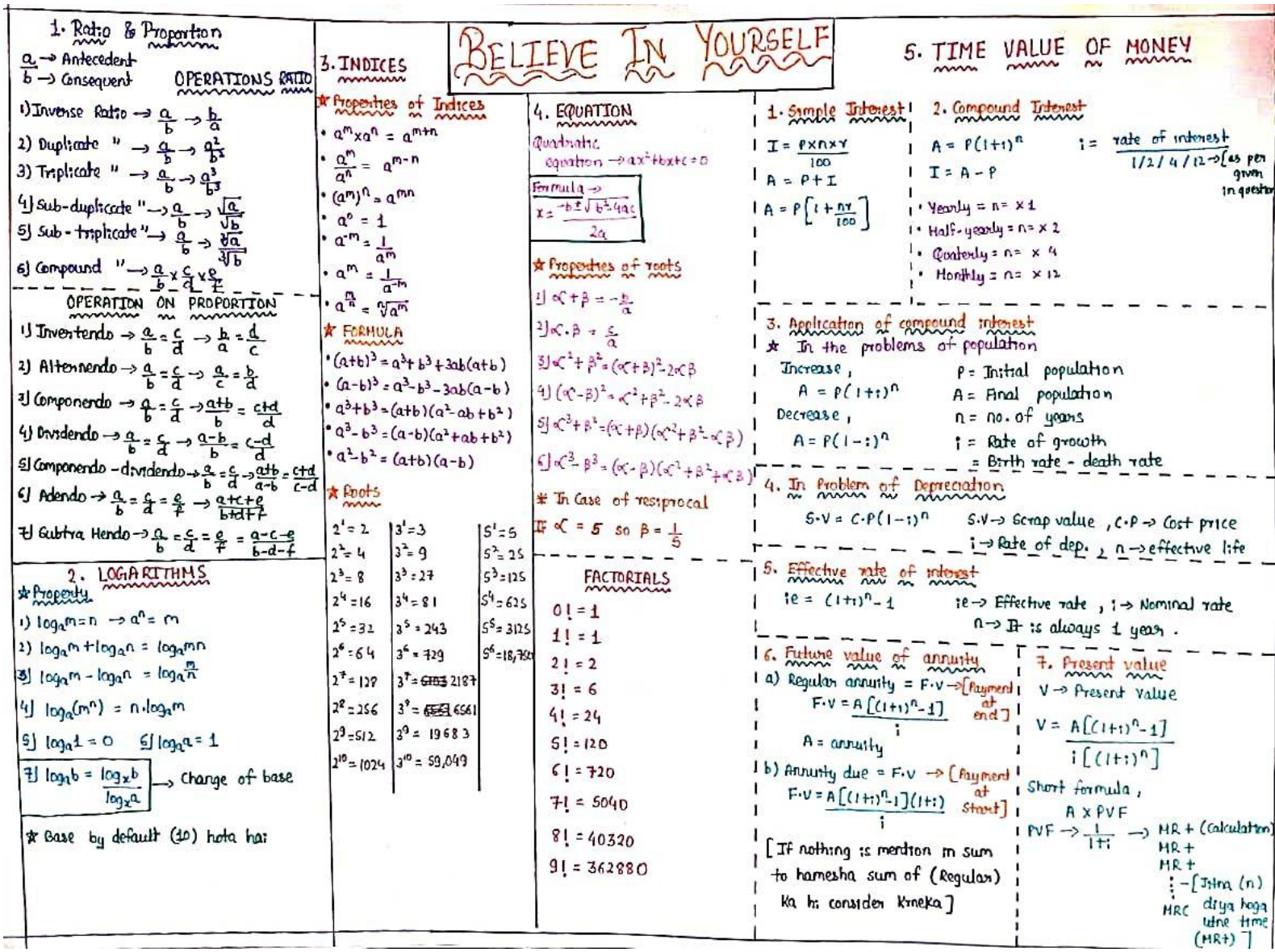
| Rahul Bhutani sir | Lakshya Gupta



Contact No: 89830 86750

CENTRAL TENDENCY

1. Arithmetic Mean
- Average

$\bar{x} = \frac{\Sigma x}{N}$ .... discrete without frequency

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ .... discrete with frequency

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ .... continuous data, x = midpoint

2. Median – (Middle most value)
a) Discrete without frequency →
- first arrange data in A·O
- Median = $\left(\frac{n+1}{2}\right)^{th}$ value

N = no. of observation

b) Discrete with frequency
- Find less than C·F
- Find $\frac{N+1}{2}$ ($N = \Sigma f$)
- Check frequency ∵ C·F $\geq \frac{N+1}{2}$
- C·F → x → median

c) Median for continuous data
- Find less than C·F.
- Find $\frac{N}{2}$ ($N = \Sigma f$)
- Check C·f. $\geq \frac{N}{2}$
- C·F → Class → median Class
- Median = $L + \left(\frac{\frac{N}{2} - c\cdot f}{f}\right) \times h$

☆ Mode
Value having maximum frequency repetition.

