CA, CMA FOUNDATION

- SUMMARY MATERIAL
- LAST MINUTE MASTRY NOTES

Covers :-

- Summary of MATHS
- All Calculator Tricks
- Summary of STATISTICS

NOTE - Refer Seperate Summary NOTES FOR LR

Summary - Time value of money.

P = Principal
I = Interest (Amount)
A.A = Accumulated Amount [Amounts to]

R = Rate of Interest (%age)
$i = R/100$ (in numbers)
Ex:- $i = \frac{R}{100}$ ⇒ R = 10% i = 0.1
R = 5% i = 0.05

* If R is missing ⇒ $R = \frac{\text{Interest for 1st year}}{\text{Principal}} \times 100$

INTEREST

Simple Interest
(Int is calculated only on principal)

Compound Interest
(Int is calculated on principal and outstanding Interest)

Simple Interest (SI):-

1) $SI = \frac{PTR}{100}$

2) $AA = P\left[1 + \frac{TR}{100}\right]$

3) R is rate of Interest
   * Sum doubles in 100/r years
   * Sum tribles in 200/r years.

4) Sum of money doubles in T years it becomes n times in (n-1)T years

5) Sum becomes AA_1 in T_1 yrs, AA_2 in T_2 yrs

then Interest (amt) = $\frac{AA_2 - AA_1}{T_2 - T_1 \text{ yrs}}$ Rate of Int = $\frac{\text{Int}}{P} \times 100$

$P = \frac{A_1 T_2 - A_2 T_1}{T_2 - T_1}$

Compound Interest (CI):-

1) $CI = P[(1+i)^n - 1]$ 2) $AA = P(1+i)^n$

3) if R is Rate of Interest:

* Sum doubles in 71/R yrs
* Sum triples in 114/R yrs

4) depriciation ⇒

Scrap value = Purchase value $(1-i)^n$

$i = \frac{\text{Rate of dep}^n}{100}$

Calculator

* $P + R\% \left.\begin{array}{l} \\ + R\% \\ + R\% \end{array}\right\} n \text{ times.}$
* $(1+i)^n$ ⇒ Calculator

$1+i \times 1 = \left.\begin{array}{l} \\ = \\ = \\ = \end{array}\right\} n \text{ times}$

5) difference B/W CI & SI

Years	Formula.
1	0
2	Pi^2
3	$Pi^2(i+3)$
4	$Pi^2(i^2+4i+6)$
n	$P[(1+i)^n - 1 - ni]$

6) $AA = P(1+i)^n$

AA also called as future value; P also called as present value.

$$\begin{tikzcd} PV \arrow[r, bend left, "\text{compounding} \times (1+i)^n"] & FV \arrow[l, bend left, "\div (1+i)^n \text{ discounting}"] \end{tikzcd}$$

* $\frac{AA}{(1+i)^n} = P$ * $FV > PV$

7) Compounding times (Int is paid more than once in a years)

Compounding times	Accumulated Amount
Yearly (1 time)	$AA = P\left(1 + \frac{R}{100}\right)^n$
half/semi annually (2 times)	$= P\left(1 + \frac{R}{200}\right)^{2n}$
quarterly (4 times)	$= P\left(1 + \frac{R}{400}\right)^{4n}$
monthly (12 times)	$= P\left(1 + \frac{R}{1200}\right)^{12n}$

8) Compound Int - follows geometric progression

Simple Interest - follows Arithmetic progression

$(CI > SI)$

Effective rate of Interest :-

$$\Rightarrow \left[\left(1+\frac{i}{K}\right)^{K}-1\right] \times 100$$

K = Compounding times

i = R/100

* Here principal is of no use.

Calculator

100 + R/K% } K times
+ R/K%
+ R/K% } − 100

ANNUITY ⇒ * Series of equal payments or Recipts for limited period of time.

* time gap b/w payments/Recipts must be same.

Hint - Annuity (or) installment (or) regular pay/Recipts

Ordinary/Regular annuity
↓
Payments/Recipts are made at End of Period.

Hint - at end of period.
- question is silent

$\boxed{\text{Ord. Annuity} \times (1+i) = \text{Annuity due.}}$

Annuity due/immediate.
↓
Payments/Recipts are made at Beginning of Period.

Hint:- Starting today
- Starting Now/Commencing now.

Ordinary Annuity

future value of ordinary Annuity (FVOA)
↓

$$FVOA = \frac{A[(1+i)^n - 1]}{i}$$

Hint:- question is silent
question is asking
Sinking fund, Reedemption of debentures.

Present value of ordinary annuity (PVOA)
↓

$$PVOA = \frac{FVOA}{(1+i)^n} \text{ or } \frac{A[(1+i)^n - 1]}{i(1+i)^n}$$

$$\text{or } \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

Hint:- loan, borrowing, Cash down, pension, question asks present value of Annuity

Calculator (Present value of Annuity)

PVAF ⇒ $1 \div (1+i) =$ } n times
$=$ } GT
$=$

$PVOA = A(PVAF)$

& $A = \frac{PVOA}{PVAF}$

Annuity due:-

Future value of Annuity due ⇒ $FVOA(1+i)$

Present value of Annuity due ⇒ $PVOA(1+i)^n$

*** Perpetuity (Annuity made for infinite period)**

without growth $= \frac{\text{Amt every year}}{i}$

with growth $= \frac{\text{Amt every year}}{i-g}$.

*** Value of Bond**

$$\left[\frac{Int_1}{(1+i)^1} + \frac{Int_2}{(1+i)^2} \cdots \frac{Int_n}{(1+i)^n}\right] + \frac{\text{Investment}}{(1+i)^n}$$

i = discount rate given in the question

*** net present value ⇒**

⇒ Present value of Cash outflow − Cash Inflow

$$\Rightarrow \left[\frac{CI_1}{(1+i)} + \frac{CI_2}{(1+i)^2} + \frac{CI_3}{(1+i)^3} \cdots\cdots\right] - \text{Investment}$$

NPV > 0 = accept
NPV < 0 = Reject.
NPV = 0 = indifferent

Leasing:-

1) Compute present value of ordinary annuity
2) if PVOA > Purchase value = don't lease.
3) if PVOA < Purchase value = lease it.

*** CAGR**

$$\Rightarrow \left[\frac{\text{Value at end}}{\text{Value at Start}}\right]^{\frac{1}{n-1}} - 1 \times 100$$

Equations – Summary Notes

Types:-

	Linear equation	Simultaneous equation	Quadratic equation	Cubic equation
form:-	$ax+b=0$ (or) $ax+by+c=0$	$a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$	$ax^2+bx+c=0$ $(a\neq 0)$	$ax^3+bx^2+cx+d=0$ $(a\neq 0)$

Simultaneous equations:-

1) Solution:-

$$x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \qquad y=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$$

Hint:- always prefer option verification

2) if $\frac{a_1}{a_2}=\frac{b_1}{b_2}$ (Simultaneous eq's cannot be solved)

3) if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (Such eq's have infinite solutions)

Quadratic equations

1) 2 roots α, β
2) if $ax^2+bx+c=0$ then $\alpha+\beta=-b/a$ $\alpha\beta=c/a$ $\alpha-\beta=\frac{\sqrt{b^2-4ac}}{a}$
3) if α, β are roots then quadratic equation is $x^2-(\alpha+\beta)x+\alpha\beta=0$
4) if roots are reciprocal $(\alpha, 1/\alpha)$ then condition is $a=c$.
5) roots are conjugate to each other if one root is $a+\sqrt{b}$ then other root is $a-\sqrt{b}$
6) if roots are in ratio $m:n$ then $mnb^2=(m+n)^2ac$.
   - roots are equal $(m:n=1:1) \Rightarrow b^2=4ac$
   - one root is double the other $(m:n=2:1) \Rightarrow 2b^2=9ac$
   - one root is triple the other $(m:n=3:1) \Rightarrow 3b^2=16ac$.
7) if one root is square of other

$$ac(a+c)+b^3=3abc$$

8) How to solve quadratic equation

- formula
  - $\alpha, \beta \Rightarrow \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$.
- factorisation
- analysis
  - * if $a+b+c=0 \Rightarrow (1, c/a)$
  - * if $a+c=b \Rightarrow (-1, -c/a)$

9) if equations $a_1x^2+b_1x+c_1=0$ & $a_2x^2+b_2x+c_2=0$ have one root in common then the root is $\Rightarrow \dfrac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$

10) If α, β are roots of $ax^2+bx+c=0$ then eq^n with roots.

* $-\alpha, -\beta$ is $f(-x)=0$	* $\alpha-k, \beta-k$ is $f(x+k)=0$
* $1/\alpha, 1/\beta$ is $f(1/x)=0$	* $k\alpha, k\beta$ is $f(x/k)=0$.
* $\alpha+k, \beta+k$ is $f(x-k)=0$	* α^2, β^2 is $f(\sqrt{x})=0$

11) nature of roots using "Δ" ($\Delta=b^2-4ac$)

- >0
  - Perfect square → real, unequal, distinct, rational
  - not a perfect square. → real, irrational, unequal.
- $=0$ → real & equal. → roots $= -b/2a$.
- <0 → imaginary & unequal.

$\boxed{b^2-4ac \geq 0 = \text{real}}$

12) $a + \cfrac{1}{a+\cfrac{1}{a+\cfrac{1}{a \cdots \infty}}}$

$\Rightarrow$ root $\Rightarrow \dfrac{a+\sqrt{a^2+4}}{2}$.

13) $\left.\begin{array}{l}\sqrt{a+\sqrt{a+\sqrt{a \cdots \infty}}} \\ \sqrt{a-\sqrt{a-\sqrt{a \cdots \infty}}}\end{array}\right\}$ Express a as multiple of 2 consecutive numbers

- for + symbol → highest number is answer
- for − symbol → smallest number is answer.

* Cubic equations:-

1) 3 roots α, β, γ.

2) sum ⇒ $\alpha+\beta+\gamma = -b/a$.
$\alpha\beta+\beta\gamma+\gamma\alpha = c/a$.

~~8~~ product ⇒ $\alpha\beta\gamma = -d/a$.

Hint = use this method to solve sums

3) one root

If roots are in A.P $= -b/3a$.

If roots are in G.P $= \sqrt[3]{-d/a}$.

4) if $a+c = b+d$, one root is "-1"

$a+b+c+d=0$, one root is "1"

5) equation with roots α, β, γ is

$$x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma=0$$

* Some less used Results.

1) for rectangle.

L = length B = bredth.

perimeter $= 2(l+b)$

area $= L \times B$

diagnal length $= \sqrt{l^2+b^2}$

2) for square

perimeter $= 4a$,

area $= a^2$

diagnal length $= \sqrt{2}a$

3) pythagorus theorm

side^2 + side^2 = hypotenuse^2

4) for triangle.

perimeter = sum of length of all 3 sides

area $= \frac{1}{2} \times$ base $\times$ height.

* Some more less used Results in equations.

