[CA] CA-Foundation

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11	Sequence & Series	4	4	4	4	3	3	3	3	4	2	3	3.36
12	Sets, Function and Relation	3	4	5	2	4	3	4	3	3	3	5	3.54
13	*Calculus (Limit & Continuity)												
14	Differential Calculus	1	2	2	3	2	1	3	3	1	3	3	2.18
15	Integration	3	2	3	3	3	1	1	1	2	3	1	2.09
16	Number Series, Coding & Decoding	3	5	4	5	4	5	5	5	5	6	5	4.72
17	Direction Tests	6	5	4	6	4	3	4	8	5	5	4	4.90
18	Seating Arrangement	5	3	4	2	5	4	4	3	4	2	4	3.63
19	Blood Relations	4	4	4	4	3	4	4	5	5	7	7	4.63
20	Description of Data	4	7	5	4	7	10	8	5	8	4	5	6.09
21	Central Tendency	5	7	5	7	11	4	4	10	6	8	8	6.81
22	Measures of Dispersion	2	4	8	8	1	5	7	1	5	8	6	5.00
23	Probability	7	6	5	3	4	6	6	4	6	7	5	5.36

S. No.	Chapter	2018		2019		2020	2021			2022		2023	Average
		J	D	J	D	D	Jan	July	D	J	D	J	
24	Probability (Theoretical) Distribution	6	5	4	7	7	4	5	8	5	4	6	5.54
25	*Sampling Theory of Estimation												
26	Correlation	8	1	4	3	3	2	1	1	4	2	2	2.81
27	Regression Analysis	5	5	2	2	-	3	4	3	1	3	3	2.81
28	Index Numbers	8	3	4	3	3	3	4	5	5	5	4	4.27

Note : J: June; D : December

***Chapter 13 & Chapter 25 are newly added Chapters in Syllabus.**

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13	Calculus (Limit & Continuity)	7
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16	Number Series, Coding & Decoding	9
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1 CHAPTER RATIO & PROPORTION

RATIO

The comparison of two or more things of same kind is called RATIO. If x and y are two values of same kind (in same units), then the ratio of x to y is written as x : y and read as x is to y.

In $\frac{x}{y}$

- Numerator "x" is called **1st term or Antecedent** and
- Denominator "y" is called **2nd term or Consequent.**
- Antecedent and Consequent must be of **same units**
- Ratio has no unit.

Remarks:

1. Normally a ratio is expressed in simplest form. As. 10 : 16 = 5 : 8.
2. The order of the terms in a ratio must be maintained. As 3 : 4 is not same as 4: 3.
3. Ratio exists only with quantities having same unit (kind).
4. (*i*) If x > y, then the ratio x : y is called of **greater inequality.**
   (*ii*) If x < y, then the ratio x : y is called of **lesser inequality.**
   (*iii*) If x = y, then the ratio a : b is called ratio of Equal **Equality**.
5. (*i*) **Duplicate ratio** of a : b is $a^2 : b^2$
   (*ii*) **Triplicate ratio** of a : b is $a^3 : b^3$
   (*iii*) **Sub-Duplicate ratio** of a : b is $\sqrt{a} : \sqrt{b} = a^{1/2} : b^{1/2}$
   (*iv*) **Sub-Triplicate ratio** of a : b is $\sqrt[3]{a} : \sqrt[3]{b} = a^{1/3} : b^{1/3}$

6. **Inverse ratio** of x : y is y : x.
7. (*i*) ***Commensurable :*** If the terms of the ratio are **integers, the ratio** is called commensurable. ***As.*** 3 : 2

   (*ii*) ***Incommensurable :*** If the terms of the ratio are not **integers, the ratio** is called Incommensurable.

   As. $\sqrt{3}:\sqrt{2}$ cannot be expressed in terms of integers. So, it is **Incommensurable.**
8. **Compound/Combined Ratio = Product of all ratios.**

PROPORTION

An equality of two ratios is called **Proportion.** Four quantities a, b, c, d are said to be in proportion if a : b = c : d.

It is also written as

a : b : : c : d

Here a, b, c, d are called 1st; 2nd; 3rd and 4th term of proportionals respectively

If, $\frac{a}{b}=\frac{c}{d}\Rightarrow ad=bc.$

∴ Product of extreme terms = Product of middle terms.

This rule is called **Cross – Product Rule.**

In a : b : : c : d. proportion, unit of a and b should be same and that of c and d should also be same.

As. ₹ 6 : ₹ 8 = 12 toffees : 16 toffees are in proportion.

Let a, b and c are of same kind (in same units).

If $\frac{a}{b}=\frac{b}{c}\ \Rightarrow a:b::b:c$

i.e. a, b, c are in proportion.

Then, this proportion is called **continuous proportion.**

Here **a** is **1st proportion** c is called **3rd proportion** and **b** is called **mean proportion.**

∴ $b=\sqrt{ac}$ = GM of a & c.

Properties of Proportion

1. **Cross - Product**

   If a : b : : c : d.

   $\Rightarrow \frac{a}{b}=\frac{c}{d}$ ∴ ad = bc.
2. **Invertendo**

   If a : b : : c : d.; Then its inverse

   b : a : : d : c also becomes in proportion.

   i.e. If $\frac{a}{b}=\frac{c}{d}$ Then, $\frac{b}{a}=\frac{d}{c}$.
3. **Componendo**

   If a : b : : c : d.

   Then a + b : b : : c + d : d.

   Proof : $\frac{a}{b}+1=\frac{c}{d}+1\Rightarrow\frac{a+b}{b}=\frac{c+d}{d}$.
4. **Dividendo**

   If a : b = c : d.

   Then a - b : b = c - d : d.

   Proof : $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a}{b}-1=\frac{c}{d}-1$

   or $\frac{a-b}{b}=\frac{c-d}{d}$
5. **Componendo and Dividendo**

   If a : b : : c : d. ; Dividing (3) by (4)

   Then, $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.
6. **Alternendo**

   If a : b : : c : d.

   Then a : c : : b ; d.

   i.e. ratio of alternate terms are also in proportion.
7. **Addendo**

   If a : b = c : d = e : f = ...

Then each ratio = $\frac{\text{Sum of antecedents of all ratios}}{\text{Sum of consequents of all ratios}}$

$\therefore \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots=\frac{a+c+e+\ldots}{b+d+f+\ldots}$

8. **Subtrahendo**

   If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots$

   Then each ratio

   $=\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots=\frac{a-c-e-\ldots}{b-d-f-\ldots}$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. What must be added to each term of the ratio 49 : 68 so that it becomes 3 : 4?

(*a*) 3 (*b*) 5

(*c*) 8 (*d*) 9

[June 2010]

Solution : Detail Method

Let x is added to each term

Then $\frac{49+x}{68+x}=\frac{3}{4}$

or 196 + 4x = 204 + 3x

or 4x - 3x = 204 - 196

or x = 8

∴ (*c*) is Correct

Tricks : Go by Choices

1st Find $\frac{3}{4}=0.75$ (By Calculator)

Then For (*a*) $\frac{49+3}{68+3}\neq 0.75$

(*b*) $\frac{49+5}{68+5}\neq 0.75$

(*c*) $\frac{49+8}{68+8}=\frac{57}{76}=0.75$

∴ (*c*) is Correct

Q.2. The students of two classes are in the ratios 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6, then the number of students in each class was :

(*a*) 30, 40 (*b*) 25, 24

(*c*) 40, 60 (*d*) 50, 70

[June 2010]

Solution : Tricks : Go by choices:

(*a*) ; (*b*) and (*c*) are not in the ratio 5 : 7

∴ (*d*) is Correct.

Q.3. If A : B = 2:5, then (10A + 3B) : (5A+2B) is equal to

(*a*) 7:4 (*b*) 7:3

(*c*) 6:5 (*d*) 7:9

[Dec. 2010]

Solution : It A : B = 2 : 5 Then

$\frac{10A+3B}{5A+2B}=\frac{10\times2+3\times5}{5\times2+2\times5}=\frac{35}{20}=\frac{7}{4}$

= 7 : 4

(*a*) is correct

Q.4. In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹1,60,000 and C gets ₹2,50,000. Find the amount received by B ?

(*a*) ₹2,00,000 (*b*) ₹2,50,000

(*c*) ₹1,00,000 (*d*) ₹1,50,000

[June 2011]

Solution : Detail Method

A : B = B : C

So, $B^2 = AC$;

So, B

$=\sqrt{AC}=\sqrt{1,60,000\times2,50,000}$

$=400\times500=2,00,000$

Q.5. The ratio compounded of 4:5 and sub-duplicate of " a" : 9 is 8:15. Then value of "a" is

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

[Dec. 2011]

Solution : (*c*) $\frac{4}{5}\times\sqrt{\frac{a}{9}}=\frac{8}{15}$

or $\frac{4}{5}\times\frac{\sqrt{a}}{3}=\frac{8}{15}$

$\therefore\sqrt{a}=2\Rightarrow a=4$

∴ (*c*) is correct

Q.6. If X varies inversely as square of Y and given that Y =2 for X = 1, then the value of X for Y = 6 will be

(*a*) 3 (*b*) 9

(*c*) 1/3 (*d*) 6

[Dec. 2011]

Solution : (*d*) is correct

$x\propto\frac{1}{y^2}\Rightarrow x=K.\frac{1}{y^2}\Rightarrow x=\frac{k}{y^2}$; where

k = proportional constant

When x = 1 Then y = 2

$\therefore 1=\frac{k}{2^2}\Rightarrow k=4\ \therefore x=\frac{4}{y^2}$

When y = 6, Then $x=\frac{4}{6^2}=\frac{1}{9}$

$\therefore x=\frac{1}{9}$

Q.7. Which of the numbers are not in proportion ?

(*a*) 6,8,5,7 (*b*) 7,14,6,12

(*c*) 18,27,12,18 (*d*) 8,6,12,9

[June 2012]

Solution : (*a*) Go by choices

For (*a*) $\frac{6}{8}=\frac{3}{4}\neq\frac{5}{7}$

∴ (*a*) is not in proportion

Q.8. Find two numbers such that mean proportional between them is 18 and third proportional between them is 144

(*a*) 9 ; 36 (*b*) 8 ; 32

(*c*) 7 ; 28 (*d*) 6 ; 14

[Dec. 2012]

Solution : (*a*) is correct

Tricks : Go by choices

For (a) Mean Proportional of 9 and 36

$=\sqrt{9\times36}=18$

It satisfies 1st condition.

If 144 is its 3rd condition.

$36^2=9\times144$

It also satisfies the 2nd Condition.

Q.9. Triplicate ratio of 4 : 5 is

(a) 125 : 64 (b) 16 : 25

(c) 64 : 125 (d) 120 : 46

[June 2013]

Solution : (c) Triplicate ratio of 4:5

$=4^3:5^3=64:125$

Q.10. The mean proportion between 24 and 54 is _____

(a) 33 (b) 34

(c) 35 (d) 36

[June 2013]

Solution : (d) Mean - Proportion

$=\sqrt{24\times54}=36$

Q.11. The ratio of numbers is 1 : 2 : 3 and sum of their squares is 504 then the numbers are

(a) 6,12,18 (b) 3,6,9

(c) 4,8,12 (d) 5,10,15

[Dec. 2013]

Solution : (a) is correct

Tricks : Go by choices

6:12:18 = 1:2:3 (True)

and $6^2+12^2+18^2=504$ (True)

Q.12. If P is 25% less than Q and R is 20% higher than Q the Ratio of R and P

(a) 5:8 (b) 8:5

(c) 5:3 (d) 3:5

[Dec. 2013]

Solution : (b) is correct

Let Q = 100, So, P = 100-025 = 75

& R = 100 + 20 = 120

$\frac{R}{P}=\frac{120}{75}=\frac{8}{5}$

Q.13. A person has assets worth ₹1,48,200. He wish to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this assets the share of his son will be

(a) ₹ 74,100 (b) ₹ 37,050

(c) ₹ 49,400 (d) ₹ 24,700

[June 2014]

Solution : (c) is correct

Share of son $=\frac{2}{3+2+1}\times1,48,200$

= ₹49,400

Q.14. If x : y = 2 : 3 then (5x+2y) : (3x - y) =

(a) 19 : 3 (b) 16 : 3

(c) 7 : 2 (d) 7 : 3

[June 2014]

Solution : (b) is correct

$\frac{5x+2y}{3x-y}=\frac{5\times2+2\times3}{3\times2-3}=\frac{16}{3}$

Q.15. The first, second and third month salaries of a person are in the ratio 2:4:5. The difference between the product of the salaries of first 2 months & last 2 months is ₹4,80,00,000. Find the salary of the second month

(a) ₹ 4,000 (b) ₹ 6,000

(c) ₹ 12,000 (d) ₹ 8,000

[Dec. 2014]

Solution : (d) is correct

Let x is common in the ratio.

1st, 2nd and 3rd month salaries of a person = 2x ; 4x ; 5x

:- From Qts.

$4x\times5x-2x\times4x=4,80,00,000.$

$or, 12x^2=4,80,00,000.$

$or, x^2=4000000$

$x=2000.$

$\therefore 2^{nd}$ month salary $=4x=4\times2000$

= ₹ 8000

Q.16. $15(2p^2-q^2)=7pq$, where p, q are positive then p : q

(a) 5:6 (b) 5:7

(c) 3:5 (d) 3:7

[June 2015]

Solution : (a) is correct

$15(2p^2-q^2)=7pq$

Tricks : Go by choices

For (a) put p = 5; q = 6 we get

$15\left|2\times5^2-6^2\right|=3\times5\times6$

or $15\times14=210$

or $210=210$

Q.17. If one type of rice of cost ₹13.84 is mixed with another type of rice of cost ₹15.54, the mixture is sold at ₹17.60 with a profit of 14.6% on selling price then in which proportion the two types of rice mixed ?

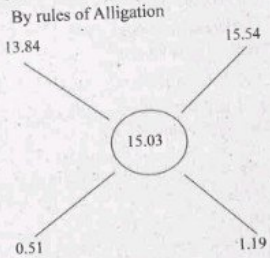
(a) 3:7 (b) 5:7

(c) 7:9 (d) 9:1

[June 2015]

Solution : Cost of mixture per kg = 17.60 - 14.6% = 15.0304 = 15.03 (approx.)

By rules of Alligation



51:119 = 3:7

Go by choices

(a) is correct (approx)

Q.18. Find the ratio of third proportional of 12 ; 30 and mean proportional of 9 ; 25 :

(a) 7 : 2 (b) 5 : 1

(c) 9 : 4 (d) None of these

[Dec. 2015]

Solution : 3rd proportional $=\frac{30^2}{12}=75$

Mean Proportional $=\sqrt{9\times25}=15$

Ratio $=\frac{75}{15}=5:1$

(b) is correct

Q.19. What must be added to each of the numbers 10, 18, 22, 38 to make them proportional:

(a) 5 (b) 2

(c) 3 (d) 9

[Dec. 2015]

Solution : (b) is correct

let x be added.

$\therefore\frac{10+x}{18+x}=\frac{22+x}{38+x}$

Tricks: Go by choices.

$\therefore x=2$ satisfies it.

Q.20. x, y, z together starts a business, if x invests 3 times as much as y invests and y invests two third of what z invests, then the ratio of capitals of x, y, z is _____

(a) 3 : 9 : 2 (b) 6 : 3 : 2

(c) 3 : 6 : 2 (d) 6 : 2 : 3

[June 2016]

Solution : (d)

Detail Method

$x=3y\Rightarrow\frac{x}{y}=\frac{3}{1}\Rightarrow x:y=3:1$

and $y=\frac{2}{3}z\Rightarrow y:z=2:3$

$x:y=3:1\ ]\times2$

$y:z=2:3$

$\Rightarrow x:y=6:2$

$y:z=2:3$

$x:y:z=6:2:3$

(d) is correct

Tricks: Go by choices

$6=3\times2$ and $2=3\times\frac{2}{3}$

Q.21. A bag contains 23 number of coins in the form of 1 rupee, 2 rupee and 5 rupee coins. The total sum of the coins is ₹43. The ratio between 1 rupee and 2 rupees coins is 3 : 2. Then the number of 1 rupee coins is

(a) 12 (b) 8

(c) 10 (d) 16

[Dec. 2016]

Solution : (a)

Tricks : Go by choices

Let option (a) is correct.

Let x is common in the ratio.

So, ₹1 coins = 3x = 12 ; So, x = 4

No. of ₹ 2 coins = 2 × 4 = 8

Hence No. of coins of ₹ 5 coins = 23-12 - 8 = 3

Total money = 12 × 1 + 8 × 2 + 3 × 5 = ₹ 43 Satisfied.

So (a) is correct

Detail Method:

Let x is common in the ratio.

∴ No. of ₹ 1 coins & ₹ 2 coins are 3x; 2x

∴ No. of ₹ 5 coins = 23 – 3x – 2x = 23 – 5x

Total Sum

$=3x\times1+2x\times2+(23-5x)\times5=43$

$7x-25x+115=43$

or 115 - 43 = 18x

or 72 = 18x

or x = 4

∴ No. of ₹ 1 coins

= 3x

= 3 × 4

= 12

Q.22. If a : b = 2 : 3, b : c = 4 : 5, c : d = 6 : 7 then a : d is _____

(a) 24 : 35 (b) 8 : 15

(c) 16 : 35 (d) 7 : 15

[June 2017]

Solution : Option (c) is correct.

Multiply all ratios.

$\frac{a}{b}:\frac{b}{c}:\frac{c}{d}=\frac{a}{d}$

$=\frac{2}{3}\times\frac{4}{5}\times\frac{6}{7}=\frac{16}{35}$

Q.23. The ratio of the number of five rupee coins to number of ten rupee coins is 8: 15. If the total value of five rupee coins is 360, then the no. of ten rupee coins is _____

(a) 72 (b) 60

(c) 150 (d) 135

[Dec. 2017]

Solution : Option (d) is correct.

Total No. of ₹5 coins = 360/5 = 72

Let x is common in the ratio.

So, ₹5 coins = 8x = 72 ; So, x = 9

No. of ₹ 10 coins = 15×9 = 135

Q.24. If $\frac{1}{2},\frac{1}{3},\frac{1}{5},\frac{1}{x}$ are in proportion then x =

(a) $\frac{15}{2}$ (b) $\frac{3}{15}$

(c) $\frac{2}{15}$ (d) $\frac{1}{15}$

[Dec. 2017]

Solution : Option (a) is correct.

Product of middle two terms

= Product of extremes

So, $\frac{1}{2x}=\frac{1}{15}$; x = 15/2

Q.25. If (a + b) : (b + c) : (c + a) = 7 : 8 : 9 and a + b + c = 18 then a : b : c =

(a) 5 : 4 : 3 (b) 3 : 4 : 5

(c) 4 : 3 : 5 (d) 4 : 5 : 3

[June 2018]

Solution : (c) is correct.

Tricks: Go by choices.

(c) Let a : b : c = 4 : 3 : 5

It is in ratio. So, it should must **satisfy given ratio** (a + b) : (b + c) : (c + a) = 7 : 8 : 9 *i.e.* (4 + 3) : (3 + 5) : (5 + 4) = 7 : 8 : 9 (True) Avoid 2nd condition.

In detail method it will take too much time.

Q.26. If p : q is the sub-duplicate ratio of $p-x^2:q-x^2$, then x^2 is :

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$

(c) $\frac{qp}{p-q}$ (d) None

[May 2018]

Solution : Detail Method:

$\frac{\sqrt{p-x^2}}{\sqrt{q-x^2}}=\frac{p}{q}$

Squaring on both side; we get

$\frac{p-x^2}{q-x^2}=\frac{p^2}{q^2}$

or $pq^2 - q^2x^2 = p^2q - p^2x^2$

or $p^2x^2 - q^2x^2 = p^2q - pq^2$

or $x^2(p^2 - q^2) = Pq(P - q)$

or x^2 (p + q)(p - q) = pq (p - q)

or $x^2 = \frac{pq}{p+q}$

$\therefore$ (*d*) is correct

Tricks : Go by choices.

Q.27. The mean proportional between 24 and 54 is :

(*a*) 33 (*b*) 34

(*c*) 35 (*d*) 36

Solution : Formula

Mean Proportion of a & b $= \sqrt{ab}$

(*d*) $= \sqrt{24 \times 54} = 36$

[May 2018]

Q.28. $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$ then find the value of x :

(*a*) 6 (*b*) 2

(*c*) 5 (*d*) 9

[Nov. 2018]

Solution : (*a*)

Given $\frac{3x-2}{5x+6} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Tricks : Go by choices

for option (*a*) putting x = 6 in LHS; we get

$\frac{3\times2-2}{5\times2+6} = \frac{4}{9}$ (R.H.S)

$\therefore$ (*a*) is correct.

Q.29. If x : y : z = 7 : 4 : 11 then $\frac{x+y+z}{z}$ is:

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

[Nov. 2018]

Solution : (*a*)

$\frac{x+y+z}{z} = \frac{7+4+11}{11} = 2$

Q.30. If the ratio of two numbers is 7 : 11. If 7 is added to each number then the new ratio will be 2 : 3 then the numbers are.

(*a*) 49, 77 (*b*) 42, 45

(*c*) 43, 42 (*d*) 39, 40

[June 2019]

Solution : Tricks:- GBC (Go by Choices)

(*a*) $\begin{bmatrix} 49+7=7 \\ 77+11=7 \end{bmatrix}$ both must be equal.

Here it is correct.

Now:

$\frac{49+7}{77+7} = \frac{56}{84} = \frac{2}{3}$

Divide 56 by numerator (2) and 84 by Denominator (3) we get same value "28"

Note:- No need to solve ; only check.

By Calculator.

Q.31. The two numbers are in ratio 3 : 4. The difference between their squares is 28. Find the greater number.

(*a*) 12 (*b*) 8

(*c*) 16 (*d*) 10

[Nov. 2019]

Solution : (*b*)

Let x is common in the ratio.

So, Numbers are 3*x* & 4*x*.

From Qts.

$(4x)^2 - (3x)^2 = 28$

or; $16x^2 - 9x^2 = 28$

or $7x^2 = 28 \Rightarrow x^2 = 4$

$\therefore x = 2$

$\therefore$ Greater No. = 4x = 4 × 2 = 8

Q.32. The ratio of No. of boys and the No. of girls in a school is found to be 15 : 32. How many boys and equal no. of girls should be added to bring the ratio to 2/3 ?

(*a*) 20 (*b*) 19

(*c*) 23 (*d*) 27

[Dec. 2020]

Solution : Tricks GBC

(*b*) $\frac{15+19}{32+19} = \frac{34}{51} = \frac{2}{3}$ (True)

$\therefore$ (*b*) is correct.

Detail Method

Let *x* is added.

$\therefore \frac{15+x}{32+x} = \frac{2}{3}$

or $45 + 3x = 64 + 2x$

or $3x - 2x = 64 - 45$

or $x = 19$

(*b*) is correct.

Q.33. If a : b = 9:4 then = $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = ?$

(*a*) 2/3 (*b*) 3/2

(*c*) 6/13 (*d*) 13/6

[Dec. 2020]

Solution : $\because$ a : b = 9 : 4

$\therefore \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \sqrt{\frac{9}{4}} + \sqrt{\frac{4}{9}}$

$= \frac{3}{2} + \frac{2}{3} = \frac{9+4}{6} = \frac{13}{6}$

$\therefore$ (*d*) is correct.

Q.34. If a : b = 3 : 7 then 3a + 2b : 4a + 5b = ?

(*a*) 27 : 43 (*b*) 23 : 47

(*c*) 24 : 51 (*d*) 29 : 53

[Dec. 2020]

Solution : Putting *a* = 3; *b* = 7

We get

$\frac{3a+2b}{4a+5b} = \frac{3\times3+2\times7}{4\times3+5\times7}$

$= \frac{23}{47}$

$\therefore$ (*b*) is correct

Q.35. In a certain business A and B received profit in a certain ratio; B and C received profits in the same ratio. If A gets ₹ 1600 and C gets ₹ 2500 then how much does B get?

(*a*) ₹ 2,000 (*b*) ₹ 2,500

(*c*) ₹ 1,000 (*d*) ₹ 1,500

[Jan. 2021]

Solution : (*a*) is given

$\frac{A}{B} = \frac{B}{C}$

or, $B^2 = AC$

or $B = \sqrt{AC}$

$= \sqrt{1600 \times 2500}$

$= 40 \times 50$

$= ₹ 2000$

Q.36. The ratio of two quantities is 15 : 17. If the consequent of its inverse ratio is 15, then the antecedent is;

(*a*) 15 (*b*) $\sqrt{15}$

(*c*) 17 (*d*) 14

[Jan. 2021]

Solution : (*c*) is correct

After inversing the ratio 15 : 17

We get $\frac{17}{15}$ i.e. 15 as consequent

Clearly its Antecedent = 17

Q.37. The salaries of A, B and C are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(*a*) 3 : 3 : 10

(*b*) 10 : 11 : 20

(*c*) 23 : 33 : 60

(*d*) Cannot be determined

[Jan. 2021 & July 2021]

Solution : (*c*) is correct

Using Calculator.

New ratio

= A : B : C

= (2 + 15%) : (3 + 10%) : (5 + 20%)

= 2.3 : 3.3 : 6.0

[**Note:** On calculator type 2 + 15% button; we get 2.3. Similarly do for rest terms]

Multiplying each term by 10; we get

= 23 : 33 : 60

Q.38. A vessel contained a solution of acid and water in which water was 64%. Four litres of the solution were taken out of the vessel and the same quantity of water was added. If the resulting solution contains 30% acid, the quantity (in litres) of the solution, in the beginning in the vessel, was

(*a*) 12 (*b*) 36

(*c*) 24 (*d*) 27

[July 2021]

Solution: (*c*) is correct

Let quantity of mixture = x

Acid = x × (100 – 64)% = 0·36x

Quantity of Acid in 4 litre mixture

= 4 × 36% = 1·44

After reduction of 4 litres of mixture; remaining Acid = 0·36x – 1·44

After adding 4 litre water, Total quantity

= x – 4 + 4 = x litre

$\therefore$ Percentage of Acid

$= \frac{0.36x - 1.44}{x} \times 100 = 30$

or,

0·36x – 1·44 = 0·3x

or 0·36x – 0·30x = 1·44

or 0·06x = 1·44

or $x = \frac{1.44}{0.06} = 24$ litres

Q.39. If A : B = 5 : 3, B : C = 6 : 7 and C : D = 14 : 9, then the value of A: B : C : D

(*a*) 20 : 14 : 12 : 9

(*b*) 20 : 9 : 12 : 14

(*c*) 20 : 9 : 14 : 12

(*d*) 20 : 12 : 14 : 9

[July 2021]

Solution : (*d*) is correct

Tricks : Go by Choices (GBC)

Checking for all ratios, we get

option (a) 20 : 14 : 12 : 9

$\frac{20}{14} \neq \frac{5}{3} = \frac{A}{B}$

So (*a*) False

Doing in this way for rest options

At the end

For option (*d*) 20 : 12 : 14 : 9

A : B : C : D

$\frac{20}{12} = \frac{5}{3} = \frac{A}{B}$ (True)

$\frac{12}{14} = \frac{6}{7} = \frac{B}{C}$ (Also True)

$\frac{14}{9} = \frac{C}{D}$ (True)

Hence (*d*) is correct

Q.40. Incomes of R and S are in the ratio 7 : 9 and their expenditures are in the ratio 4 : 5. Their total expenditure is equal to income of R. What is the ratio of their savings?

(*a*) 23 : 36 (*b*) 28 : 41

(*c*) 31 : 43 (*d*) 35 : 46

[Dec. 2021]

Solution : (*d*)

Let *x* is common in the ratio of incomes

$\therefore \frac{R}{S} = \frac{7x}{9x}$

and y is common in the ratio of expenditure = $\frac{4y}{5y}$

From Question

Total expenditure = Income of R

$\Rightarrow 4y + 5y = 7x$

or $9y = 7x \Rightarrow y = \frac{7}{9}x$

Ratio of saving = $\frac{7x-4y}{9x-5y}$

$= \frac{7x - 4\times\frac{7}{9}x}{9x - 5\times\frac{7}{9}x} = \frac{\left(\frac{63-28}{9}\right)x}{\left(\frac{81-35}{9}\right)x}$

$= \frac{35}{46}$

Q.41. A bag has 105 coins containing some 50 paise, and 25 paise coins. The ratio of the number of these coins is 4 : 3. The total value (in ₹) in the bag is

(*a*) 43.25 (*b*) 41.25

(*c*) 39.25 (*d*) 35.25

[Dec. 2021]

Solution : (*b*)

Total Numbers of 50 paise coins

$= \frac{105}{4+3} \times 4 = 60$

and 25 paise coins $= \frac{105}{4+3} \times 3 = 45$

Total Value $= 60 \times 0.50 + 45 \times 0.25$

$= 30 + 11.25$

$= ₹ 41.25$

Q.42. In a department, the number of males and females are in the ratio 3 : 2. If two males and 5 females join department, then the ratio becomes 1 : 1, initially the number of female in the department is

(*a*) 9 (*b*) 6

(*c*) 3 (*d*) 8

[Dec. 2021]

Solution : (*b*)

Let x is common in the ratio

$\therefore \frac{\text{Males}}{\text{Females}} = \frac{3x}{2x}$

From Questions

Now $\frac{3x+2}{2x+5} = \frac{1}{1} \Rightarrow 3x+2 = 2x+5$

or $x = 5 - 2 = 3$

$\therefore$ Initially No. of Females $= 2x = 2 \times 3 = 6$

Q.43. A bag contains 25 paise, 10 paise and 5 paise coins in the ratio 3:2:1. The total value is ₹ 40, the number of 5 paise coins is

(*a*) 45 (*b*) 48

(*c*) 40 (*d*) 20

[June 2022]

Solution : Tricks:

Common Factor $= \frac{\text{Total Value}}{\text{Total Value of ratio terms}}$

$= \frac{40}{3(0.25)+2(0.10)+1(0.05)} = 40$

Calculator Tricks

Type 3×0.25 = button press

2×0.10 = button press

1×0.05 = button press

Then Press GT button then M+ button then type 40 ÷ button

Then MRC button. We get 40

$\therefore$ No. of 5 paise coins $= 1 \times 40 = 40$

(*c*) is correct

Q.44. If x:y = 4:6 and 2: x = 1: 2 the y = ____

(*a*) 4 (*b*) 6

(*c*) $\frac{1}{3}$ (*d*) $\frac{3}{2}$

[June 2022]

Solution :

$\because \frac{2}{x} = \frac{1}{2} \Rightarrow x = 4$

$\because \frac{x}{y} = \frac{4}{6}$

or $\frac{\cancel{4}}{y} = \frac{\cancel{4}}{6} \therefore y = 6$

(*b*) is correct

Q.45. A sum of money is to be distributed among A, B, C, D in the proportion of 5:2:4:3. If C gets ₹ 1000 more than D, what is B's share?

(*a*) 2000 (*b*) 1500

(*c*) 2500 (*d*) 1000

[Dec. 2022]

Solution :

Let x is common in the ratio

$\therefore$ A : B : C : D $= 5x : 2x : 4x : 3x$

Given

$C - D = 4x - 3x = 1000$

So, $x = 1000$

$\therefore$ B's share $= 2x = 2 \times 1000$

$= ₹ 2000$

$\therefore$ (*a*) is correct

2 CHAPTER INDICES

If a number x is multiplied 5 times written as.

$x. x. x. x. x. = x^5$.

Here "x" is called BASE and 5 is called Power or INDEX.

Some Related Formulae.

1. $a^m = a \times a \times a \times \ldots\ldots\ldots$ to m times.
2. $a^0 = 1$ where $a \neq 0; \infty$
3. $a^{-1} = \frac{1}{a}$.
4. $a^{-m} = \frac{1}{a^m}$
5. (*i*) $a^m \times a^n = a^{m+n}$
   (*ii*) $a^m \times a^n \times a^r \times \ldots\ldots = a^{m+n+r} \ldots\ldots\ldots$
6. (*i*) $\frac{a^m}{a^n} = a^{m-n}$.
   (*ii*) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$
7. (*i*) $\left(a^m\right)^n = a^{mn}$
   (*ii*) $a^{m^n} \neq a^{mn}$
8. (*i*) If $a^m = b^m$ Then a = b
   (*ii*) If $a^m = a^n$ Then m = n
9. (*i*) $\sqrt[m]{a^n} = a^{\frac{n}{m}}$

(*ii*) $\sqrt{a} = a^{\frac{1}{2}}$

(*iii*) $\sqrt[3]{a} = a^{\frac{1}{3}}$

10. (*i*) If $a^m = k \Rightarrow a = k^{1/m}$

(*ii*) If $a^m = k^n \Rightarrow a = k^{n/m}$

(*iii*) If $a^{1/m} = k \Rightarrow a = k^m$

(*iv*) If $a^{1/m} = k^n \Rightarrow a = k^{mn}$

11. (*i*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(*ii*) $(ab....)^m = a^m . b^m$ .....

12. (*i*) $\sqrt[m]{ab.....} = \sqrt[m]{a} \cdot \sqrt[m]{b}$ .....

(*ii*) $\sqrt{ab} = \sqrt{a} . \sqrt{b}$.

13. $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$

14. If $a^b = b^a$ Then

Either (*i*) a = b

or (*ii*) If a = 2

Then b = 4

or (*iii*) If a = 4

Then b = 2

15. If a > 1 and x < y

Then $a^x < a^y$

CALCULATOR TRICKS

1. For $a^{\frac{1}{m}}$

Steps (*i*) Type a

(*ii*) Press x button.

(*iii*) Continue pressing "=" button (Power - 1 = m - 1) times.

Example

$4^{\frac{1}{5}} = 1024$

Type 4 Then Press × button Then "=" button 4 times; we get the result.

2. For a^{-m}

Steps (*i*) Type "a"

(*ii*) Press ÷ button.

(*iii*) Continue pressing "=" button (Power = m) times

Example

$2^{-4} = 0.0625$

Type base 2 then press ÷ button then continue pressing = button 4 times.

We get the required result.

3. For $(1.05)^{60}$

1st Method:

Type base 1.05

Then press "×" button 60 - 1 = 59 times.

IInd Method:

$(1.05)^{60} = [(1.05)^{12}]^5$

1st work for 1.05^{12}

Then "×" button and work for power 5.

Work As. :

Type 1.05 then press × button

Then press = button 12-1 = 11 times

Then press × button and continue

Pressing = button 5 - 1 = 4 times.

We get the required result.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If $2^x - 2^{x-1} = 4$, then the value of x is :

(*a*) 2 (*b*) 1 (*c*) 64 (*d*) 27

[Feb. 2008, June 2010]

Solution : $2^x - 2^{x-1} = 4$

or $2^{x-1}(2-1) = 4$

or $2^{x-1} \times 1 = 2^2$

or $2^{x-1} = 2^2$

∴ x - 1 = 2

∴ x = 3 ∴ $x^x = 3^3 = 27$

∴ (*d*) is Correct

Tricks : Go by choices

For (*d*) $27 = 3^3 = x^x$

x = 3

Put x = 3 in $2^x - 2^{x-1} = 4$

It satisfies it

(*d*) is correct

Q.2. If $x = y^a$, $y = z^b$ and $z = x^c$ then abc is :

(*a*) 2 (*b*) 1 (*c*) 3 (*d*) 4

[June 2008]

Solution : $x = y^a = (z^b)^a = z^{ab}$

$\Rightarrow x = (x^c)^{ab} = x^{abc}$

$\therefore x^1 = x^{abc}$

$\therefore$ abc = 1

∴ (*b*) is correct

Q.3. If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$ then find value of $3x^3 - 9x$

(*a*) 3 (*b*) 9 (*c*) 12 (*d*) 10

[June 2009]

Solution : Detail Method

It $x = 3^{1/3} + 3^{-1/3}$ _____(I)

Cubing on both sides; we get $x^3 = (3^{1/3})^3 + (3^{-1/3})^3 + 3.3^{1/3}.3^{-1/3}(3^{1/3} + 3^{-1/3})$

$= 3 + 3^{-1} + 3 \times 1 \times x$ (From (I)

or $x^3 = 3 + \frac{1}{3} + 3x$

or $x^3 - 3x = \frac{9+1}{3}$

or $3x^2 - 9x = 10$

∴ (*d*) is correct

Q.4. Find the value of: $\left[1-\left\{1-(1-x^2)^{-1}\right\}^{-1}\right]^{-\frac{1}{2}}$ **is**

(*a*) $\frac{1}{x}$ (*b*) x (*c*) 1 (*d*) none of these

[June 2009]

Solution : $\left[1-\{1-(1-x^2)^{-1}\}^{-1}\right]^{-\frac{1}{2}} \left[1-\left\{1-\frac{1}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1-\left\{\frac{1-x^2-1}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1-\left\{\frac{-x^2}{1-x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1-\frac{1-x^2}{-x^2}\right]^{-1/2} = \left[1+\frac{1-x^2}{x^2}\right]^{\frac{1}{2}}$

$= \left[\frac{x^2+1-x^2}{x^2}\right]^{-1/2}$

$= \left(\frac{1}{x^2}\right)^{-1/2} = (x^{-2})^{-\frac{1}{2}} = x$

∴ (*b*) is correct

Q.5. $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(*a*) 1/2 (*b*) 3/2 (*c*) 2/3 (*d*) 1/3

[Dec. 2009]

Solution : $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

$= \frac{2^n(2^0 + 2^{-1})}{2^n(2^1 - 2^0)} = \frac{1+\frac{1}{2}}{2-1} = \frac{3}{2}$

∴ (*b*) is correct

Tricks : Put n = 1

Q.6. If $2^x \times 3^y \times 5^z = 360$. Then what is the value of x, y, z. ?

(*a*) 3,2,1 (*b*) 1,2,3 (*c*) 2,3,1 (*d*) 1,3,2

[Dec. 2009]

Solution : If $2^x \times 3^y \times 5 = 360$

$\therefore 2^x \times 3^y \times 5 = 2^3 \times 3^2 \times 5^1$

Comparing it; we get

x = 3; y = 2 ; Z = 1 ;

$\therefore$ (*a*) is correct

Tricks : Go by choices.

For (*a*) x = 3; y = 2; Z = 1

LHS = $2^3 \times 3^2 \times 5^1 = 360$ = RHS.

$\therefore$ (*a*) is correct

Q.7. The recurring decimal 2.7777 ..... can be expressed as

(*a*) 24/9 (*b*) 22/9 (*c*) 26/9 (*d*) 25/9

[Dec. 2010]

Solution : Tricks : Go by choices.

By calculator

(*a*) $\frac{24}{9} = 2.666..... \neq 2.777.....$

(*b*) $\frac{22}{9} = 2.444..... \neq 2.777.....$

(*c*) $\frac{26}{9} = 2.888..... \neq 2.77.....$

(*d*) $\frac{25}{9} = 2.777.....$

$\therefore$ (*d*) is correct

Q.8. The value of $\frac{(3^{n+1}+3^n)}{(3^{n+3}-3^{n+1})}$ is equal to

(*a*) 1/5 (*b*) 1/6 (*c*) 1/4 (*d*) 1/9

[June 2012]

Solution : (*b*) **Tricks**

Put n = 0

$\frac{3+3^0}{3^3-3} = \frac{3+1}{27-3} = \frac{4}{24} = \frac{1}{6}$

Detail Method

$\frac{3^{n+1}+3^n}{3^{n+3}-3^{n+1}} = \frac{3^n(3+1)}{3^n(3^3-3)}$

$= \frac{4}{24} = \frac{1}{6}$

Q.9. Find the value of X, if $x.(x)^{1/3} = \left(x^{1/3}\right)^x$

(*a*) 3 (*b*) 4 (*c*) 2 (*d*) 6

[Dec. 2012]

Solution : (*b*) is correct

$x.x^{\frac{1}{3}} = x^{\frac{x}{3}}$

or $x^{1+\frac{1}{3}} = x^{x/3}$ $\therefore 1+\frac{1}{3} = \frac{x}{3}$

or $\frac{4}{3} = \frac{x}{3} \therefore x = 4$

(*b*) is correct

Q.10. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$; then find the value of $\left[\frac{a+b+c}{3}\right]^3$ =

(*a*) 9abc (*b*) $\frac{1}{9abc}$ (*c*) abc (*d*) $\frac{1}{abc}$

[Dec. 2013, June 2023]

Solution : (*c*) is correct

Detail

Let $x = \sqrt[3]{a}$; $y = \sqrt[3]{b}$; $z = \sqrt[3]{c}$

$\therefore$ $x^3 = a$; $y^3 = b$; $z^3 = c$

$\therefore$ $x + y = 2 = 0$

Formula

$x^3 + y^3 + z^3 - 3xyz = 0$

If $x + y + z = 0$

$x^3 + y^3 + z^3 = 3xyz$

or : $a + b + c = 3 \cdot \sqrt[3]{a} \cdot \sqrt[3]{b} \cdot \sqrt[3]{c}$

or; $\frac{a+b+c}{3} = \sqrt[3]{abc}$

Cubing on both sides; we get

$\left(\frac{a+b+c}{3}\right)^3 = \left(\sqrt[3]{abc}\right)^3 = abc$

$\therefore$ (*c*) is correct

Tricks : Let a = -1; b = -1 and c = 8, because $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = \sqrt[3]{-1} + \sqrt[3]{-1} + \sqrt[3]{8}$

$= -1 -1 + 2 = 0$ (R.H.S) $\therefore \left[\frac{a+b+c}{3}\right]^3 = \left[\frac{-1-1+8}{3}\right]^3 = (2)^3 = 8$

$= (-1).(-1).(8) = abc$

$\therefore$ (*c*) is correct

Q.11. The value of

$\left(\frac{y^a}{y^b}\right)^{a^2+ab+b^2} \left(\frac{y^b}{y^c}\right)^{b^2+bc+c^2} \left(\frac{y^c}{y^a}\right)^{c^2+ca+a^2}$ =

(*a*) y (*b*) -1 (*c*) 1 (*d*) None

[June 2014]

Solution : (*c*) is correct

Tricks : It is in Cyclic order

Q.12. If $p^x = q, q^y = r, r^z = p^6$, then the value of xyz is .....

(*a*) 0 (*b*) 1 (*c*) 3 (*d*) 6

[June 2015]

Solution : $q^y = r \Rightarrow (p^x)^y = r \Rightarrow r = p^{xy}$

Now $r^z = p^6 \Rightarrow (p^{xy})^z = p^6 \Rightarrow p^{xyz} = p^6$

$\therefore$ xyz = 6

Q.13. The value of $\frac{x^2-(y-z)^2}{(x+z)^2-y^2} + \frac{y^2-(x-z)^2}{(x+y)^2-z^2} + \frac{z^2-(x-y)^2}{(y+z)^2-x^2}$ =

(*a*) 0 (*b*) 1 (*c*) -1 (*d*) ∞

[June 2016]

Solution : (*b*) is correct

$\frac{(x+y-z)(x-y+z)}{(x+y+z)(x-y+z)} + \frac{(y+x-z)(y-x+z)}{(x+y+z)(x+y-z)} + \frac{(x-y+z)(z-x+y)}{(x+y+z)(y+z-x)}$

$= \frac{x+y-z+y-x+z+x-y+z}{x+y+z} = \frac{x+y+z}{x+y+z} = 1$

Tricks : Cyclic order

Q.14. If $3^x = 5^y = (75)^z$ then

(*a*) $\frac{1}{x}+\frac{2}{y}=\frac{1}{z}$ (*b*) $\frac{2}{x}+\frac{1}{y}=\frac{1}{z}$ (*c*) $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$ (*d*) None

[June 2016]

Solution :

$3^x = 5^y = (75)^z$ .....(1)

$3^1 \times 5^2 = 75^1$ .....(2)

Tricks : Power of (2) + power of (1) and put + sign at the place of "×".

We get, $\frac{1}{x}+\frac{2}{y}=\frac{1}{z}$

So, (*a*) is correct.

Detail:

Let $3^x = 5^y = 75^z = k$

$\therefore 3 = k^{1/x}$; $5 = k^{1/y}$; $75 = k^{1/z}$

$\therefore 75 = 3 \times 5^2$

or; $k^{1/z} = k^{1/x}(k^{1/y})^2 = k^{1/x}.k^{2/y}$

or; $1/z = 1/x + 2/y$

(*a*) is correct

Q.15. If $abc = 2$, then the value of $\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{b}{2}+c^{-1}} + \frac{1}{1+a^{-1}+c}$ =

(*a*) 1 (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{4}$

[June 2016]

Solution : Tricks

"Put a = 1, b = 2 & c = 1. So that abc = 2" in the given question. We get

$$\frac{1}{1+1+\frac{2}{2}}+\frac{1}{1+\frac{2}{2}+1^{-1}}+\frac{1}{1+1^{-1}+1}=1$$

Option (*a*) is correct.

Q.16. If $a=\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}, b=\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ then the value of $\frac{1}{a^2}+\frac{1}{b^2}$ is

(*a*) 486 (*b*) 484 (*c*) 482 (*d*) 500

[June 2017]

Solution : $\frac{1}{a}+\frac{1}{b}=$

$$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}+\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}=\frac{\left(\sqrt{6}-\sqrt{5}\right)^2+\left(\sqrt{6}+\sqrt{5}\right)^2}{\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)}$$

$$=\frac{6+5-2\sqrt{6}.\sqrt{5}+6+5+2.\sqrt{6}.\sqrt{5}}{6-5}$$

$$=\frac{22}{1}=22$$

$$\& \ \frac{1}{a}.\frac{1}{b}=1$$

$$\frac{1}{a^2}+\frac{1}{b^2}=\left(\frac{1}{a}+\frac{1}{b}\right)^2-2\left(\frac{1}{a}.\frac{1}{b}\right)=22^2-2=482$$

Option (*a*) is correct.

Tricks : By Calculator $\left(\text{Calculate } \frac{1}{a^2}+\frac{1}{b^2}\right)$

Q.17. If $u^{5x}=v^{5y}=w^{5z}$ and $u^2=vw$ then $xy+zx-2yz=$ _____.

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) None of these

[Dec. 2017]

Solution : (*a*) $u^{5x}=v^{5y}=w^{5z}\Rightarrow u^x=v^y=w^z$

Tricks :

$\because u^2=vw ; \therefore \frac{2}{x}=\frac{1}{y}+\frac{1}{z}=\frac{y+z}{yz}$

or ; $xy+zx=2yz$

or; $xy+zx-2yz=0$

Detail

Let $4^x=V^y=W^z=k$

$\therefore \ 4=k^{1/x}; V=k^{1/y}; W=k^{1/z}$

$\therefore \ 4^2=VW$

$(k^{1/x})^2=k^{1/y}.k^{1/z}$

or $k^{2/x}=k^{1/y+1/z}$

$\therefore \ \frac{2}{x}=\frac{1}{y}+\frac{1}{z}$

$xy+zx-2yz=0$

Q.18. $\left(x^{\sum_{n=1}^{\infty}ap^{n-1}}\right)^{1-p}\left(x^{\sum_{n=1}^{\infty}bq^{n-1}}\right)^{1-q}\left(x^{\sum_{n=1}^{\infty}cr^{n-1}}\right)^{1-r}$

(*a*) $x^{-(ap+bq+cr)}$ (*b*) x^{a+b+c} (*c*) $x^{(ap+bq+cr)}$ (*d*) x^{abc}

[June 2018]

Solution : (*b*)

$$\sum_{n=1}^{\infty}ap^{n-1}=a+ap+ap^2+\ldots\ldots=\frac{a}{1-p} \text{ [In G.P.]}$$

$$\left(x^{\sum_{n=1}^{\infty}ap^{n-1}}\right)^{1-p}=\left(x^{\frac{a}{1-p}}\right)^{1-p}=x^a$$

Similarly doing as above ; We get

$$\left(x^{\sum_{n=1}^{\infty}ap^{n-1}}\right)^{1-p}\left(x^{\sum_{n=1}^{\infty}bq^{n-1}}\right)^{1-q}\left(x^{\sum_{n=1}^{\infty}cr^{n-1}}\right)^{1-r}$$

$$=x^a.x^b.x^c$$

$$=x^{a+b+c}$$

Option (*b*) is correct.

Q.19. $\frac{2^n+2^{n-1}}{2^{n+1}-2^n}$

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{3}$

[May 2018]

Solution : (*b*)

Tricks :-

Put minimum power = n - 1 = 0 or n = 1 in the question.

$$\therefore \ \frac{2^n+2^{n-1}}{2^{n+1}-2^n}=\frac{2^1+2^{1-1}}{2^{1+1}-2^1}=\frac{2+1}{4-2}=\frac{3}{2}$$

Q.20. $\frac{2^{m+1}x3^{2m-n+3}x5^{n+m+4}x6^{2n+m}}{6^{2m+n}x10^{n+1}x15^{m+3}}$

(*a*) 3^{2m-2n} (*b*) 3^{2n-2m} (*c*) 1 (*d*) None

[Nov. 2018]

Solution : Tricks

Put m = n = 0 in this equation. $\frac{2^{m+1}x3^{2m-n+3}x5^{n+m+4}x6^{2n+m}}{6^{2m+n}x10^{n+1}x15^{m+3}}=1$

Q.21. If $2^{x^2}=3^{y^2}=12^{z^2}$ then

(*a*) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$ (*b*) $\frac{1}{x^2}+\frac{2}{y^2}=\frac{1}{z^2}$

(*c*) $\frac{2}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$ (*d*) None

[June 2019]

Solution :

$\because 2^{x^2}=3^{y^2}=12^{z^2}$ ..... (1) (Given)

Tricks:-

Factorize 12 in terms of 2 & 3. We get

$2^2\times3^1=12^1$ ..... (2)

Always write as power of base of (2) + Power on same base of 1 ; put "+" Sign at the place of "×" Sign.

So;

$\frac{2}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$

So (c) is correct.

Details:-

Let $2^{x^2}=3^{y^2}=12^{z^2}=K^1$.

So ; $2=K^{\frac{1}{x^2}}$; $3=K^{\frac{1}{y^2}}$; $12=K^{\frac{1}{z^2}}$

Now

$12=2^2\times3^1$

or $K^{\frac{1}{z^2}}=\left(K^{\frac{1}{x^2}}\right)^2\times K^{\frac{1}{y^2}}$

or $K^{\frac{1}{z^2}}=K^{\frac{2}{x^2}}.K^{\frac{1}{y^2}}$

or $K^{\frac{1}{z^2}}=K^{\left(\frac{2}{x^2}+\frac{1}{y^2}\right)}$

$\therefore \ \frac{1}{z^2}=\frac{2}{x^2}+\frac{1}{y^2}$

(*c*) is correct

Q.22. The value of $\left[\frac{9^{n+\frac{1}{4}}.\sqrt{3.3^n}}{3.\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$ is ....

(*a*) 1 (*b*) 3 (*c*) 9 (*d*) 27

[Dec. 2019]

Solution : (*d*)

Tricks:

Putting n = 1 in the Question.

We get

$$\left[\frac{9^{1+\frac{1}{4}}.\sqrt{3\times3^1}}{3.\sqrt{3^{-1}}}\right]^{\frac{1}{1}}$$

$$= \left[\frac{(3^2)^{5/4} \cdot \beta}{\beta \cdot 3^{-1/2}}\right]$$

$$= \frac{3^{\frac{5}{2}}}{3^{-1/2}} = 3^{\left(\frac{5}{2}+\frac{1}{2}\right)} = 3^{3}$$

$$= 3^3 = 27.$$

Q.23. If $x = \sqrt{3} + \frac{1}{\sqrt{3}}$ then $\left(x - \frac{\sqrt{126}}{\sqrt{42}}\right)\left(x - \frac{1}{x - \frac{2\sqrt{3}}{3}}\right) =$

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$ (c) $\frac{2}{3}$ (d) $-\frac{3}{5}$

[Dec. 2019]

Solution : (a)

$\because\ x = \sqrt{3} + \frac{1}{\sqrt{3}}$

Then

$$\left(x - \frac{\sqrt{126}}{\sqrt{42}}\right)\left(x - \frac{1}{x - \frac{2\sqrt{3}}{3}}\right)$$

$$= \left(x - \sqrt{\frac{126}{42}}\right)\left(x - \frac{1}{\sqrt{3} + \frac{1}{\sqrt{3}} - \frac{2\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}}\right)$$

$$= \left(x - \sqrt{3}\right)\left(x - \frac{1}{\sqrt{3} + \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}}}\right)$$

$$= \left(\sqrt{3} + \frac{1}{\sqrt{3}} - \sqrt{3}\right)\left(x - \frac{1}{\sqrt{3} - \frac{1}{\sqrt{3}}}\right)$$

$$= \frac{1}{\sqrt{3}}\left(x - \frac{\sqrt{3}}{3-1}\right)$$

$$= \frac{1}{\sqrt{3}}\left(\sqrt{3} + \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}\right)$$

$$= 1 + \frac{1}{3} - \frac{1}{2} = \frac{6+2-3}{6} = \frac{5}{6}$$

Q.24. Find the value of 'a' from the following

$$\left(\sqrt{9}\right)^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

(a) 13 (b) 11 (c) 15 (d) 17

[Dec. 2020]

Solution : (d)

$$\left(\sqrt{9}\right)^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

$$\text{or } 3^{-5} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

$$\text{or } 3^{\frac{-10}{2}} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

$$\text{or } \left(\sqrt{3}\right)^{-10} \times \left(\sqrt{3}\right)^{-7} = \left(\sqrt{3}\right)^{-a}$$

$$\text{or } \left(\sqrt{3}\right)^{-10-7} = \left(\sqrt{3}\right)^{-a}$$

$\therefore -a = -17 \Rightarrow a = 17$

(d) is correct.

Q.25. Find the value of $\frac{3t^{-1}}{t^{-1/3}}$

(a) $\frac{3}{t^{2/3}}$ (b) $\frac{3}{t^{3/2}}$ (c) $\frac{3}{t^{1/3}}$ (d) $\frac{3}{t^2}$

[Jan. 2021]

Solution : (a) is correct

$$3.t^{-1+\frac{1}{3}} = 3t^{-\frac{2}{3}} = 3.\frac{1}{t^{2/3}} = \frac{3}{t^{2/3}}$$

Q.26. If $xy + yz + zx = -1$, then the value of $\left(\frac{xy}{1+xy} + \frac{z+y}{1+zy} + \frac{x+z}{1+zx}\right)$ is

(a) xyz (b) $\frac{-1+y}{yz}$ (c) $\frac{1}{xyz}$ (d) $\frac{1}{x+y+z}$

[July 2021]

Solution : (c)

$$= \frac{x+y}{1+xy} + \frac{z+y}{1+zy} + \frac{x+z}{1+zx}$$

$$= \frac{z(x+y)}{z(1+xy)} + \frac{x(z+y)}{x(1+zy)} + \frac{y(x+z)}{y(1+zx)}$$

$$= \frac{zx+yz}{z(1+xy)} + \frac{zx+xy}{x(1+yz)} + \frac{xy+yz}{y(1+zx)}$$

$$= \frac{-1+xy}{z(1+xy)} + \frac{-1-yz}{x(1+yz)} + \frac{-1-zx}{y(1+zx)}$$

$$= \frac{-(1+xy)}{z(1+xy)} + \frac{-(1+yz)}{x(1+yz)} + \frac{-(1+zx)}{y(1+zx)}$$

$$= \frac{-1}{z} + \frac{-1}{x} + \frac{-1}{y}$$

$$= -\left[\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right] = -\left[\frac{yz+zx+xy}{xyz}\right]$$

$$= -\left(\frac{-1}{xyz}\right)$$

$$= \frac{1}{xyz}$$

(c) is correct

Q.27. Let $a = \left(\sqrt{5}+\sqrt{3}\right)/\left(\sqrt{5}-\sqrt{3}\right)$ and $b = \left(\sqrt{5}-\sqrt{3}\right)/\left(\sqrt{5}+\sqrt{3}\right)$. What is the Value of $a^2 + b^2$?

(a) 64 (b) 62 (c) 60 (d) 254

[Dec. 2021]

Solution : (b)

$$a = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-3} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \text{(Rationalising)}$$

$$\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2} = \frac{5+3+2.\sqrt{5}.\sqrt{3}}{5-3}$$

$$\frac{8+2.\sqrt{15}}{2}$$

$$= 4 + \sqrt{15}$$

Similarly

$$b = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 4 - \sqrt{15}$$

$$a + b = 4 + \sqrt{15} + 4 - \sqrt{15} = 8$$

$$ab = (4+\sqrt{15})(4-\sqrt{15}) = 4^2 - \left(\sqrt{15}\right)^2 = 16 - 15 = 1$$

$\therefore a^2 + b^2 = (a+b)^2 - 2ab$

$= 8^2 - 2 \times 1 = 62$

(b) is correct

Q.28. The value of $\frac{6^{n+4} + 3^{n+3} \times 2^{n+3}}{5 \times 6^n + 6^n}$ is :

(a) 232 (b) 242 (c) 252 (d) 262

[Dec. 2021]

Solution : (c)

$$= \frac{6^{n+4} + 3^{n+3} \times 2^{n+3}}{5 \times 6^n + 6^n}$$

$$= \frac{6^n.6^4 + 3^n.3^3 \times 2^n.2^3}{6^n(5+1)}$$

$$= \frac{6^n.6^4 + (3\times 2)^n.3^3.2^3}{6^n.6}$$

$$= \frac{6^n\left[6^4 + 3^3 \times 2^3\right]}{6^n.6} = \frac{1296 + 27\times 8}{6}$$

$= 252$

Tricks :-

Put $n = 0$

$$\frac{6^4 + 3^3 \times 2^3}{5+1}$$

$$\frac{1296 + 27\times 8}{6}$$

$= 252$

Q.29. If $\left(\frac{3a}{2b}\right)^{2x-4} = \left(\frac{2b}{3a}\right)^{2x-4}$, for some a and b, then the value of x is

(a) 8 (b) 6 (c) 4 (d) 2

Solution : (d)

Given

$$\left(\frac{3a}{2b}\right)^{2x-4} = \left(\frac{2b}{3a}\right)^{2x-4}$$

$$\Rightarrow \left(\frac{3a}{2b}\right)^{2x-4} = \left(\frac{3a}{2b}\right)^{-2x+4}$$

$\Rightarrow 2x - 4 = -2x + 4$

or $2x + 2x = 4 + 4$

or $4x = 8 \Rightarrow x = 2$

Q.30. The value of $\left(1 - \sqrt[3]{0.027}\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)^2\right)$ is :

(a) 11/16 (b) 13/16 (c) 15/16 (d) 1

[Dec. 2021]

Solution : (c) is correct

$$1 - \left\{(0.3)^3\right\}^{\frac{1}{3}} \frac{5}{6} \cdot \frac{1}{4}$$

$$= 1 - (0.3)\frac{5}{6} \cdot \frac{1}{4}$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

Q.31. Find the value of z from $(\sqrt{9})^{-8} \times (\sqrt{3})^{-5} = 3^z$

(a) $\frac{2}{21}$ (b) $\frac{-21}{2}$ (c) $\frac{21}{2}$ (d) $\frac{-2}{21}$

[June 2022]

Solution : $(\sqrt{9})^{-8} \times (\sqrt{3})^{-5} = 3^z$

or; $3^{z-3-8} \times (3^{1/2})^{-5}$

$= 3^{-8} \times 3^{-5/2}$

$= 3^{-8-5/2} = 3^{\frac{-16-5}{2}}$

or; $3^z = 3^{\frac{-21}{2}}$

$\therefore z = \frac{-21}{2}$

(b) is correct

Q.32. Find the value of $\frac{3t^{-1}}{t^{-1/3}}$

(a) $\frac{3}{t^{\frac{2}{3}}}$ (b) $\frac{3}{t^{\frac{3}{2}}}$ (c) $\frac{3}{t^{\frac{1}{3}}}$ (d) $\frac{3}{t^2}$

[June 2022]

Solution : $\frac{3t^{-1}}{t^{-1/3}} = 3t^{-1+\frac{1}{3}} = 3t^{-2/3}$

$= \frac{3}{t^{-2/3}}$

(a) is correct

Q.33. By simplifying $\left(2a^3b^4\right)^6 / \left(\left(4a^3b\right)^2 \times \left(a^2b^2\right)\right)$, the answer will be

(a) $4a^3b^x$ (b) $4a^2b^{2x}$ (c) $4a^{28}bw^{22}$ (d) $4a^{10}.b^{20}$

[Dec. 2022]

Solution : $\frac{\left(2a^3b^4\right)^6}{\left(4a^3b\right)^2\left(a^2b^2\right)} = \frac{64 \times a^{18} \times b^{24}}{16a^6 \times b^2 \times a^2 \times b^2}$

$= 4\,a^{10} \times b^{20}$

(d) is correct

Q.34. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then the value of $\left(\frac{a+b+c}{3}\right)^3$ is equal to:

(a) abc (b) 9abc (c) 1/(abc) (d) (1/9)bac

[June 2023]

Solution : $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

or; $\sqrt[3]{a} + \sqrt[3]{b} = -\sqrt[3]{c}$ ——(1)

Cubing on both sides; we get

$(a^{1/3} + b^{1/3})^3 = (-c^{1/3})^3$

or; $(a^{1/3})^3 + (b^{1/3})^3 + 3a^{1/3} \cdot b^{1/3}(a^{1/3} + b^{1/3}) = -(c^{1/3})^3$

or; $a + b + 3(ab)^{1/3}(-c^{1/3}) = -c$

or; $a + b + c = 3(abc)^{1/3}$

or; $\frac{a+b+c}{3} = (abc)^{1/3}$

Cubing again on both sides; we get

$$\left(\frac{a+b+c}{3}\right)^3 = abc$$

Trick

$\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

Put $\sqrt[3]{a} = 1$; $\sqrt[3]{b} = 1$; then $\sqrt[3]{c} = -2$

$\therefore a = 1$; $b = 1$ and $c = -8$

Now $\left(\frac{a+b+c}{3}\right)^3 = \left(\frac{1+1-8}{3}\right)^3 = -8$

Then Go by choices (GBC)

(a) $abc = 1.1.(-8) = -8$ (True)

$\therefore$ (a) is correct.

Q.35. If $x = y^a$, $y = z^b$, $z = x^c$, then the value of abc is:

(a) 1 (b) 2 (c) 3 (d) 4

[June 2023]

Solution :

$\therefore x = y^a = (z^b)^a = z^{ab}$

$\therefore z = x^c$

or $z^1 = (z^{ab})^c = z^{abc}$

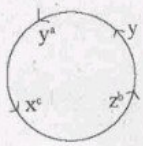
$\therefore abc = 1$.

Tricks

Product of powers = Power on last y.

$\therefore abc = 1$

$\therefore$ (a) is correct.



3 CHAPTER LOGARITHM

If $a^b = c$ Where $a \neq 1$ and a; $c > o$ (positive)

Then b is said to be the logarithm of the number c to the base "a" and expressed as

$Log_a c = b$; Where $a \neq 1$.

Types of Logarithm

(*i*) ***Natural Logarithm:***

The Logarithm of a number to base "e" is called Natural Logarithm.

i.e. $Log_e x$

where x = a number

e = 2.7183

(*ii*) ***Common Logarithm:***

Logarithm of a number to the base 10 is called common Logarithm.

i.e. $Log_{10} x$

where x = A number

Note: If base is not given then in arithmetical or commercial work; base is always taken as 10.

Remember Some Formulae

1. If $a^b = c \Leftrightarrow Log_a c = b$; Where $a \neq 1$.

2. $a^{x \log_a b} = b^x$

3. $\log_a a = 1$

4. $\log_a 1 = 0$

5. $\log_b a = \dfrac{1}{\log_a b} \Rightarrow \log_b a . \log_a b = 1$

6. (*i*) $\log_b a = \log_b x \log_x a$

$= \log_x a \log_b x$

(*ii*) $\log_b a = \log_x a . \log_y x . \log_z y .... \log_b k$

$\log_b a = \log_b x . \log_x y . \log_y z .... \log_k a$

7. (*i*) $\log_b a = \dfrac{\log_x a}{\log_x b}$

(*ii*) $\log_b a = \dfrac{\log_b x}{\log_a x}$

8. If $\log_b a = x$

Then (*i*) $\log_{\frac{1}{b}} a = -x$

(*ii*) $\log_b \dfrac{1}{a} = -x$

(*iii*) $\log_{\frac{1}{b}} \dfrac{1}{a} = +x$

9. (*i*) $\log_a (mn) = \log_a m + \log_a n$

(*ii*) $\log_a (mnr....) = \log_a m + \log_a n + \log_a r + ...$

10. $\log_a \left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

11. (*i*) $\log a^b (m^n) = \dfrac{n}{b} \log_a m.$

(*ii*) $\log_{a^n} (m^n) = n \log_a m.$

(*iii*) $\log_{a^n} m = \dfrac{1}{b} \log_a m$

12. (*i*) If $\log_a m = \log_b m \Rightarrow a = b.$

(*ii*) If $\log_a m = \log_a n \Rightarrow m = n.$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If $\log_a b + \log_a c = 0$ then

(*a*) b = c (*b*) b = -c

(*c*) b + c = 1 (*d*) b and c are reciprocals.

[June 2010]

Solution : $\log_a b + \log_a c = 0$

or $\log_a (bc) = \log_a 1$

$\therefore$ bc = 1

$\therefore b = \dfrac{1}{c}$

$\therefore$ (*d*) is correct

Q.2. The value of $2\log x + 2\log x^2 + 2\log x^3 + ... + 2\log x^n$ will be :-

(*a*) $\dfrac{n(n+1)\log x}{2}$ (*b*) n(n+1) log x

(*c*) $n^2 \log x$ (*d*) None of these

[Dec. 2010]

Solution : Detail Method

$2\log x + 2\log x^2 + 2\log x^3 + ----- + 2\log x^n$

$= 2 \log x + 2.2\log x + 2.3\log x + -------- + 2.n.\log x$

$= 2 \log x. [1 + 2 + 3 + ----------- + n]$

$= 2 \log x. \dfrac{n(n+1)}{2} = n(n+1) \log x$

= (*b*) is correct

Tricks :- Put n = 2 in options directly.

This should be equal to sum of 1st 2 terms = 2 logx + 2.2logx = 6 logx

Which gives option (*b*)

$\therefore$ (*b*) is correct.

Q.3. Solve: $\dfrac{\log_x 10 - 3}{2} + \dfrac{11 - \log_x 10}{3} = 2$

(*a*) 10^{-1} (*b*) 10^2 (*c*) 10 (*d*) 10^3

[Dec. 2010, June 2011]

Solution : $\frac{\log_x 10-3}{2}+\frac{11-\log_x 10}{3}=2$

Tricks:- Go by choices [Do Mentally]

For $(a)\ x=10^{-1}$

$L.H.S=\frac{\log_{(10^{-1})}10-3}{2}+\frac{11-\log_{10^{-1}}10}{3}$

$=\frac{-1-3}{2}+\frac{11-(-1)}{3}$

$=-2+4=2=(R.H.S)$

$\therefore$ (a) is correct

Q.4. If n = m! where ('m' is a positive integer > 2) then the value of: $\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+.....+\frac{1}{\log_m n}$ **is**

(a) 1 (b) 0 (c) -1 (d) 2

[June 2011]

Solution : Given n = m !

$\therefore \frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+.....+\frac{1}{\log_m n}$

$=\log_n 2+\log_n 3+\log_n 4+.....+\log_n m$

$=\log_n(2.3.4.....m)$

$=\log_n(1.2.3.4.....m)=\log_{(m!)}(m!)=1$

$\therefore$ (a) is correct

Q.5. If $\log_2 x+\log_4 x=6$, then the value of x is

(a) 16 (b) 32 (c) 64 (d) 128

[Dec. 2011]

Solution : (a) is correct

Tricks: Go by choices for (a) if x = 16

$L.H.S=\log_2 16+\log_4 16=4+2=6(R.H.S)$

$\therefore$ (a) is correct

Detail Method $\log_2 x+\log_4 x=6$

or $\log_2 x+\log_{2^2} x=6$

or $\log_2 x+\frac{1}{2}\log_2 x=6$

or $\left(1+\frac{1}{2}\right)\log_2 x=6$

or $\log_2 x=\frac{6\times 2}{3}=4\ \ \therefore x=2^4=16$

Q.6. If $\log_x Y=100$ and $\log_2 x=10$, then the value of 'Y' is :

(a) 2^{10} (b) 2^{100}

(c) $2^{1,000}$ (d) $2^{10,000}$

[June 2012]

Solution : (c) $\log_2 x=10\ \ \therefore x=2^{10}$

Now $\log_x y=100\ \ \therefore y=x^{100}$

$\therefore y=(2^{10})^{100}=2^{1000}$

$\therefore$ (c) is correct

Q.7. Which of the following is true. If $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=\frac{1}{abc}$

(a) log (ab + bc +ca) = abc

(b) $\log\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=abc$

(c) log (abc) = 0

(d) log (a+ b + c) = 0

[Dec. 2012]

Solution : (d) is correct

$\therefore \frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=\frac{1}{abc}$

Multiplying both sides by abc;

$\frac{abc}{ab}+\frac{abc}{bc}+\frac{abc}{ca}=\frac{abc}{abc}$

or ; c + a + b = 1

or ; a + b + c = 1

Taking log on both sides ; we get

log (a + b + c) = log1 = 0

Q.8. If $(\log_{\sqrt{x}} 2)^2=\log_x 2$ then x =

(a) 16 (b) 32

(c) 8 (d) 4

[June 2013]

Solution : (a) is correct

$(\log_{\sqrt{x}} 2)^2=\log_x 2$

or $(\log_{x^{1/2}} 2)^2=\log_x 2$

or $\left(\frac{1}{\frac{1}{2}}\log_x 2\right)^2=\log_x 2$

or $4(\log_x 2)^2-\log_x 2=0$

or $\log_x 2[4\log_x 2-1]=0$

If $\log_x 2=0$ (Invalid)

$\therefore 4\log_x 2-1=0$

or $4\log_x 2=1$

or $\log_x 2=\frac{1}{4}$

or $x^{1/4}=2\Rightarrow x=2^4=16$

Tricks :- Go by choices

For (a) LHS

$=(\log_{\sqrt{16}} 2)^2=(\log_4 2)^2$

$=\left(\frac{1}{2}\log_2 2\right)^2=\frac{1}{4}$

RHS $=\log_{16} 2=\log_{2^4} 2=\frac{1}{4}\log_2 2$

$=\frac{1}{4}$

$\therefore$ (a) is correct

Note :- Never write; check mentally.

Q.9. Find Value of $\left[\log_y x.\log_z y.\log_x z\right]^3=$

(a) 0 (b) -1

(c) 1 (d) 3

[Dec. 2013]

Solution : (c) is correct

$[\log_y x.\log_z y.\log_x z]^3$

$=[\log_x x]^3=[1]^3=1$

Q.10. Find the value of $Log_4 9.Log_3 2=$

(a) 3 (b) 9

(c) 2 (d) 1

[Dec. 2013]

Solution : (d) is correct $Log_4 9.Log_3 2=$

$\log_{(2^2)}(3^2).\log_3 2$

$=\frac{2}{2}\log_2 3.\log_3 2$

$=1\times 1=1$

Q.11. If $X=\log_{24}12$; $y=\log_{36}24$; $z=\log_{48}36$ then xyz + 1= ?

(a) 2xy (b) 2zx (c) 2yz (d) 2

[June 2014]

Solution : (c) is correct

$xyz+1=\log_{24}12.\log_{36}24.\log_{48}36+1$

$=\log_{48}12+\log_{48}48$

$=\log_{48}(12\times 48)=\log_{48}(12\times 2)^2$

$=2\log_{48}24=2\log_{36}24.\log_{48}36$

$=2yz$

Q.12. If $x^2+y^2=7xy$ then $\log\frac{1}{3}(x+y)=$

(a) logx + logy (b) $\frac{1}{2}(\log x+\log y)$ (c) $\frac{1}{3}(\log x+\log y)$ (d) $\frac{1}{3}(\log x.\log y)$

[June 2014]

Solution : (b) is correct

$\log\frac{1}{3}(x+y)=\frac{1}{2}2\log\left\{\frac{1}{3}(x+y)\right\}$

$=\frac{1}{2}\log\left\{\frac{1}{3}(x+y)\right\}^2=\frac{1}{2}\log\left(\frac{x^2+y^2+2xy}{9}\right)$

$=\frac{1}{2}\log\left(\frac{7xy+2xy}{9}\right)=\frac{1}{2}\log(xy)=\frac{1}{2}(\log x+\log y)$

Q.13. If log x = a-b; log y = a + b then $\log\left(\frac{10x}{y^2}\right)$

(a) 1-a-3b (b) a-1+3b (c) a+3b-1 (d) 1-b+3a

[Dec. 2014]

Solution : (a) is correct

$\therefore$ log x = a+b ; log y = a-b.

$\log\left(\frac{10x}{y^2}\right)=\log_{10}10+\log x-\log y^2$

$=1+a+b-2\log y=1+a+b-2(a-b)$

$=1+a+b-2a+2b=1-a+3b$

Q.14. If $x=1+\log_p qr$, $y=1+\log_q rp$ and $z=1+\log_r pq$; then the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=$ _____

(a) 0 (b) 1 (c) -1 (d) 3

[Dec. 2014]

Solution : (b) is correct

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

$=\frac{1}{1+\log_p qr}+\frac{1}{1+\log_q rp}+\frac{1}{1+\log_r pq}$

$=\frac{1}{\log_p p+\log_p qr}+\frac{1}{\log_q q+\log_q rp}+\frac{1}{\log_r r+\log_r pq}$

$=\frac{1}{\log_p pqr}+\frac{1}{\log_q pqr}+\frac{1}{\log_r pqr}$

$=\log_{pqr} p+\log_{pqr} q+\log_{pqr} r$

$=\log_{pqr} pqr=1$

Tricks:- Cyclic order

So, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

Q.15. If $\log x = m+n$; $\log y = m-n$ then $\log\left(\frac{10x}{y^2}\right)=$

(a) 1-m+3n (b) m-1+3n

(c) m+3n+1 (d) None

[June 2015]

Solution : (a)

If $\log x = m+n$; $\log y = m-n$

Then $\log\left(\frac{10x}{y^2}\right)$

$= \log 10 + \log x - \log y^2$

$= 1 + \log x - 2\log y$

$= 1 + (m+n) - 2(m-n)$

$= 1 + m + n - 2m + 2n$

$= 1 - m + 3n$

$\therefore$ (a) is correct.

Q.16. $\log_3 5 . \times \log_5 4 \times \log_2 3$:

(a) 2 (b) 5

(c) -2 (d) None of these

[Dec. 2015]

Solution : (a) is correct

$\log_3 5 . \log_5 4 . \log_2 3$

$= \log_3 4 . \log_2 3 = \log_2 4 = 2$

Q.17. The integral part of a logarithm is called_____, and the decimal part of a logarithm is called _____.

(a) Mantissa, Characteristic

(b) Characteristic, Mantissa

(c) Whole, Decimal

(d) None of these

[June 2016]

Solution : (b) is correct.

Q.18. The value of

$\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60} =$ _____

(a) 0 (b) 1

(c) 5 (d) 60

[June 2016]

Solution : (b) is correct.

$\log_{60} 3 + \log_{60} 4 + \log_{60} 5$

$= \log_{60}(3 \times 4 \times 5) = \log_{60} 60 = 1$

Q.19. If $\log_4(X^2+x) - \log_4(x+1) = 2$ then the value of x is

(a) 2 (b) 3

(c) 16 (d) 8

[June 2016]

Solution : (c) is correct.

$\log_4 \frac{(x^2+x)}{(x+1)} = 2$

or $\log_4\left\{\frac{x(x+1)}{(x+1)}\right\} = 2$

or $\log_4 x = 2 \Rightarrow x = 4^2 = 16$

Q.20. Given log 2 = 0.3010 and log 3 = 0.4771 then the value of log 24

(a) 1.3081 (b) 1.1038

(c) 1.3801 (d) 1.8301

[Dec. 2016]

Solution : (c) is correct.

Calculator Tricks:

Type 24 then $\sqrt{}$ button 19 times - 1 × 227695 = button. We will get the required value of log 24.