- Continuous data

$$\text{Mode} = L + \frac{(f_1 - f_0) \times h}{2f_1 - f_0 - f_2}$$

L = Lower Class boundary
h = UCB − LCB
f_0 = Previous Class frequency
f_1 = maximum frequency
f_2 = next class frequency.

4) Harmonic Mean
$HM = \frac{N}{\Sigma \frac{1}{x}}$ – [Discrete without frequency]

$HM = \frac{\Sigma f}{\Sigma \frac{f}{x}}$ – [Discrete with frequency]

$HM = \frac{\Sigma f}{\Sigma \frac{f}{x}}$ – [Continuous]

5) Geometric Mean
$GM = (x_1 \cdot x_2 \cdot x_3 \ldots x_n)^{\frac{1}{N}}$ – (Without frequency)

$GM = (x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \ldots)^{\frac{1}{\Sigma f}}$ – (With frequency)

$GM = (x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \ldots)^{\frac{1}{\Sigma f}}$ – (Continuous)

☆ Properties of mean, median & mode
1. If all observation are same then mean, median, mode are also same. [$\bar{x}$ = median = mode]
2. Relation between mean, median & mode

For symmetric, $\bar{x}$ = median = mode

For asymmetric, $\bar{x}$ − mode = 3 ($\bar{x}$ − median)

3. Combine arithmetic mean

$$\bar{x}_{12} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$$

4. Change of scale & change of origin

Mean, median, mode are affected by both change of scale (×, ÷) ; $y = a + bx$
change of origin (+, −) ; $\bar{y} = a + b\bar{x}$
; Key = a + b(me)x

5. Sum of deviation of all observation about Arithmetic mean is zero.
i.e. $\Sigma(x - \bar{x}) = 0$

6. Sum of absolute deviation of all observation is minimum when taken above median.

Absolute = Positive = Mod

Median = Q_2 = D_5 = P_{50} are always equal

Median
Me

a) Discrete without frequency
- first arrange in A·O
- Median = $\left(\frac{n+1}{2}\right)^{th}$ value

b) Discrete with frequency
- Find less than C·F
- Find $\frac{N+1}{2}$
- Check that CF $\geq \frac{N+1}{2}$
- Check → x → median

c) Continuous data
- Find less than c·f
- Find $\frac{N}{2}$
- C·F $\geq \frac{N}{2}$
- C·F → Class → Median Class

Median = $L + \left(\frac{\frac{N}{2} - c\cdot f}{f}\right) \times h$

Quartile
Q_1 Q_2 Q_3

a) Discrete without frequency
- first arrange in A·O
- $Q_p = \left[\frac{(N+1)P}{4}\right]^{th}$ value

b) Discrete with frequency
- Find less than C·F
- Find → $\frac{(N+1)P}{4}$
- Check that C·F $\geq \frac{(N+1)P}{4}$
- C·F → x → quartile

c) Continuous data
- Find less than C·F
- Find $\frac{NP}{4}$
- C·F $\geq \frac{NP}{4}$
- C·F → Class → Quartile class

$Q_p = L + \left(\frac{\frac{NP}{4} - c\cdot f}{f}\right) \times h$

Decile
D_1 D_2 D_3 ... D_9

a) Discrete without frequency
- first arrange in A·O
- $D_p = \left[\frac{(N+1)P}{10}\right]^{th}$ value

b) Discrete with frequency
- → Find less than C·F
- → Find $\frac{(N+1)P}{10}$
- → Check that C·F $\geq \frac{(N+1)P}{10}$
- → C·F → x → Decile

Percentile
P_1 P_2 P_3 ..... P_{99}

a) Discrete without frequency
- → First arrange in A·O
- → $P_p = \left[\frac{(N+1)P}{100}\right]^{th}$ value

b) Discrete with frequency
- → Find less than C·F
- → Find $\frac{(N+1)P}{100}$
- → Check that C·F $\geq \frac{(N+1)P}{100}$
- → C·F → x → Percentile

10B. DISPERSION

1. Range

$R = L - S$

Co-effe. of range $= \frac{L-S}{L+S} \times 100$

L = Largest Value

S = Smallest Value

2. Quartile Deviation (Q.D)

Q_1 Q_2 Q_3

Inter quartile range $= Q_3 - Q_1$

$Q.D = \frac{Q_3 - Q_1}{2}$

Co-effe. of $Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

3. Mean Deviation (M.D)

It is arithmetic mean of absolute deviation of all observation about central value.