* for Roots of Q. Eqn

$$\alpha^2+\beta^2 = \frac{b^2-2ac}{a^2}$$

$$\alpha^2-\beta^2 = \frac{-b\sqrt{b^2-4ac}}{a^2}$$

$$\alpha^3+\beta^3 = \frac{b^3+3abc}{a^3}$$

$$\alpha^3-\beta^3 = \frac{(b^2-ac)(\sqrt{b^2-4ac})}{a^3}$$

* $$\frac{\alpha^2}{\beta}-\frac{\beta^2}{\alpha} = \frac{(b^2-ac)\sqrt{b^2-4ac}}{ca^2}$$

Summary Notes - Sequence & Series

Arithmetic progression (A/P) | Geometric progression (G/P)

* Arithmetic Progression (A/P) :-

1) Progression with Common difference (d) $[d = t_2 - t_1, t_3 - t_2 \ldots t_n - t_{n-1}]$
2) general format $a, a+d, a+2d, a+3d \ldots\ldots$
3) nth term $(T_n) = a + (n-1)d$ [a = First term]
4) if t_m & t_n are two terms then $d = \frac{t_m - t_n}{m - n}$
5) Arithmetic mean $\Rightarrow \frac{a+c}{2} = b$

   (Avg of any two terms will give middle term)
6) Insert Am's b/w x & y $\Rightarrow$ It means the progression is in AP

   $\left.\begin{array}{l} t_m = x \\ t_n = y \end{array}\right\}$ find Common difference, use it to fill the A/P
7) three terms are in AP $\Rightarrow a-d, a, a+d$.

   Four terms are in AP $\Rightarrow a-3d, a-d, a+d, a+3d$.

   five terms are in AP $\Rightarrow a-2d, a-d, a, a+d, a+2d$.

   Hint:- do option verification
8) nth term from end of AP $= l + (n-1)(-d)$ [l = last term]
9) Sum of terms in AP
   - till n terms $\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$
   - if first term & last term is given $\Rightarrow S_n = \frac{n}{2}[a + l]$

☆

10) $t_1 + t_2 + t_3 \ldots t_n = S_n$

    $S_n - S_{n-1} = t_n$

11) $1 + 2 + 3 + 4 \ldots\ldots = n(n+1)/2$

    $1^2 + 2^2 + 3^2 \ldots\ldots = n(n+1)(2n+1)/6$

    $1^3 + 2^3 + 3^3 \ldots\ldots = n^2(n+1)^2/4$

    Sum of even no's $= n(n+1)$

    Sum of odd no's $= n^2$
12) if $S_n = an^2 + bn$ then

    $t_n = 2an + (b - a)$
13) $t_n + d = t_{n+1}$

Some Important Rules to Remember IN Arithmetic Progression

a) $t_m = n, t_n = m \Rightarrow t_p = m + n - p$

   $t_{m+n} = 0$

b) $t_m = 1/n, t_n = 1/m \Rightarrow t_{mn} = 1$

   $S_{mn} = mn+1/2$

c) $S_m = n, S_n = m \Rightarrow S_{m+n} = -(m+n)$

d) $S_m = S_n \Rightarrow S_{m+n} = 0$

* Geometric Progression:-

1) Progression with common ratio (r) $[r = \frac{t_2}{t_1} = \frac{t_3}{t_2} \cdots \frac{t_n}{t_{n-1}}]$
2) general format $a, ar, ar^2, ar^3 \ldots$
3) nth term $\Rightarrow t_n = ar^{n-1}$
4) if t_m & t_n are two terms then $r = \left(\frac{t_m}{t_n}\right)^{\frac{1}{m-n}}$
5) geometric mean $\Rightarrow b = \sqrt{ac}$
   (root of any two terms will give middle term)
6) Insert GM's b/w x & y $\Rightarrow$ It means the progression is in G/P
   $\left.\begin{matrix} t_m = x \\ t_n = y \end{matrix}\right\}$ find Common Ratio, use it to fill the G/P
7) three terms are in G/P $\Rightarrow a/r, a, ar.$
   four terms are in G/P $\Rightarrow a/r^3, a/r, ar, ar^3$
   five terms are in G/P $\Rightarrow a/r^2, a/r, a, ar, ar^2$

   Hint:-do option verification

8) Sum of terms in G/P (S_n)

   * till n terms $(r \neq 1)$ $= \frac{a(r^n - 1)}{r-1}$
   * till infinity $(r<1)$ $= \frac{a}{1-r}$ (S_∞)
   * if last term is given $(r \neq 1)$ $= \frac{lr - a}{r-1}$
   * if $r = 1$ then $\Rightarrow n.a.$

9) nth term from end for a finite G/P $\Rightarrow l\left(\frac{1}{r}\right)^{n-1}$
10) reciprocal of G/P - also form G/P
11) $t_n \times r = t_{n+1}$
12) $K + KK + KKK + KKKK \ldots = \frac{K}{81}(10^{n+1} - 9n - 10)$
13) $0.K + 0.KK + 0.KKK \ldots$
    $= K/81(9n - 1 + 1/10^n)$
14) if all terms are equal $\Rightarrow AM = GM = HM$
    in case of positive unequal obs $\Rightarrow AM > GM > HM$
    if AM, GM is given then two numbers $\Rightarrow AM \pm \sqrt{AM^2 - GM^2}$
15) if one AM (A) & two GM's (G_1, G_2) to be inserted b/w two numbers then $G_1^3 + G_2^3 = 2AG_1G_2$

16) If 3 positive numbers (a, b, c) are in GP then $\log a, \log b, \log c$ - AP

* If a, b, c are in AP, Substitute $a=1, b=2, c=3$
  If a^2, b^2, c^2 are in AP Substitute $a^2=1, b^2=25, c^2=49$
  $\Rightarrow a=1, b=5, c=7$
* If a, b, c are in GP Substitute $\Rightarrow a=1, b=2, c=4$
* If progressions are given & value of equations are asked, Just take their coefficients.
* If sum is given & term is asked or vice versa, go ahead & do option verification by remembering $t_1 = S_1$
  $S_2 = t_1 + t_2$

Summary - Ratios & Proportions

Ratios:- 1) A Ratio is comparison of two or more quantities of same kind. By division.

2) general form $\Rightarrow a:b \Rightarrow$ a - antecedent b - consequent (order is important) $(a:b \neq b:a)$

3) multiplication & division doesnt change a ratio

4) It must be expressed in simple form & has no units

5) In ratio $a:b$ if $a>b$ - greater inequality
$a<b$ - lesser inequality

6) multiplication of two or more ratios - compounded ratio

$a:b \times c:d = ac:bd$

$a:b \times c:d \times E:f = aCE:bdf$

7) duplicate ratio $= a^2:b^2$	triplicate ratio $= a^3:b^3$	Inverse ratio
Sub duplicate ratio $= \sqrt{a}:\sqrt{b}$	Subtriplicate ratio $= \sqrt[3]{a}:\sqrt[3]{b}$	$\hookrightarrow a:b \Rightarrow b:a$

8) divide K in ratio of $a:b:c$

$K \times a/a+b+c$

$K \times b/a+b+c$

$K \times c/a+b+c$

Calculator $\Rightarrow K \div a+b+c$
$\times \times$ a =
b =
c =

Proportions:- 1) If two Ratios are equal, they are said to be in proportion

if $a:b = c:d \Rightarrow \boxed{ac = bD}$ (Cross Product Rule)

2) 2^{nd}/mean proportion $= \sqrt{ab}$ $\quad$ 4^{th} proportion $= bc/a$

3^{rd} proportion $= b^2/a$

3) if $a:b = c:d$ then

a) $b:a = d:c$ (invertendo)	4) $a:b = c:d \Rightarrow a+c : b+d$ (addendo)
b) $a:c = b:d$ (Alternendo)	9) $a:b = c:d \Rightarrow a-c : b-d$ (Subtractendo)
c) $a+b:b = c+d:d$ (componendo)	(Short cut)
d) $a-b:b = c-d:d$ (dividendo)	4) $a:b = 3:2$, $b:c = 5:6$ } Reverse N for $a:b:c$
e) $a+b:a-b = c+d:c-d$ (componendo & dividendo)	$a:b:c = 15:10:12$

Summary- Indices & logarithms

Indices:-

1) $a \times a \times a \ldots\ldots n \text{ times} = a^n$
   $a + a + a \ldots\ldots n \text{ times} = n.a$.
2) $a^m \times a^n = a^{m+n}$
3) $(a^m)^n = a^{mn}$, $(ab)^m = a^m . b^m$
4) $\left(\frac{a}{b}\right)^m = a^m/b^m$

$(\text{anything})^0 = 1$

5) $\frac{a^m}{b^n} = a^{m-n}$
6) $a^{-m} = 1/a^m$, $a^{-1} = 1/a$
7) $\left(\frac{1}{a}\right)^{-m} = a^m$, $\left(\frac{1}{a}\right)^{-1} = a$
8) $\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$
9) if $a^m = b^m$ then $(a = b)$
   (Powers are equal, equate bases)
10) if $m^a = m^b$ then $(a = b)$
   (bases are equal, equate Powers)
11) $\sqrt[n]{a} = a^{1/n}$
   $\sqrt{a} = a^{1/2}$
12) $\sqrt[n]{a^m} = a^{m/n}$
13) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{a/b}$
14) $\sqrt[n]{a} . \sqrt[n]{b} = \sqrt[n]{a \times b}$
15) if $a^m = k$ then $a = k^{1/m}$
16) if two or more things are equal. always equate it to constant.
   Ex:- if $a^p = b^q = c^r$ then we can rewrite it as $a^p = b^q = c^r = k$.
   $a^p = k$, $a = k^{1/p}$; $b^q = k$, $b = k^{1/q}$; $c^r = k$, $c = k^{1/r}$
17) if $a^p = b^q = (ab)^r$ then
   $\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{r}}$
18) $(a+b)^2 = a^2 + b^2 + 2ab = (a-b)^2 + 4ab$
   * $(a-b)^2 = a^2 + b^2 - 2ab = (a+b)^2 - 4ab$
   * $(a+b)(a-b) = a^2 - b^2$
   * $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
   * $(a+b)^2 - (a-b)^2 = 4ab$
   * $a^2 + b^2 = (a+b)^2 - 2ab = (a-b)^2 + 2ab$
   * $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
   * $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
   * $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
   * $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
   * if $a+b+c = 0$ then $a^3 + b^3 + c^3 = 3abc$
   * $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.
19) make sure that their is no root in the denominator, you can remove the root by taking rationalisation factor of Surds.

Surd	Rationalisation factor
$\sqrt{a} + \sqrt{b}$	$\sqrt{a} - \sqrt{b}$
$\sqrt{a} - \sqrt{b}$	$\sqrt{a} + \sqrt{b}$
$\sqrt[3]{a} + \sqrt[3]{b}$	$\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}$
$\sqrt[3]{a} - \sqrt[3]{b}$	$\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}$

logarithms:-

1) if $a^x = n$ then $\log_a^n = x$.
(n≠0, negative numbers)

2) $\log_a^n = x$ (Resultant: n; $x \to$ Power; $a \to$ base)

3) if base is missing always take it as 10 (common logarithms)

4) If base is written as e we call them natural or naperian logs.

5) $\log_a^a = 1$, $\log_{10}^{10} = 1$

6) $\log_{\text{anything}}^1 = 0$

7) $a^{\log_a^n} = n$.

8) $\log m.n = \log m + n$ ☆

9) $\log m/n = \log m - \log n$.

10) $\log(A.B.C \cdots Z) = \log A + \log B \cdots \log Z$

11) $\log a^m = m \log a$.