Q.21. $\log(1^3+2^3+3^3+.....+n^3) =$ ____

(a) $2\log n + 2\log(n+1) - 2\log 2$

(b) $\log n + 2\log(n+1) - 2\log 2$

(c) $2\log n + \log(n+1) - 2\log 2$

(d) None

[June 2017]

Solution :

$\log(1^3+2^3+3^3+.....+n^3)$

$= \log\left(\frac{n(n+1)}{2}\right)^2 = 2\log\frac{n(n+1)}{2}$

$= 2\left[\log n + \log(n+1) - \log 2\right]$

$= 2\log n + 2\log(n+1) - 2\log 2$

So, (a) is correct.

Tricks :- Go by Choices

Q.22. If $\log_3[\log_4(\log_2 x)] = 0$ then X =

(a) 4 (b) 8

(c) 16 (d) 32

[Dec. 2017]

Solution : (c)

Tricks :- GBC

for option (c) $\log_3\left[\log_4(\log_2 x)\right]$

$= \log_3\left[\log_4(\log_2 16)\right]$

$= \log_3(\log_4 4) = \log_3 1 = 0$

$\therefore$ (c) is correct.

Q.23. If

$\log\left(\frac{x-y}{2}\right) = \frac{1}{2}(\log x + \log y)$ then $x^2 + y^2 =$ _____

(a) $6xy$ (b) $2xy$

(c) $3x^2y^2$ (d) $4x^2y^2$

[Dec. 2017]

Solution : $2\log\left(\frac{x-y}{2}\right) = \log xy$

or ; $\log\left(\frac{x-y}{2}\right)^2 = \log(xy)$

or ; $\frac{x^2+y^2-2xy}{4} = xy$

or ; $x^2 + y^2 - 2xy = 4xy$

or ; $x^2 + y^2 = 6xy$

$\therefore$ (a) is correct.

Q.24. If $\log_x\left(\sqrt[3]{2}\right) = \frac{1}{15}$ then x =

(a) 2 (b) 8

(c) 16 (d) 32

[June 2018]

Solution : (d) $\log_x\left(\sqrt[3]{2}\right) = \frac{1}{15}$

or $x^{1/15} = \sqrt[3]{2} = 2^{1/3}$

or $x = \left(2^{1/3}\right)^{15} = 2^5 = 32.$

Q.25. The value of the expression :

$a^{\log_a b . \log_b c . \log_c d . \log_d t}$

(a) t

(b) abcdt

(c) (a + b + c + d + t)

(d) None

[May 2018]

Solution : $a^{\log_a b . \log_b c . \log_c d . \log_d t}$

$= a^{\log_a t} = a^{1 . \log_a t} = t^1 = t$

[Formula $a^{x\log_a b} = b^x$]

Q.26. $\log_2 \log_2 \log_2 16 = ?$

(a) 0 (b) 3 (c) 1 (d) 2

[Nov. 2018]

Solution : (c) $\log_2 \log_2 \log_2 16 = \log_2 \log_2 4 = \log_2 2 = 1$

Q.27. The value of

$\log_5(1+\frac{1}{5}) + \log_5(1+\frac{1}{6}) + ... + \log_5(1+\frac{1}{624})$

(a) 2 (b) 3 (c) 5 (d) 0

[June 2019]

Solution :

$\log_5(1+\frac{1}{5}) + \log_5(1+\frac{1}{6}) + ... + \log_5(1+\frac{1}{624})$

$= \log_5\left(\frac{6}{5}\right) + \log_5\left(\frac{7}{6}\right) + ... + \log_5\left(\frac{625}{624}\right)$

$= \log_5\left(\frac{6}{5}\cdot\frac{7}{6}.....\frac{625}{624}\right)$

$= \log_5\left(\frac{625}{5}\right) = \log_5(125)$

$= \log_5 5^3 = 3.\log_5 5$ **[Calculator Tricks]**

$= 3 \times 1 = 3.$

Q.28. $\log_{2\sqrt{2}}(512) : \log_{3\sqrt{3}} 324 =$

(a) 128 : 61 (b) 2 : 3

(c) 3 : 2 (d) None

[June 2019]

Solution :

Calculator Tricks :-

$\log_{2\sqrt{2}} 512 = 5+1$

Type $2\times2\sqrt{}$ button = button.

Then press × button then continue pressing = button until to get 512

Here = button has been pressed 5 times. So log value

= (No. of = button pressings + 1)

Similarly

For $\log_{3\sqrt{2}} 324$

Type $3\times2\sqrt{}$ button = button then × = button 3 times ; we get

$\log_{3\sqrt{2}} 324$ value = 3 + 1 = 4

So; $\log_{2\sqrt{2}} 512 : \log_{3\sqrt{2}} 324$

= 6 : 4 = 3 : 2

Q.29. $\log_{0.01}(10,000) = x$; Find the value of x?

(a) 1 (b) -2

(c) -4 (d) 2

[Dec. 2019]

Solution : (b)

Calculator Tricks

Type 0.01 then press + button

then press = button 2 times; we get 10,000.

Hence ; $\log_{0.01} 10,000 = -2$

Q.30. $\log xy^2 - \log y = \log(x+y)$

Find the value of y in term of x

(a) x-1 (b) $\frac{x}{x+1}$

(c) $\frac{x}{x-1}$ (d) $x+1$

[Dec. 2019]

Solution : (c)

$\log xy^2 - \log y = \log(x+y)$

or; $\log\left(\frac{xy^2}{y}\right) = \log(x+y)$

or; $\log xy = \log(x+y)$

or; $xy = x+y$

or; $xy - y = x$

or; $y(x-1) = x$

or; $y = \frac{x}{x-1}$

$\therefore$ (c) is correct

Q.31. If $\log_a\left(\sqrt{3}\right) = \frac{1}{6}$ find the value of 'a'

(a) 81 (b) 9

(c) 27 (d) 3

[Dec. 2020]

Solution : (c)

$\log_a \sqrt{3} = \frac{1}{6}$

or $a^{\frac{1}{6}} = \sqrt{3}$

or $a = \left(\sqrt{3}\right)^6 = \left(3^{1/2}\right)^6 = 3^3 = 27$

(c) is correct.

Q.32. Log 9 + log 5 is expressed as

(a) log (9/5) (b) log 4

(c) log (5/9) (d) log 45

[Dec. 2020]

Solution : log 9 + log 5

= log (9 × 5) = log45

Q.33. If $\log_a(ab) = x$, then $\log_b(ab)$ is

(a) $1/x$ (b) $\frac{x}{t+x}$

(c) $\frac{x}{x-1}$ (d) None of these

[Jan. 2021]

Solution : (c) is correct

$$\log_a a + \log_a b = x$$

or $$1 + \log_a b = x$$

or $$\log_a b = x - 1$$

$\therefore$ $$\log_b ab = \log_b a + \log_b b$$

$$= \frac{1}{\log_a b} + 1$$

$$= \frac{1}{x-1} + 1 = \frac{1+x-1}{x-1}$$

$$= \frac{x}{x-1}$$

Q.34. If $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = 25/6$ then the value of x is

(a) 64 (b) 4 (c) 16 (d) 2

[July 2021]

Solution : $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$

$\Rightarrow$ $\log_{2^2} x^1 + \log_{2^4} x^1 + \log_{2^6} x^1 + \log_{2^8} x^1 = \frac{25}{6}$

$\Rightarrow$ $\frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x + \frac{1}{6}\log_2 x + \frac{1}{8}\log_2 x = \frac{25}{6}$

or $\log_2 x \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right] = \frac{25}{6}$

or, $\log_2 x \cdot \left[\frac{12+6+4+3}{24}\right] = \frac{25}{6}$

or $\log_2 x \times \frac{25}{24_4} = \frac{25}{6}$

or $\log_2 x = 4$

or $x = 2^4 = 16$

(c) is correct

Q.35. If $\log_{10} 3 = x$ and $\log_{10} 4 = y$, then the value of $\log_{10} 120$ can be expressed as

(a) $x - y + 1$ (b) $x + y + 1$

(c) $x + y - 1$ (d) $2x + y - 1$

[Dec. 2021]

Solution : (b)

$\log_{10} 120 = \log_{10}(10 \times 3 \times 4)$

$= \log_{10} 10 + \log_{10} 3 + \log_{10} 4$

$= 1 + x + y$

Q.36. Find the value of $\log(x^6)$ if $\log(x) + 2\log(x^2) + 3\log(x^3) = 14$.

(a) 3 (b) 4

(c) 5 (d) 6

[Dec. 2022]

Solution : (d)

$\log x + 2.2 \log x + 3.3 \log x = 14$

or $14 \log x = 14 \Rightarrow \log x = 1$

So $\log x^6 = 6 \log x = 6 \times 1 = 6$

Q.37. If $a^{\sqrt{3}} = \frac{1}{6}$, find the value of a.

(a) 9 (b) 81

(c) 27 (d) 3

[June 2022]

Solution : $\log a^{\sqrt{3}} = \frac{1}{6}$

or; $a^{\frac{1}{6}} = \sqrt{3} = 3^{\frac{1}{2}}$

or; $a = (3^{\frac{1}{2}})^6 = 3^3 = 27$

(c) is correct

Q.38. $\log \frac{p^2}{qr} + \log \frac{q^2}{pr} + \log \frac{r^2}{pq} =$

(a) pqr (b) $\frac{1}{pqr}$

(c) 1 (d) 0

[June 2022]

Solution : $\log \frac{p^2}{qr} + \log \frac{q^2}{Pr} + \log \frac{r^2}{Pq}$

$= \log\left(\frac{p^2}{qr} \cdot \frac{q^2}{pr} \cdot \frac{r^2}{pq}\right)$

$= \log(1) = 0$

(d) is correct

Q.39. If $\log_{10} 2 = y$ and $\log_{10} 3 = x$, then the value of $\log_{10} 15$ is :

(a) $x - y + 1$ (b) $x + y + 1$

(c) $x - y - 1$ (d) $y - x + 1$

[Dec. 2022]

Solution : $\because x = \log_{10} 2; y = \log_{10} 3$

$\therefore \log_{10} 15 = \log_{10}\left(\frac{30}{2}\right)$

$= \log_{10}\left(\frac{3 \times 10}{2}\right)$

$= \log_{10} 3 + \log_{10} 10 - \log_{10} 2$

$= y + 1 - x = y - x + 1.$

$\therefore$ (d) is correct

Q.40. $\log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$ equals to:

(a) 3 (b) 2

(c) 1 (d) 0

[Dec. 2022]

Solution : $\log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$

$= \log_3 9 = \log_3 3^2$

$= 2 \log_3 3 = 2 \times 1 = 2$

(b) is correct

Q.41. The value of $[\log_{10}(5\log_{10} 100)]^2$ is:

(a) 1 (b) 2

(c) 10 (d) 25

[June 2023]

Solution : $[\log_{10}(5.\log_{10} 100)]^2$

$= [\log_{10}(5 \times 2)]^2$

$= (\log_{10} 10)^2 = 1^2 = 1$

(a) is correct.

Q.42. Given that $\log_{10} x = m + n - 1$ and $\log_{10} y = m - n$, the value of $\log_{10}(100x/y^2)$ is expressed in terms of m and n as:

(a) $1 - m + 3n$

(b) $m - 1 + 3n$

(c) $m + 3n + 1$

(d) $m^2 - n^2$

[June 2023]

Solution : $\log_{10}\left(\frac{100x}{y^2}\right)$

$= \log_{10} 100 + \log_{10} x - \log_{10} y^2$

$= 2 + \log_{10} x - 2\log_{10} y$

$= 2 + m + n - 1 - 2(m - n)$

$= 2 + m + n - 1 - 2m + 2n$

$= 1 - m + 3n$

$\therefore$ (a) is correct.

4 CHAPTER LINEAR EQUATION

The mathematical statement of equality is called **Equation**.

Linear Equation

The equation having highest power on the variable one is called linear equation. This is also called Equation of 1 degree.

Example

$7x - 5 = 9$.

Simultaneous Equation

Two or more linear equations having two or more variables is called simultaneous equation.

Example

$3x + 2y = 10$ &

$2x + 3y = 2$ are jointly called Simultaneous Equation.

Quadratic Equation (Equation of degree 2):

The equation of degree 2 is called Quadratic Equation or polynomial of degree 2.

Example

$3x^2 + 5x + 6 = 0$ is a Quadratic Equation.

Cubic Equation:

The equation of degree 3 is called Cubic Equation.

Example

$4x^3 + 3x^2 + 5x - 17 = 0$ is a Cubic Equation.

Simple Equation

The Equation with one unknown variable x in the form $ax + b = 0$ is called Simple Equation. Where $a \neq 0$.

Illustrative Examples

Example 1

The Equation $-x + 1 = 5-2x$ will be satisfied for x equal to :

(*a*) 2 (*b*) 4 (*c*) 1 (*d*) None of these

Solution : (*b*) is correct.

$\because -x + 1 = 5 - 2x.$

or $-x + 2x = 5 - 1.$

or $x = 4$

Tricks

Do not solve such types of equation Go by choices. It takes minimum time.

Option (b)

Put $x = 4$ in the equation mentally and see

$LHS - 4 + 1 = -3$

$RHS = 7 - 2\times4 = -3$

$\therefore$ LHS = RHS.

So option (*b*) is correct because it satisfies the equation.

Example 2

The value of y of fraction $\frac{x}{y}$ exceeds with x by 5 and if 3 be added to both the fractions becomes $\frac{3}{4}$. Find the fraction.

(*a*) $\frac{12}{17}$ (*b*) $\frac{13}{17}$ (*c*) $-\frac{1}{3}$ (*d*) None

Solution : Option (*a*) is correct.

$\therefore y = x + 5.$

Fraction = $\frac{x}{x+5}$

From question = $\frac{x+3}{x+5+3} = \frac{3}{4}$

or $\frac{x+3}{x+8} = \frac{3}{4}$

or $4x + 12 = 3x + 24$

or $4x - 3x = 24 - 12$

or $x = 12.$

$\therefore$ Fraction = $\frac{x}{x+5} = \frac{12}{12+5} = \frac{12}{17}$

$\therefore$ Option (*a*) is correct.

Tricks

Go by choices.

Let we see option (*a*) $\frac{12}{17}$

Clearly $y = 17$ is 5 more than $x = 12$.

$\therefore$ Option (*a*) satisfies the 1st condition of the question.

If $\frac{12+3}{17+3} = \frac{15}{20} = \frac{3}{4}$

$\therefore$ Option (*a*) also satisfy 2nd condition of the equation.

Example 3

In a two digits number ; the digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number the digits are reversed. Find the number.

(*a*) 63 (*b*) 21 (*c*) 42 (*d*) None

Solution: (*c*) is correct.

Let x be the unit place digit.

So, Ten's place digit = 2x.

$\therefore$ Number = $10\times2x + x = 21x.$

By question.

$21x - 18 = 10x + 2x$

or $21x - 12x = 18$

or $9x = 18$

$\therefore x = 2.$

So, the required No. = $21x = 21\times2 = 42$

Tricks

Go by choices.

Option (*a*); (*b*) and (*c*) all satisfy 1st condition of the question.

i.e. in (*a*), (*b*), (*c*) Ten's place digit is twice of unit place digit.

Let we see for 2nd condition.

(*a*) $63 - 18 = 45 \neq 36$ (Digits not reserved)

(*b*) $21 - 18 = 3 \neq 12$ (Digits not reserved)

(*c*) $42 - 18 = 24$ (Digits reserved)

So option (*c*) also satisfies 2nd condition of the question.

$\therefore$ (*c*) is correct.

Example 4

For a certain commodity the demand equation giving "y" units for a price "p" in rupees per unit is $y = 100 (10 - p)$. The supply equation giving the supply z units for a price "p" in rupees per unit is $z = 75(p-3)$. The market price is such at which demand equals supply. Find the market price and quantity that will be brought and sold.

(*a*) ₹ 7; 300 units (*b*) ₹ 8; 400 units

(*c*) ₹ 5; 200 units (*d*) None

Solution :

(*a*) is correct.

Demand = Supply (given)

$100 (10 - P) = 75 (P - 3)$

or $40 - 4p = 3p - 9$

or $-4p - 3p = -40 - 9$

or $-7p = -49$ or $p = 7$

$\therefore y = 100 (10 - p) = 100 (10 - 7) = 300$ units

$z = 75 (p - 3) = 75 (7 - 3) = 300$ units.

$\therefore$ Price = ₹ 7 per unit.

Quantity = 300 units

$\therefore$ (*a*) is correct.

Tricks

Go by Choices

Option (*a*) satisfies given both conditions.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. For all $\lambda \in R$, the line $(2+\lambda)x + (3-\lambda)y + 5 = 0$ passing through a fixed point, then the fixed point is_____.

(*a*) (1,1) (*b*) (-1,-1)

(*c*) (1,-1) (*d*) (-1,1)

[June 2011]

Solution : (*b*)

Tricks : Go by choices

For option (*b*) Point (-1 ; -1) satisfies the eqn.

$LHS = (2+\lambda)x + (3-\lambda)y + 5$

or $(2+\lambda)(-1) + (3-\lambda)(-1) + 5$

$= -2 - \lambda - 3 + \lambda + 5 = 0 = RHS.$

$\therefore$ (*b*) is correct

Q.2. If kx - 4 = (k -1).x which of the following is true

(*a*) x = -5 (*b*) x = -4

(*c*) x = -3 (*d*) x = 4

[Dec. 2013]

Solution : (*d*) is correct

kx - 4 = (k - 1) x

or kx - 4 = kx - x

or -4 = -x $\therefore$ x = 4

Q.3. If the equations kx + 2y = 5, 3x + y = 1 has no solution then the value of k is

(*a*) 5 (*b*) 2/3

(*c*) 6 (*d*) 3/2

[Dec. 2013]

Solution : (*c*) is correct

kx + 2y = 5

3x + y = 1

They have no soln. (given)

$\frac{k}{3} = \frac{2}{1} \neq \frac{5}{1}$; $\Rightarrow -k = 6$

Q.4. The equation

$x + 5y = 33$; $\frac{x+y}{x-y} = \frac{13}{3}$

has the solution (x,y) as:

(*a*) (4,8) (*b*) (8,5)

(*c*) (4,16) (*d*) (16,4)

[Dec. 2014]

Solution : (*b*) is correct

Tricks: Go by choices

For LHS = $x + 5y = 8 + 5\times5 = 33$

and $\frac{x+y}{x-y} + \frac{8+5}{8-5} = \frac{13}{3}$

Clearly (*b*) satisfies both eqns.

Q.5. The age of a person is 8 years more than thrice the age of the sum of his two grandsons who were twins. After 8 years his age will be 10 years more than twice the sum of the ages of his grandsons. Then the age of the person when the twins were born is_

(*a*) 86 yrs (*b*) 73 yrs

(*c*) 68 yrs (*d*) 63 yrs

[June 2015]

Solution : (*b*) Let age of 1st grandson = x

$\therefore$ Person's Age = $P = 3(x+x) + 8$

$P = 6x + 8$

After 8 years

$P + 8 = 2[x + 8 + x + 8] + 10$

$= 2(2x + 16) + 10$

or $6x + 8 + 8 = 4x + 32 + 10$

or $2x = 42 - 16 = 26$

$\therefore x = 13$

$\therefore$ Age of person when grandsons were born

$= 6x + 8 - x$

$= 6\times13 + 8 - 13 = 73$

(*b*) is correct

Q.6. In a school number of students in each section is 36. If 12 new students are added, then the number of sections are increased by 4 and the number of students in each section becomes 30. The original number of section at first is

(*a*) 6 (*b*) 10

(*c*) 14 (*d*) 18

[June 2015]

Solution : (*d*) ; Let original No. of sections = x

Total students = 36x

Again Qts.

$36x + 12 = (x+4).30$

or $36x + 12 = 30x + 120$

or $6x = 108 \Rightarrow x = 18$

Tricks: Go by choices

Q.7. A person on a tour has ₹9600 for his expenses. But the tour was extended for another 16 days, so he has to cut down his daily expenses by ₹ 20. The original duration of the tour had been ?

(*a*) 48 days (*b*) 64 days

(*c*) 80 days (*d*) 96 days

[June 2015]

Solution : (*c*) ; Let No. of tour days = x

$\therefore$ Expense per day = $\frac{9600}{x}$

Now Expense per day = $\frac{9600}{x+16}$

From $\frac{9600}{x} - \frac{9600}{x+16} = 20$

Tricks Go by choices

From here we get

For (*c*) LHS

$\frac{9600}{80} - \frac{9600}{80+16} = 20$ RHS.

$\therefore$ (*c*) is correct

Do by Calculator

Q.8. If $2^{x+y} = 2^{2x-y} = \sqrt{8}$ then the respective values of x and y are _____

(*a*) $1, \frac{1}{2}$ (*b*) $\frac{1}{2}, 1$

(*c*) $\frac{1}{2}, \frac{1}{2}$ (*d*) None of these

[June 2016]

Solution : (*a*) is Correct.

$2^{x+y} = 2^{2x-y} = \sqrt{8} = \sqrt[2]{2^3} = 2^{3/2}$

$\therefore x + y = \frac{3}{2}$ (1)

$2x - y = \frac{3}{2}$ (2)

Tricks: Then Go by Choices

(*a*) satisfies (1) & (2) both.

Q.9. Let E_1, E_2 are two linear equations in two variables x and y .(0, 1) is a solution for both the equations E_1 & E_2 (2, -1) is a solution of equation E_1 only and (-2, -1) is a solution of equation E_2 only, then E_1, E_2 are _____.

(*a*) x = 0, y = 1;

(*b*) 2x - y = -1, 4x + y = 1

(*c*) x + y = 1, x - y = -1

(*d*) x + 2y = 2, x + y = 1

[June 2016]

Solution : (*c*) is correct.

Tricks: Go by Choices

(0 ; 1) satisfies E_1 & E_2 both

(2, -1) satisfies 1st Eqn. 2 - 1 = 1 (True)

But (-2 ; -1) also satisfies E_2

i.e. -2 -(-1) = -1 (True)

Q.10. Particular company produces some articles on a day. The cost of production per article is ₹2 more than thrice the number of articles and the total cost of production is ₹800 on a day then the number of articles is :

(*a*) 16 (*b*) 14

(*c*) 18 (*d*) 15

[Dec 2016]

Solution : (*a*) is correct.

Tricks : Go by choices

Let (*A*) is correct.

So, cost per unit = 800/16 = ₹ 50

It is 2 more than 3 times of 16 (as given in Qts.)

Q.11. The sides of equilateral triangle are shortened by 3 units, 4 units, 5 units respectively then a right angle triangle is formed. The side of the equilateral triangle was

(*a*) 5 (*b*) 6

(*c*) 8 (*d*) 10

[June 2017]

Solution : Tricks : Go by Choices

For option (*c*)

1st side of right angled Δ = 8 - 3 = 5

2nd side = 8 - 4 = 4

and 3rd side = 8 - 5 = 3

Here ; 5 ; 4 and 3 are making a right angled triangle.

So, $5^2 = 4^2 + 3^2$

Hence, option (*c*) is correct.

Q.12. If $\frac{3}{x+y} + \frac{2}{x-y} = -1$ and $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$ then (x, y) is

(*a*) (2, 1) (*b*) (1, 2)

(*c*) (-1, 2) (*d*) (-2, 1)

[June 2017]

Solution : Tricks: Go by Choices

Option (*B*)

$\frac{3}{1+2} + \frac{2}{1-2} = 1 - 2 = -1$ (True)

$\frac{1}{1+2} - \frac{1}{1-2} = \frac{1}{3} + 1 = \frac{4}{3}$ (True)

So ; option (*B*) is correct.

Q.13. The line $3x + 2y = 6$ intersects the line $3x - y = 12$ in _____ quadrant:

(*a*) 1st (*b*) 2nd

(*c*) 3rd (*d*) 4th

[Dec. 2017]

Solution : (*d*), Eqn. (1) - Eqn. (2) ; we get

$3x + 2y = 6$

$3x - y = 12$

$- \quad + \quad -$

$3y = -6 \Rightarrow y = -2$

From (1) ;

$3x = 6 - 2y = 6 - 2(-2) = 10$

$\therefore x = \frac{10}{3}$

$\therefore$ Co-ordinate of the point of intersection

$= (x ; y) = \left(\frac{10}{3} ; -2\right)$

It is in 4th Quadrant.

Q.14. If $2^{x+y} = 2^{2x-y} = \sqrt{8}$, then the respective values of x and y .....

(*a*) $1, \frac{1}{2}$ (*b*) $\frac{1}{2}, 1$

(*c*) $\frac{1}{2}, \frac{1}{2}$ (*d*) None

[May 2018]

Solution : (*a*)

$2^{x+y} = 2^{2x-y} = \sqrt{8} = (2^3)^{1/2} = 2^{3/2}$

Tricks:- Go by Choices (GBC)

(*a*) $2^{1+\frac{1}{2}} = 2^{\frac{3}{2}} = \text{RHS}$ and

$2^{2\times1-\frac{1}{2}} = 2^{3/2} = \text{RHS}$

$\therefore$ (*a*) is correct

[Note :- Try to do it mentally]

Q.15. If $\frac{3}{x+y} + \frac{2}{x-y} = -1$;

$\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$; then (x; y) is -

(*a*) 2 ; 1 (*b*) 1 ; 2

(*c*) -1; 2 (*d*) -2 ;1

[May 2018]

Solution : (*b*) is correct

Tricks :- Go by choices.

(*a*) $\frac{3}{2+1} + \frac{2}{2-1} = 1 + 2 = 3 \neq -1$

So(a) is False.

(*b*) $\frac{3}{1+2} + \frac{2}{1-(2)} = 1 + \frac{2}{-1} = -1$ (True)

and $\frac{1}{1+2} - \frac{1}{1-2} = \frac{1}{3} - (-1) = \frac{1}{3} + 1 = \frac{4}{3}$ (True)

Q.16. If the sides of an equilateral triangle are shortened by 3 units , 4 units and 5 units respectively and a right triangle is formed then the sides of an equilateral triangle is

(*a*) 6 units (*b*) 7 units

(*c*) 8 units (*d*) 10 units

[May 2018]

Solution : (*c*) is correct.

Tricks:- Go by Choices.

Check for option (*a*).

1st Side = 6 - 3 = 3

2nd Side = 6 - 4 = 2

3rd Side = 6 - 5 = 1

But $1^2 + 2^2 \neq 3^2$

So (*a*) is False.

(*c*) 1st Side = 8 - 3 = 5

2nd Side = 8 - 4 = 4

3rd Side = 8 - 5 = 3

But $3^2 + 4^2 = 5^2$ (True)

(Pythagoras Formula)

$\therefore$ (*c*) is correct.

Q.17. A number consists of two digits such that the digit in one's place is thrice the digit at ten's place. If 36 be added then the digits are reversed. Find the number .....

(*a*) 62 (*b*) 26

(*c*) 39 (*d*) None

[June 2019]

Solution : (*b*)

Tricks :- Go by choices

(*a*) $62 \rightarrow 2 \neq 3\times6$ (False)

and $62 + 36 = 98 \neq 26$ (False)

(*b*) 26 Clearly $6 = 3\times2$ (True)

and 26 + 36 = 62 (Orders of digits reversed)

So; (*b*) is correct.

Q.18. Find the roots of equation is $4^x, 8^y = 128$ and $3^x / 27^y = 1/3$

(*a*) 2, 1 (*b*) -2, 1

(*c*) 2, -1 (*d*) 1, 2

[Dec. 2019]

Solution : (*a*)

Trick: GBC

for option (a) x = 2 ; y = 1

$4^2 \times 8^1 = 16\times8 = 128 = \text{RHS}(\text{True})$

and

$\frac{3^2}{27^1} = \frac{9}{27} = \frac{1}{3} = \text{RHS}(\text{Also True})$

Hence ; option (a) is correct.

Q.19. The cost of 2 oranges and 3 apples is ₹ 28. If the cost of an apple is doubled then the cost of 3 oranges and 5 apples is ₹ 75 The original cost of 7 oranges and 4 apples (in ₹) is

(*a*) 59 (*b*) 47

(*c*) 71 (*d*) 63

[July 2021]

Solution : (*a*) is correct

Let cost of 1 orange = x

and cost of 1 apples = y

$\therefore \quad 2x + 3y = 28 \quad \ldots(1)$

$3x + 5(2y) = 75$

or $\quad 3x + 10y = 75 \quad \ldots(2)$

Solving (1) & (2); we get

$[2x + 3y = 28] \times 3$

$[3x + 10y = 75] \times 2$

$- \quad - \quad -$

Subtracting; or, $9y - 20y = 84 - 150$

or; $\quad -11y = -66$

or; $\quad y = 6$

From (1); $2x + 3\times6 = 28$

or $\quad x = 5$

So $\quad 7x + 4y = 7\times5 + 4\times6 = ₹\ 59.$

Q.20. In a multiple choice question paper consisting of 100 questions of 1 mark each, a candidate get 60% marks. If the candidate attempted all questions and there was a penalty of 0.25

marks for wrong answer, the difference between number of right answers and wrong answers is :

(a) 32 (b) 36
(c) 40 (d) 38

[Dec. 2021]

Solution : (b)

Let No. of right answers = x

$\therefore$ No. of Wrong answers = $100 - x$

Marks obtained = 60

or $x \times 1 - (100 - x) \times 0.25 = 60$

or; $x - 25 + 0.25x = 60$

or; $1.25x = 85$

or $x = \dfrac{85}{1.25} = 68$

No. of correct Questions = 68.

No. of incorrect questions
$= 100 - 68 = 32$

Difference between correct and incorrect questions = $68 - 32$
$= 36$

Q.21. Solve the following pair of Linear equations for x and y: $\left(\frac{b}{a}\right)x + \left(\frac{a}{b}\right)y = a^2 + b^2$

(a) $x = \frac{a}{b}, y = \frac{b}{a}$

(b) x = 3ab, y = -ab

(c) x = -ab, y = 3ab

(d) x = ab, y = ab

[June 2022]

Solution : Tricks : GBC (Go by choices)

For (d) x= ab; y = ab

LHS

$\left(\frac{b}{a}\right)x + \left(\frac{a}{b}\right)y$

$= \frac{b}{a} \times a b + \frac{a}{b} \times ab$

$= b^2 + a^2 =$ RHS.

$\therefore$ (d) is correct

Q.22. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as the shortest. What is the possible length for the shortest piece?

(a) 22 (b) 20
(c) 15 (d) 18

[June 2022]

Solution : Tricks

GBC

Let (a) is correct

Smallest length = 22

Second length = 22 + 3 =25

3rd length = 2 × smallest

= 2 × 22 = 44

Here Total Length

= 22 + 25 + 44 = 91 (True)

$\therefore$ (a) is correct

Q.23. If the cost of 3 bags and 4 pens is ₹ 257 whereas the cost of 4 bags and 3 pens is ₹ 324, then the cost of one bag is:

(a) 8 (b) 24
(c) 32 (d) 75

[Dec. 2022]

Solution : Let cost of 1 bag = x

and cost of 1 Pen = y

So,

$3x + 4y = 257$ ..... (1)

and $4x + 3y = 324$ ..... (2)

Eqn. (1) + (2); We get

$7x + 7y = 581$

Or $7(x + y) = 581$

$\therefore x + y = 83$ ..... (3)

Now

Eqn. (2) – (1); We get

$x - y = 324 - 257$

or $x - y = 67$ ..... (4)

Now Eqn. (3) + (4); We get

$x + y = 83$
$x - y = 67$
$2x = 150$
or $x = 75$

$\therefore$ Cost of 1 bag = ₹ 75

(d) is correct

Q.24. A group of 400 soldiers posted at border area had a provision for 31 days. After 28 days 280 soldiers from this group were called back. Find the number of days for which the remaining ration will be sufficient?

(a) 3 (b) 6
(c) 8 (d) 10

[Dec. 2022]

Solution : No. of soldiers after 28 days
$= 400 - 280 = 120$

Remaining No. of days = $31 - 28$
$= 3$

$\because$ 400 soldiers ration will last 3 days.

$\therefore$ 1 soldier ration will last 3×400 days

$\therefore$ 120 soldiers ration will last $\frac{3 \times 400}{120} =$ 10 days

(d) is correct

Q.25. The solution of the following system of linear equations: $2x - 5y + 4 = 0$ and $2x + y - 8 = 0$ will be

(a) (2, –3) (b) (1, –3)
(c) (3, 2) (d) (–2, 2)

[Dec. 2022]

Solution : Given eqns. are

$2x - 5y + 4 = 0$ .....(1)
$2x + y - 8 = 0$ .....(2)

Subtracting $-6y + 12 = 0$
or $6y = 12 \quad \therefore y = 12$

$\therefore y = 2$

From Eqn. (2)

$2x + 2 - 8 = 0$

or $2x = 6$

$\therefore x = 3$

$\therefore$ Soln. is (3, 2)

(c) is correct

Tricks : Go by choices

Q.26. The largest side of a triangle is 3 times the shortest side and third side is 4 cm shorter than largest side. If the

perimeter of the triangle is at least 59 cm, what is the length of shortest side?

(a) Less than 7 cm
(b) Greater than or equal to 7 cm
(c) Less than 9 cm
(d) Greater than or equal to 9 cm

[June 2023]

Solution : Let smallest side = x

$\therefore$ Largest side = $3x$

and 3rd side = $3x - 4$

Perimeter = Sum of sides ≥ 59

or; $x + 3x + 3x - 4 \geq 59$

or; $7x \geq 59 + 4$

or; $7x \geq 63$

$\therefore x \geq 9$

(d) is correct.

Q.27. The age of a man is four times the sum of the ages of his two sons and after 10 years, his age will be double the sum of their ages. The present age of the man must be:

(a) 56 Years
(b) 45 Years
(c) 60 Years
(d) 64 Years

[June 2023]

Solution : Let ages of his two sons are x and y.

Father's age = z (let)

From Qts.

$z = 4(x + y)$

$\therefore x + y = \frac{z}{4}$

After 10 yrs

$z + 10 = 2(x + 10 + y + 10)$
$= 2(x + y + 20)$
$= 2\left(\frac{z}{4} + 20\right)$

or; $z + 10 = \frac{z}{2} + 40$

or $z - \frac{z}{2} = 40 - 10$

or; $\frac{z}{2} = 30 \quad \therefore z = 60$ yrs

(c) is correct.

Tricks:– GBC (Go by choices)

(c) Sum of ages two son = $\frac{60}{4} = 15$

After 10 yrs $\Rightarrow$ Age of Father
$= 60 + 10$
$= 70$ yrs

and Sum of ages of two sons
$= 15 + 10 + 10 = 35$ yrs [10 yrs. increase for 1st and again 10 yrs. increase for 2nd son]

Clearly Father's Age = 2×35
= Twice of sum of son's age.
(True).

5 CHAPTER QUADRATIC EQUATION

$ax^2 + bx + c = 0$; where $a \neq 0$; a,b,c, are constants form equation is called Quadratic Equation or **Second degree equation.**

I. If $b = 0$ Then $ax^2 + c = 0$ is called **PURE Quadratic** Equation.

II. If $b \neq 0$ Then the equation. $ax^2 + bx + c = 0$ where $a \neq 0$ is called an **AFFECTED Quadratic** Equation.

Roots

The value of the variable "x" which satisfies the given equation is called its **Solution** or roots of the Quadratic Equation.

Discriminant

For Quad. Eqn. $ax^2 + bx + c = 0$.

Discriminant D $= b^2 - 4ac$.

Example

For Eqn. $3x^2 + 7x + 2 = 0$.

$a = 3; b = 7; c = 2$

Discriminant $D = b^2 - 4a\,c$

$= 7^2 - 4 \,.\, 3 \,.\, 2 = 49 - 24 = 25$.

III. Roots of Quad. Eqn. $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

[Remember this formula, No need to prove it.]

IV. If α and β are roots of a Quadratic Equation $ax^2 + bx + c = 0$

Then $\alpha + \beta = -\frac{b}{a}$

$\therefore$ Sum of roots $= -\frac{Co\text{-}efficient\ of\ x}{Co\text{-}efficient\ of\ x^2}$

$\alpha\beta = \frac{c}{a}$

$\therefore$ Product of roots $= \frac{\text{Constant terms}}{\text{Co-efficient of } x^2}$

V. If α and β are roots of a Quadratic Eqn. Then the eqn. is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2$ - (sum of roots) x + Product of roots = 0.

VI. Nature of Roots

Nature of roots of a Quad. Eqn. depends upon **Discriminant $D = b^2 - 4ac$.**

(A) If $D > 0$, Roots Real & Unequal

(*i*) D a perfect square then roots are Rational & unequal

As. $\frac{2}{3}; -\frac{2}{3}$.

(*ii*) D not a perfect Square.

Then roots are irrational & unequal and Conjugate *As.* $2+\sqrt{3}; 2-\sqrt{3}$.

(B) If $D = 0$, Then Roots are Real & equal.

Each root $= -\frac{b}{2a}$

(C) If $D < 0$, Then Roots are imaginary.

VII. If one root of a quadratic Eqn. is irrational then its other root is its irrational conjugate.

Example

If one root $= 3 + \sqrt{5}$

Then other root $= 3 - \sqrt{5}$

(irrational conjugate)

[To find conjugate change the sign of irrational part.]

***Note*: [Tricks]**

(*i*) If one root is reciprocal to the other Then c = a

(*ii*) If one root is equal to the other but opposite in sign. Then b = 0.

Cubic Equations

1. ***Meaning of Cubic Equation***

The equation having form.

$ax^3 + bx^2 + cx + d = 0, a \neq 0,$

Where a, b, c, d are real numbers , is called a **cubic equation.**

2. **Relation Between Roots and Coefficients**

If α, γ are the roots of the cubic equation $ax^3 + bx^2 + cx + d = 0, a \neq 0$, then

(*i*) $\alpha+\beta+\gamma = \frac{-b}{a}$

(*ii*) $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$

(*iii*) $\alpha\beta\gamma = \frac{-d}{a}$

3. ***The Cubic equation having roots α, β, γ is***

$x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha+\beta+\gamma\alpha)x - \alpha\beta = 0$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. Roots of the equation $3x^2 - 14x - k = 0$ will be reciprocal of each other if:

(*a*) k = -3 (*b*) k=0

(*c*) k = 3 (*d*) k =14

[June 2010]

Solution : (*a*) is correct.

Let one root $= \alpha$ & Another root $= \frac{1}{\alpha}$ (given)

$\because$ Product of roots = c/a

$\therefore \alpha.\frac{1}{\alpha} = \frac{-k}{3}$

$\therefore 1 = \frac{-k}{3}$ So, k = 3

$\therefore$ (*a*) is correct

Tricks: If one root is reciprocal to other then

$a = c$

$\therefore 3 = -k$

$\therefore k = -3$

Q.2. Positive value of 'k' for which the roots at equation $12x^2 + kx + 5 = 0$ are in ratio 3 : 2, is

(*a*) 5/12 (*b*) 12/5

(*c*) $\frac{5\sqrt{10}}{2}$ (*d*) $5\sqrt{10}$

[Dec. 2010]

Solution : Let α is common in the ratio.

$\therefore$ Roots are 3α and 2α

Sum of roots $= -\frac{b}{a}$

$\therefore 3\alpha + 2\alpha = 5\alpha = -\frac{k}{12}$

So, $\alpha = -\frac{k}{60}$

Product of roots $= 3\alpha \times 2\alpha = \frac{c}{a}$

$\therefore 6\alpha^2 = \frac{5}{12}$

$\Rightarrow 6.\left(-\frac{k}{60}\right)^2 = \frac{5}{12}$, So, $k^2 = 250$

$\therefore k = 5\sqrt{10}$

$\therefore$ (*d*) is correct

Q.3. If one root of the equation $x^2 - 3x + k = 0$ is 2, then value of k will be

(*a*) -10 (*b*) 0

(*c*) 2 (*d*) 10

[Dec. 2010]

Solution : (*c*) is correct

2 is a root of given eqn.

$\therefore 2^2 - 3\times 2 + k = 0$

or -2 + k = 0

$\therefore k = 2$

$\therefore$ (*c*) is correct

Q.4. It roots of equation $x^2 + x + r = 0$ are 'α' and 'β' and $\alpha^3 + \beta^3 = -6$. Find the value of 'r'?

(*a*) $\frac{-5}{3}$ (*b*) $\frac{7}{3}$

(*c*) $\frac{-4}{3}$ (*d*) 1

[June 2011]

Solution : (*a*) is correct.

$\alpha + \beta = \frac{-1}{1} = -1$ & $\alpha\beta = \frac{r}{1} = r$

$\therefore \alpha^3 + \beta^3 = -6$

or $(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -6$

or $(-1)^3 - 3r(-1) = -6$

or -1 + 3r = -6; or 3r = -5

$\therefore$ r = -5/3

$\therefore$ (*a*) is correct

Q.5. If one root of the Equation $px^2 + qx + r = 0$ is r then other root of the Equation will be

(*a*) 1/q (*b*) 1/r

(*c*) 1/p (*d*) $\frac{1}{p+q}$

[Dec. 2011]

Solution : (*c*) Let α is another root.

$\therefore r\alpha = \frac{r}{p} \therefore \alpha = \frac{1}{p}$

Q.6. If the ratio of the root of the Equation $4x^2 - 6x + p = 0$ is 1: 2 then the value of p is

(*a*) 1 (*b*) 2

(*c*) -2 (*d*) -1

[Dec. 2011]

Solution : (*b*) Let α is common in the ratio

$\therefore \alpha + 2\alpha = \frac{-(-6)}{4} \Rightarrow \alpha = \frac{1}{2}$

$\therefore \alpha.2\alpha = \frac{p}{4} \therefore p = 8\alpha^2 = 8.\frac{1}{4} = 2$

Q.7. If p & q are the root of the Equation x^2 - bx + c = 0, then what is the Equation whose roots are (pq + p + q) and (pq - p - q) ?

(*a*) $x^2 - 2cx + c^2 - b^2 = 0$

(*b*) $x^2 - 2cx + c^2 + b^2 = 0$

(*c*) $8cx^2 - 2(a+c)x + c^2 = 0$

(*d*) $x^2 + 2bx - (c^2 - b^2) = 0$

[Dec. 2011]

Solution : (*a*) Eqn is

$x^2 - (p+q)x + pq = 0$

$\therefore$ b = p + q ; C = pq

New roots are

pq + (p + q) = c + b

& pq - (p + q) = c - b

$\therefore$ Eqn is

$x^2 - (c+b+c-b)x + (c+b)(c-b) = 0$

or $x^2 - 2cx + c^2 - b^2 = 0$

Q.8. If arithmetic mean between roots of a quadratic equation is 8 and the geometric mean between them is 5, the equation is -

(*a*) $x^2 - 16x - 25 = 0$

(*b*) $x^2 - 16x + 25 = 0$

(*c*) $x^2 - 16x + 5 = 0$

(*d*) None of these

[June 2012]

Solution : (*b*) Let α and β are two roots.

$(\alpha+\beta)/2 = 8$ and $\sqrt{\alpha\beta} = 5$

$\Rightarrow \alpha + \beta = 16$ & $\alpha\beta = 25$

Eqn. is $x^2 - 16x + 25 = 0$

Q.9. The minimum value of the function $x^2 - 6x + 10$ is _____

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 10

[June 2012]

Solution : (*a*) coeff. of $x^2 = 1 > 0$; function is minimum (Formula)

$\therefore$ Minimum value $= \frac{4ac - b^2}{4a}$

$= \frac{4.1.10 - (-b)^2}{4\times 1} = \frac{4}{4} = 1$

Q.10. If one of the roots of the equation $x^2 + px + a$ is $\sqrt{3}+2$, then the value of 'p' and 'a' is

(*a*) -4,-1 (*b*) 4,-1

(*c*) -4,1 (*d*) 4,1

[June 2012]

Solution : (*c*) Roots are $2+\sqrt{3}$ *and* $2-\sqrt{3}$

[$\because$ If one root is an irrational No. then its other root is its irrational conjugate]

(conjugate of $2+\sqrt{3}$)

$\therefore$ Eqn is

x^2 - (Sum of roots) x + product of roots = 0

$x^2 - 4x + (4 - 3) = 0$

$x^2 + px + a = 0 \therefore P = -4 ; a = 1$

Q.11. Roots of equation $2x^2+3x+7=0$ are α and β. The value of $\alpha\beta^{-1}+\beta\alpha^{-1}$ is

(a) 2 (b) 3/7

(c) 7/2 (d) -19/14

[Dec. 2012]

Solution : (d) $\alpha+\beta=-\frac{3}{2};\alpha\beta=7/2$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\frac{9}{4}-2.\frac{7}{2}}{\frac{7}{2}}=\frac{9-28}{4}\times\frac{2}{7}=\frac{-19}{14}$$

Q.12. The quadratic equation $x^2-2kx+16=0$ will have equal roots when the value of 'k' is -

(a) ±1 (b) ±2

(c) ±3 (d) ±4

[Dec. 2012]

Solution : (d) Let roots are $\alpha;\alpha$

$$\therefore \alpha+\alpha=\frac{-(-2k)}{1}\Rightarrow\alpha=k$$

$\therefore \alpha\alpha = k.k = 16 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4$

Q.13. If α,β are roots of $x^2+7x+11=0$ then the equation whose roots as $(\alpha+\beta)^2$ & $(\alpha-\beta)^2$ is___

(a) $x^2-54x+245=0$

(b) $x^2-14x+49=0$

(c) $x^2-24x+144=0$

(d) $x^2-50x+49=0$

[June 2013]

Solution : (a) is correct

$$\alpha+B=-\frac{b}{a}=-\frac{7}{1}=-7$$

$$\alpha\beta=\frac{c}{a}=\frac{11}{1}=11$$

$$\therefore(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$$

$$=(-7)^2-4\times11=5$$

Required eqn. is

$$x^2-\{(\alpha+\beta)^2+(\alpha-\beta)^2\}x+(\alpha+\beta)^2(\alpha-\beta)^2=0$$

or $x^2-(49+5)x+49\times5=0$

or $x^2-54x+245=0$

$\therefore$ (a) is correct

Q.14. If b^2-4ac is perfect square but not equal to zero then the roots of the equation $ax^2+bx+c=0$ are

(a) Real and equal

(b) Real irrational and equal

(c) Real rational and unequal

(d) Imaginary

[Dec. 2013]

Solution : (c) $b^2-4ac>0$ & perfect square

$\therefore$ Roots are real rational and unequal

Q.15. Divide 80 into two parts so that their products is maximum then the numbers are

(a) 15,65 (b) 25,55

(c) 35,45 (d) 40,40

[Dec. 2013]

Solution : (d) is correct

Let 1st part = x

$\therefore$ 2nd part = 80 -x

Let $y = x(80 - x) = -x^2 + 80x$

Here co. eff. of $x^2 < 0$

$\therefore$ y is maximum at

$$x=\frac{-b}{2a}=-\frac{80}{2(-1)}=40$$

$\therefore$ Numbers are (40 ; 40)

Tricks : Go by choices.

Q.16. The roots of equation $y^3+y^2-y-1=0$ are

(a) 1,1,-1 (b) -1,-1,1

(c) 1,1,1 (d) None

[June 2014]

Solution : (b) is correct

$y^3+y^2-y-1=0$

or $y^2(y+1)-1(y+1)=0$

or $(y+1)(y^2-1)=0$

or $(y+1)(y+1)(y-1)=0$

$\therefore y=-1;-1;1.$

Tricks : Go by choices.

Q.17. If α,β are the roots of the quadratic equation $2x^2-4x=1$ then the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=$

(a) -11 (b) 22

(c) -22 (d) 11

[June 2015]

Solution : $2x^2-4x-1=0$

Let α and β are its roots

$$\alpha+\beta=\frac{-b}{a}=\frac{-(-4)}{2}=2$$

$$\alpha\beta=\frac{c}{a}=\frac{-1}{2}$$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{2^3-3.(-\frac{1}{2}).(2)}{(-\frac{1}{2})}=\frac{8+3}{-\frac{1}{2}}$$

$$=11\times\left(\frac{-2}{1}\right)=-22$$ So, (c) is correct

Q.18. If α,β be the roots of a quadratic equation if $\alpha+\beta=-2,\alpha\beta=-3$ Find quadratic equation:

(a) $x^2+2x-7=0$

(b) $x^2+2x-3=0$

(c) $x^2-2x-3=0$

(d) $x^2-2x+7=0$

[Dec. 2015]

Solution : (b) is correct

Quadratic Eqn. is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-(-2)x+(-3)=0$

$\therefore x^2+2x-3=0$

Q.19. Value of k for which roots are equal of given equation $4x^2-12x+k=0$:

(a) 144 (b) 9

(c) 5 (d) None of these

[Dec. 2015]

Solution : (b) is correct

$4x^2-12x+k=0$

$D=b^2-4ac=0$

$=(-12)^2=4\times4.k$

Or, $144=16k \therefore k=9$

Q.20. If difference between the roots of the equation $x^2-kx+8=0$ is 4 then the value of K is

(a) 0 (b) ±4

(c) $\pm8\sqrt{3}$ (d) $\pm4\sqrt{3}$

[June 2016]

Solution : (d) is correct.

let $\alpha;\beta$ are roots of $x^2-kx+8=0$

$\therefore \alpha+\beta=-\frac{b}{a}=-\frac{(-k)}{1}=k$ &

$\alpha.\beta=\frac{c}{a}=\frac{8}{1}=8$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=4^2$

$\Rightarrow k^2-4\times8=16$

or $k^2=48\Rightarrow k=\pm\sqrt{16\times3}$

$\Rightarrow k=\pm4\sqrt{3}$

(d) is correct.

Q.21. If α,β be the roots of $x^2+x+5=0$ then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=$ ___

(a) $\frac{16}{5}$ (b) 2

(c) 3 (d) $\frac{14}{5}$

[June 2017, May 2018]

Solution : $\alpha+\beta=-\frac{b}{a}=-\frac{1}{1}=-1$

$\alpha\beta=\frac{c}{a}=\frac{5}{1}=5$

$$\therefore \frac{\alpha}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{(-1)^3-3.5.(-1)}{5}$$

$$=\frac{-1+15}{5}=\frac{14}{5}$$

$\therefore$ (d) is correct.

Q.22. If the sum of two numbers is 13 and the sum of their squares is 85 then the numbers are :

(a) 6, 7 (b) 4, 9

(c) 10, 3 (d) 5, 8

[Dec. 2017]

Solution : (a)

Tricks : GBC (Go by choices)

6 + 7 = 13 (True)

& $6^2+7^2=36+49=85$ (True)

Q.23. The difference between the roots of the equation $x^2-7x-9=0$ is _____:

(a) 7 (b) $\sqrt{85}$

(c) 9 (d) $2\sqrt{85}$

[Dec. 2017]

Solution : (b)

Let α and β are roots.

$$\therefore \alpha+\beta=-\frac{b}{a}=-\frac{-7}{1}=7$$

$$\alpha\beta=\frac{c}{a}=\frac{-9}{1}=-9.$$

$\therefore (\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=7^2-4(-9)=85$

$\therefore \alpha-\beta=\sqrt{85}$

Q.24. The roots of the cubic equation $x^3+7x^2-21x-27=0$ is ____:

(a) -1, 3, 9 (b) 1, -3, 9

(c) -1, 3, -9 (d) -1, -3, 9

[Dec. 2017]

Solution : (c)

Tricks: - Go by choices.

Q.25. If the roots of the equation $kx^2-3x-1=0$ are the reciprocal of the roots of the equation $x^2+3x-4=0$ then K =

(a) 4 (b) -4

(c) 3 (d) -3

[June 2018]

Solution :

Tricks : Eqn. having roots the reciprocal of the roots of $ax^2+bx+c=0$ is $cx^2+bx+a=0$ *i.e.* 1st and last term interchanges.

Eqn. is

$-4x^2+3x+1=0$

Multiplying it by (-1); we get

$4x^2-3x-1=0$

Comparing it with

$Kx^2-3x-1=0$

$\therefore K=3$

(a) is correct

Q.26. If the roots of the equation $x^3-15x^2+kx-45=0$ are in A.P., find value of k:

(a) 56 (b) 59

(c) -56 (d) -59

[June 2018]

Solution : $\because$ Roots are in A.P.

Let roots are $a-d\ ;\ a\ ;\ a+d$

So, $(a-d)+a+(a+d)=15$

or ; $3a=15$

or ; $a=5$

And Product of roots

$(a-d).a.(a+d)=45$

or $(5-d).5.(5+d)=45$

or $25-d^2=9$

or ; $d^2=25-9=16$

or ; $d=\sqrt{16}=4$

Hence ; roots are

$a-d\ ;\ a\ ;\ a+d=5-4\ ;\ 5\ ;\ 5+4$

$=1\ ;\ 5\ ;\ 9.$

The value of K

= Sum of product of two roots in a order

$=(1\times5)+(5\times9)+(9\times1)$

$=5+45+9=59$

$\therefore$ (b) is correct.

Tricks : If $\alpha\ ;\ \beta$ and γ are the roots of a cubic Eqn.

So ; Cubic Eqn. is

$x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma=0$

Given Eqn. is

$x^3-15x^2+k.x-45=0$

Comparing it,

$\alpha+\beta+\gamma=15\Rightarrow1+5+9=15$

& $\alpha\beta\gamma=45\Rightarrow1\times5\times9=45$

[Apply Hit and Trial method which can satisfy both]

Hence ; we can say ;

$\alpha=1\,;\beta=5\,;\gamma=9\,;$

$\therefore\ k=\alpha\beta+\beta\gamma+\gamma\alpha=1\times5+5\times9+9\times1=59$

Q.27. If $\alpha+\beta=-2$ and $\alpha\beta=-3$ where α and β are the roots of the equation, which is

(a) $x^2-2x-3=0$

(b) $x^2+2x-3=0$

(c) $x^2+2x+3=0$

(d) $x^2-2x+3=0$

[May 2018]

Solution : (b)

Quadratic Eqn. having roots α and β is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

or ; $x^2-(-2)x+(-3)=0$

or ; $x^2+2x-3=0$

(b) is correct.

Q.28. When two roots of quadratic equation are $\alpha,\dfrac{1}{\alpha}$ then what will be the quadratic equation:

(a) $\alpha x^2-(\alpha^2+1)x+\alpha=0$

(b) $\alpha x^2-\alpha x^2+1=0$

(c) $\alpha x^2-(\alpha^2+1)x+1=0$

(d) None of these

[Nov. 2018]

Solution : (a)

Tricks : For (a) $\alpha.\dfrac{1}{\alpha}=1=\dfrac{c}{a}=\dfrac{\alpha}{\alpha}=1$ (True)

Q.29. Let α and β be the roots of $x^2+7x+12=0$. Then the value of $\left(\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}\right)$ will be

(a) $\dfrac{49}{144}+\dfrac{144}{49}$

(b) $\dfrac{7}{12}+\dfrac{12}{7}$

(c) $-\dfrac{91}{12}$

(d) None of the above

[Nov. 2018]

Solution : (c)

$x^2+7x+12=0$

or $x^2+4x+3x+12=0$

or $x(x+4)+3(x+4)=0$

or $(x+4)(x+3)=0$

$\therefore\ x=-3\,;-4$

let $\alpha=-3\,;\beta=-4$

$\therefore\ \dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}=\dfrac{9}{-4}+\dfrac{16}{-3}$

$=-\left[\dfrac{9}{4}+\dfrac{16}{3}\right]=-\dfrac{91}{12}$

IInd Method

$\alpha+\beta=\dfrac{-7}{1}=-7$

$\alpha\beta=\dfrac{c}{a}=\dfrac{12}{1}=12$

$\therefore\ \dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}=\dfrac{\alpha^3+\beta^3}{\alpha\beta}=\dfrac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$=\dfrac{(-7)^3-3\times12(-7)}{12}=-\dfrac{91}{12}$

Q.30. Find the condition that one root is double the of $ax^2+bx+c=0$

(a) $2b^2=3ac$ (b) $b^2=3ac$

(c) $2b^2=9.ac$ (d) None

[June 2019]

Solution : (c)

Tricks:-

Let 1st root = 1

Then 2nd root = 2

The Eqn. is

$x^2-(1+2)x+1\times2=0$

or $x^2-3x+2=0$

Comparing it with

$ax^2+bx+c=0$

we get ;

$a=1\,;\ b=-3\,;\ c=2$

Go by choices (GBC)

(a) $2b^2=3ac$

$2.(-3)^2=3.1.2$ (False)

(c) $2b^2=9.ac$

$\therefore\ 2.(-3)^2=9.1.2$

$\Rightarrow\ 18=18$ (True)

Hence, Option (c) is (true)

Q.31. If $x=\dfrac{1}{5+2\sqrt{6}}$ then the value of the expression $x^2-10x+1$ is

(a) 0 (b) 10 (c) $26-12\sqrt{2}$ (d) $\sqrt{15}+\sqrt{3}$

[Dec. 2019]

Solution : (a)

$\because\ x=\dfrac{1}{5+2\sqrt{6}}=\dfrac{1}{5+2\sqrt{3}}\times\dfrac{5-2\sqrt{6}}{5-2\sqrt{6}}$ [rationalising]

$=\dfrac{5-2\sqrt{6}}{25-24}=5-2\sqrt{6}$

If one root $=5-2\sqrt{6}$

Then other root $=5+2\sqrt{6}$ [Irrational Conjugate of first]

$\therefore$ The Quadratic Eqn. is ——

x^2 - (sum of roots) x + product of roots = 0

So, $x^2-\left(5+2\sqrt{6}+5-2\sqrt{6}\right)x+\left(5+2\sqrt{6}\right)\left(5-2\sqrt{6}\right)=0$

or $x^2-10x+1=0$

(a) is correct.

Q.32. The three roots of equation is $x^3+9x^2-x-9=0$

(a) 1, -1, -9 (b) 1, -1, 9

(c) 1, 1, 9 (d) -1, -1, -9

[Dec. 2019]

Solution : (a) is correct.

Tricks: GBC

* Co-efficient of $x^3=1$ (Should be)

For (a)

* Sum of roots = Co-efficient of x^2 but opposite in sign.

$\Rightarrow\ 1+(-1)+(-9)=-9$

Which should be equal to the multiple of x^2 but its sign should be opposite.

i.e. +9 of the question (True)

** Product of roots = The constant term but opposite sign.

Here, Product of roots

$=1.(-1).(-9)=+9$

Which is equal to constant term "-9" but opposite in sign.

Hence ; option (a) satisfies these conditions. No. other condition is required.

So; option (a) is correct.

Detail: (a)

$x^3+9x^2-x-9=0$

or $x^2(x+9)-1(x+9)=0$

or $(x+9)(x^2-1)=0$

If $x+9=0\Rightarrow x=-9$

and if $x^2-1=0\Rightarrow x^2=+1$

$\therefore\ x=\pm1$

Hence ; roots $=-1\,;1\,;-9$.

Q.33. Find the value of K so that x = 2 is a root of the equation $3x^2-2kx+5=0$

(a) 17/4 (b) 4/17

(c) -17/4 (d) -4/17

[Dec. 2019]

Solution : (a)

$\because\ x=2$ is a root.

$\therefore\ 3.2^2-2.K.2+5=0$

or, $-4K=-17$

or, $K=\dfrac{17}{4}$

Q.34. If $2x^2-(a+6)2x+12a=0$ then roots are

(a) 4 & a^2 (b) 6 & a

(c) 3 & 2a (d) 6 & 3a

[Dec. 2020]

Solution : Tricks : GBC

* Product of roots $=\dfrac{c}{a}=\dfrac{12a}{2}$

$=6a.$

It is satisfied by option (b) & (c)

* sum of roots $=\dfrac{-b}{a}=-\dfrac{-(a+6)2}{2}$

$=a+6$

It is satisfied by option (b) only.

$\therefore$ option (b) is correct.

Q.35. Solving equation $3g^2-14g+16=0$, we get roots as

(a) 0 (b) ± 5

(c) 8 and 2/3 (d) 2 and 8/3

[Dec. 2020]

Solution : Tricks : GBC

* Product of roots $=\dfrac{c}{a}=\dfrac{16}{3}$.

Options (c) & (d) satisfy it.

* sum of roots $=\dfrac{-b}{a}=-\dfrac{-14}{3}=\dfrac{14}{3}$

Only option (d) satisfies it.

because $2+\dfrac{8}{3}=\dfrac{6+8}{3}=\dfrac{14}{3}$ (True)

$\therefore$ (d) is correct.

Q.36. Solving equations $m+\sqrt{m}=6/25$ the value of 'm' works out to

(a) 2/25 (b) 1/25

(c) 3/25 (d) 1

[Dec. 2020]

Solution : Tricks GBC

In $m+\sqrt{m}=\dfrac{6}{25}$

* For $\sqrt{m}$; we should select that option, of which roots can be obtained.

So option (b) & (d) are suitable

but for (d) $m+\sqrt{m}$

$=1+\sqrt{1}=1+1=2\neq\dfrac{6}{25}$

$\therefore$ (b) should be correct.

For (b)

$m+\sqrt{m}=\dfrac{1}{25}+\sqrt{\dfrac{1}{25}}$

$=\dfrac{1}{25}+\dfrac{1}{5}=\dfrac{1+5}{25}$

$=\dfrac{6}{25}$ RHS (True)

$\therefore$ (b) is correct.

Q.37. The rational root of the equation $0=2p^3-p^2-4p+2$ is

(a) - 2 (b) 2

(c) ½ (d) -1/2

[Dec. 2020]

Solution :

$2p^3-p^2-4p+2=0$

or ; $p^2(2p-1)-2(2p^2-1)=0$

or $(2p-1)(p^2-2)=0$

If $2p-1=0\Rightarrow p=\dfrac{1}{2}$ * [Rational]

If $p^2-2=0\Rightarrow p^2=2$

$\therefore\ p=\pm\sqrt{2}$ [Irrational]

Clearly option (c) is correct.

Tricks : GBC

Q.38. The value of P for which the difference between the root of equation $x^2 + px + 8 = 0$ is 2 is

(a) ± 2 (b) ± 4
(c) ± 6 (d) ± 8

[Jan. 2021]

Solution : (c) is correct

Let α and β are roots of Eqn.

$$x^2 + px + 8 = 0$$

$$\therefore \quad \alpha + \beta = -\frac{b}{a} = -\frac{p}{1} = -p$$

and $$\alpha\beta = \frac{c}{a} = \frac{8}{1} = 8$$

Given $\alpha - \beta = 2$

Formula $(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$

$$\Rightarrow \quad p^2 = 2^2 + 4 \times 8$$
$$= 36$$
$$\therefore \quad p = \pm\sqrt{36} = \pm 6$$

Q.39. If the quadratic equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root then $p + q = ?$

(a) 0 (b) 1
(c) -1 (d) 2

[Jan. 2021]

Solution : (c) is correct

Let α be a common root.

$$\therefore \quad \alpha^2 + p\alpha + q = 0 \qquad (1)$$

$$\alpha^2 + q\alpha + p = 0 \qquad (2)$$

Eqn. (1)—Eqn (2); we get

$$p\alpha - q\alpha + q - p = 0$$

or $$\alpha(p - q) = p - q$$

or $$\alpha = 1$$

$\Rightarrow$ Putting $\alpha = 1$ in (1);

We get

$$1^2 + p \times 1 + q = 0$$

$$\therefore p + q = -1$$

Q.40. The harmonic mean of the roots of the equation $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

(a) 2 (b) 4
(c) 6 (d) 8

[Jan. 2021]

Solution : (b) is correct Let α and β are roots quadratic

Equation. $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$

Comparing it with standard form $ax^2 + bx + c = 0$

$\therefore a = 5 + \sqrt{2}$, $b = -(4 + \sqrt{5})$ and $c = 8 + 2\sqrt{5}$

We have to find H.M. of α & β

$$= \frac{2\alpha\beta}{\alpha + \beta} = \frac{2\frac{c}{a}}{-\frac{b}{a}}$$

$$= -\frac{2c}{b} = -\frac{2(8 + 2\sqrt{5})}{-(4 + \sqrt{5})}$$

$$= \frac{2.2(4 + \sqrt{5})}{(4 + \sqrt{5})} = 4$$

Q.41. If α and β are the roots of the equation $2x^2 + 5x + k = 0$, and $4(\alpha^2 + \beta^2 + \alpha\beta) = 23$, then which of the following is true?

(a) $k^2 + 3k - 2 = 0$
(b) $k^2 - 2k + 3 = 0$
(c) $k^2 - 2k - 3 = 0$
(d) $k^2 - 3k + 2 = 0$

[July 2021]

Solution: (d) is correct

$$\therefore \quad 2x^2 + 5x + k = 0$$

$$\therefore \quad \alpha + \beta = \frac{-5}{2} \text{ and } \alpha\beta = \frac{K}{2}$$

[Let α & β are roots]

$$\because \quad 4(\alpha^2 + \beta^2 + \alpha\beta) = 23$$

or $$4[(\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta] = 23$$

or $$4\left[\left(\frac{-5}{2}\right)^2 - \alpha\beta\right] = 23$$

or $$4\left[\frac{25}{4} - \frac{K}{2}\right] = 23$$

or $$25 - 2k = 23$$

or $$2k = 2 \Rightarrow k = 1$$

Go by choices (GBC)

Putting $k = 1$ in option (d). It satisfies.

$\therefore$ (d) is correct

Q.42. The value of 'K' is ....., if 2 is a root of the following cubic equation: $x^3 - (k + 1)x + k = 0$

(a) 2 (b) 6
(c) 1 (d) 4

[July 2021]

Solution : (b) is correct

Given Equation is

$$x^3 - (K + 1)x + K = 0$$

$\because$ $x = 2$ is its roots

So, $$2^3 - (K + 1)2 + K = 0$$

or $$8 - 2K - 2 + K = 0$$

or $$-K + 6 = 0$$

or $$K = 6$$

Q.43. If one root is half of the other of a quadratic equation and the difference in roots is a then the equation is

(a) $x^2 + ax + 2a^2 = 0$
(b) $x^2 - 3ax - 2a^2 = 0$
(c) $x^2 - 3ax + 2a^2 = 0$
(d) $x^2 + 3ax - 2a^2 = 0$

[Dec. 2021]

Solution: (c)

Let 1st root = α ; $\therefore$ 2nd root = $\alpha/2$

From question

$$\alpha - \frac{\alpha}{2} = a \Rightarrow \frac{\alpha}{2} = a \Rightarrow \alpha = 2a$$

$\therefore$ 1st root = $\alpha = 2a$.

2nd root $= \frac{\alpha}{2} = \frac{2a}{2} = a$.

The equation is

$$x^2 - (2a + a)x + 2a \times a = 0$$

$$\Rightarrow x^2 - 3ax + 2a^2 = 0$$

Q.44. If the square of a number exceeds twice of the number by 15, then number that satisfies the condition is

(a) -5 (b) 3
(c) 5 (d) 15

[Dec. 2021]

Solution: (c) is Correct.

Tricks GBC C $5^2 = 2 \times 5 + 15$ (True)

Detail

Let No. = x;

Given

$$\therefore x^2 = 2x + 15$$

or $$x^2 - 2x - 15 = 0$$

or $$x^2 - 5x + 3x - 15 = 0$$

or $$x(x - 5) + 3(x - 5) = 0$$

or $$(x - 5)(x + 3) = 0$$

If $x - 5 = 0 \Rightarrow x = 5$

If $x + 3 = 0 \Rightarrow x = -3$ (Invalid)

$\therefore$ No. = 5; (C) is True

Q.45. If one root of $5z^2 + 13z + y = 0$ be reciprocal of the other then the value of y is

(a) $\frac{1}{5}$ (b) $\frac{-1}{5}$
(c) 5 (d) -5

[June 2022]

Solution: Tricks

If one root of $ax^2 + bx + c = 0$

is reciprocal of other root

Then $a = c$

Here one root of $5z^2 + 13z + y = 0$

is reciprocal of other

So $5 = y$

(c) is correct.

Q.46. If the roots of the equation $x^3 - px + q = 0$ are in the ratio 2:3, then

(a) $p^2 = 25q$ (b) $p^2 = 6q$
(c) $6p^2 = 5q$ (d) $6p^2 = 25q$

[Dec. 2022]

Solution: *Given*. Eqn.

$$1x^2 - px + q = 0$$

Let α is common in the ratio 2 : 3.

$\therefore$ Roots = 2α and 3α

Now Sum of roots $= 2\alpha + 3\alpha = -\frac{b}{a}$

$$\Rightarrow 5\alpha = -\frac{-p}{1} = p$$

or $$\alpha = \frac{P}{5}$$

And product of roots

$$= 2\alpha \times 3\alpha = 6\alpha^2 = \frac{c}{a}$$

$$\Rightarrow 6 \times \left(\frac{p}{5}\right)^2 = \frac{q}{1}$$

or $$\frac{6p^2}{25} = q$$

$$\therefore 6p^2 = 25q$$

(d) is correct

Tricks

Let roots = 2; 3

Q. Eqn. is $x^2 - (2 + 3)x + 2 \times 3 = 0$

or; $x^2 - 5x + 6 = 0$

Comparing it with

$$x^2 - px + q = 0$$

$$\therefore p = 5; q = 6$$

GBC

(d) $6p^2 = 25q$

$6 \times 5^2 = 25 \times 6$

$150 = 150$ (True)

$\therefore$ (d) is correct

Q.47. What will be the value of k, if the roots of the equation $(k - 4)x^2 - 2kx + (k + 5) = 0$ are equal?

(a) 18 (b) 20
(c) 19 (d) 21

[Dec. 2022]

Solution:

Quad. Eqn. is

$$(k - 4)x^2 - 2kx + (k + 5) = 0$$

$\because$ Roots equal

$\therefore D = b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac$

Or; $(-2k)^2 = 4 \times (k - 4) \times (k + 5)$

Or; $4k^2 = 4(k^2 + k - 20)$

Or; $k^2 = k^2 + k - 20$

$\therefore k = 20$ Ans.

$\therefore$ (b) is correct

Q.48. If α and β are roots of the quadratic equation $x^2 - 2x - 3 = 0$, then the equation whose roots are $\alpha + \beta$ and $\alpha - \beta$ is:

(a) $x^2 - 6x - 8 = 0$
(b) $x^2 - 6x + 8 = 0$
(c) $x^2 + 6x + 8 = 0$
(d) $x^2 + 6x - 8 = 0$

[June 2023]

Solution: $1 \times x^2 - 2x - 3 = 0$

Let α and β are its roots.

$$\therefore \alpha + \beta = -\frac{b}{a} = -\frac{-2}{1} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{-3}{1} = -3$$

$$\therefore (\alpha - \beta)^2 = (\alpha - \beta)^2 - 4\alpha\beta$$
$$= 2^2 - 4 \times (-3)$$
$$= 4 + 12$$
$$= 16$$

$$\therefore \quad \alpha - \beta = \sqrt{16} = 4$$

Given roots are:

$\alpha + \beta = 2$ and $\alpha - \beta = 4$

$\therefore$ Eqn. is

$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$

$$\therefore x^2 - (2 + 4)x + 2 \times 4 = 0$$

$$\therefore x^2 - 6x + 8 = 0$$

(b) is correct.

Q.49. If α and β are roots of the equation $x^2 - (n^2 + 1)x + \frac{1}{2}(n^4 + n^2 + 1) = 0$, then the value of $\alpha^2 + \beta^2$ is:

(a) $2n$
(b) n^2
(c) $2n^2$
(d) n^3

[June 2023]

Solution:

$$\alpha + \beta = \frac{-b}{a} = -\frac{-(n^2 + 1)}{1} = n^2 + 1$$

$$\alpha\beta = \frac{c}{a} = -\frac{\frac{1}{2}(n^4+n^2+1)}{1}$$

$$= \frac{1}{2}(n^4+n^2+1)$$

$\therefore \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$= (n^2+1)^2 - 2\frac{1}{2}(n^4+n^2+1)$$

$$n^4 + 2n^2 \times 1 + 1^2 - (n^4+n^2+1)$$

$$n^4 + 2n^2 \times 1 - n^4 - n^2 - 1$$

$$= n^2$$

(*b*) is correct.

6 CHAPTER INEQUALITIES

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. The solution of the inequality $\frac{(5-2x)}{3} \le \frac{x}{6} - 5$ is

(*a*) $x \ge 8$ (*b*) $x \le 8$

(*c*) x=8 (*d*) none of these

[June 2010]

Solution : (*a*) is correct.

$$\frac{5-2x}{3} \le \frac{x-5}{6}$$

or $\frac{5-2x}{3} \le \frac{x-30}{6}$

or $5-2x \le \frac{x-30}{2}$

or $10-4x \le x-30$

or $10+30 \le x+4x$

or $5x \ge 40$

or $x \ge 8$

$\therefore$ option (*a*) is correct

Q.2. Solution space of inequalities $2x+y \le 10$ and $x-y \le 5$:

(*i*) includes the origin.

(*ii*) includes the point (4,3) which one is correct?

(*a*) Only (*i*)

(*b*) Only (*ii*)

(*c*) Both (*i*) and (*ii*)

(*d*) none of the above

[June 2011]

Solution : (*a*) is correct

Tricks : Go by choices

(0, 0) satisfies both ineqns. but (4 ; 3) does not satisfy 1st

$\therefore$ (*a*) is correct

Q.3. On the average, experienced person does 5 units work while a fresh one 3 units work daily but the employer have to maintain the output of atleast 30 units of work per day.

The situation can be expressed as.

(*a*) $5x+3y \le 30$ (*b*) $5x+3y \ge 30$

(*c*) 5x+3y=30 (*d*) None of these

[Dec. 2011 & 12]

Solution : (*b*) Let No. of experienced persons = x and No. of Freshers = y

$\therefore 5x+3y \ge 30$

Q.4. Find the range of real of x satisfying the inequalities 3x-2 > 7 and 4x-13 > 15

(*a*) x > 3 (*b*) x > 7

(*c*) x < 7 (*d*) x < 3

[June 2012]

Solution : (*b*) is correct.

$3x-2>7 \Rightarrow 3x>9 \therefore x>3$ _____(1)

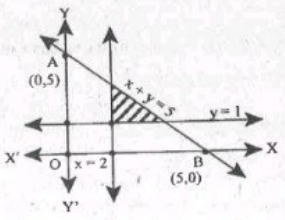
$4x>15+13 \Rightarrow 4x>28 \therefore x>7$ ____(2)

Clearly From (1) and (2) ;

x > 7 satisfies both

$\therefore$ (*b*) is correct.

Q.5. The shaded region represents :



(*a*) $x+y \le 5, x \ge 2, y \le 1$

(*b*) $x+y \le 5, x \ge 2, y \ge 1$

(*c*) $x+y \ge 5, x \ge 2, y \ge 1$

(*d*) None of these

[Feb. 2008]

Solution : Tricks: Go by choices.

option (*b*)

Q.6. The union forbids the employer to employ less than 2 experienced person (x) to each fresh person (y). This situation can be expressed as

(*a*) $x \le y/2$ (*b*) $y \le x/2$

(*c*) $y \ge x/2$ (*d*) None

[June 2013]

Solution : (*b*) is correct

No. of Fresh persons for x Experienced person $= \frac{x}{2}$

$\therefore \frac{x}{2} \ge y$ (given) $\therefore y \le \frac{x}{2}$.

Q.7. The solution of the inequality $8x + 6 < 12x + 14$ is

(*a*) (-2, 2) (*b*) (-2, 0)

(*c*) $(2, \infty)$ (*d*) $(-2, \infty)$

[Dec. 2013]

Solution : (*d*) is correct

$8x + 6 < 12x + 14$

or $-8 < 4x$

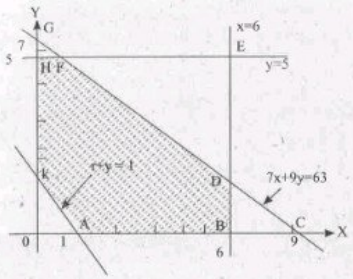
or $-2 < x$

$x > -2$

$\therefore$ Soln. is $(-2; \infty)$

Q.8. The graph of linear inequalities

$7x + 9y \le 63$; $x + y \ge 1$; $0 \le x \le 6$ *and* $0 \le y \le 6$ has been given below



(*a*) BCDB and DEFD (*b*) Unbounded

(*c*) HFGH (*d*) ABDFHKA

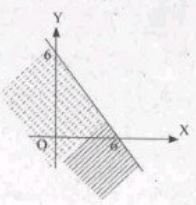
[June 2014]

Solution : (*d*) Clearly common region is ABDFHKA.

