

- › **Class Limits (CL)** : CL is the minimum and maximum value the class interval may contain

$$\text{Class Interval} = \text{UCL} - \text{LCL}$$

UCL : Upper Class Limit

LCL : Lower Class Limit

Class Boundary (CB) : These are actual class limits of a class interval

- › In case of Mutually Exclusive Classification

Class Boundary = Class Limit

$$UCB = UCL \text{ and } LCB = LCL$$

- › In case of Mutually Inclusive Classification

$$UCB = UCL + 0.5$$

$$LCB = LCL - 0.5$$

Mid Point of Class Interval : average of LCL and UCL or average of LCB and UCB

$$\frac{LCL+UCL}{2}$$

$$\frac{LCB+UCB}{2}$$

Mid point is also called as Mid Value of a class or Class Mark

Class Length / Width of Class / Size of Class

UCB - LCB

π

Frequency Density

$$\frac{\text{Frequency of a class}}{\text{Class length of that class}}$$

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Relative Frequency

$$\frac{\text{Frequency of class}}{\text{Total Frequency of distribution}}$$

Percentage Frequency

$$\frac{\text{Frequency of class}}{\text{Total Frequency of distribution}} \times 100$$

Arithmetic Mean of Discrete Distribution:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$\bar{x} = \frac{\sum x}{n}$$

Arithmetic Mean in case of Frequency Distribution :

$$\bar{x} = \frac{\sum fx}{N}$$

Here,

x will be equal to mid point of class interval when frequency distribution is grouped.

N is number of observations which is also equal to total of all frequencies.

AM using Assumed Mean / Step Deviation Method

$$\bar{x} = A + \frac{\sum fd}{N} \times C$$

where $d = \frac{x - A}{C}$

$A = \text{assumed mean}, C = \text{class length}$

Property of AM

the algebraic sum of deviations of a set of observations from their AM is zero

$$\Sigma(x - \bar{x}) = 0$$

Property of AM

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Combined AM

- If there are two groups containing n_1 & n_2 observations with AMs as \bar{x}_1 & \bar{x}_2 respectively then the combined AM is given by

$$\bar{x}_c = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

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Median in case of discrete distribution

- If $n = \text{odd}$, then middle term
- If $n = \text{even}$, average of two middle terms

Same formula will be used for *Simple Frequency distribution*

Median in case of Grouped Frequency Distribution

- Step 1: Prepare a less than type cumulative frequency distribution.
- Step 2: Calculate $N/2$ and check between which class boundaries it falls and call it as Median Class.
- Step 4: Find $l_1 = \text{LCB of Median Class}$, N_u as Cum Freq. of Median Class, N_l as Cum. Freq. of Pre Median Class, $C = \text{Class length of Median Class}$
- Step 3: Apply the below formula

$$Me = l_1 + \left(\frac{\frac{N}{2} - N_l}{N_u - N_l} \right) \times C$$

Property of Median

*For a set of observations, the sum of **absolute** deviations is **minimum**, when the deviations are taken from the median.*

Quartiles (in case of discrete observations)

Quartiles
$Q_1 = \left((n+1) \times \frac{1}{4} \right)^{th} \text{ term}$
$Q_2 = \left((n+1) \times \frac{2}{4} \right)^{th} \text{ term}$
$Q_3 = \left((n+1) \times \frac{3}{4} \right)^{th} \text{ term}$

Deciles (in case of discrete observations)

Deciles
$D_1 = \left((n+1) \times \frac{1}{10} \right)^{th} \text{ term}$
$D_2 = \left((n+1) \times \frac{2}{10} \right)^{th} \text{ term}$
$Q_9 = \left((n+1) \times \frac{9}{10} \right)^{th} \text{ term}$

Percentiles (in case of discrete observations)

Percentiles
$P_1 = \left((n+1) \times \frac{1}{100} \right)^{th} \text{ term}$
$P_2 = \left((n+1) \times \frac{2}{100} \right)^{th} \text{ term}$
$P_{99} = \left((n+1) \times \frac{99}{100} \right)^{th} \text{ term}$

Quartiles (Grouped FD)

- For first Quartile Q_1

- › Find Q_1 class
- › Use Formula :

$$Q_1 = l_1 + \left(\frac{\frac{N}{4} - N_l}{N_u - N_l} \right) \times C$$

- For third Quartile Q_3

- › Find Q_3 class
- › Use Formula :

$$Q_3 = l_1 + \left(\frac{\frac{3N}{4} - N_l}{N_u - N_l} \right) \times C$$

› Deciles (Grouped FD)