12) $\log_{b^n}^a = \frac{1}{n} \log_b^a$

13) $\log_{b^n}^{a^m} = \frac{m}{n} \log_b^a$

14) $\log_b^a = \frac{\log a}{\log b} = \frac{1}{\log_a^b}$.
(change of base-rule)

15) $\log_b^a \cdot \log_c^b \cdot \log_a^c = 1$

16) if $\log^x = \log y$ then $\underline{x = y}$

17) if $a = \log_{2x}^x$, $b = \log_{3x}^{2x}$, $c = \log_{4x}^{3x}$.
then $1 + abc = 2bc$

18) Remember
$\log 2 = 0.3010$, $\log 3 = 0.4771$
$\log 5 = 0.6990$

19) $[1 - \{1 - (1 - x^p)^{-1}\}^{-1}]^{q} = x^{-pq}$.

20) log of any number has two parts.
whole part - characteristic
integral part - mantissa.

Calculator trick.

$\log_b^a = \frac{a\sqrt{\ }\ 19 \text{ times} - 1}{b\sqrt{\ }\ 19 \text{ times} - 1}$

Summary – Permutations & Combinations

Addition Rule :- Connecting word 'or' $\Rightarrow$ m or n means $m+n$

multiplication Rule - Connecting word and $\Rightarrow$ m & n means $m \times n$

factorial :-

$0! = 1$ $\quad 3! = 6 \quad 6! = 720 \quad 10! = 5,18,400$

$1! = 1 \quad 4! = 24 \quad 8! = 5760 \quad n! = (n)(n-1)(n-2)\ldots\ldots(3)(2)(1)$

$2! = 2 \quad 5! = 120 \quad 9! = 51,840 \quad$ or $n(n-1)!$

Permutations :- (arrangement)

1) no of ways of arranging r things from n things is ${}^nP_r$ ways

$${}^nP_r = (n)(n-1)(n-2)\ldots\ldots(n-r+1) = n!/(n-r)!$$

Restriction
$n \geq r$
n, r - Positive integers

2) ${}^nP_1 = n \quad {}^nP_2 = n(n-1)$, ${}^nP_3 = (n)(n-1)(n-2)$

3) ${}^nP_r = n \cdot {}^{n-1}P_{r-1}$

4) $\dfrac{{}^nP_r}{{}^nP_{r-1}} = (n-r+1)$ $\qquad$ 5) ${}^nP_r + r \cdot {}^nP_{r-1} = {}^{n+1}P_r$

5) n persons seated in a Row $= n!$

6) no of r digited number from n digits.

* without zero $= {}^nP_r$
* with zero $= {}^nP_r - {}^{n-1}P_{r-1}$

7) Sum of n digits from r digits

* without zero $= {}^{n-1}P_{r-1}$ (Sum of digits)(1111.... r times)
* with zero $= {}^{n-1}P_{r-1}$ (Sum)(111... r times) $- {}^{n-2}P_{r-2}$ (Sum)(1111.... r-1 times)

8) Circular / Round table arrangements $= (n-1)!$

Circular arrangements in which clock wise & anticlock wise is same Ex:- Ring, garland, where there are no two same neighbours $\Big\}$ $\dfrac{(n-1)!}{2}$ ways

9) Circular permutations of r things from n things $\Rightarrow \dfrac{{}^nP_r}{r!}$

10) Circular permutations of r things from n things when clock wise & anticlock wise is same $\Rightarrow \frac{^nP_r}{r!} \times \frac{1}{2}$

11) n articles are arranged in such a way that 2 particular articles never come together $= (n-2)(n-1)!$

12) no of ways n boys & n girls be seated around a round table alternatively $= (n!)(n-1)!$

13) no of ways in which n things can be equally divided among p, q, r this where $(p+q+r=n)$ is $= n! / p! \times q! \times r!$

14) no of arrangements if there are repetitions in given things

$\Rightarrow \frac{n!}{a! \times b! \times c! \ldots}$

where a, b, c --- are repetitions

Ex:- ACCOUNTANT - arrangements

$\frac{10!}{2! \times 2! \times 2! \times 2!}$

Combinations:- (Selection)

1) no of ways of selecting r things from n things is ^nC_r ways

$^nC_r = \frac{n!}{r!(n-r)!} = \frac{^nP_r}{r!} = \frac{(n)(n-1)(n-2)\ldots(n-r+1)}{r!}$

2) $^nC_n = {^nC_0} = 1$

$^nC_1 = n, \ {^nC_{n-1}} = n$

$^nC_2 = n(n-1)/2$

$^nC_3 = \frac{(n)(n-1)(n-2)}{3!}$

$\frac{^nP_r}{^nC_r} = r!$

$^nC_r = \frac{n}{r} {^{n-1}C_{r-1}}$

$\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$

$^nC_r = {^nC_{n-r}}$

if $^nC_x = {^nC_y} \Rightarrow n = x+y$ & $x = y$.

3) $^nC_0 + {^nC_1} + {^nC_2} \ldots {^nC_n} = 2^n$

4) no of ways of selecting atleast 1 thing (one or more) from n things

(or) no of ways of failing in a test with n subjects (or) $^nC_1 + {^nC_2} + {^nC_3} \ldots {^nC_n}$

$\Bigg\} \ 2^n - 1$

5) $^nC_{r-1} + {^nC_r} = {^{n+1}C_r}$, $\quad {^nC_r} + 2\,{^nC_{r-1}} + {^nC_{r-2}} = {^{n+2}C_r}$

6) no of ways of selecting one or more questions from n questions where each question has an alternative $= 3^n - 1$

7) factors of a number:-

* Express it as a multiple of Prime numbers (Prime factorisation)

  Ex:- $2^a \cdot 3^b \cdot 5^c \cdot 7^d \dots$

  a) positive divisors $= (a+1)(b+1)(c+1)\dots$

  b) proper divisors = positive divisors $- 2$

8) no of hand shakes, if each person shakes hands with Remaining people $= {}^nC_2$

no of gifts, if each person gives a gift to other $= {}^nP_2$

no of Railway tickets in n Railway stations $= {}^nP_2$.

9) Selection of 4 letters from		arrangement of 4 letters from	
word: Combination	136	word: Combination	2456
mathematics		mathematics	
Examination		Examination	

10) geometrical applications

* no of diagnals in a polygon $= n(n-3)/2$.

* n points in a plane in which no points are collinear	n points in a plane in which m points are collinear
- no of lines $= {}^nC_2$	- no of lines $= {}^nC_2 - {}^mC_2 + 1$
- no of triangles $= {}^nC_3$	- no of triangles $= {}^nC_3 - {}^mC_3$
- no of quadrilaterals $= {}^nC_4$	- no of quadrilaterals $= {}^nC_4 - {}^mC_4$

* n×n type square board.

  no of squares $= n(n+1)(2n+1)/6$

  no of Rectangles $= \left(\frac{(n)(n+1)}{2}\right)^2$

* no of parllelograms in m points are intersecting n points

  ${}^mC_2 \times {}^nC_2$ ways

Summary - Sets, Relations & functions

Sets :-

1) Collection of well defined objects is called as sets.
2) * Represented by capital letters,
* enclosed in Flower Brackets.
* elements cannot Repeat more than once
* no of elements in a set is called Cardinal number ($n(A)$)

Types :-

1) finite set - countable elements
2) infinite set - uncountable elements
3) null set = { }, ϕ, no elements.
4) Single ton set = one element.
5) Equal sets = two sets have exactly same elements.
6) Equivalent set = two sets have same number of elements.

All equal sets are equivalent but all equivalent sets are not equal sets

7) Subset - A set which has few elements than main set ($\subset$)
* $\phi \subset$ All sets
* A set is subset to it self.
8) Superset = if $A \subset B$ then $A \supset B$ ($\supset$ superset)
9) universal set = A set which has all elements in it [U (or) E]
10) Disjoint sets → A & B have no elements in common.
$\Rightarrow A \cap B = \phi$

* Few Formulas & Rules :-

1) if $n(A) = k$.
no of subsets $= 2^k$.
Proper subsets $= 2^{k-1}$
2) if $A \subset B$, $B \subset A$ then $A = B$
3) $A \cup A = A$, $A \cup \phi = A$, $A \cup U = U$.
$A \cap A = A$, $A \cap \phi = \phi$, $A \cap U = U$.
4) Compliment ($'$ or ^{c}) :- other than given set ($A' = U - A$)
$(A')' = A$, $U' = \phi$, $\phi' = U$
$A \cup A' = U$, $A \cap A' = \phi$
5) In word sum - look for connecting word

word	operation
or/atleast	union - '$\cup$'
And, both	Intersection - '$\cap$'
But not	difference - '$-$'

6) * $A \cup B = A + B - A \cap B$
* $A \cap B = A + B - A \cup B$
* $A \cup B \cup C = A + B + C - A \cap B - B \cap C - C \cap A + A \cap B \cap C$
* $A - B = A - A \cap B$, $B - A = B - B \cap A$.
* $A \Delta B = (A - B) \cup (B - A)$
(Symmetric difference of sets)
* $A - B' = B - A' = A \cap B$

7) A & B but not C

$n(A\cap B - C) \Rightarrow n(A\cap B) - n(A\cap B\cap C)$

8) $n(A-B-C)$ or only A·

$\Rightarrow n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$

9) if "$\cap$" symbol & complement comes together then "$\cap$" symbol becomes "−"

$A\cap B' = A-B,\quad B\cap A' = B-A$

$\left.\begin{array}{l}(A\cup B)' = A'\cap B' \\ (A\cap B)' = A'\cup B'\end{array}\right\}$ demorgan laws

10) Natural numbers $\subset$ whole numbers $\subset$ Integers $\subset$ Rational numbers $\subset$ Real numbers.

Relations

1) Relation is a subset of Cartesian Product of sets.

2) Cartesian product $= A\times B$

$A\times B = \{(x,y) \mid x\in A, y\in B\}$ (Ordered Pair)

3) $A\times B \neq B\times A$, $n(A\times B) = n(B\times A)$

$n(A\times B) = n(A)\times n(B)$

no of Relations in $A\times B = 2^{n(A).n(B)}$

Types of Relations

1) Identity Relation $\Rightarrow R = \{(x,x)/x\in A\}$

2) Reflexive Relation (R) $\Rightarrow$ Identity + Some Extra order Pairs

3) Symmetric (S) $\Rightarrow$ if $(x,y)\in A$ then $(y,x)\in A$

4) transitive (T) $\Rightarrow$ if $(x,y),(y,z)\in A$ then $(x,z)\in A$

5) Equivalence (E) $\Rightarrow$ Reflexive + Symmetric + transitive.

Examples :- Questions on above types

name	R	S	T	E
1) equal to	✓	✓	✓	✓
2) greater than	×	×	✓	×
3) lesser than	×	×	✓	×
4) Parallel to	✓	✓	✓	✓
5) Perpendicular to	×	✓	×	×
6) Reciprocal ($\neq 1$)	×	✓	×	×
7) Square ($\neq 1$)	×	×	×	×
8) has a same father	✓	✓	✓	✓

6) Any Relation which is Symmetric & transitive must be Reflexive also.