$M.D = \frac{\Sigma|d|}{N}$ (Without frequency)

$M.D = \frac{\Sigma f|d|}{\Sigma f}$ (With frequency)

$M.D = \frac{\Sigma f|d|}{\Sigma f}$ (Continuous)

$\therefore d = x - A$ (A = AM/Median/Mode)

$\therefore$ Co-effe. of $MD = \frac{MD}{A} \times 100$

4. Standard Deviation (S.D)

It is root of mean of squares of deviation of all obser. about AM.

$S.D \rightarrow \sigma$

$S.D = \sigma = \sqrt{\frac{\Sigma d^2}{N}}$ – without frequency $\boxed{d = x - \bar{x}}$

$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f}}$ – with frequency

$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f}}$ – continuous

Variance $= \sigma^2$

C.V = Co.eff. of variation $= \frac{\sigma}{\bar{x}} \times 100$

* Properties of Dispersion

1) If all observation are same then Range = QD = MD = SD are also same.

$[\bar{x} = \text{med} = \text{mode} = GM = HM$

$R = MD = S.D = Q.D = 0]$

2) For 2 numbers a and b

Range $= b - a$

$\sigma = \frac{b-a}{2} = \frac{R}{2}$

3) Measures of Dispersion

- Absolute measure → has some unit → R, MD, SD, QD
- Relative measure → No unit → co-eff. of range, co-eff. of MD, co-eff. of SD, co-eff. of QD

4) Application of C.V

Less C.V → more consistent, more stable

5) Combined S.D

Steps

① $\bar{x}_{12} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$

② $d_1 = \bar{x}_{12} - \bar{x}_1$

$d_2 = \bar{x}_{12} - \bar{x}_2$

③ $\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$

6) Change of Scale & Change of Origin :-

Central tendency	Dispersion
AM, Median, Mode	Range, S.D, M.D, Q.D
$(\times, \div)$ Scale → ✓	Scale → ✓
$(+, -)$ origin → ✓	Origin → ✗
$y = a + bx$	
$\bar{y} = a + b\bar{x}$	$R_y = \lvert b\rvert . R_x$
$me_y = a + b\, me_x$	$\sigma_y = \lvert b\rvert . \sigma_x$
$mo_y = a + b\, mo_x$	$M.D_y = \lvert b\rvert . M.D_x$
	$S.D_y = \lvert b\rvert . S.D_x$
	$Q.D_y = \lvert b\rvert . Q.D_x$

I CENTRAL TENDENCY

1. If all obser. are same then AM, GM & HM are also same.

$AM = GM = HM$

2. Relationship between AM, GM & HM

Case A :- for 2 numbers

$AM = \frac{a+b}{2}$

$GM = \sqrt{ab}$

$HM = \frac{2ab}{a+b}$ $\quad GM^2 = AM \times HM$

Case B :- for any numbers

$AM \geq GM \geq HM$

3. Application of GM & HM

GM & HM are used for finding average rate & average interest.

4. Problems of average speed

Speed $= \frac{\text{Distance}}{\text{Time}}$

[Case I] Distance is same → Use HM

[Case II] Time is same → Use AM

[Case III] Distance & time both are different → Average speed $= \frac{\text{Total Dist}}{\text{Total time}}$

5. Combine Harmonic mean

$H_{12} = \frac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$

8. DERIVAITIVE & INTEGRATION

	f(x)		f'(x)
1)	x^n	→	$n.x^{n-1}$
2)	a^x	→	$a^x.\log a$
3)	x^2	→	$2x$
4)	x	→	1
5)	e^x	→	e^x
6)	$\log x$	→	$\frac{1}{x}$
7)	$\frac{1}{x}$	→	$-\frac{1}{x^2}$
8)	$\sqrt{x}$	→	$\frac{1}{2\sqrt{x}}$
9)	k	→	0

10) $y = u \cdot v$

$\frac{dy}{dx} = \frac{d}{dx}(u).v + u\frac{d}{dx}(v)$

11) $y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{\frac{d}{dx}u.(v) \overset{(-)}{\bullet} u\frac{d}{dx}(v)}{v^2}$

Note :- Agr variable ke upr variable hai to both the side log leneka.