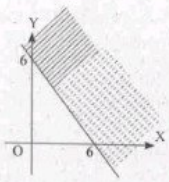
Q.9. Which of the following graph represents the in equality $x + y \le 6$ is

[Dec. 2014]

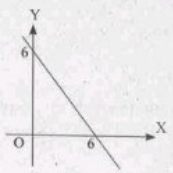
(*a*)



(*b*)

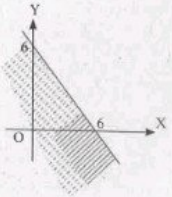


(*c*)

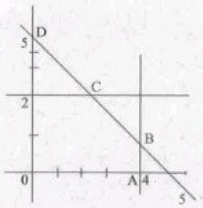


(*d*) None of these

Solution : (*a*) is correct. The graphical representation of $x+y \le 6$ is as follows :



Q.10. The graph of linear inequalities $x + y \geq 5; x + y \leq 5; 0 \leq x \leq 4$ and $0 \leq y \geq 2$ is given below:



The common region of the inequalities will be:

(*a*) OABCEO

(*b*) ECDE

(*c*) Line Segment DC

(*d*) Line Segment BC

[Dec. 2014]

Solution : (*c*)

Q.11. The common region represented by the inequalities $2x + y \geq 8, x+y \geq 12, 3x+2y \leq 34$ is

(*a*) Unbounded

(*b*) In feasible

(*c*) Feasible and bounded

(*d*) Feasible and unbounded

[June 2015]

Solution : (*c*) is correct.

$2x + y = 8 \Rightarrow$

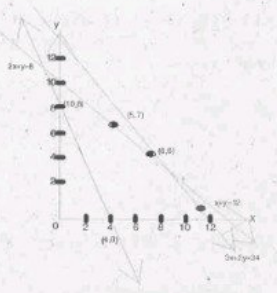
x	0	4
y	8	0

$x + y = 12 \Rightarrow$

x	6	5
y	6	7

and $3x + 2y = 34$;

x	10	8
y	2	5



clearly It is Feasible and bounded.

Q.12. By lines $x + y = 6, 2x - y = 2$, the common region shown is the diagram refers to:

(*a*) $x + y \geq 6, 2x - y \leq 2, x \geq 0, y \geq 0$

(*b*) $x + y \leq 6, 2x - y \leq 2, x \geq 0, y \geq 0$

(*c*) $x + y \leq 6, 2x - y \geq 2, x \geq 0, y \geq 0$

(*d*) None of these

[Dec. 2015]

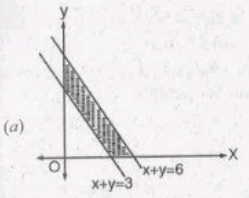
Solution : (*b*) is correct

Tricks: Go by choices

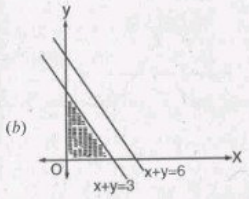
A point (1,1) (let) satisfies all inequations of (*b*).

Q.13. The common region of $x + y \leq 6; x + y \geq 3$, is for shown by shaded region

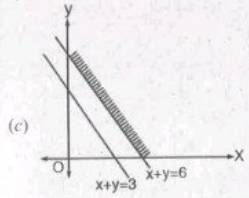
(*a*)



(*b*)



(*c*)



(*d*) None of these

[June 2016]

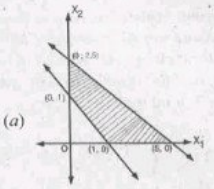
Solution : (*a*) is correct.

Tricks : Go by choices.

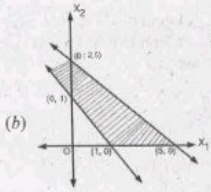
Clearly a point of the common region of option (*a*) satisfy all given constraints $x+y \leq 6$ & $x+y \geq 3$.

Q.14. The inequalities $x_1 + 2x_2 \leq 5; x_1 + x_2 \geq 1; x_1 \geq 0; x_2 \geq 0$ represents the region.

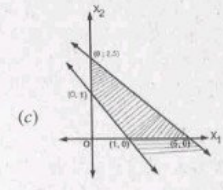
(*a*)



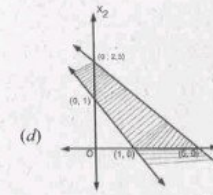
(*b*)



(*c*)



(*d*)



[Dec. 2016]

Solution : (*a*) is correct

Tricks : Go by choices.

Q.15. A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is atleast 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per kg. of each food is shown below:

	A	B	C	D
Food I :	2	1	1	2
Food II :	1	1	2	3

Assuming x kgs of food I is to be mixed with y kgs of food II the situation can be expressed as

(*a*) $2x + y \leq 9$; $x + y \leq 7$;
$x + 2y \leq 10$;
$2x + 3y \leq 12$; $x > 0, y > 0$

(*b*) $2x + y \geq 30$; $x + y \leq 7$;
$x + 2y \geq 10$;
$x + 3y \geq 12$; $x \geq 0$; $y > 0$

(*c*) $2x + y \geq 9$; $x + y \leq 7$;
$x + y \leq 10$;
$x + 3y \geq 12$; $x \geq 0, y \geq 0$

(*d*) $2x + y \geq 9$; $x + y \geq 7$;
$x + 2y \geq 10$;
$2x + 3y \geq 12$; $x \geq 0$; $y \geq 0$

[June 2017]

Solution : Atleast → Minimum

So, use $\geq$ Sign here.

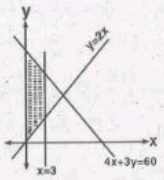
Constraints are :

$2x + y \geq 9$;
$x + y \geq 7$
$x + 2y \geq 1$
$2x + 3y \geq 12$
$\therefore$ (*d*) is correct.

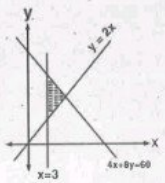
Q.16. The shaped region represented by the inequalities

$4x + 3y \leq 60$, $y \geq 2x$, $x \geq 3$, $x \geq 0$, $y \geq 0$

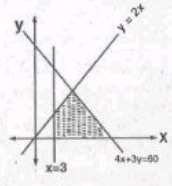
(*a*)



(*b*)



(*c*)



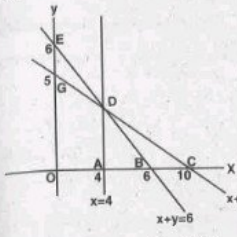
(*d*) None

[June 2017]

Solution : Tricks : Go by choices

Option (*b*) is correct.

Q.17. In the following diagram, the region represented by the inequalities $x + 2y \leq 10, x + y \leq 6, x \leq 4$ & $x \geq 0, y \geq 0$ is :



(*a*) OADGO (*b*) ADC

(*c*) ACD (*d*) DEG

[June 2018]

Solution : (*a*)

Tricks : Go by choices

Q.18. The linear relationship between two variables in an inequality

(*a*) $ax + by \leq c$

(*b*) $ax \,.\, by \leq c$

(*c*) $axy + by \leq c$

(*d*) $ax + bxy \leq c$

[May 2018]

Solution : (*a*)

Standard form of Linear Eqn. is

$ax + by = c$.

So; $ax + by \leq c$ is a Linear Ineqn.

Q.19. On Solving the Inequalities $5x + y \leq 100, x + y \leq 60, x \geq 0, y \geq 0$, we get the following situation:

(*a*) (0, 0), (20, 0), (10, 50) & (0, 60)

(*b*) (0, 0), (60, 0), (10, 50) & (0, 60)

(*c*) (0, 0), (20, 0), (0, 100) & (10, 50)

(*d*) None of these *[Nov. 2018]*

Solution : (*a*)

Tricks : Go by choices

Q.20. An employer recruits experienced (x) and fresh workmen (y) under the condition that he cannot employ more than 11 people. x and y can be related by the inequality

(*a*) $x + y \neq 11$;

(*b*) $x + y \leq 11, x \geq 0, y \geq 0$

(*c*) $x + y \geq 11, x \geq 0, y \geq 0$

(*d*) None of these

[June 2019]

Solution : (*b*)

Clearly $x + y \leq 11$.

and $x \; ; \; y \geq 0$.

Q.21. The solution set of the inequations $x + 2 > 0$ and $2x - 6 > 0$ is

(*a*) (-2 , ∞); (*b*) (3 , ∞)

(*c*) ($-\infty$, -2) (*d*) ($-\infty$, -3)

[June 2019]

Solution : $\because \; x + 2 > 0 \Rightarrow x > -2$

$\Rightarrow x = \{-1 ; 0, 1, 2, 3, 4, ...\}$ (1)

and $2x - 6 > 0 \Rightarrow \quad x > 3$

$\Rightarrow x = \{4 ; 5 ; 6 ; 7 ; .....\}$ (2)

From (1) and (2) ; we get $x = \{4, 5, 6, ...\}$ satisfies both conditions.

$\therefore$ Solution Set = $(3 ; \infty)$

Q.22. The solutions of the set of inequations $2x+y \geq 12, 5x+8y \geq 74, x+6y \geq 24, x \geq 0, y \geq 0$ are

(*a*) $(24,.0), \left(\frac{126}{11}, \frac{23}{11}\right), (2, 8)\ (0, 12)$

(*b*) $(0, 24), (2, 8), (0, 12), \left(\frac{126}{11}, \frac{23}{11}\right)$

(*c*) (8, 4), (2, 8), (0, 12), (0, 24)

(*d*) (8, 4), (0, 0) (0, 6) (2, 0)

[Nov. 2019]

Solution : Tricks : GBC

* In option (*a*) & (*b*) ; all points are common except (0 ; 24).

putting $x = 0$; $y = 24$ in all eqns.; No Eqn. is satisfied by (0 ; 24).

Hence option (*b*) is not the answer.

* In option (*a*) & (*c*) ; point (8 ; 4) does not satisfy any eqn. ;

So option (*c*) is not the answer.

* In option (*a*) & (*d*) ; No point is common. Point (8 ; 4) is also present in option (*d*)

which does not satisfy any eqn.

Hence ; option (*b*) ; (*c*) and (*d*) are eliminated.

So ; option (*a*) is correct.

Q.23. The solution of the inequality $\frac{(5-2x)}{3} \leq \frac{x}{6} - 5$ is

(*a*) $x \geq 8$ (*b*) $x \leq 8$

(*c*) $x = 8$ (*d*) None of these

[Dec. 2020]

Solution : (*a*) is correct.

$$\frac{5-2x}{3} \leq \frac{x-5}{6}$$

or $\frac{5-2x}{3} \leq \frac{x-30}{6}$

or $5-2x \leq \frac{x-30}{2}$

or $10-4x \leq x-30$

or $10+30 \leq x+4x$

or $5x \geq 40$

or $x \geq 8$

$\therefore$ option (*a*) is correct

Q.24. The common region in the graph of the inequalities $x+y \leq 4, x-y \leq 4, x \geq 2$ is

(*a*) Equilateral triangle (*b*) Isosceles triangle

(*c*) Quadrilateral (*d*) Square

[Jan. 2021]

Solution : For $x+y \leq 4$

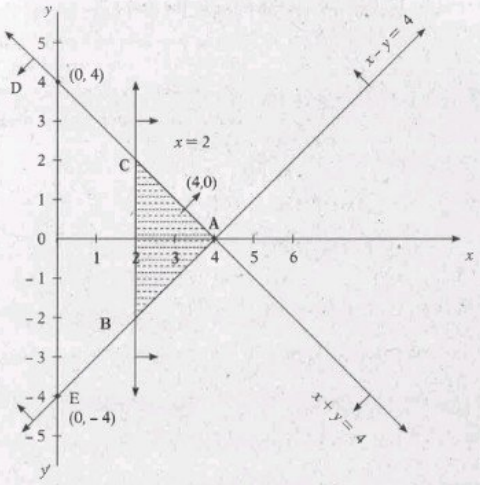
x	0	4
y	4	0

For $x-y \leq 4$

x	0	4
y	–4	0

Clearly ABC is common region. Here AB = AC

$\therefore$ Isosceles Triangle (*b*) correct



Q.25. If $y = 4+9 \sin 5x$ then which holds good?

(*a*) $-5 \leq y \leq 13$ (*b*) $-4 \leq y \leq 8$ (*c*) $0 < y < 1$ (*d*) $-5 < y < 5$

[July 2021]

Solution : (*a*) is correct

We know the value of $\sin\theta$ always his between –1 and + 1

So, Minimum value of $\sin 5x = -1$

and Maximum value of $\sin 5x = +1$

$\therefore$ Minimum value of $y = 4 + 9 \sin 5x$

$= 4 + 9(-1) = -5$

And Maximum value of $4 + 9 \sin 5x$

$= 4 + 9 \times 1 = 13$

$\therefore -5 \leq y \leq 13$

Solution: Let No. of bags manufactured are x_1 & x_2 of Grade I and Grade II respectively.

$\therefore$ Total hrs taken

$= 4x_1 + 10x_2$

Available Time = 180 hrs.

i.e. Maximum Time

$\therefore 4x_1 + 10x_2 \leq 180$

$\therefore$ (*d*) is correct.

CHAPTER 7

SIMPLE INTEREST

SIMPLE INTEREST

When we borrow money from a bank or co-operative society or an individual institution, for a period of time, we pay some extra money in addition to the money borrowed, to the lender for using his money. **This extra money paid is called Interest (I),** the amount borrowed is called **Principal (P)** and the time period for which money is borrowed is called **Time (t).** The money with interest paid back to the lender is called of **Amount (A or S).** In other words

Amount = Principal + Interest

A = P + I

Simple Interest

The interest computed on the principal only (*i.e.* not on interest earned) for the entire period of borrowing is called **SIMPLE INTEREST.**

Type - I

Some Important Formulae.

(*i*) $I = \frac{P.r.t}{100}$ [when r in %]

(*ii*) I = p.r.t. [when r in decimal form]

(*iii*) $r = \frac{I \times 100}{pt}$

(*iv*) $t = \frac{I \times 100}{pr}$

(*v*) $P = \frac{I \times 100}{rt}$

(*vi*) $A = P + I$

(*vii*) $I = A - P$

(*viii*) $A = P\left(1+\frac{rt}{100}\right)$

Where

A = Accumulated amount
[Final value of investment]

P = Principal. [Initial value of an investment]

r = Rate of interest

t = time (years.)

I = Amount if interest

Illustrative examples

Example 1

How much interest will be earned on ₹ 2000 at 6% simple interest for 2 years.

Solution: $I = \frac{Prt}{100} = \frac{2000\times6\times2}{100} = ₹\,240$

Example 2

Sarita deposited ₹ 50,000 in a bank for 2 years with the interest rate of 5.5% p.a. what will be the final value of Investment ?

Solution: $A = P\left(1+\frac{rt}{100}\right)$

$= 50{,}000\left[1+\frac{5.5\times2}{100}\right] = ₹55{,}500$

Calculator Tricks:-

rate for 2 years = 2 × 5.5 = 11%

Amount = 50000 + 11% button = ₹55000

Example 3

Find the rate of interest if the amount owed after 6 months is ₹1050 borrowed amount being ₹ 1000.

(*a*) 5% (*b*) 10% (*c*) 15% (*d*) None

Solution :

SI = ₹ (1050 - 1000) = ₹ 50.

$r = \frac{I\times100}{pt} = \frac{50\times100\times12}{1000\times6} = 10\%$

Tricks: Go by choices

For (*b*) A = 1000 + 1000 × rate of interest of 6 months

$= 1000 + 1000\times\frac{10}{2}(\%) = ₹1050$

∴ option (*b*) is correct

Calculator Tricks:- GBC

For option (b)

$r = \frac{10}{2} = 5\%$

∴ A = 1000 + 5% button (press) = 1050 (True)

Example 4

Kapil deposited some amount in a bank for $7\frac{1}{2}$ years at the rate of 6% p.a. simple interest.

Kapil received ₹ 1,01,500 at the end of the term. Compute initial deposit of Kapil.

(*a*) ₹ 70,000 (*b*) ₹ 60,000 (*c*) ₹ 80,000 (*d*) None

Solution: Tricks : Go by Choices

For option (*a*)

= 70,000 + [6 × 7.5] %

= ₹ 1,01,500 (True)

∴ Option (*a*) is correct.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. ₹ 8, 000 becomes ₹ 10,000 in two years at simple interest. The amount that will become ₹ 6,875 in 3 years at the same rate of interest is :

(*a*) ₹ 4,850 (*b*) ₹ 5,000

(*c*) ₹ 5,500 (*d*) ₹ 5,275

[Nov. 2006]

Solution : Tricks

(*b*) S.I./year = $\frac{10000-8000}{2} = ₹1000$

$r = \frac{1000\times100}{8000} = 12.5\%$

$P = \frac{Amt}{Amt\ at\ ₹1} = \frac{6875}{1+0.125\times3} = ₹5000$

∴ (*b*) is correct.

Q.2. The rate of simple: interest on a sum of money is 6% p.a. for first 3 years, 8% p.a. for the next five years and 10% p.a. for the period beyond 8 years. If the simple interest accrued by the sum for a period for 10 years is ₹ 1,560. The sum is :

(*a*) ₹ 1,500 (*b*) ₹ 2,000

(*c*) ₹ 3,000 (*d*) ₹ 5,000

[Feb. 2007]

Solution : (*b*) is correct.

Single S.I For 1 yrs

= (6×3+8×5+10×2)% = 78%

Tricks:

$P = \frac{Total\ S.I}{S.I\ on\ Rs1} = \frac{1560}{0.78} = ₹\,2000$

(*b*) is correct

Q.3. A sum of money doubles itself in 10 years. The number of years it would treble itself is :

(*a*) 25 years (*b*) 15 years

(*c*) 20 years (*d*) None

[Feb. 2007]

Solution : (*c*) is correct.

Tricks :

$\frac{t_2}{t_1} = \frac{x_2-1}{x_1-1}$

or $\frac{t_2}{10} = \frac{3-1}{2-1}$ or; $t_2 = 20$ yrs.

(*c*) is correct

Q.4. A certain sum of money amounts to ₹ 6,300 in two years and ₹7,875 in three years nine months at simple interest. Find the rate of interest per annum :

(*a*) 20% (*b*) 18%

(*c*) 15% (*d*) 10%

[May 2007]

Solution : (*a*) is correct.

Tricks:

S.I. Per year =

$\frac{\text{Amount for 3.75 yrs} - \text{Amount for 2yrs}}{(3.75-2)\text{ yrs}}$

= ₹ 900

P = 6300 - 2 × 900 = ₹ 4500

$r = \frac{900\times100}{4500\times1} = 20\%$

Q.5. A person borrows ₹ 5,000 for 2 years at 4% p.a. simple interest. He immediately lends to another person at $6\frac{1}{4}\%$ p.a. for 2 years. Find his gain in the transaction per year:

(*a*) ₹ 112.50 (*b*) ₹ 125

(*c*) ₹ 225 (*d*) ₹ 167.50

[Nov. 2007]

Solution : (*a*) % Gain = $6\frac{1}{4}\% - 4 = 2.25\%$

S.I for 2 yrs. = 5000 × 2.25% × 2 = ₹ 225

S.I per yr. = 5000 × 2.25% = ₹ 112.50

∴ (*a*) is correct

Q.6. Two equal sums of money were lent at simple interest at 11% p.a. for $3\frac{1}{2}$ years and $4\frac{1}{2}$ years respectively. If the difference in interests for two periods was ₹ 412.50, then each sum is:

(*a*) ₹ 3,250 (*b*) ₹ 3,500

(*c*) ₹ 3,750 (*d*) ₹ 4,350

[Feb. 2008]

Solution : (*c*) **Tricks** : Difference in interest is due to time

∴ rate of interest for the whole duration = (11×4.5−11×3.5) = 11%

$\therefore P = \frac{\text{Total S.I}}{\text{Interest on ₹1}} = \frac{412.50}{0.11} = ₹3750$

(*c*) is correct

Q.7. In how much time would the simple interest on a certain sum be 0.125 times the principal at 10% per annum ?

(*a*) $1\frac{1}{4}$ years (*b*) $1\frac{3}{4}$ years

(*c*) $2\frac{1}{4}$ years (*d*) $2\frac{3}{4}$ years

[June 2008]

Solution : (*a*) is correct

Tricks : $t = \frac{I/P}{r\%} = \frac{0.125}{0.10} = 1.25\,yrs$

Detail: $S.I = \frac{p.r.t}{100}$

or; $0.125P = \frac{P.10\times t}{100}$

or $t = 0.125\times10 = 1.25\,yrs$

∴ (*a*) is correct

Q.8. Find the numbers of years in which a sum doubles itself at the rate of 8% per annum.

(*a*) $11\frac{1}{2}$ (*b*) $12\frac{1}{2}$

(*c*) $9\frac{1}{2}$ (*d*) $13\frac{1}{2}$

[Dec. 2008]

Solution : (*b*) is correct

Tricks : $t = \frac{(x-1)\times100}{r}$

$= \frac{(2-1)\times100}{8} = 12.5\,yrs$

Q.9. The time by which a sum of money is 8 times of itself if it double itself in 15 years.

(*a*) 42 years (*b*) 43 years

(*c*) 45 years (*d*) 46 years

[June 2009]

Solution : (*c*) is correct

It is Compound Interest Qts.

Tricks : $2^{t/2} = 8^{15}$

or $2^{t/2} = (2^3)^{15}$: $t_2 = 45\,yrs$

Q.10. What is the rate of simple interest if a sum of money amount ₹ 2,784 in 4 years and ₹2,688 in 3 years ?

(*a*) 1% p.a. (*b*) 4% p.a.

(*c*) 5% p.a. (*d*) 8% p.a.

[June 2009]

Solution : (*b*) is correct

S.I. pa = $\frac{\text{Difference in S.I}}{\text{Difference in time}}$

$= \frac{SI_2 - SI_1}{t_2 - t_1} = \frac{2784-2688}{4-3} = ₹\,96$

Principal = ₹ (2688 − 3×96) = ₹ 2400

$r = \frac{I\times100}{p\times t} = \frac{96\times100}{2400\times1} = 4\%$

Q.11. If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a. The ratio will be:

(a) 2:15 (b) 7:15
(c) 15:7 (d) 1:7

[June 2011]

Solution : (c) is correct

$$P_1.\frac{6.7}{100}=2\times\frac{P_2.9.5}{100}$$

$$\text{or } \frac{P_1}{P_2}=2\times\frac{9\times5}{6\times7}=\frac{15}{7}\Rightarrow\frac{P_1}{P_2}=\frac{15}{7}$$

Tricks:- GBC

Q.12. By mistake a clerk, calculated the simple interest on principal for 5 months at 6.5% p.a. instead of 6 months at 5.5% p.a. If the error in calculation was ₹ 25.40. The original sum of principal was_____.

(a) ₹ 60,690 (b) ₹ 60,960
(c) ₹ 90,660 (d) ₹ 90,690

Solution : (b) is correct

$$P=\frac{25.40}{\frac{5.5}{100}\times\frac{6}{12}-\frac{6.5}{100}\times\frac{5}{12}}$$

$$=\frac{25.40\times1200}{5.5\times6-6.5\times5}=₹\,60,960$$

Q.13. If the Simple Interest on ₹1,400 for 3 years is less than the simple interest on ₹ 1,800 for the same period by ₹ 80, then the rate of interest is:

(a) 5.67% (b) 6.67%
(c) 7.20% (d) 5.00%

[Dec. 2011]

Solution : (b) is correct.

Tricks : $r=\frac{80\times100}{(1800-1400)\times3}$

$=6.67\%$

Q.14. The S.I. on a sum of money is $\frac{4}{9}$ of the principal and the No. of years is equal to the rate of interest per annum. Find the rate of interest per annum?

(a) 5% (b) 20/3%
(c) 22/7% (d) 6%

[June 2012]

Solution : (b)

$$\text{S.I.}=\frac{p.r.r}{100}\Rightarrow\frac{4}{9}p.=p.\left(\frac{r}{10}\right)^2$$

$$\therefore\frac{r}{10}=\frac{2}{3}\therefore r=\frac{20}{3}\%$$

Q.15. Simple interest on ₹2,000 for 5 months at 16% p.a. is_____.

(a) ₹133.33 (b) ₹133.26
(c) ₹134.00 (d) ₹132.09

[June 2012, Dec. 2012]

Solution : (a)

$$\text{S.I.}=2000\times\frac{5}{12}\times\frac{16}{100}=₹\,133.33$$

Q.16. How much investment is required to yield an Annual income of ₹420 at 7% p.a. Simple interest.

(a) ₹ 6,000 (b) ₹ 6,420
(c) ₹ 5,580 (d) ₹ 5,000

[Dec. 2012]

Solution : (a) $P=\frac{420\times100}{7\times1}=₹\,6000$

Calculator Tricks II GBC :

P = 420 ÷ 7% button = ₹ 6000

Q.17. Mr. X invests ₹ 90,500 in post office at 7.5% p.a. simple interest. While calculating the rate was wrongly taken as 5.7% p.a. The difference in amounts at maturity is ₹ 9,774. Find the period for which the sum was invested.

(a) 7 years (b) 5.8 years
(c) 6 years (d) 8 years

[Dec. 2012]

Solution : (c) **Tricks**

$$t=\frac{9774\times100}{90,500\times(7.5-5.7)}=6\text{ yrs}$$

Q.18. If the sum of money when compounded annually become ₹1140 in 2 years and ₹1710 in 3 years at rate of interest

(a) 30% (b) 40%
(c) 50% (d) 60%

[June 2013]

Solution : (c) Interest in 3rd

yr = ₹ 1710 - ₹1140

= ₹ 570

Tricks Note : For 3rd yr ; it will be like S.I

$$r=\therefore\frac{I\times100}{Pt}=\frac{570\times100}{1140\times1}=50\%$$

Tricks II Go by choices.

For (c) A = 1140 + 50% (Calculator)

= ₹ 1710

∴ (c) is correct

Q.19. In what time will a sum of money double itself at 6.25% p.a. at simple interest

(a) 5 yrs (b) 8 year
(c) 12 yrs (d) 16 yrs

[Dec. 2013]

Solution : (d) is correct.

Tricks :

$$t=\frac{(2-1)\times100}{6.25}=16\text{ years}$$

Q.20. What principal will amount to ₹370 in 6 years at 8% p.a. at simple interest

(a) ₹ 210 (b) ₹ 250
(c) ₹310 (d) ₹ 350

[Dec. 2013]

Solution : (b) is correct

Tricks : $P=\frac{370}{1+6\times0.08}=₹\,250$

Calculator Tricks :- GBC

(b) Amt = 250 + (6 × 8)% button press = 370

Q.21. If a sum triples in 15 yrs at Simple rate of interest then the rate of interest per annum will be

(a) 13.0% (b) 13.3%
(c) 13.5% (d) 18%

[June 2014]

Solution : (b) is correct

Tricks $r=\frac{(3-1)\times100}{1\times15}=13.3\%$

Calculator Tricks :- GBC

(b) r = 15 × 13.333% = 200%

A = 1 + 200% (button) = 3

∴ (b) is correct

Q.22. A certain sum of money was invested at simple rate of interest for three years. If it was invested at 7% higher, the interest have been ₹ 882 more, then sum has been invested at that rate was

(a) ₹ 12,600 (b) ₹ 6,800
(c) ₹ 4,200 (d) ₹ 2,800

[Dec. 2014]

Solution : (c) is correct

S.I = ₹882 for r = 7%

t = 3 years.

$$P=\frac{I\times100}{rt}=\frac{882\times100}{7\times3}=₹\,4200$$

Calculator Tricks :- GBC

Q.23. A sum of money will be doubled itself in 8 years at S.I. In how many years the sum will be tripled itself ?

(a) 20 years (b) 12 years
(c) 16 years (d) None

[June 2015]

Solution : (c) is correct.

Tricks : $\frac{t_2}{8}=\frac{3-1}{2-1}$

$t_2=16$ yrs.

Q.24. A sum of 44,000 is divided into 3 parts such that the corresponding interest earned after 2 years, 3 years and 6 years may be equal at the rate of simple interest are 6% p.a. 8% p.a. & 6% p.a., respectively. Then the smallest part of the sum will be.

(a) ₹ 4,000 (b) ₹ 8,000
(c) ₹ 10,000 (d) ₹ 12,000

[June 2015]

Solution : (b) is correct.

Tricks: $p_1:p_2:p_3=\frac{1}{r_1t_1}:\frac{1}{r_2t_2}:\frac{1}{r_3t_3}$

$=\frac{1}{2\times6}:\frac{1}{8\times3}:\frac{1}{6\times6}$

$=\left[\frac{1}{12}:\frac{1}{24}:\frac{1}{36}\right]\times72$ LCM of denominators

= 6:3:2

So, Smallest principal

$=\frac{44000}{6+3+2}\times2=₹8000$

Q.25. No. of years a sum of money becomes 4 times itself at 12% p.a. at simple interest:

(a) 20 (b) 21
(c) 25 (d) 30

[Dec. 2015]

Solution : (c) is correct

Tricks: $t=\frac{(4-1)\times100}{1\times12}=25\,yrs.$

Q.26. If a person lends ₹ 6,000 for 4 years and ₹ 8,000 for 3 years at S.I. The total interest earned is ₹ 2400 then the rate of interest is.....

(a) 5% (b) 6%
(c) 7% (d) 8%

[Dec. 2016]

Solution : (a) is correct.

Tricks: Go by choices.

For (a) ;

Total SI = 6000×4×5% + 8000×3×5%

= ₹2400

So, (a) is correct.

Q.27. In simple interest, a certain sum becomes ₹ 97,920 in 3 years, and ₹ 1,15,200 in 5 years, then the rate of interest is:

(a) 10% (b) 11.2%
(c) 12% (d) 13.6%

[June 2018]

Solution : (c)

Tricks :

$$\text{S.I p.a.}=\frac{1,15,200-97,920}{5-3}$$

= ₹8640

Principal = 97,920 - 3 yrs interest = 97,920 - 3 × 8640 = ₹72,000

$$r=\frac{8640\times100}{72000}=12\%$$

Calculator Tricks :

Amounts = 72000 + (12×3 = 36) % button = ₹ 97,920 (True)

So, option (C) is correct.

Q.28. A person borrows ₹ 5,000 for 2 years at 4% per annual simple interest. He immediately lends to another person at $6\frac{1}{4}\%$. Per annual for 2 years find his gain in the transaction .

(a) ₹ 112.50 (b) ₹ 225
(c) ₹ 125 (d) ₹ 107.50

[May 2018]

Solution : (b)

$$\text{Interest Gain}=\left(6\frac{1}{4}-4\right)$$

$$=2\frac{1}{4}=2.25\%$$

So, Interest Gain

$$=\frac{5000\times2\times2.25}{100}=₹\,225.$$

Q.29. A certain money doubles itself in 10 years when deposited on simple interest. It would triple itself in

(a) 30 years (b) 20 years
(c) 25 years (d) 15 years

[Nov. 2018]

Solution : (b)

Tricks : See Simple Interest (Quicker BMLRS)

$$\frac{t_2}{t_1}=\frac{x_2-1}{x_1-1}$$

$$\Rightarrow\frac{t_2}{10}=\frac{3-1}{2-1}\Rightarrow t_2=20\,yrs.$$

Q.30. A certain sum of money Q was deposited for 5 year and 4 months at 4.5% simple interest and amounted to ₹ 248, then the value of Q is

(a) ₹ 240 (b) ₹ 200
(c) ₹ 220 (d) ₹ 210

[Nov. 2018]

Solution : (b)

$$t=5\text{ yrs }4\text{ months}=5+\frac{4}{12}=\frac{16}{3}\,yrs$$

$$A=Q\left(1+\frac{rt}{100}\right)$$

$$\text{or } 248=Q\left[1+\frac{4.5}{100}\times\frac{16}{3}\right]$$

$$\therefore Q=\frac{248\times300}{372}=\text{Rs. }200$$

Tricks : GBC

Rates for 5 yrs 4 Months = 5 × 4.5 %+ one third of 4.5% = 24%

Note : 4 months means one third of one year, so rate for 4 months = one third of one year interest rate.

(*b*) Amounts = 200 + 24% = 248 (True)

So, (*b*) is correct.

Q.31. The certain sum of money became ₹ 692 in 2 yrs. and ₹ 800 in 5 yrs. then the principle amount is

(*a*) ₹ 520 (*b*) ₹ 620

(*c*) ₹ 720 (*d*) ₹ 820

[June 2019]

Solution : (*b*)

Tricks:- If a certain sum of money becomes A_1 in t_1 years and A_2 in t_2 years then

$$\text{S.I. per annum} = \frac{A_2 - A_1}{t_2 - t_1}$$

$$\therefore \text{S.I. p.a} = \frac{800 - 692}{5 - 2}$$

$= ₹ 36.$

$\therefore$ Principal = A - Interest

= 692 - Interest of 2 yrs.

$= 692 - 2 \times 36 = ₹ 620.$

(*b*) is correct.

Q.32. A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years as per S.I. then the principal is

(*a*) ₹ 3,000 (*b*) ₹ 3,500

(*c*) ₹ 3,800 (*d*) None

[June 2019]

Solution : (*c*)

Tricks:- S.I. p.a $= \frac{7400 - 6200}{3 - 2}$

$= ₹ 1200.$

$\therefore$ Principal $= 6200 - 2 \times 1200$

$= ₹ 3800.$

Q.33. P = ₹ 5,000; R = 15%; T = $4\frac{1}{2}$ using $I = \frac{PTR}{100}$ then I will be $I = \frac{Prt}{100}$

(*a*) ₹ 3,375 (*b*) ₹ 3,300

(*c*) ₹ 3,735 (*d*) None

[June 2019]

Solution : (*a*)

$$I = \frac{5000 \times 15 \times 4.5}{100} = 3375$$

[Use Calculator ; Never Write]

Q.34. In simple interest if the principal is ₹ 2,000 and the rate and time are the roots of the equation $x^2 - 11x - 30 = 0$ then simple interest is

(*a*) ₹ 500 (*b*) ₹ 600

(*c*) ₹ 700 (*d*) ₹ 800

[June 2019]

Solution : (*b*)

$\because x^2 - 11x + 30 = 0$

or $x^2 - 5x - 6x + 30 = 0$

or $x(x-5) - 6(x-5) = 0$

or $(x-5)(x-6) = 0$

$\therefore x = 5 ; 6$

If r = 5% then t = 6 yrs.

$$\therefore \text{S.I} = \frac{Prt}{100} = \frac{2000 \times 5 \times 6}{100}$$

$= ₹ 600.$

(*b*) is correct.

Q.35. If the interest of a money is equal to its one by nine, the rate of interest and time are equal then find rate of interest is.

(*a*) $3\frac{1}{3}\%$ (*b*) 4½%

(*c*) 3% (*d*) 3.5%

[Dec. 2019]

Solution : (*a*)

Let Principal = ₹ 1

$\text{S.I} = \frac{1}{9}$

Given ; r = t

$\therefore \text{S.I} = \frac{\text{P.r.t}}{100}$

or $\frac{1}{9} = 1.\frac{\text{r.r.}}{100}$

or ; $r^2 = \frac{100}{9} \Rightarrow r = \frac{10}{3} = 3\frac{1}{3}\%$

Q.36. 1/7 of a money is deposited at 4% per annum, 1/2 of a money deposited at 5% per annum and the remaining at the rate of 6%, then total interest gained ₹ 730 find deposit amount is

(*a*) ₹ 14000 (*b*) ₹ 215500

(*c*) ₹ 212800 (*d*) ₹ 214500

[Dec. 2019]

Solution : (*a*)

Calculator Tricks : GBC

For (*a*) S.I. =

$\left(\frac{1}{7} \times 14000 = 2000\right) \times 4\%$ button *i.e.* type 2000 then press multiply button then type 4 then percentage button [never press any other button]

then $\left(\frac{1}{2} \times 14000 = 7000\right) \times 5\%$ button *i.e* type 7000 then press multiply button then type 5 then percentage button and $(14000 - 2000 - 7000) \times 6\%$ button *i. e.* type 5000 then press multiply button then type 6 then percentage button

then Press GT button ; we get ₹ 730.

$\therefore$ (*a*) is correct.

Q.37. Ram deposited ₹ 12000 in a bank at 10% per annum and remaining amount deposit in other bank at 20% per annum, if he received interest according to 14% per annum find the Ram's amount.

(*a*) ₹ 20000 (*b*) ₹ 22000

(*c*) ₹ 30000 (*d*) ₹ 25000

[Dec. 2019]

Solution : (*a*)

Tricks : GBC

For (*a*) S.I on ₹ 20,000

$= 20,000 \times 14\%$

$= ₹ 2800.$

S.I. on ₹ 12,000 = 12000 × 10%

$= ₹ 1200.$

Rest principal = 20000 - 12000

$= ₹ 8000.$

S.I. on rest money = 8000× 20%

= 1600.

Sum of these S.I.

= 1200 + 1600 = 2800 (True)

$\therefore$ (*a*) is correct.

Q.38. If the difference between interest received by two persons A and B on the same sum of ₹1500 for 3 years is ₹ 18. Then what is the difference between the two rates of interest.

(*a*) 1% (*b*) 2.5%

(*c*) 3% (*d*) 0.4%

[Dec. 2019]

Solution : (*d*)

Principal same ; time same but interest differ by ₹ 18.

It means rate differs.

$$\therefore r = \frac{I \times 100}{p \times r}$$

Here ; r = difference between rates.

$$\therefore r = \frac{18 \times 100}{1500 \times 3} = 0.4\%$$

Detail : Let their rates are r_1 and r_2

$$\therefore \frac{1500 \times r_1 \cdot 3}{100} - \frac{1500 \times r_2 \cdot 3}{100} = 18$$

or $\frac{1500 \times 3}{100}(r_1 - r_2) = 18$ (is)

or $r_1 - r_2 = \frac{18}{15 \times 3} = 0.4\%$

Q.39. If the compound interest on a certain sum for 2 years at 3% p.a. is ₹ 1015. What would be the simple interest on the sum at the same rate and same time is

(*a*) 1005 (*b*) 1010

(*c*) 1000 (*d*) 1003

[Dec. 2019]

Solution : (*c*)

Tricks : GBC

For (*c*) S.I. for 2 yrs.

$= ₹ 1000.$

$\therefore$ S.I for 1 yr. = ₹ 500. *i.e.* $(1000 \div 2)$

C.I for 1st yr. = S.I of 1st yr. = ₹ 500.

C.I for 2nd yr. = S.I for 1st yr + Interest on this S.I

$$= 500 + \frac{500 \times 3 \times 1}{100} = \text{Rs. } 515.$$

$\therefore$ Total C.I = 500 + 515 = ₹ 1015 [True]

Calculator Tricks for C.I

C.I. = 500 + (500 + 5% button)

= ₹ 1015 **(True)**

Q.40. What sum of money will produce ₹ 42,800 as an interest in 3 years and 3 months at 2.5% p.a simple interest ?

(*a*) ₹ 3,78,000 (*b*) ₹ 5,26,769

(*c*) ₹ 4,22,000 (*d*) ₹ 2,24,000

[Dec. 2020]

Solution : time = *t* = 3 years & 3 months

$= 3 + \frac{3}{12} = 3.25$ years

Formula

$$P = \frac{1 \times 100}{t \times r} = \frac{42,800 \times 100}{(3.25) \times (2.5)}$$

$= ₹ 5,26,769.23 = ₹ 5,26,769/-$

(*b*) is correct.

Calculator Tricks

P = 42,800 ÷ 2.5% button ÷ (3.25)

= ₹ 5,26,769

Q.41. A certain sum amounted to ₹ 575 at 5% in a time in which ₹ 750 amounted to ₹ 840 at 4%. If the rate of interest is simple, find the sum—

(*a*) 525 (*b*) 550

(*c*) 515 (*d*) 500

[Jan. 2021]

Solution : (*d*) is correct S.I. = A – P = 840 – 750 = ₹ 90

$$t = \frac{I \times 100}{p.r.} = \frac{90 \times 100}{750 \times 4} = 3 \text{ years}$$

TRICKS

Then use GBC (Go by choices) with calculator.

[$\therefore$ 1 year interest rate = 5%

$\therefore$ 3 years interest rate = 3 × 5 = 15%]

(*a*) A = 525 + (3 × 5)% = 603.75 ≠ 575

(*a*) wrong

Amount ₹ 603.75 is more than 575

So, principal must be less than ₹ 525.

For option (*c*)

A = 515 + (3 × 5)% = 592.25 ≠ 575

So option (*d*)

A = 500 + 15% = 575 (True)

(*d*) is correct.

Q.42. A man invested one-third of his capital at 7% one fourth at 8% and the remainder at 10%. If the annual income is ₹ 561. The capital is -

(*a*) ₹ 4,400 (*b*) ₹ 5,500

(*c*) ₹ 6,600 (*d*) ₹ 5,800

[Jan. 2021]

Solution : (*c*) is correct

Tricks

Remainder

$$= 1 - \frac{1}{3} - \frac{1}{4} = \frac{12 - 4 - 3}{12} = \frac{5}{12}$$

Single rate of interest

$$= \left(7 \times \frac{1}{3} + 8 \times \frac{1}{4} + 10 \times \frac{5}{12}\right)\%$$

[Do by using calculator]

= 8.5%

Annual income = Annual interest = ₹ 561

Capital = Interest ÷ rate of interest

= 561 ÷ 8.5% (button)

= ₹ 6600

$\therefore$ (*c*) is correct

Q.43. Certain sum amounts to ₹ 15748 in 3 years at simple interest at r% p.a. The same sum amounts to ₹ 16,510 at (r+2)% p.a. simple interest in the same time. What is the value of r?

(*a*) 10% (*b*) 8%

(*c*) 12% (*d*) 6%

[July 2021]

Solution : (*b*) is correct

Due to increase in interest by 2% in 3 years

Interest increase

= 16510 – 15748

= ₹ 762

$$\text{So, } P = \frac{I \times 100}{r.t.} = \frac{762 \times 100}{2 \times 3} = ₹ 12,700$$

Then apply tricks GBC

(*a*) A = 12700 + (3 × 10% = 30%) button ≠ 15748

So (*a*) Wrong

(*b*) A = 12700 + (3 × 8% = 24%) button = ₹ 15748 (True)

So (*b*) correct

Q.44. Two equal amounts of money are deposited in two banks each at 15% p.a. S.I. fix 3.5 year in the bank and fix 5 years respectively. The difference between the interest amount from the bank is ₹ 144. Find the sum

(*a*) ₹ 620 (*b*) ₹ 640

(*c*) ₹ 820 (*d*) ₹ 840

[Jan. 2021]

Solution : (*b*) is correct

Let Each deposit = P

Tricks

Rate of interest for 3.5 years

= 3.5 × 15 = 52.5% pa S.I.

Rate of interest for 5 years = 5 × 15 = 75% pa S.I.

Difference in rate of interest

= 75 – 52.5% = 22.5% pa S.I.

∴ P = Interest difference ÷ Rate of interest difference

= 144 ÷ 22.5% (button) = ₹ 640.

Detail each

Let Principal money = P

From Question

$$\frac{P\cdot 15\times 5}{100}-\frac{P\cdot 15\times 3.5}{100}=144$$

or $\frac{P}{100}[75-52.5]=144$

or $P=\frac{144\times 100}{22.5}=₹\,640$

(*b*) is correct

Q.45. A sum of money in simple interest doubles itself in 7 years. How many years will it take to triple itself?

(*a*) End of 12 years

(*b*) End of 14 years

(*c*) End of 18 years

(*d*) End of 16 years ***[Dec. 2021]***

Solution : (*b*)

Tricks

A certain sum of money becomes x_1 times in t_1 years and x_2 times in t_2 years at same rate of S.I. The relationship is

$$\frac{t_2}{t_1}=\frac{x_2-1}{x_1-1}$$

$$\therefore \frac{t_2}{7}=\frac{3-1}{2-1}=t_2=7\times 2=14 \text{ years.}$$

Q.46. Simple interest on a sum of money is amount to ₹ 59,000 in 3 years and ₹ 62,000 in 4 years at same rate of interest. What are the principal amount and rate of interest?

(*a*) ₹ 50,000, 6%

(*b*) ₹ 45,000, 5.5%

(*c*) ₹ 55,000, 5%

(*d*) ₹ 52,000, 7% ***[Dec. 2021]***

Solution : (*a*)

Tricks

$$\text{S.I. Pa}=\frac{62000-59000}{4-3}=₹\,3,000/-$$

Principal

P = 59000 -3 × SI Pa

= 59000-3 × 3000 = ₹ 50,000/-

Rate of Interest

$$=r=\frac{I\times 100}{Pt}=\frac{3000\times 100}{50,000\times 1}$$

= 6%

Q.47. An amount is lent at R% simple interest for R years and the simple interest amount was one fourth of the principal amount. Then it is

(*a*) 5 (*b*) 6

(*c*) 5½ (*d*) 6½

[Dec. 2021]

Solution : (*a*)

Let Principal = P

$$\text{Interest}=\frac{1}{4}P$$

$$\therefore \text{rate}=\frac{1\times 100}{P.t}$$

$$\text{or } R=\frac{\frac{1}{4}P\times 100^{25}}{P\times R}$$

or; $R^2=25\Rightarrow R=5\%$

Q.48. An investor is saving to pay off an obligation of ₹ 15,250 which will be due in seven years, if the investor is earning 7.5% simple interest rate per annum, he must deposit ₹ _____ to meet the obligation.

(*a*) 8,000 (*b*) 9,000

(*c*) 10,000 (*d*) 11,000

[June 2022]

Solution : FV = ₹ 15,250

S.I rate for 1 yr = 7.5%

∴ S.I for 7 yrs = 7 × 7.5

= 52.5 %

Tricks

$$PV=P=\frac{A}{1+\frac{rt}{100}}=\frac{15250}{1+\frac{52.5}{100}}$$

$$=\frac{15250}{1.525}$$

= ₹ 10,000

∴ (*c*) is correct

Q.49. The annual rate of simple interest is 12.5%. In how many years does the principal double?

(*a*) 11 years (*b*) 9 years

(*c*) 8 years (*d*) 7 years

[June 2022]

Solution : Tricks

$$t=\frac{(m-1)\,100}{r}$$

$$\frac{(2-1)\times 100}{12.5}=8 \text{ yrs.}$$

Calculator Tricks

(2–1) ÷ 12.5% button = 8 yrs

(*c*) is correct.

Q.50. A farmer borrowed ₹ 3600 at the rate of 15% simple interest per Annum. At the end of 4 years. He cleared this account by paying ₹ 4000 and a cow. The cost of the cow is :

(*a*) ₹ 1000 (*b*) ₹ 1200

(*c*) ₹ 1550 (*d*) ₹ 1760

[Dec. 2022]

Solution: *Given :*

Let *P* = Principle = ₹ 3600

r = rate of interest = 15%

t = time = 4 years

Simple Interest

$$=\frac{p\times r\times t}{100}=\frac{3600\times 15\times 4}{100}$$

= ₹ 2160

Total amount payable after 4 years

= 3600 + 2160 = ₹ 5760

Which is equal to ₹ 4000 + cost of 1 cow

∴ Cost of 1 cow = 5760 – 4000

= ₹ 1760

∴ (*d*) is correct.

Q.51. Mr. Ram invested a total of ₹ 1,00,000 in two different banks for a fixed period. The first bank yields an interest of 9% per annum and second, 11% per annum. If the total interest at the end of one year is 9.75% per annum, then the amount invested in these banks are respectively:

(*a*) ₹ 52,500, ₹ 47,500

(*b*) ₹ 62,500, ₹ 37,500

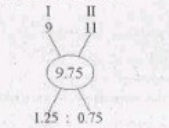
(*c*) ₹ 57,500, ₹ 42,500

(*d*) ₹ 67,500, ₹ 32,500

Solution :

Trick-I

Investment ratio =



125 : 75 + 25

5 : 3

∴ Investment in 1st bank

$$=\frac{1,00,000}{5+3}\times 5=62,500$$

$$\text{in 2nd bank}=\frac{1,00,000}{5+3}\times 3=37,500$$

(*b*) is correct.

Trick-II

GBC

Total interest = 1,00,000 × 9.75%

= ₹ 9750

(*b*) Total Interest (by Calculator)

62,500 × 9% =

37,500 × 11% = (GT button)

Press = ₹ 9,750 (True)

∴ (*b*) is correct.

8 CHAPTER COMPOUND INTEREST

Definition :- Compound interest (or compounding interest) is interest calculated on the initial principal and which also includes all of the accumulated interest of previous periods of a deposit or loan, the computed interest is called **Compound Interest.**

Conversion Period:- The period at the end of which the interest is computed is called **Conversion period.**

Description	Conversion period	No. of Conversion periods in 1 year = m	Rate of interest of a conversion period $i=\frac{r}{100m}$
10% compounded yearly	1 year	m = 1	i = 10/100
10% compounded half-yearly	6 months	m = 2	i = 10/200
10% compounded Quarterly	3 months	m = 4	i = 10/400
10% compounded monthly	1 months	m = 12	i = 10/1200

Formula Compound Amount = $A=P\left(1+\frac{r}{100m}\right)^{mt}$

$=P(1+i)^n$

Where $i=\frac{r}{100m}$ & n = mt

Compound Interest = $P\left[(1+i)^n-1\right]$

Type - I (To find Amount & Compound Interest)

Working Rule:-

(*i*) If rate of interest compounded yearly then divide r by 100 *i.e.* $i = \frac{r}{100}$.

(*ii*) If rate of interest compounded $\frac{1}{2}$ yearly Then divide r by 200 *i.e.* $i = \frac{r}{200}$.

(*iii*) If rate of interest compounded $\frac{1}{4}$ yearly then $i = \frac{r}{400}$

(*iv*) If rate of interest compounded monthly then $i = \frac{r}{1200}$ and so on.

CALCULATOR TRICKS

Step-I Type r then push ÷ button then type 100 if r is yrly.; 200 if r is half yearly and so on.

Step-II Press + 1.

Step-III Press × button

Step-IV Continue pressing "=" button (n-1 = power - 1) times.

Step -V Press × button then type principal value then push = button.

We will get the result of amount on calculator screen.

Step -VI Press - button then type principal value, we will get the value of compound Interest on the screen.

Ex.1 ₹ 2000 is invested at annual rate of interest of 10%. What is the amount after 2 years if compounding is done.

(*a*) Annually (*b*) Semi-annually

(*c*) Quarterly (*d*) Monthly.

Solution : (*a*) Given that P = ₹ 2000; r = 10% yrly.; t = 2 years.

$$i = \frac{r}{100} = \frac{10}{100} = 0.10$$

$$n = mt = 1 \times 2 = 2$$

[$\because$ m = No. of conversion periods in a year = 1]

$$\therefore A = P(1+i)^n$$

$$= 2000\left(1+\frac{10}{100}\right)^{1\times 2} = 2000(1.10)^2$$

= ₹2420.

Calculator Trick

Work As

Type 10 ÷ 100 + 1 × button then press = button (2-1=1) time then press × button type 2000 then press = button. (This is the required result).

(*b*) $i = \frac{10}{200}; m = 2$

$n = mt = 2 \times 2 = 4$

$$A = 2000\left[1+\frac{10}{200}\right]^{2\times 2} = 2000(1+0.05)^4$$

$= 2000 \times 1.21550625 = Rs.\ 2431.0125 = Rs.\ 2431.$

Calculator Trick

Type 10 ÷ 200 then + 1 then × button then press = button (4 - 1 = 3) times then type × 2000 = button (we get the required result)

[**This Trick** will take minimum 5 seconds and saves lots of time]

(*c*) $n = mt = 4 \times 2 = 8$

$$\therefore A = P(1+i)^n = 2000\left(1+\frac{10}{400}\right)^{4\times 2}$$

$$= 2000 \times [1+0.025]^8 = 2000 \times 1.2184$$

= ₹ 2436.81

Calculator Trick

Type 10 ÷ 400 + 1 × Then press = button (8-1 = 7) times then × 2000 = button (we get the result)

(*d*) $i = \frac{10}{1200}; m = 12; n = mt = 12 \times 2 = 24$

Calculator Trick

Type 10 ÷ 1200 + 1 × button then press = button (24-1 = 23) times then press × 2000 = button (we get the result) = ₹ 2440.78.

Ex. 2 Determine the compound amount and compound interest on ₹1000 at 6% compounded semi-annually for 6 years. Given that $(1+i)^n = 1.42576$ for $i = 3\%$ and n = 12.

(*a*) ₹ 1425.76; ₹425.76 (*b*) ₹1420.76 ; ₹420.76

(*c*) ₹1525.76 ; ₹525.76 (*d*) None

Solution : (*a*) is correct.

Given that P = ₹ 1000; t = 6 years

r = 6 % compounded half -yearly

$$\therefore m = 2\ ;\ i = \frac{6}{200} = 0.03$$

$$n = mt = 2 \times 6 = 12$$

$$\therefore A = 1000(1+0.03)^{12} = 1000 \times 1.42576$$

= ₹ 1425.76

C.I = A - P = 1425.76 - 1000 = ₹ 425.76

[Note:- If $(1+i)^n$ value is given in the question then use that given value]

Calculator Trick [If values not given]

Type 6 ÷ 200 + 1 × Then press = button (12 - 1 = 11) time then × 1000 = button ; we will get the required result = ₹1425.76 = Amount. Press - 1000 = button we will get compound interest value.

EFFECTIVE RATE OF INTEREST

Type - II

The equivalent annual rate of interest compounded annually if interest is compounded more than once in a year is called EFFECTIVE RATE of INTEREST. It is denoted by E or r_e.

Formula

$$r_e = E = \left[\left(1+\frac{r}{100m}\right)^m - 1\right] \times 100 = (1+i)^m - 1$$

where r = Nominal rate of interest, m = No. of conversion periods in a year.

Calculator Trick

Type r ÷ 100m then push × button then push = button (m - 1) times then - 1 then push × 100 = we get r_e in %.

Ex- Find effective rate of interest of the nominal rate of interest 6% compounded quarterly.

Solution : r = 6% compounded quarterly.

m = 4

$$\therefore r_e = \left[\left(1+\frac{6}{400}\right)^4 - 1\right] \times 100 = 6.136355062 \quad \% = 6.14\ \%$$

Calculator Trick

Type 6 ÷ 400 + 1 × button then press = button (4 - 1 = 3) times then - 1 then × 100 then press = button we get the required result.

Ex- Find effective rate of interest of 12% compounded monthly.

Solution : r = 12% compounded monthly, m = 12

$$\therefore r_e = \left[\left(1+\frac{12}{1200}\right)^{12} - 1\right] \times 100\% = 12.6825\%$$

Type 12 ÷ 1200 + 1 × button then press = button 11 times then - 1 then × 100 = button. [we get the required result]

Type-III (To find Present Value)

$\because\ A = P(1+i)^n$

$$or\ P = \frac{A}{(1+i)^n}$$

$$or\ P = A(1+i)^{-n}$$

Calculator Trick

Step-I Type i = r ÷ 100 m + 1

Step-II Press ÷ button

Step-III Press = buttons n times *i.e.* (power times)

Step-IV Press × button then type A value then press = button (we get the required result)

Ex- A certain sum invested at 4% per annum compounded semi annually amounts to ₹78030 at the end of one year. Find the sum.

(*a*) ₹75000 (*b*) ₹70,000 (*c*) ₹72000 (*d*) None

Solution : option (*a*) is correct

A = ₹78030; r = 4% compounded semi - annually

$m = 2,\ t = 1$ yr.

$n = mt = 2 \times 1 = 2$

$$P = A(1+i)^{-n} = 78030\left(1+\frac{4}{200}\right)^{-2}$$

$= 74999.999.... = ₹\ 75000$

Calculator Trick

Type 4 ÷ 200 + 1 then press ÷ button then press = button (power = 2) times then press × button then type A value = 78030 then press = button we get the required result.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. The difference between the simple and compound interest on a certain sum for 3 year at 5% p.a. is ₹ 228.75. The compound interest on the sum for 2 years at 5% p.a. is :

(*a*) ₹ 3,175 (*b*) ₹ 3,075
(*c*) ₹ 3,275 (*d*) ₹ 2,975

[Nov. 2006]

Solution : Tricks

$$P = \frac{Difference \times (100)^3}{r^2(300+r)}$$

[For 3 years only]

$$= \frac{228.75 \times (100)^3}{5 \times 5(300+5)} = ₹\ 30{,}000$$

A = 30000 + 5 % + 5% buttons = ₹ 33075

∴ C.I = A - P = ₹ 33075 - 30000

= ₹ 3,075

∴ (*b*) is Correct

Q.2. In what time will ₹ 3,90,625 amount to ₹ 4,56,976 at 8% per annum, when the interest is compounded semi-annually ? [Given: $(1.04)^4 = 1.16986$]

(*a*) 2 years (*b*) 4 years
(*c*) 5 years (*d*) 7 years

[Feb. 2007]

Solution : (*a*)

$$A = P\left(1+\frac{r}{100m}\right)^{mt}$$

$$\frac{4,56,976}{3,90,625} = \left(1+\frac{8}{200}\right)^{2t}$$

or $1.16985856 = (1.04)^{2t}$

or $1.16966 = (1.04)^{2t}$

or $(1.04)^4 = (1.04)^{2t}$

∴ $2t = 4$ ∴ $t = 2$ years

∴ (a) is correct

Q.3. How long will ₹12,000 take to amount to ₹14,000 at 5% p.a. converted quarterly? [Given: $(1.0125)^{12.4} = 1.1666$]

(*a*) 3 years (*b*) 3.1 years
(*c*) 13.5 years (*d*) 12.4 years.

[May 2007]

Solution : (*b*) $\frac{A}{p} = \left(1+\frac{5}{400}\right)^{4t}$

or $\frac{14000}{12000} = (1.0125)^{4t}$

or $1.16666..... (1.0125)^{4t}$

or $(1.0125)^{12.4} = (1.0125)^{4t}$ [Note :- Always use values given in question]

or $4t = 12.4$ ∴ $t = 3.1$yrs.

∴ (*b*) is correct

Q.4. If ₹ 1,000 be invested at interest rate of 5% and the interest be added to the principal every 10 years, then the number of years in which it will amount to ₹ 2,000 is :

(*a*) $16\frac{2}{3}$ years (*b*) $\frac{1}{10}$ years

(*c*) 16 years (*d*) $6\frac{2}{3}$ years

[Aug. 2007]

Solution : Given P = ₹ 1000; m = $\frac{1}{10}$;

$n = mt = \frac{1}{10} \times t = 0.1t$

r = 5% p.a.

$$A = P\left(1+\frac{5}{0.1\times100}\right)^{0.1t}$$

$$\frac{2000}{1000} = (1.50)^{0.1t}$$

or $2 = (1.5)^{0.1t}$

or $0.1t = \frac{\log 2}{\log(1.5)}$

or $0.1t = 1.709$ or $t = \frac{1.709}{0.1}$

$= 17.09 = 16\frac{2}{3}$, (a) is correct

Q.5. The annual birth and death rates per 1000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is :

(*a*) 35 years (*b*) 30 years
(*c*) 25 years (*d*) None of these

[Aug. 2007]

Solution : (*a*) is correct

Birth rate	Per 1000	Per 100
	39.4	3.94
Death rate	19.4	1.94

Population increase 2.00%

Population increase = 2.%

$$\frac{A}{P} = 2 = (1+0.02)^t$$

Calculator Trick

Type 1.02 Then push × button then continue pressing = button until to get 2. No. of pressings of = button is 34 times. So, t = 35 years (approx.)

Q.6. The effective rate equivalent to nominal rate of 6% compounded monthly is:

(*a*) 6.05 (*b*) 6.16
(*c*) 6.26 (*d*) 6.07

[Aug. 2007]

Solution : (*b*)

$$r_e = \left[\left(1+\frac{r}{100m}\right)^m - 1\right] \times 100$$

$$= \left[\left(1+\frac{6}{1200}\right)^{12} - 1\right] \times 100 = 6.16\%$$

option (b) is correct.

Q.7. A person deposited ₹ 5,000 in a bank. The deposit was left to accumulate at 6% compounded quarterly for the first five years and at 8% compounded semiannually for the next eight years. The compound amount at the end of 13 years is :

(*a*) ₹ 12621.50 (*b*) ₹ 12613.10
(*c*) ₹ 13613.10 (*d*) None

[Nov. 2007]

Solution :

Calculator Tricks :-

$$A = 5000\left(1+\frac{6}{400}\right)^{5\times4}\left(1+\frac{8}{200}\right)^{8\times2}$$

₹12613.17 = ₹ 12610.00 (approx)

∴ (*b*) is correct.

Q.8. Anshul's father wishes to have ₹ 75,000 in a bank account when his first college expenses begin. How much amount his father should deposit now at 6.5% compounded annually if Anshul is to start college in 8 years hence from now?

(*a*) ₹ 45,320 (*b*) ₹ 46,360
(*c*) ₹ 55,360 (*d*) ₹ 48,360.

[Feb. 2008]

Solution : (*a*)

Calculator Tricks

$P = A(1+i)^n$

$$= 75000\left(1+\frac{6}{100}\right)^{-8}$$

Calculator Tricks:

Type 6 ÷ 100 + 1 ÷ button

then press = button 8 times

× 75000 = button

we get ₹ 45,317 ≅ 45320

(*a*) is correct.

Q.9. The difference between compound interest and simple interest on a certain sum for 2 years @ 10% p.a. is ₹ 10. Find the sum:

(*a*) ₹ 1,010 (*b*) ₹ 1,095
(*c*) ₹ 1,000 (*d*) ₹ 990

[June 2008]

Solution : (*c*) **Tricks**

$$P = \frac{\text{Actual (C.I - S.I.)}}{\text{(C.I - S.I.) at Re.1}}$$

$$P = \frac{10}{[(1.10)^2 - 1] - 0.2} = ₹\ 1000$$

∴ (*c*) is correct

Calculator Tricks :- For 2 years. P = 10 ÷ 10% ÷ 10% button = ₹ 1000.

Q.10. A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,000 :

(*a*) 5 years 6 months
(*b*) 5 years 7 months
(*c*) 5 years 5 months
(*d*) None

[June 2008]

Solution : (*a*) is correct

Tricks :

$$t = \frac{\log(2,00,000/4,90,740)}{\log(1-15/100)}$$

= 5.5 years (approx.)

= 5 yrs. 6 months

Note:

Calculator Tricks : Type 0.85 √button 19 times −1 × 227695 = button then press M+ then type 200,000 ÷ 4,90,740 = button then √button 19 times −1 × 227695 = button then ÷ MRC button = button we get answer.

Q.11. If the difference between simple interest and compound interest is ₹ 11 at the rate of 10% for two years, then find the sum:

(*a*) ₹ 1,200 (*b*) ₹ 1,100
(*c*) ₹ 1,000 (*d*) None of these

[Dec. 2008]

Solution : (*b*) is correct

Tricks $P = \frac{Difference \times (100)^2}{(rate)^2}$

$$= \frac{11\times(100)^2}{(10)^2} = ₹\ 1100$$

Calculator Tricks :- P = 11 ÷ 10% ÷ 10% button = ₹ 1100

Q.12. In how many years, a sum will become double at 5% p.a. compound interest.

(*a*) 14.0 years (*b*) 14.1 years
(*c*) 14.2 years (*d*) 14.3 years

[June 2009]

Solution : (*c*) is correct

Tricks $t = \frac{Log(A/P)}{me\log(1+r/100m)}$

$$= \frac{\log 2}{\log(1.05)} = 14.2 yrs.\ (approx)$$

Q.13. A sum amount to ₹ 1331 at a principal of ₹1,000 at 10 % compounded annually. Find the time.

(*a*) 3.31 years (*b*) 4 years
(*c*) 3 years (*d*) 2 years

[June 2009; Dec. 2009]

Solution : (*c*) is correct

Tricks :- Go by choices

For (*c*) ; $A = 1000\left(1+\frac{10}{100}\right)^3 = ₹\ 1331.$

So; t = 3 yrs.

Calculator Tricks :- GBC

(*c*) A = 1000 + 10% + 10% + 10% button = ₹ 1,331

Q.14. The compound interest for a certain sum @ 5% p.a. for first years is ₹25. The S-I for the same money @ 5% p.a. for 2 years will be.

(*a*) ₹40 (*b*) ₹50
(*c*) ₹60 (*d*) ₹70

[Dec. 2009]

Solution : (*b*) is correct

Tricks :-

S.I For 1st yrs. = C . I for 1st yrs. = ₹25

S.I For 2 yrs. For same 'p' = 2 × 25 = ₹ 50

Q.15. At what % rate of compound interest corresponding (C.I) will a sum of money became 16 times in four years, if interest is being calculated compounding annually:

(*a*) r = 100% (*b*) r = 10%
(*c*) r = 200% (*d*) r = 20%

[June 2010]

Solution : (*a*) is correct

Tricks :- Go by choices

For (*a*) Let P = 1; $A = 1\left(1+\frac{100}{100}\right)^4 = (2)^4 = 16$

∴ (*a*) is correct

Q.16. If the simple interest on a sum of money at 12% p.a. for two years is ₹3,600. The compound interest on the same sum for two years at the same rate is:

(a) ₹ 3,816 (b) ₹ 3,806
(c) ₹ 3,861 (d) ₹ 3,860

[June 2010]

Solution : (a) is correct

$$P = \frac{3600\times100}{12\times2} = ₹\ 15000$$

$$\therefore c.i = 15000\left(1+\frac{12}{100}\right)^2 - 15000$$

$$= ₹\ 3816.$$

Tricks:-

CI for 1st yr. = SI for 1st year = 3600 ÷ 2 = ₹ 1800

CI for 2nd year = 1800 + 1800 × 12% = ₹ 2016

∴ C.I for 2 years = 1800 + 2016 = ₹ 3816.

Q.17. The effective annual rate of interest corresponding to nominal rate 6% p.a. payable half yearly is

(a) 6.06% (b) 6.07%
(c) 6.08% (d) 6.09%

[Dec. 2010]

Solution : (d) is correct

$$r_e = \left[\left(1+\frac{6}{200}\right)^2 - 1\right]\times100 = 6.09\%$$

Q.18. The cost of Machinery, is ₹1,25,000/- If its useful life is estimated to be 20 years and the rate of depreciation of its cost is 10% p.a., then the scrap value of the Machinery is (given that $(0.9)^{20} = 0.1215$)

(a) 15,187 (b) 15,400
(c) 15,300 (d) 15,250

[Dec. 2010]

Solution : (a) is correct

$$S\ (\text{Scrap Value}) = P\left(1-\frac{d}{100}\right)^t$$

where P = Principal;

d = rate of depreciation

$$\therefore S = 1,25,000\left(1-\frac{10}{100}\right)^{20}$$

$$= ₹15,187.50$$

Q.19. Mr. X invests 'P' amount at Simple Interest rate 10% and Mr. Y invests 'Q' amount at Compound Interest rate 5% compounded annually. At the end of two years both get the same amount of interest, then the relation between two amounts P and Q is given by:

(a) $p = \frac{41Q}{80}$ (b) $p = \frac{41Q}{40}$

(c) $p = \frac{41Q}{100}$ (d) $p = \frac{41Q}{200}$

[Dec. 2010]

Solution : (a) is correct

$$S.I = \frac{P.10\times2}{100} = \frac{P}{5}$$

$$C.I = Q\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$$

$$= 0.1025.Q$$

From Question

S.I = C.I

$$\frac{P}{5} = 0.1025Q$$

or $P = 5\times0.1025Q = 0.5125Q$

$$\therefore P = \frac{5125}{10000}Q = \frac{205Q}{400} = \frac{41Q}{80}$$

$$\therefore P = \frac{41Q}{80}$$

Tricks :- GBC

Q.20. If the difference of S.I and C.I is ₹ 72 at 12% for 2 years. Calculate the amount.

(a) 8,000 (b) 6,000
(c) 5,000 (d) 7,750

[June 2011]

Solution : (c) is correct

Tricks: $P = \frac{(C.I - S.I)\times(100)^2}{r^2}$

$$= \frac{72\times100\times100}{12\times12} = ₹\ 5000$$

Calculator Tricks :- P = 72 ÷ 12% ÷ 12% = 5000

Q.21. Nominal rate of interest is 9.9% p.a. If interest is Compounded monthly, What will be the effective rate of interest

(Given $\left(\frac{4033}{4000}\right)^{12} = 1.1036$ (approx))?

(a) 10.36% (b) 9.36%
(c) 11.36% (d) 9.9%

[Dec. 2011, June 2012]

Solution : (a) is correct.

Tricks:-

$$r_e = \left[\left(1+\frac{9.9}{1200}\right)^{12} - 1\right]\times100$$

= 10.36 %

Q.22. The difference between CI and SI on a certain sum of money for 2 years at 4% per annum is ₹1. The sum is

(a) 625 (b) 630
(c) 640 (d) 635

[June 2013]

Solution : (a) is correct

Tricks :- For 2 yrs

$$\text{Sum of Money} = \frac{Diff.(100)^2}{r^2}$$

$$= \frac{1\times(100)^2}{4^2} = ₹\ 625$$

Calculator Tricks:- P = 1 ÷ 4% ÷ 4% button = ₹ 625.

Q.23. If the sum of money when compounded annually become 1140 in 2 years and 1710 in 3 years at rate of interest

(a) 30% (b) 40%
(c) 50% (d) 60%

[June 2013]

Solution : (c) is correct.

Interest in 3rd yr. = ₹ 1710 - ₹ 1140 = ₹ 570

Tricks Note :- For 3rd yr ; it will be like S.I

$$r = \therefore \frac{1\times100}{PI} = \frac{570\times100}{1140\times1} = 50\%$$

Tricks II Go by choices.

For (c) A = 1140 + 50% (Calculator)

= ₹ 1710

∴ (c) is correct

Q.24. The difference between and C.I & S.I at 7% p.a. for 2 years is ₹29.4 then principal is

(a) ₹5,000 (b) ₹5,5000
(c) ₹6,000 (d) ₹6,500

[Dec. 2013]

Solution : (c) is correct

Tricks $P = \frac{Difference\times(100)^2}{r^2}$

$$= \frac{29.4\times(100)^2}{(7)^2} = ₹\ 6000.$$

Calculator Tricks :- P = 29.4 ÷ 7% ÷ 7% button = ₹ 6000

Q.25. The Partners A & B together lent ₹3903 at 4% p.a. interest compounded annually. After a spam of 7 years, A gets the same amount as B gets after 9 years. The share of A in the sum of ₹3903 would have been

(a) ₹1875 (b) ₹2280
(c) ₹2028 (d) ₹2820

[June 2014]

Solution : (c) is correct

$$A\left(1+\frac{4}{100}\right)^7 = B\left(1+\frac{4}{100}\right)^9$$

$$\text{or } \frac{A}{B} = \left(1+\frac{4}{100}\right)^2 = \left(\frac{26}{25}\right)^2$$

$$= \frac{676}{625}$$

A : B = 676 : 625

$$A = \frac{676}{676+625}\times3903 = ₹\ 2028$$

Tricks:- GBC

Q.26. A certain sum of money double itself in 4 years at C.I. In how many years it will become 32 times to itself

(a) 15 years (b) 24 years
(c) 20 years (d) None

[Dec. 2014]

Solution : (c) is correct

Tricks:- $2^{t_2} = 32^4$

$= 2^{t_2} = (2^5)^4 = 2^{20}$

$= t_2 = 20$ yrs.

Q.27. On a certain sum rate of interest @ 10% p.a., S.I = ₹ 90 Term = 2 year, Find Compound interest for the same :

(a) ₹ 544.5 (b) ₹ 94.5
(c) ₹ 450 (d) ₹18

[Dec. 2015]

Solution : (b) is correct

$$S.I.\ p.a = \frac{90}{2} = ₹45$$

Tricks: Compound interest

= 45+(45+10%) = ₹ 94.5

Q.28. If an amount is kept at simple interest, it earns ₹ 600 in first 2 years but when kept at Compound interest it earns at interest of ₹ 660 for the same period; then the rate of interest and principle amount respectively are

(a) 20%; ₹1200 (b) 10%; ₹1200
(c) 20%; ₹1500 (d) 10%; ₹1500

[June 2016]

Solution : (c)

Tricks:- Go by choices

(c) $S.I. = \frac{1500\times2\times20}{100} = ₹\ 600$ (True)

$$C.I = 1500\left[\left(1+\frac{20}{100}\right)^2 - 1\right] = ₹\ 660$$ (also True)

(c) is correct

Q.29. Mr. X bought an electronic item for ₹1000. What would be the future value of the same item after two years, if the value is compounded semi-annually at the rate of 22% per annum ?

(a) ₹1488.40 (b) ₹1518.07
(c) ₹2008.07 (d) ₹2200.00

[June 2016]

Solution : (b) is correct

$$FV = P\ (1+i)^n$$

$$= 1000\left(1+\frac{22}{200}\right)^{2\times2}$$

= ₹ 1518.07 (approx.)

Q.30. The difference between the simple interest and compound interest on a certain sum of money invested for 2 years at 5% p.a. is ₹30. Then the sum =

(a) 10,000 (b) 12,000
(c) 13,000 (d) None

[Dec. 2016]

Solution : (b)

Calculator Tricks:

P = 30 ÷ 5% ÷ 5% button = ₹12,000

Q.31. A sum of money amounts to ₹7803 for one year at the rate of 4% compounded semi-annually then the sum invested is

(a) 7,000 (b) 7,500
(c) 7,750 (d) 8,000

[Dec. 2016]

Solution : (b)

$$P = 7803\left[1+\frac{4}{200}\right]^{-2}$$

Calculator Tricks:

P = (4 ÷ 200 +1) ÷ = button 2 times × 7803 = button

= ₹ 7500

Tricks : (b) (GBC) → A = 7500 + 2% + 2% button = 7803.

Q.32. The difference between simple and compound interest on a sum of ₹ 10000 for 4 years at the rate of interest 10% per annum is _____

(a) 650 (b) 640
(c) 641 (d) 600

[June 2017]

Solution : C.I – S.I

$$= \left[10,000\left(1+\frac{10}{100}\right)^4 - 10,000\right] - \left[\frac{10,000\times10\times4}{100}\right]$$

= 4641 - 4000 = ₹ 641.

option (c) is correct. [Note :- Do by Calculator]

Q.33. If the compound interest on a sum for two year at the rate 5% p.a. is ₹512.50, then the principal is _____:

(a) 4,000 (b) 3,000
(c) 5,000 (d) None of these

[Dec. 2017]

Solution : (c)

Tricks:- GBC

Amount = 5000 + 5% + 5% button = 5512.50.

C.I = 5512.50 - 5000 = ₹ 512.50.

Q.34. Find effective rate of interest corresponding to the nominal rate of interest 7% compounded monthly is _____ :

(a) 7.26 % (b) 7.22 %

(c) 7.02 % (d) 7.20 %

[Dec. 2017]

Solution : (b)

$$r_e = \left[\left(1+\frac{7}{1200}\right)^{12} - 1\right] \times 100\%$$

$= 7.229\% = 7.22\%$.

Q.35. In compound interest, if the amount is 9 times to its principle in two years then the rate of interest is ?

(a) 300% (b) 200%

(c) 150% (d) 100%

[June 2018]

Solution : (b) Given,

$$A = P\left(1+\frac{r}{100}\right)^t$$

$$\text{or; } 9P = P\left(1+\frac{r}{100}\right)^2$$

$$\text{or; } 9 = \left(1+\frac{r}{100}\right)^2$$

$$\text{or; } 3^2 = \left(1+\frac{r}{100}\right)^2 \Rightarrow 3 = 1+\frac{r}{100}$$

$$\Rightarrow 2 = \frac{r}{100} \Rightarrow r = 200\%$$

Tricks : - 1 + 200 % + 200 % = 9

So, (b) is correct.

Q.36. If difference between Compound Interest and Simple Interest for 3 years is ₹ 912 at the rate 4 % p.a., the principal is

(a) ₹1,87,500 (b) ₹1,87,000

(c) ₹1,87,550 (d) ₹1,85,700

[June 2018]

Solution : (a)

Tricks :-

$P = 912 \div 4\% \div 4\% \div (300 + 4)\%$

= ₹ 1,87,500

Q.37. If Rs. 1,000 be invested at interest at interest rate of 5% and the interest be added to the principal every 10 years, than the number of years in which it will amount to Rs. 2,000 is :

(a) $16\frac{2}{3}$ years (b) $6\frac{1}{4}$

(c) 16 years (d) $6\frac{2}{3}$

[May 2018]

Solution : (a)

∵ Interest is added to the principal every 10 years. So, within 10 years ; simple interest will apply.

So, Amount after 10 yrs.

$$= 1000 + 1000 \times \frac{10 \times 5}{100}$$

= Rs. 1500.