- For first Decile D_1

- › Find D_1 class
- › Use Formula:

$$D_1 = l_1 + \left(\frac{\frac{N}{10} - N_l}{N_u - N_l} \right) \times C$$

- For ninth Decile D_9

- › Find D_9 class
- › Use Formula:

$$D_9 = l_1 + \left(\frac{\frac{9N}{10} - N_l}{N_u - N_l} \right) \times C$$

› Percentiles (Grouped FD)

– For first Percentile P_1

- › Find P_1 class
- › Use Formula:

$$P_1 = l_1 + \left(\frac{\frac{N}{100} - N_l}{N_u - N_l} \right) \times C$$

– For 99th Percentile P_{99}

- › Find P_{99} class
- › Use Formula:

$$P_{99} = l_1 + \left(\frac{\frac{99N}{100} - N_l}{N_u - N_l} \right) \times C$$

Mode in case of discrete observation

Observation repeating for maximum no. of times/ max frequency

- › *When all observations have same frequency : No Mode*
- › *When two observations have max frequency : Bimodal Observation*
- › *In general, if we have two or more modes : Multi Modal Observation*

- › Mode in case of grouped frequency distribution
 - First find modal class i.e. class with highest frequency
 - Now, Find
 - › f_0 = frequency of the modal class,
 - › f_{-1} = frequency of pre modal class,
 - › f_1 = frequency of the post modal class
 - Apply formula

$$Mo = l_1 + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \right) \times C$$

Relationship between Mean, Median and Mode

In case of symmetrical Distribution

$$\text{Mean} = \text{Median} = \text{Mode}$$

Relationship between Mean, Median and Mode

In case of moderately skewed distribution

$$\text{Mean} - \text{Mode} = 3 (\text{Mean} - \text{Median})$$

Geometric Mean (in case of discrete positive observations)

$$G = \left(x_1 \times x_2 \times \dots \times x_n \right)^{1/n}$$

Geometric Mean (in case of frequency distribution)

$$G = (x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \times \dots \times x_n^{f_n})^{1/N}$$

Harmonic Mean (in case of discrete observations)

$$H = \frac{n}{\Sigma(1/x)}$$

Harmonic Mean (in case of frequency distribution)

$$H = \frac{N}{\Sigma(f/x)}$$

Combined HM

$$\bar{x}_c = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$

› Relationship between AM, GM and HM

When to use	Relationship
When all the observations are same	$AM = GM = HM$
When all the observations are distinct	$AM > GM > HM$
In General	$AM \geq GM \geq HM$

Range (in case of Discrete Observations)

$$L - S$$

L = Largest Observation

S = Smallest Observation

Range (in case of Grouped FD)

$$L - S$$

L = UCB of last class interval

S = LCB of first class interval

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Coefficient of Range

$$\frac{L - S}{L + S} \times 100$$

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Mean Deviation (in case of discrete observations)

$$MD_A = \frac{1}{n} \sum |x - A|$$

Here A is mean or median or mode as given in question

Mean Deviation (in case of grouped frequency distributions)

$$MD_A = \frac{1}{N} \sum f |x - A|$$

Here A is mean or median or mode as given in question

Coefficient of Mean Deviation

$$\frac{\text{Mean Deviation about } A}{A} \times 100$$

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Standard Deviation (in case of discrete observations)

$$\sigma_x = SD_x = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$\sigma_x = SD_x = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

Standard Deviation (in case of grouped frequency observations)

$$\sigma_x = SD_x = \sqrt{\frac{\sum f(x - \bar{x})^2}{N}}$$

$$\sigma_x = SD_x = \sqrt{\frac{\sum fx^2}{N} - (\bar{x})^2}$$

Coefficient of Variation

$$\frac{SD_x}{\bar{x}} \times 100$$

For any two numbers, SD = half of range

$$SD = \frac{|a - b|}{2}$$

- › Standard Deviation of first n natural numbers

$$s = \sqrt{\frac{n^2 - 1}{12}}$$

Combined SD

$$SD_c = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$d_1 = \bar{x}_c - \bar{x}_1$$

$$d_2 = \bar{x}_c - \bar{x}_2$$

Property of SD

If all the observations are constant, SD is **Zero**

Property of SD

- › *No effect of change of origin but affected by change of scale in the magnitude (ignore sign)*