7) Same is for Reflexive & transitive

8) no of identity relations $= 2^n - 1$

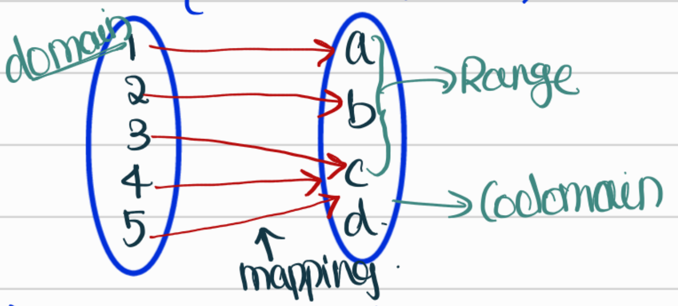
no of Reflexive relations $= 2^{n(n-1)}$

no of symmetric relations $= 2^{n(n+1)/2}$

Functions

1) $A = \{1,2,3,4,5\}$ $B = \{a,b,c,D\}$

$A\times B = \{(1,a),(2,b),(3,c),(4,c),(5,c)\}$



2) A Relation is function if ·

* every element in domain has a mapping & has only one mapping.

3) All functions are relations but all Relations are not functions

4) Range $\subset$ Codomain

Types of functions:-

1) one one function :- one element maps with one element only
2) onto function ⇒ Range = codomain
3) Bijective function ⇒ one one + onto
4) into function ⇒ Range < codomain
(onto & into are complimentary)

functions - Represented by $f(x)$

odd function ⇒ $f(-x) = -f(x)$
even function ⇒ $f(-x) = f(x)$
composite function

$fog(x) \rightarrow f(g(x))$

$gof(x) \rightarrow g(f(x))$

Summary - Linear Inequalities

1) general format:
$ax + by \geq k$, $ax + by > k$
$ax + by \leq k$, $ax + by < k$.

2)
I $(+,+)$ $(x \geq 0, y \geq 0)$
II $(-,+)$ $(x \leq 0, y \geq 0)$
III $(-,-)$ $(x \leq 0\ y \leq 0)$
IV $(+,-)$ $(x \geq 0, y \leq 0)$

x^-, x, y, y^-, 0

4 quadrants

3) method to solve word sums
atleast, minimum - Symbol $\geq$
Starting point.
at most, maximum, available.
- Symbol $\leq$ [ending point]

4) if linear inequality is multiplied by "-" Symbol the inequality sign changes
Ex:- $5 > 3$, $-5 < -3$

5) Solution for graph sums:-
without constant $(k = 0)$
- line always passes through origin
- Substitute points
with constant $(k \neq 0)$

$\geq$ → Shading will be against the origin

$\leq$ → Shading will be towards the origin.

6) types of shading
feasible solution - Shading will touch the line $(\geq, \leq)$
non feasible solution - shading will not touch the line $(>, <)$

Summary - Differentiation, Integration

* Differentiation :-

1) Differentiation / derivative / gradient / Slope - all are same.

2) Basic Formulas :-

Variable	Differentiation
* Constant	0
* x^n	$n \cdot x^{n-1}$
* x	1
* Constant $\cdot x$	Constant
* $\sqrt{x}$	$1/2\sqrt{x}$
* $1/x$	$1/x^2$
* $1/\sqrt{x}$	$-1/2x\sqrt{x}$
* $\log x$	$1/x$.
* x^x	$x^x(1+\log x)$
* a^x	$a^x \log a$
* e^x	e^x

3) Standard Results :-

* $\frac{d}{dx} f(x)^n = n \cdot f(x)^{n-1} \cdot f'(x)$

* $\frac{d}{dx} \sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$

* $\frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x)$

* $\frac{d}{dx} \log f(x) = \frac{1}{f(x)} \cdot f'(x)$

* $\frac{d}{dx} a^{f(x)} = a^{f(x)} \cdot \log a \cdot f'(x)$

* $y = \frac{ax+b}{cx+d} \Rightarrow \frac{dy}{dx} = \frac{ad-bc}{(cx+d)^2}$

4) Suppose u & v are two functions

* $\frac{d}{dx} u \cdot v = uv' + u'v$ (' = differentiation)

* $\frac{d}{dx} \frac{u}{v} = \frac{u'v - uv'}{v^2}$

* $\frac{d}{dx} u \pm v = u' \pm v'$

5) $\frac{d}{dx}[\log(x+\sqrt{x^2+a^2})] = 1/\sqrt{x^2+a^2}$

6) $\frac{d}{dx}[\log(x+\sqrt{x^2-a^2})] = 1/\sqrt{x^2-a^2}$

7) Implicit function (x & y are together)

$$\frac{-[\text{diff of } x \text{ assuming } y \text{ as constant}]}{-[\text{diff of } y \text{ assuming } x \text{ as constant}]}$$

8) $y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) \cdots \infty}}}$

$dy/dx = f'(x)/2y - 1$

9) $y = f(x)^{f(x)^{f(x) \cdots \infty}}$

$$\frac{dy}{dx} = \frac{y^2}{1 - y\log f(x)} \left[\frac{f'(x)}{f(x)}\right]$$

10) if $\text{variable}^{\text{variable}}$ (Powers in variable) is given always solve by taking log on Both sides.

11) differential equations value = 0.

12) Cost & revenue functions.

* $C(x) = V(x) + F(x)$

$C(x)$ = total cost $V(x)$ = variable cost

$F(x)$ = fixed cost

* Avg Cost $= \dfrac{\text{total Cost}}{\text{Output}} = \dfrac{C(x)}{x}$

* Avg variable Cost $= \dfrac{\text{total Variable Cost}}{\text{Output}}$
$= TVC/x$.

* Avg fixed Cost $= \dfrac{\text{total fixed Cost}}{\text{Output}}$
$\Rightarrow TFC/x$.

* marginal Cost / additional Cost equation $= \dfrac{d}{dx}C(x)$ & $\dfrac{d}{dx}V(x)$

* maximum & minimum value for any function:-

Step 1:- do differentiation of given function & calculate the value of x by equating it to zero.

Step 2:- *) is the above value minimum & maximum??

* do double differentiation
↓
if constant value-

+ve (minimum value)

-ve (maximum value)

---

* Break even point:- that point where their is no profit / loss
here Revenue $(R(x)) =$ Cost $(C(x))$
$\Rightarrow R(x) - C(x) = 0$.

* Profit:- Revenue $-$ Cost $\Rightarrow R(x) - C(x)$

Integration $\left[\int f(x).dx\right]$

1) $f(x) \xrightarrow{\text{differentiation}} f'(x)$, $f'(x) \xrightarrow{\text{Integration}} f(x)$.

2) Basic formulas

functions	Integration
* x^n	$x^{n+1}/n+1 + C$
* constant	$\text{constant}.x + C$
* x	$x^2/2 + C$
* $\sqrt{x}$.	$2x^{3/2}/3 + C$
* $1/\sqrt{x}$	$2\sqrt{x} + C$.
* e^x	$e^x + C$
* e^{ax}.	$e^{ax}/a + C$.
* $1/x$.	$\log x + C$.
* a^x	$a^x/\log a + C$

3) $\int f(x^n).f'(x).dx = \dfrac{f(x)^{n+1}}{n+1} + C$

4) $\int \dfrac{f'(x)}{f(x)}.dx = \log f(x) + C$

5) $\int e^x[f(x)+f'(x)]\,dx = e^x f(x) + C$

6) $\int e^{f(x)}.f'(x).dx = e^{f(x)} + C$.

7) $\int \dfrac{f'(x)}{\sqrt{f(x)}}.dx = 2\sqrt{f(x)} + C$

8) $\int a^{f(x)}.f'(x)\,dx = \dfrac{a^{f(x)}}{\log a} + C$.

9) $\int \dfrac{1}{x(x^n+1)}\,dx = \dfrac{1}{n}.\log\left|\dfrac{x^n}{x^n+1}\right| + C$

10) $\int \frac{1}{x(x^n-1)} dx = \frac{1}{n} \log\left|\frac{x^n}{x^n-1}\right| + C$

11) $\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \log\left|\frac{x+a}{x+b}\right| + C$

12) $\int \log x \, dx = x[\log x - 1] + C$

13) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$

14) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$

15) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left|x + \sqrt{x^2 \pm a^2}\right| + C$

16) $\int \sqrt{x^2 \pm a^2} \, dx$

$= \frac{x}{2}\sqrt{x^2 \pm a^2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2 \pm a^2}\right| + C$

17) Integration by parts (Product rule)

Model 1:- $\int$ Algebric x [exp & a^x] dx

Step 1:- Algebric & its differentiation

Step 2:- Start with Integration of exp & a^x

Step 3:- + − + − + − ----

Step 4:- Combine Step 1, 2, 3.

Model 2:-

$\int x^n \cdot \log x \, dx = \frac{x^{n+1}}{n+1}\left[\log x - \frac{1}{n+1}\right] + C$

Model 3:- $\int u \cdot v \cdot dx$

$\Rightarrow u \cdot \int v \cdot dx - \int u' \int v \cdot dx \cdot dx + C$

Order
L - log
A - algebric
E - EXP

18) definite Integrals:- after solving the sum substitute upper limit - lower limit $\int_a^b f(x) \, dx$

a = lower limit b = upper limit
(don't add constant)

Rules:-

* $\int_a^b f(x) \, dx = -\int_b^a f(x) \, dx$
* $\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \quad (a \le c \le b)$
* $\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$
* $\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$
* $\int_{-a}^a f(x) \, dx = 2\int_0^a f(x) \, dx$

* Calculator method for $x^{1/n}$.

Step 1 ⇒ $x\sqrt{\ }$ 13 times

Step 2 ⇒ − 1

Step 3 ⇒ ÷ n

Step 4 ⇒ + 1

Step 5 ⇒ x = 13 times

Maths and Stats – Calculator TRics

1) <u>How to use m+, m−, MRC</u>

m+ = memory plus

m− = memory minus

MRC = memory Recall.

* look function before the number & Press m+ or m−
* for final ans press MRC

Ex:- $(6\times5)+(7\times3)-(10\times2)+(30\div10)$

Ans:- 6x5 m+

7x3 m+

10x2 m−

80÷10 m+ MRC

⇒ 34

2 a^n

$a \times 1 =$ } n times
=
=

Ex:- 2^{10}

$2\times1 =$ } 10 times
=
=

⇒ 1024

3) $1 \div a^n$ (or) $\frac{1}{a^n}$

$1 \div a =$ } n times
=
=

4) $a^{1/n}$

a√ − 13 times

−1

÷n

+1

x = 13 times

Ex:- $\sqrt[5]{243}$

243 √ 13 times

−1

÷5

+1

x = 13 times

Ans = 3

* for $a^{m/n}$

replace step 3 with x m/n

5) <u>divison in terms of Ratio</u>

$x \longrightarrow a:b:c$

$x \div a+b+c$ x x

a = , b = , c =

Ex:- $1536 \longrightarrow 5:4:3$

1536 ÷ 12 x x

5 =	4 =	3 =
640	512	384

6) $\log_b^a \Rightarrow$ b√ 19 times − 1 m+

a√ 19 times − 1 ÷ MRC =

Ex:- $\log_2^{128} \Rightarrow$

2√ 19 times − 1 m+

128 √ 19 times − 1 ÷ MRC =

7) <u>grand total</u>

⇒ Adds all the values that we get after pressing "="

used in → AP, GP, Present value & future value of Annuity.