Total amount = Rs. 2000

Extra Interest needed = 2000 - 1500 = Rs. 500.

$$\text{Time} = \frac{500 \times 100}{1500 \times 5} = \frac{20}{3}$$

$$= 6\frac{2}{3} \text{ yrs.}$$

So; Total time $= 10 + 6\frac{2}{3}$

$$= 16\frac{2}{3} \text{ yrs.}$$

Q.38. If an amount is kept at S.I. it earns an interest of Rs. 600 in first two years but when kept at compound interest it earns an interest of Rs. 660 for the same period, then the rate of interest and principal amount respectively are :

(a) 20%., Rs. 1,200

(b) 20%, Rs. 1,500

(c) 10%, Rs. 1,200

(d) 10%., Rs. 1,500

[May 2018]

Solution : (b)

Tricks:- Go by choices (GBC)

$$\text{(a) S.I} = \frac{1200 \times 2 \times 20}{100} = 480 \neq 600$$

So; (a) is false.

$$\text{(b) S.I} = \frac{1500 \times 2 \times 20}{100} = \text{Rs. } 600$$

C.I = (−1500 + 20% + 20%) (button)

= 660.

So; (b) is True.

Q.39. If ₹10,000 is invested at 8% per year compound quarterly, then the value of the investment after 2 years is [Given $(1 + 0.2)^8 = 1.171659$]

(a) ₹ 10,716.59 (b) ₹ 11,716.59

(c) ₹ 117.1659 (d) None of these

[Nov. 2018]

Solution : (b)

$$FV = 100000\left(1+\frac{8}{400}\right)^{2\times4}$$

= ₹ 11716.59

Q.40. A bank pays 10% rate of interest, interest being calculated half yearly. A sum of ₹ 400 is deposited in the bank. The amount at the end of 1 years will be

(a) ₹ 439 (b) ₹ 440

(c) ₹ 442 (d) ₹ 441

[Nov. 2018]

Solution : (d)

$$FV = 400\left(1+\frac{10}{200}\right)^2 = 441$$

Calculator Tricks :-

FV = 400 + 5 % + 5 % = 441

Q.41. A men deposited ₹ 8,000 in a bank for 3 years at 5% per annum compound interest, after 3 years he will get

(a) ₹ 9,000 (b) ₹ 8,800

(c) ₹ 9,200 (d) ₹ 9,261

[Nov. 2018]

Solution : (d)

$$FV = 8000\left(1+\frac{5}{100}\right)^3 = ₹\ 9261.$$

Calculator Tricks :

FV = 8000 + 5 % + 5 % + 5 % buttons = 9261

Q.42. If in two years time a principal of ₹ 100 amounts to ₹ 121 when the interest at the rate of r % is compounded annually, then the value of r will be

(a) 14 (b) 10.5

(c) 15 (d) 10

[Nov. 2018]

Solution : (d)

Details :-

$$121 = 100\left(1+\frac{r}{100}\right)^2 \Rightarrow \frac{121}{100} = \left(1+\frac{r}{100}\right)^2$$

$$\text{or } \left(\frac{11}{10}\right)^2 = \left(1+\frac{r}{100}\right)^2 \Rightarrow 1+\frac{r}{100} = \frac{11}{10}$$

$$\text{or } \frac{r}{100} = \frac{11}{10} - 1 = \frac{1}{10}$$

$\therefore r = 10\%$

I Tricks :- GBC

$$\text{for FV} = 100\left(1+\frac{10}{100}\right)^2 = 121$$

(True)

II Calculator Tricks :-

FV = 100 + 10 % + 10 % buttons = 121

Q.43. The effective rate of interest for one year deposit corresponding to a nominal 7% rate of interest per annum convertible quarterly is

(a) 7% (b) 7.4%

(c) 7.5% (d) 7.18%

[Nov. 2018]

Solution : (d)

$$r_e = \left[\left(1+\frac{7}{400}\right)^4 - 1\right] \times 100 = 7.18\%$$

Q.44. How much will ₹ 25,000 amount to in 2 years at compound interest if the rates for the successive years are 4% and 5% per year

(a) ₹ 27,000 (b) ₹ 27,300

(c) ₹ 27,500 (d) ₹ 27,900

[Nov. 2018]

Solution : (b)

$$FV = 25000\left(1+\frac{4}{100}\right)\times\left(1+\frac{5}{100}\right)$$

= ₹ 27,300/-

Calculator Tricks:- 25000 + 4% + 5% buttons

= ₹27,300/-

Q.45. ₹ 8,000/- at 10% per annum interest compounded half yearly will become at the end of one year

(a) ₹ 8,800 (b) ₹ 8,900

(c) ₹ 8820 (d) ₹ 9,600

[Nov. 2018]

Solution : (c)

$$FV = 8000\left(1+\frac{10}{200}\right)^2 = ₹\ 8,820$$

Calculator Tricks :-

FV = 8000 + 5% + 5% buttons = 8820

Q.46. The value of furniture depreciates by 10% a year, if the present value of the furniture in an office is ₹ 21870, calculate the value of furniture 3 years ago

(a) ₹ 30,000 (b) ₹ 40,000

(c) ₹ 35,000 (d) ₹ 50,000

[Nov. 2018]

Solution : (a)

Calculator Tricks :- GBC

(a) 30000 - 10% - 10% - 10% button = 21870.

Details Method

$$21870 = P\left(1+\frac{10}{100}\right)^3$$

$$\therefore P = \frac{21870}{(0.9)^3} = ₹\ 30,000$$

Q.47. If compound interest on a sum for 2 years at 4% per annum is ₹ 102, then the simple interest on the same period at the same rate will be

(a) ₹ 90 (b) ₹ 100

(c) ₹ 101 (d) ₹ 93

[Nov. 2018]

Solution : (b)

Details :- $C.I = P\left(1+\frac{4}{100}\right)^2 - P = 102$

$$\text{or } P\left[(1.04)^2 - 1\right] = 102$$

$$\text{or } P \times 0.0816 = 102$$

$$\text{or } P = \frac{102}{0.0816} = 1250$$

$$\therefore \text{S.I.} = \frac{p.r.t}{100} = \frac{1250 \times 4 \times 2}{100} = ₹\ 100$$

Tricks :- Go by choices

For option (b)

S.I. for 2 years = ₹ 100

∴ S.I. for 1 years = ₹ 50

S.I. of 1st yr. = C.I. of 1st yr. = ₹ 50

C.I. for 2nd yr. = 50 + 4% = ₹ 52

Total C.I. for 2 yrs = 50 + 52 = ₹ 102 (True)

∴ Option (b) is correct

Q.48. If the difference between the compound interest compounded annually and simple interest on a certain amount at 10% per annum for two years is ₹ 372, then the principal amount is

(a) ₹ 37,000

(b) ₹ 37,200

(c) ₹ 37,500

(d) None of the above

[Nov. 2018]

Solution : (b)

Tricks :-

$P = 372 \div 10\% \div 10\% = ₹\ 37,200$

Q.49. What is the net present value of piece of property which would be valued at ₹ 2 lakh at the end of 2 years? (Annual rate of increase = 5%)

(a) ₹ 2.00 lakh

(b) ₹ 1.81 lakh

(c) 2.01 lakh

(d) None of the above

[Nov. 2018]

Solution : (b)

$NPV = 2\left(1+\frac{5}{100}\right)^{-2} = 1.81$ lakh (approx)

Q.50. A sum was invested for 3 years as per C.I. and the rate of interest for first year is 9%, 2nd year is 6% and 3rd year is 3% p.a. respectively. Find the sum if the amount in three years is ₹ 550 ?

(a) ₹ 250 (b) ₹ 300
(c) ₹ 462.16 (d) ₹ 350

[June 2019]

Solution :

Tricks :- GBC

(c) 462.16 on Calculator, do as Type 462.16 + 9% + 6% + 3% button we get 550. So (c) is correct.

Q.51. The effective rate of interest does not depend upon

(a) Amount of Principal
(b) Amount of Interest
(c) Number of Conversion Periods
(d) None of these

[June 2019]

Solution : (a) is correct

Q.52. If p.i^2 = 96 , and R = 8% compounded annually then P =

(a) ₹ 14,000 (b) ₹ 15,000
(c) ₹ 16,000 (d) ₹ 17,000

[June 2019]

Solution : (b)

Tricks :- Given , $p.i^2 = 96$

Means interest of two periods (yrs. here) is 96.

So; GBC (Calculator Tricks)

(a) I = 14000×8%×8%

= 89.6 ≠ 96

So; (a) is False.

(b) Type 15000×8%×8% button

We get 96.

So, (b) is correct.

Tricks II

P = 96 ÷ 8% ÷ 8% buttons.

= ₹ 15,000.

Q.53. The present value of a scooter is ₹ 7290. The rate of depreciation is 10%. What was its value 3 years ago?

(a) 10,000 (b) 10010
(c) 9990 (d) 12000

[Dec. 2019]

Solution : (a)

Calculator Tricks : GBC

after 3 years;

PV. = 7290.

For option (a), Type 10,000 - 10% button - 10% button -10% button ;

we get ₹ 7290.

∴ (a) is correct.

Q.54. The difference between compound interest, compounded semi annually and simple interest on ₹ 400 at 10% p.a. for one year.

(a) ₹ 1 (b) ₹ 28
(c) ₹ 35 (d) ₹ 40

[Dec. 2019]

Solution : (a)

In C.I. 10% p.a compounded semi-annually ⇒ (10/2) = 5% interest in 6 months.

There are 2 periods in 1 yr.

∴ FV = 400 + 5% + 5% button = 441

∴ C.I = 441 - 400 = 41.

S.I = 400×10% = 40 for 1 yr.

∴ C.I - S.I = 41 - 40

= ₹ 1.

[**Another Trick:** Difference = 400 × 5% × 5% = ₹ 1]

Q.55. In how much time the S.I. on a certain sum becomes 0.125 times to its principle at 10% p.a. is

(a) 1.00 yrs (b) 1.25 yrs
(c) 1.50 yrs (d) 2.00 yrs

[Dec. 2019]

Solution : (b)

Let principal = ₹ 1

∴ S.I = 0.125

$\therefore t = \frac{I\times100}{p.r} = \frac{0.125\times100}{1\times10}$

= 1.25 yrs

Q.56. In what time will a sum ₹ 800 amounts to ₹ 882 at 5% p.a. compounded annually

(a) 1 yrs (b) 2 yrs
(c) 3 yrs (d) 4 yrs

[Dec. 2019]

Solution : (b)

Tricks : GBC

for (b) FV = Amounts = 800 + 5% + 5% button = ₹ 882

∴ (b) is correct.

Q.57. Find the effective rate of interest if an amount of ₹ 30,000 deposited in a bank. For 1 year at the rate of 10% p.a. compounded semi annually.

(a) 10.05% (b) 10.10%
(c) 10.20% (d) 10.25%

[Dec. 2019]

Solution : (d)

Here, No need of Principals value.

Formula

∴ Effective rate of interest

$= \left[(1+1)^m - 1\right]\times100$

$r_e = \left[\left(1+\frac{10}{200}\right)^2 - 1\right]\times100$

Calculator Tricks

Type 10 + 200 + 1×button = button 1 time -1× 100 = button

We get 10.25%

∴ r_e = 10.25%

Q.58. The present population of a town is 25,000. If it grows at the rate of 4%, 5%, 8% during 1st year, 2nd year, 3rd year respectively. Then find the population after 3 years.

(a) 29,484 (b) 29,844
(c) 29,448 (d) 28,944

[Dec. 2019]

Solution : (a)

Tricks : Population after 3 yrs

= 25000 + 4% + 5% + 8% buttons

= 29484.

Q.59. An amount ₹ 35000 with the rate of interest is 7% per annum, it is compounded on a monthly basis, then tell the effective rate of interest.

(a) 7.22% (b) 7.64%
(c) 7.0% (d) 7.5%

[Dec. 2019]

Solution : (a)

$r_e = \left[\left(1+\frac{7}{1200}\right)^{12} - 1\right]\times100$

Calculator Tricks

Type 7 ÷ 1200 + 1×button = button 11 times -1× 100 = button

We get 7.229% = 7.22% (approx.)

Q.60. On what sum will the compound interest at 5% p.a for 2 years compounded annually be ₹ 3,280

(a) ₹ 16,000 (b) ₹ 32,000
(c) ₹ 48,000 (d) ₹ 64,000

[Dec. 2020]

Solution : (b)? Formula

C.I. = P[(1 + i)ⁿ - 1]

$3280 = P\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$

= P[1.1025 - 1]

3280 = P(0.1025)

$\therefore P = \frac{3280}{0.1085} = 32000$

Calculator Tricks :- GBC

(a) CI = 16000 + 5% + 5% - 16000

= ₹ 1640

≠ 3280

∴ (a) is incorrect.

(b) C.I. = 32000 + 5% + 5% button - 32000

= ₹ 3280 (Correct)

∴ (b) is correct.

Note : No Need to write anything else

Only on Calculator in seconds.

Q.61. An amount P becomes ₹ 5,100.5 and ₹ 5,203 after second and fourth years respectively, at r% of interest per annum compounded annually. Thus, values of P and r are

(a) ₹ 5,000 and 1 (b) ₹ 4,000 and 1.5
(c) ₹ 6,000 and 2 (d) ₹ 5,500 and 3

[Dec. 2020]

Solution : Tricks GBC (Calculator Tricks)

(a) FV after 2 yrs = 5000 + 1% + 1% button (Press) = ₹ 5100.5 (True)

FV after 4 yrs = 5000 + 1% + 1% + 1% + 1% button = ₹ 5203 (True)

∴ (a) is correct.

Q.62. The useful life of a machine whose cost is ₹ 10,000 is 10 years. If it depreciates at 10% p.a. then the scrap value of the machine is.

(a) 3486.70 (b) 3158.30
(c) 3500 (d) 7033

[Dec. 2019]

Solution : (a)

Scrap Value

$= 10,000\left(1-\frac{10}{100}\right)^{10} = 10,000(0.9)^{10}$

Calculator Tricks:

Type 0.9× = button 9 times × button 10,000 = button, we get ₹ 3486.78

Q.63. A certain sum invested at 4% per annum compounded semi-annually amounts to ₹ 1,20,000 at the end of one year. Find the sum

(a) 1,10,120 (b) 1,15,340
(c) 1,12,812 (d) 1,13,113

[Dec. 2020]

Solution : Tricks GBC

Calculator Tricks Interest for 1st 6 months = 2%.

(a) FV = 1,10,120 + 2% + 2%

≠ 1,20,000

(a) is incorrect.

(b) FV = ₹ 1,15,340 + 2% + 2% button

= ₹ 1,19,999.736 ≈ ₹ 120,000

(b) is correct.

II. Calculator Tricks

P = A (1 + i)ⁿ

$= 1,20,000\left(1+\frac{4}{200}\right)^{-2}$

on Calculator → Type 4 ÷ 200 + 1 ÷ button

then Press = button 2 times

then press "×" button then type 1,20,000 = button = ₹ 1,15,340

Q.64. The ratio of principal and the compound interest value for three years (compounded annually) is 216 : 127. The rate of interest is

(a) 0.1567 (b) 0.1777
(c) 0.1666 (d) 0.1588

[Dec. 2020]

Solution : Tricks GBC

Use calculator [No need to write anything else]

(a) Let r = 0.1567 = 15.67%

$C.I = 216\left(1+\frac{15.67}{100}\right)^3 - 216 \neq 127$

(b) $C.I = 216\left(1+\frac{17.77}{100}\right)^3 - 216$

≠ 127

(c) r = 0.1666 = 16.66%

$\therefore C.I = 216\left(1+\frac{16.66}{100}\right)^3 - 216.$

Cal. 16.66 ÷ 100 + 1 × = 2 times - 216

= 126.9412 ≅ ₹ 127.

∴ (c) is correct.

Detail

$\frac{P}{P\left[\left(1+\frac{r}{100}\right)^3 - 1\right]} = \frac{216}{127}$

or $\left[\left(1+\frac{r}{100}\right)^3 - 1\right] = \frac{127}{216}$

or $\left[\left(1+\frac{r}{100}\right)^3\right] = \frac{127}{216} + 1$

$= \frac{343}{216} = \left(\frac{7}{6}\right)^3$

$\therefore 1 + \frac{r}{100} = \frac{7}{6}$

$\therefore \frac{r}{100} = \frac{7}{6} - 1 = \frac{1}{6}$

$\therefore r = \frac{100}{6} = 16.66\%$

$r = 0.1666$

(*c*) is correct.

Q.65. Find the present value of ₹ 1,00,000 be required after 5 years if the rate of interest is 9% given that $(1.09)^5 = 1.5386$

(*a*) 78,995.98 (*b*) 64,994.20

(*c*) 88,992.43 (*d*) 93,902.12

[Dec. 2020]

Solution : Calculator Tricks

$PV = A(1+i)^{-n}$

$= 100{,}000\left(1+\frac{9}{100}\right)^{-5}$

On Calculator Type 9 ÷ 100 + 1 ÷ = 5 times × 100000

= buttons = ₹ 64,994.20

OR $PV = \frac{A}{(1+1)^n}$

$= \frac{1{,}00{,}000}{\left(1+\frac{9}{100}\right)^5} = \frac{1{,}00{,}000}{(1.09)^5}$

$= \frac{1{,}00{,}000}{1.5386} = 64{,}994.15$ [Given $(1.09)^5$ = 1.5386]

∴ (*b*) is correct.

Q.66. An amount is lent at a nominal rate of 4.5% per annum compounded quarterly. What would be the gain in rupees over when compounded annually.

(*a*) 0.56 (*b*) 0.45

(*c*) 0.076 (*d*) 0.85

[Dec. 2020]

Solution : Let P = ₹ 1

Effective rate

$= \left[1\left(1+\frac{4.5}{400}\right)^4 - 1\right] \times 100$

= 4.576 % (approx) yearly.

If 4.5% compounded yearly

Then Gain % = 4.576 – 4.5% = 0.076%

(*c*) is correct.

Q.67. Find the amount of compound interest, if an amount of ₹ 50,000 is deposited in a bank for one year at the rate of 8% per annum compounded semiannually

(*a*) 3080 (*b*) 4080

(*c*) 5456 (*d*) 7856

[Jan. 2021]

Solution : Tricks (*b*) is correct.

Principal = ₹ 50,000

Semi annually rate of interest

$r = \frac{8}{2} = 4\%$

Amount = 50,000 + 4% + 4% (button)

(By using calculator)

= ₹ 54,080

C.I. = A – P = 54080 – 50000

= ₹ 4080

(*b*) is correct

Q.68. The population of a town increase by 2% of the population at the beginning of the year. The number of year by which the total increases in population would be 40% is :

(*a*) 7 years

(*b*) 10 years

(*c*) 17 years

(*d*) 19 years (approx)

[Jan. 2021]

Solution : (*c*) is correct

% Increase in population = r = 2%

Let after t years population = 1 + 40% = 1.40

In population increase case, always use compound Interest. Formula.

$\therefore A = P(1+1)^n$

or $1.40 = 1\left(1+\frac{2}{100}\right)^{t\times1}$

$= 1.40 = (1{\cdot}02)^t$

Calculator

Type 1·02 × = button (Press)

until to get 1.40

After pressing = button 16 times, we get 1.40 approx,

∴ t = 16 + 1 = 17 years

Q.69. The simple on sum at 4% p.a. for 2 years is ₹ 80. Find the CI on the same sum for the same period.

(*a*) ₹ 81.6 (*b*) ₹ 80.3

(*c*) ₹ 83.2 (*d*) ₹ 82.3

[Jan. 2021]

Solution : Tricks (*a*) is correct

S.I. for 2 year = ₹ 80

∴ S.I. for each year = $\frac{80}{2}$ = ₹ 40

S.I. of 1st year = C.I. of 1st year = ₹ 40

C.I. for 2nd years = Interest on Principal + Interest on interest earned in 1st year.

= S.I. + Interest on S.I.

= 40 + 40 × 4% = ₹ 41.6

[Calculator 40 + 4% = 41.6]

∴ Total Compound Interest = 40 + 41.6 = ₹ 81.6

i.e. C.I. = 40 + 4% + 40 = ₹ 81.6

Q.70. Which is a better investment 9% p.a. compounded quarterly or 9.1% p.a. simple interest?

(*a*) 9% compounded

(*b*) 9.1% S.I.

(*c*) Both are same

(*d*) Cannot be said

[Jan. 2021]

Solution : (*a*) is correct

$r_e = [(1+i)^n - 1] \times 100\%$

$= \left[\left(1+\frac{9}{400}\right)^4 - 1\right] \times 100$

Where m = No. of conversion periods in 1 year = 4

Calculator

[9 ÷ 400 + 1 × = button 3 times – 1]

× 100 (button) = 9.308%

= 9.31% p.a.

Clearly 9.31% is more than 9.1%

9% compounded quarterly is better.

Q.71. The effective rate of interest corresponding to a nominal rate of 7% p.a. compounded quarterly is

(*a*) 7.5% (*b*) 7.6%

(*c*) 7.7% (*d*) 7.185%

[Jan. 2021]

Solution : (*d*) is correct

$r_e = \left[\left(1+\frac{7}{400}\right)^4 - 1\right] \times 100$

Calculator

7 ÷ 400 + 1 × = button 3 times –1 × 100 = button

We get 7.185% approx

Q.72. A sum of money is lent at C.I. Rate 20% p.a. 2 years. It would fetch ₹ 482 more if the interest is compounded half yearly. The sum is :

(*a*) ₹ 19,800 (*b*) ₹ 19,900

(*c*) ₹ 20,000 (*d*) ₹ 20,100

[Jan. 2021]

Solution : Tricks (*c*) is correct

Rate of interest difference in 2 years

$= \left(1+\frac{20}{200}\right)^{2\times2} - \left(1+\frac{20}{100}\right)^2$

$= (1{\cdot}1)^4 - (1{\cdot}2)^2 = 0.0241 = 2.41\%$

[Calculator → 1·1 × = button 3 times M + button 1.2 × button = button 1 time M – button (press)

Then press MRC button we get 0.0241]

P = Difference ÷ rate of interest difference

= 482 ÷ 2.41% button, we get

= ₹ 20,000

Q.73. What 'I' denote the actual rate of interest in decimal, and n denote the number of conversion periods, the formula for computing the effective rate of interest E is given by.

(*a*) $(1+i)^n$ (*b*) $(1+i)^n - 1$

(*c*) $1-(1+i)^n$ (*d*) $(1+i)^{-n}$

[Jan. 2021]

Solution : (*b*) is correct

$r_e = E = (1+i)^n - 1$

Here, $r_e = E = (1+i)^n - 1$

Q.74. A sum of ₹ 7500 amounts to ₹ 9075 at 10% p.a., interest being compounded yearly in a certain time. The simple interest (in ₹) on the same sum for the same time and the same rate is

(*a*) 1000 (*b*) 1250

(*c*) 1800 (*d*) 1500

[July 2021]

Solution : (*d*) is correct

Calculator Tricks

Type 7500 + 10% + 10% = ₹ 9075 (We get) (True)

Hence t = 2 years

$S.I. = \frac{P.r.t.}{100} = \frac{7500\times10\times2}{100} = ₹\ 1500$

or; on calculator S.I. = 7500 × 20% = 1500

Q.75. If the desired future value after 5 years with 18% interest rate is ₹ 1,50,000, then the present value (in ₹) is (Given that $(1.18)^5 = 2.2877$)

(*a*) 63,712 (*b*) 65,568

(*c*) 53,712 (*d*) 41,712

[July 2021]

Solution : (*b*) is correct

$PV = P = A(1+i)^{-n}$

$= 150{,}000\left(1+\frac{18}{100}\right)^{-5} = ₹\ 65{,}568$

Calculator Tricks

Type 2.2877 (given in question)

Then ÷ = button then × button type

150,000 Press = button

We get ₹ 65568

Q.76. What is the compound interest (in ₹) on a sum of ₹ 12,600 for 1½ years at 20% per annum if the interest is compounded half yearly? (Nearest to a Rupee)

(*a*) 4271 (*b*) 4171

(*c*) 4711 (*d*) 4117

[July 2021]

Solution : (*b*) is correct

C.I. = P[(1 + i)^n – 1]

$= 12{,}600\left[\left(1+\frac{20}{200}\right)^{2\times\frac{3}{2}} - 1\right]$

= ₹ 4171

Calculator Tricks

Type 20 ÷ 200 + 1 × = button 2 times

–1 × 12,600 (Type) = button

We get ₹ 4171

Q.77. A sum of ₹ x amounts to ₹ 27,900 in 3 years and to ₹ 41,850 in 6 years at a certain rate per cent per annum, when the interest is compounded yearly. The value of x is

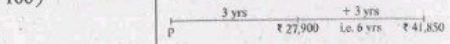
(*a*) 16080 (*b*) 18600

(*c*) 18060 (*d*) 16800

[July 2021]

Solution : (*b*) is correct

Tricks :



Let = Principal = P

It means in 3 years it becomes

$\frac{41{,}850}{27{,}900} = 1{\cdot}5$ times

Means in 3 years principal becomes 1.5 times

Calculator Tricks

∴ P = 27900 ÷ 1.5 = button

= ₹ 18600

Hence (*b*) is correct

Q.78. The effective rate of return for 24% per annum convertible monthly is given as

(*a*) 24% (*b*) 26.82%

(*c*) 18% (*d*) 24.24%

[July 2021]

Solution : (*b*) is correct

Effective rate of return

$= \left[\left(1+\frac{24}{1200}\right)^{12} - 1\right] \times 100 = 26.82\%$

Calculator Tricks

Type 24 ÷ 1200 + 1 × = button 11 times then –1 × 100 = button. We get 26·82%

Q.79. What is the difference (in ₹) between the simple interest and the compound interest on a sum of ₹ 8,000

for $2\frac{2}{5}$ years at the rate of 10% p.a., when the interest is compounded yearly?

(a) 135.75 (b) 129.50
(c) 151.75 (d) 147.20

[July 2021]

Solution : (d) is correct

$$S.I. = \frac{Prt}{100} = \frac{800\cancel{0} \times 1\cancel{0} \times \frac{12}{5}}{1\cancel{00}}$$

= ₹ 1920

For compound interest

Rate of interest for $\frac{2}{5}$ years

$= \frac{2}{5} \times 10 = 4\%$

Tricks

Hence

C.I. = 8000 + 10% + 10% + 4% button – 8000 (Type)

= 2067·20

So, Difference between C.I. & S.I.

= 2067.20 – 1920 = ₹ 147·20

Q.80. S deposits an amount in bank which gives 10% compound interest, compounded annually for 5 years. What is effective rate of simple interest?

(a) 12.21 (b) 11.11
(c) 13.21 (d) 12.81

[Dec. 2021]

Solution : (a)

Given

r = 10% Compounded yearly

t = Time = 5 years

Let P = ₹ 100

∴ Compound Interest

$$= P\left[\left(1+\frac{r}{100}\right)^t - 1\right]$$

$$= 100\left[\left(1+\frac{10}{100}\right)^5 - 1\right] = ₹\ 61.051$$

Let R be @ single rate of interest which gives ₹ 61.051 in 5 years

∴ S.I = C.I.

$$\therefore \frac{P.R.T}{100} = 61.051$$

$$\text{or } \frac{100 \times R \times 5}{100} = 61.051.$$

$$\text{or } R = \frac{61.051}{5} = 12.2102\%$$

$$= 12.21\%$$

Q.81. Cost of a laptop is ₹ 1,10,000 and its value depreciate 12% annually its life is 6 years its scrap value.....times its cost

(a) 0.44 (b) 0.42
(c) 0.45 (d) 0.48

[Dec. 2021]

Solution : (c)

Scrap Value

$$S = P\left(1-\frac{d}{100}\right)^t$$

$$\text{or } \frac{S}{P} = \left(1-\frac{12}{100}\right)^6$$

$= (0.88)^6 = 0.4644$

S = 0.46

$P \Rightarrow S = 0.46P$

Calculator

Type 0.88 × = 5 times

Q.82. If the compound interest earned at *i*% p.a. in n years is to be earned at *s*% simple interest rate for n years, the s =

(a) i (b) $i\frac{1}{n}$
(c) $\frac{(1+i)^n - 1}{n}$ (d) $\frac{1-(1+i)^n}{n}$

[Dec. 2021]

Solution : (c)

Compound Interest = Simple Interest

$\therefore P[(1+i)^n - 1] = P.n.s$

[Where S in decimal form]

$$\text{or } S = \frac{(1+i)^n - 1}{n}$$

Q.83. A company needs ₹ 10,000 in five years to replace as equipment. How much (in ₹) must be invested now at the interest rate of 8% p.a. is order to provide for the equipment ?

(a) ₹ 6,606 (b) ₹ 6,806
(c) ₹ 10,500 (d) ₹ 11,500

[Dec. 2021]

Solution : (b)

$PV = A(1+i)^{-n}$

$$= 10000\left(1+\frac{8}{100}\right)^{-5}$$

$= 1000(1.08)^{-5} = ₹\ 6806$

Calculator Tricks → Type 8 ÷ 100 + 1 + = button

5 times × 10000 = button = **₹ 6805.8** = ₹ 6806

Q.84. It needs to pay ₹ 5,00,000 after 10 years. He invested a sum in a scheme at 9% rate of interest compounded half-yearly. How much amount (in ₹) he invested? ($1.045^{19} = 2.41171$)

(a) 3.97.321 (b) 2.70.321
(c) 2.97.321 (d) 3.40.321

[Dec. 2021]

Solution : (c)

Given

FV = A = ₹ 5,00,000

t = 10 years

r = 9% compounded half yearly m = 2;

n = mt = 2×10 = 20

$$PV = A\left(1+\frac{r}{100\,m}\right)^{-n}$$

$$= 5,00,000\left(1+\frac{9}{200}\right)^{-20}$$

$$= \frac{500,000}{(1.045)^{20}} = \frac{5,00,000}{2.41171}$$

= ₹ 2,07,321

Q.85. A sum of money is put at 20% compound interest rate p.a. At which year the aggregated amount just exceeds the double of the original sum?

(a) 5 (b) 6
(c) 4 (d) 3

[Dec. 2021]

Solution : (c)

I. Calculator Tricks

1 + 20% + 20% + 20% + 20% button

= 2.0736 > 2

∴ t = 4 yrs.

2nd

Method

$$\frac{A}{P} = \left(1+\frac{20}{100}\right)^t$$

$= (1.20)^t$

On Calculator

Press 1.20 × = button 3 times

It becomes greater than 2.

So t = 3+1 = 4 years.

Q.86. An investment is earning compound interest, ₹ 100 invested in the year 2 accumulated to ₹ 105 by year 4. If ₹ 500 invested in the year 5, will become ₹ ..... by year 10.

(a) 364.80 (b) 564.80
(c) 464.80 (d) 664.80

[June 2022]

Solution : 1st Condition P = ₹ 100; A = ₹ 105; n = 2; r = ?

Now $\frac{A}{P} = (1+i)^n$

$$\text{or } \frac{105}{100} = (1+i)^2$$

$\Rightarrow (1+i)^2 = 1.05$ .....(1)

Secondly ∴ P = ₹ 500/-; A = ?

t = 5 yrs; n = 5

∴ $A = P(1+i)^n$

$=500(1+i)^5$

$=500(1+i)^4(1+i)$

$=500\{(1+i)^2\}^2 . \sqrt{(1+i)^2}$

$=500(1.05)^2 . \sqrt{1.05}$

= ₹ 564.86

∴ (a) is correct

Q.87. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of ₹ 12,000 after 3 years at the same rate?

(a) ₹ 3,972 (b) ₹ 2,160
(c) ₹ 3,120 (d) ₹ 3,742

[June 2022]

Solution :

Let P = ₹ 100

A = ₹ 160

S.I = A – P = 160 – 100 = ₹ 60

$$r = \frac{I \times 100}{P.t} = \frac{60 \times 100}{100 \times 6} = 10\%$$

Now

As CI A = 12,000 + 10% + 10% + 10% button

= ₹ 15,972

C.I = A–P = ₹ 15,972–12,000

= ₹ 3,972

∴ (a) is correct

Q.88. The present value of ₹ 2,000, after 8 years at the rate of 6% per annum, is_____. ($1.06^8 = 1.59385$)

(a) ₹ 1,054 (b) ₹ 1,254
(c) ₹ 3,054 (d) ₹ 2,054

[June 2022]

Solution : $PV = A(1+i)^{-n}$

$$=2000\left(1+\frac{6}{100}\right)^{-8}$$

Calculator Tricks

Type 6 + 100 + 1 ÷ = button 8 times × 2000 = button

∴ PV = ₹ 1254 (Approx)

(b) is correct.

Q.89. A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,750.

(a) 5 years 5 months
(b) 5 years 6 months
(c) 5 years 7 months
(d) 5 years 8 months

[Dec. 2022]

Solution:

Tricks :

$$t = \frac{\log\left(\frac{S}{P}\right)}{\log\left(1-\frac{d}{100}\right)}$$

$$= \left[\frac{\log\left(\frac{2,00,750}{4,90,740}\right)}{\log\left(1-\frac{15}{100}\right)}\right]$$

$$= \frac{\log(0.409076)}{\log(0.85)} = 5.4998 = 5.5 \text{ yrs}$$

= 5 yrs 6 months

Calculator Tricks

I. Type 0.85 press √ button 19 times –1 × 2,27,695 = button then press M+ button.

II. Type 2,00,750 ÷ 4,90,740 = button

Then press √ button 19 times

–1 × 2,27,695 = button

III. Then press ÷ button then press MRC = button

[It is time value in decimal form]

IV. We get 5.4998 yrs

= 5.5 yrs

Means 5 years and 6 months

(b) is correct.

Q.90. If ₹ 64 Amount to ₹ 83.20 in 2 years, what will ₹ 86 Amount to in 4 years at the same. Rate per cent per annum?

(a) ₹ 127.60 (b) ₹ 147.60
(c) ₹ 145.34 (d) ₹ 117.60

[Dec. 2022]

Solution : ₹ 64 Amounts to ₹ 83.20 in 2 yrs.

It means ₹ 64 becomes 83.20 ÷ 64

= 1.3 times in 2 years.

Hence, ₹ 86 will become 86 × 1.3 (times) in 2 yrs

and ₹ 86 × 1.3 will become (86 × 1.3) × 1.3 (times)

= ₹ 145.34 in next 2 yrs.

So ₹ 86 will become ₹ 145.34 in 4 years.

(c) is correct

Q.91. The effective annual rate of interest corresponding to a normal rate of 6% per annum payable half yearly is :

(a) 6.06% (b) 6.07%
(c) 6.08% (d) 6.09%

[Dec. 2022]

Solution: $r_e = \left[(1+i)^m - 1\right] \times 100\%$

$= \left[\left(1+\frac{6}{200}\right)^2 - 1\right] \times 100\% = 6.09\%$

[Note : Calculator :– Type 6 ÷ 200 + 1 × = button 1 time –1 × 100 = button]

(We get result)

∴ (*d*) is correct.

Q.92. 10 years ago the Earning Per Share (EPS) of ABC Ltd. was ₹ 5 share. Its EPS for this year is ₹ 22. Compute at what rate, EPS of the company grow annually?

(*a*) 15.97% (*b*) 16.77%

(*c*) 18.64% (*d*) 14.79%

[Dec. 2022]

Solution: ∵ EPS growing annually

So compound interest applies

Let C.I *pa* = *r*

₹5 10 years ₹22 Now

Go by choices

(*a*) *r* = 15.97% (let)

$\therefore Fv = 5\left(1+\frac{15.97}{100}\right)^{10}$

= 22.00019 .....

= 22 (True)

[Note : Calculator:-

Type 15.97 ÷ 100 + 1 × = button 9 times × 5 = button.

We get 22]

∴ (*a*) is correct.

Q.93. Mr. Prakash invested money in two schemes 'A' and 'B' offering compound interest at the rate of 8% and 9% per annum respectively. If the total amount of interest accrued through these two schemes together in two years was ₹ 4818.30 and total amount invested was ₹ 27,000. What was the amount invested in Scheme 'A'?

(*a*) ₹ 12,000 (*b*) ₹ 12,500

(*c*) ₹ 13,000 (*d*) ₹ 13,500

[Dec. 2022]

Solution:

₹27,000/- (Invested)
Schemes

A 8% B 9%

Time 2 years 2 years

Go by choices

(*a*) Let $P_1 = 12000$ $P_2 = 27,000 - 12,000$

$= ₹\ 15,000$

Calculator : CI = 12,000 + 8% + 8% – 12,000 = M + button

Then 15,000 + 9% + 9% – 15,000 = M + button

Then press MRC button

We get ₹ 4818.30 (True)

∴ (*a*) is correct

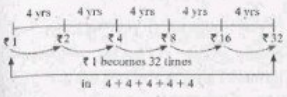
Q.94. A sum of money invested of compound interest doubles itself in four years. In how many years it becomes 32 times of itself at the same rate of compound interest.

(*a*) 12 years (*b*) 16 years

(*c*) 20 years (*d*) 24 years

[Dec. 2022]

Solution: Tricks :



$= 4 \times 5 = 20$ years

∴ (*c*) is correct

Q.95. The difference between compound interest and simple interest on an amount of ₹ 15,000 for 2 years is ₹ 96. What is the rate of interest per annum?

(*a*) 9% (*b*) 8%

(*c*) 11% (*d*) 10%

[Dec. 2022]

Solution: *Given,*

P = ₹ 15000

r = 9; *t* = 2 yrs

C.I – S.I = ₹ 96

Tricks

P = D ÷ *r*% ÷ *r*% button

GBC (*a*) 96 ÷ 9% ÷ 9% button

≠ 15000 (False)

So (*a*) is wrong.

(*b*) For 8%

P = 96 ÷ 8% ÷ 8% button (Press)

= ₹ 15000 (True)

∴ (*b*) is correct

Q.96. A sum of money doubles itself in 4 years at certain compound interest rate. In how many years this sum will become 8 times at the same compound interest rate?

(*a*) 12 years (*b*) 14 years

(*c*) 16 years (*d*) 18 years

[Dec. 2022]

Solution : Tricks

∴ ₹ 1 becomes ₹ 8 in 4 + 4 + 4 = 12 yrs.

(*a*) is correct

Q.97. The Nominal rate of interest is 10% per annum. The interest is compounded quarterly. The effective rate of interest per annum will be:

(*a*) 10 % (*b*) 10.10 %

(*c*) 10.25 % (*d*) 10.38 %

[June 2023]

Solution : Here r = 10% Quarterly ∴ m = 4

$r_e = \left[(1+i)^m - 1\right] \times 100\%$

$= \left[\left(1+\frac{10}{400}\right)^4 - 1\right] \times 100\% = 10.38\%$

Calculator

10 ÷ 400 + 1 × = button 3 times

–1 × 100% (button)

= 10.38%

(*d*) is correct.

Q.98. The difference between compound interest and simple interest on a certain sum of money invested for 3 years at 6% per annum is ₹ 110.16. The principle is:

(*a*) ₹ 3,000 (*b*) ₹ 3,700

(*c*) ₹ 12,000 (*d*) ₹ 10,000

[June 2023]

Solution : Calculator Tricks

P = D÷ r%÷ r% + (300 + r)%

= 110.16÷ 6%÷ 6%÷ (300 + 6)% button. (Press)

= 10,000/-

(*d*) is correct.

Q.99. A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being ₹ 23.240 and ₹ 9,000 respectively. Approximately, for how many years the machine is put to use?

(*a*) 7 (*b*) 8

(*c*) 9 (*d*) 10

[June 2023]

Solution :

$\because S = P\left(1-\frac{d}{100}\right)^t$

Where P = Present Value = Cost

S = Scrap Value

d = Rate of depreciation

t = Time

$9000 = 23240\left(1-\frac{10}{100}\right)^t$

or $(0.9)^t = \frac{9000}{23240}$

or $(0.9)^t = 0.387...$

By Calculator

Press 0.9 x = button 8 times

So t = 8 + 1 = 9 years.

∴ (*c*) is correct.

Q.100. The population of a town increases every year by 2% of the population at the beginning of that year. The approximate number of years, by which the total increase of population will be 40%, is _____ (Given $1.02^8 = 1.17166$)

(*a*) 15 (*b*) 17

(*c*) 19 (*d*) 20

[June 2023]

Solution : Let Present population P = 100

Let after "t" yrs; population

A = 100 + 40 = 140

$\therefore \frac{A}{O} = \left(1-\frac{r}{100}\right)^t$

$\frac{140}{100} = 1.40 = \left(1+\frac{2}{100}\right)^t$

or $1.40 = (1.02)^t$.

Calculator Tricks

Press 1.02 x = button 16 times

We get 1.4000 Approx.

∴ t = 16 + 1 = 17 yrs.

Q.101. The Compound interest on ₹ 15,625 for 9 months at 16% per annum compounded quarterly is:

(*a*) ₹ 1,851 (*b*) ₹ 1,941

(*c*) ₹ 1,951 (*d*) ₹ 1,961

[June 2023]

Solution :

P = ₹ 15,625

t = 9 months = $\frac{9}{12}$ year

$n = mt = 4 \times \frac{9}{12} = 3$

r = 1.6% $\frac{1}{4}$ yearly

∴ C.I. = P[(1 + i)^n – 1]

$= 15,625\left[\left(1+\frac{16}{400}\right)^3 - 1\right]$

Calculator Tricks

Press 16 ÷ 400 + 1 x = button

2 times – 1 x 15625 = button; we get ₹ 1951

∴ (*c*) is correct.

9 CHAPTER ANNUITY

Definition:

A sequence of payments, generally equal in size, made at equal intervals of times is called an **annuity.**

Monthly Rent ; premiums of LIC; deposit into a recurring account in a bank; equal monthly payments got by a retired government servant as pension and loan instalments to houses or automobiles etc.

Some terms related with annuities

Periodic Payment: The size of each payment of an annuity is called the periodic payment of the annuity.

Annual Rent: The sum of all payments of an annuity made in one year is called its annual rent.

Payment Period/Interval : The duration between two successive payments of an annuity is called the payment period (or payment interval) of the annuity

Term: The total duration from the beginning time of the first payment period to the end of the last payment period is called the **term** of the annuity.

Amount of an Annuity: The total Value of all the payments at the maturity time of an annuity is called the amount (or future value) of the annuity.

Present Value of an Annuity: Sum of the present values of all the payments of an annuity is called the present value or capital value of the annuity.

TYPES OF ANNUITIES

Ordinary Annuity: If the payments of an annuity are made at the end of payment interval is called An Ordinary annuity or Regular annuity.

Annuity Due: If the payments of an annuity are made at the beginning of payment interval is called An Annuity Due or Annuity Immediate.

Perpetuity: A perpetuity is an annuity whose payments continue forever.

Note. In what is to follow, it is understood that the payment interval coincides with the interest period unless statement to the contrary is made.

ORDINARY ANNUITY OR ANNUITY REGULAR

Definition: Payments of an annuity are made at the end of payment interval.

Type-I

(TO Find Amount)

$$S = A\left[\frac{(1+i)^n - 1}{r}\right] \times 100m.$$

Where S = Amount of an Annuity

A= Value of each instalment

r = rate of interest

m = No. of conversion periods in a year

n = m.t = No. of instalments made in t yrs.

$i = \frac{r}{100m}$ = Rate of interest of one conversion Period

Calculator Trick

Step - I Find $(1 + i)^n$ by calculator *i.e.* Type r ÷ 100 m + 1 Then push × button then push = button (n - 1) times.

Step - II Then - 1

Step - III ÷ r× 100m

Step - IV Then× A push = button (We get the required value of Amount)

Ex-1. Find the future value of an annuity of ₹500 is made annually for 7 years at interest rate of 14% compounded annually. [Given that $(1.14)^7 = 2.5023$]

(*a*) ₹5365.25 (*b*) ₹5265.25 (*c*) ₹5465.25 (*d*) none

Solution : option (*a*) is correct

Calculator Trick

$$S = A\left[\frac{(1+i)^n - 1}{r}\right] \times 100m. = ₹\ 5365.25$$

$$= 500\left[\frac{\left(1+\frac{14}{100}\right)^7 - 1}{14}\right] \times 100 = ₹\ 5365.25$$

Step - I Find $\left(\frac{14}{100}+1\right)^7$ *As* Type 14 ÷ 100 + 1× Push = button 6 times.

Step - II Type - 1 ÷ 14 then ×100 (Because it is annually)

Step - III Then× 500 = (we get the result)

Ex-2. ₹200 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $(1.005)^{10} = 1.0511$

(*a*) ₹2544 (*b*) ₹2144 (*c*) ₹2544 (*d*) None

Solution : (*a*) is correct.

Here A = 200 ; r = 6% compounded monthly

n = 10 = No. of payments.

$$FV = S = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 200\left[\frac{\left(1+\frac{6}{1200}\right)^{10} - 1}{6}\right] \times 1200 = ₹2045.60$$

Calculator Trick

Step- I Type 6 ÷ 1200 + 1 Then push × button then push = button 9 times.

Step-II Type - 1 Then ÷ 6× 1200

Step-III Then Type× 200 = buttons we get the required amount.

Note: If $(1 + i)^n$ value is given in the question then use given value in the question otherwise answer may vary.

Type - II

To find the **Value of EACH INSTALMENT**

Ex: If a bank pays 6% interest compounded quarterly what equal deposit have to be made at the end of the each quarter for 3 years if you want to have ₹1500 at the end of 3 years?

(*a*) ₹117.86 (*b*) ₹115.01 (*c*) ₹150.50 (*d*) None of these

Solution : (*b*) is correct

$$FV = S = A\left[\frac{(1+i)^n - 1}{r}\right] \times 100m$$

$$1500 = A\left[\frac{\left(1+\frac{6}{400}\right)^{12} - 1}{6}\right] \times 400$$

A = ₹150.01

Calculator Trick

Step-I Type 6 ÷ 400 + 1 Then push × button then push = buttons 11 times

Step-II Then push -1 ÷ 6× 400 buttons

Step-III Then push M+ button to save the typed value.

Step-IV Then type 1500 then ÷ button then push "MRC" button 2 times then push = button.

[we get the required result]

Type-III

(To find Present Value for Ordinary Annuity)

$$\text{PV = Present value} = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Calculator Trick

Step-I Type (1 + i) value then push ÷ button

Step-II Then push = buttons "n" times

Step-III Push GT button

Step-IV Then type× A (value) then push = button

we get the required result.

Ex- Find the present value of an annuity which pays 200 at the end of each 3 months for 10 years assuming money to be worth 5% converted quarterly?

(*a*) ₹3473.86 (*b*) ₹3108.60 (*c*) ₹6265.38 (*d*) None of these

Solution : option (*c*) is correct

Here A = 200 ; m = 4 ; r = 5% 1/4 yrly.

t = 10 years $\Rightarrow$ n = mt = 4 × 10 = 40 year PV=?

Calculator Trick

Step-I Type 5 ÷ 400 + 1 then push ÷ button

Step-II Then push = buttons 40 times

Step-III Then Push GT button

Step-IV Then type× 200 = buttons

[We get the resulting value]

Type-IV

(To find instalment value if PV is given).

Ex- Mr. A borrows 5,00,000 to buy a house.

If he pays equal instalments for 20 years and 10% interest on outstanding balance what will be the equal annual instalment?

(*a*) ₹58239.84 (*b*) ₹58729.84 (*c*) ₹68729.84 (*d*) None of these

Solution : (*b*) is correct

Here PV = ₹5,00,000 ; r = 10% yrly.

t = 20 years ∴ n = 20; A = ?

$$5,00,000 = A\left[\frac{1-\left(1+\frac{10}{100}\right)^{-20}}{0.10}\right] = ₹\,58729.84$$

Calculator Trick

Step-I Type 10 ÷ 100 + 1 then push ÷ button

Step-II Push = buttons 20 times

Step-III Then Push GT button

Step-IV Then M+ buttons to save the result.

Step-V Type 5,00,000 then push ÷ button then- MRC button 2 time and then = button.

(We get the required result)

Annuity Immediate/Due

Definition: An annuity due is an annuity the first payment of which is made at the beginning of the first payment interval

Type-V

(To find Amount)

$$\text{FV = Amount } S = A\left[\left\{\frac{(1+i)^{n+1}-1}{r}\right\} \times 100m - 1\right]$$

Calculator Trick (work as ordinary annuity)

Step-I Type r ÷ 100 m + 1 then push× button

Step-II Push = buttons n + 1 - 1 = n times then push - 1 button then push ÷ button then push r value then push × 100m value buttons.

Step-III Push - 1 button then × button and then type A value & then push = button (we get the required result)

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. Mr. X Invests ₹ 10,000 every year starting from today for next 10 years suppose: interest rate is 8% per annum compounded annually. Calculate future value of the annuity:

(Given that $(1 + 0.08)^{10} = 2.15892500$]

(*a*) ₹ 156454.88 (*b*) ₹ 144865.625

(*c*) ₹ 156554.88 (*d*) None of these

[Nov. 2006]

Solution : (*a*) It is Annuity Due Question

$$A = FV = R\left[\left\{\frac{(1+i)^{n+1}-1}{r}\right\} \times 100m - 1\right]$$

$$= 10,000\left[\frac{(1+0.08)^{10+1}-1}{8} \times 100 - 1\right]$$

= ₹ 1,56,454.88.

(*a*) is correct

Q.2. The present value of an annuity of ₹ 3,000 for 15 years at 4.5% p.a. C.I. is:

[Given that $(1.045)^{15}$,= 1.935282]

(*a*) ₹ 23,809.67 (*b*) ₹ 32,218.67

(*c*) ₹ 32,908.67 (*d*) None of these

[Nov. 2006]

Solution : $\text{PV} = R\left[\frac{1-(1+1)^{-n}}{i}\right]$

$$= 3000\left[\frac{1-(1.045)^{-15}}{0.045}\right]$$

Tricks = ₹ 32,218.67

Q.3. A machine can be purchased for ₹ 50,000. Machine will contribute ₹ 12,000 per year for the next five years. Assume borrowing cost is 10% per annum. Determine whether machine should be purchased or not:

(*a*) Should be purchased

(*b*) Should not be purchased

(*c*) Can't say about purchase

(*d*) None of the above

[Feb. 2007]

Solution : (*b*) $\text{PV} = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$$\text{PV} = 12000\left[\frac{1-(1.10)^{-5}}{0.10}\right]$$

= ₹ 45,489.44

But it costs ₹ 50,000

∴ It should not be purchased

∴ (*b*) is correct

Q.4. How much amount is required to be invested every year so as to accumulate ₹ 3,00,000 at the end of 10 years, if interest is compounded annually at 10 % ?

[Give $(1.1)^{10} = 2.5937$]

(*a*) ₹ 18,823.65 (*b*) ₹18,828.65

(*c*) ₹ 18,832.65 (*d*) ₹18,882.65

[Feb. 2007]

Solution : (*a*) $\text{FV} = R\left[\frac{(1+i)^n - 1}{r} \times 100m\right]$

$$3,00,000 = R\left[\frac{(1+\frac{10}{100})^{10}-1}{10} \times 100\right]$$

$$R = \frac{3,00,000}{\left[\frac{(1.1)^{10}-1}{10} \times 100\right]} = ₹\,18,823.65$$

(*a*) is correct

Q.5. A company is considering proposal of purchasing a machine either by making full payment of ₹ 4,000 or by leasing it for four years at an annual rate of ₹ 1,250. Which course of action is preferable, if the company can borrow money at 14% compounded annually ? [Given: (1.14) = 1.68896]

(*a*) Leasing is preferable

(*b*) Should be purchased

(*c*) No difference

(*d*) None of these

[May 2007]

Solution : (*a*) ₹ 4000 = Present value

$$\text{PV} = R\left[\frac{1-(1-i)^{-n}}{r} \times 100m\right]$$

$$= 1250\left[\frac{1-(1+\frac{14}{100})^{-4}}{14} \times 100\right] = ₹\,3642.14$$

It is less than real cost price.

∴ Leasing is better

∴ (*a*) is correct

Q.6. Vipul purchases a car for ₹ 5,50,000. He gets a loan of ₹ 5,00,000 at 15% p.a. from a Bank and balance ₹ 50,000 he pays at the time of purchase. He has to pay the whole amount of loan in 12 equal monthly instalments with interest starting from the end of the first month. The money he has to pay at the end of every month is :

[Given $(1.0125)^{12} = 1.16075452$]

(*a*) ₹ 45,130.43 (*b*) ₹ 45,230.43

(*c*) ₹ 45,330.43 (*d*) None of these

[May 2007]

Solution : Loan Value = ₹ 5,00,000 = PV

R = Instalment value = ?

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$5,00,000 = R\left[\frac{1-\left(1+\frac{15}{1200}\right)^{-12}}{i}\right]$$

R = 45,130.43.

Q.7. A company establishes a sinking fund to provide for the payment of ₹ 2,00,000 debt maturing in 20 years. Contributions to the fund are to be made at the end of every year. Find the amount of each annual deposit if interest is 5% per annum :

(a) ₹ 6,142 (b) ₹ 6,049
(c) ₹ 6,052 (d) 6,159

[Aug. 2007]

Solution : A = ₹ 200,000

$$200,000 = R\left[\frac{\left(1+\frac{5}{100}\right)^{20}-1}{5}\times 100\right]$$

$$\text{or } R = \frac{2,00,000\times 5}{[(1.05)^{20}-1)]\times 100}$$

= ₹ 6049 (Approx)

Q.8. Raja aged 40 wishes his wife Rani to have ₹ 40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a. How much should he invest annually?

(a) ₹ 84,077 (b) ₹ 81,628
(c) ₹ 84,449 (d) ₹ 84

[Nov. 2007]

Solution : (b) is correct.

R = value of instalment

$$= \frac{40,00,000}{\left[\left\{\frac{(1+0.03)^{30+1}-1}{0.03}\right\}-1\right]}$$

= ₹ 81,628.19

Calculator Trick

Type 1.03 × = button 30 times – 1 ÷ 0.03 – 1 = M + button (Press) Then type 40,00,000 ÷ MRC button = button we get answer.

Q.9. A company may obtain a machine either by leasing it for 5 years (useful life) at an annual rent of ₹ 2,000 or by purchasing the machine for ₹ 8,100. If the company can borrow money at 18% per annum, which alternative is preferable?

(a) Leasing (b) Purchasing
(c) Can't say (d) None of these

[Feb. 2008]

Solution : (a) PV = ₹8100

It is an ordinary Annuity

$$PV = 2000\left[\frac{1-\left(1+\frac{8}{100}\right)^{-5}}{18}\times 100\right]$$

= ₹6254.34

It is less than ₹8100.

∴ (a) is correct

Q.10. A sinking fund is created for redeeming debentures worth ₹ 5 lacs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?

(a) 12,006 (b) 12,040
(c) 12,039 (d) 12,035

[June 2008]

Solution : (a) is correct

Tricks : $₹ 5,00,000 = R\left[\frac{(1.04)^{25}-1}{0.04}\right]$

∴ R = 12006.00 approx

Q.11. Future value of an ordinary annuity :

(a) $A(n,i) = A\left[\frac{(1+i)^n-1}{i}\right]$

(b) $A(n,i) = A\left[\frac{(1+i)^n+1}{i}\right]$

(c) $A(n,i) = A\left[\frac{1-(1+i)^n}{i}\right]$

(d) $A(n,i) = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$

[Dec. 2008]

Solution : (a) is correct.

It is Formulae.

Q.12. Paul borrowers ₹20,000 on condition to repay it with compound interest at 5% p.a. in annual instalment of ₹2,000 each. Find the number of years in which the debt would be paid off.

(a) 10 years (b) 12 years
(c) 14 years (d) 15 years

[June 2009]

Solution : (d) is correct

$$20,000 = 2000\left[\frac{1-\left(1+\frac{5}{100}\right)^{-t}}{5}\times 100\right]$$

$$\text{or } 10 = \left[\frac{1-(1.05)^{-t}}{5}\right]\times 100$$

$$\text{or } \frac{10\times 5}{100} = 1-(1.05)^{-t}$$

$$\text{or } 0.5-1 = (1.05)^{-t}$$

$$\text{or } 0.5-1 = -(1.05)^{-t}$$

$$\text{or } (1.05)^{t} = \frac{1}{0.5} = 2$$

$$\text{or } t = \frac{\log 2}{\log(1.05)} = 15 \text{ yrs. approx.}$$

Tricks : Go by choices

Q.13. Find the present value of an annuity of ₹1,000 payable at the end of each year for 10 years. If rate of interest is 6% compounding per annum. (given $(1.06)^{-10} = 0.5584$):

(a) ₹7,360 (b) ₹ 8,360
(c) ₹ 12,000 (d) None of these

[June 2010]

Solution : (a) is correct

$$PV = 1000\left[\frac{1-(1.06)^{-10}}{0.06}\right]$$

= ₹7360

(a) is correct.

Q.14. The future value of an annuity of ₹ 5,000 is made annually for 8 years at interest rate of 9% compounded annually

[Given that $(1.09)^8 = 1.99256$ _____]

(a) ₹ 55,142.22 (b) ₹ 65,142.22
(c) ₹ 65,532.22 (d) ₹57,425.22

[Dec. 2010]

Solution : (a) is correct

$$FV = 5000\left[\frac{(1.09)^8-1}{0.09}\right] = ₹55,142.22$$

∴ (a) is correct

Q.15. How much amount is required to be invested every year as to accumulate ₹6,00,000 at the end of 10th year, if interest is compounded annually at 10% rate of interest?

(a) ₹37,467 (b) ₹37,476
(c) ₹37,647 (d) ₹37,674

[June 2014]

Solution : (c) is correct

Let amount invested annually = R

$$R = \frac{6,00,000}{\left[\frac{\left(1+\frac{0.1}{100}\right)^{10}-1}{10}\times 100\right]}$$

= ₹ 37,647 (approx)

Q.16. The future value of an annuity of ₹1,000 made annually for 5 years at the rate of interest 14% compound annually is

(a) ₹5610 (b) ₹6610
(c) ₹6160 (d) ₹5160

[Dec. 2014]

Solution : (b)

$$FV = 1000\left[\frac{\left(1+\frac{14}{100}\right)^{5}-1}{14}\times 100\right]$$

= ₹ 6610.104 = ₹ 6610.

Q.17. Suppose your mom decides to gift you ₹10,000 every year starting from today for the next sixteen years. You deposit this amount in a bank as and when you receive and get 8.5% per annum interest rate compounded annually. What is the present value of this money: [Given that P (15, 0.085) = 8.304236]

(a) 83,042 (b) 90,100
(c) 93,042 (d) 10,100

[Dec. 2015]

Solution : (c) is correct

PV = 10,000

$$\left[\frac{1-\left(1+\frac{8.5}{100}\right)^{(-16-1)}}{8.5}\times 100+1\right]$$

= 10,000 (8.304236+1)

= ₹ 93,042

Q.18. The future value of an annuity of ₹1500 made annually for 5 years at an interest rate of 10% compounded annually is ___

[Given that $(1.1)^5 = 1.61051$]

(a) 9517.56 (b) 9157.65
(c) 9715.56 (d) 9175.65

[June 2017]

Solution :

$$FV = 1500\left[\frac{\left(1+\frac{10}{100}\right)^{5}-1}{10}\times 100\right]$$

Use Calculator tricks

= ₹ 9157.65

option (b) is correct.

Q.19. What sum should be invested at the end of every year so as to accumulate an amount of ₹ 796870 at the end of 10 years at the rate of interest 10% compounded annually, [given that A(10 ; 0.1) = 15.9374]

(a) 40,000 (b) 4,50,000
(c) 4,80,000 (d) 50,000

[June 2017]

Solution : Calculator Tricks:

$$R = \frac{796870}{\left[\frac{\left(1+\frac{10}{100}\right)^{10}-1}{10}\times 100\right]}$$

= ₹ 50,000

option (d) is correct.

Q.20. A person invests ₹2,000 at the end of each month @ of interest 6% compounding monthly, find the amount of annuity after the 10th payment is :

(a) ₹20,456 (b) ₹20,156
(c) ₹20,256 (d) ₹20,356

[June 2018]

Solution : (a)

$$FV = 2000\left[\frac{\left(1+\frac{6}{1200}\right)^{10}-1}{6}\times 1200\right]$$

= ₹ 20, 456

Type 6 ÷ 1200 + 1 then press × button then = button 9 times - 1 ÷ 6 × 1200 × 2000 = button ; we will get the required result.

Q.21. Determine the present value of perpetuity of ₹ 50,000 per month @ Rate of interest 12% p.a. is _____

(a) ₹ 45,00,000
(b) ₹ 50,00,000
(c) ₹ 55,00,000
(d) ₹ 60,00,000

[June 2019]

Solution : (b)

$$i = \frac{12}{1200} = 0.01$$

Formula

$$PV = \frac{R}{i} = \frac{50,000}{0.01}$$

= ₹ 50,00,000

(b) is correct.

Q.22. A person wants to lease out a machine costing ₹ 5,00,000 for a 10 year period. It has fixed a rental of

₹ 51,272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum, compounded annually on which money can be invested. To whom this agreement is favourable?

(*a*) Favour for lessee

(*b*) Favour for lessor

(*c*) Not for both

(*d*) Can't be determined

[June 2019]

Solution : (*a*)

Cost = ₹ 5,00,000.

So; GST = PV of Instalments made

$$= PV = 51{,}272\left[\frac{1-\left(1+\frac{10}{100}\right)^{-10}}{i}\right]$$

Calculator Tricks

Type 10 ÷ 100 + 1 ÷ = button 10 times then press GT button then × 51,272 = button = ₹ 3,15,044.25.

Which is less than ₹ 5,00,000.

So, Leasing is preferable.

(*a*) is correct.

Q.23. Let a person invest a fixed sum at the end of each month in an account paying interest 12% per year compounded monthly. It the future value of this annuity after the 12th payment is Rs. 55,000 then the amount invested every month is?

(*a*) ₹ 4,837 (*b*) ₹ 4,637

(*c*) ₹ 4,337 (*d*) ₹ 3337

[June 2019]

Solution : (*c*)

Calculator Tricks

Value of each instalments

$$= R = \frac{FV}{FV \text{ for } ₹ 1}$$

$$= \frac{55{,}000}{\left[\frac{\left(1+\frac{12}{1200}\right)^{12}-1}{12}\times 1200\right]}$$

= ₹ 4337

* Type 12 ÷ 1200 + 1 × = 11 times

−1 ÷ 12 × 1200 = button.

Then press (m+) button.

* Type 55000 ÷ button

then press MRC button

then = button.

We get ₹ 4337.

Q.24. Find the future value of annuity of ₹ 500 is made annually for 7 years interest rate of 14% compound at annually. Given that $(1.14)^7 = 2.5023$

(*a*) ₹ 15635.35 (*b*) ₹ 10,730.71

(*c*) ₹ 16535.35 (*d*) ₹ 16355.35

[Dec. 2019]

Solution : (*b*)

$$FV = 500\left[\frac{\left(1+\frac{14}{100}\right)^{7}-1}{7}\times 100\right]$$

$$= 500\times\left[\frac{2.5023-1}{7}\times 100\right]$$

= ₹ 10,730.71

Q.25. Determine the present value of perpetuity ₹ 10 per month for infinite period at an effective rate of interest of 14% p.a.?

(*a*) ₹ 657 (*b*) ₹ 757

(*c*) ₹ 857 (*d*) ₹ 957

[Dec. 2020]

Solution : $i = \left[\frac{14}{1200}\right]$

$$\therefore PV = \left[\frac{R}{i}\right] = \frac{10}{\frac{14}{1200}} = \frac{10}{14}\times 1200$$

= ₹ 857.14 = ₹ 857.

(*c*) is correct.

Q.26. Which of the following statement is true?

(*a*) F.V of ordinary annuity < F.V of annuity due

(*b*) F.V of ordinary annuity > F.V of annuity due

(*c*) P.V of ordinary annuity > P.V of annuity due

(*d*) None of these

[Dec. 2020]

Solution : (*a*) is correct.

Q.27. Suppose you deposit ₹ 900 per month into an account that pays 14.8% interest compounded monthly. How much money will you get after 9 months?

(*a*) ₹ 8,511 (*b*) ₹ 9,000

(*c*) ₹ 9,200 (*d*) ₹ 1,000

[Dec. 2020]

Solution :

$$FV = R\left[\frac{(1+i)^n-1}{r}\times 100m\right]$$

$$= 900 = \left[\frac{\left(1+\frac{14.8}{1200}\right)^{9}-1}{14.8}\times 1200\right]$$

= ₹ 8511.31 = ₹ 8511

Calculator Trick

14.8 ÷ 1200 + 1 × = button 8 times -1

÷ 14.8 × 1200 × 900 = button. We get FV ₹ 8511.

(*a*) is correct.

Q.28. ₹ 2,500 is paid every year for 10 years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually?

(*a*) 13,040.27 (*b*) 15,847.90

(*c*) 14,674.21 (*d*) 16,345.11

[Dec. 2020]

Solution : Calculator Tricks

Loan amount

$$= PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 2500\left[\frac{1-\left(1+\frac{4}{100}\right)^{-10}}{i}\right]$$

Calculator Tricks

Type 14 ÷ 100 + 1 ÷ = button 10 times (Press)

Then press GT button. then × button.

Type 2500 then = button. (Press)

We get PV = ₹ 13,040.28

(*a*) is correct.

Q.29. Assuming that the discount rate is 7% p.a. how much would pay to receive ₹ 200 growing at 5% annually for ever?

(*a*) ₹ 2,500 (*b*) ₹ 5,000

(*c*) ₹ 7,500 (*d*) ₹ 10,000

[Jan. 2021]

Solution : (*d*) is correct

Discount rate = i = 7% = $\frac{7}{100}$ = 0.07

Growing rate = g = 5% = 0.05

R = Value of each payment received = ₹ 200

$$\therefore PVA = \frac{R}{i-g} = \frac{200}{0.07-0.05}$$

= ₹ 10,000

Q.30. ₹ 800 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annually after 10th payment?

(*a*) ₹ 4,444 (*b*) ₹ 8,756

(*c*) ₹ 3,491 (*d*) ₹ 8,182

[Jan. 2021]

Solution: (*d*) is correct.

Monthly Instalment = A = ₹ 800

rate of interest = r = 6% p.a. compounded monthly

n = No. of Payments = 10

$$\therefore FV = A_{(n,i)} = A\left[\frac{(1+r)^n-1}{r}\times 100m\right]$$

$$= 800\left[\frac{\left(1+\frac{6}{1200}\right)^{10}-1}{6}\times 1200\right]$$

= ₹ 8182

[Calculator Tricks 6 ÷ 1200 + 1 × = 9 times −1 ÷ 6 × 1200 × 800 = button; we get ₹ 8182]

Q.31. The present value of an Annuity immediate is the same as

(*a*) Annuity regular for (n - 1) year plus the initial receipt in the beginning of the period

(*b*) Annuity regular for (n - 1) years

(*c*) Annuity regular for (n + 1) years

(*d*) Annuity regular for (n + 1) years plus the initial receipt in the beginning of the period

[Jan. 2021]

Solution : (*a*) is correct

$$\therefore PV = R\left[\frac{1-(1+i)^{-(n-1)}}{i}+1\right]$$

$$= R\left[\frac{1-(1+i)^{-(n-1)}}{i}\right]+R$$

= PV of Annuity Regular + Value of 1st instalment

(*a*) is correct

Q.32. Find the future value of annuity of ₹ 1,000 made annually for 7 year at interest rate of 14% compounded annually (Given that $1.14^7 = 2.5023$)

(*a*) ₹ 10,730.7 (*b*) ₹ 5,365.35

(*c*) ₹ 8,756 (*d*) ₹ 9892.34

[Jan. 2021]

Solution : (*a*) is correct

$$FV = A\left[\frac{(1+1)^n-1}{r}\times 100m\right]$$

Where m = No. of conversion periods in 1 year = 1

n = No. of payments made = mt

= n = 1 × 7 = 7

r = 14% yearly

A = Value of each instalment

= ₹ 1000

$FV = A_{(n,i)} = 1000$

$$\left[\frac{\left(1+\frac{14}{100}\right)^{7}-1}{14}\times 100\right]$$

Calculator Tricks

Type 14 ÷ 100 × = button 6 times −1 ÷ 14 × 100 × 1000 = button

We get 10,730.5 which is nearest to option (*a*) ₹ 10,730.70

∴ (*a*) is correct

Q.33. A loan of ₹ 1,02,000 is to be paid back in two equal annual instalments. If the rate of interest is 4% p.a., compounded annually, then the total interest charged (in ₹) under this instalment plan is

(*a*) 6160 (*b*) 8120

(*c*) 5980 (*d*) 7560

[July 2021]

Solution : (*a*) is correct

Value of one Instalment

$$= \frac{PV}{\left[\frac{1+(1+i)^{-(n-1)}}{i}+1\right]}$$

$$= \frac{1{,}02{,}000}{\left[\frac{1-(1\cdot04)^{-1}}{i}+1\right]}$$

Calculator Tricks

Type 1·04 ÷ = 1 time press GT button + 1 = button then press M + button

Then type 1,02,000 ÷ MRC button then = button.

We get

R = ₹ 52,000

1st year, C.I. = ₹ 1,02,000 × 4% = ₹ 4080

2nd year, C.I. = ₹ 52000 × 4% = ₹ 2080

Total compound Interest = ₹ 6160

∴ (*a*) is correct

Q.34. If the nominal rate of growth is 17% and inflation is 9% for the five years. Let P be the Gross Domestic Product (GDP) amount at the present

year then the projected real GDP after 6 years is

(*a*) 1.587 P (*b*) 1.921 P
(*c*) 1.403 P (*d*) 2.51 P

[July 2021]

Solution : Real rate of return = Nominal rate of return – Inflation = 17 - 9 = 8%

Expected Real GDP after 6 years

$= P\left(1+\frac{8}{100}\right)^6$

$= 1{\cdot}587\ P$

Where P = Gross Domestic Product (GDP) Amount

(*a*) is correct

Q.35. If a person bought a house by paying ₹ 45,00,000 down payment and ₹ 80,000 at the end of each year till the perpetuity, assuming the rate of interest as 16%, the present value of house (in ₹) is given as

(*a*) 47,00,000 (*b*) 45,00,000
(*c*) 57,80,000 (*d*) 50,00,000

[July 2021]

Solution : (*d*) is correct

Present value of House

$= 45,00,000 + \frac{R}{i}$

$= 45,00,000 + \frac{80000}{0{\cdot}16}$

Here $i = \frac{r}{100m} = \frac{16}{100\times1} = 0{\cdot}16$

$= 45,00,000 + 5,00,000$

$= 50,00,000$

Q.36. Let the operating profit of a manufacturer for five years is given as:

Year	1	2	3	4	5	6
Operating profit (in lakh ₹)	90	100	106.4	107.14	120.24	157.35

Then the operating profit of Compound Annual Growth Rate (CAGR) for year 6 with respect to year 2 is given at

(*a*) 9% (*b*) 12% (*c*) 11% (*d*) 13%

[July 2021]

Solution : (*b*) is correct

Compound Annual Growth

Rate (CAGR) for year 6 with respect to year 2

$= \left[\left(\frac{V_6}{V_2}\right)^{\frac{1}{6-2}} - 1\right] \times 100\%$

$= \left[\left(\frac{157{\cdot}35}{100}\right)^{\frac{1}{4}} - 1\right] \times 100\%$

$= \sqrt[4]{1.5735} - 1 \times 100\%$

$= 11{\cdot}999\% = 12\%$

Calculator Tricks

Type 1·5735 press $\sqrt{\text{button}}$ two times –1 × 100 = button

We get 12%

Q.37. If discount rate is 14% per annum, then how much a company has to pay to receive ₹ 280 growing at 9% annually forever.

(*a*) ₹ 5,600 (*b*) ₹ 2,800
(*c*) ₹ 1,400 (*d*) ₹ 4,200

[July 2021]

Solution : (*a*) is correct

$PVA = \frac{R}{i-g} = \frac{280}{0{\cdot}14-0{\cdot}03}$

= ₹ 5600

Q.38. If the cost of capital be 12% per annum, then the net present value (in nearest ₹) from the given cash flow is given as

Year	0	1	2	3
Operating profit (in thousands ₹)	(100)	60	40	50

(*a*) 31048 (*b*) 34185 (*c*) 51048 (*d*) 24187

Solution : (*d*) is correct

Year	Operating profit 1	PVIF @ 12% 2	Discounted Cash Flows = 1 ₹ 2
0	(100,000)	$\left(1+\frac{12}{100}\right)^0 = 1$	(1,00,000)
1	60,000	$\left(1+\frac{12}{100}\right)^{-1} = 0{\cdot}893$	53,580
2	40,000	$\left(1+\frac{12}{100}\right)^{-2} = 0{\cdot}797$	31,880
3	50,000	0·712	35,600
Net present Value (NPA)			₹ 21,060

Its nearest value in option is (*d*)

So (*d*) is correct

Formula
PVIF = $A(1+i)^{-n} = (1+i)^{n}$
When A = 1

Q.39. The future value of annuity of ₹ 2,000 for 5 years at 5% compounded annually is given (in nearest ₹) as

(*a*) 51051 (*b*) 21021
(*c*) 15624 (*d*) 61254

[July 2021]

Solution : (*c*) is correct

$FV = R\left[\frac{(1+i)^n-1}{r}\times100m\right]$

$= 2000\left[\frac{(1+\frac{5}{100})^5-1}{5}\times100\right] = ₹\ 11051{\cdot}26$

Calculator Tricks

Type 5 ÷ 100 + 1 × = 4 times –1

÷ 5 × 100 × 2000 = button

We get ₹ 11,051.26

Which is nearest to smallest value in option (c)

Q.40. Mr. X wants to accumulate ₹ 50,00,000 at the end of 10 years. Then how much amount is required to be invested every year if interest is compounded annually at 10% (Given that P(10,0.10) = 15.9374298)

(*a*) ₹ 3,13,726.87
(*b*) ₹ 4,13,726.87
(*c*) ₹ 3,53,726.87
(*d*) ₹ 4,53,726.87 ***[Dec. 2021]***

Solution : (*a*)

$R = \frac{50,00,000}{\left[\frac{\left(1+\frac{10}{100}\right)^{10}-1}{10}\times100\right]}$

[Calculator Tricks: Type 10 ÷ 100 + 1 × = button 9 Times – 1 ÷ 10 × 100 (M+) button (Press) Then type 50,00,000 ÷ button then MRC button = button]. We get the Ans.

= ₹ 3,13,726.87

Q.41. The present value of an annuity of ₹ 25,000 to be received after 10 years at 6% per annum compounded annually is ₹ _____. (1.06^5 = 1.33823)

(*a*) ₹ 15,960 (*b*) ₹ 13,960
(*c*) ₹ 11,960 (*d*) ₹ 17,960

[Dec. 2021]

Solution : (*b*)

Note : - Rectification : Delete word "an Annuity" from the question,

Then Answer will match.

Formula

$PV = A\left(1+\frac{r}{100}\right)^{-t}$

$= 25,000\left(1+\frac{6}{100}\right)^{-10} = ₹\ 13,960$

On Calculator.

Type 6 ÷ 100 + 1 ÷ = button 10 times × 25,000 = button

we get 13,959.8 = ₹ 13,960.

Q.42. ₹ 800 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $1.005^{10} = 1.0511$

(*a*) ₹ 4,444 (*b*) ₹ 8,766
(*c*) ₹ 3,491 (*d*) ₹ 8,176

[June 2022]

Solution : Given that

R = ₹ 800 = Value of one instalment

r = 6% Compounded Monthly

n = 10

$\therefore FV = R\left[\frac{(1+i)^n-1}{r}\times100m\right]$

$= 800\left[\frac{\left(1+\frac{6}{1200}\right)^{10}-1}{6}\times1200\right]$

$= 800\left[\frac{1.0511-1}{6}\times1200\right]$

= ₹ 8176/-

(*d*) is correct

Q.43. Lokesh deposits ₹ 3,000 at the start of each quarter in his savings account. If the account earns interest 5.75% per annum compounded quarterly, how much money (in ₹) will he have at the end of 4 years? (1.014375^{16} = 1.25696)

(*a*) ₹ 53,624.4 (*b*) ₹ 58,353.6
(*c*) ₹ 68,353.6 (*d*) ₹ 63,624.4

[June 2022]

Solution : Given

R = ₹ 3000/- = value of one instalment

r = 5.75% compounded quarterly

t= 4 years; n = mt = 4×4=16

$FV = 3000\left[\frac{\left(1+\frac{5.75}{400}\right)^{16}-1}{5.75}\times400\right]$

$= 3000\left[\frac{1.25696-1}{5.75}\times400\right]$

= ₹ 53,626.43 ≅ ₹ 53,624.40

(*a*) is correct

Q.44. Find the future value of annuity of ₹ 1,000 made annually for 7 years at interest rate 14% compounded annually. Given that $(1.14)^7$ = 2.5023

(*a*) ₹ 10,730.71 (*b*) ₹ 5,365.35
(*c*) ₹ 8,756 (*d*) ₹ 9,892.34

[June 2022]

Solution : $FV = R\left[\frac{(1+i)^n-1}{i}\right]$

$=1000\left[\frac{\left(1+\frac{14}{100}\right)^7-1}{14}\times100\right]$

Where

$R = ₹\ 1000;\ i = \frac{14}{100};\ n = m\,t = 1\times7 = 7$

$=1000\left[\left(\frac{2.5023-1}{14}\right)\times100\right]$

= ₹ 10,730.71

∴(*a*) is correct

Q.45. Assuming that the discount rate is 7% p.a. How much would you pay to receive ₹ 200. Growing at 5% annually forever?

