Index Numbers

Denoted by P_{01}

0 => Base year

1 => Current year

- Statistical tool
- measure changes in variable
  - Price
  - Quantity
  - Income
  - Expenses
- Compare changes over time or places
- Current year value is compared with Base year value
- Expressed in percentage

Price index

⇓

measure changes in Price

Quantity index

⇓

measure changes in volume consumption, Sales.

Challenges in construction of index number

- Selection of Base year
- Selection of Product
- Selection of Prices
- Selection of formula
- Selection of weight

methods of construction of index number

- Unweighted
  - Aggregative
  - Relative
- weighted
  - Aggregative
  - Relative

Simple Aggregative method

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

Simple Relative method

$$P_{01}(AM) = \frac{\Sigma \left(\frac{P_1}{P_0} \times 100 \right)}{N}$$

$$P_{01}(GM) = AL\left[\frac{\Sigma \log \left(\frac{P_1}{P_0} \times 100 \right)}{N} \right]$$

Weighted Aggregative method

$$P_{01} = \frac{\Sigma P_1 (w)}{\Sigma P_0 (w)} \times 100$$

weight
↓
→ quantity is taken as weight
→ Different statisticians take different year quantity

Laspeyre (Base year quantity)

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

Paasches (current year quantity)

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

Fishers (Geometric mean of L & P)

$$P_{01} = \sqrt{L \times P}$$

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

Marshall Edgeworth (Average of Base year & current year quantity)

$$P_{01} = \frac{\Sigma p_1 \left(\frac{q_0 + q_1}{2}\right)}{\Sigma p_0 \left(\frac{q_0 + q_1}{2}\right)}$$

Dorbish & Bowley (Average of L & P)

$$P_{01} = \frac{L + P}{2} = \left[\frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2}\right] \times 2$$

Weighted Relative method

$$P_{01}(AM) = \frac{\Sigma w_i \left(\frac{p_1}{p_0} \times 100\right)}{\Sigma w_i} \quad \& \quad P_{01}(GM) = A.L.\left[\frac{\Sigma w_i \log\left(\frac{p_1}{p_0} \times 100\right)}{\Sigma w_i}\right]$$

Quantity Index (volume index)

measure changes in consumptions, productions or sales over a period of time or some other character

Method

- Simple
  - Aggregative
    $$Q_{01} = \frac{\Sigma q_1}{\Sigma q_0} \times 100$$
  - Relative
    $$Q_{01} = \frac{\Sigma \left(\frac{q_1}{q_0} \times 100 \right)}{N}$$
- weighted
  - Aggregative
    - Laspeyre
      $$Q_0 = \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times 100$$
    - Paasche
      $$Q_{01} = \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}$$
    - Fisher's
      $$\sqrt{L \times P}$$
    - Bowley
      $$\frac{L + P}{2}$$
    - marshall
      $$Q_{01} = \frac{\Sigma q_1 \left(\frac{p_0 + p_1}{2} \right)}{\Sigma q_0 \left(\frac{p_0 + p_1}{2} \right)}$$
  - Relative
    $$Q_{01} = \frac{\Sigma \left[\frac{q_1}{q_0} \times 100 \right]}{N}$$

Value index

$$V_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100$$

Group index

$$\frac{\Sigma w_i I_i}{\Sigma w_i}$$ where I_i = index of particular item

Consumer Price index (cost of Living index)

changes in cost of Living

→ Representative market Basket of goods & services

→ Food, clothing, Rent, Health & other Daily expenses

$$CPI = \frac{\text{Total exp. in C.Y.}}{\text{Total exp in B.Y.}}$$ (Laspeyre formula)

or $$CPI = \frac{\Sigma w_i I_i}{\Sigma w_i}$$ (weighted Relative)

Deflated value (Real value)

$$\text{Deflated value} = \frac{\text{current value}}{\text{Price index}}$$

Eg.

Year	Index	wages	Real wages
2010	100	500	$\frac{500}{100} \times 100 = 500$
2011	110	600	$\frac{600}{110} \times 100 = 545$
2012	140	800	$\frac{800}{140} \times 100 = 571$
2013	200	900	$\frac{900}{200} \times 100 = 450$

Shifting of Base year

Eg.

Year	index (Base-2010)	New index (Base-2012)
2010	100	$\frac{100}{120} \times 100 = 83$
2011	110	$\frac{110}{120} \times 100 = 92$
2012	120	$\frac{120}{120} \times 100 = 100$
2013	140	$\frac{140}{120} \times 100 = 117$

-> when old Base year is not relevant

-> Changes in economic policies, global & Domestic market structure

$$\text{Shifted Price index} = \frac{\text{original Price index}}{(\text{index of the year where it has to be shifted})} \times 100$$

Splicing of Two index series

→ Combining Two index series with different Base year

→ used when * changes in quantity weight
* change in method of calculation
* New Product introduction

Eg.

Year	(Base-2010) Old index	(Base-2012) Revised index	(Base-2010) Spliced index	(Base-2012) Spliced index
2010	100		100	$\frac{100}{150} \times 100 = 67$
2011	120		120	$\frac{100}{150} \times 120 = 80$
2012	150	100	150	100
2013		130	$\frac{150}{100} \times 130 = 195$	130
2014		160	$\frac{150}{100} \times 160 = 240$	160

Test of Adequacy

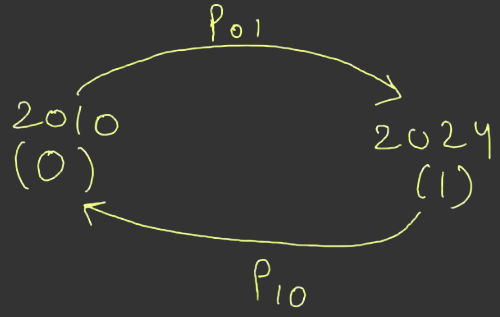
#1] Unit Test

formula should be independent of units of price & quantity, Due to change in units index number should not change

→ This test is satisfied by all methods except "simple aggregative method"

#2] Time Reversal Test

formula should work both ways "forward" & "Backward"



$$P_{01} \times P_{10} = 1$$

→ Satisfied by
* fishers (um)
* simple Relative (um)
* weighted Relative (um)
* marshall edgeworth

#3] Factor Reversal Test

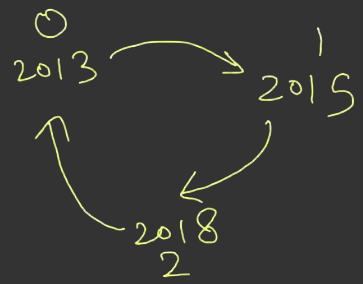
$$P_{01} \times Q_{01} = V_{01}$$

or

$$P_{01} \times Q_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0}$$

Satisfied by "fishers"

#4] Circular Test → Extension of Time reversal



$$P_{01} \times P_{12} \times P_{21} = 1$$

Satisfied by
→ Simple price Relative (um)
→ weighted aggregative (fixed weight)