Quartile Deviation

$$QD_x = \frac{Q_3 - Q_1}{2}$$

Coefficient of Quartile Deviation

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

Relationship between SD, MD, QD

For a moderately skewed distribution

$$4SD = 5MD = 6QD$$

$$SD : MD : QD = 15 : 12 : 10$$

Basic formula of Probability

$$P(A) = \frac{\text{No. of favorable events to } A}{\text{Total no. of events}}$$

π

Odds in Favor of Event A

$$\frac{\text{no. of favorable events}}{\text{no. of unfavorable events}}$$

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π

› Odds against an Event

$$\frac{\text{no. of unfavorable events}}{\text{no. of favorable events}}$$

50

- › No. of total outcome of a random experiment

If an experiment results in p outcomes and if it is repeated q times then

Total number of outcomes is

$$p^q$$

Relative Frequency Probability

Relative Frequency = $\frac{\text{no. of times the event occurred during experimental trials}}{\text{total no. of trials}} = \frac{f_A}{n}$

Probability by this method is defined as

$$P(A) = \lim_{n \rightarrow \infty} \frac{f_A}{n}$$

› Set Based Probability

$$P(A) = \frac{\text{no.of sample points in } A}{\text{no.of sample points in } S} = \frac{n(A)}{n(S)}$$

Addition Theorem 1

In case of two mutually exclusive events A and B

$$P(A \cup B) = P(A+B) = P(A \text{ or } B) = P(A) + P(B)$$

Addition Theorem 2

In case of two or more mutually exclusive events

$$P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$$

Addition Theorem 3

For any two events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Addition Theorem 4

In case of any three events

$$P(A \cup B \cup C)$$

$$\begin{aligned} &= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) \\ &\quad - P(A \cap C) + P(A \cap B \cap C) \end{aligned}$$

*Conditional Probability of Event B
knowing that Event A is already occurred*

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

provided $P(A) \neq 0$

*Conditional Probability of Event A
knowing that Event B is already occurred*

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

provided $P(B) \neq 0$

Compound Theorem

In case of two dependent events

$$P(A \cap B) = P(B) \times P(A/B) \text{ or}$$

$$P(A \cap B) = P(A) \times P(B/A)$$

Compound Theorem

In case of two independent events

$$P(A \cap B) = P(A) \times P(B)$$

Same thing applies for two or more independent events

Expected value of a Probability Distribution

$$E(x) = \sum p_i x_i$$

$$E(x) = \mu$$

Variance of Probability Distribution

$$V(x) = \sigma^2$$

$$E(x - \mu)^2 = E(x)^2 - \mu^2$$

Probability Mass Function in case of Binomial Distribution

$$f(x) = P(X = x) = {}^nC_x p^x q^{n-x}$$

Mean and Variance in case of Binomial Distribution

$$\mu = np$$

$$\sigma^2 = npq$$

Mode in case of Binomial Distribution

Calculate $(n + 1)p$

*if the resulting value is integer
then Bi-modal*

$\mu_0 = (n + 1)p$ and
 $[(n + 1)p - 1]$

*If the resulting value is non-
integer then Uni-modal*

$\mu_0 =$ largest integer contained in
 $(n + 1)p$

Probability Mass Function in case of Poisson Distribution

$$f(x) = P(X = x) = \frac{(e^{-m} \cdot m^x)}{x!}$$

Mean, Variance and SD in case of Poisson Distribution

$$\mu = m$$

$$\sigma^2 = m$$

$$\sigma = \sqrt{m}$$

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Mode in case of Poisson Distribution

*if the value of m is integer
then Bi-modal*

$$\mu_0 = m \text{ and } [m - 1]$$

If the value of m is non-integer then Uni-modal

$\mu_0 =$ largest integer contained in m

Probability Density Function in case of Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\left(\frac{x-\mu}{\sigma}\right)^2 \times \frac{1}{2}}$$

Mean Deviation in case of Normal Distribution

$$MD = 0.8 \sigma$$

Quartiles in case of Normal Distribution

$$Q_1 = \mu - 0.675\sigma$$

$$Q_3 = \mu + 0.675\sigma$$

Quartile Deviation in case of Normal Distribution

$$QD = 0.675\sigma$$

Points of Inflexion

$$\mu - \sigma \text{ \& } \mu + \sigma$$

π

Ratio between

$$QD : MD : SD = 10:12:15$$

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Conditions of Standard Normal Distribution