(*a*) ₹ 2,500 (*b*) ₹ 5,000
(*c*) ₹ 7,500 (*d*) ₹ 10,000

[June 2022]

Solution : Given

R = ₹ 200; r = 7% yearly

∴ i = 0.07

g = 5%, = 0.05

$\therefore$ Present Value = PV = $\frac{R}{i-g}$

$= \frac{200}{0.07-0.05} = \frac{200}{0.02}$

= ₹ 10,000

∴ (*d*) is correct

Q.46. ₹ 2,500 is paid every year for 10 years to pay off a loan. What is the loan amount if interest rate be 14% p.a. compounded annually?

(*a*) ₹ 15,847.90 (*b*) ₹ 13,040.27
(*c*) ₹ 14,674.21 (*d*) ₹ 16,345.11

[June 2022]

Solution : Given that

Each instalment = R = ₹ 2500

time = 10 yrs

r = 14% compounded yearly

m = 1

∴ n = mt = 1 × 10 = 10

$\text{Loan} = PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$= 2500\left[\frac{1-\left(1+\frac{14}{100}\right)^{-10}}{i}\right]$

Calculator Tricks

Type 14 ÷ 100 + 1÷ = button 10 times then press GT button then × button

Type 2500 = button.

∴ Loan = ₹ 13,040.27

∴ (*b*) is correct

Q.47. Raj made an investment of ₹ 15,000 in a scheme and at the time of maturity the amount was ₹ 25,000. If Compound Annual Growth Rate (CAGR) for this investment is 8.88%. Calculate the approximate number of years for which he has invested the amount.

(*a*) 6 (*b*) 7.7
(*c*) 5.5 (*d*) 7

[June 2022]

Solution : Given

Vn = ₹ 25000

Vo = ₹ 15000

CAGR = r = 8.88%

Formula

$\left[\left(\frac{Vn}{Vo}\right)^{\frac{1}{n-0}}-1\right]\times 100 = CAGR$

or $\left[\left(\frac{25000}{15000}\right)^{\frac{1}{n}}-1\right]\times 100 = 8.88$

or $\left(\frac{25000}{15000}\right)^{1/n} - 1 = \frac{8.88}{100}$

or $\left(\frac{25000}{15000}\right)^{1/n} = 1 + 0.0888$

or $(1.0888)^n = \frac{25000}{15000} = 1.666.... \approx 1.67$

GBC

@ n = 6

∴ $(1.0888)^6 \Rightarrow$ By calculator

1.0888 × = 5 times

= 1.666605 = 1.67 Approx. (True)

∴ t = n = 6 (True)

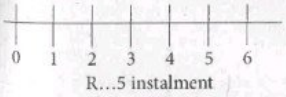
(*a*) is correct

Q.48. Madhu takes a loan of ₹ 50,000 from XYZ Bank. The rate of interest is 10% per annum. The first instalment will be paid at the end of year 5. Determine the amount (in ₹) of equal instalments, if Madhu wishes to repay the amount in five instalments.

(*a*) ₹ 19,510 (*b*) ₹ 19,430
(*c*) ₹ 19,310 (*d*) ₹ 19,630

[June 2022]

Solution :



R = Value of one instalment.

It is a Question of Deferred Annuity

1st Payment made at the end of 5 years.

So Deferred Period = m-1 = 5-1 = 4

No. of instalments = n = 5

$i = \frac{r}{100\,m} = \frac{10}{100} = 0.10$

Loan = PV = ₹ 50, 000

Formula

PV for Deferred Annuity

$= \frac{R[1-(1+i)^{-n}]}{i(1+i)^{m-1}}$

$\Rightarrow 50{,}000 = R\frac{\left[1-(1+0.1)^{-5}\right]}{(0.10)(1+0.1)^4}$

$= R\left[\frac{1-(1.1)^{-5}}{0.1}\right](1.1)^{-4}$

[On Calculator 1+ =Type 1.1 ÷ = button 5 times than press GT button then on + button

Then 1.1+ = button 4 times × MRC button = button]

∴ 50,000 = R [2.589158]

$\therefore R = \frac{50{,}000}{2.589158}$

= ₹ 19311/-

∴ (*c*) is correct.

Q.49. Ramesh invests ₹ 20,000 per year in a stock index fund, which earns 9% per year, for the next ten years. What would be the closest value of the accumulated value of the investment upon payment of the last instalment? ($1.09^{10} = 2.36736$)

(*a*) ₹ 3,88,764.968
(*b*) ₹ 3,03,858.594

(*c*) ₹ 2,68,728.484
(*d*) ₹ 4,08,718.364

[June 2022]

Solution : Given

R = Volume of one instalment = ₹ 20,000/-

r = 9% yrly ; t =10 yrs

n = 1 × 10 = 10

$\therefore FV = 20{,}000\left[\frac{\left(1+\frac{9}{100}\right)^{10}-1}{0.09}\right]$

$= 20000\left[\frac{(1.09)^{10}-1}{0.09}\right]$

$= 20000\left[\frac{2.36736-1}{0.09}\right]$

= 3,03,857.77

≅ 3,03,858.594 (approx.)

∴ (*b*) is correct

Q.50. A company creates a sinking fund of ₹ 2,00,000 in a bank account for 15 years bank offers interest rate 6% per annum the yearly payment to be paid by company is approximately ↔ (if need, use: $1.06^{14} = 2.209$)

(*a*) ₹ 8,945 (*b*) ₹ 8,145
(*c*) ₹ 9,345 (*d*) ₹ 9,645

[Dec. 2022]

Solution : Given that

FV = ₹ 2,00,000

T = 15 yrs; r = 6% yearly

n = mt = 1× 15 = 15

∴ Value of one instalment

$= R = \frac{FV}{\left[\frac{(1+i)^n-1}{r}\times 100\,m\right]}$

$= \frac{2{,}00{,}000}{\left[\frac{\left(1+\frac{6}{100}\right)^{15}-1}{6}\times 100\right]}$

$= \frac{2{,}00{,}000}{\frac{(1.06)^{14}(1.06)-1}{6}\times 100}$

[Note:- We have to use given value $(1.06)^{14} = 2.209$]

$= \frac{200{,}000}{\frac{(2.209\times 1.06-1)}{6}\times 100}$

= ₹ 8944.94 = ₹ 8945/-

∴ (*a*) is correct

Q.51. How much amount is required to be invested every year so as to accumulate ₹ 5,00,000 at the end of 12 years if interest is compounded annually at 10%? (Where A (12, 0.1) = 21.384284).

(*a*) ₹ 23381.65 (*b*) ₹ 24385.85
(*c*) ₹ 26381.65 (*d*) ₹ 28362.75

[Dec. 2022]

Solution: FV = ₹ 5,00,000

R. A(n, i) = FV

Where

R = value of 1 instalment (yearly)

∴ R. A(12, 0.10) = 5,00,000

or $R = \frac{5{,}00{,}000}{A(12, 0.10)}$

$= \frac{5{,}00{,}000}{21.38165}$

= ₹ 23,381.65

∴ (*a*) is correct.

Q.52. Raju invests ₹ 20,000 every year in a deposit scheme starting from today for next 12 years. Assuming that interest rate on this deposit is 7% per annum compounded annually. What will be the future value of this annuity? Given that $(1 + 0.07)^{12} = 2.25219159$.

(*a*) ₹ 5,40,526 (*b*) ₹ 3,82,813
(*c*) ₹ 6,43,483 (*d*) ₹ 3,57,769

[Dec. 2022]

Solution: *Given :*

Value of 1 instalment = R = ₹ 20,000 (yearly)

Starting from today means annuity due

r = 7% compounded yearly; *t* = 12 yrs

$FV = (1+i)\times R\left[\frac{(1+i)^n-1}{i}\right]$

$= \left(1+\frac{7}{100}\right)\times 20{,}000\left[\frac{\left(1+\frac{7}{100}\right)^{12}-1}{7}\times 100\right]$

$= (1.07)\times 20{,}000\left[\frac{2.25219159-1}{7}\times 100\right]$

= ₹ 3,82,813

∴ (*b*) is correct

Q.53. Mr. A invested ₹ 10,000 every year for next 3 year at the interest rate of 8 per cent per annum compounded annually. What is future value of the annuity?

(*a*) 32,644 (*b*) 32,464
(*c*) 34,4264 (*d*) 36,442

[Dec. 2022]

Solution: *Given :*

Value of 1 instalment = R = ₹ 10,000

Time = *t* = 3 years

r = 8% p.a. compounded yearly

∴ *m* = 1

n = *mt* = 3 ×1 = 3

FV = ?

$FV = R\left[\frac{(1+i)^n-1}{r}\times 100\,m\right]$

$= 10{,}000\left[\frac{\left(1+\frac{8}{100}\right)^3-1}{8}\times 100\right]$

= ₹ 32,464

∴ (*b*) is correct

Q.54. ₹ 5,000 is invested every month end in an account paying interest @ 12% per annum compounded monthly. What is the future value of this annuity just after making 11th payment? (Given that $(1.01)^{11} = 1.1156$)

(*a*) ₹ 57,800 (*b*) ₹ 56,100
(*c*) ₹ 56,800 (*d*) ₹ 57,100

[Dec. 2022]

Solution: *Given*

R = ₹ 5000;

r = 12% p.a. compounded monthly

n = 11 (ordinary Annuity)

$$\therefore FV = R\left[\frac{(1+i)^n - 1}{r} \times 100m\right]$$

$$= 5000\left[\frac{\left(1+\frac{12}{1200}\right)^{11} - 1}{12} \times 1200\right]$$

$$= 5000\left[\frac{1.1156 - 1}{12} \times 1200\right]$$

$= ₹\ 57,800$

(*a*) is correct

Q.55. Sinking fund factor is the reciprocal of :

(*a*) Present value interest factor of a single cash flow

(*b*) Present value interest factor of an annuity

(*c*) Future value interest factor of an annuity

(*d*) Future value interest factor of a single cash flow

[Dec. 2022]

Solution: (*c*) $FV = A.A\ (n\ ;\ i) = A = \frac{Fv}{A\ (n; i)}$

Where A = Value of one instalment

A[*n*; *i*] = Annuity factor

(*a*) is correct

Q.56. Suppose you have decided to make a Systematic Investment Plan (SIP) in a mutual fund with ₹ 1,00,000 every year from today for next 10 years where you get return at the rate of 10% per annum compounded annually. What is the future value of this annuity? Given $1.1^{10} = 2.59374$

(*a*) ₹ 17,35,114

(*b*) ₹ 17,53,411

(*c*) ₹ 17,35,411

(*d*) ₹ 17,53,114

[June 2023]

Solution: Given

R = ₹ 1,00,000/- Today ⇒ Annuity Due

r = 10% pa. yearly; n = 10

$$FV = (1 + i) \times A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\left(1+\frac{1}{100}\right) \times 1,00,0000\left[\frac{\left(1+\frac{10}{100}\right)^{10} - 1}{0.10}\right]$$

$$= (1.10) \times (1,00,000) \times \left[\frac{2.59374 - 1}{0.10}\right]$$

$= ₹\ 17,53,114$

(*d*) is correct.

Q.57. Ms. Paul invested ₹ 1,00,000 in a mutual fund scheme in January 2018. After one year in January, 2019, she got a dividend amounting to ₹ 10,000 for first year. ₹ 12,000 for second year, ₹ 16,000 for third year, ₹ 18,000 for fourth year and ₹ 21,000 for fifth year in January 2023. What is Compounded Annual Growth Rate (CAGR) of dividend return? Given $1.2038^4 = 2.1$.

(*a*) 20.38%

(*b*) 18.59%

(*c*) 16.36%

(*d*) 15.89%

[June 2023]

Solution: Given

Investment = ₹ 1,00,000/- (in 2018)

Years	2019	2020	2021	2022	2023
	10,000	12,000	16,000	18,000	21,000

Formula

$$CAGR = \left[\left(\frac{V_n}{V_o}\right)^{\frac{1}{n_n - i_o}} - 1\right] \times 100$$

$$= \left[\left(\frac{21,000}{10,000}\right)^{\frac{1}{2023-2019}} - 1\right] \times 100$$

$$= \left[(2.1)^{\frac{1}{4}} - 1\right] \times 100 \qquad \left[\because (1.2038)^4 = 2.1 \;\; \therefore (2.1)^{\frac{1}{4}} = 1.2038\right]$$

$= [1.2038 - 1] \times 100 = 20.38\%$

$\therefore$ (*a*) is correct.

Q.58. A company want to replace its existing tool room machine at the end of 10 years, the expected cost of machine would be ₹ 10,00,000. If management of the company creates a sinking fund, how much provision needs to be made out of revenue each year which can earn at the rate of 10% compounded annually? Given A(10,0.10) = 15.937425

(*a*) ₹ 74,625

(*b*) ₹ 72,514

(*c*) ₹ 62,745

(*d*) ₹ 67,245

[June 2023]

Solution: Value of 1st instalment

$$= \frac{FV}{A(n; i)} = \frac{10,00,000}{A(10; 0.10)}$$

$$\frac{10,00,000}{15.937425} = ₹\ 62,745.39$$

$= ₹\ 62,745$

(*c*) is correct.

Q.59. A car is available for ₹ 4,98,200 cash payment or ₹ 60,000 cash down payment followed by three equal annual instalments. If the rate of interest charged is 14% per annum compounded yearly, then total interest charged in the instalment plan is (Given P(3,0.14) = 2.32163):

(*a*) ₹ 1,46,314

(*b*) ₹ 1,46,137

(*c*) ₹ 1,28,040

(*d*) ₹ 1,58,040

[June 2023]

Solution:

Cost of Car	=	₹ 4,98,200
Cash Down	=	₹ 60,000
Rest Amount for instalments	=	₹ 4,38,200 = PV

Value of each Instalment

$$= \frac{PV}{PV\ for\ ₹\ 1}$$

$$= \frac{4,38,200}{P(3,0.14)}$$

$$= \frac{4,38,200}{2.32163}\ ₹\ 188,746.65$$

Total Value of 3 Instalments

$= 188,746.65 \times 3$

$= ₹\ 5,66,240.$

Total Interest = 5,66,240 - 4,38,200

$= ₹\ 1,28,040$

(*c*) is correct.

Q.60. Govinda's mother decides to gift him ₹ 50,000 every year starting from today for the next five years. Govinda deposits this amount in a bank as and when he receives and gets 10% per annum interest rate, compounded annually. What is the present value of this annuity? Given P(4,0.10) = 3.16987.

(*a*) ₹ 2,80,493.5

(*b*) ₹ 2,08,493.5

(*c*) ₹ 2,08,943.5

(*d*) ₹ 2,58,493.5

[June 2023]

Solution: Given

Value of one instalment

= R = ₹ 50,000/- yearly

t = 5 years; r = 10% yearly

n = 5 × 1 = 5

(Annuity Due Qts.)

$$PV = R\left[\frac{1-(1+i)^{-(n-1)}}{i} + 1\right]$$

$$= 50,000\left[\frac{1-\left(1+\frac{10}{100}\right)^{-(5-1)}}{i} + 1\right]$$

$= 50,000[P(4, 0.10) + 1]$

$= 50,000[3.16987 + 1]$

$= 2,08,493.5$

$\therefore$ (*b*) is correct.

Q.61. If the discount rate is 10% per annum, how much amount would you pay to receive ₹ 2,500 growing at 8%, annually forever?

(*a*) ₹ 1,25,000

(*b*) ₹ 2,50,000

(*c*) ₹ 1,50,000

(*d*) ₹ 2,00,000

[June 2023]

Solution:

Given

R = ₹ 2,500/-

$$PVA = \frac{R}{i-g} = \frac{2500}{\frac{10}{100} - \frac{8}{100}}$$

$$= \frac{2500}{\frac{2}{100}} = 2500 \times \frac{100}{2}$$

$= ₹\ 1,25,000$

$\therefore$ (*a*) is correct.

Q.62. Mr. Sharad got his retirement benefits amounting to ₹ 50,00,000. He want to receive a fixed monthly sum of amount for his rest of life, starting after one month and thereafter he want to pass on the same to future generation. He expects to earn an interest of 9% compounded annually. Determine how much perpetuity amount he will receive every month?

(*a*) ₹ 39,500

(*b*) ₹ 38,500

(*c*) ₹ 37,500

(*d*) ₹ 36,600

[June 2023]

Solution:

Given PVA = ₹ 50,00,000

Value of 1st instalment = R (let)

i = 9% monthly,

$$\therefore PVA = \frac{R}{i} \Rightarrow R = PVA \times i$$

$$\therefore \quad R = 50,00,000 \times \frac{9}{1200}$$

$= ₹\ 37,500$

$\therefore$ (*c*) is correct.

Tricks

P ∪ A = R ÷ (r − g)% button

= 2500 ÷ 2% button

= ₹ 1,25,000

Q.63. Jonny wants to have ₹ 2,00,000 in his saving account after three year. The rate of interest offered by bank is 8% per annum compounded annually. How much should he invest today to achieve his target amount?

(*a*) ₹ 1,47,489.10

(*b*) ₹ 1,58,766.44

(*c*) ₹ 1,71,035.59

(*d*) ₹ 1,84,417.96

[June 2023]

Solution: Given

FV = ₹ 2,00,000

r = 8% yearly; t = 3 yrs; n = 3

Annuity Due Qts.;

Value of one instalment = R = ?

$$R = \left[\frac{FV}{\frac{(1+i)^{n+1} - 1}{i}}\right]$$

$$= \left[\frac{2,00,000}{\frac{\left(1+\frac{8}{100}\right)^{3+1} - 1}{0.08} - 1}\right]$$

$$= \frac{2,00,000}{\left[\frac{(1.80)^4 - 1}{0.08} - 1\right]} = ₹\ 57043.24$$

Calculator Tricks

Type 1.08 x = button 3 times – 1 + 0.08 – 1 = button then press (M+) button. After this

Type 2,00,000 ÷ MRC button 2 times (press) then press = button. We get R Value

Now 1st investment value

$$= PV = 57043.24\left[\frac{1-\left(1+\frac{8}{100}\right)^{-(3-1)}}{0.08}+1\right]$$

$= ₹\ 1,58,766.44$ approx.

$\therefore$ (*b*) is correct.

10 CHAPTER PERMUTATIONS AND COMBINATIONS

Permutations and combinations :

There are various ways in which objects from a set may be selected, generally without replacement, to form subsets. This selection of subsets is called a **Permutation** when the order of selection is a factor and this selection of subsets is called a **Combination** when order is not a factor.

The concepts of and differences between permutations and combinations can be illustrated by examination of all the different ways in which a pair of objects can be selected from five distinguishable objects such as the letters A, B and C. If both the letters selected and the order of selection are considered, then the following 6 outcomes are possible they are AB ; AC ; BC; BA; CA ; CB.

Each of these 6 different possible selections is called a **permutation**. In particular, they are called the **permutations of three objects taken two at a time,** and the number of such permutations possible is denoted by the symbol ${}^3\mathbf{P}_2$, read "3 permute 2." In general, if there are n objects available from which to select, and permutations (P) are to be formed using r of the objects at a time, the number of different permutations possible is denoted by the symbol ${}^nP_r$ & formulated as

$${}^nP_r = \frac{n!}{(n-r)!}$$

$${}^3P_2 = \frac{3!}{(3-2)!} = 3! = 3.2.1 = 6$$

For **Combinations,** r objects are selected from a set of n objects to produce subsets without ordering. Contrasting the previous permutation example with the corresponding combination, the AB and BA subsets are no longer distinct selections; by eliminating such cases there remain only 3 different possible subsets—AB, AC, BC.

The number of such subsets is denoted by ${}^nC_r$, read "n choose r" & is formulated as ${}^nC_r = \frac{n!}{r!(n-r)!}$

Fundamental Rule of Counting

(*i*) The SUM Rule/Addition Rule

Ex- 7 buses

CP ⟶ Laxmi Nagar

5 Autoes

There are 7 buses and 5 Autoes to go from CP to Laxmi Nagar . In how many ways a person can reach Laxmi Nagar from CP by buses or autoes. We have two options/ choices of vehicles Bus or Auto. There are 7 options to reach by bus or 5 options to reach by auto.

$\therefore$ Total ways to reach Laxmi Nagar from CP

$= 7 + 5 = 12$ ways.

Rule: If there are two alternative jobs, one can be done by "p" ways and another by "q" ways, either of these two jobs can be done by (p + q) ways.

(*ii*) Multiplication (Product) Rule:

If one things is done by "p" different ways and when it has been done a second thing can be done by "q" different ways then total number of ways of doing both things simultaneously

$= p \times q$. ways

Ex-1 Suppose, there are 7 different buses to reach Jaipur from Delhi. In how many ways a person can go to Jaipur by a bus and can come back by different bus. There are 7 options to go to Jaipur by bus but 6 options to come back by a different bus.

$\therefore$ Total No. of ways to reach Jaipur and come back

$= 7 \times 6 = 42$ ways

Ex-2 There are 5 roads to reach LUCKNOW from Delhi; 4 roads to reach Gorakhpur from Lucknow and 6 roads for Patna from Gorakhpur. In how many different ways can a person reach Patna from Delhi.

Solution : Total No. of ways $= 5 \times 4 \times 6 = 120$

Note:

(*i*) In case of either or ; or ; one of them ; atleast one ; or options then use addition rule

(*ii*) In case of AND ; then use product rule.

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. Six points are on a circle. The number of quadrilaterals that can be formed are:

(*a*) 30 (*b*) 360

(*c*) 15 (*d*) None

[June 2010]

Solution : (*c*) is correct

No. of Quadrilaterals

$${}^6C_4 = \frac{6!}{4!.2!} = \frac{6.5.4!}{4!.2!} = 15$$

Q.2. The number of ways of arranging 6 boys and 4 girls in a row so that all 4 girls are together is:

(*a*) 6!.4! (*b*) 2(7!.4!)

(*c*) 7!.4! (*d*) 2.(6!.4!)

[June 2010]

Solution : (*c*) is correct

No. of arrangements of 6 boys and 4 girls so that all 4 girls are together

$= (6+1)!\ .4!$

$= 7!.\ 4! = 120960.$

Q.3. How many numbers not exceeding 1000 can be made from the digits 1,2, 3, 4, 5, 6, 7, 8, 9 if repetition is not allowed.

(*a*) 364 (*b*) 585

(*c*) 728 (*d*) 819

[June 2010]

Solution : (*b*) is correct

Given digits = 1, 2, 3, ...;9

Numbers less than 1000 will be of 1 digit 2 digits and of 3 digits.

$\therefore$ Total No. of Numbers $= {}^9P_1 + {}^9P_2 + {}^9P_3$

$= 9 + 9 \times 8 + 9 \times 8 \times 7 = 585$

Q.4. A garden having 6 tall trees in a row. In how many ways 5 children stand, one in a gap between the trees in order to pose for a photograph?

(*a*) 24 (*b*) 120

(*c*) 720 (*d*) 30

[Dec. 2010]

Solution : (*b*) is correct

Treas

$1 * 2 * 3 * 4 * 5 * 6$

Clearly there will be 5 positions for children

$= {}^5P_5 = 120$

Q.5. ${}^{15}C_3 + {}^{15}C_3$ is equal to:

(*a*) ${}^{16}c_3$ (*b*) ${}^{30}c_{16}$

(*c*) ${}^{15}c_{16}$ (*d*) ${}^{15}c_{15}$

[Dec. 2010]

Solution : (*a*) is correct

${}^{15}C_3 + {}^{15}C_2 = {}^{16}C_3$

Q.6. How many ways a team of 11 players can be made out of 15 players if one particular player is not to be selected in the team.

(*a*) 364 (*b*) 728

(*c*) 1,001 (*d*) 1,234

[Dec. 2010]

Solution : No. of ways to make a 11 - member teams

$= {}^{15-1}C_{11} = {}^{14}C_{11} = \frac{14!}{11!3!}$

$= 364$

Q.7. Find the number of arrangements of 5 things taken out of 12 things, in which one particular thing must always be included.

(a) 39,000 (b) 37,600
(c) 39,600 (d) 36,000

[June 2011]

Solution : (c) is correct

No. of arrangements of 5 things

$= {}^{12-1}C_{5-1}.5! = {}^{11}C_4.5!$

$= \frac{11!}{4!7!} \times 120 = 330 \times 120 = 39600$

Q.8. In how many ways 3 prizes out of 5 can be distributed amongst 3 brothers Equally ?

(a) 10 (b) 45
(c) 60 (d) 120

[Dec. 2011]

Solution : (c) is correct

No. of ways $= {}^5C_1.{}^4C_1.{}^3C_1 = 5.4.3 = 60$

Q.9. There are 12 question are to be answered to be Yes or No. How many ways can these be answered ?

(a) 1024 (b) 2048
(c) 4096 (d) None

[Dec. 2011]

Solution : (c) No. of ways $= 2^{12} = 4096$

Q.10. The letters of the word VIOLENT are arranged so that the vowels occupy even place only. The number of permutations is

(a) 144 (b) 120
(c) 24 (d) 72

[June 2012]

Solution : (a) Vowels = I , O, E

Consonants = V, L, N, T

No. of perms. of vowels

$= {}^3P_3 = 3! = 6$

$\therefore$ Total no. of words

$= 4! \times 3! = 24 \times 6 = 144$

Q.11. If ${}^nP_4 = 20({}^nP_2)$ then the value of 'n' is_____

(a) -2 (b) 7
(c) -2 and 7 both (d) None of these

[June 2012]

Solution : (b) Note : n is always positive Go by choices.

Q.12. A man has 3 sons and 6 schools within his reach, in how many ways, he can send them to school, if no two of his sons are to read in the same school ?

(a) ${}^6P_2$ (b) ${}^6P_3$
(c) 6^3 (d) 3^6

[Dec. 2012]

Solution : (b) No. of ways $= {}^6P_3$

Q.13. How many permutations can be formed from the letters of the word "DRAUGHT", if both vowels may not be separated?

(a) 720 (b) 1,440
(c) 140 (d) 1,000

[Dec. 2012]

Solution : (b) Total Perms. of DRAUGHT

$= 6!.2! = 720 \times 2 = 1440.$

Q.14. If ${}^{13}C_6 + 2^{13}C_5 + {}^{13}C_4 = {}^{15}C_x$ then, x = _____

(a) 6 (b) 7
(c) 8 (d) 9

[Dec. 2012]

Solution : (a)

${}^{13}C_6 + {}^{13}C_5 + {}^{13}C_5 + {}^{13}C_4 = {}^{15}C_x$

or ${}^{14}C_6 + {}^{14}C_5 = {}^{15}C_x$

or ${}^{15}C_6 = {}^{15}C_x \therefore x = 6$

Q.15. The total number of shake hands in a group of 10 persons to each other are_____

(a) 45 (b) 54
(c) 90 (d) 10

[June 2013]

Solution : (a) Total No. of hand shakes

$= {}^{10}C_2 = 45$

Q.16. A regular polygon has 44 diagonals then the No. of sides are_____

(a) 8 (b) 9
(c) 10 (d) 11

[June 2013]

Solution : (d) No. of Diagonals in a polygon of "n" sides $= {}^nC_2 - n = 44$

Tricks : Go by choices.

For.- (a) ${}^8C_2 - 8 = 28 - 8 \neq 44$

(b) ${}^9C_2 - 9 = 36 - 9 \neq 44$

(c) ${}^{10}C_2 - 10 = 45 - 10 \neq 44$

(d) ${}^{11}C_2 - 11 = 55 - 11 = 44$

$\therefore$ (d) is correct

Q.17. In how many ways the word "ARTICLE" can be arranged in a row so that vowels occupy even places?

(a) 132 (b) 144
(c) 72 (d) 160

[June 2013]

Solution : (b) In word ARTICLE

Vowels = A,I,E

Positions For vowels = 2, 4, 6

$\therefore$ Vowels can be arranged in 3! = 6 ways

Rest letters can be arranged in 4! = 24 ways

$\therefore$ Total No. of such arrangements

$= 6 \times 24 = 144$

Q.18. How many different words can be formed with the letters of the word "LIBERTY"

(a) 4050 (b) 5040
(c) 5400 (d) 4500

[Dec. 2013]

Solution : (b) is correct

LIBERTY

No. of words = 7! = 5040

Q.19. In how ways can a family consist of 3 children have different birthday in a leap year

(a) $366 \times 365 \times 364$

(b) ${}^{366}C_6$

(c) ${}^{365}C_3$

(d) ${}^{366}C_3 - 3$

[Dec. 2013]

Solution : (a) is correct

1 Leap year = 366

No. of ways $= {}^{366}C_1.{}^{365}C_1.{}^{364}C_1$

$= 366 \times 365 \times 364$

Q.20. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$ then r =

(a) 2 (b) 3
(c) 4 (d) 5

[Dec. 2013]

Solution : (b) is correct

${}^{15}C_{3r} = {}^{15}C_{r+3}$

Either $3r = r + 3$

$2r = 3$; So, $r = \frac{3}{2}$ (In fraction; so invalid)

Or $3r + r + 3 = 15$

or $4r = 12$; So, $r = 3$

Q.21. If 6 times the No. of permutations of n items taken 3 at a times is equal to 7 times the No. of permutations of (n-1) items taken 3 at a time then the value of n will be

(a) 7 (b) 9
(c) 13 (d) 21

[June 2014]

Solution : (d) is correct

$6.{}^nP_3 = 7.{}^{(n-1)}P_3$ (given)

or; $6.\frac{n!}{(n-3)!} = 7.\frac{(n-1)!}{(n-1-3)!}$

or; $\frac{6.n.(n-1)!}{(n-3)(n-4)!} = \frac{7(n-1)!}{(n-4)!}$

or $\frac{6n}{n-3} = 7$

or $6n = 7n - 21$

or $n = 21$

Tricks : By Calculator (Go by choices)

Q.22. If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^xC_{901}$ then the value of x will be

(a) 999 (b) 998
(c) 997 (d) None

[June 2014]

Solution : (a) is correct

Tricks : Go by choices

For (a) RHS $= {}^{999}C_{97} + {}^{999}C_{901}$

$= {}^{999}C_{97} + {}^{999}C_{98} \left[\because {}^nC_r = {}^nC_{n-r}\right]$

$= {}^{1000}C_{98} (L.H.S)$

$\left[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$

Q.23. ${}^6P_r = 360$ then find r

(a) 4 (b) 5
(c) 6 (d) None

[Dec. 2014]

Solution : (a) is correct

Tricks: Go by choices:

${}^6P_r = 360$

LHS at r = 4

$= {}^6P_4 = 6.5.4.3 = 360.$ R.H.S.

Q.24. If 5 books of English 4 books of Tamil and 3 books of Hindi are to be arranged in a single row so that books of same language come together

(a) 1,80,630 (b) 1,60,830
(c) 1,03,680 (d) 1,30,680

[Dec. 2014]

Solution : (c) is correct

Total No. of ways so that same language

books remain together

$= 5!.4!.3!.3!.$

$= 120 \times 24 \times 6 \times 6 = 1,03,680$

Q.25. 5 Boys and 4 girls are to be seated in row. If the girls occupy even places then the No. of such arrangements

(a) 288 (b) 2808
(c) 2008 (d) 2880

[Dec. 2014]

Solution : (d) is correct

Total No. of students = 9

For girls positions may be 2,4,6,8,

Total no. of arrangements of girls

${}^4P_4 = 4! = 24$

For boys = 5 ! = 120

Total ways $= 5!.4! = 120 \times 24 = 2880$

Q.26. A person has 10 friends of which 6 of them are relatives. He wishes to invite 5 persons so that 3 of them are relatives. In how many ways he can invites?

(a) 450 (b) 600
(c) 120 (d) 810

[June 2015]

Solution : Friends = 10

Relatives = 6 ; So, Rest = 4 friends

Selection of 5 in which 3 are relatives

$= {}^6c_3.{}^4c_2 = 20 \times 6 = 120$

(c) is correct

Q.27. A student has 3 books on computer, 3 books on Economics, 5 on Commerce. If these books are to be arranged subject wise then these can be placed on a shelf in the__ number, of ways.

(a) 25.290 (b) 25,920
(c) 4,230 (d) 4,320

[June 2015]

Solution : Total ways $= 3! \times 3! \times 5! \times 3!$

$= 6 \times 6 \times 120 \times 6$

$= 25920$

Q.28. The number of 4 digit numbers that can be formed from seven digits 1, 2, 3, 5, 7, 8, 9 such that no digit being repeated in any number, which are greater than 3000 are

(a) 120 (b) 480
(c) 600 (d) 840

[June 2015]

Solution :

5	6	5	4

At 1st place 3,5,7,8,9 these 5 digits are suitable.

So, permutation of 1st place = 5

One of them will be used at that place.

Remaining 6 digits are suitable for next place.

Similarly doing as above,

Total ways = 5.6.5.4 = 600

(c) is correct.

Q.29. A question paper consist 10 questions, 6 in math and 4 in stats. Find out number of ways to solve question paper if at least one question is to be attempted from each section.

(a) 1024 (b) 950
(c) 945 (d) 1022

[Dec. 2015]

Solution : (c) is correct

No. of ways to attempt at least one from each $= (2^6-1)(2^4-1) = 945$.

Q.30. There are 6 gents and 4 ladies. A committee of 5 is to be formed if it include at least two ladies.

(a) 64 (b) 162 (c) 102 (d) 186

[Dec. 2015]

Solution : (d) is correct

	Gents	Ladies	Combs
	(6)	(4)	
A Committee of 5	(i) 3	2	$= {}^6c_3.{}^4c_2 = 120$
	(ii) 2	3	$= {}^6c_2.{}^4c_3 = 60$
	(iii) 1	4	$= {}^6c_1.{}^4c_4 = 6$
	Total	= 186	

Q.31. $^nP_r = 720$ **and** $^nC_r = 120$ **, Find r ?**

(a) 6 (b) 4
(c) 3 (d) 2

[Dec. 2015]

Solution : (c) is correct

$^nP_r = {}^nC_r\, r! = 720$

$120r! = 720$

$r! = 6 = 3!$ $\therefore r = 3$

Q.32. There are 10 students in a class, including 3 girls. The number of ways to arrange them in a row, when any two girls out of them never come together

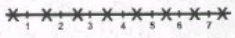
(a) $8_{P_3} \times 7!$ (b) $3_{P_3} \times 7!$
(c) $8_{P_3} \times 10!$ (d) None

[June 2016]

Solution : (a)

Rest Students = 7

Total no. of permutations of rest 7 stds. = 7 !



Total no. of places for girls = 7 +1 = 8

Total ways to arrange girls = $^8P_3$

Total No. of required ways = $^8P_3 \cdot (7!)$

Q.33. In how many ways can a selection of 6 out of 4 teachers and 8 students be done so as to include atleast two teachers ?

(a) 220 (b) 672 (c) 896 (d) 968

[June 2016]

Solution : (b)

		Teachers	Stds.	Combs.
		4	8	
Selection of 6 Persons =	(i)	2	4	$= {}^4C_2.{}^8C_4 = 420$
	(ii)	3	3	$= {}^4C_3.{}^8C_3 = 224$
	(iii)	4	2	$= {}^4C_4.{}^8C_2 = 28$ Total = 672

Q.34. The maximum number of points of inter section of 10 circles will be

(a) 2 (b) 20
(c) 90 (d) 180

[June 2016]

Solution : (c)

To intersect 2 circles out of 10 are required

$= {}^{10}C_2 \times 2 = 45 \times 2 = 90$ Nos. of ways

(Each circle will cut at 2 points)

Q.35. How many numbers between 1,000 and 10,000 can be formed with the digits 1, 2, 3, 4, 5, 6

(a) 720 (b) 360
(c) 120 (d) 60

[Dec. 2016]

Solution : (b) is correct.

Total no. of numbers = $^6p_4 = 6.5.4.3 = 360$

Q.36. If $^{n+1}C_{r+1} : {}^nC_r : {}^{n-1}C_{r-1} = 8:3:1$ **; then find the value of** ***n.***

(a) 14 (b) 15
(c) 16 (d) 17

[Dec. 2016]

Solution : (b) is correct.

Formula, $^nC_r : {}^{n-1}C_{r-1} = \frac{n}{r} = \frac{3}{1}$;

So, n = 3r

& $^{n+1}C_{r+1} : {}^nC_r = \frac{n+1}{r+1} = \frac{8}{3}$;

So, $3n + 3 = 8r + 8$

or, $3 \times 3r + 3 = 8r + 8$

or, $r = 8 - 3 = 5$

Hence, $n = 3r = 3 \times 5 = 15$

Q.37. In how many ways 4 members can occupy 9 vacant seats in a row

(a) 3204 (b) 3024
(c) 4^9 (d) 9^4

[Dec. 2016]

Solution : (b) is correct.

Total ways = $^9p_4 = 9.8.7.6 = 3024$

Q.38. The number of arrangements that can be formed from the letters of the word "ALLAHABAD"

(a) 7560 (b) 3780
(c) 30240 (d) 15320

[June 2017]

Solution : $\frac{9!}{(4!).(2!)}$

$= \frac{9.8.7.6.5(4!)}{(4!).2.1.} = 7560$

$\therefore$ Option (a) is correct

Q.39. If $^{10}c_3 + 2.{}^{10}c_4 + {}^{10}c_5 = {}^nc_5$ **then the value of n =** _____

(a) 10 (b) 11 (c) 12 (d) 13

[June 2017]

Solution : $^{10}C_3 + 2.{}^{10}C_4 + {}^{10}C_5 = {}^nC_5$

or ; $^{10}C_3 + {}^{10}C_4 + {}^{10}C_4 + {}^{10}C_5 = {}^nC_5$

or ; $^{11}C_4 + {}^{11}C_5 = {}^nC_5$

or ; $^{12}C_5 = {}^nC_5 \Rightarrow n = 12$

$\therefore$ Option (c) is correct

Q.40. The number of parallelograms that can be formed by a set of 6 parallel lines intersected by the another set of 4 parallel lines is _____

(a) 360 (b) 90 (c) 180 (d) 45

[June 2017]

Solution : No. of parallelograms = $^6C_2 \cdot {}^4C_2$

$= 15 \times 6 = 90$

$\therefore$ Option (b) is correct

Q.41. If $^n p_{13} : {}^{(n+1)}p_{12} = 3:4$ **then 'n' is** _____**:**

(a) 13 (b) 15 (c) 18 (d) 31

[Dec. 2017]

Solution : (b) is correct

Tricks : Go by choices (Test from beginning)

OR $\frac{{}^nP_{13}}{{}^{n+1}P_{12}} = \frac{3}{4}$

or ; $\frac{n!}{(n-13)!} \div \frac{(n+1)!}{(n-11)!} = \frac{3}{4}$

or ; $\frac{n!}{(n-13)!} \times \frac{(n+11)!}{(n-1)!} = \frac{3}{4}$

or ; $\frac{n!}{(n-13)!} \cdot \frac{(n+11)(n-12).(n-13)!}{(n-1).n!} = \frac{3}{4}$

or ; $\frac{(n-11)(n-12)}{(n-1)} = \frac{3}{4}$

Tricks : GBC

for (b) ; $\text{LHS} = \frac{(15-11)(15-12)}{(15+1)} = \frac{4 \times 3}{16}$

$= \frac{3}{4}(\text{RHS})$.

Q.42. In how many ways that 3 commerce books, 3 computer books and 5 economics books be arranged along a row, so that books of same subjects are come together is _____ **:**

(a) 29,950 (b) 25,940 (c) 25,920 (d) None of these

[Dec. 2017]

Solution : (c)

From Qts. ; Total no. of ways

$= (3!) \cdot (3!) \cdot (5!) \cdot (3!)$

$= 6 \times 6 \times 120 \times 6$

$= 25{,}920.$

Q.43. If $^{12}C_3 + 2.{}^{12}C_4 + {}^{12}C_5 = {}^{14}C_x$ **,The value of x**

(a) 3 or 5 (b) 5 or 9 (c) 7 or 1 (d) 9 or 12

[June 2018]

Solution : (b)

$^{12}C_3 + 2\ {}^{12}C_4 + {}^{12}C_5 = {}^{14}C_x$

$\Rightarrow$ $^{12}C_3 + {}^{12}C_4 + {}^{12}C_4 + {}^{12}C_5 = {}^{14}C_x$

$\Rightarrow {}^{13}C_4 + {}^{13}C_5 = {}^{14}C_x$

$\Rightarrow {}^{14}C_5 = {}^{14}C_x \Rightarrow x = 5$

and ${}^{14}C_5 = {}^{14}C_{14-5} = {}^{14}C_x \Rightarrow x = 14 - 5 = 9$

$\therefore x = 5$ or 9.

Q.44. The number of ways in which a man can invite one or more of his 7 friends to dinner is

(a) 64 (b) 128

(c) 127 (d) 63

[June 2018]

Solution : (c)

No. of ways to invite at least one friend

$= 2^7 - 1 = 128 - 1 = 127.$

Q.45. The number of words from the letters of the word BHARAT, in which B and H will never come together, is

(a) 120 (b) 360

(c) 240 (d) None

[Nov. 2018]

Solution : (c)

Use Gap rule.

*A*R*A*T*

Total words $= \frac{4!}{2!} \times {}^5P_2$

$= \frac{24}{2} \times 5 \times 4 = 240$

Q.46. The value of *N* in $\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$ is

(a) 81 (b) 64

(c) 78 (d) 89

[Nov. 2018]

Solution : (a)

$\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$

or ; $\frac{1}{7!} + \frac{1}{8 \times 7!} = \frac{N}{9 \times 8 \times 7!}$

or ; $\frac{1}{7!}\left(1 + \frac{1}{8} =\right) = \frac{N}{9 \times 8 \times 7!}$

$\Rightarrow \frac{9}{8} = \frac{N}{9 \times 8}$

$\Rightarrow$ N=81.

Q.47. If ${}^nP_r = 720$ and ${}^nC_r = 120$ then *r* is

(a) 4 (b) 5

(c) 3 (d) 6

[Nov. 2018]

Solution : (c)

$\because {}^nP_r = {}^nC_r \cdot r!$

or ; $720 = 120 \cdot (r!)$

or $6 = r! \Rightarrow r! = 3!$

$\therefore r = 3$

Q.48. A bag contains 4 red, 3 black and 2 white balls. In how many ways 3 balls can be drawn from this bag so that they include at least one black ball ?

(a) 46 (b) 64

(c) 86 (d) None

[Nov. 2018]

Solution : (b)

		Black balls	Rest balls	
Selection of 3 balls		3	4 + 2 = 6	Combs.
	(i)	1	2	$= {}^3C_1 \cdot {}^6C_2 = 45$
	(ii)	2	1	$= {}^3C_2 \cdot {}^6C_1 = 18$
	(iii)	3	0	$= {}^3C_3 \cdot {}^6C_0 = 1$

Total ways = 64

Q.49. If ${}^{11}C_x = {}^{11}C_{2x-4}$ and $x \neq 4$ than the value of ${}^7C_x =$

(a) 20 (b) 21

(c) 22 (d) 23

[June 2019]

Solution : (b)

Formula ; ${}^nC_r = {}^nC_k \Rightarrow r = k$

or $r + k = n.$

$\therefore {}^{11}C_x = {}^{11}C_{2x-4}$

$\Rightarrow x = 2x - 4 \Rightarrow x = 4$

but $x \neq 4$

Then, ${}^{11}C_x = {}^{11}C_{2x-4}$

$\therefore x + 2x - 4 = 11$

or; $3x = 15 \Rightarrow x = 5$

$\therefore {}^7C_x = {}^7C_5 = \frac{7!}{5! \cdot 2!} = \frac{7 \cdot 6 \cdot 5!}{5! \cdot 2 \cdot 1}$

$= 21$

(b) is correct.

Q.50. Which of the following is a correct statement

(a) ${}^nP_n = {}^nP_{n-1}$

(b) ${}^nP_n = {}^{2n}P_{n-2}$

(c) ${}^nP_n = {}^{3n}P_{n-3}$

(d) ${}^nP_n = {}^{n(n+1)}P_{n-1}$

[June 2019]

Solution : (a)

Tricks:-Put n = 2 and then GBC.

(a) ${}^nP_n = {}^nP_{n-1} \Rightarrow {}^2P_2 = {}^2P_1$

$\Rightarrow 2 + 1 = 2$ (True)

(b) ${}^nP_n = {}^{2n}P_{n-2} \Rightarrow {}^2P_2 = {}^4P_0$

$\Rightarrow 2 \times 1 \neq 1$

$(\because {}^4P_0 = 1)$

(c) Put n = 3

${}^3P_3 = {}^9P_0$

$\Rightarrow 3 \cdot 2 \cdot 1 \neq 1$ (False)

(d) Put n = 2

${}^2P_2 = 2 \cdot (2 + 1)$

$\Rightarrow {}^2P_2 = {}^{2(2+1)}P_{2-1}$

$\Rightarrow 2 \times 1 = {}^6P_1$

$\Rightarrow 2 = 6$ (False)

(a) is correct.

Q.51. If these are 40 guests in a party. If each guest takes a shake hand with all the remaining guests. Then the total number of hands shake is _____ :

(a) 780 (b) 840

(c) 1,560 (d) 1,600

[June 2019]

Solution : (a)

No. of guests = 40

To shake hands 2 persons are required.

$\therefore$ Total No. of hand shakes

$= {}^{40}C_2 = \frac{40!}{(2!)(38!)}$

$= \frac{40.39.38!}{2 \times 1 \times 38!} = 780.$

Q. 52. In how many ways that the crew of an eight oared be arranged so that if 3 of crew can row only on a stoke side and 2 row on the other side is _____ :

(a) 1728 (b) 256

(c) 164 (d) 126

[June 2019]

Solution : (a)

3 this side || 2 this side

Total No. of rest persons

= 8 - 3 - 2 = 3

$\therefore$ Total No. of ways =

$= \left[{}^3C_1 \cdot (4!)\right] \cdot \left[{}^2C_2 \cdot (4!)\right]$

$= (3 \times 24) \times (1 \times 24)$

$= 1728$

Q.53. How many numbers divisible by 5 of 6 digit can be made from the digit 2, 3, 4, 5, 6, 7

(a) 120 (b) 600

(c) 240 (d) none

[Dec. 2019]

Solution : (a)

Given digits are 2, 3, 4, 5, 6, 7 for being divisible by 5 means 5 should be at unit place.

$\therefore$ Permutation of 5 at unit place = 1

Remaining digits are 2, 3, 4, 6, 7.

They can be arranged in 5! = 120 ways

$\therefore$ Total No. of Nos. divisible by 5 = $120 \times 1 = 120$

Q.54. 5 boys and 3 girls are to be seated together such that no two girls are together

(a) 14,400 (b) 2400

(c) 720 (d) None of these

[Dec. 2019]

Solution : (a)

Use Gap Rule

Arrangement of 5 boys = 5! = 120.

$*B_1 *B_2 *B_3 *B_4 *B_5 *$

After arranging boys; we get 6 places for girls can be arranged in

${}^6P_3 = 6.5.4 = 120$ ways

$\therefore$ Total Arrangements =

$120 \times 120 = 14400$

Q.55. Out of 6 Boys & 4 girls, find the number of ways for selecting 5 members committee in which there is exactly two girls ?

(a) 120 (b) 1440

(c) 720 (d) 71

[Dec. 2019]

Solution : (a)

Total members in 1 committee = 5.

In this committee, girls should be 2

$\therefore$ Boys should be 3

$\therefore$ Total no. of ways to select 3 boys and 2 girls

$= {}^6C_3 \cdot {}^4C_2 = 20 \times 6 = 120$

Q.56. If ${}^nP_5 : {}^nP_3$ is 2:1 than value of n is

(a) 2 (b) -5

(c) -2 (d) 5

[Dec. 2015]

Solution : (d)

Tricks: GBC

In ${}^nP_r$; n & r are always positive.

So options (b) & (c) are wrong.

In ${}^nP_r$; $n \geq r$

So; in ${}^nP_5 \Rightarrow n \geq 5$

$\therefore$ (a) is rejected.

Hence (d) is correct.

Q.57. If ${}^nP_4 = 20 \cdot {}^nP_2$ where P denotes the number of permutations n =.....

(a) 4 (b) 2

(c) 5 (d) 7

[Dec. 2020]

Solution : ${}^nP_4 = 20 \cdot {}^nP_2$

Tricks : GBC

(c) For $n = 5 \Rightarrow {}^5P_4 = 20 \cdot {}^5P_2$

$= 5.4.3.2 \neq 20 \times 5 \times 4$

(c) is incorrect.

(d) For $n = 7$

$\therefore {}^nP_4 = 20 \cdot {}^nP_2$

$\Rightarrow {}^7P_4 = 20 \cdot {}^7P_2$

$\Rightarrow 7.6.5.4 = 20 \times 7 \times 6$ (Correct)

Detail.

${}^nP_4 = 20 \cdot {}^nP_2$

$\Rightarrow \frac{\lfloor n}{\lfloor n-4} = 20 \cdot \frac{\lfloor n}{\lfloor n-2}$

$\Rightarrow \lfloor n-2 = 20 \lfloor n-4$

$\Rightarrow (n-2)(n-3) \lfloor n-4 = 20 \lfloor n-4$

$\therefore (n-2)(n-3) = 20$

Then **GBC** (d) (7 - 2) (7 - 3)

$= 5 \times 4 = 20$ (RHS)

$\therefore$ (d) is correct.

Q.58. A fruit basket contains 7 apples, 6 bananas and 4 mangoes. How many selections of 3 fruits can be made so that all 3 are apples?

(a) 120 ways (b) 35 ways

(c) 168 ways (d) 70 ways

[Dec. 2020]

Solution : No. of selections of 3 apples

$= {}^7C_3 = \frac{\lfloor 7}{\lfloor 3 \lfloor 7-3}$

$= \frac{7.6.5 \cancel{(4!)}}{(3!) \cancel{(4!)}}$

$= \frac{7.6.5}{3 \times 2 \times 1} = 35$

(b) is correct.

Q.59. Out of 7 boys and 4 girls a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is.....

(*a*) 429 (*b*) 439 (*c*) 419 (*d*) 441

[Dec. 2020]

Solution :

	7 Boys	4 Girls		Combinations
Team of 5 (*i*)	4	1	=	$^7C_4 \cdot {}^4C_1 = \frac{7.6.5.4}{\lfloor 4} \times 4 = 140$
(*ii*)	3	2	=	$^7C_3 \cdot {}^4C_2 = \frac{7.6.5}{\lfloor 3}\ 6 = 210$
(*iii*)	2	3	=	$^7C_2 \cdot {}^4C_3 = \frac{7\times6}{\lfloor 2} \times 4 = 84$
(*iv*)	1	4	=	$^7C_1 \cdot {}^4C_4 = 7 \times 1 = 7$
				Total = 441

$\therefore$ (*d*) is correct.

Q.60. From a group of 8 men and 4 women, 4 persons are to be selected to form a committee so that at least 2 women are there on the committee. In how many ways can it be done?

(*a*) 201 (*b*) 168 (*c*) 202 (*d*) 220

[Dec. 2020]

Solution :

	8 Men	4 Women		Combinations
Selection of 4 (*i*)	2	2	=	$^8c_2 \cdot {}^4c_2 = \frac{8\times7}{\lfloor 2} \times \frac{4\times3}{\lfloor 2} = 168$
(*ii*)	1	3	=	$^8c_1 \cdot {}^4c_3 = 8 \times 4 = 32$
(*iii*)	0	4	=	$^8c_0 \cdot {}^4c_4 = 1 \times 1 = 1$
				Total ways = 201

$\therefore$ (*a*) is correct.

Q.61. Eight Chairs are numbered from 1 to 8. Two women and three men are to be seated by allowing one Chair for each. First, the women choose the chairs from the chairs numbered 1 to 4 and then men select the chairs from the remaining.

The number of possible arrangement is

(*a*) 120 (*b*) 288

(*c*) 32 (*d*) 1440

[Jan. 2021]

Solution : (*d*) is correct

Total No. of possible arrangements

= (Arrange of women) (Arrangement of men)

$= {}^4P_2 \cdot {}^{(8-2)}P_3$

$= (4 \times 3) \times (6 \times 5 \times 4)$

$= 12 \times 120 = 1440$

$\therefore$ (*d*) is correct

Q.62. 'n' locks and 'n' corresponding keys are available but the actual combination is not known. The maximum number of trails that are needed to assigns the keys to the corresponding locks is

(*a*) $(n-1)C_2$ (*b*) $(n+1)C_2$

(*c*) $\sum_{k=2}^{n} k$ (*d*) $\sum_{k=2}^{n} (k-1)$

[Jan. 2021]

Solution : (*c*) is correct

Total No. of locks = n and Total no. of keys = n

1st Step: For 1st key, only one key will match with any one lock. It means in 'n' trails, (n – 1) trial will fail and nth trial will match.

2nd Step : Now available locks = n – 1 and available keys = n – 1

Similarly as above, in (n – 1) trials, (n–2) trials will fail and $(n-1)^{th}$ trial will match with the lock.

3rd Step : Similarly doing as above Maximum No. of trials

$= n + (n-1) + (n-2) + \ldots + 3 + 2$

It can be written as $= \sum_{k=2}^{n} k$

Q.63. There are ten flights operating between city A and city B. The number of ways in which a person can travel from city A to city B and return by different flight is

(*a*) 90 (*b*) 95

(*c*) 8C (*d*) 78

[Jan. 2021; June 2022]

Solution : (*a*) is correct

There are 10 options to reach city B from city A.

$\therefore$ Permutations of going = 10

The person has to return by different flight: So, there are 9 options of returning.

Total ways of doing so = $10 \times 9 = 90$.

Q.64. How many odd numbers of four digit can be formed with digit 0, 1, 2, 3, 4, 7 and 8?

(*a*) 150 (*b*) 300

(*c*) 120 (*d*) 210

[Jan. 2021]

Solution : (*b*) is correct

Given digits are

0, 1, 2, 3, 4, 7 and 8

We have to make 4 - digits No.

So, make Four boxes as

Th	H	T	U
5	5	4	3

$5 \times 5 \times 4 \times 3 = 300$

There are 3 options for unit place.

They are 1, 3, 7

So, Permutation of unit place = 3

Out of 7 digits, 1 digit will be used at unit place.

Total No. of remaining digits = 7 – 1 = 6 in which zero (0) is included

But at thousand place, zero (0) cannot be used. So No. of no. of options for thousand place = 5

Similarly, No. of options for hundred places = 7 – 1 –1 = 5

and for Tens place = 7 – 1 –1 – 1 = 4 options

Hence, Total no. of 4- digit odd Nos.

= Product of permutations of each box.

$5 \times 5 \times 4 \times 3 = 300$

$\therefore$ (*b*) is correct

Q.65. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd numbered positions

(*a*) 32 (*b*) 36

(*c*) 48 (*d*) 60

[Jan. 2021; Dec. 2021]

Solution : (*b*) is correct

In word "DETAIL"

No. of letters = 6

No. of vowels (E, A, I) = 3

Places

1	2	3	4	5	6
V		V		V	

Suitable places for vowels = 1, 3, 5

So Permutation of vowels $= {}^3P_3$

$= 3 \times 2 \times 1 = 6$

Permutations of consonants

$= (6-3)! = 3!$

$= 3 \times 2 \times 1 = 6$

Total no. of permutations (i.e. such words) $= 6 \times 6 = 36$

$\therefore$ (*b*) is correct

Q.66. $^nC_p + 2\,{}^nC_{p-1} + {}^nC_{p-2}$**?**

(*a*) $^{n+1}C_p$ (*b*) $^{n+2}C_p$

(*c*) $^{n+1}C_{p+1}$ (*d*) $^{n+2}C_{p-1}$

[Jan. 2021]

Solution : $^nC_p + 2\,{}^nC_{p-1} + {}^nC_{p-2}$

$= {}^nC_p + {}^nC_{p-1} + {}^nC_{p-1} + {}^nC_{p-2}$

$= ({}^nC_p + {}^nC_{p-1}) + ({}^nC_{p-1} + {}^nC_{p-2})$

$= n + {}^{1}C_p + {}^{n+1}C_{p-1}$

($\because$ Formula $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$= {}^{n+2}C_p$

$\therefore$ (*b*) is correct

Q.67. A business houses wishes to simultaneously elevate two of its six branch heads. In how many ways these elevation can take place?

(*a*) 12 (*b*) 3

(*c*) 6 (*d*) 15

[Jan. 2021]

Solution : (*b*) is correct

Total no. of branches = 6

No. of ways to elevate 2 out of 6 branches

$= {}^6C_2 = \frac{\lfloor 6}{\lfloor 2 \lfloor 4} = \frac{6 \cdot 5 \cdot \lfloor 4}{\lfloor 2 \lfloor 4} = 15$

$\therefore$ (*b*) is correct.

Q.68. If $^np_6 = 20\ {}^np_4$ **then the value of n is given by**

(*a*) n = 5 (*b*) n = 3

(*c*) n = 9 (*d*) n = 8

[July 2021]

Solution : (*c*) is correct

$^nP_6 = 20\ {}^nP_4$

or $\frac{n!}{(n-6)!} = 20\ \frac{n!}{(n-4)!}$

$\Rightarrow (n-4)! = 20\,(n-6)!$

or $(n-4)(n-5)(n-6)! = 5 \times 4\,(n-6)!$

or $(n-4)(n-5) = 5 \times 4$

Equating on both sides; we get

Largest factor of LHS = Largest factor of RHS

$\Rightarrow n - 4 = 5$

$\therefore n = 9$

Tricks : Go by choices (GBC)

(*c*) is correct

Q.69. How many numbers of seven digit numbers which can be formed from the digits 3, 4, 5, 6, 7, 8, 9 no digits being repeated are not divisible by 5?

(*a*) 4320 (*b*) 4690

(*c*) 3900 (*d*) 3890

[July 2021]

Solution : (*a*) is correct

Total no. of 7 digit numbers not divisible by 5 = Total Nos. — Nos. divisible by 5

$= \lfloor 7 - \lfloor 6 \times 1$

where 1 is the permutation of unit place

$= 5040 - 720 \times 1$

$= 4320$

Q.70. A person can go from place 'A' to 'B' by 11 different modes of transport but is allowed to return back to "A" by any mode other than the one earlier. The number of different ways, the entire journey can be complete is

(*a*) 110 (*b*) 10^{10}

(*c*) 9^5 (*d*) 10^9

[July 2021]

Solution : (*a*) is correct

There are 11 choices to go from A to B and 10 choices to return from B to A.

So, Total No. of ways to complete the journey = $11 \times 10 = 110$

Q.71. The number of ways 5 boys and 5 girls can be seated at a round table, so no two boys are adjacent is

(*a*) 2550 (*b*) 2880

(*c*) 625 (*d*) 2476

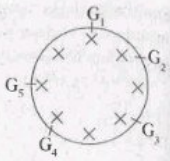
[July 2021]

Solution : (*b*) is correct

Using Gap rule

Permutation of Girls

$= \lfloor 5-1 = \lfloor 4 = 24$



There are 5 places suitable for boys.

They can be arranged in ${}^5P_5 = \lfloor 5 = 120$ ways

Hence, Total no. of arrangements

$= 24 \times 120 = 2880$

Q.72. The number of four letter words can be formed using the letters of the word DICTIONARY is

(*a*) 5040 (*b*) 720 (*c*) 90 (*d*) 30240

[Dec. 2021]

Solution : (*a*)

Total No. of words

$= {}^{10}P_4 = 10.9.8.7 = 5040$

Q.73. The number of words that can be formed using the letters of "PETROL" such that the words do not have "P" in the first position, is

(*a*) 720 (*b*) 120 (*c*) 600 (*d*) 540

[Dec. 2021]

Solution : (*c*)

Total No. of Words = | 5 | 5 | 4 | 3 | 2 | 1 |

E, T, R, O, L (Eligible letters for 1st place).

$= 5.5.4.3.2.1 = 600$

OR

Total words = $\lfloor 6 - \boxed{1 \cdot \lfloor 5}$

P(Fix)

$= 720 - 1 \times 120$

$= 600$

Q.74. If ${}^nP_2 = 12$, then the value of n is

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 6

[Dec. 2021]

Solution : (*c*)

${}^nP_2 = 12$

GBC (C) ${}^4P_2 = 4 \times 3 = 12$ (True)

$\therefore n = 4$

Detail

${}^nP_2 = n(n-1) = 4 \times 3$

Comparing; we get

$n = 4$

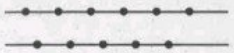
Q.75. Six points are marked on a straight line and five points are marked on another line which is parallel to the first line, how many straight lines, including the first two, can be formed with these points?

(*a*) 28 (*b*) 30

(*c*) 32 (*d*) 34

[June 2022]

Solution :



Total No. of Points = 6 + 5 = 11

Total No. of Lines =

${}^{11}C_2 - {}^6C_2 - {}^5C_2 + 2$

$= \frac{(11)!}{(2)!(11-2)!} - \frac{6!}{2!(6-2)!} - \frac{5!}{2!(5-2)!} + 2$

$= \frac{11.\overset{5}{\cancel{10}}.(9)!}{\underset{1}{\cancel{2}} \times 1.(9)!} - \frac{\overset{3}{\cancel{6}}.5.(4)!}{\underset{1}{\cancel{2}} \times 1.(4)!} - \frac{5.\overset{2}{\cancel{4}}.(3)!}{\underset{1}{\cancel{2}} \times 1 \times 3!} + 2$

$= 55 - 15 - 10 + 2$

$= 32$

(*c*) is correct.

Q.76. How many 4 letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

(*a*) 7,020 (*b*) 5,040

(*c*) 1,480 (*d*) 2,520

[June 2022]

Solution : LOGARITHMS

Total Letters = 10

Total No. of 4 Letters word

$= {}^{10}P_4 = 10.9.8.7$

$= 5040$

(*b*) is correct.

Q.77. Out of 7 boys and 4 girls, a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is

(*a*) 4439 (*b*) 429 (*c*) 419 (*d*) 441

[June 2022]

Solution :		7 Boys	4 girls	Combinations
Selection of 5	(*i*)	4	1=	${}^7C_4 . {}^4C_1 = 35 \times 4 = 140$
	(*ii*)	3	2=	${}^7C_3 . {}^4C_2 = 35 \times 6 = 210$
	(*iii*)	2	3=	${}^7C_2 . {}^4C_3 = 21 \times 4 = 84$
	(*iv*)	1	4=	${}^7C_1 . {}^4C_4 = 7 \times 1 = 7$
				Total = 441

$\therefore$ (*d*) is correct.

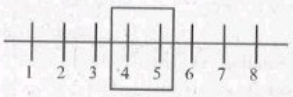
Q.78. The total number of sitting arrangements of 8 members of a board on a row with the president and the vice-president occupying central places is

(*a*) 7! × 2! (*b*) 6! × 2!

(*c*) 6! (*d*) 7!

[June 2022]

Solution :



Central Place

Arrangement of rest 6 persons = 6!

Arrangement of 2 persons

President and Vice-President at Central Place = 2!

Total Ways = (6!). (2!)

$\therefore$ (*b*) is correct.

Q.79. The solution for $\frac{n!}{10} = {}^{n-1}P_{n-3}$ then n =

(*a*) 5 (*b*) 6

(*c*) 7 (*d*) 8

[June 2022]

Solution :

$\frac{n!}{10} = {}^{n-1}P_{n-3}$

or; $\frac{n!}{10} = \frac{(n-1)!}{(n-1-n+3)!}$

or; $\frac{n.(n-1)!}{10} = \frac{(n-1)!}{2!}$

or; $\frac{n}{\cancel{10}_5} = \frac{1}{\cancel{2}}$ $\therefore n = 5$

$\therefore$ (*a*) is correct.

Q.80. A multiple - choice test contains five questions and each has four possible options. How many different answer keys are possible?

(*a*) 20 (*b*) 120

(*c*) 256 (*d*) 1024

[June 2022]

Solution : There are 4 options in each question.

Total No. of ways to answer 1st question = 4

Similarly Total No. of ways to answer 2nd question = 4

Similarly Total No. of ways to answer 3rd question = 4

Similarly Total No. of ways to answer 4th question = 4

Similarly Total No. of ways to answer 5th question = 4

$\therefore$ Total ways to answer 5 questions

$= 4.4.4.4.4. = 1024$

(*d*) is correct.

Q.81. There are 20 points in a plane area. How many triangles can be formed by these points if 5 points are collinear?

(*a*) 550 (*b*) 560

(*c*) 1130 (*d*) 1140

[Dec. 2022]

Solution : No. of Triangles made

$= {}^{20}C_3 - {}^5C_3 = \frac{20!}{(3!)(17!)} - \frac{5!}{(3!)(2!)}$

$= \frac{20 \times 19 \times 18 \times (17!)}{3 \times 2 \times 1 \times (17!)} - \frac{5 \times 4 (3!)}{(3!) \times 2!}$

$= 1140 - 10 = 1130$

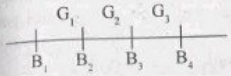
(*c*) is correct

Q.82. The number of ways 4 boys and 3 girls can be seated in a row so that they are alternate is:

(*a*) 12 (*b*) 288 (*c*) 144 (*d*) 256

[Dec. 2022]

Solution : 4 boys and 3 girls are to be arranged in = $\lfloor 4 \times \lfloor 3 = 24 \times 6 = 144$ ways



(Use GAP rule)

(*c*) is correct.

Q.83. If ${}^nP_r = 3024$ and ${}^nC_r = 126$, then find n and r

(*a*) 9, 4 (*b*) 10, 3

(*c*) 12, 4 (*d*) 11, 4

[Dec. 2022]

Solution : $\because {}^nP_r = {}^nC_r \times \lfloor r$ (Formula)

or; $3024 = 126 \times \lfloor r$

or $\lfloor r = 24 = \lfloor 4$

$\therefore r = 4$

Then Go by choices

(*a*) ${}^9P_4 = 9 \times 8 \times 7 \times 6 = 3024$ (True)

$\therefore$ (*a*) is correct.

Q.84. How many 3 digit odd numbers can be formed using the digits 5, 6, 7, 8, 9, if the digits can be repeated?

(*a*) 55 (*b*) 75

(*c*) 65 (*d*) 85

[Dec. 2022]

Solution: Given digits are 5, 6, 7, 8, 9

No. of 3-digit odd Nos. = 5×5×3 = 75

5	5	3

↑ 5, 7, 9

[Note : Only 5, 7, 9 digits are suitable for unit place. So, permutation of unit place = 3. Permutation of Rest Places = 5]

Total No. of numbers = 5×5×3 = 75

Q.85. A committee of 3 women and 4 men is to be formed out of 8 women and 7 men. Mrs. Kajal refuses to serve in a committee in which Mr. Yash is a member. The number of such committees can be:

(*a*) 1530 (*b*) 1500 (*c*) 1520 (*d*) 1540

[June 2023]

Solution:

		Mrs. Kajal	Mr. Yash	Rest Men	Rest Women	Comb.
Committee		1	1	6	7	
of 4 men	(*i*)	0	0	4	3	$= {}^1C_0 \cdot {}^1C_0 \cdot {}^6C_4 \cdot {}^7C_3 = 15 \times 35$
& 3 women	(*ii*)	1	0	4	2	$= {}^1C_1 \cdot {}^1C_0 \cdot {}^6C_4 \cdot {}^7C_2 = 15 \times 21$
or	(*iii*)	0	1	3	3	$= {}^1C_0 \cdot {}^1C_1 \cdot {}^6C_3 \cdot {}^7C_3 = 20 \times 35$

Total ways $= 15 \times 35 + 15 \times 21 + 20 \times 35$

$= 525 + 315 + 700 = 1540$

(*d*) is correct.

Q.86. If ${}^6P_{2r} = 12 \times {}^6P_r$, then r is equal to:

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

[June 2023]

Solution:

${}^6P_{2r} = 12 \times {}^6P_r$

or; $\frac{\lfloor 6}{\lfloor 6-2r} = 12 \times \frac{\lfloor 6}{\lfloor 6-r}$

$\frac{\lfloor 6-r}{\lfloor 6-2r} = 12 = \frac{4 \times 3 \times \lfloor 2}{\lfloor 2}$

$= \frac{\lfloor 4}{\lfloor 2}$

Comparing it; we get

$\lfloor 6-r = \lfloor 4 \Rightarrow 6-r = 4$

$\therefore r = 2$

OR $\lfloor 6-2r = \lfloor 2 \Rightarrow 6-2r = 2$

or $2r = 4$

$r = 2$

Tricks GBC

(*b*) LHS $= {}^6p_{2\times2} = {}^6p_4$

$= 6 \times 5 \times 4 \times 3$

$= 360$

RHS $= 12 \times {}^6p_2$

$= 12 \times 6 \times 5 = 360$

$= LHS$

$\therefore$ (*b*) is correct.

Q.87. In how many different ways can the letters of the word "SOFTWARE" be arranged so that the vowels always come together?

(*a*) 720

(*b*) 144

(*c*) 2880

(*d*) 4320

[June 2023]

Solution: Word = SOFTWARE

Vowels = O, A, E

Total No. of words as per Qts.

$= \boxed{O, A, E}, S, F, T, W, R$

[Vowels will be taken as one unit]

$= \lfloor 6 \times \lfloor 3 = 720 \times 6$

$= 4320$

Note:— All vowels are put in a box.

$\therefore$ (*d*) is correct.

Q.88. In the next world cup of cricket, there will be 12 teams divided equally into two equal groups. Team of each group will play a match against other teams of the group. From each group, 3 top teams will qualify for next round. In this round, each team will play against each other. Four top teams of this round will qualify for semi-finals and play against each other and then two top teams will go to final, where they play the best of three matches. How much minimum number of matches in the next world cup will be?

(*a*) 54 (*b*) 53

(*c*) 38 (*d*) 43

[June 2023]

Solution: Total teams = 12

They are divided into 2 groups.

Each group has 6 teams.

Each team of each group will play with every team.

So in 1st round

Total no. of matches played

$= 6_{C_2} + 6_{C_2} = 15 + 15 = 30$

For second round 6 teams are selected. Each team of this round will play with other.

So total matches played in this second round

$= 6_{C_2} = 15$

In semi-final round top 4 teams are selected. Total matches played in this round

$= 4_{C_2} = 6$

For final round, top 3 teams are selected.

In this round, if a team win 1st and 2nd match than there is no need of 3rd match. Hence in this round, minimum no. of matches played = 2.

$\therefore$ Minimum total no. of matches played $= (15 + 15) + 15 + 6 + 2 = 53$

$\therefore$ (*b*) is correct.

11 CHAPTER SEQUENCE & SERIES

PREVIOUS YEAR EXAM QUESTIONS (MEMORY BASED)

Q.1. If a_1, a_2, a_3 represent first, second and third term of an A.P respectively, the first term is 2 and $(a_1 + a_2)a_3$ is minimum, then the common difference is equal to

(*a*) 5/2 (*b*) -5/2

(*c*) 2/5 (*d*) -2/5

[June 2010]

Solution : (*b*) Given $a_1 = 2$

Let c.d = x

$\therefore a_2 = 2 + x$ *and* $a_3 = 2 + 2x$

The Function is $(a_1 + a_2)a_3$

$= (2 + 2 + x)(2 + 2x)$

$= (4 + x)(2 + 2x)$

$= 8 + 8x + 2x + 2x^2$

$= 2x^2 + 10x + 8$ (a Quadratic Function)

Formula ;

$ax^2 + bx + c$ is minimum when

$a > 0$ *i.e.* + ve at $x = -b/2a$

$= \frac{-10}{2 \times 2} = \frac{-5}{2}$

$\therefore$ (*b*) is correct

Q.2. Divide 144 into three parts which are in AP. and such that the largest is twice the smallest, the smallest of three numbers will be :

(*a*) 48 (*b*) 36

(*c*) 13 (*d*) 32

[June 2010]

Solution : (*d*) Let $t_1 = a$ and $cd = d$

$\therefore a + a + d + a + 2d = 144$

or $3a + 3d = 144$

or $3(a + d) = 144$

or $a + d = \frac{144}{3} = 48$

$\therefore a + d = 48$ _____(1)

$\therefore$ Largest $= 2\times$ Smallest

$\therefore a + 2d = 2a$

$2d = a$

$d = a/2$

From (1)

$a + \frac{a}{2} = 48$

or $\frac{3}{2}a = 48 \therefore a = 48 \times \frac{2}{3}$

$\therefore$ a = 32

$\therefore$ (*d*) is correct

Tricks:- GBC

Q.3. Sum of series $\frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ...\infty$

(*a*) 15/36 (*b*) 35/36 (*c*) 35/16 (*d*) 15/16

[June 2010]

Solution : (*c*) $S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ... \text{ to } \infty$ _____ (1)

$\frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + ... \text{ to } \infty$ _____ (2)

(1) - (2); we get

$S - \frac{1}{5}S = 1 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + ... \text{ to } \infty$

$\frac{4}{5}S = 1 + \frac{\frac{3}{5}}{1 - 1/5} = 1 + \frac{3}{5} \times \frac{5}{4}$

$\frac{4}{5}S = 1 + \frac{3}{4}$

$\frac{4}{5}S = \frac{7}{4}; \quad \therefore S = \frac{35}{16}$

(*c*) is correct

Q.4. If G be geometric mean between a & b, then the value of $\frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2}$ **is equal to**

(*a*) G^2 (*b*) $3G^2$ (*c*) $1/G^2$ (*d*) $2/G^2$

[Dec. 2010]

Solution : (*c*) **Tricks** :- 1, 2, 4 are in GP

$\therefore$ 2 is the GM of 1 & 4

$\therefore$ a = 1; b = 4 and G = 2

$\therefore \frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2} = \frac{1}{4-1} + \frac{1}{4-16} = \frac{1}{3} - \frac{1}{12}$

$= \frac{4-1}{12} = \frac{3}{12} = \frac{1}{4}$

Then Go by choices

For (*a*) $G^2 = 2^2 = 4 \neq \frac{1}{4}$

(*c*) $\frac{1}{G^2} = \frac{1}{2^2} = \frac{1}{4}$

$\therefore$ (*c*) is correct

Q.5. If the sum of n terms of an A.P. is $2n^2 + n$. What is the difference between its 10th term & 1st term

(*a*) 207 (*b*) 36 (*c*) 90 (*d*) 63

[June 2011]

Solution : (*b*) $S_n = 2n^2 + n$

$\therefore t_1 = s_1 = 2 \times 1^2 + 1 = 3$

$s_2 = 2 \times 2^2 + 2 = 10$

$\therefore d = s_2 - 2s_1 = 10 - 2 \times 3 = 4$

$\therefore t_{10} - t_1 = a + 9d - a = 9d = 9 \times 4$

$= 36$ $\therefore$ (*b*) is correct

Q.6. Find the product of $243.243^{\frac{1}{6}}.243^{\frac{1}{36}}.....$ **to** ∞

(*a*) 1024 (*b*) 27 (*c*) 729 (*d*) 246

[June 2011]

Solution : (*c*) is correct $P = (243).(243)^{\frac{1}{6}}.(243)^{\frac{1}{36}}.....\text{to } \infty$

$= 243^{\frac{1}{1-1/6}} = 243^{6/5} = (3^5)^{6/5}$

$= 3^6 = 729$

Q.7. Insert two arithmetic means between 68 and 260

(*a*) 132, 196 (*b*) 130, 194 (*c*) 70, 258 (*d*) None

[June 2011]

Solution : (*a*) Trick I

$A_1 + A_2 + A_3 + ... + A_n = n \times \text{AM of a \& b}$

$= n\left(\frac{a+b}{2}\right)$

$132 + 196 = 2\left(\frac{68+260}{2}\right)$

328 = 328 (True)

Trick II

Go by choices

For (*a*) 68, 132, 196, 260 marks an AP

$\therefore$ (*a*) is correct

Q.8. Geometric mean of p, p^2, p^3.....p^n will be

(*a*) p^{n+1}

(*b*) $p^{\left(\frac{1+n}{2}\right)}$

(*c*) $p^{\frac{n(n+1)}{2}}$

(*d*) None of the above

[June 2011]

Solution : (*b*)

$GM = \left(p.p^2.p^3.....p^n\right)^{1/n}$

$= \left(p^{1+2+3+.....+n}\right)^{1/n}$

$= \left[p^{\frac{n(n+1)}{2}}\right]^{1/n} = p^{(n+1/2)}$

Tricks :- Put n = 3

$Gm = \left(p.p^2.p^3\right)^{1/3} = p^2$

For (*a*) $GM = p^{3+1} \neq p^2$

(*b*) $GM = p^{\frac{1+3}{2}} = p^2$

$\therefore$ (*b*) is correct.