* use of GT in AP, GP to calculate sum of n terms.

Arithmetic progression

Ex:- 3, 5, 7, 9 ---- S_{10}

21

a + =

+d = = = = (n-1) times

GT Ans = 120

t_n = a + d = = = (n-1) times

geometric progression

Ex:- 3, 6, 12 ---- S_{10}

a × 1 =

r × a = = = = ---(n-1 times)

GT Ans = 3069.

t_n = r × a = = = (n-1) times

8) Harmonic mean Calculation

$$\frac{n}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}---\frac{1}{z}}$$

⇒ 1/a m+

1/b m+

1/c m+ ---- MRC

n ÷ MRC =

Ex:- HM of 10, 18, 24.

1 ÷ 10 m+

1 ÷ 18 m+

1 ÷ 24 m+ MRC

3 ÷ MRC =

Ans = 15.21

9) Compound Interest.

* AA ⇒ (1+i) × 1 = $\left.\begin{matrix} \\ = \\ = \end{matrix}\right\}$ n times × P =

(or) P + R% $\left.\begin{matrix} \\ +R\% \\ +R\% \end{matrix}\right\}$ n times

10) Effective rate of Interest.

1 + R/K% $\left.\begin{matrix} \\ +R/K\% \end{matrix}\right\}$ K times − 1 × 100

11) NPV Calculation:-

1 ÷ (1+i) × ×

Amount$_1$ = m+ , Amount$_2$ = = m+

Amount$_3$ = = = m+ ...... MRC

− Initial investment.

12) Present value of Annuity

PV. ordinary Annuity

* PV.OA ⇒ A × PVAF

* A = PVOA ÷ PVAF.

$\boxed{PVAF}$ ⇒ 1 ÷ (1+i)

= ⇒ n times GT

Present value of Annuity due.

* PVOA × (1+i) = PV AD

STATISTICS

Basic Theory of STATISTICS

Statistics:- Science of counting, Science of averages (other names)

Definition
- Collection of data (olden days)
- Collection, organisation, presentation, Analysis, interpretation & Communication of data (modern days)

Data:- quantitative information about some particular characteristic under consideration.

Ex:- marks of 5 students - 5, 20, 15, 18, 17
no of accidents in a road - 30 accidents

The quantitative information is also called as Variable.

- qualitative information can be data by converting it into quantitative information

Ex:-

Dikshu	Karthik	Aseri	Museri
II	I	III	IV

Though Honesty cannot be measured, we can Rank it and convert qualitative info into quantitative information

data

quantitative data (variable)
Ex:- height, weight, sale, profit etc

qualitative data (attribute)
ex:- gender of a baby, nationality of person, colour of a House

Discreate data

if a variable assumes a finite value

Ex:- petals of a flower, number of misprints in a book, no of road accidents in a day

Continuous Data

if data assumes a value in an ① interval or ② if continuously changes

ex: height, weight, age, sales, profit, salary temparature

unifold, Bifold, trifold, & multifodal classification (manifold classification)

History: 1) Kautilya → Arthashastra → Kept the record of Birth & death (4^{th} BC)

2) Akbar → 16^{th} Century AD

3) Abu Fazl → Ain-i-Akbari → Agriculture records

4) Pharaoh → Egypt → 300BC to 200BC → Census

Origin of word Statistics:-

- latin → Status
- italian word → Statista
- german word → Statistik
- french word → Statistique

Types of statistics:-

DESCRIPTIVE STATISTICS

if we are summarising the entire data

ex:- Average, median

Inferential Statistics

if the character of population is judged through sample

ex:- sampling theory

LIMITATIONS OF STATISTICS:-

1) It deals with aggregates, An individual to a statistician is of no significance

2) It is concerned only with quantitative/numerical data.

3) Statistics helps in projection/Estimation of sales, price, prod^n etc which may ultimately be inaccurate or go wrong.

4) the theory of inferential statistics is based on random sampling these

are chances where this sample can be wrong eventually leading to wrong estimates

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Collection of data — 1st step in statistics

Primary method

Primary data

data collected first time by the investigator or agency

- **Observation method**
  - data collected through observation
  - Best method
  - time consuming, laborious, small area
- **Mailed questionnaire method**
  - Set of questions are prepared & sent
  - Non responsiveness is maximum
  - Very wide area
- **Questionnaire sent through enumerators**
  - Set of questions are prepared & sent with a person
- **Interview method**
  - **Personal interview method**
    - investigator directly meets the respondents & collects the data
    - Natural calamities, Job interview etc
    - accurate method
  - **Indirect interview method**
    - investigator interviews the person who is associated with the incident
    - accidents
  - **Telephonic interview method**
    - quick & non expensive method
    - call & collect the information
    - inaccurate

Secondary method

Secondary data

If the investigator uses the data collected by others

Ex:- news papers, International agencies, govt agencies etc

→ Organisation or Classification of data :- (Grouping)

- neat and is in condensed form
- Statistical analysis is possible only for classified data

Chronological (or) Temporal (or) Time series data

divide the data on basis of time

Ex:-

Year	Passing rate
2018	20%
2019	25%
2020	30%

Geographical or Spatial data

divide the data on the basis of area

In 2020 no of students passed in south, north, east, west, central

Qualitative or ordinal classification

Quantitative or cardinal classification

Presentation of DATA

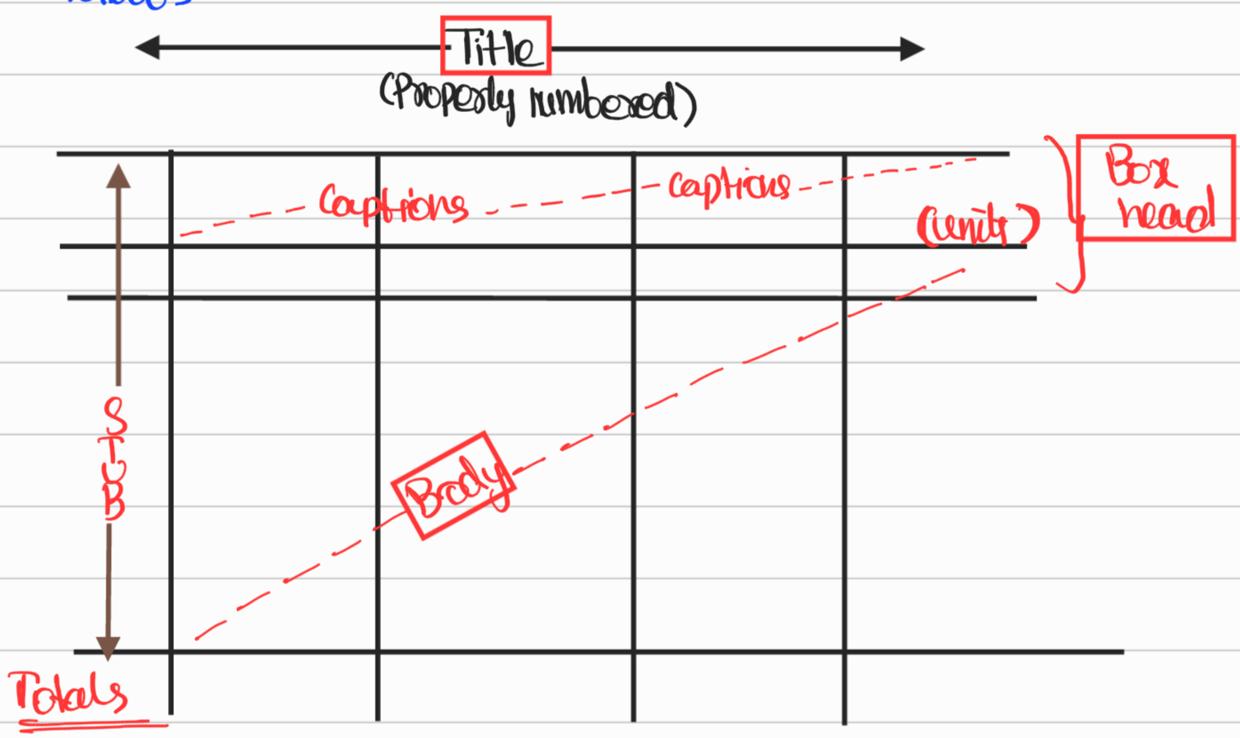
- Textual Presentation
- Tabular presentation or tabulation
- Diagrammetic Presentation

⇒ Textual presentation :-

- In this method we present the data in paragraph or number of paragraphs
  Ex:- official report of enquiry Commission
- It is very Simple & even a layman Can understand this method
- It is first mode of presentation
- It is not preferred by Statisticians simply because it is dull, monotonous, Comparision is not possible, manifold classification Cannot be presented using this method

⇒ Tabular presentation or tabulation :-

- It is a Systematic presentation of data with the help of Statistical tables having no of rows & columns
- There are several parts in the table which are presented as follows



- Source note : from where we got the table
- foot note : any special highlighted points

(b) Tabular presentation or Tabulation

Tabulation may be defined as systematic presentation of data with the help of a statistical table having a number of rows and columns and complete with reference number, title, description of rows as well as columns and foot notes, if any.

We may consider the following guidelines for tabulation :

I A statistical table should be allotted a serial number along with a self-explanatory title.

II The table under consideration should be divided into caption, Box-head, Stub and Body. Caption is the upper part of the table, describing the columns and sub-columns, if any. The Box-head is the entire upper part of the table which includes columns and sub-column numbers, unit(s) of measurement along with caption. Stub is the left part of the table providing the description of the rows. The body is the main part of the table that contains the numerical figures.

III The table should be well-balanced in length and breadth.

IV The data must be arranged in a table in such a way that comparison(s) between different figures are made possible without much labour and time. Also the row totals, column totals, the units of measurement must be shown.

V The data should be arranged intelligently in a well-balanced sequence and the presentation of data in the table should be appealing to the eyes as far as practicable.

VI Notes describing the source of the data and bringing clarity and, if necessary, about any rows or columns known as footnotes, should be shown at the bottom part of the table.