Mean μ	0
Standard Deviation σ	1

$$Z = \frac{x - \mu}{\sigma}$$

Area under Normal Curve

<i>From</i>	<i>To</i>	<i>Area/Probability</i>
μ	$+\sigma$	34.135%
$+\sigma$	$+2\sigma$	13.59%
$+2\sigma$	$+3\sigma$	2.14%
3σ	∞	0.135%

For a bivariate frequency distribution

No. of marginal distributions = 2

No. of conditional distributions = $m + n$

Karl Pearson's Product Moment Correlation Coefficient

$$r_{xy} = \frac{Cov(x, y)}{\sigma_x \cdot \sigma_y}$$

Covariance

$$Cov(x, y) = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{n} \text{ or } \frac{\Sigma xy}{n} - \bar{x} \cdot \bar{y}$$

Spearman's Rank Correlation Coefficient

$$r_R = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

here d means difference in ranks

Adjustment value in case of Tie

$$\sum \frac{(t^3 - t)}{12}$$

Coefficient of Concurrent Deviations

$$r_c = \pm \sqrt{\pm \frac{2c - m}{m}}$$

Regression Coefficients

$$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} \text{ or } b_{yx} = \frac{\text{Cov}(x,y)}{\sigma_x^2}$$

$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} \text{ or } b_{xy} = \frac{\text{Cov}(x,y)}{\sigma_y^2}$$

Regression Property

π

$$r = \pm \sqrt{b_{xy} \times b_{yx}}$$

b_{xy} , b_{yx} and r all will have same sign

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Regression Property

› Change of Origin and Scale

- *Origin: No Impact*
- *Scale: If original pair is x, y and modified pair is u, v*

then,

$$b_{vu} = b_{yx} \times \frac{\text{change of scale of } y}{\text{change of scale of } x}$$

$$b_{uv} = b_{xy} \times \frac{\text{change of scale of } x}{\text{change of scale of } y}$$

Regression Property

- › Intersection of two regression lines:
 - *Two regression lines (if not identical) will intersect at the point [means]*

$$(\bar{x}, \bar{y})$$

π

Coefficient of determination

Explained Variance

Accounted Variance

$$r^2$$

89

π

Coefficient of non-determination

Unexplained Variance

Unaccounted Variance

$$1 - r^2$$

90

Probable Error

π

$$0.6745 \times \frac{1-r^2}{\sqrt{N}}$$

- where, r = correlation coefficient, N = no. of pairs of observation

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Limits of Population Correlation Coefficient

π

$$r \pm PE$$

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Index Relatives

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<i>Price Relative</i>	$Price\ Relative = \frac{P_n}{P_0}$
<i>Quantity Relative</i>	$Quantity\ Relative = \frac{Q_n}{Q_0}$
<i>Value Relative</i>	$Value\ Relative = \frac{V_n}{V_0} = \frac{P_n Q_n}{P_0 Q_0}$
<i>Link relative</i>	$\frac{P_1}{P_0}, \frac{P_2}{P_1}, \frac{P_3}{P_2}, \dots, \frac{P_n}{P_{n-1}}$ <p>Same can be created for quantities also</p>
<i>Chain relatives</i>	<p>When the above relatives are in respect to a fixed base period these are also called the chain relatives</p> $\frac{P_1}{P_0}, \frac{P_2}{P_0}, \frac{P_3}{P_0}, \dots, \frac{P_n}{P_0}$

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Simple Aggregative Method of Index

Price Index is expressed as total of commodity prices in a given year as a percentage of total of commodity prices in the base year

› Formula

$$\frac{\Sigma P_n}{\Sigma P_0} \times 100$$

Simple Average of Relatives – Method of Index

- › *Under this method, we invert the actual price for each variable into percentage of the base period. These percentages are called relatives.*
- › Formula

$$\frac{\sum \frac{P_n}{P_0}}{N}$$

Weighted Index Methods

› *Laspeyres' Index*

$$\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$$

Weighted Index Methods

π

› *Passche's Index*

$$\frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n} \times 100$$

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Weighted Index Methods

π

› *Marshall-Edgeworth Index*

$$\frac{\Sigma P_n(Q_0 + Q_n)}{\Sigma P_0(Q_0 + Q_n)} \times 100$$

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Weighted Index Methods

› *Fisher's Index*

(GM of Laspeyres' and Paasche's)

$$\sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{\sum P_n Q_n}{\sum P_0 Q_n}} \times 100$$

Deflated Value

π

$$\text{Deflated Value} = \frac{\text{Current Value}}{\text{Price Index of the current year}}$$

100



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