e.g. $x^x = \log x^x = x.\log x$

6. Set, Function & Relation

- Cardinal number = $n(A) = xx$
- Types of set →

Singleton Set: $A = \{7\}$, $B = \{8\}$

Null Set / Empty Set / Void set: $\phi\{\}$

- Subset

e.g. $A = \{2,3,5\}$, $B = \{3,5\}$, $B \subset A$

No. of subset $= 2^n$

- Subset $= 2^n$
  - Proper subset: 2^{n-1}
  - Improper subset: 1

- Operation of Set

1) Union → $(A \cup B)$
2) Intersection → $(A \cap B)$
3) Subtraction → $A = \{2,3,7,8,9\}$, $B = \{1,5,7,9,10\}$, $A - B = \{2,3,8\}$

★ Theorem of addition :-

1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(B \cap C) + n(A \cap B \cap C)$
3) $n(A \cap B') = n(A) - n(A \cap B)$
4) $n(A' \cap B) = n(B) - n(A \cap B)$
5) Total = Atleast + Nothing.

★ Formula for three sets :-

① $n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C)$
② $n(A \cap B' \cap C) = n(A \cap C) - n(A \cap B \cap C)$
③ $n(A' \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C)$
④ $n(A \cap B' \cap C') = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
⑤ $n(A' \cap B \cap C') = n(B) - n(B \cap A) - n(B \cap C) + n(A \cap B \cap C)$

★ Domain & Co-domain

e.g. $\{(3,2)(5,2)(7,5)(6,1)\}$

Domain = input = $\{3,5,7,6\}$

Co-domain = Output = $\{8,2,5,1\}$

PROBABILITY

Formula = $\frac{\text{Special way}}{\text{Normal way}}$

52 card
- Red 26
  - Heart 13
  - Diamond 13
- Black 26
  - Spade 13
  - Club 13

- $P(A) = 0.3$, $P'(A) = 1 - 0.3 = 0.7$
- $P(B) = 0.45$, $P'(B) = 1 - 0.45 = 0.55$
- $P(C) = 0.75$, $P'(C) = 1 - 0.75 = 0.25$

★ ODD's in favour of event & ODD's against event.

- ODD's in favour of event = $\frac{\text{Favourable}}{\text{Unfavourable}}$
- ODD's against = $\frac{\text{Unfavourable}}{\text{Favourable}}$
- $P(A) = \frac{F}{F+U} = \frac{3}{10}$

★ Types of event

1) Sure event → $P(A) = 1$
2) Impossible event → $P(A) = 0$
3) Exclusive event → $P(A \cap B) = 0$
4) Exhaustive event → $P(A \cup B) = 1$
5) Equally likely event → $P(A) = P(B)$
6) Dependent event = $P(A/B) = \frac{P(A \cap B)}{P(B)}$, $P(B/A) = \frac{P(A \cap B)}{P(B)}$
7) Independent event = $P(A \cap B) = P(A) \cdot P(B)$

★ Problems of expected value

Expected value → [avg. value] [mean]

$E(x) = \Sigma x \cdot P$

$E(x^2) = \Sigma x^2 \cdot P$

Variance = $E(x^2) - [E(x)]^2$

INDEX NUMBER

1) Price Index Number :-
$P_{on} = \frac{P_n}{P_o} \times 100$

2) Quantity Index Number :-
$Q_{on} = \frac{Q_n}{Q_o} \times 100$

3) Value Index Number :-
$V_{on} = \frac{V_n}{V_o} \times 100$

4) Simple Aggregative method :-
$P_{on} = \frac{\Sigma P_n}{\Sigma P_o} \times 100$

5) Simple relative method :-
$P_{on} = \frac{\Sigma \frac{P_n}{P_o}}{N} \times 100$

6) Weighted relative method :-
$P_{on} = \frac{\Sigma \frac{P_n}{P_o} \cdot W}{\Sigma W} \times 100 = \frac{\Sigma I \cdot W}{\Sigma W}$

7) Weighted aggregative method :-
$P_{on} = \frac{\Sigma P_n \cdot W}{\Sigma P_o \cdot W} \times 100$

8) Laspayre's method = $\frac{\Sigma P_n \cdot Q_o}{\Sigma P_o \cdot Q_o} \times 100$

9) Paasche's method = $\frac{\Sigma P_n \cdot Q_n}{\Sigma P_o \cdot Q_n} \times 100$

10) Fisher = $P_{on} = \sqrt{L \times P}$

11) Bowley = $P_{on} = \frac{L+P}{2}$

12) Marshall-Edgeworth method → $P_{on} = \frac{\Sigma P_n \cdot \left[\frac{Q_n + Q_n}{2}\right]}{\Sigma P_o \cdot \left[\frac{Q_o + Q_o}{2}\right]} \times 100$

13) CLI = $\frac{\Sigma I \cdot W}{\Sigma W}$