Diagrametic presentation

- It is an attractive representation of statistical data provided with charts, diagramms, patterns
- Both educated and uneducated society can use this method
- Hidden trend if any can be found in diagrammetic presentation
- It is less accurate as compared to tabulation

Types

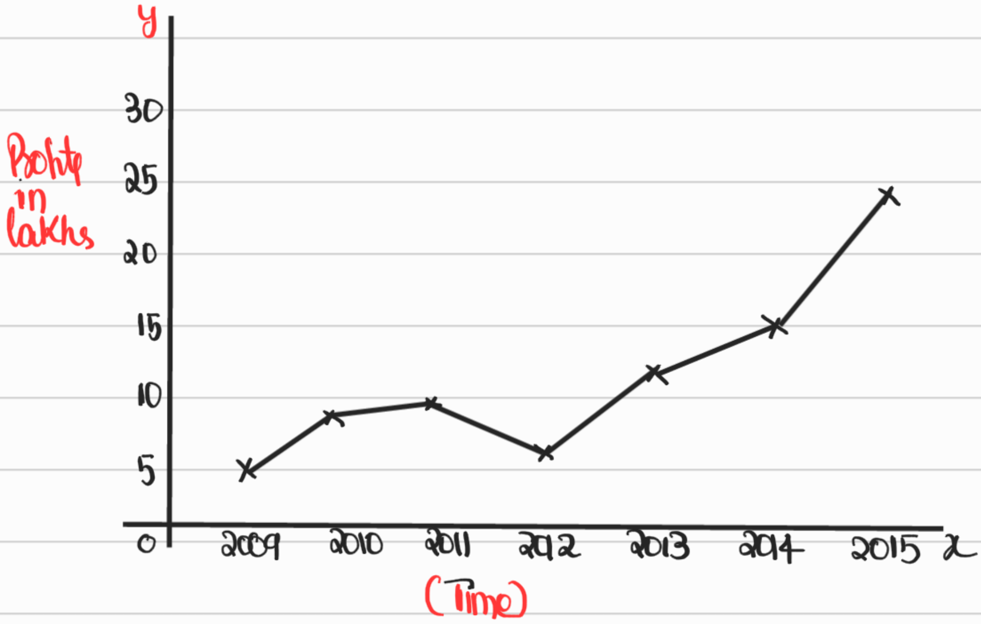
- Line diagram or Historiagram
- Bar diagram
- Pie chart

Line diagram or Historiagram

:- if data varies over time we use line diagram

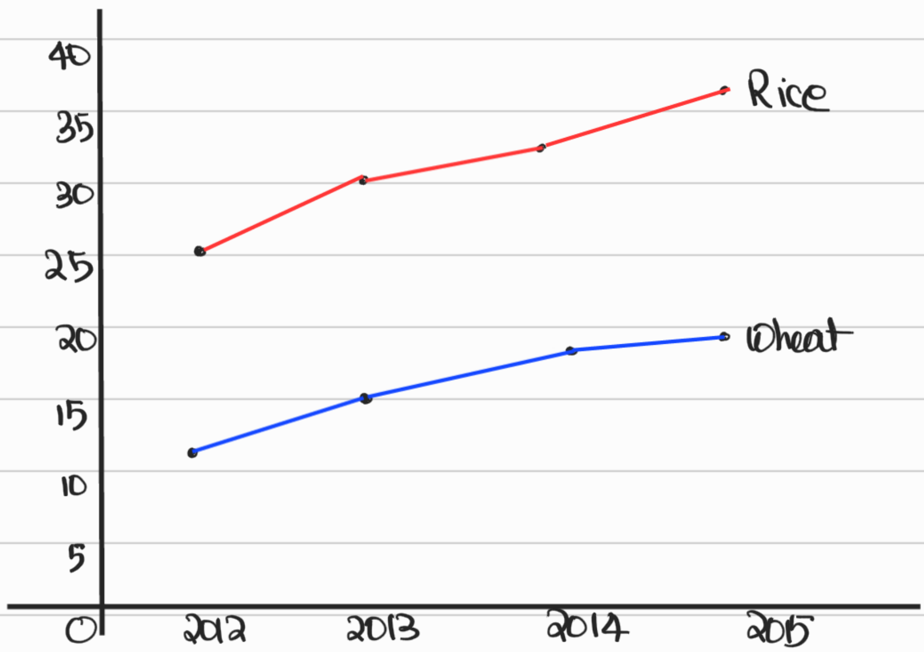
- **Simple line diagram** → one variable is written on y axis & time on x axis
- **logarithmic/ratio chart** → when there is wide range of fluctuation in time
- **multiple line charts** → To represent two or more related time series data expressed in same units
- **multiple axis chart** → To represent two or more related time series data expressed in different units

Ex:- The profits in lakhs of Rupees of an Industrial house for 2009, 2010, 2011, 2012, 2013, 2014, 2015 are 5, 8, 9, 6, 12, 15, 24 respectively



Ex:- the production of wheat and Rice in an area is given as follows

Years	Production in Tones: Wheat	Rice
2012	12	25
2013	15	30
2014	18	32
2015	19	36



⇒ **Bar diagram**:- If rectangles of equal width is used to determine data we call it bar diagram

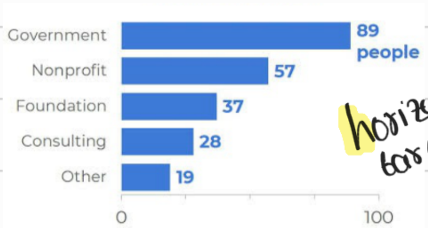
Horizontal Bar diagram

Vertical Bar diagram

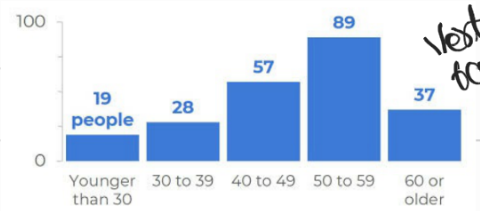
To present qualitative data or data varying over space

To present quantitative data or data varying over time

Simple Bar diagram

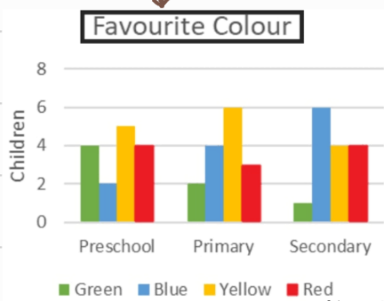


horizontal bar diagram



vertical bar diagram

multiple or grouped Bar diagram



(Similar to multiple line chart)

Sub divided bar diagram

further classification in Bar diagram

Pie chart :- if we want to express data in percentages which is used for comparison we use Pie chart

Source	Amt (million)	angle $\left(\frac{Amt}{Total} \times 360^\circ\right)$
Customs	80	$\frac{80}{540} \times 360 = 53.33^\circ$
Excise	190	$\frac{190}{540} \times 360 = 126.67^\circ$
Inco-Tax	160	$\frac{160}{540} \times 360 = 106.67^\circ$
Corporate tax	75	$\frac{75}{540} \times 360 = 50^\circ$
Other taxes	35	$\frac{35}{540} \times 360 = 23.33^\circ$
	540	360°

Frequency Distribution

number of times an observation is Repeated.
Representation of data.

Individual data / ungrouped data — we use individual data to form grouped data → **freq distribution / grouped data**

Individual data / ungrouped data:
- if every obs in the data is given importance.
- Ex:- marks of 5 students
  10m, 15m, 20m, 30m, 15m

freq distribution / grouped data:

- **discreate data** — if frequencies are assigned to discreate variables.
- **continuous data** — if frequencies are assigned to values in an interval.

Continuous data:

Exclusive data / overlapping data

upper limit of a class is Excluded to next class.

Ex:-

class	freq
0-10	5
10-20	7
20-30	11
30-40	12
	35

class intervals

lower class boundries (LCB), upper class boundries (UCB)

Inclusive / non overlapping data

upper limit of a class is included in same class.

Ex:-

class	freq
0-9	5
10-19	7
20-29	11
30-39	12
	35

class intervals

lower class limits (LCL), upper class limits (UCL)

* inclusive data = used for discreate observations
* Exclusive data = used for continuous observations
* Midvalue = $\frac{LCL+UCL}{2}$ (or) $\frac{LCB+UCB}{2}$ = Class mark

* we always use exclusive data for analysis
  if inclusive is given convert it into exclusive. → Exception "mean"

How to convert inclusive to exclusive data:-

$LCL \xrightarrow{-0.5} LCB$

$UCL \xrightarrow{+0.5} UCB$

(+0.5 or −0.5 is nthg but avg of diff b/w LCL & UCL of previous class & current class)

Inclusive data	Exclusive data	class mark
9 – 19	8.5 – 19.5	14
20 - 29	19.5 - 29.5	24.5
30 - 39	29.5 - 39.5	34.5
40 - 49	39.5 - 49.5	44.5

mid value (class mark/mid point) $\Rightarrow \frac{UCB+LCB}{2} = \frac{UCL+LCL}{2}$

---

*Cummulative frequency:-

less than Cummulative frequency (LCF)

↓

How many obs are less than given class interval.

↓

$\boxed{\text{Start from Top.}}$

↓

$(LCf_n - LCf_{n-1} = f)$

Ex:-

obss	freq	LCF	
0-10	7	7	– less than 10
10-20	9	16	– less than 20
20-30	13	29	– less than 30
30-40	11	40	– less than 40
40-50	10	50	– less than 50
	50		

greater than cummulative frequency (GCF)

↓

How many obs are more than given class interval.

↓

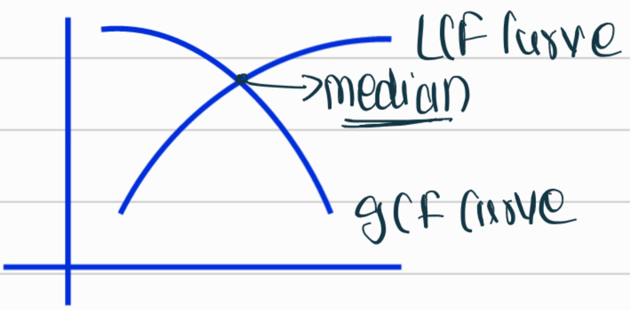
$\boxed{\text{Start from bottom}}$

↓

$(gcf_n - gcf_{n+1} = f)$

Ex:-

obss	freq	gcf	
0-10	7	50	more than 0
10-20	9	43	– more than 10
20-30	13	34	– more than 20
30-40	11	21	– more than 30
40-50	10	10	– more than 40



" ogive curves "

⇓

LCF & GCF curves

LCF	data	class	freq
50	less than 10	0-10	50
60	less than 20	10-20	10
90	less than 30	20-30	30
110	less than 40	30-40	20
115	less than 50	40-50	5
			115

Frequency density ⇒ ratio of frequency of class interval & class length.

relative frequency ⇒ $\frac{\text{Class frequency}}{\text{total frequency}}$ (Σ relative frequency $= 1$)

Percentage frequency ⇒ relative frequency $\times 100$ ($\Sigma PF = 100$)

Central Tendency & Dispersion.

Central tendency :- we are trying to locate one observation which helps us to judge entire data. Such value is called as central tendency value (leader value)

- Measures of averages, 1^{st} order averages.

Types (3 types)

- mean → Algebric measure
  - Arithmetic mean (A.m) ($\bar{x}$)
  - geometric mean (g.m)
  - Harmonic mean (H.m)
- Position Values → Positional measures
  - median (me)
  - quartiles (Q)
  - deciles (d)
  - percentiles (P)
- mode or Norm ($\bar{z}$)

Comparitive analysis.

