

10/7/28

Ch- Equation

* Equation:-

$$\text{eg:- } x^2 + 2x + 3 = 5x + 7$$

* Linear Equation:- Eqⁿ in which degree of variable is highest power

$$\text{Eq:- } 2x + 3y = 0 ; \text{ HP} = 1 ; 2 \text{ variable}$$

$$2x + 3y^2 = 0 ; \text{ HP} = 2 ; \text{ No L.E}$$

$$2x + 3^y = 0 ; \text{ HP} = 1 ; \text{ One variable / Simple eqⁿ}$$

Standard form of Single Eqⁿ:-

$$ax + b = 0$$

$$a \neq 0$$

Standard form of linear eqⁿ in two variable

$$ax + by + c = 0$$

$$\text{Eq :- } 2x + 3y + 5 = 0 ; \text{ Infinite Solution, Represent line}$$

Pair of linear eqⁿ in two variable

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Unique

~~One~~ Solution

No Solution

Infinite Solution

* Solution of linear eqⁿ in two variable

• Substitution

• Elimination

• Cross-Multiplication

* Quadratic Equation:-

Degree of variable is 2

Eq:- $x^2 + 2x + 3 = 0 \rightarrow$ Q.E in one variable

$x^2 + 2y + 3 = 0 \rightarrow$ Q.E in two variable

Standard form:- $ax^2 + bx + c = 0$
 $a \neq 0$

* Solution of Quadratic Equation:-

1. Factorisation

- Direct

- Middle Term Splitting (MTS)

2. Quadratic Formula

$$ax^2 + bx + c = 0; a \neq 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac =$ Discriminant (D)

Eq:- $x^2 + 2x - 4 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 + 16}}{2} \Rightarrow \frac{-2 \pm \sqrt{20}}{2}$$

* Nature of roots:- Solutions or zeroes

$$ax^2 + bx + c = 0$$

equal roots $\Rightarrow b^2 - 4ac = D \rightarrow$ Discriminant

- i) If $D > 0$ (+ve) $\rightarrow$ Real and unequal roots
- ii) If $D = 0 \rightarrow$ Real and equal roots
- iii) If $D < 0$ (-ve) $\rightarrow$ No real roots
- iv) If $D \geq 0 \rightarrow$ Real roots

* Roots and Coefficients:-

$$ax^2 + bx + c = 0$$

Two roots $\rightarrow \alpha, \beta$

$\downarrow \quad \downarrow$
alpha Beta

* Sum of roots: $\alpha + \beta = -\frac{b}{a}$

* Product of roots: $\alpha\beta = \frac{c}{a}$

* If one root is reciprocal of other $\Rightarrow \boxed{a = c}$

* If one root is negative of other $\Rightarrow \boxed{b = 0}$

* Identities

- i) $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$
- ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$\text{iii)} \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$\text{iv)} \quad (\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$$

$$\text{v)} \quad \alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

* Formation of Quadratic Equations:-
If α and β are roots.

$$\text{quadratic Eqn} \Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Cubic Equation:-

Standard form: $ax^3 + bx^2 + cx + d = 0$; $a \neq 0$
roots = α ; β ; γ

$$1. \quad \text{Sum of roots} = \alpha + \beta + \gamma = -\frac{b}{a}$$

$$2. \quad \text{Sum of product of roots} = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$3. \quad \text{Product of roots} = \alpha\beta\gamma = -\frac{d}{a}$$