of every year for 8 years. If money is worth 12, what is the capital value of the house?				

8.	Y bought a TV costing Rs. 13,000 by making a down payment of Rs. 3000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually?				
9.	A certain sum invested at 4% per annum compounded semi-annually amounts to Rs78030 at the end of one year. Find the sum.				
10.	At the Beginning of each Period Consisting of 6-months, Rs 500 is deposited into saving account that pays 5% compounded half-yearly. Find the balance in the account at the end of 5 year.				
11.	Rs 8000 at 10% per annum interest compounded half yearly will become at the end of one year				
12.	Find the present value of Rs 1,00,000 to be required after 5 years if the interest rate be 9%.				
13.	Rs 2,500 is paid every year for 10 Years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually?				
14.	Assuming that discount rate is 7% p.a. how much would pay to receive Rs 200 growing at 5% annually forever?				
15.	A company needs Rs 10,000 in five years to replace an equipment. How much should be invested now at an rate of 8% p.a. in order to provide for this equipment?				
16	R needs money to pay Rs 5,00,000 in 10 years. He invested a sum in a scheme at 9% rate of compounded half-yearly. How much amount he invested?				
17.	If the desired future value after 5 Years with 18% interest rate is Rs 1,50,000, then the present value is				

Formula Practice

Compound Interest

Effective Rate of Interest

Present Value

Future Value

Important Conclusions

Other Useful Concepts