Particulars	Arithmetic mean (AM) → $\bar{x}$	Geometric mean → (gm)	Harmonic mean → HM
definition	Sum of observations ÷ no of observations	n^{th} root of product of observations	Reciprocal of A.m of Reciprocal of observations
formulas: Individual data	Simple/unweighted mean: $\frac{\Sigma x}{n}$; x = observations, n = no of obss.	$\sqrt[n]{x_1 \times x_2 \times x_3 \ldots x_n}$	$\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3} \ldots \frac{1}{x_n}}$
discreate data	$\frac{\Sigma f x}{\Sigma f}$; f = frequency	$(x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \ldots x_n^{f_n})^{1/\Sigma f}$	$\frac{\Sigma f}{\frac{f_1}{x_1}+\frac{f_2}{x_2} \ldots \frac{f_n}{x_n}}$
Continuous data	$\frac{\Sigma m f}{\Sigma f}$; m = mid value $\left(\frac{UCB+LCB}{2}\right)$	$(m_1^{f_1} \times m_2^{f_2} \ldots m_n^{f_n})^{1/\Sigma f}$	$\frac{\Sigma f}{\frac{f_1}{m_1}+\frac{f_2}{m_2} \ldots \frac{f_n}{m_n}}$
Combined / grouped / Pooled	$\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$	$\text{Antilog}\left(\frac{\Sigma \log x}{n}\right)$	$\frac{n_1+n_2}{\frac{n_1}{Hm_1}+\frac{n_2}{Hm_2}}$
weighted formula (Similar to discreate data)	$\frac{\Sigma wx}{\Sigma w}$; w = weights, x = obss.	$\text{Antilog}\left(\frac{\Sigma w \cdot \log x}{\Sigma w}\right)$	$\frac{w_1+w_2 \ldots w_n}{\frac{w_1}{Hm_1}+\frac{w_2}{Hm_2} \ldots \frac{w_n}{Hm_n}}$
if one obss = 0	Possible to calculate	gm = 0	Hm = undefined.
if one obss = -ve.	possible to calculate	gm = undefined.	Hm = possible to calculate.

Particulars	Arithmetic mean (AM) $\rightarrow \bar{x}$	Geometric mean $\rightarrow$ (gm)	Harmonic mean. $\rightarrow$ HM
importance to what observations	highest observations	all observations	lowest observations
use.	whose gm & HM is not use. Ex:- Average length of Rod calculation.	* Index numbers * Avg of Rates, Ratios, Percentages * Constant increase/ decrease.	* Avg of speed, time, distance. * one value is constant in several places
if all observations are same (=K)	then AM=K	then gm=K	then HM=K.
if only two observations are given.	$AM = \frac{a+b}{2}$	$gm = \sqrt{ab}$	$HM = \frac{2ab}{a+b}$
In progressions.	A.P = mid value = AM	G.P = mid value = g.m	H.P = mid value = H.m.

*Relation B/w Am, gm, HM:-

① if all obs = same ⇒ Am = gm = HM

② in positive unequal observations = $Am > gm > HM$

③ $AM \times HM = gm^2$

④ if Am & gm is given then the two observations = $Am \pm \sqrt{Am^2 - gm^2}$

*Other points in Arithmetic mean:-

1) The Sum of deviations (difference) from Am = 0

2) The Sum of Squared deviations $\Sigma(x-\bar{x})^2$ from Am are always minimum

3) Corrected Arithmetic mean:-

$$\frac{(\text{old Am} \times \text{old obs}) - \text{Inc value} + \text{Correct value}}{\text{new observations}}$$

generally old obs = new obs

but if there are any additions or deletions then

old obs ± New obs

4) Am is simple, Easy to calculate, is a algebraic measure, (if all obs are considered to calculate avg) Rigidly defined, Best measure, Stable measure.

5) Am is highly affected by Extreme observations – obs which are different from normal obs

Ex:- 5, 10, 1500

10,000, 12,000, 8500

6)

Case	Sum	Am
1st n natural numbers	$\frac{n(n+1)}{2}$	$\frac{n+1}{2}$
1st n^2 natural numbers	$\frac{n(n+1)(2n+1)}{6}$	$\frac{(n+1)(2n+1)}{6}$
1st n^3 natural numbers	$\frac{n^2(n+1)^2}{4}$	$\frac{n(n+1)^2}{4}$
1st n even numbers	$n(n+1)$	$n+1$
1st n odd numbers	n^2	n

7) $\boxed{\text{deviation formulas}}$:-

Individual data

$\boxed{A+\frac{\Sigma d}{n}}$ $d=(x-A)$

Discrete data

$\boxed{A+\frac{\Sigma fd}{\Sigma f}}$ $d=(x-A)$

Continuous data

$\boxed{A+\frac{\Sigma fd}{\Sigma f}}$ $d=(m-A)$

A = Assumed mean

d = deviation from Assumed mean

* $\boxed{\text{other points in geometric mean}}$:-

$gm(x\times y)=gm(x)\times gm(y)$ $gm(x/y)=\frac{gm(x)}{gm(y)}$

* Calculator trics:-

gm:- $\sqrt[2]{a}$ → a √

$\sqrt[4]{a}$ → a √ √

$\sqrt[8]{a}$ → a √ √ √

any other number:-

1) √ – 13 times
2) – 1
3) ÷ n
4) + 1
5) × = 13 times

Hm:-

1 ÷ x_1 m+

1 ÷ x_2 m+

1 ÷ x_3 m+ ---- then directly

n ÷ MRC =

*** Entire Central Tendency is affected by Scale & Origin(+, −)
(×, ÷)

Ex:- $ax \pm by \pm c = 0$
a ↳ scale, c ↳ origin

In entire central tendency if Am, median, mode of x is given & Am, median, mode of y is to be calculated, then Put the value, get the answer.

Partition values:- The partition values divide the data into many parts

Individual data

Particulars	Step-1	Step 2	Step-3
1) median (me) (2 Parts − 1)	Ascending order	even data → avg of middle two obs; odd data → middle obs	
2) quartiles (Q) (4 Parts − 3)	Ascending order	$\frac{J(n+1)}{4}$	These formulas will not give you answers ⇓ They will give the location ⇓ The location must be used to identify the observations
3) deciles (d) (10 Parts − 9)	Ascending order	$\frac{J(n+1)}{10}$	
4) percentiles (P) (100 Parts − 99)	Ascending order	$\frac{J(n+1)}{100}$	

n = number of observations
J = Required value

Ex:- $Q_1 = J = 1$ $P_{27} = J = 27$ $Q_3 = J = 3$

Remember
* $Q_2 = D_5 = P_{50} = \text{Median}$
* $Q_1 = P_{25},\ Q_2 = P_{50},\ Q_3 = P_{75}$
* $Q_1 < Q_2 < Q_3$
* $P_1 < P_2 < P_3 \text{---} P_{99}$
* $D_1 < D_2 < D_3 \text{---} D_9$

* if question asks to Compute Rank of any Partition value then write Step 2 of Continuous data formula.

* **Discrete data**:- apply Individual data formula, locate in LCF.

Continuous data

Particulars	Step 1	Step 2	Step 3	Step 4
median	LCF	$\frac{N}{2}$	locate the step 2	$L + \frac{N/2 - CF}{f} \times CI$
quartile	LCF	$\frac{JN}{4}$	value in LCF in order	$L + \frac{JN/4 - CF}{f} \times CI$
decile	LCF	$\frac{JN}{10}$	to get partition values class. If	$L + \frac{\frac{JN}{10} - CF}{f} \times CI$
percentile	LCF	$\frac{JN}{100}$	exact value is not available	$L + \frac{JN/100 - CF}{f} \times CI$

L = lower class boundary

f = frequency of same class

CF = Cummulative frequency of previous class

$N = \Sigma f$

CI = class interval (UCB−LCB)

* **Other points of median** :-

1) median will be used in qualitative data.
2) Absolute deviations are always minimum from median

deviations where negative signs are ignored

Mode or Norm (Z) :- Most repeated observation (or) observation with highest frequency

→ unimodal - one mode → Bimodal - two modes → Trimodal - 3 modes

→ multimodal - >3 mods

mode is used in fashion items & zoology

Formulas

Individual data
- most repeated obs

Discrete data
- obs with highest frequency

Continuous data

1) locate the modal class (class with highest frequency)

2) $L+\dfrac{f_1-f_0}{2f_1-f_0-f_2}\times CI$

or $L+\dfrac{\Delta_1}{\Delta_1+\Delta_2}\times CI$ $\quad \Delta_1=f_1-f_0$, $\Delta_2=f_1-f_2$

L = lower class boundary, f_0 = preceding frequency, f_2 = succeeding frequency

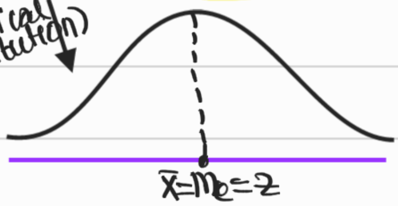
f_1 = frequency of modal class, CI = class interval

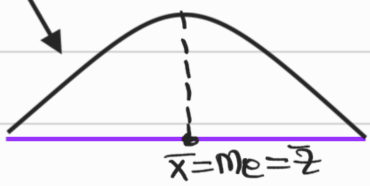
Relation B/w mean, median and mode :-

1) If all observations are same $\boxed{\text{mean}=\text{median}=\text{mode}}$

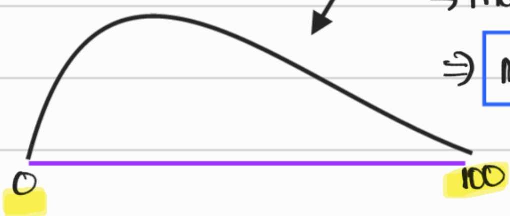
2) In case of normal distribution and perfectly skewed data $\boxed{\text{mean}=\text{median}=\text{mode}}$

(theoretical distribution)





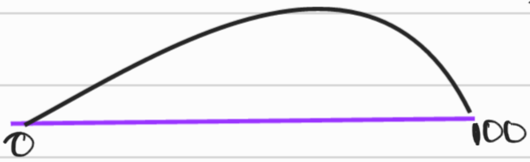
3) In case of positive skewed data



⇒ marks obtained by students in toughest exam

⇒ $\boxed{\text{mean}>\text{median}>\text{mode}}$

4) In case of negatively skewed data



⇒ marks obtained by students in easy exam

⇒ $\boxed{\text{mean}<\text{median}<\text{mode}}$

5) Emperical relation (Karl pearson) (Assymetric distribution)

⇒ mode = 3 median − 2 mean (Both sides substract mean)

(or) mode − mean = 3(median − mean) (multiply − on B/S)

(or) mean − mode = 3(mean − median)

Measures Of Dispersion :-

- Dispersion means calculation of deviation or spread from central tendency value
- It is also called as averages of second order | measures of deviation | measures of spread.

Types of Dispersion

Absolute measures of Dispersion	Relative measures of Dispersion
⇒ Here Dispersion is calculated in terms of units	⇒ Here dispersion is calculated in terms of percentages / number / degree
⇒ Comparision only between similar units is possible but comparison between different units is not possible	⇒ There is no restriction on comparison
	⇒ Relative measures are difficult to compute

absolute measures are used to calculate Relative measures

Types (Absolute):
- Positional measures: Range (R), Quartile deviation (Q·D)
- algebric measure: mean deviation (M·D), standard deviation (S·D) (Best method)

Types (Relative):
- Coefficient of Range
- Coefficient of Q·D
- Coeff of MD
- Coeff of SD (or) Coefficient of variation

[degree of variation → Relative measures
amount of variation → absolute measures]

---

1) **Range** (R) :- In Range we calculate the difference b/w largest observation and smallest observation.