Q.9. Find the number whose arithmetic mean is 12.5 and geometric mean is 10.

(*a*) 20 and 5 (*b*) 10 and 5

(*c*) 5 and 4 (*d*) None of these

[Dec. 2011]

Solution : (*a*) is correct

Tricks :- Go by choices

For (*a*) $AM = \frac{20+5}{2} = 12.5$

and $GM = \sqrt{20 \times 5} = 10$

$\therefore$ 20 & 5 satisfy both given condition in qts.

$\therefore$ (*a*) is correct.

Q.10. If sum 3 arithmetic mean between "a" and 22 is 42, then "a"=

(*a*) 14 (*b*) 11

(*c*) 10 (*d*) 6

[Dec. 2011]

Solution : (*d*) is correct

Tricks :- It $A_1; A_2; A_3; .....$; An are "n" AMS

between a and b

$A_1 + A_2 + A_3 + ... + A_n = n\left(\frac{a+b}{2}\right)$

= n.(AM of a and b)

$\therefore 3\left(\frac{a+22}{2}\right) = 42 \therefore a = 6$

Q.11. If each month ₹100 increases in any sum then find out the total sum after 10 months, if the sum of first month is ₹2,000.

(*a*) ₹24,500 (*b*) ₹24,000

(*c*) ₹50,000 (*d*) ₹60,000

[Dec. 2011]

Solution : (*a*) Sum =

$\frac{10}{2}[2 \times 2000 + (10-1).100]$

= ₹24,500.

Q.12. If 5th term of a G.P. is $\sqrt[3]{3}$, then the product of first nine terms is

(*a*) 8 (*b*) 27

(*c*) 243 (*d*) 9

[Dec. 2011]

Solution : (*b*) ; $t_5 = ar^4 = \sqrt[3]{3} = 3^{1/3}$

Product of 1st 9 terms

$= a.ar.ar^2 .....ar^8$

$= a^9 .r^{1+2+3+.....+8}$

$= a^9 .r^{\frac{8}{2}(1+8)} = a^9 .r^{36}$

$= (ar^4)^9 = (3^{1/3})^9 = 3^3 = 27$

Q.13. The sum of the third and ninth term of an A.P. is 8. Find the sum of the first 11 terms of the progression.

(*a*) 44 (*b*) 22

(*c*) 19 (*d*) 11

[Dec. 2011]

Solution : (*a*) is correct

$\because t_3 + t_9 = 8$

a + 2d + a + 8d

or 2a + 10d = 8

$\therefore S_{11} = \frac{11}{2}[2a + (11-1)d]$

$= \frac{11}{2}[2a + 10d] = \frac{11}{2} \times 8 = 44$

Q.14. 8th term of an A.P is 15, then sum of its 15 terms is

(*a*) 15 (*b*) 0

(*c*) 225 (*d*) 225/2

[June 2012]

Solution : (*c*) is correct

$t_8 = a + 7d = 15$

$S_{15} = \frac{15}{2}[2a + (15-1)d] = \frac{15}{2} \times 2(a + 7d)$

$= 15 \times 15 = 225$

Q.15. Find the sum of the infinite terms

2, $\frac{4}{y}, \frac{8}{y^2}, \frac{16}{y^3}$.....; If y>2

(*a*) $\frac{2y}{y-2}$ (*b*) $\frac{4y}{y-2}$

(*c*) $\frac{3y}{y-2}$ (*d*) None of these

[June 2012]

Solution : (*a*) is correct

$S = \frac{a}{1-r} = \frac{2}{1 - \frac{2}{y}} = \frac{2y}{y-2}$

Q.16. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term 'a' and common difference 'd'.

(a) a=3,d=2 (b) a=4,d=3
(c) a=5,d=4 (d) a=6,d=5

[June 2012]

Solution : (a) is correct

$t_4 = 3t_1 \Rightarrow a+3d = 3a \therefore 2a = 3d;\ a = \frac{3d}{2}$

$\because t_7 = 2t_3 + 1$

or $a + 6d = 2(a + 2d) + 1$

or $a + 6d = 2a + 4d + 1$

or $2d - a = 1$

or $2d - \frac{3}{2}d = 1 \Rightarrow \frac{d}{2} = 1 \therefore d = 2$

and $a = \frac{3}{2} \times 2 = 3$

Tricks :- Go by choices.

Q.17. In an A.P., if common difference is 2, Sum of n terms is 49, 7th term is 13 then n = _____

(a) 0 (b) 5
(c) 7 (d) 13

[Dec. 2012]

Solution : (c) is correct.

$t_7 = a + 6\times 2 = 13 \therefore a = 1$

$s_n = \frac{n}{2}[2\times 1 + (n-1).2] = 49$

or $\frac{n}{2}.2[1+n-1] = 49$

or $n^2 = 49 \therefore n = 7$

Q.18. The first term of a G.P. where second term is 2 and sum of infinite term is 8 will be

(a) 6 (b) 3
(c) 4 (d) 1

[Dec. 2012]

Solution : (c) is correct

$t_2 = ar = 2 \Rightarrow r = \frac{2}{a}$

$S_\infty = \frac{a}{1-r} = 8$

or $a = 8\ (1 - r)$

or $a = 8\left(1 - \frac{2}{a}\right)$

or $a^2 = 8(a-2)$

or $a^2 - 8a + 16 = 0$

or $(a-4)^2 = 0 \Rightarrow a = 4$

Tricks :- Go by choices

For (c) $4r = 2 \therefore r = \frac{1}{2}$

$S = \frac{9}{1-r} = \frac{4}{1-1/2} = 8$ (Which is correct)

$\therefore$ (c) is correct

Q.19. If the sum of n terms of an A.P be $2n^2 + 5n$, then its 'n^{th}' term is

(a) 4n-2 (b) 3n-4
(c) 4n+3 (d) 3n+4

[Dec. 2012]

Solution : (c) is correct

$\therefore S_n = 2n^2 + 5n$

$\therefore S_1 = t_1 = 2\times 1^2 + 5\times 1 = 7 = a$

$d = S_2 - 2S_1$

$= 2\times 2^2 + 5\times 2 - 2\times 7 = 4$

$t_n = a + (n-1)d = 7 + (n-1)4 = 4n + 3$

Tricks :- Go by choices

For (a) $S_1 = t_1 = 4\times 1 - 2 = 2 \neq 7$

(c) $t_1 = 4\times 1 + 3 = 7$

$t_2 = 4\times 2 + 3 = 11$

$s_2 = t_1 + t_2 = 7 + 11 = 18$

and $S_2 = 2\times 2^2 + 5\times 2 = 18$

$\therefore$ (c) Satisfies it

$\therefore$ (c) is correct.

Q.20. In an A.P. if $s_n = 3n^2 - n$ and its common difference is 6 then first term is_____

(a) 2 (b) 3
(c) 4 (d) 6

[June 2013]

Solution : (a) is correct.

$s_n = 3n^2 - n$

$s_1 = 3\times 1^2 - 1 = 2 = t_1$

$\therefore$ 1st term =2

Q.21. In an A.P if the sum of 4th & 12th term is 8 then sum of first 15 term is_____

(a) 60 (b) 120
(c) 110 (d) 150

[June 2013]

Solution : (a) is correct.

Given, $t_4 + t_{12} = 8$

or $a + 3d + 1 + 11d = 8$

or $2a + 14d = 8$

$\therefore s_{15} = \frac{15}{2}[2a + (15-1)d]$

$= \frac{15}{2} \times 8 = 60$

$\therefore$ (a) is correct

Q.22. There are 'n' AMs between 7 & 71 and 5th AM is 27 then 'n' = _____

(a) 15 (b) 16
(c) 17 (d) 18

[June 2013]

Solution : (a) is correct.

$c.d = \frac{b-a}{n+1}$ **(Tricks)**

$= \frac{71-7}{n+1} = \frac{64}{n+1}$

$A_5 = a + 5d$ (Tricks)

$= 7 + 5\times \frac{64}{n+1} = 27$

or $\frac{5\times 64}{n+1} = 20$

or $20n + 20 = 320$

or $20n = 300 \therefore n = 15$

$\therefore$ (a) is correct

Q.23. In a G.P the 6th term is 729 and the common ratio is 3 then is_____

(a) 2 (b) 3
(c) 4 (d) 7

[June 2013]

Solution : (b) is correct .

$t_1 = a; cr = r = 3$

$t_6 = 729$

or $ar^5 = 729$

or $a\times 3^5 = 3^6$

$\therefore a = 3$

Q.24. An AP has 13 terms whose sum is 143. The third term is 5, then first term is

(a) 4 (b) 7
(c) 9 (d) 2

[Dec. 2013]

Solution : (d) is correct

$\therefore t_3 = a + 2d = 5$ _____(1)

$\therefore 2d = 5 - a$

$s_{13} = \frac{13}{2}[2a + (13-1)d] = 143$

or $2a + 12d = \frac{143\times 2}{13} = 22$

or $a + 6d = 11$

or $a + 3\times 2d = 11$

or $a + 3\ (5 - a) = 11$

or $a + 15 - 3a = 11$

or $4 = 2a \therefore a = 2$

Tricks :- Go by choices

[Solve mentally by calculator]

Q.25. GM of a, b, c, d is 3 then GM of $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ is

(a) $\frac{1}{3}$ (b) 3
(c) $\frac{1}{81}$ (d) 81

[Dec. 2013]

Solution : (a) is correct

$G = 3 = (abcd)^{1/4}$ _____(1)

New GM $= \left(\frac{1}{a}.\frac{1}{b}.\frac{1}{c}.\frac{1}{d}\right)^{1/4} = \frac{1}{3}$

Tricks :- GM of a,b,c,d = 3

GM of their Reciprocals $= \frac{1}{3}$

Q.26. The value of $1^3 + 2^3 + 3^3 + \ldots\ldots + m^3$ is equal to

(a) $\left[\frac{m(m+1)}{2}\right]^3$

(b) $\frac{m(m+1)(2m+1)}{6}$

(c) $\left[\frac{m(m+1)}{2}\right]^2$

(d) None

[June 2014]

Solution : (c) is correct

Formula $= \left\{\frac{m(m+1)}{2}\right\}^2$

Q.27. The sum of the infinite GP $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\ldots\infty$ is equal to

(a) 1.95 (b) 1.5
(c) 1.75 (d) None

[June 2014]

Solution : (b) is correct

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} = 1.5$

Q.28. The sum of m terms of the series is 1 + 11 + 111+..... is equal to

(a) $\frac{1}{81}\left[10^{m+1} - 9m - 10\right]$

(b) $\frac{1}{2}\left[10^{m+1} - 9m - 10\right]$

(c) $[10^{m+1} - 9m - 10]$

(d) None of these

[June 2014, June 2015]

Solution : (a) is correct

Tricks :- Go by choices

For (a) put m = 1 ; we get

$s = \frac{1}{81}\left[10^{1+1} - 9\times 1 - 10\right] = 1 = 1$st term

Put m=2; $S = \frac{1}{81}\left[19^{2+1} - 9\times 2 - 10\right] = 12$

=1 +11 = Sum of 1st 2 terms

So; (a) is correct.

Q.29. If the sum of first 'n' terms of an A.P is $6n^2 + 6n$, then the fourth term of the series:

(a) 120 (b) 72
(c) 48 (d) 24

[Dec. 2014]

Solution : (c) is correct

S_n = Sum of 1st n terms of as AP.

$= 6n^2 + 6n$

$a = t_1 = s_1 = 6\times 1^2 + 6\times 1 = 12$

$s_2 = 6\times 2^2 + 6\times 2 = 36$

$c.d = d = s_2 - 2s_1 = 36 - 2\times 12 = 12$

$\therefore$

$t_4 = a + (4-1)d = 12 + 3\times 12 = 48$

Q.30. If $S_n = n^2p$ and $S_m = m^2p$; $(m \neq n)$ is the sum of A.P., then S_p = _____

(a) p^2 (b) p^3
(c) $2p^3$ (d) p^4

[Dec. 2014]

Solution : (b) is correct

$\because s_n = n^2 p$

$s_m = m^2 p$

$\therefore s_p = p^2.p = p^3$

Q.31. If x, y, z are the terms in G.P then the terms $x^2 + y^2, xy + yz, y^2 + z^2$ are in:

(a) A.P (b) G.P
(c) H.P (d) None of these

[Dec. 2014]

Solution : (b) is correct

$\because x; y; z$ are in G.P

Tricks:- Let x = 1 ; y = 2; z = 4 make a G.P

$\therefore x^2 + y^2 = 1^2 + 2^2 = 5$

$xy + yz = 1\times 2 + 2\times 4 = 10$

$y^2 + z^2 = 2^2 + 4^2 = 20$

$\therefore x^2 + y^2; xy + yz; y^2 + z^2 =$

5,10,20..... clearly are in G.P.

Q.32. Let S be the sum, P be the product and R be the sum of reciprocals of n terms of a G.P then P^2R^n =

(a) S^{2n} (b) S^{-n}
(c) S^n (d) S^{-2n}

[June 2015]

Solution : Let n = 3

Let S = 1+ 2 + 4 ----- a GP. = 7

Tricks:- P = $1\times 2\times 4 = 8$

$R = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} = \frac{4+2+1}{4} = \frac{7}{4}$

Let n = 3

$\therefore P^2R^n = P^2.R^3 = 8^2 \times \left(\frac{7}{4}\right)^3$

$= 64 \times \frac{349}{64} = 7^3$

$= s^3$

$= s^n$

$\therefore$ (c) is correct.

Q.33. The sum of n terms of an AP is $3n^2 + 5n$, which last term is 164.

(a) 25 (b) 27

(c) 29 (d) 31

[Dec. 2015]

Solution : (b) is correct

$S_n = 3n^2 + 5n$

$a = t_1 = S_1 = 3\times1^2 + 5\times1 = 8$

$S_2 = 3\times2^2 + 5\times2 = 22$

$d = S_2 - 2S_1 = 22 - 2\times8 = 6$

$n = \frac{t_n - a}{d} + 1 = \frac{164-8}{6} + 1 = 27$

Q.34. Three No's a,b,c are in A.P find a - b + c

(a) a (b) -b

(c) b (d) c

[Dec. 2015]

Solution : (c) is correct

let $a = 1; b = 2; c = 3$ makes an A.P.

$\therefore a - b + c = 1 - 2 + 3 = 2 = b.$

Q.35. Find the numbers whose GM is 5 and AM is 7.5:

(a) 12 and 13

(b) 13.09 and 1.91

(c) 14 and 11

(d) 17 and 19

[Dec. 2015]

Solution : (b) is correct

Tricks: Go by Choices

$GM = \sqrt{13.09\times1.51} = 5.$ (approx.)

$AM = \frac{13.09+1.91}{2} = 7.5$

Q.36. If $\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}$ are in Arithmetic Progression then a^2, b^2, c^2 are in _____.

(a) Arithmetic Progression

(b) Geometric Progression

(c) Both A.P & G.P

(d) None of these

[June 2016]

Solution : (a)

Tricks:- a^2, b^2, c^2 are in AP.

$a = 1, b = 5, c = 7$ Make it in AP.

let

$\therefore \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ in AP

$\frac{1}{5+7}, \frac{1}{7+1}, \frac{1}{1+5}$

$\left[\frac{1}{12}, \frac{1}{8}, \frac{1}{6}\right] \times 24$

2, 3, 4 is also in AP.

$\therefore$ Our assumption is correct.

Q.37. The sum of n terms of the series

If $\log(x) + \log\left(\frac{x^2}{y}\right) + \log\left(\frac{x^3}{y^2}\right) + \ldots$ is

(a) $\frac{n}{2}\left[2n\log\left(\frac{x}{y}\right) + \log xy\right]$

(b) $\frac{n}{2}\left[n\log xy + \log\left(\frac{x}{y}\right)\right]$

(c) $\frac{n}{2}\left[n\log\left(\frac{x}{y}\right) - \log xy\right]$

(d) $\frac{n}{2}\left[n\log\left(\frac{x}{y}\right) + \log xy\right]$

[June 2016]

Solution : (d) is correct.

Tricks : Go by Choices

put $n = 1; 2$ Then

(d) Should be equal to first term *i.e.* $\log x$

$\therefore$ option (d)

$= \frac{1}{2}\left[1.\log\frac{x}{y} + \log xy\right]$

$= \frac{1}{2}\left[\log\left(\frac{x}{y}xy\right)\right]$

$= \frac{1}{2}\log x^2 = \frac{1}{2}2\log x = \log x$

if $n = 2$

then option (d) should be equal to

$\log x + \log\frac{x^2}{y} = \log\frac{x^3}{y}$

Option (d)

Q.38. A G.P (Geometric Progression) consists of 2n terms. If the sum of the terms occupying the odd places is S_1 and that of the terms in even places is S_2. The common ratio of the progression is

(a) n (b) $2S_1$

(c) $\frac{S_2}{S_1}$ (d) $\frac{S_1}{S_2}$

[June 2016]

Solution : (c) **Tricks:-**

let $S = \underline{1} + 2 + \underline{4} + 8 + \underline{16} + \ldots$ to $2n$ terms

$S_1 = 1 + 4 + 16 + \ldots$ to n terms

$S_2 = 2 + 8 + 32 + \ldots$ to n terms

for $n = 1$

$S_1 = 1; S_2 = 2$

c.r. $= \frac{S_2}{S_1} = \frac{2}{1} = 2$ (True)

for $n = 2$

c.r. $= \frac{2+8}{1+4} = 2$ (Also True)

$\therefore$ (c) is correct.

Q.39. 2.353535 ..... = 2.3$\dot{5}$

(a) $\frac{233}{99}$ (b) $\frac{234}{99}$

(c) $\frac{232}{99}$ (d) None

[Dec. 2016]

Solution : (a) is correct.

Tricks : Go by choices [use calculator]

Divide 233 by 99 we get 2.3535...

Q.40. The number of terms of the series needed for the sum of the series 50 + 45 + 40 + ... becomes zero

(a) 22 (b) 21

(c) 20 (d) None

[Dec. 2016]

Solution : (b) is correct.

Tricks : Go by choices

Let (b) is correct.

$S_{21} = \frac{21}{2}[2\times50 + (21-1)\times(-5)]$

$= 0$

So, (b) is correct.

Q.41. A person received the salary for the 1st year is ₹5,00,000 per year and he received an increment of ₹15,000 per year then the sum of the salary he taken in 10 years

(a) ₹56,75,000 (b) ₹72,75,000 (c) ₹63,75,000 (d) None of these

[Dec. 2016]

Solution : (a) is correct.

$S_{10} = \frac{10}{2}[2\times5,00,000 + (10-1)\times15000]$

$= ₹\ 56,75,000.$

Q.42. If a, -3, b, 5, c are in A.P then the value of c is

(a) -7 (b) 1 (c) 9 (d) 13

[June 2017]

Solution : a, -3 ; b ; 5, are in AP.

So; $b = \frac{-3+5}{2}$ [$\because$ b is A.M. of -3 & 5]

b = 1

Similarly ;

$5 = \frac{b+c}{2} \Rightarrow b + c = 10$

or c = 10 - b = 10 - 1 = 9

$\therefore$ (c) is correct.

Q.43. The sum of n terms of the series 1+(1+3)+(1+3+5)+.....

(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\frac{n(n+1)(2n+1)}{3}$

(c) $\frac{n(n+1)(n+2)}{6}$ (d) None

[June 2017]

Solution : Tricks :- Go by Choices

For (a) put n = 1 $\Rightarrow$ It should be equal

$\frac{1(1+1)(2\times1+1)}{6} = 1$ (True)

If n = 2 Then it should be equal to sum of 1st 2 term

So; $\frac{2(2+1)(2\times2+1)}{6} = 1 + (1+3)$

or $\frac{30}{6} = 5$ (True)

$\therefore$ Option (a) is correct.

Q.44. The sum of first 20 terms of a G.P is 1025 times the sum of first 10 terms then the common ratio is _____

(a) 2 (b) $2\sqrt{2}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$

[June 2017]

Solution : Given

$S_{20} = 1025.\ S_{10}$

$\frac{a(r^{20}-1)}{r-1} = 1025 . \frac{a(r^{10}-1)}{r-1}$

or; $r^{20} - 1 = 1025\ (r^{10} - 1)$

or; $(r^{10})^2 - 1^2 = 1025\ (r^{10} - 1)$

or; $(r^{10} + 1)(r^{10} - 1) = 1025\ (r^{10} - 1)$

or; $r^{10} + 1 = 1025$

or; r = 2

$\therefore$ Option (a) is correct.

Q.45. Find the sum of all natural numbers between 100 and 1000 which are divisible by 11 is :

(a) 44,550 (b) 66,770 (c) 55,440 (d) 33,440

[Dec. 2017]

Solution : (a)

Series

S = 110 + 121 + 132 + ... + 990

$$n=\frac{l-a}{d}+1=\frac{990-110}{11}+1=81$$

$$S=\frac{n}{2}(a+l)=\frac{81}{2}(110+990)=44,550$$

Q.46. If pth, qth, rth terms of a G.P. be a, b, c respectively, then (q - r) log a + (r - p) log b + (p - q) log c =

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) None

[June 2018]

Solution : (*a*)

Tricks :- It is in cyclic order.

Q.47. If a, b, c, d are in GP then $(b-c)^2+(c-a)^2+(d-b)^2$ = ?

(*a*) $(a-b)^2$ (*b*) $(a-d)^2$

(*c*) $(c-d)^2$ (*d*) 0

[June 2018]

Solution : (*b*)

a, b, c, d $\rightarrow$ in GP

let a = 1 ; b = 2 ; c = 4 ; d = 8 in G.P.

$\therefore (b-c)^2+(c-a)^2+(d-b)^2$

$=(2-4)^2+(4-1)^2+(8-2)^2$

$=4+9+36=49=7^2$

GBC

For (*b*) $(a-d)^2=(1-8)^2=7^2=49.$

$\therefore$ (*b*) is correct.

Q.48. If the n th term of a series, $a_n = 3^n - 2^n$ then S_n = ?

(*a*) $\frac{3}{2}(3^n-1)+1(n+1)$ (*b*) $\frac{3}{2}(3^n+1)-1(n+1)$

(*c*) $\frac{3}{2}(3^n-1)-n(n+1)$ (*d*) $\frac{3}{2}(3^n+1)-1(n-1)$

[June 2018]

Solution : (*c*)

$\because a_n=3^n-2^n$

$a_1=3^1-2^1=1$

$a_2=3^2-2^2=5$

$s_2=a_1+a_2=1+5=6$

Tricks :- Go by choices (GBC)

for (*c*) let

$$s_n=\frac{3}{2}(3^n-1)-n(n+1)$$

$$s_1=\frac{3}{2}(3^1-1)-1(1+1)=\frac{3}{2}2-2=1=a_1 \text{ (True)}$$

Now $s_2=\frac{3}{2}(3^2-1)-2(2+1)$

$=\frac{3}{2}\times8-6=12-6=6=a_1+a_2$ (True)

$\therefore$ (*c*) is correct.

Q.49. The sum to m terms of the series 1 + 11 + 111 + ... upto m terms, is equal to :

(*a*) $\frac{1}{81}(10^{m+1}-9m-10)$ (*b*) $\frac{1}{27}(10^{m+1}-9m-10)$

(*c*) $10^{m+1}-9m-10$ (*d*) None of these

[May 2018]

Solution : (a)

Tricks:-GBC

If m = 1 $\Rightarrow$ S_1 = Sum of 1st 1 term = 1

If m = 2 $\Rightarrow$ S_2 = Sum of 1st 2 term

= 1 + 11 = 12

Note:- Check only for m = 1 and m = 2

(a) If m = 1

$$S_1=\frac{1}{81}(10^{1+1}-9\times1-10)=\frac{1}{81}(100-9-10)=1$$

(a) If m = 2

$$S_2=\frac{1}{81}\left[10^{2+1}-9\times2-10\right]=12$$

So; Option (a) is true for m = 1 & m = 2.

So; (a) is correct.

Q.50. A person pays Rs. 975 in monthly instalments, each instalment is less than former by Rs. 5. The amount of Ist instalment is ₹ 100. In what time will be entire amount be paid?

(*a*) 26 months

(*b*) 15 months

(*c*) Both (*a*) & (*b*)

(*d*) 18 months

[May 2018]

Solution : (*b*)

Tricks:- Go by choices (GBC)

Series

S = 100 + 95 90 +.....to n months (let)

= 975.

1st check for n = 15 months

$$S=\frac{15}{2}\left[2\times100+(15-1)\cdot(-5)\right]$$

If loan is paid off in n = 15 months, then no need of other instalments. So (b) is correct.

Q.51. If the sum of n terms of an AP is $(3n^2-n)$ and its common difference is 6, then its first term is :

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 1

[May 2018]

Solution : (b)

$S_n=3n^2-n$

Tricks:-

$\because t_1=S_1=3\times1^2-1=2$

= sum of 1st 1 term.

Q.52. Insert two arithmetic means between 68 and 260.

(*a*) 132, 196 (*b*) 130, 194

(*c*) 70, 258 (*d*) None

[May 2018]

Solution : (a)

Tricks:-

Go by choices

(a) 68; 132 ; 196; 260 are in AP.

Hence ; 132 ; 196 are

A.Ms. b/w 68 and 260.

Hence (a) is correct.

Q.53. If the P^{th} term of an A.P. is 'q' and the q^{th} term is 'p', then its r^{th} term is

(*a*) p + q + r (*b*) p + q - r

(*c*) p - q - r (*d*) p + q

[Nov. 2018]

Solution : (b)

Tricks:- c.d $=\frac{q-p}{p-q}=\frac{(p-q)}{p-q}=-1$

$\therefore t_r=t_p+(r-p)d$

$=q+(r-p)\cdot(-1)$

$=q+p-r$

Q.54. The 3rd term of a G.P. is $\frac{2}{3}$ and the 6th term is $\frac{2}{81}$, then the 1st term is

(*a*) 2 (*b*) 6

(*c*) 9 (*d*) $\frac{1}{3}$

[Nov. 2018]

Solution : (*b*)

$t_3=ar^2=\frac{2}{3}$; $t_6=ar^5=\frac{2}{81}$

or $ar^2\cdot r^3=\frac{2}{81}$

or $\frac{2}{3}r^3=\frac{2}{81}\Rightarrow r^3=\left(\frac{1}{3}\right)^3$

$\because ar^2=\frac{2}{3}$

or $a\cdot\left(\frac{1}{3}\right)^2=\frac{2}{3}$

or a = 6

Q.55. The sum of the series –8, –6, –4,...n terms is 52. The number of terms n is

(*a*) 10 (*b*) 11

(*c*) 13 (*d*) 12

Solution : (*c*)

Series S = - 8 - 6 - 4 ..... to n terms

first term = -8 ; c.d = d = 2

Tricks :- Go by choices (Use calculator)

option (*c*)

$$S_{13}=\frac{13}{2}\left[2\times(-8)+(13-1)\times2\right]=52$$

Q.56. The value of K, for which the terms 7K + 3, 4K - 5, 2K + 10 are in A.P., is

(*a*) –13 (*b*) –23

(*c*) 13 (*d*) 23

[Nov. 2018]

Solution : (*b*)

Formula 2A = a + b

$\therefore$ 2(4k - 5) = 7k + 3 + 2k + 10

or 8k - 10 = 9k + 13

or k = -23

Q.57. The ratio of sum of n terms of the two AP's is (n+1) then the ratio of their m^{th} terms is

(*a*) (m + 1) : 2m

(*b*) (m + 1) : (m - 1)

(*c*) (2m - 1) : (m + 1)

(*d*) m : (m - 1)

[June 2019]

Solution : Given that

$$\frac{S_n^I}{S_n^{II}}=\frac{n+1}{n-1}$$

Tricks:-

To find the ratio of r^{th} term ;

put n = 2r - 1

$\therefore$ Put n = 2m - 1

Ratio of m^{th} term

$=\frac{2m-1+1}{2m-1-1}=\frac{2m}{2m-2}$

$=\frac{2m}{(2m-2)}=\frac{m}{m-1}$

(d) is correct.

Q.58. In a G.P. if the fourth term is '3' then the product of first seven terms is

(*a*) 3^5 (*b*) 3^7

(*c*) 3^6 (*d*) 3^8

[June 2019]

Solution :(*b*)

Tricks:-

Product of 1st (2r-1) terms of a

G.P = $(t_r)^{2r-1}$

$\therefore t_4 = 3$

So; Product of 1st $2\times4-1=7$ terms

$= (t_4)^{2\times4-1} = 3^7$

(b) is correct.

Details:-

$\because$ Product of 1st 7 terms

$= a.ar.ar^2.ar^3.....ar^6$

$= a^7 . r^{1+2+3+.....+6}$

$= a^7 . r^{\frac{6}{2}(6+1)} = a^7 . r^{21}$

$= (ar^3)^7 = 3^7$

Q.59. If 2 + 6 + 10 + 14 + 18 + ... + x = 882 then the value of x

(a) 78 (b) 80

(c) 82 (d) 86

[June 2019]

Solution : (c)

S = 2 + 6 + 10 + 14 ..... + x(to n terms) = 882

$\therefore \frac{n}{2}[2+x] = 882$ (1)

Where x = Last term

Last term $= x = 2+(n-1)\times4$

$x = 4n-2$

or $4n = x+2$

or $n = \frac{x+2}{4}$

$\therefore$ From (1) ; we get

$\frac{(x+2)}{4\times2}(x+2) = 882$

or $(x+2)^2 = 8\times882 = 84^2$

$\therefore x+2 = 84 \Rightarrow x = 82$.

Tricks:-

Let $t_n = x$

or $2+(n-1)\cdot4 = x$

or $4n-2 = x$

or $n = \frac{x+2}{4}$

For GBC

(c) If $x = 82 \Rightarrow n = \frac{82+2}{4} = 21$

$\therefore S = \frac{n}{2}(a+l) = \frac{21}{2}(2+x)$

$= \frac{21}{2}(2+82) = 882$

$\therefore$ (c) is correct.

Q.60. If $y = 1+x+x^2+.....\infty$ then x =

(a) $\frac{y-1}{y}$ (b) $\frac{y+1}{y}$

(c) $\frac{y}{y+1}$ (d) $\frac{y}{y-1}$

[June 2019]

Solution : (a)

$y = 1+x+x^2+..... \infty$ are in G.P.

$\therefore y = \frac{1}{1-x}$ Where c.r = x

or $1-x = \frac{1}{y}$

or $x = 1-\frac{1}{y} = \frac{y-1}{y}$. $\left[\because S_\infty = \frac{a}{1-r}\right]$

Q.61. In the series 25, 5, 1, ... 1/3125 which term is 1/3125?

(a) 8th term (b) 9th term

(c) 15th term (d) None of these

[Dec. 2019]

Solution : (a)

Let $t_n = \frac{1}{3125}$.

$\therefore 25\cdot\left(\frac{1}{5}\right)^{n-1} = \frac{1}{5^5}$

or $5^2\cdot\frac{1}{5^{n-1}} = \frac{1}{5^5}$

or $5^{n-1} = 5^7 \Rightarrow n-1 = 7$

$\therefore n = 8$

Q.62. The sum of five terms of AP is 75 find the 3rd term is.

(a) 20 (b) 30

(c) 15 (d) None of these

[Dec. 2019]

Solution : (c)

$t_3 = a+(3-1)d = a+2d$.

$S_5 = \frac{5}{2}[2a+(5-1)d] = 75$

or $\frac{5}{2}\cdot2[a+2d] = 75$

or $a+2d = \frac{75}{5} = 15$

So, $t_3 = 15$.

Q.63. (c+a-b)/b, (a+b-c)/c, (b+c-a)/a are in AP then a,b,c are in

(a) AP (b) GP

(c) HP (d) None of these

[Dec. 2019]

Solution : (d)

Adding 2 to each term; we get

$\frac{c+a-b}{b}+2; \frac{a+b-c}{c}+2; \frac{b+c-a}{a}+2$

are also in AP

$\Rightarrow \frac{a+b+c}{b}; \frac{a+b+c}{c}; \frac{a+b+c}{a}$

are in AP.

Dividing all terms by (a + b + c) ; we get

$\frac{1}{b}, \frac{1}{c}; \frac{1}{a}$ are also in AP.

$\Rightarrow$ b ; c ; a are in HP.

OR a, c ; b are in HP.

but a ; b ; c are not in HP.

Q.64. The sum of series 1/2+1/3²+1/2³+1/3⁴ + ... up to infinity is

(a) 25/24 (b) 19/24

(c) 1/12 (d) None of these

[Dec. 2019]

Solution : (b)

$S = \frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+.....$to ∞

$= \left[\frac{1}{2}+\frac{1}{2^3}+.....\text{to }\infty\right]+\left[\frac{1}{3^2}+\frac{1}{3^4}+.....\text{to }\infty\right]$

$= \frac{\frac{1}{2}}{1-\frac{1}{2^2}} + \frac{\frac{1}{3^2}}{1-\frac{1}{3^2}}$

$= \frac{1/2}{3/4} + \frac{1/9}{8/9}$

$= \frac{1}{2}\times\frac{4}{3}+\frac{1}{8} = \frac{2}{3}+\frac{1}{8}$

$= \frac{16+3}{24} = \frac{19}{24}$

Q.65. The 20th term of arithmetic progression whose 6th term is 38 and 10th term is 66 is.....

(a) 136 (b) 118

(c) 178 (d) 210

[Dec. 2020]

Solution : Tricks

Common difference

$= d = \frac{t_{10}-t_6}{10-6}$

$= \frac{66-38}{4} = 7$

Tricks

$t_{20} = a + 19d$

$= [a + (6 - 1) d] + 14d$

$= 38 + 14 \times 7$

$= 136$

$\therefore$ (a) is correct.

Q.66. Three numbers in G.P with their sum is 130 and their product is 27,000 are.....

(a) 90, 30, 10

(b) 10,30,90

(c) (a) & (b) Both

(d) 10,20,30

[Dec. 2020]

Solution : (c)

Tricks : GBC (Go by choices)

* (a) & (b) both follow G.P.
* sum of terms = 90 + 30 +10 = 130 (also follows)
* Their product = 90 × 30 × 10 = 27000/-

Which is also satisfied

$\therefore$ option (c) is correct.

Q.67. Divide 69 into 3 parts which are in A.P and are such that the product of first two parts is 460

(a) 20, 23, 26 (b) 21, 23, 25

(c) 19, 23, 27 (d) 22, 23, 24

[Dec. 2020]

Solution : Tricks : GBC (Go by choices)

* All options are in A.P.
* Only in option (a)

Product of 1st two terms

= 20 × 23 = 460 (True)

$\therefore$ (a) is correct.

Q.68. The nth terms of the series 3 + 7 + 13 + 21 + 31 +.....is

(a) 4n - 1 (b) $n^2 + 2n$

(c) $n^2 + n + 1$ (d) $n^3 + 2$

[Jan. 2021]

Solution : Tricks

In such type of Questions always find answer by GBC (Go by choices).

For $n = 1 \Rightarrow t_1 = 3$

for $n = 2 \Rightarrow t_2 = 7$

and $n = 3 \Rightarrow t_3 = 13$

Putting n = 1 in all options, we get $t_1 = 3$

So, Here, we cannot decide any option.

Now putting n = 2 in all options we get in

(a) $t_2 = 4 \times 2 - 1 = 7 = t_2$ (True)

(b) $t_2 = 2^2 + 2 \times 2 = 8 \neq t_2$ (False)

(c) $t_2 = 2^2 + 2 + 1 = 7 = t_2$ (True)

(d) $t_2 = 2^3 + 2 = 10 \neq t_2$ (False)

Hence, we conclude that option (a) or (c) should be answer. (Both same)

So check for n = 3 in (a) & (c); we get

(a) $t_3 = 4 \times 3 - 1 = 11 \neq 13 = t_3$ (False)

(c) $t_3 = 3^2 + 3 + 1 = 13 = t_3$ (True)

Hence (c) should be correct.

Q.69. The number of integers from 1 to 100 which are neither divisible by 3 nor by 5 nor by 7 is

(a) 67 (b) 55 (c) 45 (d) 33

[Jan. 2021]

Solution : (c) is correct.

Let U = Set of integers from 1 to 100 = 1, 2, 3, ———; 100

$\therefore$ n (U) = 100

Let A = Integers divisible by 3

= {3, 6, 9, ...; 99} in A.P.

Let

B = Integers divisible by 5.

= {5, 10, 15, ...; 100} in A.P.

and C = Integers divisible by 7

= {7, 14, 21, ...; 98} in A.P.

We have to find No. of integers from 1 to 100 which are neither divisible by 3, nor divisible by 5, nor divisible by 7

$= n(A \cup B \cup C)' = n(U) - N(A \cup B \cup C)$

$= 100 - [n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)] = ?$ (1)

$\therefore n(A) = \frac{1-a}{d} + 1 = \frac{99-3}{3} + 1 = 33$

$n(B) = \frac{100-5}{5} + 1 = 20$

$n(C) = \frac{98-7}{7} + 1 = 14$

Now $(A \cap B)$ = Integers divisible by 3 & 5 both i.e., divisible by 15.

$= \{15, 30, 45, \ldots; 90\}$

$n(A \cap B) = \frac{90-15}{15} + 1 = 6$

$(B \cap C)$ = Integers divisible by 5 and 7 i.e. divisible by LCM of 5 & 7 = 35

$= \{35, 70\}$

$n(B \cap C) = 2$

$(C \cap A)$ = Integers divisible by 3 and 7 i.e. divisible by 21.

$= \{21, 42, 63, 84\}$

$n(C \cap A) = 4$

And $(A \cap B \cap C)$ = Integers divisible by 3, 5 and 7 all i.e. LCM of 3, 5 & 7 = 105

$= \{\}$

$n(A \cap B \cap C) = 0$

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 33 + 20 + 14 - 6 - 2 - 4 + 0$

$= 55$

$\therefore$ From (1)

$n(A \cup B \cup C)' = 100 - n(A \cup B \cup C)$

$= 100 - 55 = 45.$

$\therefore$ (*c*) is correct.

Q.70. In a geometric progression the 3rd and 6th terms are respectively 1 and -1/8. The first term (*a*) and common ratio are respectively.

(*a*) 4 and $\frac{1}{2}$ (*b*) 4 and $\frac{-1}{4}$ (*c*) 4 and $\frac{-1}{2}$ (*d*) 4 and $\frac{1}{4}$

[Jan. 2021]

Solution : (*c*) is correct.

Tricks GBC [Go by choices]

From (*a*) $t_3 = a r^{3-1} = 4\left(\frac{1}{2}\right)^2 = 1$ (True)

and $t_6 = a r^{6-1} = \left(\frac{1}{2}\right)^5 = \frac{1}{8} \neq -\frac{1}{8}$ (False)

So (*a*) is False

(*b*) $t_2 = a r^2 = 4\left(-\frac{1}{4}\right)^2 = 4\frac{1}{16} = \frac{1}{4} \neq 1$

(It is also False)

(*c*) $t_3 = a r^2 = 4\left(-\frac{1}{2}\right)^2 = 4\frac{1}{4} = 1$ (True)

$t_6 = a r^5 = 4\left(-\frac{1}{2}\right)^5 = 4\left(-\frac{1}{32}\right) = -\frac{1}{8}$ (True)

$\therefore$ (*c*) is correct

Q.71. The number of terms of the series: 5 + 7 + 9 + … must be taken so that the sum may be 480

(*a*) 20 (*b*) 10

(*c*) 15 (*d*) 25

[July 2021]

Solution : (*a*) is correct

Let S = 5 + 7 + 9 + … to "n" terms

= 480

Tricks : Go by choices (GBC)

For (*a*) at n = 20

$S = \frac{20}{2}[2 \times 5 + (20-1)\cdot 2]$

$= 10(10 + 38) = 480$ (True)

Hence, (*a*) is correct

Q.72. If the sum of 'n' terms of an AP (Arithmetic Progression) is $2n^2$, the fifth term is _____

(*a*) 20 (*b*) 50

(*c*) 18 (*d*) 25

[July 2021]

Solution : (*c*) is correct

$Q\ t_5 = S_5 - S_4$ [i.e. sum of 1st 5 terms – sum of 1st 4 terms]

$= 2 \times 5^2 - 2 \times 4^2$

$= 50 - 32 = 18$

Q.73. The sum of three numbers in a geometric progression is 28. When 7,2 and 1 are subtracted from the first, second and the third numbers respectively, then the resulting numbers are in arithmetic progression. What is the sum of squares of the original three numbers?

(*a*) 510 (*b*) 456 (*c*) 400 (*d*) 336

[July 2021]

Solution : (*d*) is correct

Let first term = a and common ratio = r

3 Numbers are a, ar, ar^2

$\because \quad a + ar + ar^2 = 28 \quad (1)$

From question a – 7; ar – 2 and $ar^2 - 1$ are in AP

$\therefore \quad 2(ar - 2) = (a - 7) + (ar^2 - 1)$

or; $\quad 2ar - 4 = a + ar^2 - 8$

Adding "ar" on both sides; we get

$\Rightarrow \quad 2ar - 4 + ar = a + ar + ar^2 - 8$

or $\quad 3ar - 4 = 28 - 8$

(From (*i*))

or; $\quad 3ar = 20 + 4 = 24$

or $\quad ar = \frac{24}{3} = 8 \quad (2)$

From (1); $\quad a + ar + ar^2 = 28$

or ; $\quad a + ar + ar.r = 28$

or $\quad a + 8 + 8r = 28$

or ; $\quad a = 20 - 8r$

From (2)

$ar = 8$

or $\quad (20 - 8r)r = 8$

or $\quad 4(5 - 2r)r = 8$

or $\quad 5r - 2r^2 = 2$

or $\quad 2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

or $\quad 2r(r-2) - 1(r-2) = 0$

or $\quad (r-2)(2r-1) = 0$

$\therefore \quad r = 2; \frac{1}{2}$

From (2) $ar = 8$

or $\quad a \cdot 2 = 8 \Rightarrow a = 4$

Sum of squares of these numbers

$= a^2 + a^2 r^2 + a^2 r^4$

$= 4^2 + 4^2 \times 2^2 + 4^2 \times 2^4$

$= 16 + 64 + 256 = 336$

Q.74. The sum of square of any real positive quantities and its reciprocal is never less that

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

[July 2021]

Solution : (*b*) is correct

Let a positive no. = x

From question,

Two nos. are x^3 & $\frac{1}{x^2}$

Its Arithmetic mean

$= A = \frac{x^2 + \frac{1}{x^2}}{2}$

and Its Geometric mean

$G = \sqrt{x^2 \frac{1}{x^2}} = \sqrt{1} = 1$

We know that

$A \geq G$

or $\frac{x^2 + \frac{1}{x^2}}{2} \geq 1$

or $x^2 + \frac{1}{x^2} \geq 2$

$\therefore$ Minimum value of $x^2 + \frac{1}{x^2}$ is 2

Q.75. If the sum and products of three numbers in G.P. are 7 and 8 respectively, then 4th term of the series is

(*a*) 6 (*b*) 4

(*c*) 8 (*d*) 16

[Dec. 2021]

Solution : (*c*) If the sum and product of three numbers in G.P. are 7 and 8 respectively, then 4th term of the series is

(*a*) 6 (*b*) 4

(*c*) 8 (*d*) 16

Tricks

Product of 3 terms in G.P. = 8-1.2.4

[Note - Find 3 Factors of 8 such that the factors in G.P.]

So Nos. are = 1, 2, 4

And their sum = 7

Here 1 + 2 + 4 = 7 (True)

$\therefore T_4 = 4 \times CR$

$= 4 \times 2$

$= 8$

Q.76. The sum of series 7+14+21+….. to 17th term is:

(*a*) 1071 (*b*) 971

(*c*) 1171 (*d*) 1271

[Dec. 2021]

Solution : (*a*)

S = 7 + 14 + 21… to 17 terms

= 7 [1 + 2 + 3 …. to 17 terms]

$= 7.\frac{17(17+1)}{2} = 1071$

[$\because$ 1 + 2 + 3 + ….. to *n* terms]

$= \frac{n(n+1)}{2}$

Q.77. The sum of first n terms of an AP is $3n^2 + 5n$. The series is:

(*a*) 8, 14, 20, 26.....

(*b*) 8, 22, 42, 68,.....

(*c*) 22, 68, 114,.....

(*d*) 8, 14, 28, 44,.....

[Dec. 2021]

Solution : (*a*)

$\because\ S_n = 3n^2 + 5n$

$S_1 = 3 \times 1^2 + 5 \times 1 = 8$

$S_2 = 3 \times 2^2 + 5 \times 2 = 22$

$S_3 = 3 \times 3^2 + 5 \times 3 = 42$

GBC

(A)

$S_1 = 8$ (True)

$S_2 = 8 + 14 = 22$ (True)

$S_3 = 8 + 14 + 20 = 42$ (True)

$\therefore$ **a** is correct.

Details

$a = t_1 = S_1 = 3 \times 1^2 + 5 \times 1 = 8$

$S_2 = 3 \times 2^2 + 5 \times 2 = 22$

$c.d = d = S_2 - 2S_1 = 22 - 2 \times 8 = 6$

$t_n = a + (n-1)\,d$

$= 8 + (n-1).6 = 8 + 6n - 6 = 6n + 2$

$\therefore\ t_1 = 6 \times 1 + 2 = 8$

$t_2 = 6 \times 2 + 2 = 14$

$t_3 = 6 \times 3 + 2 = 20$

So (a) is Correct.

Q.78. The largest value of n for which $\frac{1}{2} + \frac{1}{2^2} + \ldots \frac{1}{2^n} < 0.998$ is _____.

(*a*) 9 (*b*) 6

(*c*) 7 (*d*) 8

[Dec. 2021]

Solution : (*a*)

$S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^n} < 0.998$

$S = \frac{\frac{1}{2}\left[1 - (½)^n\right]}{1 - ½} = \frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{½} < 0.998$

$= 1 - \frac{1}{2^n} < 0.998$

or $1 - 0.998 < \frac{1}{2^n}$

or $0.002 < 2^{-n}$

Calculator Trick

Press 2 ÷ button = button 9 times

= 0.00195

(Makes True)

$\therefore\ n = 9$

Q.79. If the nth term of the arithmetic progression 9, 7, 5 ... is same as the nth term of the arithmetic progression 15, 12, 9 ..., then n will be

(*a*) 7 (*b*) 9

(*c*) 15 (*d*) 11

[June 2022]

Solution : t_n of 1^{st} AP = t_n of 2^{nd} AP

$\therefore\ 9 + (n-1)(-2) = 15 + (n-1)(-3)$

or; $9 - 2n + 2 = 15 - 3n + 3$

or $3n - 2n = 18 - 11$

or $n = 7$

$\therefore$ (a) is correct

Q.80. In a geometric progression, the second term is 12 and the sixth term is 192. Find the 11^{th} term.

(*a*) 3,072 (*b*) 1,536

(*c*) 12,288 (*d*) 6,144

[June 2022]

Solution : Given

$t_2 = ar = 12$.....(1)

$t_6 = ar^5 = 192$.....(2)

Eqn. (2) ÷ (1); we get

$\frac{t_6}{t_2} = \frac{ar^5}{ar} = \frac{192}{12}$

or; $r^4 = 16 = 2^4$

$\therefore\ r = 2$

Now $t_{11} = ar^{11-1} = ar^{10}$

$= ar^5.r^5$

$= 192 \times 2^5$ (From (2))

$= 6144$

$\therefore$ (d) is correct

Q.81. The first and last terms of an arithmetic progression are 5 and 905. Sum of the terms is 45,955. The number of terms is

(*a*) 99 (*b*) 100

(*c*) 101 (*d*) 102

[June 2022]

Solution : Let No. of terms = n.

$S_n = \frac{n}{2}(a + l) = 45,955$

Where a = 1^{st} term;

l = last term.

$\frac{n}{2}(5 + 905) = 45955$

or $\frac{n}{2} \times \frac{455}{910} = 45955$

or $455n = 45955$

or; $n = \frac{45955}{455} = 101$

(*c*) is correct.

Q.82. The sum of first eight terms of geometric progression is five times the sum of the first four terms. The common ratio is

(*a*) $\sqrt{2}$ (*b*) $\sqrt{3}$

(*c*) 4 (*d*) 2

[June 2022]

Solution : Given

Sum of 1st 8 terms

= 5 (sum of 1st 4 terms)

or; $\frac{a\left(r^8 - 1\right)}{r - 1} = \frac{5.a\left(r^4 - 1\right)}{r - 1}$

or; $r^8 - 1 = 5\,(r^4 - 1)$

or; $(r^4)^2 - 1^2 = 5(r^4 - 1)$

or; $(r^4 - 1).\,(r^4 + 1) = 5\,(r^4 - 1)$

or; $r^4 + 1 = 5$

or; $r^4 = 5 - 1 = 4 = 2^2$

or, $(r^2)^2 = 2^2$

or $r^2 = 2\ \therefore r = \sqrt{2}$

(*a*) is correct.

Q.83. If *p*th term of an AP is q and its *q*th term is *p*, then what will be the value of (*p* + *q*)th term?

(*a*) 0 (*b*) 1

(*c*) $p + q - 1$ (*d*) $2(p + q - 1)$

[Dec. 2022]

Solution: *Detail :*

Let $t_1 = a$ and common difference = d

$\therefore\ c \times d = d = \frac{t_p - t_q}{p - q} = \frac{q - p}{-(q - p)}$

$= -1$

Tricks :

$t_{p+q} = t_p + (p + q - p)d$

$= q + (q)\,(-1)$

$= q - q = 0$

$\therefore$ (*a*) is correct.

Q.84. In a G P, 5th term is 27 and 8th term is 729. Find its 11th term.

(*a*) 729 (*b*) 6,561

(*c*) 2,187 (*d*) 19,683

[Dec. 2022]

Solution: Let $t_1 = a$ and $c \times r = r$

$\therefore\ \frac{t_8}{t_5} = \frac{ar^7}{ar^4} = \frac{729}{27}$

or; $r^3 = 27 = 3^3$

$\therefore\ r = 3$

$\therefore\ t_{11} = t_8 \times r^3 = 729 \times 3^3 = 729 \times 27$

$= 19,683.$

$\therefore$ (*d*) is correct.

Q.85. How many number between 74 and 25,556 are divisible by 5?

(*a*) 5090 (*b*) 5097

(*c*) 5095 (*d*) 5075

[June 2023]

Solution: Series

S = 75 + 80 + 85 + ... + 25,555

Total No. of Nos. divisible by 5

$= \frac{1 - a}{d} + 1 = \frac{25,555 - 75}{5} + 1 = 5097$

Where a = 1st term

l = last term

(*b*) is correct.

Q.86. If 9th and 19th term of an Arithmetic Progression are 35 and 75, respectively, then its 20th term is:

(*a*) 78 (*b*) 79

(*c*) 80 (*d*) 81

[June 2023]

Solution: Tricks

Common difference

$= d = \frac{A_{19} - A_9}{19 - 9} = \frac{75 - 35}{10}$

$= \frac{40}{10} = 4$

$t_{20} = t_{19} + cd = 75 + 4 = 79$

(*b*) is correct.

Q.87. If 4th, 7th and 10th terms of a Geometric Progression are *p*, *q* and *r*, respectively then:

(*a*) $p^2 = q^2 + r^2$

(*b*) $p^2 = qr$

(*c*) $q^2 = pr$

(*d*) $pqr + pq + 1 = 0$

Solution: Let $t_1 = a$ and $c.r = x$

$t_4 = ax^3 = p$

$t_7 = ax^6 = q$

$t_{10} = ax^9 = r$

Clearly ; $q^2 = pr$

$(ax^6)^2 = ax^3 \times ax^9$

$\Rightarrow a^2x^{12} = a^2 \times x^{3+9}$

$= a^2\,x^{12}$

(True)

$\therefore$ (*b*) is correct.

12 CHAPTER SETS, FUNCTION AND RELATION

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If $A=\{X : X^2-3X+2=0\}$ $B=\{X : X^2-4X+12=0\}$, Then B-A is equal to

(a) $\{-6\}$ (b) $\{1\}$

(c) $\{1,2\}$ (d) $\{2,-6\}$

[June 2010]

Solution : (a) $\because\ x^2-3x+2=0$

or $x^2-2x-x+2=0$

or $x(x-2)-1(x-2)=0$

or $(x-2)(x-1)=0$

$\therefore\ x=1;2$

$\therefore\ A=\{1,2\}$

And $x^2+4x-12=0$

or $x^2+6x-2x-12=0$

or $x(x+6)-2(x+6)=0$

or $(x+6)(x-2)=0$

$\therefore\ x=-6\ ;\ 2$

$\therefore\ B=\{-6\ ;\ 2\}$

$\therefore\ B-A=\{-6\ ;\ 2\}-\{1\ ;\ 2\}$

$=\{-6\}$

$\therefore$ (a) is correct

Q.2. If F: A → R is a real valued function defined by $f(x)=\frac{1}{x}$ then

(a) R (b) $R-\{1\}$

(c) $R-\{0\}$ (d) R-N

[June 2010]

Solution : (c)

$f(x)=\frac{1}{x}$ is defined at all $x \in R$ except $x=0$

$\therefore\ A=R-\{0\}$

$\therefore$ (c) is correct

Q.3. In the set N of all natural numbers the relation R defined by a R b "if and only if, a divide b", then the relation R is:

(a) Partial order relation

(b) Equivalence relation

(c) Symmetric relation

(d) None of these

[June 2010]

Solution : (a) It is transitive relation. *i.e.* partial order relation

Q.4. For any two sets A and B, $A\cap(A'\cup B)=$_____, where A' represent the compliment of the set A

(a) $A\cap B$ (b) $A\cup B$

(c) $A\cup B$ (d) None of these

[Dec. 2010]

Solution : Tricks : Take an example and then decide the answer

Let $U=\{0,1,2,3,4,5\}$

$A=\{0,1,2,3\}$

$B=\{2,3,4,5\}$

$\therefore A^1=\cup-A$

$=\{4;5\}$

$A^1\cup B=\{4,5\}\cup\{2,3,4,5\}$

$=\{2,3,4,5\}$

$\therefore A\cap(A^1\cup B)$

$=\{0,1,2,3\}\cap\{2,3,4,5\}$

$=\{2,3\}$

$=A\cap B$

$\therefore$ (a) is correct

IInd method $=(A\cap A')\cup(A\cap B)=\{\ \}\cup(A\cap B)=A\cap B$

Q.5. If $f : R\to R$, $f(x)=x+1$, $g : R\to R$ $g(x)=x^2+1$ then fog(-2) equals to

(a) 6 (b) 5

(c) -2 (d) None

[Dec. 2010]

Solution : (a) $\because$ f(x) = x + 1

$g(x)=x^2+1 \Rightarrow g(-2)=(-2)^2+1=5$

$fog(-2)=f\{g(-2)\}=f(5)$

$=5+1=6$

$\therefore$ (a) is correct

Q.6. If A ⊂ B, then following is true

(a) $A\cap B=B$ (b) $A\cup B=B$

(c) $A\cap B=A'$ (d) $A\cap B$

[Dec. 2010]

Solution : (b)

Q.7. If $f(x-1)=x^2-4x+8$, then $f(x+1)=$

(a) x^2+8 (b) x^2+7

(c) x^2+4 (d) x^2-4x

[Dec. 2010]

Solution : (c); $f(x-1)=x^2-4x+8$

$=(x-1+1)^2-4(x-1+1)+8$

$\therefore f(x+1)$

$=(x+1+1)^2-4(x+1+1)+8$

$=(x+2)^2-4(x+2)+8$

$=x^2+4x+4-4x-8+8$

$=x^2+4$

$\therefore$ (c) is correct.

Q.8. There are 40 students, 30 of them passed in English, 25 of them passed in Maths and 15 of them passed in both. Assuming that every Student has passed atleast in one subject. How many student's passed in English only but not in Maths.

(a) 15 (b) 20

(c) 10 (d) 25

[June 2011]

Solution : (a) Total students = 40

n(E) = 30 ; n(M) = 25

$(E\cap M)=15$

No. of stds. passed in English only

$n(E)-n(E\cap M)$

$=30-15=15$

$\therefore$ (a) is correct

Q.9. If $A=\{\pm2,\pm3\}$, $B=\{1,4,9\}$ AND $F=\{(2,4),(-2,4),(3,9),(-3,4)\}$ then 'F' is defined as:

(a) One to one function from A into B

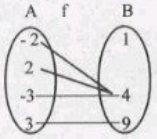
(b) One to one function from A onto B

(c) Many to one function from A onto B.

(d) Many to one function from A into 8.

[June 2011]

Solution : (c)



$\therefore$ (c) is correct

Q.10. If f(x) $=\frac{x}{\sqrt{1+x^2}}$ and $g(x)=\frac{x}{\sqrt{1-x^2}}$ Find fog ?

(a) x (b) $\frac{1}{x}$

(c) $\frac{x}{\sqrt{1-x^2}}$ (d) $x\sqrt{1-x^2}$

[June 2011]

Solution : (a) $fog=f\{g(x)\}=f\left(\frac{x}{\sqrt{1-x^2}}\right)$

$=\dfrac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{1+\frac{x^2}{1-x^2}}}$

$=\dfrac{\frac{x}{\sqrt{1-x^2}}}{\sqrt{\frac{1-x^2+x^2}{1-x^2}}}$

$=\frac{x}{\sqrt{1-x^2}}\times\frac{\sqrt{1-x^2}}{\sqrt{1}}=x$

$\therefore$ (a) is correct

Q.11. f(x) = 3+x, for $-3<x<0$ and 3-2x for $0<x<3$, then value of f(2) will be

(a) -1 (b) 1

(c) 3 (d) 5

[Dec. 2011]

Solution: (a) is correct

$\because$ f(x) = 3 + 2x ; when $-3<x<0$

$=3-2x$; when $0<x<3$

$f(x=2)=3-2\times2=-1$

$\therefore$ 2 lies in 2nd condition

Q.12. If A = (1,2,3,4,5), B = (2,4) and C = (1,3,5) then (A -C) × B is

(a) $\{(2,2),(2,4),(4,2),(4,4),(5,2),(5,4)\}$

(b) $\{(1,2),(1,4),(3,2),(3,4),(5,2),(5,4)\}$

(c) $\{(2,2),(4,2),(4,4),(4,5)\}$

(d) $\{(2,2),(2,4),(4,2),(4,4)\}$

[Dec. 2011]

Solution : (d) is correct

$(A-C)\times B=\{2,4\}\times\{2,4\}=\{(2,2),(2,4);(4;2);(4;4)\}$

Q.13. For any two sets A and B the set $(A\cup B')'$ is Equal to (where' denotes compliment of the set)

(a) B-A (b) A-B
(c) A'-B' (d) B'-A'

[Dec. 2011]

Solution: (a) is correct

By De-Morgan's formula

$(A \cup B')'$
$= A' \cap (B')'$
$= A' \cap B$
$= B - A \cap B$
$= B - A$

Q.14. The number of proper sub-set of the set $\{3,4,5,6,7\}$ is

(a) 32 (b) 31
(c) 30 (d) 25

[June 2012]

Solution : (b) No. of proper sub-sets $= 2^n - 1$
$= 2^5 - 1 = 31$

Q.15. On the set of lines, being perpendicular is a _____

(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

[June 2012]

Solution : (b) is correct

It is symmetric relation Because it x is perpendicular to y

Then y is also perpendicular to x

Q.16. The range of the function $f: N \to N$; $f(x) = (-1)^{x-1}$, is

(a) $\{0,-1\}$ (b) $\{1,-1\}$
(c) $\{1,0\}$ (d) $\{1,0,-1\}$

[June 2012]

Solution : (b) is correct

$\therefore f(x) = (-1)^{x-1}$

If x = odd No.

f(x) = 1

It x = 0; even No.

f(x) = -1

∴ Range = {1 ; -1}

Domain = {any real No.}

Q.17. For a group of 200 persons, 100 are interested in music, 70 in photography and 40 in swimming, Further more 40 are interested in both music and photography, 30 in both music and swimming, 20 in photography and swimming and 10 in all the three. How many are interested in photography but not in music and swimming ?

(a) 30 (b) 15 (c) 25 (d) 20

[Dec. 2012]

Solution : (d) is correct

Let A = No. of persons interested in Music

B = No. of persons interested in photography

C = No. of persons interested in Swimming

n(A) = 100; n(B) = 70 ; n(C) = 40;

$n(A \cap B) = 40; n(A \cap C) = 30;$

$n(B \cap C) = 20; n(A \cap B \cap C) = 10.$

$\therefore n(B \cap A' \cap C') = n(B) - n(B \cap A) - n(B \cap C) + n(A \cap B \cap C)$

$= 70 - 40 - 20 + 10$

$= 20$

Q.18. If $f: R \to R$ is a function, defined by f(x) = 10x-7, if $g(x) = f^{-1}(x)$, then the value of g(x) is equal to

(a) $\frac{1}{10x-7}$ (b) $\frac{1}{10x+7}$
(c) $\frac{x+7}{10}$ (d) $\frac{x-7}{10}$

[Dec. 2012]

Solution : (c) is correct

Let $y = f(x) = 10x - 7$

or $10x = y + 7$

$\therefore x = \frac{y+7}{10}$

$\therefore f^{-1}(x) = \frac{x+7}{10}$

$\therefore g(x) = \frac{x+7}{10}$

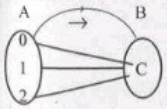
Q.19. The No. of elements in range of constant function is

(a) One (b) Zero
(c) Infinite (d) None

[Dec. 2012]

Solution : (a) is correct

Let f(x) = c (where c = constant)



Domain = {x/x∈R}

Range = {c}

Q.20. If f(x) = x+2, g(x) = 7^x then go f(x)=

(a) $7^x.x + 2.7^x$ (b) 7^{x+2}
(c) $(7^x)+2$ (d) none

[June 2013]

Solution : f(x) = x+2 ; g(x) = 7^x

$g\ of\ (x) = g\{f(x)\} = g(x+2) = 7^{x+2}$

∴ (b) is correct

Q.21. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then $f\left(\frac{2x}{1+x^2}\right)$

(a) f(x) (b) 2 f(x)
(c) 3 f(x) (d) -f(x)

[June 2013]

Solution :

(b) $f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$

$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$

$= \log\frac{(1+x)^2}{(1-x)^2} = \log\left(\frac{1+x}{1-x}\right)^2$

$= 2\log\left(\frac{1+x}{1-x}\right) = 2f(x)$

∴ (b) is correct

Q.22. If A = {1,2,3} then the relation R={(1, 1), (2, 3), (2, 2), (3, 3), (1, 2)} on A is:

(a) Reflexive (b) Symmetric
(c) Transitive (d) Equivalence

[June 2013]

Solution : (a) is correct.

Reflexive Relation.

xRx ; (x ; x) ∈ R

Here (1, 1), (2, 2), (3, 3)∈R

So; It is Reflexive

Q.23. Of the 200 candidates who were interviewed for a position at call center, 100 had a two wheeler, 70 had a credit card and 140 had a mobile phone 40 of them had both a two wheeler and a credit card, 30 had both a credit card and mobile phone, 60 had both a two wheeler and a mobile phone and 10 had all the three. How many candidates had none of them ?

(a) 0 (b) 20
(c) 10 (d) 18

[Dec. 2013]

Solution : (c) is correct

Let n(A) = No. of Candidates having two wheeler

n(B) = No. of candidates having credit cards

n(C) = No. of candidates having mobile phone.

Given

n(A) = 100 ; n(B) = 70 ; n(c) = 140

$n(A \cap B) = 40;\ n(B \cap C) = 30;\ n(C \cap A) = 60\ n(A \cap B \cap C) = 10.$

$\therefore n(A \cup B \cup C) = 100 + 70 + 140 - 40 - 30 - 60 + 10 = 190$

No. of candidates having none = 200 -190 = 10

Q.24. If $f(x) = \frac{x^2-25}{x-5}$ then f(5)

(a) 0 (b) 1
(c) 10 (d) Undefined

[Dec. 2013]

Solution : (d) is correct

$f(5) = \frac{x^2-25}{x-5} = \frac{5^2-25}{5-5} = \frac{0}{0}$

∴ Undefined

Q.25. $f(x) = (a - x^n)^{\frac{1}{n}}$, a > 0 and n is positive integer then f [f(x)]=

(a) x (b) a
(c) $x^{1/n}$ (d) $a^{1/n}$

[Dec. 2013]

Solution : (a) is correct

$f\{f(x)\} = f\left\{(a-n^n)^{1/n}\right\}$

$= \left[a - \left\{(a-x^n)^{\frac{1}{n}}\right\}^n\right]^{\frac{1}{n}}$

$= \left[a-(a-x^n)\right]^{\frac{1}{n}} = \left[x^n\right]^{\frac{1}{n}} = x.$

Q.26. In a class of 50 students 35 opted for Maths, 37 opted for commerce. The number of such student who opted for both maths and commerce is

(a) 13 (b) 15
(c) 22 (d) 28

[June 2014]

Solution : (c) is correct

n(M) = No. of students opted for Maths = 35

n(C) = No. of Student opted for Commerce = 37

So; $(M \cup C) = 50$

$n(M \cap C) = 35 + 37 - 50 = 22$

Q.27. The range of the relation {(1, 0) (2, 0) (3, 0) (4, 0) (0, 0)} is

(a) {1, 2, 3, 4, 0}
(b) {0}
(c) {1, 2, 3, 4}
(d) None

[June 2014]

Solution : (b) is correct

Range = {0}

Q.28. If A = {1, 2, 3} and B = {4, 6, 7} then the relation R = {(2, 4) (3, 6)} is

(a) A function
(b) A function from A to B
(c) Both (a) and (b)
(d) Not a function

[June 2014]

Solution : (d) is correct.

Note:- 1 has no image

Q.29. A = (2, 3), B=(4, 5), C=(5, 6) then $A \times (B \cap C)$

(a) [(5, 2), (5, 3)] (b) [(2, 5), (3, 5)]
(c) [(2, 4), (5, 3)] (d) [(3, 5), (2, 6)]

[Dec. 2014]

Solution : (b) is correct

$B \cap C = \{4,5\} \cap \{5,6\} = \{5\}$

$\therefore A \times (B \cap C) = \{2,3\} \times \{5\}$
$= \{(2,5);(3,5)\}$

Q.30. If a relation S = ((1,1), (2,2), (1,2), (2,1)) is symmetric and

(a) Reflexive but not transitive
(b) Reflexive as well as transitive
(c) Transitive but not reflexive
(d) Neither transitive nor reflexive

[Dec. 2014]

Solution : If S = {1,2,3} then

Then relation {(1,1); (2,2); (1,2); (2,1)} is symmetric and transitive but not Reflexive.

(b) is correct

Q.31. If

$f(x) = \frac{x}{x-1}$, then $\frac{f(x/y)}{f(y/x)} =$ _____

(a) x/y (b) y/x
(c) -x/y (d) -y/x

[Dec. 2014]

Solution : $f(x) = \frac{x}{x-1}$

$f\left(\frac{x}{y}\right) = \frac{\frac{x}{y}}{\frac{x}{y}-1} = \frac{\frac{x}{y}}{\frac{x-y}{y}} = \frac{x}{x-y}$

$$f\left(\frac{y}{x}\right)=\frac{\frac{y}{x}}{\frac{y}{x}-1}=\frac{\frac{y}{x}}{\frac{y-x}{x}}=\frac{y}{-(x-y)}$$

$$\therefore -\frac{f\left(\frac{x}{y}\right)}{f\left(\frac{y}{x}\right)}=\frac{\frac{x}{x-y}}{\frac{y}{-(x-y)}}=-\frac{x}{y}$$

Q.32. Let N be the set of all Natural number ; E be the set of all even natural numbers then the function

f : N → E defined as f(x) =2x ; ∨ x ∈ N is :

(*a*) One-one into

(*b*) One-one onto

(*c*) Many-one into

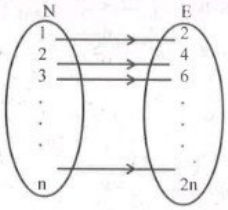
(*d*) Many-one onto

[Dec. 2014]

Solution : (*b*) is correct

N = {1,2,3,.....;n}

E = {2,4,6,.....;2n}



Clearly it is one-one onto mapping

Q.33. Which of these is a function from A → B; A= {x,y,z}; B={a,b,c,d}

(*a*) {(x,a) (x,b) (y,c)}

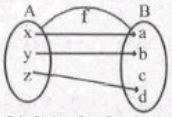
(*b*) {(x,a) (x,b) (y,c) (z,d)}

(*c*) {(x,a) (y,b) (z,d)}

(*d*) {(a,x) (b,z) (c,y)}

[Dec. 2015]

Solution : (*c*) is correct.



Q.34. f(x) = 2x+2, g(x) = x^2, fog(4) = ?

(*a*) 100 (*b*) 10

(*c*) 34 (*d*) None of these

[Dec. 2015]

Solution : (*c*) is correct

$fog(x)=f\{g(x)\}$

$=f(x^2)=2.x^2+2$

$fog(4)=2\times4^2+2=34$

Q. 35. In a class of 80 students, 35% play only cricket, 45% only Tennis. How many play Cricket?

(*a*) 86 (*b*) 54

(*c*) 36 (*d*) 44

[Dec. 2015]

Solution : (*d*) is correct

Given $n(C-T)=n(C)-n(C\cap T)=35\%$

$n(T-C)=n(T)-n(C\cap T)=45\%$

$n(CUT)=n(C)+n(T)-n(C\cap T)=100$

or, $35+n(C\cap T)+45+n(C\cap T)-n(C\cap T)=100$

or $80+n(C\cap T)=100$

$n(C\cap T)=20\%$

$\therefore n(C)=35+n(C\cap T)=35+20=55\%$

$=80\times55\%=44$

Q.36. If set $A=\left\{x:\frac{x}{2}\in Z, 0\le x\le 10\right\}$

$B=\{x : x \text{ is one digit prime number}\}$ and

$C=\left\{x:\frac{x}{3}\in N, x\le 12\right\}$ **then**

$A\cap(B\cap C)=$

(*a*) ϕ (*b*) Set A

(*c*) Set B (*d*) Set C

[June 2016]

Solution : (a)

$A=\{2,4,6,8,10\}$

$B=\{2,3,5,7\}$

$C=\{3,6,9,12\}$

$A\cap(B\cap C)$

$=A\cap(B\cap C)=\varphi$

No Common element in all 3 sets.

Q.37. The domain (D) and range (R) of the function

f (x) =2−|x+1| is

(*a*) D = Real numbers, R = (2, ∞)

(*b*) D = Integers, R = (0, 2)

(*c*) D = Integers, R = (- ∞, ∞)

(*d*) D = Real numbers, R = (- ∞, 2)

[June 2016]

Solution : (*d*) is correct.

let $y=f(x)=2-|x+1|$

For any real values of x ; $f(x)$ is defined.

∴ Domain = D Real numbers

Minimum value of $|x+1|$ is Zero

∴ Maximum value of Range

$=2-0=2$

∴ Range $=-\infty<y\le2$

$=(-\infty;2]$

Q.38. Let A be the set of the squares of natural numbers and $x\in A, y\in A$. **Then _____.**

(*a*) $x+y\in A$ (*b*) $x-y\in A$

(*c*) $\frac{x}{y}\in A$ (*d*) $xy\in A$

[June 2016]

Solution : (*d*) is correct.

$A=\{x/x$ is the squares of natural Nos.$\}$

$=\{1,4,9,16,25,.....\}$

Tricks: then Go by Choices

let $x=1; y=4\in A$.

$\therefore x+y=1+4=5\notin A$.

$x-y=1-4=-3\notin A$.

$\frac{x}{y}=\frac{1}{4}\notin A$.

But $xy=1\times4=4\in A$.

∴ (*d*) is correct.

Q.39. The number of sub-sets formed from the letters of the word "ALLAHABAD".

(*a*) 128 (*b*) 16

(*c*) 32 (*d*) None

[June 2016]

Solution : (c) is correct.

Let X = {Letters of word ALLAHABAD}

= { A, L, H, B, D }

No. of sub-sets = $2^5=32$

Q.40. If $f(x)=100x$ **then** $f^{-1}(x)=$

(*a*) $\frac{x}{100}$ (*b*) $\frac{1}{100x}$

(*c*) $\frac{1}{100}$ (*d*) None of these

[June 2016]

Solution : (a) is correct.

Let y = f (x) = 100x

$x=\frac{y}{100}$; So, $f^{-1}(x)=\frac{x}{100}$

Q.41. f : R → R is defined by f (x) = 2^x then f is

(*a*) One - one and onto

(*b*) One - one and into

(*c*) Many to one

(*d*) One to many

[June 2016]

Solution : (*b*) is correct.

Q.42. In a class, 80 students speak Hindi, 60 students speak English and 40 students speak both Hindi and English then the number of students in the class is _____.

(*a*) 100 (*b*) 120

(*c*) 140 (*d*) 180

[June 2017]

Solution : Let H = Students speak Hindi

E = Students speak English

Given

n(H) = 80 ; n (E) = 60

and $n(H\cap E)=40$

$n(H\cup E)=n(H)+n(E)-n(H\cap E)$

= 80 + 60 - 40 = 100.

∴ Option (a) is correct

Q.43. If $f(x)=\frac{x-1}{x}$ **and** $g(x)=\frac{1}{1-x}$ **then fog(x)=** ∪

(*a*) x-1 (*b*) x

(*c*) 1-x (*d*) -x

[June 2017]

Solution : $fog(x)=f\{g(x)\}=$

$\frac{g(x)-1}{g(x)}$

$=\frac{\frac{1}{1-x}-1}{\frac{1}{1-x}}=\left(\frac{1-1+x}{1-x}\right)\times\frac{(1-x)}{1}$

$=x$

∴ Option (b) is correct

Q.44. The Range of the function f is defined by $f(x)=\frac{x}{x^2+2}$ **is**

(*a*) $\left\{x:\frac{-1}{2}<x<\frac{1}{2}\right\}$

(*b*) $\left\{x:\frac{-1}{2}\le x<\frac{1}{2}\right\}$

(*c*) $\left\{x:\frac{-1}{2}\le x\le\frac{1}{2}\right\}$

(*d*) $\left\{x:x>\frac{-1}{2}\text{ or }x<\frac{-1}{2}\right\}$

[June 2017]

Solution : Let $y=\frac{x}{x^2+2}=f(x)$

or ; $yx^2+2y=x$

or $yx^2-x+2y=0$

It is a quadratic equation in terms of x.

∴ Discriminant = D = b^2-4ac

$=(-1)^2-4.y.2y=1-8y^2$

To be Real solutions ;

$D\ge0\Rightarrow1-8y^2\ge0$

or $1\ge8y^2\Rightarrow8y^2\le1$

or ; $y^2\le\frac{1}{8}$

or $-\frac{1}{2\sqrt{2}}\le y\le\frac{1}{2\sqrt{2}}$

$\left[\because \text{If } y^2=\frac{1}{8}\Rightarrow y=\pm\frac{1}{2\sqrt{2}}\right]$

∴ (c) is correct.

Q.45. In a class of 35 students, 16 students play football and 24 students play cricket. Assume that each one play atleast one game, then number of students who play both the games is _____ :

(*a*) 5 (*b*) 11

(*c*) 12 (*d*) 17

[June 2017]

Solution : $n(F\cap C)$ = n(F) + n(C) - n(F ∪ C)

= 16 + 24 - 35 = 5

option (*a*) is correct.

Q.46. If $f(x)=\frac{x+1}{x+2}$ **= then**

$f[f\left(\frac{1}{x}\right)]=$ _____ :

(*a*) $\frac{2x+3}{3x+5}$ (*b*) $\frac{2x+5}{3x+2}$

(*c*) $\frac{3x+2}{5x+3}$ (*d*) $\frac{5x+2}{2x+3}$

[Dec. 2017]

Solution : (*c*)

$f\left(\frac{1}{x}\right)=\frac{1/x+1}{1/x+2}=\frac{1+x}{x}\times\frac{x}{1+2x}$

$=\frac{1+x}{1+2x}$

Now ; $f\left[f\left(\frac{1}{x}\right)\right]=f\left(\frac{1+x}{1+2x}\right)$

$$= \frac{\frac{1+x}{1+2x}+1}{\frac{(1+x)}{1+2x}+2} = \frac{\frac{3x+2}{1+2x}}{\frac{5x+3}{1+2x}}$$

$$= \frac{3x+2}{5x+3}.$$

Q.47. If $A = \{\phi, \{\phi\}\}$ then the Power Set of A is

(a) $\{\phi\}, \{0\}$

(b) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

(c) A

(d) $\{A\}; \{\phi\}$

[June 2018]

Solution : (b)

$A = \{\phi; \{\phi; \}\}$

$\therefore P(A) = \{\{\ \}\}; \{\phi\}; \{\{\phi\}\}; \{\phi; \{\phi\}\}$

$= \{\phi; \{\phi\}; \{\{\phi\}; A\}\}$

Q.48. If $A = \{x / x = 3^n - 2n - 1$, where $n \in N\}$, $B = \{x/x = 4(n-1)$, where $n \in N\}$. Then

(a) $A \subset B$ (b) $B \subset A$

(c) $A = B$ (d) None

[June 2018]

Solution : (a)

Putting n = 1, 2, 3,..... ; we get

$A = \{x / x = 3^n - 2n - 1\}$

$= \{0 ; 4 ; 20 ; \ldots\ldots\}$

$B = \{x / x = 4(n-1)\}$

$= \{0 ; 4 ; 8 ; 12 ; 16 ; 20 ; \ldots\ldots\}$

Clearly ; $A \subset B$

Q.49. The range of the function $\frac{x^6}{x^{12}+1}$ is:

(a) $(0, \infty)$

(b) $\left[0, \frac{1}{2}\right]$

(c) $(-\infty, 0) \cup [2, \infty)$

(d) $\left(0, \frac{1}{2}\right)$

[June 2018]

Solution : (b)

Let $y = \frac{x^6}{x^{12}+1}$

or $yx^{12} + y = x^6$

let $z = x^6$;

$\therefore yz^2 + y = z \Rightarrow yz^2 - z + y = 0$

It is a Quadratic Eqn. in terms of Z. for real solns.

$D = b^2 - 4ac = (-1)^2 - 4.y.y$

$= 1 - 4y^2$

$D \geq 0$

or ; $1 - 4y^2 \geq 0 \Rightarrow 1 \geq 4y^2$

or ; $4y^2 \leq 1$

or $y^2 \leq \frac{1}{4}$ If $y^2 = \frac{1}{4} \Rightarrow y \pm \frac{1}{2}$

$\therefore -\frac{1}{2} \leq y \leq \frac{1}{2}$

From Qts. $\left[0, \frac{1}{2}\right]$ [$\because$ y is always positive.]

Q.50. Let N be the set of all natural numbers; E be the set of all even natural numbers then the function;

(a) One-one-into

(b) Many-one-into

(c) One-one-onto

(d) Many-one-onto

[May 2018]

Solution : (c)

$\because$ N = set of Natural Numbers

$= \{1, 2, 3, \ldots\ldots\ldots\}$

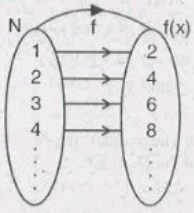
$f\{x\} = 2x$; $\forall X \in N$

So; $f(1) = 2 \times 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(3) = 2 \times 3 = 6$

So;



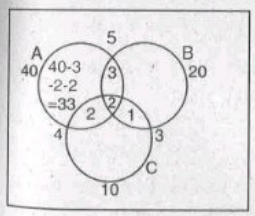
Clearly ; It is one-one and onto mapping.

Q.51. In a town of 20,000 families it was found that 40% families buy newspaper. A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C if 2% families buy all the three newspapers, then the number of families which buy A only is :

(a) 6600 (b) 6300 (c) 5600 (d) 600

[May 2018]

Solution : (a)



Given that

$n(A) = 40\%$; $n(B) = 20\%$

$n(C) = 10\%$; $n(A \cap B) = 5\%$

$n(B \cap C) = 3\%$; $n(C \cap A) = 4\%$

$n(A \cap B \cap C) = 2\%$

$\therefore n(A \cap \overline{B} \cap \overline{C}) =$ Only A

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= 40 - 5 - 4 + 2 = 33\%$

$= 20000 \times 33\%$

$= 6600.$

Q.52. The numbers of proper sub-sets of the set {3, 4, 5, 6, 7} is :

(a) 32 (b) 31

(c) 30 (d) 25

[May 2018]

Solution : (b)

Formula

No. of proper sub-sets = $2^n - 1$

$= 2^5 - 1 = 31.$

Q.53. A is {1,2,3,4} and B is {1,4,9,16,25} if a function f is defined from set A to B where $f(x) = x^2$ then the range of f is:

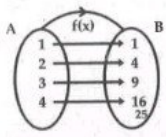
(a) {1,2,3,4} (b) {1,4,9,16}

(c) {1,4,9,16,25} (d) None of these

[Nov. 2018]

Solution : (b)

$\because f(x) = x^2$



$\therefore$ Range = {1, 4, 9, 16}

Q.54. If A = {1, 2} and B = {3, 4}. Determine the number of relations from A and B :

(a) 3 (b) 16

(c) 5 (d) 6

[Nov. 2018]

Solution : (b)

No. of Relations $= 2^{n(A \times B)}$

$= 2^{(2\times2)} = 16.$

Q.55. If A = {1,2,3,4,5,6,7} and B = {2, 4,6,8}. Cardinal number of A - B is:

(a) 4 (b) 3

(c) 9 (d) 7

[Nov. 2018]

Solution :

$A - B = \{1, 2, 3, 4, 5, 6, 7\} - \{2, 4, 6, 8\}$

$= \{1, 3, 5, 7\}$

$\therefore n(A - B) = 4$

(a) is correct.

Q.56. Identify the function from the following:

(a) {(1,1), (1,2), (1,3)}

(b) {(1,1), (2,1), (2,3)}

(c) {(1,2), (2,2), (3,2), (4,2)}

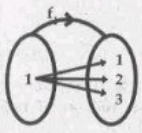
(d) None of these

[Nov. 2018]

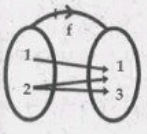
Solution : (c)

Go by choices

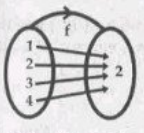
(a)



(b)



(c)



Q.57. If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}; B = {1, 3, 4, 5, 7, 8}; C = {2, 6, 8}

Then find $(A - B) \cup C$

(a) {2, 6} (b) {2, 6, 8}

(c) {2, 6, 8, 9} (d) None

[June 2019]

Solution : (c)

$A - B = A - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\{1, 3, 4, 5, 7, 8\}$

$= \{2, 6, 9\}$

$\therefore (A - B) \cup C = \{2;6;9\} \cup \{2;6;8\}$

$= \{2 ; 6 ; 8 ; 9\}$

(c) is correct.

Q.58. A = {1, 2, 3, 4 ..... 10} a relation on A, R = {(x, y)/x + y = 10, x $\in$ A, Y $\in$ A, x $\geq$ Y} then domain of R^{-1} is

(a) {1, 2, 3, 4, 5}

(b) {0, 3, 5, 7, 9}

(c) {1, 2, 4, 5, 6, 7}

(d) None

[June 2019]

Solution : (a)

Given ; A = {1, 2, 3, ... ; 10}

R = {x ; y) / x + y = 10 ; $X \in A$; $Y \in A$; $x \geq y$}

$\Rightarrow$ R = (5 ; 5) ; (6 ; 4) ; (7 ; 3) ; (8 ; 2) ; (9 ; 1)

$\Rightarrow$ R^{-1} = (5 ; 5) ; (4 ; 6) ; (3 ; 7) ; (2 ; 8) ; (1; 9)

$\therefore$ Domain of R^{-1} = (5 ; 4 ; 3 ; 2 ; 1)

Q.59. The no. of sub-sets of the set {3, 4, 5} is :

(a) 4 (b) 8

(c) 16 (d) 32

[June 2019]

Solution : (b)

No. of sub-sets = $2^n = 2^3 = 8.$

Q.60. If $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(a) go f(3) = 3

(b) go f(-3) = 9

(c) go f(9) = 3

(d) go f(-9) = 3

[June 2019]

Solution : (a)

$\because f(x) = x^2 ; g(x) = \sqrt{x}$

$\therefore go\ f(x) = g\{f(x)\}$

$\sqrt{f(x)} = \sqrt{x^2}$

$= x$

$\therefore go\ f(3) = 3$

$\therefore$ (a) is correct.

Q.61. If A = {a, b, c, d}; B = {p, q, r, s} which of the following relation is a function from A to B

(a) R_1 = {(a, p), (b, q), (c, s)}

(b) R_2 = {(p, a), (b, r), (d, s)}

(c) R_3 = {(b, p), (c, s), (b, r)}

(d) R_4 = {(a, p), (b, r), (c, q), (d, s)}

[June 2019]

Solution : (d)

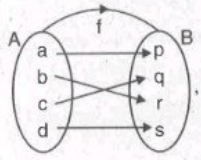
GBC

(a) All have one and only one solution of A in B except d i.e. d has no solution in B

$\Rightarrow$ Clearly R_1 is not Function.

Similarly (b) & (c) are not functions or mappings.

(d)



$\therefore R_4$ is a function or mapping from A to B.

Q.62. $f(x) = f(x-1) + f(x-2)$ if $f f(0) = 0, f(1) = 1, x = 2,3,4,.....$ then what is $f(7)$

(a) 8 (b) 13

(c) 3 (d) 5

[Dec. 2019]

Solution : (b)

$f(2) = f(2-1) + f(2-2)$

$= f(1) + f(0) = 1 + 0 = 1$

$f(3) = f(3-1) + f(3-2)$

$= f(2) + f(1) = 1 + 1 = 2$

$f(4) = f(4-1) + f(4-2) = f(3) + f(2)$

$= 2 + 1 = 3$

$f(5) = f(4) + f(3) = 3 + 2 = 5$

$f(6) = f(5) + f(4) = 5 + 3 = 8$

$f(7) = f(6) + f(5) = 8 + 5 = 13.$

Q.63. $f(x) = 2x^3 + 1$ then what is $f^{-1}(x)$ options

(a) $\frac{1}{2}(x-1)^{1/3}$

(b) $\left(\frac{x-1}{2}\right)^{1/3}$

(c) $\left(\frac{x-1}{2}\right)^{1/2}$

(d) None of these

[Dec. 2019]

Solution : (b)

Let $y = f(x) = 2x^3 + 1$

or ; $2x^3 = y - 1$

or $x^3 = \frac{y-1}{2}$

or $x = \left(\frac{y-1}{2}\right)^{1/3}$

$\therefore f^{-1}(x) = \left(\frac{x-1}{2}\right)^{1/3}$

[Replacing y by x]

Q.64. Two finite sets respectively have x and y number of elements. The total number of subsets of the first is 56 more than the total no. of subsets of the second. The values of x, y are respectively.....

(a) 4 and 2 (b) 6 and 3

(c) 2 and 4 (d) 3 and 6

[Dec. 2020]

Solution : (b) **Tricks :- Go By Choices (GBC)**

Total No. of Subsets = 2^n

(a) $2^4 - 2^2 = 16 - 4 = 12 \neq 56$

$\therefore$ (a) is in correct.

(b) $2^{n(x)} - 2^{n(y)} = 56$

$\Rightarrow 2^6 - 2^3 = 64 - 8 = 56$

(b) is correct

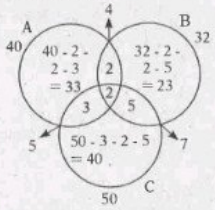
Q.65. The number of items in the set A is 40 in the Set B is 32; in the Set C is 50; in both A and B is 4; in both A and C is 5; in both B and C is 7; in all the set is 2. How many are in only one set?

(a) 65 (b) 110

(c) 96 (d) 84

[Dec. 2020]

Solution : Solution (c).



Given

$n(A) = 40$

$n(B) = 32$

$n(C) = 50$

$n(A \cap B) = 4$

$n(A \cap C) = 5$

$n(B \cap C) = 7$

and $n(A \cap B \cap C) = 2$

No. of items in only one set.

= only A + only B + only C

= (40 - 2 - 2 - 3) + (32 - 2 - 2 - 5)

+ (50 - 3 - 2 - 5)

= 43 + 23 + 40 = 96

Q.66. The set of cubes of natural numbers is

(a) Null set

(b) Finite set

(c) Infinite set

(d) A finite set of three numbers

[Dec. 2020]

Solution : (c) is correct.

Q.67. The inverse function $f^{-1}(y)$ of $f(y) = 3y$ is ....

(a) 1/3y (b) y/3

(c) -3y (d) 1/y

[Dec. 2020]

Solution : (b) Let $x = f(y) = 3y$

$\therefore y = \frac{x}{3}$

$\therefore f^{-1}(y) = \frac{y}{3}$ ($\because$ Replace x by y)

(b) is correct.

Q.68. The set of cubes of natural number is

(a) Null set (b) A finite set

(c) An infinite set (d) Singleton Set

[Jan. 2021]

Solution : Let A = Set of cubes of natural Numbers

$= \{1^3, 2^3, 3^3 \ldots\}$

$n(A) = \infty$

$\therefore$ (c) is correct

Q.69. In the set of all straight lines on a plane which of the following is Not 'TRUE'?

(a) Parallel to an equivalence relation

(b) Perpendicular to is a symmetric relation

(c) Perpendicular to is an equivalence relation

(d) Parallel to a reflexive relation

[Jan. 2021]

Solution : (c) is correct

Q.70. Let F : R R be defined by

$$f(x) = \begin{cases} 2x & \text{for } x > 3 \\ x^2 & \text{for } 1 < x \leq 3 \\ 3x & \text{for } x \leq 1 \end{cases}$$

The value of f(-1) + f(2) + f(4) is

(a) 9 (b) 14

(c) 5 (d) 6

[Jan. 2021]

Solution : (c) is correct

Let R = being a relation of perpendicular

Reflexive Relation

Let x be a straight line on a plane.

x R x i.e. x is perpendicular to itself is not true.

Hence R is not Equivalence.

(c) is correct.

Q.71. Let U be the universal set, A and B are the subsets of U. If n(U) = 650, n(A) = 310, n(A$\cap$B). = 95 and n(B) = 190, then $n(\bar{A} \cap \bar{B})$ is equal to ($\bar{A}$ and $\bar{B}$ are the complement of A and B, respectively)

(a) 400 (b) 200

(c) 300 (d) 245

[July 2021]

Solution :

$\because n(\bar{A} \cap \bar{B}) = n(A \cup B)'$

$= n(\cup) - n(A \cup B) = ?$ (1)

Now

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 310 + 190 - 95 = 405$

From (1)

$n(\bar{A} \cap \bar{B}) = n(\cup) - n(A \cup B)$

$= 650 - 405 = 245$

(d) is correct.

Q.72. The range of the function f defined by $f(x) = \sqrt{16 - x^2}$ is

(a) [-4, 0] (b) [-4,4]

(c) [0, 4] (d) (-4, 4)

[July 2021]

Solution : (c) is correct

Given $f(x) = \sqrt{16 - x^2}$

For Domain $(16 - x^2)$ should not be negative.

i.e. $16 - x^2 \geq 0$

or $16 \geq x^2$

Hence $x \leq 4$ or $x \geq -4$

$\Rightarrow -4 \leq x \leq 4$

$\because$ Domain = [- 4, 4]

For Range : $f(x)$ is maximum

if $x = 0 \Rightarrow f(x) = 4$

and $f(x)$ is minimum at $x = \pm 4$

i.e. $f(x) = 0$.

Hence Range $= 0 \leq f(x) \leq 4$

$= [0, 4]$

Q.73. Let A = R - {3} and B = R - {1}. Let f : A $\rightarrow$ B defined by $f(x) = \frac{x-2}{x-3}$

What is the value of $f^{-1}\left(\frac{1}{2}\right)$?

(a) 2/3 (b) 3/4

(c) 1 (d) -1

[July 2021]

Solution : (c) is correct

Net $y = f(x) = \frac{x-2}{x-3}$

or $xy - 3y = x - 2$

or $xy - x = 3y - 2$

or $x(y-1) = 3y - 2$

or $x = \frac{3y-2}{y-1}$

$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$

Now $= f^{-1}\left(\frac{1}{2}\right) \frac{3x\frac{1}{2} - 2}{\frac{1}{2} - 1} = \frac{-\frac{1}{2}}{-\frac{1}{2}}$

$= 1$

Q.74. If $f(x) = x^2 - 1$ and $g(x) = |2x + 3|$, then fog(3) - g of(-3) =

(a) 71 (b) 61

(c) 41 (d) 51

[July 2021]

Solution : (b) is correct

$g(3) = |2 \times 3 + 3| = 9$

$f(-3) = (-3)^2 - 1 = 8$

$\therefore fog(3) - g\ of(-3)$

$= f\{g(3)\} - g\{f(-3)\}$

$= f(9) - g(8)$

$= (9^2 - 1) - |2 \times 8 + 3|$

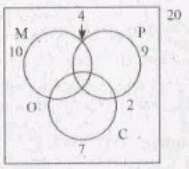
$= 80 - 19 = 61$

Q.75. Out of a group of 20 teachers in school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry, 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics, how may teach only Physics?

(*a*) 2, 3 (*b*) 3, 2 (*c*) 4, 6 (*d*) 6, 4

[Dec. 2021]

Solution : (*a*)



Given $n(M \cup P \cup C) = 20$;

$n(m) = 10$; $n(P) = 9$

$n(c) = 7$; $n(M \cap P) = 4$

$n(M \cap C) = 0 \Rightarrow n(M \cap P \cap C)$

$\therefore n(M \cup P \cup C) = n(M) + n(P) + n(C)$

$-n(M \cap P) - n(M \cap C) - n(P \cap C)$

$+ n(M \cap P \cap C)$

or $20 = 10 + 9 + 7 - 4 - 0 - x(\text{let}) + 0$

or; $x = 22 - 20$

$= 2$

$\therefore n(P \cap C) = 2$

only physics $= n(P) - n(P \cap C)$

$-n(M \cap P)$

$= 9 - 4 - 2 = 3$

$\therefore$ (*a*) is correct.

Q.76. If a is related to b if and only if the difference in a and b is an even integer. This relation is

(*a*) symmetric, reflexive but not transitive

(*b*) symmetric, transitive but not reflexive

(*c*) transitive, reflexive but not symmetric

(*d*) equivalence relation

[Dec. 2021]

Solution : (*d*)

(*i*) $\because a - a = 0$ and 0 is an even integer.

$\Rightarrow (a, a) \in R$ (True)

$\therefore$ R is reflexive

(*ii*) If $a - b$ = even integer

$\Rightarrow b - a$ = Also even integer

It means

If $(a - b) \in R \Rightarrow (b - a) \in R$.

$\therefore$ The relation is symmetric

(*iii*) Let $a = 1, b = 3, c = 5$.

$(1, 3) \in R$; $(3, 5), \in R$

$\Rightarrow$ (1–3) even integer

(3–5) also even integer

Then (1–5) also even integer

i.e. If $(a, b) \in R$, $(b, c) \in R$

$\Rightarrow (a, c) \in R$

$\therefore$ R is Transitive relation.

Since R is Reflexive, Symmetric and Transitive. So it is an Equivalence relation.

Q.77. If $u(x) = \dfrac{1}{1-x}$, then $u^{-1}(x)$ is :

(*a*) $\dfrac{1}{x-1}$ (*b*) $1 - x$

(*c*) $1 - \dfrac{1}{x}$ (*d*) $\dfrac{1}{x} - 1$

[Dec. 2021]

Solution : (*c*)

Net $y = u(x) = \dfrac{1}{1-x}$

or $\dfrac{1}{y} = 1 - x$

or $x = 1 - \dfrac{1}{y}$

$\therefore u^{-1}(x) = 1 - \dfrac{1}{x}$

Q.78. If $f(p) = \dfrac{1}{1-p}$, then f^{-1} is

(*a*) 1-p (*b*) $\dfrac{p-1}{p}$

(*c*) $\dfrac{p}{p-1}$ (*d*) $\dfrac{1}{p}$

[June 2022]

Solution : Let $y = f(p) = \dfrac{1}{1-p}$

$\therefore 1 - p = \dfrac{1}{y}$

or $p = 1 - \dfrac{1}{y} = \dfrac{1-1}{y}$

So $f^{-1}(p) = \dfrac{P-1}{P}$

(replace y by p).

(b) is correct.

Q.79. Let R = {(3, 3) (6, 6), (9, 9) ,(12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. Then relation is

(*a*) an equivalence relation

(*b*) reflexive and transitive only

(*c*) reflexive only

(*d*) reflexive and symmetric only

[June 2022]

Solution : Given that

$A = \{3, 6, 9, 12\}$

A relation R = {(3,3); (6,6); (9,9); (12,12); (6,12); (3,9); (3,12); (3,6)}

Reflexive Relation :

If x R x $\Rightarrow (x; x) \notin R$

Then R is Reflexive

Here (3, 3); (6, 6); (9, 9), (12, 12) $\notin$ R

So R is Reflexive

Symmetric Relation

$\because$ if (x; y) $\in$ R then (y, x) $\in$ R

Here (3, 9) $\in$ R but (9, 3) $\notin$ R

So R is not Symmetric

Transitive Relation::

if (x; y) $\in$ R; (y, z) $\in$ R

Then (x; z) $\in$ R

Here (3, 6) $\in$ R; (6, 12) $\in$ R

$\Rightarrow$ (3, 12) $\in$ R (True)

$\therefore$ R is Reflexive and Transitive

(*b*) is correct

Q.80. Two finite sets have m and n elements. The total number of sub-sets of the first set is 56 more than the total number of sub-sets of the second set. The values of m and n are

(*a*) 6, 3 (*b*) 7, 6

(*c*) 5, 1 (*d*) 8, 7

[June 2022]

Solution : Formula

Let A has n elements

$\therefore$ Total No. of sub-sets $= 2^n$

Now Difference between No. of sub-sets of both sets

$= 2^m - 2^n$

Go by choices

Let (a) is correct

So, $2^m - 2^n = 2^6 - 2^3$

$= 64 - 8 = 56$ (True)

$\therefore$ (*a*) is correct

Q.81. If A = {1,2,3,4,5,7,8,9} and B = {2, 4, 6, 7, 9} then how many proper subset of $A \cap B$ can be created.

(*a*) 16 (*b*) 15

(*c*) 32 (*d*) 31

[Dec. 2022]

Solution: A = {1, 2, 3, 4, 5, 7, 8, 9}

B = {2, 4, 6, 7, 9}

$A \cap B = \{2, 4, 7, 9\} \Rightarrow n(A \cap B) = 4$

No. of proper subsets of $(A \cap B)$

$= 2^n - 1$

$= 2^4 - 1 = 15$

(*b*) is correct.

Q.82. Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}, Then R is:

(*a*) Symmetric and transitive

(*b*) Reflexive but not transitive

(*c*) Reflexive but not symmetric

(*d*) Neither symmetric nor transitive

[Dec. 2022]

Solution: *Given :*

$A = \{1, 2, 3\}$

Relation = R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

Reflexive : $(x, x) \in R$ (True)

Symmetric : If $(x, y) \in R \Rightarrow (y, x) \in R$

$\therefore$ If $(1, 2) \in R$ but $(2, 1) \notin R$

Not symmetric.

(*c*) is correct.

Q.83. The number of subjects of the set {0, 1, 2, 3} is:

(*a*) 2 (*b*) 4

(*c*) 8 (*d*) 16

[Dec. 2022]

Solution: Let A = {0, 1, 2, 3} be a set.

No. of Subsets = n(Power set of A)

$= nP(A) = 2^n = 2^4 = 16$

(*d*) is correct

Q.84. Given the relation $R = \{(1, 2), (2, 3)\}$ on the set $A = \{1, 2, 3\}$, the minimum number of ordered pairs which when added to R make it equivalence relation is:

(*a*) 5 (*b*) 7

(*c*) 6 (*d*) 8

[June 2023]

Solution:

$A \times A = \{1, 2, 3\} \times \{1, 2, 3\}$

$= \{(1, 1)(1, 2)(1, 3)(2, 1), (2, 2), (2, 3), (3, 1)(3, 2), (3, 3)\}$

Minimum No. of ordered pairs added to R to make it equivalence

$= n(A \times A) - n(R)$

$= 9 - 2 = 7$

(*b*) is correct.

Q.85. A survey shows that 74% of the Canadian like grapes, whereas 68% like bananas. What percentage of the Canadians like both grapes and bananas, if everybody like either of two?

(*a*) 32% (*b*) 26%

(*c*) 6% (*d*) 42%

[June 2023]

Solution:

Let n (A) = 74% = Grapes

n (B) = 68% = Banana

and $n(A \cup B) = 100\%$

$\therefore n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 74 + 68 - 100 = 42\%$

$\therefore$ (*d*) is correct.

Q.86. If R be a relation defined, on the set of Natural numbers as "$x\ Ry \Leftrightarrow (x - y)$ is divisible by 5" $\forall\ x, y \in N$, then the relation R is:

(*a*) Equivalence

(*b*) Anti-symmetric

(*c*) Symmetric but not transitive

(*d*) Symmetric but not reflexive

[June 2023]

Solution: Given

$xRy \Rightarrow (x - y)$ is divisible by 5

Let $x - y = 5m$

where m = An integer

Reflexive Relation:—

$xRx \Rightarrow x - x = 5m$

$= 0 \times 5$

Hence it is Reflexive.

Symmetric Relation:—

Let $xRy \Rightarrow x - y$ is divisible by 5.

Let $x - y = 5m$

where m is an integer.

$\therefore y - x = -(x - y)$

$= -5m = 5(-m)$

Here $-m$ is also an integer.

Hence $y - x$ is also divisible by 5.

$\therefore xRy$ is also Symmetric Relation.

Transitive Relation:—

Let $xRy \Rightarrow x - y = 5m$

and $yRz \Rightarrow y - z = 5n$

i.e. $y - z$ is also divisible by 5.

So, $x - z = x - y + y - z$
$= 5m + 5n$
$= 5(m + n)$

Here $m + n$ is also an integer.

$\therefore x - z$ is also divisible by 5

$\therefore xRy$ is also Transitive.

Hence $x - y$ divisible by 5 is an equivalence relation.

$\therefore$ (*a*) is correct.

Q.87. If $A = (a, b, c)$, $B = (b, c, d)$ and $C = (a, d, c)$, then $(A - B) \times (B \cap C)$ is equal to:

(*a*) $\{(a, d), (c, d)\}$
(*b*) $\{(a, c), (a, d)\}$
(*c*) $\{(c, a), (d, a)\}$
(*d*) $\{(a, c), (a, d), (b, d)\}$

[June 2023]

Solution:

$A - B = \{a, b, c\} - \{b, c, d\}$
$= \{a\}$

Now $B \cap C = \{b, c, d\} \cap \{a, d, c\}$
$= \{c, d\}$
$\therefore (A - B) \times (B \cap C)$
$= \{a\} \times \{c, d\}$
$= \{(a, c)\}; \{a, d\}$
$\therefore$ (*b*) is correct.

Q.88. If $f(x)$: $N \to R$ is a function defined as $f(x) = 4x + 3$, $\forall x \in N$, then $f^{-1}(x)$ is:

(*a*) $4 + \frac{x+3}{4}$

(*b*) $\frac{x+3}{4}$

(*c*) $\frac{x-3}{4}$

(*d*) $\frac{3x+4}{4}$

[June 2023]

Solution:

Let $f(x) = 4x + 3 = y$

or $4x = y - 3$

or $x = \frac{y-3}{4}$

Replacing y by x. We get

$\therefore f^{-1}(x) = \frac{x-3}{4}$

$\therefore$ (*c*) is correct.

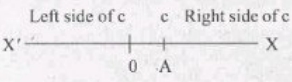
13 CHAPTER CALCULUS

LIMITS AND CONTINUITY

Limits

Meaning of $x \to c$

1. It is known that all real numbers can be plotted on a straight line and every point on it, represents a real number. Let c (zero or non-zero) be any real number. Let it corresponds to the point A on the line as shown below:



We can move to the point A, along the line (X′ OX), from the left side of A as well as from the right side of A.

2. To be definite, let c = 1. Consider two sequences of number:

(*i*) 0.9, 0.99, 0.999, 0.9999,....

(*ii*) 1,1, 1.01, 1.001, 1.0001,...

As one can easily see, the difference of 1 from these numbers is becoming smaller and smaller as we move nearer to 1; we say that these numbers are approaching (or tending to) 1 and we write this in symbols as $x \to 1$.

3. It may further be noted that the numerical difference between x and 1 i.e., $|x, - 1|$ can be made less than any pre-assigned positive number, howsoever small, when we keep moving along these sequences of numbers.

4. Notation

If we reach to c through the right side, we shall write; $x \to c + (\text{or } x \to c + 0)$ and if we reach to c through the left side, we shall write: $x \to c - (\text{or } x \to c - 0)$.

LIMIT OF A FUNCTION

1. Before defining this concept, let us consider two examples. Let f be any function with domain and codomain = R.

Example 1: Let $f(x) = x + 2$. Let us consider: $x \to 1$, i.e., x approaches 1, i.e., x takes values nearer and nearer to 1 from either side. We prepare the following table:

x	$f(x) = x + 2$	$\lvert f(x) - 3 \rvert$
0.9	2.9	0.1

0.99	2.99	0.01
0.999	2.999	0.001
-	-	-
1.0001	3.0001	0.0001
1.001	3.001	0.001

Looking at the above table. we find that as x goes on taking values nearer and nearer to 1, the function. f(x), takes values nearer and nearer to 3. Thus, as $x \to 1$ (from the left or from the right), $f(x) \to 3$. This means, we can bring f(x) as near to 3 as we please by taking x sufficiently near to 1, i.e., the difference, $|f(x) - 3|$ can be made as small as we please, by taking x sufficiently nearer to 1. In such a case, we say that 3 is the limit of f(x) as x tends to 1.

Example 2:

Let $f(x) = \frac{x^2 - 9}{x - 3} (x \neq 3)$. Let x approach 3 and we prepare the following table:

x	f(x)	$\lvert f(x) - 6 \rvert$
2.9	5.9	0.1
2.99	5.99	0.01
2.999	5.999	0.001
2.9999	5.9999	0.0001
-	-	-
-	-	-
3.0001	6.0001	0.0001
3.001	6.001	0.001
3.01	6.01	0.01
3.1	6.1	0.1

We observe from the table that as x aproaches 3, from the left or from the right, f(x) approaches 6, i.e., the difference, $|f(x) - 6|$ can be made as small as we please, by taking x sufficiently close to 3.

Hence, 6 is the limit of f(x) as x tends to 3.

These illustrations lead us to define the concept of limit as follows:

2. Definition

If there exists a finite real number l such that the difference, $|f(x) - l|$ can be made as small as we please, by taking x sufficiently close to c (but not equal to c), then ***l is said to be the limit of f(x) as x tends to c.*** we shall express this in symbols as:

$$\lim_{x \to c} f(x) = l$$

or $f(x) \to l$ as $x \to c$

More precisely: Let f(x) be any function defined in a neighbourhood of c, not necessarily at c. Then f(x) is said to tend to a limit l (a finite real number) as $x \to c$, if for every number $\in > 0$, there exists a number $\delta > 0$ (depending upon $\in$) such that $|f(x) - l| < \in$ whenever $0 < |x - c| < \delta$.

3. ONE SIDE LIMITS

4. When $x \to c$ from the left side (i.e., through values of x less than c) and f(x) is very close to l_1: then we say that l_1 is **the Left Hand Limit (L.H.L.)** of $f(x)$ and we shall write it as:

$$\lim_{x \to c-} f(x) = l_1.$$

5. When $x \to c$ from the right side (i.e., through values of x greater than c) and f(x) is very close to l2: then we say that

l2 is the right hand limit (R.H.L.) of f(x) and we shall write it as:

$$\lim_{x \to c+} f(x) = l_2.$$

Note: If the left hand and the right hand limits of f(x) as x tends to c, both exit and are equal then we say that limit of f(x) as $x \to c$ exists and $\lim_{x \to c-} f(x) = \lim_{x \to c+} f(x) = \lim_{x \to c} f(x)$: otherwise we say that $\lim_{x \to c} f(x)$ does not exist.

SOME RESULTS ON LIMITS

The calculation of limits is based on the following results:

1. $\lim_{x \to c}(f(x) \pm g(x)) = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$.
2. $\lim_{x \to c}[kf(x)] = k \lim_{x \to c} f(x)$ where k is a scalar.
3. $\lim_{x \to c}(f(x).g(x)) = \lim_{x \to c} f(x).\lim_{x \to c} g(x)$.
4. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c}} (provided \lim_{x \to c} g(x) \neq 0)$.

SOME IMPORTANT LIMITS

1 $\lim_{x \to 0} \frac{(e^x - 1)}{x} = 1$

2 $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a (a > 0)$

3 $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

4 $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$

5 $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

6 $\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x = e^a$

7 $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

8 $\lim_{x \to 0} \frac{(1 + x)^n - 1}{x} = n$

9 $e^x \to \infty, as\ x \to \infty$

10 $e^{-x} \to 0, as\ x \to \infty$

11 $\lim_{x \to 0} \frac{1}{x}$ does not exist.

COMPUTATION OF LIMITS

Suppose we have to find $\lim_{x\to a}\frac{f(x)}{g(x)}$.

Direct Substitution Method :-

If f. g are polynomial, Logarithmic or Exponential function such that $g(a)\neq 0$.

Then provided f(a) and g(a) are finite.

Type-I

ILLUSTRATION 1

Evaluate $\lim_{x\to 2}\frac{x^2+x+2}{x^3+1}$

Solution

$$\lim_{x\to 2}\frac{x^2+x+2}{x^3+1}=\frac{2^2+2+2}{2^3+1}=\frac{8}{9}.$$

ILLUSTRATION 2

Evaluate

(*i*) $\lim_{x\to 4}\sqrt[3]{5x+7}$

(*ii*) $\lim_{x\to 1}\frac{x^2-3x+2}{x^2+5}$.

Solution

(*i*) $\lim_{x\to 4}(5x+7)^{1/3}=(5\times 4+7)^{1/3}=(27)^{1/3}=3.$

(*ii*) $\lim_{x\to 1}\frac{x^2-3x+2}{x^2+5}=\frac{1^2-3\times 1+2}{1^2+5}=0.$

Indeterminate Form

After putting the limit value in the function; if we get the result $\frac{0}{0};\frac{\infty}{\infty};\infty-\infty;1^{\infty}$ etc. form then these forms are called indeterminate form.

Type-II L' Hospetal Rule.

Tricks:-

If the Limits are in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form then

I we use L' Hospetal Rule. **working steps** 1st we put x = given limiting value and find the form.

II If the form is $\frac{0}{0}$ then we differentiate numerator and denominator separately.

III After defferentiation ; we again put the limit value then find the result.

If the result is not equal to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then this result is the answer.

IV If we again get the result $\frac{0}{0}$ form then we again differentiate numerator and denominator separately and then put the limit value we continue this process untill not to get $\frac{0}{0}$ form. The found result is the answer.

Ex-1 $\lim_{x\to 1}\frac{x^3-1}{x^2-1}$ is equal to

(*a*) $\frac{3}{2}$ (*b*) $-\frac{3}{2}$ (*c*) $\frac{4}{5}$ (*d*) None

Soln.:- (*a*) is correct.

Tricks:- **L' Hospetal Rule**

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}\left[Form\ \frac{1^3-1}{1^2-1}=\frac{0}{0}\right]$$

Differentiate numerator and denominator with respect to x. ; we get

$$\lim_{x\to 1}\frac{3x^2-0}{2x-0}=\lim_{x\to 1}\frac{3x}{2}=\frac{3\times 1}{2}=\frac{3}{2}$$

Ex-2 $\lim_{x\to 2}\frac{2x^2-7x+6}{5x^2-11x+2}$ is equal to

(*a*) 1/9 (*b*) 9 (*c*) -1/9 (*d*) None

Soln.:- (*a*) is correct

$$\lim_{x\to 2}\frac{2x^2-7x+6}{5x^2-11x+2}\ \left(\text{Form }\frac{0}{0}\right)$$

Using L' Hospetal Rule; we get

$$\lim_{x\to 2}\frac{2\times 2x-7\times 1+0}{5\times 2x-11\times 1+0}=\lim_{x\to 2}\frac{4x-7}{10x-11}$$

$$=\frac{4\times 2-7}{10\times 2-11}=\frac{1}{9}$$

Ex-3 $\lim_{x\to 1}\frac{x^3-5x^2+2x+2}{x^3+2x^2-6x+3}$ is equal to

(*a*) 5 (*b*) -5 (*c*) $\frac{1}{5}$ (*d*) None

Soln.:- (*b*) is correct.

$$\lim_{x\to 1}\frac{x^3-5x^2+2x+2}{x^3+2x^2-6x+3}\ \left(\text{Form }\frac{0}{0}\right)$$

Using L' Hospetal Rule ; we get

$$\lim_{x\to 1}\frac{3x^2-10x+2}{3x^2+4x-6}=\frac{3\times 1^2-10\times 1+2}{3\times 1^2+4\times 1-6}=\frac{3-10+2}{3+4-6}$$

$$=\frac{-5}{1}=-5\ \text{Ans}$$

Type - III (Rationalisation Method)

1st Rationalise then put value.

Ex-1 $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ is equal to

(*a*) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 2 (*d*) None

Soln.:- (*b*) is correct

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\quad\left(form\ \frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}\right)^2-1^2}{x\left(\sqrt{1+x}+1\right)}=\lim_{x\to 0}\frac{1+x-1}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\left(\sqrt{1+x}-1\right)}$$

$$=\frac{1}{\sqrt{1+0}+1}=\frac{1}{1+1}=\frac{1}{2}\ \text{Ans}$$

Ex-2 $\lim_{x\to 0}\frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2}$ is equal to

(*a*) 2 (*b*) -2 (*c*) 2 (*d*) None

Soln.:- (*a*) is correct.

$$\lim_{x\to 0}\frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2}\times\frac{\sqrt{1+2x^2}+\sqrt{1-2x^2}}{\sqrt{1+2x^2}+\sqrt{1-2x^2}}$$

$$=\lim_{x\to 0}\frac{(1+2x^2)-(1-2x^2)}{x^2\left(\sqrt{1+2x^2}+\sqrt{1-2x^2}\right)}$$

$$=\lim_{x\to 0}\frac{1+2x^2-1+2x^2}{x^2\left(\sqrt{1+2x^2}+\sqrt{1-2x^2}\right)}$$

$$=\lim_{x\to 0}\frac{4x^2}{x^2\left(\sqrt{1+2x^2}+\sqrt{1-2x^2}\right)}$$

$$=\lim_{x\to 0}\frac{4}{x^2\left(\sqrt{1+2x^2}+\sqrt{1-2x^2}\right)}$$

$$=\frac{4}{\sqrt{1+0}+\sqrt{1-0}}=\frac{4}{2}=2$$

Type - IV From $\frac{\infty}{\infty}$

Working Rule

(*i*) First of all simplify the given erpression.

(*ii*) Then divide each term of numerator and denominator by x^m

Where x = In dependent variable.

m = maximum power of x.

(*iii*) Then put expression of the form $\frac{a}{x^k}=o$; where a = constant ; a = a positive number

When $x\to\infty$ or $x\to-\infty;\frac{c}{x^k}\to o;k>o$

Ex-1 $\lim_{x\to\infty}\frac{2x^2+7x+5}{4x^2+3x+1}$ is equal to l where l is

(*a*) $-\frac{1}{2}$ (*b*) $\frac{1}{2}$ (*c*) 2 (*d*) None

Soln.:- (*b*) is correct.

$$\lim_{x\to\infty}\frac{2x^2+7x+5}{4x^2+3x+1}\ \left[Form\ \frac{\infty}{\infty}\right]$$

$$= \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{7x}{x^2}+\frac{5}{x^2}}{\frac{4x^2}{x^2}+\frac{3x}{x^2}+\frac{1}{x^2}}$$

[Dividing each term of numerator and denominator by x^2]

$$= \lim_{x\to\infty} \frac{2+\frac{7}{x}+\frac{5}{x^2}}{4+\frac{3}{x}+\frac{1}{x^2}} = \frac{2+\frac{7}{\infty}+\frac{5}{\infty}}{4+\frac{3}{\infty}+\frac{1}{\infty}}$$

$$= \frac{2+0+0}{4+0+0} = \frac{2}{4} = \frac{1}{2} \quad Ans.$$

Ex-2 $\lim_{n\to\infty}\left(\frac{1^2}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+.....+\frac{n^2}{n^3}\right)$ is

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{4}$ (*c*) $-\frac{1}{2}$ (*d*) None

Soln.:- (*a*) is correct.

$$\lim_{n\to\infty}\left[\frac{1^2+2^2+3^2+.....+n^2}{n^3}\right]$$

$$= \lim_{n\to\infty}\frac{\sum n^2}{n^3} = \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}$$

$$= \lim_{n\to\infty}\frac{2n^2+3n+1}{6n^2} \quad \left[Form \frac{\infty}{\infty}\right]$$

$$= \lim_{n\to\infty}\frac{2+\frac{3}{n}+\frac{1}{n^2}}{6} = \frac{2+\frac{3}{\infty}+\frac{1}{\infty}}{6}$$

[Dividing each term of numerator and denominator by n^2]

$$= \frac{2+0+0}{6} = \frac{1}{3} \quad Ans.$$

Ex-3 Find $\lim_{n\to\infty}(2^n-2)(2^n+1)^{-1}$

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) None

Soln.:- (*c*) is correct.

$$\lim_{n\to\infty}\frac{2^n-2}{2^n+1} \quad \left[Form \frac{\infty}{\infty}\right] = \lim_{n\to}\frac{2^n\left[1-\frac{2}{2^n}\right]}{2^n\left[1+\frac{1}{2^n}\right]}$$

$$= \lim_{n\to\infty}\frac{1-\frac{2}{2^n}}{1+\frac{1}{2^n}} = \frac{1-0}{1+0} = 1.$$

Type - V Form $[\infty-\infty]$

(*i*) First of all simplify the given expression.

(*ii*) Then rationalise the expression.

(*iii*) Then take x max. power common and simplify it.

(*iv*) Then put the lim. value as $x\to\infty$ *or* $x\to-\infty$ and find the answer.

Ex-1 $\lim_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)$ is

(*a*) 0 (*b*) 1 (*c*) -1 (*d*) None

Soln. (*a*) is correct.

$$\lim_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)$$

$$= \lim_{x\to\infty}\frac{\sqrt{x+1}-\sqrt{x}}{1}\times\frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}$$

$$= \lim_{x\to\infty}\frac{x+1-x}{\sqrt{x+1}+\sqrt{x}} = \lim_{x\to\infty}\frac{1}{\sqrt{x+1}+\sqrt{x}}$$

$$= \frac{1}{\sqrt{\infty+1}+\sqrt{\infty}} = \frac{1}{\infty+\infty} = \frac{1}{\infty} = 0$$

Ex-2 $\lim_{x\to\infty}\left(\sqrt{x^2+7x}-x\right)$ is

(*a*) 5/3 (*b*) 7/2 (*c*) -7/2 (*d*) None

Soln.:- (*b*) is correct $\lim_{x\to\infty}\frac{\left(\sqrt{x^2+7x}-x\right)}{1}\times\frac{\sqrt{x^2+7x}+x}{\sqrt{x^2+7x}+x}$

$$= \lim_{x\to\infty}\frac{x^2+7x-x^2}{\sqrt{x^2+7x}+x} = \lim_{x\to\infty}\frac{7x}{\sqrt{x^2\left(1+\frac{7}{x}\right)}+x}$$

$$= \lim_{x\to\infty}\frac{7x}{x\sqrt{\left(1+\frac{7}{x}\right)}+x} = \lim_{x\to\infty}\frac{7x}{x\left[\sqrt{1+\frac{7}{x}}+1\right]}$$

$$= \frac{7}{\sqrt{1+0}+1} = \frac{7}{2}$$

Type - VI Form $[1^\infty]$

Formula

(*i*) $\lim_{x\to0}(1+x)^{\frac{1}{x}} = e$ (*ii*) $\lim_{x\to0}(1+mn)^{\frac{1}{mx}} = e$

(*iii*) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$ (*iv*) $\lim_{x\to\infty}\left(1+\frac{m}{x}\right)^{\frac{x}{m}} = e$

Ex-1 $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{7x}$ *is*

(*a*) e^7 (*b*) 1 (*c*) 0 (*d*) None

Soln.:- (*a*) is correct.

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^x\right]^7 = (e)^7 = e^7$$

Ex-2 Find $\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x$.

(*a*) 0 (*b*) 1 (*c*) - 1 (*d*) None

Soln.:- (*b*) is correct.

$$\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x = \lim_{x\to\infty}\left[\left(1+\frac{1}{x^2}\right)^{x^2}\right]^{\frac{1}{x}}$$

$$= e^{\frac{1}{\infty}} = e^0 = 1$$

Ex-3 The value of the limit when x tends to zero of the expression $(1+n)^{\frac{1}{n}}$ is

(*a*) e (*b*) 0 (*c*) 1 (*d*) -1

Soln.:- (*a*) is correct.

$$\lim_{n-\infty}(1+n)^{\frac{1}{n}} = e.$$

Type - VII Form $\left[\frac{0}{0}\right]$

Formula $\lim_{x\to0}\frac{e^{3x}-1}{x} = 1$

Ex-1 $\lim_{x\to0}\frac{e^{3x}-1}{x}$ is

(*a*) 0 (*b*) 3 (*c*) 1 (*d*) None

Soln.:- (*b*) is correct.

Tricks

Use L' Hospetal Rule.

$$\lim_{x\to0}\frac{e^{3x}-1}{x} \left[\text{Form } \frac{0}{0}\right]$$

Differentiating numerator and denominater with respect to x ; we get

$$\lim_{x\to 0}\frac{3e^{3x}-0}{1}=3\times e^{0}=3\times 1=3$$

Ex-2 $\lim_{x\to\infty}\frac{e^{x}+1}{e^{x}+2}$ is evaluated to be

(*a*) 0 (*b*) -1 (*c*) 1 (*d*) None

Soln.:- (*c*) is correct.

$$\lim_{x\to\infty}\frac{e^{x}+1}{e^{x}+2}\quad\left[\text{Form}\frac{\infty}{\infty}\right]$$

Using L' Hospetal rule ; we get

$$\lim_{x\to\infty}\frac{e^{x}+0}{e^{x}+0}$$

[Differentiating numerator and denominator with respect to x]

$$=\lim_{x\to\infty}1=1.$$

Type - VIII

Formula.

(*i*) $\lim_{x\to 0}\frac{\log_e(1+x)}{x}=1$ (*ii*) $\lim_{x\to 0}\frac{x}{\log_e(1+x)}=1$

Ex-1 $\lim_{x\to 0}\frac{\log(1+3x)}{x}$ *is*

(*a*) 1 (*b*) 3 (*c*) -3 (*d*) None

Soln.:- (*b*) is correct

$$\lim_{x\to 0}\frac{\log_e(1+3x)}{x}\lim_{x\to 0}\frac{\log(1+3x)}{3x}\times 3=1\times 3=3$$

Ex-2 $\lim_{x\to 0}\frac{\log(1-x)}{x}$ is eual to

(*a*) 1 (*b*) -1

(*c*) cannot be determined (*d*) none

Soln.:- (*b*) is correct

$$\lim_{x\to 0}\frac{\log(1-x)}{x}=\lim_{x\to 0}\frac{\log[1+(-x)]}{-x}\times(-1)$$

$$=1\times(-1)=-1$$

Ex-3 $\lim_{x\to 0}\frac{\log(1+px)}{e^{3x}-1}$ is eual to

(*a*) $\frac{p}{3}$ (*b*) p (*c*) $\frac{1}{3}$ (*d*) None

Soln.:- (*a*) is correct

$$\lim_{x\to 0}\frac{\left(\frac{\log(1+px)}{px}\right)\times px}{\left(\frac{e^{3x}-1}{3x}\right)\times 3x}=\frac{\lim_{x\to 0}\frac{\log(1+px)}{px}}{\lim_{x\to 0}\frac{e^{3x}-1}{3x}}$$

$$\times\frac{p}{3}=\frac{1\times p}{1\times 3}=\frac{p}{3}$$

Ex-4 The value of $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}$ is

(*a*) 0.5 (*b*) 2 (c) -0.5 (*d*) None

Soln.:- (*a*) is correct.

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$=\lim_{x\to 0}\frac{1+x-1}{\log(1+x).(\sqrt{1+x}+1)}$$

$$=\lim_{x\to 0}\frac{x}{\log(1+x)}\times\frac{1}{\sqrt{1+x}+1}$$

$$=1\times\frac{1}{\sqrt{1+0}+1}=1\times\frac{1}{1+1}=\frac{1}{2}=0.5$$

Type - IX

Formula

$$\lim_{x\to 0}\frac{a^{x}-1}{x}=\log_e a$$

Ex-1 $\lim_{x\to 0}\frac{3^{x}-1}{x}$ is equal

(*a*) $10^3.\log_{10}3$ (*b*) $\log_{10}e$ (*c*) $\log_e 3$ (*d*) None

Soln. (*c*) is correct.

Ex-2 $\lim_{x\to 0}\frac{5^{x}+3^{x}-2}{x}$ will be equal to

(*a*) $\log_e 15$ (*b*) $-\log_e 15$ (*c*) log e (*d*) None

Soln.:-

$$\lim_{x\to 0}\frac{5^{x}-1+3^{x}-1}{x}=\lim_{x\to 0}\left[\frac{5^{x}-1}{x}+\frac{3^{x}-1}{x}\right]$$

$$=\log_e 5+\log_e 3=\log_e 15$$

Ex-3 $\lim_{x\to 0}\frac{10^{x}-5^{x}-2^{x}+1}{x^{2}}$

(*a*) $\log_e 2+\log_e 5$ (*b*) $\log_e 2.\log_e 5$

(*c*) $\log_e 10$ (*d*) None

Soln.:- (*b*) is correct

$$\lim_{x\to 0}\frac{10^{x}-5^{x}-2^{x}+1}{x^{2}}$$

$$=\lim_{x\to 0}\frac{(2\times 5)^{x}-5^{x}-2^{x}+1}{x^{2}}$$

$$=\lim_{x\to 0}\frac{2^{x}\times 5^{x}-5^{x}-2^{x}+1}{x^{2}}$$

$$=\lim_{x\to 0}\frac{5^{x}(2^{x}-1)-1(2^{x}-1)}{x^{2}}$$

$$=\lim_{x\to 0}\frac{(2^{x}-1)(5^{x}-1)}{x\times x}$$

$$=\log_e 2\times\log_e 5$$

Type - X

(Differentiation by 1st principle method)

Formula

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)=\frac{df(x)}{dx}$$

Ex-1 If $f(x)=ax^2+bx+c$ then

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is eual to

(a) ax + b (*b*) ax + 2b (*c*) 2ax + b (*d*) None

Soln.:- (*c*) is correct

$$\because\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)$$

$$=\frac{d}{dx}(ax^2+bx+c)=2ax+b\times 1+0=2ax+b.$$

Ex-2 If $f(x)=5x^2+7x+2$ then

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is equal to

(a) $15x^2 + 7$ (b) $5x^2 + 7$

(c) $15x^2 + 7x + 2$ (d) None

Soln.:- (a) is correct.

$$\because \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x)$$

$$= \frac{d}{dx}(5x^3+7x+2) = 5\times 3x^2 + 7\times 1 + 0 = 15x^2 + 7.$$

Type - XI

(For Two or more than two functions)

Formula.

$$\lim_{x\to a} f(x) \text{ exists if } \lim_{x\to a+} f(x) = \lim_{x\to a} f(x)$$

or RHL = LHL

i.e. Right hand limit = Left hand limit

Ex-1 Evaluate limits of the function defined by

$$f(x) = \begin{cases} 1+x^2 \ ; \ if\ 0 \le x \le 1 \\ 2-x \ \ ; \ if\ x > 1 \end{cases}$$

at x = 1. Then $\lim_{x\to 1} f(x)$ is

(a) 2 (b) 1 (c) does not exist (d) None

Soln.:- (c) is correct.

$$\text{LHL} = \lim_{x\to 1-} \frac{f(x)}{n<1} = \lim_{x\to 1}(1+x^2) = 1+1^2 = 2$$

$$\text{RHL} = \lim_{x\to 1+} \frac{f(x)}{x>1} = \lim_{x\to 1}(2-x) = 2-1 = 1$$

Clearly $\lim_{x\to 1-} f(x) \neq \lim_{x\to 1+} f(x)$

$\therefore \lim_{x\to 1} f(x)$ does not exist.

Ex-2 Find $\lim_{x\to 0}\frac{3x+|x|}{7x-5|x|}$

(a) 2 (b) $\frac{1}{6}$

(c) Limit does not exist. (d) None

Soln.:- (c) is correct

$$\therefore f(x) = \frac{3x+x}{7x-5x} = 2 \ if\ x > 0$$

$$= \frac{3x-x}{7x+5x} = \frac{1}{6} \ if\ x < 0$$

$$\therefore RHL = \lim_{x\to 0+} f(x) = \lim_{x\to 0}(2) = 2$$

$$LHL = \lim_{x\to 0-} f(x) = \lim_{x\to 0} \frac{1}{6} = \frac{1}{6}$$

Clearly $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

$\therefore \lim_{x\to 0} f(x)$ **does not exist**

Ex-3 If $f(x) = \begin{cases} 2x-1; \ where\ x < 2 \\ \frac{3}{2}x; \ where\ x \ge 2 \end{cases}$ then $\lim_{x\to 2} f(x) = ?$

(a) 3 (b) $\frac{3}{2}$

(c) 2 (d) Does not exist

Soln.:- (a) is correct.

$$LHL = \lim_{n\to 2^-} f(x) = \lim_{x\to 2}(2x-1) = 2\times 2 - 1 = 3$$

$$RHL = \lim_{n\to 2^+} f(x) = \lim_{x\to 2}\left(\frac{3}{2}x\right) = \frac{3}{2}\times 2 = 3$$

$$\therefore \lim_{x\to 2^-}(x) = \lim_{x\to 2^+} f(x) = 3$$

$$\therefore \lim_{x\to 2} f(x) \text{ exists } = 3$$

CONTINUITY

Continuity : - Something which goes on without interruption and without abrupt changes is called **Continuous**.

A function f(x) is said to be continuous at x = a

if and only if

(i) f(x) is defined at x = a

(ii) $\lim_{x\to a-} f(x) = \lim_{x\to a+} f(x)$

(iii) $\lim_{x\to a} f(x) = f(a)$

If *f(x)* is not continuous at x = a. we say that it is discontinuous at x = a.

Notes:

(i) The sum, difference and product of two continuous functions is a continuous function. This property holds good for any finite number of functions.

(ii) The quotient of two continuous functions is a continuous function provided the denominator is not equal to zero.

(iii) Constant functions are continuous every where.

(iv) Polynomial functions are continuous every where.

(v) A rational function is continuous at all points except at those points where the denominator is zero.

(vi) Exponential and logarithmic functions are continuous at every point of their respective domains.

Ex-1 A function g (x) is defined as follows.

g(x) = x when 0 < x < 1

= 2 - x when x ≥ 1.

Is g(x) continuous at x = 1. ?

(a) continuous (b) Discontinuous

(c) undefined at x = 1 (d) None

Soln.:- (a) is correct

$\because g(x) = x$; when $0 < x < 1$

$= 2 - x$; when $x \ge 1$

$\therefore g(x=1) = 2-1 = 1.$

$$\lim_{x\to 1^+} g(x) = \lim_{x\to 1}(2-x) = 2-1 = 1$$

$$\lim_{x\to 1^-} g(x) = \lim_{x\to 1}(x) = 1.$$

$$\therefore g(x=1) = \lim_{x\to 1^+} g(x) = \lim_{x\to 1^-} g(x) = 1$$

$\therefore g(x)$ is continuous at x = 1

Ex-2 *If* $f(x) = \frac{1}{2} - x$; when $0 < x < \frac{1}{2}$

$= \frac{3}{2} - x$; when $\frac{1}{2} < x < 1$

$= \frac{1}{2}$; when $x = \frac{1}{2}$

Discuss the continuity of *f(x)* at $x = \frac{1}{2}$

(a) continuous at $x = \frac{1}{2}$

(b) discontinuous and defined at x = 1/2

(c) discontinuous and undefined at $x = \frac{1}{2}$

(d) None of these

Soln.:- (*b*) is correct

$$f\left(x=\frac{1}{2}\right)=\frac{1}{2}(defined)$$

$$\lim_{x\to\frac{1}{2}^+} f(x)=\lim_{x\to\frac{1}{2}}\left(\frac{3}{2}-x\right)=\frac{3}{2}-\frac{1}{2}=1$$

$$\lim_{x\to\frac{1}{2}^-} f(x)=\lim_{x\to\frac{1}{2}}\left(\frac{1}{2}-x\right)=\frac{1}{2}-\frac{1}{2}=0.$$

$$\therefore\ f\left(x=\frac{1}{2}\right)\neq\lim_{x\to\frac{1}{2}^+} f(x)\neq\lim_{x\to\frac{1}{2}^-} f(x)$$

Hence f(x) is discontinuous at $x=\frac{1}{2}$ but defined at $x=\frac{1}{2}$.

Ex-3 Find the points of discontinuity of the function $f(x)=\frac{x^2+2x+5}{x^2-3x+2}$

(*a*) 1; (*b*) 2;
(c) (*a*) and (*b*) both (d) None of these

Soln.:- (*c*) is correct.

For discontinuity of *f(x)* ; denominator should be equal to zero.

$\therefore\ x^2-3x+2=0$
or $x^2-2x-x+2=0$
or $x(x-2)-1(x-2)=0$
or $(x-2)(x-1)=0$
$\therefore\ x=2\ ;\ 1.$
$\therefore$ f(x) is discontinuous at x = 1 ; 2

Ex-4 The function $f(x)=\frac{x^2-9}{x-3}$ is undefined at x = 3, what value must be assigned to f(3) if f(x) is to be continuous at x = 3 ?

(*a*) 6 (*b*) -6
(*c*) cannot be decided (*d*) none

Soln.:- (*a*) is correct.

$\because f(x)=\frac{x^2-9}{x-3}$

$\because f(x)$ *is undefined at* $x=$.

$\therefore x-3\neq 0.$

$$\therefore\ \lim_{x\to 3} f(x)=\lim_{x\to 3}\frac{(x+3)(x-3)}{(x-3)}$$

$$\lim_{x\to 3}(x+3)=3+3=6$$

$\because f(x)$ *is continuous at* $x=3$

$$\therefore f(3)=\lim_{x\to 3} f(x)=6$$

MODEL EXAM QUESTIONS (FOR PRACTICE)

Choose the most appropriate option (a) (b) (c) or (d)

1. $\lim_{x\to 0} f(x)$ when $f(x)=6$ is
(*a*) 6 (*b*) 0 (*c*) 1/6 (*d*) none of these

2. $\lim_{x\to 2}(3x+2)$ is equal to
(*a*) 6 (*b*) 4 (*c*) 8 (*d*) none of these

3. $\lim_{x\to -2}\frac{x^2-4}{x+2}$
(*a*) 4 (*b*) -4 (*c*) does not exist (*d*) none of these

4. $\lim_{x\to\infty}\left(\frac{3}{x^2}+2\right)$
(*a*) 0 (*b*) 5 (*c*) 2 (*d*) none of these

5. $\lim_{x\to 1}\log e^x$ is evaluated to be
(*a*) 0 (*b*) *e* (*c*) 1 (*d*) none of these

6. The value of the limit of f (x) as x $x\to 3$ when $f(x)=e^{x^2+2x+1}$ is
(*a*) e^{15} (*b*) e^{16} (*c*) e^{10} (*d*) none of these

7. $\lim_{x\to 1/2}\left(\frac{8x^3-1}{6x^2-5x+1}\right)$ is equal to
(*a*) 5 (*b*) -6 (*c*) 6 (*d*) none of these

8. $\lim_{x\to 1}\frac{x^3-5x^2+2x+2}{x^3+2x^2-6x+3}$ is equa to
(*a*) 5 (*b*) -5 (*c*) 1/5 (*d*) none of these

9. $\lim_{x\to t}\frac{x^3-t^3}{x^2-t^2}$ evaluated to be
(*a*) -1 (*b*) 1 (*c*) 0 (*d*) none of these

10. $\lim_{x\to 0}\frac{4x^4+5x^3 7x^2+6x}{5x^5+7x^2+x}$ is equal to
(*a*) 6 (*b*) 5 (*c*) -6 (*d*) none of these

11. $\lim_{x\to 2}\frac{(x^2-5x+6)(x^2-3x+2)}{x^3-3x^2+4}$ is equal to
(*a*) 1/3 (*b*) 3 (*c*) -1/3 (*d*) none of these

12. $\lim_{x\to\infty}\frac{\sqrt{3x^4+5x^2+7x+5}}{4x^2}$ is evaluated
(*a*) $\frac{\sqrt{3}}{4}$ (*b*) $\sqrt{3}$ (*c*) -1/4 (*d*) none of these

13. $\lim_{x\to 1}\frac{(1-x^{-1/3})}{(1-x^{-2/3})}$ is equal to
(*a*) - 1/2 (*b*) 1/2 (*c*) 2 (*d*) none of these

14. $\lim_{x\to 4}\frac{(x^2-16)}{(x-4)}$ is evaluated as
(*a*) -3 (*b*) 1/3 (*c*) 3 (*d*) none of these

15. If $f(x)=\frac{(1+x)^6-1}{(1+x)^2-1}$ then $\lim_{x\to 0} f(x)$ exists and is equal to
(*a*) 3 (*b*) -1/3 (*c*) -3 (*d*) none of these

16. $\lim_{x\to 2}\frac{4-x^2}{3-\sqrt{x^2+5}}$ is equal to
(*a*) 6 (*b*) 1/6 (*c*) -6 (*d*) none of these

17. $\lim_{x\to\sqrt{2}}\frac{x^{3/2}-2^{3/4}}{\sqrt{x}-2^{1/4}}$ exists and is equal to a finite value where is
(*a*) -5 (*b*) 1/6 (c) 6 (*d*) none of these

18. $\lim_{x\to 0}\left(\frac{x^2-\sqrt{x}}{\sqrt{x}-1}\right)$ is equal to
(*a*) 3 (*b*) -3 (*c*) 1/3 (*d*) none of these

19. $\lim_{x\to 0}\frac{e^{x^2}-1}{x^2}$ is evaluated to be
(*a*) 1 (*b*) 1/2 (*c*) -1 (*d*) none of these

20. If $\lim_{x\to 2}\frac{e^n-2^n}{x-2}=80$ and n E N then
(*a*) n = 5 (*b*) n = 4 (*c*) n = 0 (*d*) none of these

21. $\lim_{x\to\sqrt{2}}\frac{x^{5/2}-2^{5/4}}{\sqrt{x}-2^{1/4}}$ is equal to
(*a*) 1/10 (*b*) 10 (*c*) 20 (*d*) none of these

22. $\lim_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{x}{x^3-1}\right)$ is evaluated to be
(*a*) 1/9 (*b*) 9 (*c*) -1/9 (*d*) none of these

23. $\lim_{x\to 0}\frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2}$ is equal to

24. $\lim_{x\to p}\frac{\sqrt{x-q}-\sqrt{p-q}}{x^2-p^2}(p>q)$ is evaluated as
(*a*) $\frac{1}{p\sqrt{p-q}}$ (*b*) $\frac{1}{p\sqrt{p-q}}$ (*c*) $\frac{1}{2p\sqrt{p-q}}$ (*d*) none of these

25. $\lim_{x\to 1}\frac{x^2-1}{\sqrt{3x+1}-\sqrt{5x-1}}$ is evaluated to be

(a) 4 (b) 1/4 (c) -4 (d) none of these

26. $\lim(\sqrt{x+h}-\sqrt{x})/h$ where $h\to 0$ is equal to

(a) $1/2\,x$ (b) $1/2x$ (c) $x/2$ (d) none of these

27. $\lim_{x\to 0}\frac{(e^{2x}-1)}{x}$ is equal to

(a) 1/2 (b) 2 (c) 0 (d) none of these

28. $\lim_{x\to 0}\frac{e^{x^2}-1}{x^2}$ is evaluated to be

(a) 1 (b) 1/2 (c) -1 (d) none of these

29. $\lim_{x\to\infty}\frac{e^x+1}{e^x+2}$ is evaluated to be

(a) 1/2 (b) 2 (c) 0 (d) none of these

30. $\lim_{x\to 0}\frac{(e^x+e^{-x}-2)(x^2-3x+2)}{(x-1)}$ is equal to

(a) 1 (b) 0 (c) -1 (d) none of these

31. $\lim_{x\to 0}\frac{(3^x-1)}{x}$ is equal to

(a) $10^3\log_{10}3$ (b) $\log_3 e$ (c) $\log_e 3$ (d) none of these

32. $\lim_{x\to 0}\frac{5^x+3^x-2}{x}$ will be equal to

(a) $\log_e 15$ (b) $\log(1/15)$ (c) $\log e$ (d) none of these

33. $\lim_{x\to 0}\frac{10^x-5^x-2^x+1}{x^2}$ equal to

(a) $\log_e 2+\log_e 5$ (b) $\log_e 2\log_e 5$

(c) $\log_e 10$ (d) none of these

34. If $f(x)=\frac{e^{\log x}-1}{e^{x-1}-1}$ then $\lim_{x\to 0} f(x)$ exists and is equal to

(a) -1 (b) 1 (c) 0 (d) none of these

35. $\lim_{x\to 0}\log\frac{(1+px)}{e^{3x}-1}$ is equal to

(a) p/3 (b) P (c) 1/3 (d) none of these

36. The value of $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}$

(a) 0.5 (b) 2 (c) -0.5 (d) none of these

37. If $f(x)=ax^2+bx+c$ then $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is equal to

(a) $ax+b$ (b) $ax+2b$ (c) $ax+b$ (d) none of these

38. $\lim_{x\to\infty}\frac{(2x^2+7x+5)}{4x^2+3x+1}$ is equal to 1 where 1 is

(a) -1/3 (b) 1/2 (c) 2 (d) none of these

39. $\lim_{x\to\infty}\frac{(x\sqrt{x}-m\sqrt{m})}{1-x^{-2/3}}$ is equal to

(a) 1 (b) -1 (c) 1/2 (d) none of these

40. $\lim_{x\to 0}$ 1/x log (1- x /2) is equal to

(a) -1 / 2 (b) 1 / 2 (c) 2 (d) none of these

41. A function f(x) is defined as follows: f(x)= x when $x \le 1 = 1 + x$ when $x > 1 = 3/2$ when $x = 1$ Then f(x) is

(a) continuous at x = 1/2 (b) discontinuous at x = 1

(c) undefined at x = 1,2 (d) none of these

42. A function f(x) is defined by f(x) = (x - 2) + 1 over all real values of x. Now f(x) is

(a) continuous at x = 2 (b) discontinuous at x = 2

(c) undefined at x = 2 (d) none of these

43. Let f(x) = x/ | x |. Now f(x) is

(a) continuous at x = 0 (b) discontinuous at x = 0

(c) defined at x = 0 (d) none of these

44. $f(x) = x - 1$ when $x > a = -1/2$ when $x = 0 = x + 1$ when $x < a$ f(x) is

(a) continuous at x = 0 (b) undefined at x

(c) discontinuous at x = 0 (d) none of these

45. f(x) = (x² - 1) / (x³ - 1) is undefined at x = 1 the value of f(x) at x = 1, such that it is continuous at x = 1 is

(a) 3/2 (b) 2/3 (c) -3/2 (d) none of these

46. f(x) = 2x - | x | is

(a) undefined at x = 0 (b) discontinuous at x = 0

(c) continuous at x = 0 (d) none of these

47. $f(x)=\frac{x^2-3x+2}{x-1}$ $x\neq 1$ becomes continuous at x = 1. Then the value of f(1) is

(a) 1 (b) -1 (c) 0 (d) none of these

48. $f(x)=\frac{(x^2-3x-2}{(x+1)}$ $x\neq -1$ and the function f(x) is continuous at x = - 1. The value of k will be

(a) -1 (b) 1 (c) -4 (d) none of these

49. If $f(x) = 3x < 2 = kx^2$ when $x \ge 2$ is continuous at x = 2 then the value of k is

(a) 3/4 (b) 4/3 (c) 1/3 (d) none of these

ANSWERS

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(a)	**6.**	(b)	**7.**	(c)
8.	(b)	**9.**	(c)	**10.**	(a)	**11.**	(c)	**12.**	(a)	**13.**	(b)	**14.**	(a)
15.	(c)	**16.**	(a)	**17.**	(c)	**18.**	(d)	**19.**	(d)	**20.**	(a)	**21.**	(b)
22.	(c)	**23.**	(a)	**24.**	(b)	**25.**	(c)	**26.**	(d)	**27.**	(b)	**28.**	(d)
29.	(c)	**30.**	(b)	**31.**	(c)	**32.**	(a)	**33.**	(b)	**34.**	(b)	**35.**	(a)
36.	(a)	**37.**	(c)	**38.**	(b)	**39.**	(d)	**40.**	(a)	**41.**	(a)	**42.**	(a)
43.	(b)	**44.**	(c)	**45.**	(b)	**46.**	(c)	**47.**	(b)	**48.**	(c)	**49.**	(a)

14 CHAPTER DIFFERENTIAL CALCULUS

DIFFERENTIATON

Concept of Differentiation

We are familiar with the expressions like kilometres/hour, price/metre, etc. All such expressions (or functions) represent rates. The rate of change of function of one variable with respect to another on which it depends is called the **derivative of the function.** The process of finding the derivative in terms of a limit involving the increments of the independent and the dependent variables is called **differentiation.**

Let $y = f(x)$ be a function of x. If values of a variable x is changed from one value x_0 to another value x_1, then the difference $x_1 - x_0$ is called the increment of x (which may be positive or negative) and we denote it by Δx (or h). In the similar manner, the increment *of y or f(x)* is denoted by Δy *or* $\Delta f(x)$, *i.e.*, $f(x + \Delta x) - f(x)$.

∴ The rate of change is given by

$$\frac{\Delta y}{\Delta x} = \frac{\text{increment in the value of } y(\text{dependent variable})}{\text{increment in the value of } x(\text{independe nt variabl e})} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

If this ratio tends to a definite finite limit as Δx tends to zero from either side, then this limit is called the differential coefficient (or derivative) of $f(x)$ with respect to x. Symbolically, the differential coefficient of y with respect to x is denoted by

$$\frac{dy}{dx} \text{ or } f'(x) y_1 \text{ or } \frac{d}{dx}[f(x)] \text{ or } Df(x).$$

Thus, $\frac{dy}{dx}$ or $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

The process of finding the differential coefficient is known as **differentiation.**

Remarks I. For the differential coefficient f'(x) to exist, the right-hand and left-hand different coefficient must both exists and equal, i.e.,

$$\lim_{\Delta x \to 0^+} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

2. The differential coefficient of f(x) at any fixed value 'c' of x is given by

$$f'(c) = \lim_{\Delta x \to 0} \frac{f(c+\Delta x)-f(c)}{\Delta x}$$, provided this limit exists.

Illustration. Consider the function, $y = f(x) = x^2$.

By definition,

$$\frac{dy}{dx} = \frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2+2x\Delta x+(\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x+\Delta x) = 2x+0 = 2x.$$

FORMULAE OF DIFFERENTIATION

Let f(x) and g(x) be differentiable function and $\alpha \in R$

1. Sum and Difference Rule

$$\frac{d}{dx}(f(x) \pm g(g)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

2. Scalar Multiple Rule

$$\frac{d}{dx}(\alpha f(x)) = \alpha \frac{d}{dx} f(x)$$

3. $\frac{dc}{dx} = 0$

Where c = Constant

4. $\frac{dx^n}{dx} = nx^{n-1}$

5. $\frac{da^x}{dx} = a^x.\log e^a$

$= a^x.\log a$

Where a = Constant

6. $\frac{de^x}{dx} = e^x$

7. $\frac{d}{dx}\log x = \frac{d}{dx}\log e^x = \frac{1}{x}$

8. $\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$

9. $\frac{d}{dx}\log a^x = \frac{1}{x \log e^a} = \frac{1}{x \log a}$

10. $\frac{dx^x}{dx} = x^x(1+\log x)$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. If $f(x) = {}^xC_3$; then $f'(1) = ?$

(a) $\frac{1}{6}$ (b) $-\frac{1}{6}$

(c) $\frac{5}{6}$ (d) $-\frac{5}{6}$

[Dec. 2010]

Solution : (b) is correct.

Given $f(x) = x_{C_3} = \frac{x!}{3!(x-3)!}$

$= \frac{x(x-1)(x-2).(x-3)!}{6.(x-3)!}$

$\therefore f(x) = \frac{(x^3-3x^2+2x)}{6}$

Differentiating w.r.t(x) both the sides, we get

$f'(x) = \frac{1}{6}(3x^2-6x+2)$

$\therefore f'(1) = \frac{1}{6}(3\times1^2-6\times1+2)$

$\therefore f'(1) = \frac{-1}{6}$

Q.2. $\frac{d}{dx}\left[2^{\log_2 x}\right] =$ _____

(a) 1 (b) 0

(c) 1/2 (d) $2^x.\log_2 x$

[Dec. 2011]

Solution : (a) is correct.

$\frac{d}{dx}2^{\log_2 x} = \frac{d(x)}{dx} = 1$

Formula ($\because a^{\log_a x} = x$)

Q.3. If $Y = X^x$ then $\frac{d^2y}{dx^2} =$ _____

(a) $z\frac{dY}{dx}(1+\log x)+Y\frac{d}{dx}(1+\log x)$

(b) $\frac{dY}{dx}(1+\log x)+\frac{d}{dx}(1+\log x)$

(c) $\frac{dY}{dx}(1+\log x)-Y\frac{d}{dx}(1+\log x)$

(d) $\frac{dY}{dx}(1+\log x)-\frac{d}{dx}(1+\log x)$

[Dec. 2011]

Solution : (a) is correct.

If $y = x^x$

taking log on both side

$\log y = \log x^x$

$\log y = x \log x$

Diff w.r.t (x)

$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x} + \log x.1$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1+\log x$

$\frac{dy}{dx} = y(1+\log x)$

Again differentiating w.r.t 'x'

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\{y(1+\log x)\}$

$\frac{d^2y}{dx^2} = y.\frac{d}{dx}(1+\log x)+(1+\log x)\frac{dy}{dx}$

$= \frac{dy}{dx}(1+\log x)+y.\frac{d}{dx}(1+\log x)$

Q.4. If x = c t, y = c/t, then $\frac{dy}{dx}$ is equal to :

(a) 1/t (b) $t.e^t$

(c) $-1/t^2$ (d) None of these

[June 2012]

Solution : (c) is correct.

$\frac{dx}{dt} = C \times 1 = C$

$\frac{dy}{dt} = C \times \frac{d}{dt}\left(\frac{1}{t}\right)$

$= C\left(-\frac{1}{t^2}\right) = -\frac{c}{t^2}$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-c/t^2}{C}$

$= -\frac{1}{t^2} = -t^{-2}$

Q.5. If $y = e^{a\log x} + e^{x\log a}$, then $\frac{dy}{dx} =$

(a) $X^a + a^x$

(b) a. $X^{a-1} + a^x \log a$

(c) $a X^{a-1} + Xa^{x-1}$

(d) $X^x + a^a$

[June 2012]

Solution : (b) is correct.

$y = e^{a\log x} + e^{x\log a}$

$\Rightarrow y = x^a + a^x$ [$\because e^{\log m} = m$]

Diff. w.r.t. x on both side ; we get

$\frac{dy}{dx} = ax^{a-1} + a^x \log a$

Q.6. For the functions $y = x^3 - 3x$, the value of $\frac{d^2y}{dx^2}$ at which $\frac{dy}{dx}$ is zero, is

(a) ±1 (b) ±3

(c) ±6 (d) None of these

[Dec. 2012]

Solution : (c) is correct.

Given $y = x^3 - 3x$

Diff. w.r.t. 'x'

$\frac{dy}{dx} = 3x^2 - 3$ _____(1)

$0 = 3(x^2-1)$

$x^2 - 1 = 0$

$x^2 = 1$; so $x = \pm1$

Diff. (1) w.r.t 'x'

$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2-3) = 6x$

$\left(\frac{d^2y}{dx^2}\right)_{(x=\pm1)} = 6(\pm1) = \pm6$

Q.7. The equation of the tangent to the curve, $x^3 - 2x + 3$, at the point (2,7) is

(a) y = 2x-13 (b) y = 10x

(c) y = 10x -13 (d) y = 10

[Dec. 2012]

Solution : (c) is correct. Given that

$f(x) = x^3 - 2x + 3$

i.e. $y = x^3 - 2x + 3$

$\frac{dy}{dx} = 3x^2 - 2$

$\left(\frac{dy}{dx}\right)_{(2,7)} = 3(2)^2 - 2$

$= 12 - 2$

$\left(\frac{dy}{dx}\right)_{(2,7)} = 10$

Slope of tangent $m = \left(\frac{dy}{dx}\right)_{(2,7)} = 10$

The equation of tangent at (2,7)

$y - y_1 = m(x - x_1)$

y - 7 = 10 (x - 2)

y - 7 = 10x - 20

y = 10x - 20 + 7

y = 10x - 13 **Tricks** : GBC.

Q.8. If $y = \log\left[\frac{5-4x^2}{3+5x^2}\right]$, then $\frac{dy}{dx} =$ _____

(a) $\frac{8}{4x-5} - \frac{10}{3+5x}$

(b) $(4x^2-5)-(3+5x^2)$

(c) $\frac{8x}{4x^2-5} - \frac{10x}{3+5x^2}$

(d) 8x-10

[Dec. 2012]

Solution : (c) is correct.

Since, $y = \log\left(\frac{5-4x^2}{3+5x^2}\right)$

$y = \log(5-4x^2) - \log(3+5x^2)$

Diff. w.r.t. x

$\frac{dy}{dx} = \frac{d}{dx}\log(5-4x^2) - \frac{d}{dx}\log(3+5x^2)$

$= \frac{1}{(5-4x^2)}\frac{d}{dx}(5-4x^2) - \frac{1}{(3+5x^2)}\frac{d}{dx}(3+5x^2)$

$= \frac{1}{(5-4x^2)}(0-8x) - \frac{1}{(3+5x^2)}(0+10x)$

$= \frac{8x}{4x^2-5} - \frac{10x}{3x+5x^2}$

Q.9. If $y = \log_y x$ then $\frac{dy}{dx} =$

(a) $\frac{1}{x\log y}$ (b) $\frac{1}{x+x\log y}$

(c) $\frac{1}{1+x\log y}$ (d) $\frac{1}{y+\log x}$

[June 2013]

Solution : (b) Let $y = \log_y x = \frac{\log_e x}{\log_e y}$

or $\log_e x = y\log_e y$ _____(1)

Diff. on both sides w.r.t.y; we get

$\frac{1}{x}\cdot\frac{dx}{dy} = y.\frac{1}{y} + 1.\log_e y = 1 + \log_e y$

or $\frac{dx}{dy} = x + x\log_e y$

$\therefore \frac{dy}{dx} = \frac{1}{x+x\log_e y}$ $\therefore$ (b) is correct

Q.10. $y = e^t$ & $x = \log t$; then $\frac{dy}{dx} =$

(a) $\frac{1}{t}$ (b) te^t

(c) $-\frac{t}{t^2}$ (d) none

[June 2013]

Solution : (b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t}{\frac{1}{t}} = te^t$

$\therefore$ (b) is correct.

Q.11. The points on the curve $y = x^3 - x^2 - x + 1$. Where the tangent is parallel to x-axis are:

(a) $(1,0)\left(\frac{-1}{3},\frac{32}{27}\right)$

(b) (1,0) (1,1)

(c) $\left(\frac{-1}{3},\frac{21}{37}\right)(0,0)$

(d) (0,0) (1,0)

[Dec. 2013]

Solution : (a) is correct.

$\because y = x^3 - x^2 - x + 1$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

$\because$ Tangent is parallel to x - axis

$\therefore \frac{dy}{dx} = 0$

$3x^2 - 2x - 1 = 0$

or $3x^2 - 3x + x - 1 = 0$

or $3x(x-1)+1(x-1)=0$

or $(x-1)(3x+1)=0$

$\therefore x=1; x=-1/3$

At x = 1

$y = 1^3 - 1^2 - 1 + 1 = 0$

$\therefore$ Point is (1;0)

and At x = -1/3

Then $y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(\frac{-1}{3}\right) + 1$

$= \frac{32}{27}$

Or; Another Point is $= \left(-\frac{1}{3};\frac{32}{27}\right)$

Tricks : Go by choices

Q.12. A seller makes an offer of selling certain articles that can be described by the equation x = 25 - 2y where x is price per unit and y denotes the No. of units. The cost price of the article is ₹10 per unit. The maximum quantity that can be offered in single deal to avoid loss is

(a) 6 (b) 7

(c) 8 (d) 9

[Dec. 2013]

Solution : (b) is correct.

$\because x = 25 - 2y$

Total cost = Cost price per unit $\times$ No. of unit sold

$= 10y$

Total sale = Selling price per unit $\times$ No. of units sold

$= x.y = (25 - 2y)y = 25y - 2y^2$

Profit = sale - Cost

$= 25y - 2y^2 - 10y = 15y - 2y^2$

For No loss; Profit > 0

$15y - 2y^2 > 0$

or $15 - 2y > 0$

or $\frac{15}{2} > y \Rightarrow y < 7.5$

y = No. of units (a whole No.)

$\therefore y = 7$

$\therefore$ Maximum Quantity sold y = 7

Tricks : Go by choices

Q.13. If $y = ae^{nx} + be^{-nx}$ then $\frac{d^2y}{dx^2}$

(a) n^2y (b) $-n^2y$

(c) ny (d) None

[June 2014]

Solution : (a) is correct.

$\therefore y = ae^{nx} + be^{-nx}$

So; $\frac{dy}{dx} = a.e^{nx}.n + b.e^{-nx}(-n)$

$= n\left[ae^{nx} - be^{-nx}\right]$

$\therefore \frac{d^2y}{dx^2} = n\left[a.e^{nx}.n - b.e^{-nx}(-n)\right]$

$= n.n\left[ae^{nx} + be^{-nx}\right]$

$= n^2.y$

Q.14. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots\ldots + \frac{x^n}{n!} + \ldots\ldots$, then the value of $\frac{dy}{dx} - y =$ ____

(a) 1 (b) 0

(c) -1 (d) None

Solution : (b) is correct.

$\because y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots + \frac{x^n}{n!} + \ldots\ldots$

$\therefore y = e^x$ _____(1)

$\therefore \frac{dy}{dx} = \frac{de^x}{dx} = e^x = y$ [From (1)]

$\therefore \frac{dy}{dx} - y = 0$

Q.15. If $e^{xy} - 4xy = 4$ then $\frac{dy}{dx} =$

(a) $\frac{y}{x}$ (b) $\frac{-y}{x}$ (c) $\frac{x}{y}$ (d) $\frac{-x}{y}$

[June 2015]

Solution : If $e^{xy} - 4xy - 4 = 0$

Tricks : $\frac{dy}{dx} = -\frac{D.C.w.r.t.x \text{ keeping y constant}}{D.C.w.r.t.y \text{ keeping x constant}}$

$= -\frac{e^{xy}.y - 4y - 0}{e^{xy}.x - 4x - 0}$

$= -\frac{(4xy+4)y - 4x}{(4xy+4)x - 4x}$

$= -\frac{4xy^2 + 4y - 4y}{4x^2y + 4x - 4x} = -\frac{4xy^2}{4x^2y}$

$= -y/x$

$\therefore$ (b) is correct.

Q.16. If $x^p.y^q = (x+y)^{p+q}$ then $\frac{dy}{dx} =$

(a) $\frac{y}{x}$ (b) $\frac{-y}{x}$ (c) $\frac{p}{q}$ (d) $\frac{-p}{q}$

[June 2015]

Solution : $x^p.y^2 = (x+y)^{p+q}$

Tricks $\frac{dy}{dx} = \frac{y}{x}$

(a) is correct

Q.17. Find slope of tangent of curve $y = \frac{x-1}{x+2}$ at x = 2.

(a) 3/16 (b) 5/17

(c) 9/11 (d) None of the above

[Dec. 2015]

Solution : Slope - $m = \frac{dy}{dx} = \frac{\frac{d}{dx}(x-1)(x+2) - (x-1)\frac{d}{dx}(x+2)}{(x+2)^2}$

$= \frac{(1-0)(x+2) - (x-1)(1+0)}{(x+2)^2}$

$= \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$

Slope at $(x=2) = \frac{3}{(2+2)^2} = \frac{3}{16}$

(a) is correct

Q.18. $u = 5t^4 + 4t^4 + 2t^4 + 4$ at t=-1 find du/dt

(a) -11 (b) 11

(c) -16 (d) 16

[Dec. 2015]

Solution : (a) is correct.

$u = 5t^4 + 4t^3 + 2t^2 + t + 4$

$\frac{du}{dt} = 5\times4t^3 + 4\times3t^2 + 2\times2t + 1 + 0$

$= 20t^3 + 12t^2 + 4t + 1$

$\frac{du}{dt}$ at $= -1$

$= 20(-1)^3 + 12(-1)^2 + 4(-1) + 1$

$= -20 + 12 - 4 + 1 = -11$

Q.19. $Y = \sqrt{\frac{1-x}{1+x}}$ then $\frac{dy}{dx}$ is equal to _____

(a) $\frac{y}{x^2-1}$ (b) $\frac{y}{1-x^2}$

(c) $\frac{y}{1+x^2}$ (d) $\frac{y}{y^2-1}$

[June 2016]

Solution : (a) is correct.

$\therefore y = \sqrt{\frac{1-x}{1+x}} = \left(\frac{1-x}{1+x}\right)^{1/2}$

Taking log on both sides; we get

$\log y = \log\left(\frac{1-x}{1+x}\right)^{1/2} = \frac{1}{2}\log\left(\frac{1-x}{1+x}\right)$

$= \frac{1}{2}[\log(1-x) - \log(1+x)]$

Differentiating *w.r.t. x; we.get*

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1-x}(0-1) - \frac{1}{1+x}(0+1)\right]$

$= \frac{1}{2}\left[\frac{1}{x-1} - \frac{1}{x+1}\right]$

$= \frac{1}{2}\left(\frac{x+1-x+1}{x^2-1^2}\right)$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x^2-1}$

$\frac{dy}{dx} = \frac{y}{x^2-1}$

Q.20. $\frac{d}{dx}\left(\log\left(\sqrt{x-1} + \sqrt{x+1}\right)\right) =$

(a) $\frac{1}{2\sqrt{x^2-1}}$

(b) $\frac{1}{2\sqrt{x^2+1}}$

(c) $\frac{1}{\sqrt{x-1}+\sqrt{x+1}}$

(d) None of these

[Dec. 2016]

Solution : (a) is correct.

Soln. : $\frac{d}{dx}\left\{\log\left(\sqrt{x-1}+\sqrt{x+1}\right)\right\}=\frac{1}{\sqrt{x-1}+\sqrt{x+1}}\times\left[\frac{d\sqrt{x-1}}{dx}+\frac{d\sqrt{x+1}}{dx}\right]$

$=\frac{1}{\sqrt{x-1}+\sqrt{x+1}}\cdot\left[\frac{1}{2\sqrt{x-1}}+\frac{1}{2\sqrt{x+1}}\right]$

$=\frac{1}{\sqrt{x-1}+\sqrt{x+1}}\times\frac{1}{2}\left(\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x-1}.\sqrt{x+1}}\right)$

$=\frac{1}{2\sqrt{(x-1)(x+1)}}=\frac{1}{2\sqrt{x^2-1}}$

$\therefore$ (a) is correct

Q.21. $f(x)=\log_e\left(\frac{x-1}{x+1}\right)$ **and** $f^1(x)=1$ **then the value of** $x=$

(a) 1 (b) 0 (c) $\pm\sqrt{3}$ (d) $\pm\sqrt{2}$

[Dec. 2016]

Solution : (c) is correct.

Soln.

$f(x)=\log_e\left(\frac{x-1}{x+1}\right)=\log_e(x-1)-\log_e(x+1)$

$f^1(x)=\frac{d\log(x-1)}{dx}-\frac{d\log(x+1)}{dx}$

$=\frac{1}{x-1}-\frac{1}{x+1}$

$=\frac{x+1-(x-1)}{(x-1)(x+1)}$

$=\frac{x+1-x+1}{x^2-1}=\frac{2}{x^2-1}\Rightarrow 1=\frac{2}{x^2-1}$

$\Rightarrow x^2-1=2$

$\Rightarrow x^2=3$

$\therefore x=\pm\sqrt{3}$

Q.22. The equation of the curve which passes through the point (1, 2) and has the slope 3x - 4 at any point (x, y) is

(a) $2y=3x^2-8x+9$

(b) $y=6x^2-8x+9$

(c) $y=x^2-8x+9$

(d) $2y=3x^2-8x+c$

[June 2017]

Solution : Tricks: Go by choices for option (a) point (1, 2) satisfies

$2y=3x^2-8x+9$

and its slope is

$2.\frac{dx}{dx}=3\times2x-8\times1+0$

$=6x-8=2(3x-4)$

$\therefore \frac{dx}{dx}=\text{slope}=3x-4\,(\text{True})$

$\therefore$ Option (a) is correct.

Q.23. If $y=1+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots\ldots$ **then** $\frac{dy}{dx}=$ _____ :

(a) x (b) y (c) 1 (d) 0

[Dec. 2017]

Solution : $y=1+\frac{1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots$

$\Rightarrow y=e^x$ (Formula)

$\frac{dy}{dx}=\frac{de^x}{dx}=e^x=y$

$\therefore \frac{dy}{dx}=y$.

(b) is correct.

Q.24. If $x=at^2$ **and** $y=2at$ **then** $\left(\frac{dy}{dx}\right)_{t=2}=$ _____ :

(a) 2 (b) 4 (c) 1/2 (d) $\frac{1}{4}$

[Dec. 2017]

Solution : (c) is correct.

$\frac{dy}{dx}=a\times2t=2at\Rightarrow\frac{dy}{dx}=2a\times2=4a$ *at* $t=2$

$\frac{dy}{dx}=2a\times1=2a\Rightarrow\frac{dy}{dx}=2a$. *at* $t=2$

$\therefore\left(\frac{dy}{dx}\right)at\,t=2=\frac{\left(dy/dt\right)at\ t=2}{\left(dx/dt\right)at\ t=2}$

$=\frac{2a}{4a}=\frac{1}{2}$

Q.25. If $x^y=e^{x-y}$ **then**

$\frac{dy}{dx}=$ _____ :

(a) $\frac{2\log x}{(1+\log x)^2}$

(b) $\frac{\log x}{1+\log x}$

(c) $\frac{\log x}{(1+\log x)^2}$

(d) None of these

[Dec. 2017]

Solution : (c)

$\because\ x^y=e^{x-y}$

Taking log on both sides ; we get

$y\log x=(x-y).\log e=x-y$

or ; $y+y\log x=x$

or ; $y(1+\log x)=x$

or ; $y=\frac{x}{1+\log x}$

$\therefore \frac{dy}{dx}=\frac{1.(1+\log x)-x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$

$=\frac{1+\log x-1}{(1+\log x)^2}$

$=\frac{\log x}{(1+\log x)^2}$

Q.26. If $y=\log x^x$ **then**

$\frac{dy}{dx}=$ _____ :

(a) log (ex) (b) log (e/x)

(c) log (x/e) (d) 1

[Dec. 2017]

Solution : (a)

$\because\ y=\log x^x=x\log x$

$\frac{dy}{dx}=1.\log x+x\times\frac{1}{x}$

$=1+\log x$

$=\log_e e+\log_e x$

$=\log_e(ex)$

$=\log(ex)$

Q.27. The cost function for the production of x units of a commodity is given by

$C(x)=2x^3-15x^2+36x+15$

The cost will be minimum when x = ?

[May 2018]

(a) 3 (b) 2

(c) 1 (d) 4

Solution : (a)

Let

$\because c(x)=y=2x^3-15x^2+36x+15$

$\therefore \frac{dy}{dx}=6x^2-30x+36$

$\frac{d^2y}{dx^2}=12x-30$

If $\frac{dy}{dx}=0\Rightarrow 6x^2-30x+36=0$

or $6(x^2-5x+6)=0$

or ; $x^2-3x-2x+6=0$

or ; $x(x-3)-2(x-3)=0$

or $(x-2)(x-3)=0$

$\therefore x=2;3.$

Case-I $\frac{d^2y}{dx^2}at\ x=2$

$=12\times2-30=-6<0.$

$\therefore c(x)$ is maximum at $x=2$.

Case-II $\frac{d^2y}{dx^2}at\ x=3$

$=12\times3-30=6>0.$

$\therefore y=c(x)$ is minimum at $x=3$.

$\therefore$ (a) is correct.

Q.28. Let $x=at^3, y=\frac{a}{t^2}$, **Then** $\frac{dy}{dx}$

(a) $\frac{-3a}{t^6}$ (b) $\frac{-1}{t^6}$

(c) $\frac{1}{3at^2}$ (d) None

[Nov. 2018]

Solution : (*d*)

$$\frac{dx}{dt} = a \times 3t^2 = 3at^2$$

$$\frac{dy}{dt} = a \cdot \frac{dt^{-2}}{dt} = a(-2)t^{-3} = -2at^{-3}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2at^{-3}}{3at^2} = \frac{-2}{3}t^{-5}$$

Q.29. xy = 1 then $y^2 + \frac{dy}{dx} = ?$

(*a*) 1 (*b*) 0 (*c*) 2 (*d*) None

[Nov. 2018]

Solution : (*b*)

$$\because xy = 1 \Rightarrow y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = -\frac{1}{x^2} = -\left(\frac{1}{x}\right)^2$$

$$\Rightarrow \frac{dy}{dx} = -y^2$$

$$\Rightarrow y^2 + \frac{dy}{dx} = 0$$

Q.30. If the given cost function of commodity is given by $C = 150x - 5x^2 + \frac{x^3}{6}$, where C stands for cost and x stands for output, if the average cost is equal to the marginal cost then the output x = _____.

(*a*) 5 (*b*) 10

(*c*) 15 (*d*) 20

[June 2019]

Solution :

$$\because c(x) = 150x - 5x^2 + \frac{x^3}{6}$$

$$AC = \frac{c(x)}{x} = 150 - 5x + \frac{x^2}{6}$$

$$MC = \frac{dc}{dx} = 150 \times 1 - 5 \times 2x + \frac{3x^2}{6}$$

$$= 150 - 10x + \frac{3x^2}{6}$$

$\because$ AC = MC (given)

$$\because 150 - 5x + \frac{x^2}{6} = 150 - 10x + \frac{3x^2}{6}$$

$$\Rightarrow \frac{2x^2}{6} - 10x + 5x = 0$$

$$\text{or } \frac{x^2}{3} - 5x = 0$$

$$\text{or } x\left(\frac{x}{3} - 5\right) = 0$$

$$\text{or } x = 0 ; \frac{x}{3} - 5 = 0$$

$$\text{or } x = 0 ; \frac{x}{3} - 5 = 0$$

$$\text{or } \frac{x}{3} = 5 \quad \therefore x = 15.$$

$\therefore$ (c) is correct.

Q.31. If $2^x - 2^y = 2^{x-y}$ then $\frac{dy}{dx}$ at x = y = 2

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 5

[June 2019]

Solution : (*a*)

Given $2^x - 2^y - 2^{x-y} = 0$.

Tricks:- It is implicit function.

So;

$$\frac{dy}{dx} = -\frac{\text{D.C of } f(x;y) \text{ w.r.t.x keeping } y \text{ constant}}{\text{D.C of } f(x;y) \text{ w.r.t.y keeping } x \text{ constant}}$$

$$= -\frac{2^x \cdot \log 2 - 0 - 2^{x-y} \cdot \log 2 \cdot (1-0)}{0 - 2^y \cdot \log 2 - 2^{x-y} \cdot \log 2 \cdot (0-1)}$$

$$= \frac{\log 2 \cdot \left[2^x - 2^{x-y}\right]}{\log 2 \cdot \left[-2^y + 2^{x-y}\right]}$$

$$\therefore \frac{dy}{dx} \text{ at } x = y = 2 = -\frac{2^2 - 2^0}{-2^2 + 2^0}$$

$$= -\frac{4-1}{-4+1} = -\frac{3}{-3} = +1$$

(*a*) is correct.

Q.32. $f(x) = \begin{cases} x, & x < 0 \\ 0, & x = 0 \\ x^2, & x > 0 \end{cases}$ **then find** $\lim_{x \to 0} f(x)$

(*a*) 1 (*b*) 0

(*c*) -2 (*d*) 2

[Dec. 2019]

Solution : (*b*) $LHL = \lim_{x \to 0^-} f(x)$

$$= \lim_{x \to 0} (x) = 0$$

$$RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0} (x^2)$$

$$= 0^2 = 0$$

Here, LHL = RHL = 0

$\therefore$ Limit exists

$$\therefore \lim_{x \to 0} f(x) = 0$$

Q.33. Find the value of dy/dx if $y = x^x$

(*a*) $x^x \log x$ (*b*) $1 + \log x$

(*c*) $y \log x$ (*d*) none of these

[Dec. 2019]

Solution : (*a*)

$\because y = x^x$ ——————(1)

Taking log on both sides ; we get

$\log y = \log x^x$

$= x \log x.$

Differentiating on both sides with respect to x ; we get

$$\frac{d \log y}{dx} = \frac{dx}{dx} \cdot \log x + x \cdot \frac{d \log x}{dx}$$

$$\text{or } \frac{1}{y} \cdot \frac{dy}{dx} = 1 \cdot \log x + x \cdot \frac{1}{x}$$

$$\text{or } \frac{dy}{dx} = y[\log x + 1]$$

from (1) ; we get

$$\frac{dy}{dx} = x^x (\log_e x + \log_e e)$$

$$= x^x \cdot \log_e (ex) = x^x \cdot \log ex^x$$

Q.34. If $f(x) = a(x^2 + x + 1)^2$ and $f^1(-1) = -6$ then the value of a =

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

[Dec. 2019]

Solution : (*c*)

$$\because f(x) = a(x^2 + x + 1)^2$$

$$f^1(x) = a \cdot 2(x^2 + x + 1)^{2-1} \cdot \frac{d}{dx}(x^2 + x + 1), \text{ [by chain Rule]}$$

$$= 2a(x^2 + x + 1) \cdot (2x + 1)$$

$$f^1(-1) = 2a[(-1)^2 + (-1) + 1] \cdot [2 \cdot (-1) + 1]$$

$$\text{or; } -6 = 2a[1 - 1 + 1](-2 + 1)$$

$$\text{or; } -6 = 2a \cdot 1 \cdot (-1)$$

$$\text{or } -2a = -6 \Rightarrow a = 3$$

Q.35. If Y = x(x - 1)(x - 2) then dy/dx is

(*a*) -6x (*b*) $3x^2 - 6x + 2$

(*c*) 6x+4 (*d*) $3x^2 - 6x$

[Dec. 2020]

Solution : $y = x(x - 1)(x - 2)$

$$= (x^2 - x)(x - 2)$$

$$= x^3 - 2x^2 - x^2 + 2x$$

$$= x^3 - 3x^2 + 2x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^3 - 3x^2 + 2x)$$

$$= 3x^2 - 6x + 2$$

(*b*) is correct.

Q.36. The cost function of a good is $2Q + 6 + \frac{13}{Q}$ where Q is the quantity produced.

The approx. cost at Q = 15 is

(*a*) 42 (*b*) 36

(*c*) 66 (*d*) 130

[Dec. 2020]

Solution : Cost function $= 2Q + 6 + \frac{13}{Q}$

Cost at Q = 15

$$= 2 \times 15 + 6 + \frac{13}{15}$$

$$= 36.86 = \cong ₹ 36$$

$\therefore$ (*b*) is correct.

Q.37. The cost function of production is given by $C(x) = \frac{x^3}{2} - 15x^2 + 36x$ where x, denotes the number of items produced.

The level of output for which marginal cost is minimum and the level of output for which the average cost is minimum are given by, respectively.

(*a*) 10 and 15 (*b*) 10 and 12

(*c*) 12 and 15 (*d*) 15 and 10

[Jan. 2021]

Solution : $C(x) = \frac{x^3}{2} - 15x^2 + 36x$

MC = Marginal Cost = $\frac{dC}{dx}$

$$= \frac{1}{2} 3x^2 - 15 \times 2x + 36 \times 1$$

$$= \frac{3}{2} x^2 - 30x + 36$$

$$\text{Now } \frac{dMC}{dx} = \frac{3}{2} \times 2x - 30 \times 1 + 0$$

$$= 3x - 30$$

$$\text{And } \frac{d^2MC}{dx^2} = 3 \times 1 - 0 = 3 > 0 \text{ (+ Ve)}$$

For Maximum/Minimum

$$\frac{dMC}{dx} = 0 \Rightarrow 3x - 30 = 0 \Rightarrow x = 10$$

$$\because \frac{d^2MC}{dx^2} > 0$$

$\therefore$ MC is minimum at $x = 10$

$$\text{Now } AC = \frac{C(x)}{x} = \frac{x^2}{2} - 15x + 36$$

$$\frac{dAC}{dx} = \frac{1}{2} 2x - 15 \times 1 + 0 = x - 15$$

For maximum/Minimum

$$\frac{dAC}{dx} = 0 \Rightarrow x - 15 = 0$$

$\therefore x = 15$

$$\text{Now } \frac{d^2AC}{dx^2} = 1 - 0 = +1 > 0 \text{ (+ ve)}$$

$\therefore$ AC is minimum at $x = 15$.

$\therefore$ (*a*) is correct.

Q.38. The cost function $C(x) = 125 + 500x - x^2 + x^3/3$, $0 \le x \le 100$ and the demand function for the items is given by, $p(x) = 1500 - x$, then the marginal profit when 18 items are sold is

(*a*) 751 (*b*) 571

(*c*) 676 (*d*) 875

[July 2021]

Solution : (*c*) is correct

Revenue Function = R(x) = Px = (1500 − x)x

$= 1500x - x^2$

Profit Function = P(x) = R(x) − C(x)

$P(x) = 1500x - x^2 - 125 - 500x + x^2 - \frac{-x^3}{3}$

$= \frac{-x^3}{3} + 1000x - 125$

Marginal Profit = MP = $P'(x) = \frac{-1}{3} \times 3x^2 + 1000 \times 1 - 0$

$= -x^2 + 1000$

MP at $x = 18 = -18^2 + 1000$

$= 676$

(c) is correct

Q.39. If $f(x) = 3e^{x^4}$ then $f'(x) - 4x^3 f(x) + \left(\frac{1}{3}\right) f(0) - f'(0)$ is equal to

(a) 0 (b) e^{x^2}

(c) 1 (d) -1

[July 2021]

Solution : (c) is correct

$\because \quad f(x) = 3e^{x^4}$

$f'(x) = 3.e^{x^4}$

$\frac{dx^4}{dx} = 3e^{x^4}.4x^3$

$= 4x^3.(3e^{x^4})$

$f'(x) = 4x^3.f(x)$

Now $f(0) = 3.e^{0^4} = 3.e^0 = 3 \times 1 = 3$

$f'(0) = 4.0^3.(3.e^{0^4})$

$= 0$

Hence

$f'(x) - 4x^3 f(x) + \frac{1}{3} f(0) - f'(0)$

$= 4x^3.f(x) - 4x^3 f(x) + \frac{1}{3} \times 3 - 0$

$= 0 + 1 - 0 = 1$

(c) is correct

Q.40. The cost for producing x units is $500 - 20x^2 + x^3/3$. The material cost is minimum at $x =$ _____

(a) 5 (b) 10

(c) 20 (d) 50

[Dec. 2021]

Solution : (c)

$C(x) = 500 - 20x^2 + \frac{x^3}{3}$

$MC = \frac{dc}{dx} = 0 - 20 \times 2x + \frac{3x^2}{3}$

$= -40x + x^2$

Now $\frac{dMC}{dx} = -40 \times 1 + 2x = 2x - 40$

$\frac{d^2MC}{dx^2} = 2 \times 1 - 0 = +2 > 0$

For Maxima/Minima.

$\frac{dMC}{dx} = 0 \Rightarrow 2x - 40 = 0$

$\therefore x = 20$

So MC is minimum at $x = 20$

$\therefore$ (c) is Correct.

Q.41. If $y = \frac{x^4}{e^2}$ then $\frac{dy}{dx}$ is equal to :

(a) $x^3 (4 - x)/(e^x)^2$

(b) $x^3 (4 - x)/e^x$

(c) $x^2 (4 - x)/e^x$

(d) $x^3 (4x - 1)/e^x$ ***[Dec. 2021]***

Solution : (b)

$\because y = \frac{x^4}{e^x}$

$\frac{dy}{dx} = \frac{\frac{dx^4}{dx}.e^x - x^4.\frac{de^x}{dx}}{(e^x)^2}$

$= \frac{4x^3.e^x - x^4.e^x}{e^x.e^x}$

$\frac{x^3.(4-x)e^x}{e^x.e^x} = \frac{x^3(4-x)}{e^x}$

$\therefore$ (b) is Correct

Q.42. The speed of a train at a distance x (from the starting point) is given by $3x^2 - 5x + 4$. What is the rate of change (of distance) at $x = 1$?

(a) -1 (b) 0

(c) 1 (d) 2

[Dec. 2021]

Solution : (d)

Rate of Change of distance is called speed.

Given $\frac{dx}{dt} = 3x^2 - 5x + 4$

When $x = 1$

Then Rate of change of distance

$= 3 \times 1^2 - 5 \times 1 + 4$

$= 3 - 5 + 4$

$= 2$

(d) is correct.

Q.43. Which of the following is the differentiation of $e^t.\log_e t$ with respect to 't'?

(a) $e^t(t \log_e t)$

(b) $\frac{e^t(1 + t \log_e t)}{t}$

(c) $\frac{e^t}{t}$

(d) $e^t(1 - \log_e t)$

[June 2022]

Solution : Let $y = e^t \log_e t$

$\therefore \frac{dy}{dt} = \frac{de^t}{dt}.\log_e t + e^t.\frac{d\log_e t}{dt}$

$= e^t.\log_{e^t} + e^t.\frac{1}{t}$

$= e^t\left[\log_e t + \frac{1}{t}\right]$

$= e^t\left(\frac{t.\log_e t + 1}{t}\right)$

(b) is correct

Q.44. The maxima and minima of the function $y = 2x^3 - 15x^2 + 36x + 10$ occurs respectively at

(a) $x = 2$ and $x = 3$

(b) $x = 1$ and $x = 3$

(c) $x = 3$ and $x = 2$

(d) $x = 3$ and $x = 1$

[Dec. 2022]

Solution: $\because y = 2x^3 - 15x^2 + 36x + 10$

$\frac{dy}{dx} = 2.3x^2 - 15 \times 2x + 36 \times 1 + 0$

$= 6x^2 - 30x + 36$

$\frac{dy^2}{dx^2} = 6 \times 2x - 30 \times 1 + 0$

$= 12x - 30$

If $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 30x + 36 = 0$

or $6(x^2 - 5x + 6) = 0$

or; $x^2 - 3x - 2x + 6 = 0$

or $x(x - 3) - 2(x - 3) = 0$

or $(x - 3)(x - 2) = 0$

$\therefore x = 2; 3$

For Maxima/Minima :

At $x = 2$

$\frac{dy^2}{dx^2} = 12x - 30$

$= 12 \times 2 - 30 = -6$ (-ve)

$\therefore y$ is max. at $x = 2$

Now at $x = 3$

$\frac{dy^2}{dx^2} = 12 \times 3 - 30 = 6$ (+ve)

$\therefore y$ is minima at $x = 3$

$\therefore$ Max. at $x = 2$ and

Minima at $x = 3$

$\therefore$ (a) is correct.

Q.45. If $y = x^3$, then dy/dx at $x = 1$ is equal to

(a) 0 (b) 1

(c) -1 (d) 2

[Dec. 2022]

Tricks : It is an Implicit function

$\frac{dy}{dx} = -\frac{\text{D.C of } f(x;y) \text{ w.r.t. } x \text{ keeping } y \text{ constant}}{\text{D.C of } f(x;y) \text{ w.r.t. } y \text{ keeping } x \text{ constant}}$

[D.C means differential coefficient]

$\therefore \frac{dy}{dx} = -\frac{\frac{dx^5}{dx} + \frac{dy^5}{dx} - \frac{d}{dx}(5xy)}{\frac{dx^5}{dy} + \frac{dy^5}{dy} - \frac{d}{dy}(5xy)}$

Solution : $\because y = x^x = e^{x.\log_e x}$

$\therefore \frac{dy}{dx} = \frac{de^{(x.\log_e x)}}{dx}$

$= e^{x.\log_e x} \times \frac{d(x.\log_e x)}{dx}$

$= x^x\left[\frac{dx}{dx} \times \log_e x + x \times \frac{d\log_e x}{dx}\right]$

$= x^x\left[1 \times \log_e x + x \times \frac{1}{x}\right]$

$= x^x[\log x + 1]$

$\therefore \left(\frac{dy}{dx}\right)_{(\text{at } x = 1)} = 1^1(\log 1 + 1)$

$= 1 \times (0 + 1) = 1$

$\therefore$ (b) is correct

Q.46. If $x^5 + y^5 - 5xy = 0$ then $\frac{dy}{dx}$ is

(a) $\frac{y + x^4}{x + y^4}$ (b) $\frac{y - x^4}{y^4 - x}$

(c) $\frac{x - y^4}{x^4 + y}$ (d) $\frac{x + y^4}{x^4 + y}$

[Dec. 2022]

Solution : *Given* Eqn. is

$x^5 + y^5 - 5xy = 0$

$= -\frac{5x^4 + 0 - 5y \times \frac{dx}{dx}}{0 + 5y^4 - 5x \times \frac{dy}{dy}}$

$= -\frac{5x^4 - 5y \times 1}{5y^4 - 5x \times 1}$

$= -\frac{5(x^4 - y)}{5(y^4 - x)}$

$= \frac{x^4 - y}{x - y^4} = \frac{y - x^4}{y^4 - x}$

(b) is correct

Q.47. For a given curve $y = 2 - x^2$, when 'x' increases at the rate of 3 units/s, then the slope of curve will:

(a) Increase at 6 units/s

(b) Increase at 3 units/s

(c) Decrease at 6 units/s

(d) Decrease at 3 units/s

[June 2023]

Solution : $\because y = 2 - x^2$

Slope of curve $= \frac{dy}{dx} = 0 - 2x$

at $x = 3$

$= -2 \times 3 = -6$

i.e. decreases at 6 units/sec.

(c) is correct.

Q.48. If $y = \frac{x}{x+5}$, then $\frac{dx}{dy}$ is equal to:

(a) $\frac{5}{(1-y)^2}$

(b) $\frac{5}{(1+y)^2}$

(c) $\frac{3}{(1-y)^2}$

(d) $\frac{3}{(1+y)^2}$

[June 2023]

Solution : $\because y = \frac{x}{x+5}$

$\therefore xy + 5y = x$

or $xy + 5y - x = 0$

$\Rightarrow x = \frac{5y}{1-y}$

Difference on both sides w.r.t. x;

we get

$\frac{dx}{dx} \times y + x \times \frac{dy}{dx} + 5\frac{dy}{dx} - \frac{dx}{dx}$

$= 0$

or $1 \times y + x\frac{dy}{dx} + 5\frac{dy}{dx} - 1 = 0$

or $(x + 5)\frac{dy}{dx} = 1 - y$

or $\left(\frac{5y}{1-y} + 5\right)\frac{dy}{dx} = 1 - y$

or; $\left(\frac{5y + 5 - 5x}{1-y}\right)\frac{dy}{dx} = 1 - y$

or; $\frac{dy}{dx} = \frac{(1-y)^2}{5}$

$\therefore \frac{dx}{dy} = \frac{5}{(1-y)^2}$

(a) is correct.

Q.49. If $xy = 1$, then $y^2 + dy/dx$ is equal to:

(a) 1 (b) 0

(c) −1 (d) 1/2

[June 2023]

Solution : $\because \quad xy = 1$

$$y = \frac{1}{x} \quad \text{.....(1)}$$

$$\frac{dy}{dx} = \frac{d\left(\frac{1}{x}\right)}{dx} = -\frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{1}{x}\right)^2 = -y^2$$

$$\therefore \frac{dy}{dx} + y^2 = 0$$

$\therefore$ (b) is correct.

15 CHAPTER INTEGRATION

FORMULA

1. $\int o\,dx = c$ (Constant)
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$; Where $n \neq -1$

   Where c = Integration Constant (I.C.)
3. $\int a^x\,dx = \frac{a^x}{\log a} = \frac{a^x}{\log e^a} + c$; Where c = I.C.
4. $\int e^x\,dx = e^x + c$; Where c = I.C.
5. $\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \log e^x + c$; Where c = I.C.
6. $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)^a} + c$; Where c = I.C.
7. $\int a^{bx+c}\,dx = \frac{a^{bx+c}}{b.\log a} + K$; Where K = I.C.
8. $\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + c$; Where c = I.C.
9. $\int \frac{1}{a^{cx+b}}\,dx = \log e\,(ax+b) + c$; Where c = I.C.

10. $\int_a^b f(x)\,dx = [F(x)]_a^b = f(b) - f(a)$; Where $\int f(x)\,dx = F(x)$

PAST EXAM QUESTIONS WITH SOLUTIONS (MEMORY BASED)

Q.1. $\int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}}$ **Equal to**

(a) $\frac{2}{27}\left[(3x+4)^{3/2} - (3x+1)^{3/2}\right] + c$ (b) $\frac{2}{27}\left[(3x+4)^{3/2} + (3x+1)^{3/2}\right] + c$

(c) $\frac{2}{3}\left[(3x+4)^{3/2} - (3x+1)^{3/2}\right] + c$ (d) None of these

[June 2010]

Solution : (b) is correct.

$$\int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}}$$

$$= \int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}} \times \frac{\sqrt{3x+4}+\sqrt{3x+1}}{\sqrt{3x+4}+\sqrt{3x+1}}$$

$$= \int \frac{\sqrt{3x+4}+\sqrt{3x+1}dx}{3x+4-3x-1}$$

$$= \frac{1}{3}\int \sqrt{3x+4}dx + \frac{1}{3}\int \sqrt{3x+1}dx$$

$$= \frac{1}{3}\cdot\frac{(3x+4)^{3/2}}{3/2}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{(3x+1)^{3/2}}{3/2}\cdot\frac{1}{3}$$

$$= \frac{2(3x+4)^{3/2}}{27} + \frac{2(3x+1)^{3/2}}{27}$$

$$= \frac{2}{27}\left[(3x+4)^{3/2} + (3x+1)^{3/2}\right]$$

Q.2. $\int_1^2 \frac{xdx}{x^2+2} =$ ———

(a) $\log\sqrt{2}$ (b) $\log\sqrt{3}$ (c) $\log\frac{1}{\sqrt{2}}$ (d) $\log\frac{1}{\sqrt{3}}$

[June 2010]

Solution : (a) is correct.

$$\int_1^2 \frac{xdx}{x^2+2}$$

Let $x^2 + 2 = t$

$\rightarrow$ Differentiating both sides w.r.t. x

$2x\,dx = dt$

$$x\,dx = \frac{dt}{2}$$

$$\int_3^6 \frac{dt}{2t}\text{; when, } x = 1,\ t = 3;\ x = 2,\ t = 6$$

$$= \left[\frac{\log|t|}{2}\right]_3^6$$

$$= \frac{1}{2}[\log 6 - \log 3] = \frac{1}{2}\log\frac{6}{3} = \frac{1}{2}\log 2$$

$$= \log\sqrt{2}$$

Q.3. $\int \frac{6x+4}{(x-2)(x-3)}dx$ **is equal to**

(a) 22log (x-3)-16(x-2) (b) 11log(x-3)-8(x-2)

(c) 22log (x-3) -16log(x-2) (d) 232log (x-3) +16log (x-2)

[Dec. 2010]

Solution : (c) is correct

$$\int \frac{6x+4}{(x-2)(x-3)}dx$$

Let, $\frac{6x+4}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$ (Partial Fractions)

$6x + 4 = A(x - 3) + B(x - 2)$

When x = 3 ; B = 22

& When x = 2; A = - 16

$\int \frac{-16}{x-2}dx + \int \frac{22}{x-3}dx$

$= -16\log(x-2) + 22\log(x-3) + c$

$= 22\log(x-3) - 16\log(x-2) + c$

Q.4. $\int \frac{1}{x(1+\log x)^2}dx$ **is equal to**

(a) $-\frac{1}{2(1+\log x)^2}+c$ (b) $\frac{1}{(1+\log x)}+c$

(c) $-\frac{1}{(1+\log x)}+c$ (d) None of these

[Dec. 2010]

Solution : (c) is correct .

$\int \frac{1}{x(1+\log x)^2}dx$

Let $1 + \log x = t$

$\frac{1}{x}dx = dt$

$\int \frac{dt}{t^2} = -\frac{1}{t}+c$

$= -\frac{1}{(1+\log x)}+c$

Q.5. $\int_{-1}^{1} (e^x - e^{-x})dx$ **= ?**

(a) 0 (b) 1 (c) 12 (d) None

[Dec. 2010]

Solution : (a) is correct.

Let $f(x) = e^x - e^{-x}$

then $f(-x) = -(e^x - e^{-x}) = -f(x)$

So, the function is a "ODD" function.

so; $\int_{-1}^{1} (e^x - e^{-x})dx = 0$

Q.6. $\int \frac{(\log x^x)^2}{x^3}dx$ **=?**

(a) $\frac{3}{2}(\log x)^3 + c$ (b) $\frac{1}{3}(\log x)^3 + c$

(c) $\frac{1}{6}(\log x)^3 + c$ (d) $\frac{3}{7}(\log x)^3 + c$

[Dec. 2010]

Solution : (b) is correct.

$\int \frac{(\log x^x)^2}{x^3}dx = I$ (say)

$\therefore I = \int \frac{(x\log x)^2}{x^3}dx = \int \frac{(\log x)^2}{x}dx$

Let $\log x = t$

$\therefore \frac{1}{x}dx = dt$

$\therefore I = \int t^2 dt = \frac{t^3}{3} + C = \frac{(\log x)^3}{3} + C = \frac{1}{3}(\log x)^3 + C$

Q.7. Given $y = \int (e^{a\log x} + e^{x\log a})dx$ **then** $\frac{dy}{dx}$

(a) $x^a\ a^x$ (b) $x^a + a^x$

(c) $ax^{x-1} + a^x \log a$ (d) None

[Dec. 2010]

Solution : (b) Since, $e^{\log x^a} + e^{\log a^x} = x^a + a^x$ .....(1)

$\therefore Y = \int (e^{a\log x} + e^{x\log a})dx$

or $Y = \int (x^a + a^x)dx$ using (1)

So, $\frac{dy}{dx} = x^a + a^x$

Differentiation is just reverse of Integration.

Q.8. If $f'(x) = 3x^2 - \frac{2}{x^3}, f(1) = 0$ *and* $f(x) =$_____.

(a) $\frac{x^3}{3} - x^{-2} - 2$ (b) $x^3 + x^2 + 2$ (c) $x^3 + x^{-2} - 2$ (d) None of these.

[June 2011]

Solution : (c) is correct

Given : $f'(x) = 3x^2 - \frac{2}{x^3}$

Formula, $f(x) = \int f'(x)dx = \int (3x^2 - \frac{2}{x^3})dx$

$\therefore f(x) = x^3 + x^{-2} + c$ .....(1)

Given $f(1) = 0$

$\Rightarrow f(1) = (1)^3 + (1)^{-2} + c$

$0 = 1 + 1 + c$

$\Rightarrow c = -2$

$\therefore$ from (1); $f(x) = x^3 + x^{-2} - 2$

Q.9. $\int_{-1}^{1} \frac{|x|}{x}dx =$_____

(a) -1 (b) 0 (c) 1 (d) 2

[Dec. 2011]

Solution : (b) is correct

$\int_{-1}^{1} \frac{|x|}{x}dx$

Let $f(x) = \frac{|x|}{x}$

$f(-x) = \frac{|-x|}{-x} = \frac{|x|}{-x} = \frac{-|x|}{x}$

$f(-x) = -f(x)$

So, it is an odd function

$\int_{-1}^{1} \frac{|x|}{x}dx = 0$

$\because \int_{-a}^{a} f(x)dx = 0$; if f(x) is odd function.

$= 2\int_{0}^{a} f(x)dx$; if f(x) an Even function

Q.10. $\int \frac{e^x}{(1+x)^3}dx - \int \frac{e^x}{2(1+x)^2}dx =$ ____

(a) 0 (b) $\frac{e^x}{2(1+x)^2}+c$ (c) $-\frac{e^x}{2(1+x)^2}+C$ (d) $\frac{e^x}{(1+x)^2}+C$

[Dec. 2011]

Solution : (c) is correct.

$= \int \frac{e^x}{(1+x)^3}dx - \int \frac{e^x}{2(1+x)^2}dx$

$= \int \left\{\frac{e^x}{(1+x)^3} - \frac{e^x}{2(1+x)^2}\right\}dx$

$= \int e^x \left\{\frac{1}{(1+x)^3} - \frac{1}{2(1+x)^2}\right\}dx$

$= \int e^x \left(-\frac{1}{2}\right)\left\{\frac{1}{(1+x)^2} - \frac{2}{(1+x)^3}\right\}dx$

$= \frac{-1}{2}\int e^x \left\{\frac{1}{(1+x)^2} - \frac{2}{(1+x)^3}\right\}dx$

$= \frac{-1}{2}e^x \frac{1}{(1+x)^2} + c$

$[\because \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c]$

Q.11. $\int_0^1 \frac{dx}{[ax + b(1-x)]^2} =$ _____

(a) a/b (b) b/a (c) ab (d) 1/ab

[June 2012]

Solution : (d) $\int_0^1 \frac{dx}{[ax + b(1-x)]^2} =$.....

Let t = ax + b (1 -x)

$= dt = [a + b(-1)]\,dx$

$= dt = (a - b)\,dx$

or $dx = \dfrac{dt}{(a-b)}$

Let $I = \int_0^1 \dfrac{dx}{[ax+b(1-x)]^2}$

$= \int_b^a \dfrac{dt}{(a-b).t^2}$ [if x = 0, t = b & x = 1 then t = a]

$= \dfrac{1}{(a-b)}\int_b^a t^{-2}dt$

$= \dfrac{1}{(a-b)}\left[\dfrac{t^{-2+1}}{-2+1}\right]_b^a$

$= -\dfrac{1}{(a-b)}\left[\dfrac{1}{a}-\dfrac{1}{b}\right] = \dfrac{1}{ab}$

Q.12. $\int 2^{3x}.3^{2x}.5^x dx =$ ____

(a) $\dfrac{2^{3x}.3^{2x}.5^x}{\log(720)}+c$ (b) $\dfrac{2^{3x}.3^{2x}.5^x}{\log(360)}+c$ (c) $\dfrac{2^{3x}.3^{2x}.5^x}{\log(180)}+c$ (d) $\dfrac{2^{3x}.3^{2x}.5^x}{\log(90)}+c$

[Dec. 2012, June 2013]

Solution : (b) is correct

$\int 2^{3x}.3^{2x}.5^x dx$

$= \int 8^x.9^x.5^x dx$

$= \int (8\ .9\ .5)^x dx$

$= \int (360)^x dx$

$= \dfrac{(360)^x}{\log 360}+c$

$= \dfrac{2^{3x}.3^{2x}.5^x}{\log 360}+c$

Q.13. $\int_1^2 \dfrac{[\log_e (ex)]^n}{x}dx =$ **(where n ≠ -1)**

(a) $\dfrac{[\log 2e]^{n+1}}{n+1}$ (b) $\dfrac{[\log 2e]^{n+1}}{n+1}-\dfrac{\log e}{n+1}$

(c) $\dfrac{[\log 2e-\log 2]^{n+1}}{n+1}$ (d) None

[June 2013]

Solution : (b) $\int_1^2 \dfrac{[\log_e e+\log_e x]^n}{x}dx$

$= \int_1^2 \dfrac{(1+\log x)^n}{x}dx$

$= \left[\dfrac{(1+\log x)^{n+1}}{n+1}\right]_1^2$

$= \dfrac{1}{n+1}\left[(1+\log 2)^{n+1}-(1+\log 1)^{n+1}\right]$

$= \dfrac{1}{n+1}\left[\log 2e)^{n+1}-\log e\right]$

$= \dfrac{(\log 2e)^{n+1}}{n+1}-\dfrac{\log e}{n+1}$

Q.14. $\int a^{2x}dx =$

(a) $\dfrac{a^{2x}\log a}{2}$ (b) $\dfrac{2.a^{2x}}{\log a}$ (c) $\dfrac{a^{2x}}{2\log a}$ (d) None

[Dec. 2013]

Solution : (c) is correct

$\int a^{2x}dx = \dfrac{a^{2x}}{2\log_e a}$ (Formula)

Q.15. $\int_0^5 \dfrac{x^2}{x^2+(5-x)^2}dx$ **is equal to**

(a) 5 (b) 5/2 (c) 1 (d) None

[June 2014]

Solution : (b) is correct

Let $I = \int_0^5 \dfrac{x^2}{x^2+(5-x)^2}dx$

$I = \int_0^5 \dfrac{(5-x)^2}{(5-x)^2+x^2}dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right] = 0$

Adding Them ; We get

$2I = \int_0^5 \dfrac{x^2+(5-x)^2}{x^2+(5-x)^2}dx$

$= \int_0^5 dx = [x]_0^5 = 5-0 = 5$

$\therefore I = \dfrac{5}{2}$

Q.16. $\int_0^2 |1-x|dx =$

(a) $\dfrac{3}{2}$ (b) $\dfrac{1}{2}$ (c) 0 (d) 1

[June 2014]

Solution : (d) is correct

$\int_0^2 |1-x|dx = \int_0^1 (1-x)dx + \int_1^2 -(1-x)dx$

$= \left[x-\dfrac{x^2}{2}\right]_0^1 - \left[x-\dfrac{x^2}{2}\right]_1^2$

$= \left[\left(1-\dfrac{1^2}{2}\right)-(0-0)\right]-\left[\left(2-\dfrac{2^2}{2}\right)-\left(1-\dfrac{1^2}{2}\right)\right]$

$= \dfrac{1}{2}-\left(0-\dfrac{1}{2}\right) = \dfrac{1}{2}+\dfrac{1}{2} = 1$

Q.17. $\int_0^{1/2} \dfrac{1}{\sqrt{3-2x}}dx =$

(a) 1 (b) $1-\dfrac{\sqrt{3}}{2}$ (c) $\sqrt{3}-\sqrt{2}$ (d) $2-\sqrt{3}$

[June 2015]

Solution : $\int_0^{1/2} \dfrac{1}{\sqrt{3-2x}}dx = \int_0^{1/2} (3-2x)^{-1/2}dx$

$\left[\dfrac{(3-2x)^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)(-2)}\right]_0^{1/2}$

$= \left[\dfrac{\sqrt{3-2x}}{\frac{1}{2}.(-2)}\right]_0^{1/2}$

$= -\left[\sqrt{3-2.\dfrac{1}{2}}-\sqrt{3-2\times 0}\right]$

$= -\left[\sqrt{2}-\sqrt{3}\right] = \sqrt{3}-\sqrt{2}$

(c) is correct

Q.18. $\int_0^1 xe^{x^2}dx =$

(a) 1 (b) e-1 (c) $\dfrac{e}{2}-1$ (d) $\dfrac{1}{2}(e-1)$

Solution : $I = \dfrac{1}{2}\int_0^1 2xe^{x^2}dx$

$= \dfrac{1}{2}\int_0^1 e^{x^2}2xdx = \dfrac{1}{2}\left[e^{x^2}\right]_0^1$

$= \dfrac{1}{2}\left[e^1-e^0\right] = \dfrac{1}{2}(e-1)$

(d) is correct

Q.19. $\int_1^2 \dfrac{1-x}{1+x}$:

(a) 2 log 3/2 - 1 (b) 2 log 3 + 1 (c) ½ log 3/2 - 1 (d) 2 log 2 – 1 + k

[Dec. 2015]

Solution : (a) is correct $\int_1^2 \dfrac{1-x}{1+x}dx = \int_1^2 \dfrac{2-1-x}{1+x}dx = \int_1^2 \dfrac{2-(1+x)}{1+x}dx$

$$= \int_1^2 \left(\frac{2}{1+x} - 1\right) dx = \left[2\log(1+x) - x\right]_1^2$$

$$= 2\left[\log(1+x)\right]_1^2 - \left[x\right]_1^2$$

$$= 2\left[\log 3 - \log 2\right] - \left[2-1\right] = 2\log\frac{3}{2} - 1$$

Q.20. $\int \frac{x}{(x^2+1)(x^2+2)} dx$ **is equal to**

(a) $\log\left|\frac{x^2+1}{x^2+2}\right| + c$ (b) $\frac{1}{2}\log\left|\frac{x^2+1}{x^2+2}\right| + c$

(c) $\frac{1}{2}\log\left|\frac{x^2+1}{x^2+1}\right| + c$ (d) $-\log\left|\frac{x^2+1}{x^2+2}\right| + c$

[June 2016]

Solution : (b) is correct.

$$I = \frac{1}{2}\int \frac{2x}{(x^2+1)} \frac{dx}{(x^2+2)}$$

let $t = x^2$

$dt = 2xdx$

$$\therefore I = \frac{1}{2}\int \frac{dt}{(t+1)(t+2)}$$

$$= \frac{1}{2}\int\left(\frac{1}{t+1} - \frac{1}{t+2}\right)dt$$

$$= \frac{1}{2}\left[\log(t+1) - \log(t+2)\right] + C$$

$$= \frac{1}{2}\log\frac{t+1}{t+2} + C$$

$$= \frac{1}{2}\log\left(\frac{x^2+1}{x^2+2}\right) + C$$

where C = Integration Constant.

(b) is correct.

Q.21. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$ **is equal to**

(a) $\frac{3\sqrt{2}}{\log_e 3}$ (b) 0 (c) $\frac{2}{\log 3}\left(3^{\sqrt{2}} - 1\right)$ (d) None

[June 2016]

Solution : (c)

let $I = 2\int \frac{3^{\sqrt{x}}}{2\sqrt{x}} dx$

let $t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}} dx$

$I = 2\int 3^t dt = 2 \times \frac{3^t}{\log_e 3} + C$

$$\therefore \int_0^2 \frac{3^{\sqrt{x}}}{2\sqrt{x}} dx = 2\left[\frac{3^{\sqrt{x}}}{\log_e 3}\right]_0^2$$

$$= 2\left[\frac{3^{\sqrt{2}} - 1}{\log 3}\right] = \frac{2}{\log 3}\left(3^{\sqrt{2}} - 1\right)$$

Q.22. $\int_1^e \frac{e^x\left(x\log_e x + 1\right)}{x} dx =$

(a) $e-1$ (b) e^e (c) $e^e - 1$ (d) None

[Dec. 2016]

Solution : (b) is correct.

$$\int_1^e \frac{e^x\left(x\log_e x + 1\right)}{x} dx =$$

$$= \int_1^e e^x\left(\frac{x\log_e x}{x} + \frac{1}{x}\right)dx$$

$$= \int_1^e e^x\left(\log x + \frac{1}{x}\right)dx = \left[e^x \log_e x\right]_1^e$$

$$= = e^e \log_e e - e^1 \log_e 1$$

$$= e^e \times 1 - 0 = e^e$$

Q.23. Evaluate $\int_1^2 \frac{x}{x^2+1} dx$

(a) $\log_{f(x)}$ (b) $\frac{1}{2}\log_{f(x)}$ (c) $\log_{f'} - \log_{f}$ (d) None

[June 2017]

Solution : $I = \int_1^2 \frac{x}{x^2+1} dx = \frac{1}{2}\int_1^2 \frac{2x}{x^2+1} dx$

$$= \frac{1}{2}\left[\log\left(x^2+1\right)\right]_1^2 = \frac{1}{2}\left[\log 5 - \log 2\right]$$

$$= \frac{1}{2}\log \frac{5}{2}$$

∴ Option (b) is correct.

Q.24. $\int e^x\left[f(x) + f'(x)\right]dx =$:

(a) $e^x.f(x) + c$ (b) $e^x.f'(x) + c$

(c) $\frac{f'(x)}{f(x)} + c$ (d) $e^x.\frac{f(x)}{f'(x)} + c$

[Dec. 2017]

Solution : (a)

It is formula

Q.25. $\int x.e^{x^2}.dx =$ _____

(a) $2.e^{x^2} + c$ (b) $\frac{1}{3}e^{x^2} + c$ (c) $\frac{1}{2}e^{x^2} + c$ (d) None of these

[Dec. 2017]

Solution : (c)

$$I = \frac{1}{2}\int 2x.e^{x^2} dx = \frac{1}{2}\int e^{x^2}.2x\, dx$$

let $t = x^2$

$dt = 2xdx$

$$\therefore I = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + C = \frac{1}{2}e^{x^2} + C$$

Where C = Integration constant

Q.26. $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$ **is equal to :**

(a) 0 (b) 0 (c) 2 (d) -1

[June 2018]

Solution : (b)

$$I = \int_0^1 \left[\log(1-x) - \log x\right] dx \text{ - (1)}$$

$$I = \int_0^1 \left[\log\left\{1-(1-x)\right\} - \log(1-x)\right] dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$I = \int_0^1 \left[\log x - \log(1-x)\right] dx \text{ - (2)}$$

Eqn. (1) + Eqn. (2) ; we get

$$2I = \int_0^1 0.dx = 0$$

$I = 0$

Q.27. If $f(x) = \frac{x}{x + \sqrt{1+x^2}}$ **then** $\int_0^1 f(x)\,dx$:

(a) $\frac{2}{3}\left(\sqrt{2} - 1\right)$ (b) $\frac{2}{3}\left(\sqrt{2} - 1\right)$ (c) $\frac{2}{3}\left(1 - \sqrt{2}\right)$ (d) $\frac{3}{2}\left(1 - \sqrt{2}\right)$

[June 2018]

Solution : (a)

$$f(x) = \frac{x}{x + \sqrt{1+x^2}} \times \frac{x - \sqrt{1+x^2}}{x - \sqrt{1+x^2}}$$

$$= \frac{x^2 - x\sqrt{1+x^2}}{x^2 - 1 - x^2} = x\sqrt{1+x^2} - x^2$$

$$\therefore I = \int_0^1 f(x)dx = \int_0^1 \left\{\sqrt{1+x^2}.\frac{2x}{2} - x^2\right\} dx$$

$$= \left[\frac{1}{2}\frac{(1+x^2)^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{3}\left[(1+x^2)^{3/2} - x^3\right]_0^1$$

$$= \frac{1}{3}\left[\left(2^{3/2}-1\right)-\left(1^{3/2}-0\right)\right] = \frac{1}{3}\left[2\sqrt{2}-1-1\right]$$

$$= \frac{1}{3}\left(2\sqrt{2}-2\right) = \frac{2}{3}\left(\sqrt{2}-1\right)$$

Q.28. $\int_0^5 \frac{x^2}{x^2+(5-x)^2}$ **dx is equal to :**

(*a*) 0 (*b*) 5/2 (*c*) 1 (*d*) -1

Solution : (*b*)

$$I = \int_0^5 \frac{x^2}{x^2+(5-x)^2}dx$$

$$I = \int_0^5 \frac{(5-x)^2}{(5-x)^2+x^2}dx$$

Adding them ; we get

$$2I = \int_0^5 \frac{x^2+(5-x)^2}{x^2+(5-x)^2}dx$$

$$\int_0^5 dx = [x]_0^5 = 5-0 = 5$$

$$\therefore 2I = 5 \Rightarrow I = 5/2$$

Q.29. The value of $\int_1^2 \frac{1-x}{1+x}dx$ **is equal to :**

(*a*) $\log\frac{3}{2}-1$ (*b*) $2\log\frac{3}{2}-1$ (*c*) $\frac{1}{2}\log\frac{3}{2}-x$ (*d*) $\frac{1}{2}\log\frac{2}{3}-1$

[May 2018]

Solution : (*b*)

$$I = \int_1^2 \frac{1-x}{1+x}dx$$

$$= \int_1^2 \frac{2-1-x}{1+x}dx - \int_1^2 \frac{2-(1+x)}{1+x}dx$$

$$= \int_1^2 \left(\frac{2}{1+x} - \frac{1+x}{1+x}\right)dx$$

$$= \left[2\cdot\log(1+x)-x\right]_1^2$$

$$= \left[2\log(1+2)-2\right]-\left[2\log(1+1)-1\right]$$

$$= [\log 3 - \log 2] - 2 + 1$$

$$= 2\log\frac{3}{2} - 1$$

(*b*) is correct.

Q.30. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}dx$ **is equal to**

(*a*) $\frac{2\sqrt{2}}{\log_e 3}$ (*b*) 0 (*c*) $\frac{2(3\sqrt{2}-1)}{\log_e 3}$ (*d*) $\frac{3\sqrt{2}}{\sqrt{2}}$

[May 2018]

Solution : (*c*)

$$I = \int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}dx = 2\int_0^2 3^{\sqrt{x}}\frac{1}{2\sqrt{x}}dx$$

let $t = \sqrt{x}$

$$dt = \frac{1}{2\sqrt{x}}dx$$

If $x = 0 \Rightarrow t = \sqrt{0} = 0$

and if $x = 2 \Rightarrow t = \sqrt{2}$

$$\therefore I = 2\int_0^{\sqrt{2}} 3^t \cdot dt = 2\left[\frac{3^t}{\log_e 3}\right]_0^{\sqrt{2}}$$

$$= \frac{2}{\log_e 3}\left[3^t\right]_0^{\sqrt{2}} = \frac{2}{\log 3}\left[3^{\sqrt{2}} - 3^0\right]$$

$$= \frac{2\left(3^{\sqrt{2}}-1\right)}{\log 3}$$

∴ (c) is correct.

Q.31. The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx$ **is :**

(*a*) 0 (*b*) 3 (*c*) 2 (*d*) 1

[May 2018]

Solution : (*d*)

Tricks:-

$$\int_a^b \frac{x^n}{x^n+(a+b-x)^n}dx = \frac{b-a}{2}$$

$$\Rightarrow \int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx = \frac{2-0}{2} = 1.$$

Q.32. $\int x\left(x^2+4\right)^5 dx$ **is equal to**

(*a*) $\frac{1}{12}\left(x^2+4\right)^6 + c$ (*b*) $\left(x^2+4\right)^6 + c$

(*c*) $\frac{1}{6}\left(x^2+4\right)^6 + c$ (*d*) None

[Nov. 2018]

Solution : (*a*)

$$I = \int x\left(x^2+4\right)^5 dx$$

$$= \frac{1}{2}\int\left(x^2+4\right)^5 \cdot 2x\,dx$$

$$= \frac{1}{2}\frac{\left(x^2+4\right)^6}{6} + C$$

$$= \frac{\left(x^2+4\right)^6}{12} + C$$

Q.33. $\int_{-1}^3 (1+3x-x^3)dx$ **is equal to**

(*a*) –3 (*b*) –4 (*c*) 3 (*d*) 4

Solution : (*b*)

$$\int_{-1}^3 \left(1+3x-x^3\right)dx$$

$$= \left[x + 3\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^3$$

$$= [x]_{-1}^3 + \frac{3}{2}\left[x^2\right]_{-1}^3 - \frac{1}{4}\left[x^4\right]_{-1}^3$$

$$= 3-(-1) + \frac{3}{2}\left[3^2-(-1)^2\right] - \frac{1}{4}\left[3^4-(-1)^4\right]$$

$$= 3+1+\frac{3}{2}(9-1) - \frac{1}{4}(81-1)$$

$$= 4+12-20 = -4$$

Q.34. $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx =$

(*a*) 1 (*b*) 1/2 (*c*) 2 (*d*) 3/2

[June 2019]

Solution : (*b*)

Tricks:-

$$\int_2^3 \frac{\sqrt{x}}{\sqrt{3+2-x}+\sqrt{x}}dx = \frac{3-2}{2} = \frac{1}{2}.$$

Q.35. $\int e^x\left(x^2+2x\right)dx =$

(*a*) $x^2\cdot e^x + c$ (*b*) $e^x\cdot x + c$

(*c*) $-e^x\cdot x^2 + c$ (*d*) $-e^x\cdot x + c$

[June 2019]

Solution : (*a*)

Formula

$$I = \int e^x\cdot\left\{f(x)+f'(x)\right\}dx$$

$$= e^x\cdot f(x) + c.$$

$$\therefore \int e^x\cdot\left(x^2+2x\right)dx = e^x\cdot x^2 + c$$

Q.36. $\int \log\left(a^x\right)dx =$

(a) $\log a\left(\frac{x^2}{2}\right)+c$ (b) $\log a\left(\frac{x}{2}\right)+c$

(c) $x\log a^x - x + c$ (d) $x\log a^x + c$

[June 2019]

Solution : (a)

$\int \log(a^x)dx$

$= \int x.\log a\,dx$

$= \log a.\int x.dx$

$= \log a.\left(\frac{x^2}{2}\right)+c$

Where C = Integration Constant

(a) is correct.

Q.37. Find the value of $\int xe^x dx$

(a) $e^x(x-1)+c$ (b) $e^x(2x-1)+c$

(c) $e^x(x-1)$ (d) None of these

[Dec. 2019]

Solution : (a)

$I = \int xe^x dx$

By Parts ; we get

$I = x\int e^x dx - \int\left\{\frac{dx}{dx}\int e^x dx\right\}dx$

$= xe^x - \int 1.e^x dx$

$= xe^x - e^x + c$

$= e^x.(x-1)+c$

Q.38. Find the value of $\int(4x+5)^6 dx$ **is equal to**

(a) 1/7(4x+5)^7 + c (b) 1/28(4x+5)^7 + C

(c) 1/4(4x+5)^7 + c (d) None of these

[Dec. 2019]

Solution : (b)

$\int(4x+5)^6 dx = \frac{(4x+5)^7}{7\times 4}+c$

$= \frac{1}{28}(4x+5)^7+c$

Where C = Integration constant.

Q.39. $\int_{-1}^{1}\left(2x^2-x^3\right)dx =$

(a) 14 (b) 104 (c) $\frac{2x^3}{3}-\frac{x^4}{4}$ (d) $\frac{4}{3}$

[Dec. 2019]

Solution : (d)

$I = \int_{-1}^{1}\left(2x^2-x^3\right)dx$

$= \left[2\cdot\frac{x^3}{3}-\frac{x^4}{4}\right]_{-1}^{+1}$

$= \frac{2}{3}\left[x^3\right]_{-1}^{+1} - \frac{1}{4}\left[x^4\right]_{-1}^{+1}$

$= \frac{2}{3}\left[1^3-(-1)^3\right]-\frac{1}{4}\left[1^4-(-1)^4\right]$

$= \frac{2}{3}(1+1)-\frac{1}{4}(1-1)$

$= \frac{4}{3}-0=\frac{4}{3}$

Q.40. $\int_{-1}^{1}(e^x - e^{-x})dx = ?$

(a) 0 (b) 1 (c) 12 (d) None

[Dec. 2020]

Solution : (a) is correct.

Let $f(x) = e^x - e^{-x}$

then $f(-x) = -(e^x - e^{-x}) = -f(x)$

So, the function is a "ODD" function.

so; $\int_{-1}^{1}(e^x - e^{-x})dx = 0$

Q.41. $\int_{1}^{2}\frac{1-x}{1+x}dx$

(a) 2 log 3/2 - 1 (b) 2 log 3 + 1 (c) ½ log 3/2 - 1 (d) 2 log 2 – 1 + k

[Dec. 2020]

Solution : (a) is correct $\int_{1}^{2}\frac{1-x}{1+x}dx = \int_{1}^{2}\frac{2-1-x}{1+x}dx = \int_{1}^{2}\frac{2-(1+x)}{1+x}dx$

$= \int_{1}^{2}\left(\frac{2}{1+x}-1\right)dx = \left[2\log(1+x)-x\right]_{1}^{2}$

$= 2\left[\log(1+x)\right]_{1}^{2} - [x]_{1}^{2}$

$= 2[\log 3 - \log 2] - [2-1] = 2\log\frac{3}{2} - 1$

Q.42. The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx$ **is :**

(a) 0 (b) 3 (c) 2 (d) 1

[Dec. 2020]

Solution : (d)

Tricks:-

$\int_a^b \frac{x^n}{x^n+(a+b-x)^n}dx = \frac{b-a}{2}$

$\Rightarrow \int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx = \frac{2-0}{2} = 1.$

Q.43. $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx =$

(a) $e\left(\frac{e}{2}-1\right)$ (b) e (e - 1) (c) a (d) e^2 (e - 1)

[Jan. 2021]

Solution : (a) is correct

$I = \int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

$\int e^x[f(x)+f'(x)]dx = e^x f(x)+c$

$\therefore \quad I = \int e^x\frac{1}{x}-\frac{1}{x^2}dx$

$= e^x\frac{1}{x}$

Now $I = \int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx = \left[e^x\frac{1}{x}\right]_1^2$

$= \frac{e^2}{2}-\frac{e^1}{1} = e\left(\frac{e}{2}-1\right)$

$\therefore$ (a) is correct

Q.44. The value of $\int_{-2}^{2} f(x)\,dx$, **where f(x) = 1 + x, ≤ 0; f(x) = 1-2x, x ≥ 0 is**

(a) 20 (b) -2 (c) -4 (d) 0

[July 2021]

Solution : (b) is correct

$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)dx + \int_{0}^{2} f(x)dx$

$= \int_{-2}^{0}(1+x)dx + \int_{0}^{2}(1-2x)dx$

$= \left[x+\frac{x^2}{2}\right]_{-2}^{0} + \left[x-2\frac{x^2}{2}\right]_{0}^{2}$

$$= 0 + 0 - \left[(-2) + \frac{(-2)^2}{2}\right] + [(2 - 22) - 0 - 0]$$

$$= -[-2 + 2] + (-2)$$

$$= -2$$

Q.45. Integrate with respect to x, $1/[x(\log x)^2]$

(a) $-1/\log x + k$ (b) $1/\log x + k$ (c) $\log x$ (d) x

[Dec. 2021]

Solution : (a)

$$I = \int \frac{1}{x(\log x)^2} dx$$

Let $t = \log x$

$$\Rightarrow dt = \frac{1}{x} dx$$

$$I = \int \frac{1}{(\log x)^2} \frac{1}{x} dx$$

$$= \int \frac{1}{t^2} . dt = \int t^{-2} dt$$

$$= \frac{t^{-2+1}}{-2+1} + K = \frac{t^{-1}}{-1} + K$$

$$= \frac{1}{-t} + K$$

$$= = \frac{-1}{\log x} + K$$

Where

K = Integration constant

Q.46. $\int_0^1 xe^x dx$

(a) -1 (b) 1 (c) e^1 (d) e^{-1}

[June 2022]

Solution : $I = \int_0^1 xe^x dx$

By Parts ; we get

$$\int xe^x dx = x\int e^x dx - \int \left\{\frac{dx}{dx}\int e^x dx\right\} dx$$

$$= xe^x - \int 1 . e^x dx$$

$$= xe^x - e^x$$

$$= e^x (x - 1)$$

$$\therefore I = \int_0^1 xe^x dx = \left[e^x (x-1)\right]_0^1$$

$$= e^1 (1-1) - e^0 (0 - 1)$$

$$= 0 - 1 . (-1) = +1$$

$\therefore$ (b) is correct

Q.47. Determine f(x), given that $f'(x) = 12x^2 - 4x$ and $f(-3) = 17$.

(a) $f(x) = 4x^3 - 2x^2 + 143$ (b) $f(x) = 6x^2 - x^4 + 137$

(c) $f(x) = 3x^4 - x^3 - 137$ (d) $f(x) = 4x^3 - 2x^2 - 143$

[June 2022]

Solution : $f(x) = \int f'(x) dx$

$$\int (12x^2 - 4x)\, dx$$

$$= 12 . \frac{x^3}{3} - 4 . \frac{x^2}{2} + c$$

where c = integration constant.

$\therefore f(x) = 4x^3 - 2x^2 + C$.....(1)

Now $f(-3) = 4(-3)^3 - 2(-3)^2 + c$

or; $17 = -108 - 18 + C$

$\therefore C = 143$

$\therefore$ From (1)

$f(x) = 4x^3 - 2x^2 + 143$

(a) is correct

Q.48. Find the area under the curve $f(x) = x^2 + 5x + 2$ with the limits 0 to 1.

(a) 3.833 (b) 4.388 (c) 4.833 (d) 3.338

[Dec. 2022]

Solution: Required Area $= \int_0^1 f(x) dx$

$$= \int_0^1 (x^2 + 5x + 2) dx$$

$$= \left[\frac{x^3}{3} + 5 \times \frac{x^2}{2} + 2x\right]_0^1$$

$$= \frac{1}{3}\left[x^3\right]_0^1 + \frac{5}{2}\left[x^2\right]_0^1 + 2[x]_0^1$$

$$= \frac{1}{3}[1^3 - 0] + \frac{5}{2}[1^2 - 0] + 2[1 - 0]$$

$$= \frac{1}{3} + \frac{5}{2} + 2 = \frac{2 + 15 + 12}{6} = \frac{29}{6} = 4.833$$

$\therefore$ (c) is correct.

Q.49. $\int (2x - 3)^5 dx$ **is**

(a) $\frac{(2x-3)^6}{6}$ (b) $\frac{(2x-3)^6}{2}$ (c) $\frac{(2x-3)^6}{12}$ (d) $\frac{(2x-3)^6}{3}$

[Dec. 2022]

Solution: $\int (2x - 3)^5 dx$

$$= \frac{(2x-3)^{5+1}}{(5+1) \times 2} + C$$

$$= \frac{(2x-3)^6}{12} + C$$

(c) is correct

Q.50. $\int_2^4 \frac{x dx}{x^2 + 1}$ **is**

(a) $\frac{1}{2}\log\left(\frac{17}{5}\right)$ (b) $2\log\left(\frac{17}{5}\right)$ (c) $\frac{1}{2}\log\left(\frac{5}{17}\right)$ (d) $2\log\left(\frac{5}{17}\right)$

[Dec. 2022]

Solution: $I = \int_2^4 \frac{x}{x^2 + 1} dx$

$$= \frac{1}{2}\int_2^4 \frac{2x}{x^2 + 1} dx$$

$$= \frac{1}{2}\left[\log|x^2 + 1|\right]_2^4$$

$$\left[Tricks: -\int \frac{f^1(x)}{f(x)} dx = \log|f(x)|\right]$$

$$= \frac{1}{2}\left[\log(4^2 + 1) - \log(2^2 + 1)\right]$$

$$= \frac{1}{2}\log\frac{17}{5}$$

So, (c) is correct.

Q.51. Evaluate the integral $\int \frac{1}{(x-1)(x-2)} dx$

(a) $\log\left(\frac{x-2}{x-1}\right) + C$

(b) $\log[(x-2)(x-1)] + C$

(c) $\log\left(\frac{x-1}{x-2}\right) + C$

(d) $\log[(x-2)(x+1)] + C$

[June 2023]

Solution:

$$I = \int \frac{1}{(x-1)(x-2)} dx$$

$$= \int \left(\frac{1}{x-2} - \frac{1}{x-1}\right) dx$$

$= \log(x - 2) - \log(x - 1) + C$

where C = Integration constant

$$= \log\left(\frac{x-2}{x-1}\right) + C$$

(a) is correct.