Individual data / discreate data / continuous data →

$$\text{Range} = L - S$$

$$\text{Coefficient of Range} = \frac{L-S}{L+S} \times 100$$

For Range frequencies of distribution are of no use

Range is used in Industries & profit calculations

2) Quartile deviation :- Semi Inter Quartile Range

Individual data
discreate data
Continuous data

$QD = \frac{Q_3 - Q_1}{2}$

Coefficient of Q.D $= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$ (or) $\frac{QD}{Me} \times 100$

Q_1 - lower quartile

Q_2 - middle quartile

Q_3 - upper quartile

$(Q_3 > Q_2 > Q_1)$

* quartile deviation is based on middle 50% of observations

25% | 25% | 25% | 25%

Q_1 Q_2 Q_3

below $Q_1 = 25\%$
above $Q_1 = 75\%$
in b/w Q_1 & $Q_3 = 50\%$

Note:- Quartile deviation is Best measure for open end series

(In central tendency median is best measure)

Mean Deviation :- The Arithmetic mean of absolute deviations (don't take -ve signs) taken from mean, median, mode is called mean deviation.

Particulars	taken from mean	Taken from median	Taken from mode
Individual data	$\frac{\Sigma \lvert x-\bar{x} \rvert}{n}$ $\bar{x}$ = mean	$\frac{\Sigma \lvert x-Me \rvert}{n}$ Me = median	$\frac{\Sigma \lvert x-Z \rvert}{n}$ Z = mode
Discreate data	$\frac{\Sigma f \lvert x-\bar{x} \rvert}{\Sigma f}$	$\frac{\Sigma f \lvert x-Me \rvert}{\Sigma f}$	$\frac{\Sigma f \lvert x-Z \rvert}{\Sigma f}$
Continuous data	$\frac{\Sigma f \lvert m-\bar{x} \rvert}{\Sigma f}$ $\bar{x} = \frac{\Sigma mf}{\Sigma f}$	$\frac{\Sigma f \lvert m-Me \rvert}{\Sigma f}$ $Me = L + \frac{\frac{N}{2} - Cf}{f} \times CI$	$\frac{\Sigma f \lvert m-Z \rvert}{\Sigma f}$ $Z = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times CI$
Coeff of mean deviation	$\frac{MD}{\bar{x}} \times 100$	$\frac{MD}{Me} \times 100$	$\frac{MD}{Z} \times 100$

MD of 1st n natural no $= \frac{n^2 - 1}{4n}$

4) Standard deviation – Root mean square deviation from mean

	Individual data	discrete data	Continuous data
① normal formulas	$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$	$\sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$	$\sqrt{\frac{\Sigma f(m-\bar{x})^2}{\Sigma f}}$
② easy formula	$\sqrt{\frac{\Sigma x^2}{n}-(\bar{x})^2}$	$\sqrt{\frac{\Sigma f x^2}{\Sigma f}-(\bar{x})^2}$	$\sqrt{\frac{\Sigma f m^2}{\Sigma f}-(\bar{x})^2}$

Coefficient of SD (or) Coeff of variation $= \frac{SD}{\bar{x}} \times 100$

Use of Coefficient of variation :- The data which has less coeff of variation has more consistency

	Mr A	Mr B	Mr C
S/D	17	13	15
$\bar{x}$	25	46	34
Coeff of variation	68%	28.26%	44.11%

∵ coeff of variation is less for Mr B, he is more consistent player

* Standard deviation of only 2 numbers $= \frac{\text{Range}}{2} = \frac{\text{highest obs} - \text{lowest obs}}{2}$

* Variance :- Square of standard deviation is called variance.

$$\text{Variance} = (\sigma)^2 = (SD)^2$$

* standard deviation of 1st natural no $\Rightarrow \sqrt{\frac{n^2-1}{12}}$

⇒ Combined/grouped/Pooled Standard deviation :-

$$\text{Combined SD} = \sqrt{\frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}}$$

n_1, n_2 = no of observation

σ_1, σ_2 = Standard deviation

d_1, d_2 = deviations from Arithmetic mean

x_{12} = Combined A.mean.

$d_1 = |\bar{x}_1 - x_{12}|$ $d_2 = |\bar{x}_2 - x_{12}|$

* Corrected SD:-

1) identify correct obs, incorrect obs
2) Calculate new Σx^2 = old obs(old σ^2 + old Am^2) − Inc.value^2 + Correct value^2
3) Correct SD = $\sqrt{\frac{\text{Step 2}}{n} - (\text{correct Am})^2}$ n = new obs.

*Common Properties of Entire dispersion

1) If all observations are constant then dispersion = 0
2) If all observations are increased or decreased by a constant then dispersion is unaffected {[change of origin]}
3) If all observations are multiplied or divided by a constant then Range is also multiplied or divided by same constant {[change of scale]}
4) Dispersion can never be negative

Ex:- $\underbrace{ax \pm by}_{\text{change of scale}} \pm \underbrace{c}_{\cancel{\text{change of origin}}} = 0 \Rightarrow$ linear equation

⇒ The Relation B/w x and y is given as $\boxed{3x+4y-92=0}$ if Range of x is 8 what is Range of y

$$3x+4y-\cancel{92}=0$$
$$3x+4y=0$$
$$3(8)+4y=0$$
$$4y=3\times 8$$
$$R_y=\frac{3\times 8}{4}=6$$

⇒ dont consider constant in linear equations
⇒ final ans can never be negative

*Relation B/w QD, MD, SD

⇒ $6QD = 5MD = 4SD$

Correlation and Regression

Correlation :- mathematical relationship, degree of relation
Expressed in numbers, unit free measures, relative measure

Range of r :- -1 to $+1$ (both inclusive) ($r_{xy} = r_{yx}$)

- -1 → Perfect negative → $x\uparrow$ $y\downarrow$, $x\downarrow$ $y\uparrow$
- 0 → Spurious or bad correlation
- $+1$ → Perfect positive → $x\uparrow$ $y\uparrow$, $x\downarrow$ $y\downarrow$

* if variables are independent to each other then $r=0$
* Simple correlation = r for 2 variables
  multiple correlation = r for more than 2 variables
* linear correlation = x & y change in same ratio
  Curvilinear correlation = x & y change in different ratio.

How to calculate (4 methods)

- Scatter diagram (graphical method)
- Concurrent deviation method. (easiest method)
- Rank Correlation (qualitative data)
- Karl pearson Correlation Coefficient. (Best method)

I) Scatter diagram method :-

* first method.
* gives nature ignores degree of correlation.
* plot x & y in graph & check the nature.

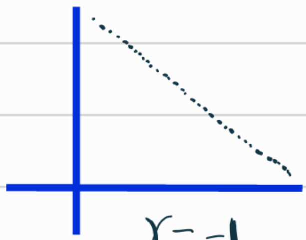
Ex:- $+0.69$, -0.74

degree of correlation (the numbers 0.69, 0.74)

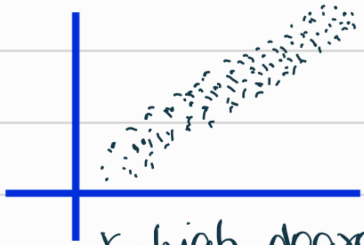
nature of correlation (the signs $+$, $-$)

Ex:-

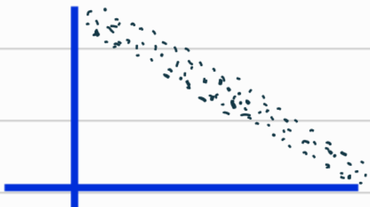
$r = +1$



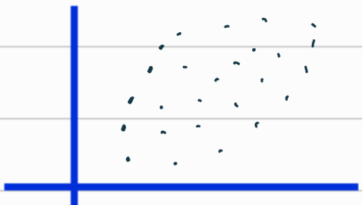
$r = -1$



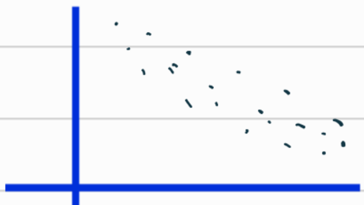
r = high degree of positive correlation



r = high degree of negative correlation

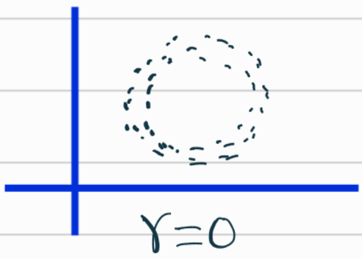


r = low degree of positive correlation



r = low degree of negative correlation

$r = 0$



$r = 0$

left to right.
upwards = positive
downwards = negative

II) Concurrent deviation method

$$r = \pm\sqrt{\pm\frac{2c-m}{m}}$$

c = no of concurrents

m = n − 1 obs

if $2c - m > 0$ outside → +

if $2c - m < 0$ outside → −

III) Spearman Rank Correlation -

without repetition ⇒ $r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

with repetition

$$\Rightarrow r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m_1^3 - m_1) \text{ - - - - }\right]}{n(n^2-1)}$$

$m_1 = 2 \Rightarrow \frac{1}{12}(m_1^3 - m_1) = 0.5$

$m_2 = 3 \Rightarrow$ 〃 $\Rightarrow 2$

$m_2 = 4 \Rightarrow$ 〃 $\Rightarrow 5$

$\boxed{\text{always } \Sigma d = 0}$

IV) $\boxed{\text{Karl pearson / product moment Correlation Coefficient}}$:- Best method.

Cant use for Curvilinear data.

r = ratio of Covariance & Standard deviations.

a) $r = \dfrac{Cov(x,y)}{\sigma_x \cdot \sigma_y}$

Covariance of (x,y)

$=$ Joint variance of x, y

$= \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$

$=$ limits $\Rightarrow -\infty$ to $+\infty$.

b) $r = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \cdot \Sigma(y-\bar{y})^2}}$

c) $\dfrac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{n \cdot \Sigma x^2 - (\Sigma x)^2}\sqrt{n \cdot \Sigma y^2 - (\Sigma y)^2}}$

Ex: $Cov(x,y) = 40$ $\sigma_x = 4$

$\sigma_y = 16$ $r = ??$

$r = \dfrac{Cov(x,y)}{\sigma_x \cdot \sigma_y} = \dfrac{40}{16 \times 4} = 0.625$

notes:-

① $Cov(x,y) \leq \sigma_x \cdot \sigma_y$.

Ex:- $Cov(x,y) = 15$

restriction on product of SD's

SD's must be ≥ 15

② if $y = a + bx$

$b > 0 \rightarrow r = +1$

$b < 0 \rightarrow r = -1$

Ex: $2x + 3y + 4 = 0$

$r = ??$ $\therefore r = -1$

$3y = -4 - 2x$

$y = -\frac{4}{3} - \frac{2}{3}x$

③ Coefficient of determination $= r^2 = \dfrac{\text{\%age of accounted variation}}{\text{Total variation}}$

(%age of dependency)

④ Coefficient of non determination $= 1 - r^2 = \dfrac{\text{\%age of unaccounted variation}}{\text{Total variation}}$

(%age of independency)

⑤ Impact of Scale & origin $\Rightarrow$ No impact

$\Rightarrow$ only impact is of origin.

Ex:- $r_{xy} = 0.58$ $r_{uv} = ??$

$u + 5x = 6$ $-7v + 3y = 20$

$\hookrightarrow$ u & v Symbols are opposite

$\therefore r_{uv} = -r_{xy} = -0.58$

* Compare Symbols of u & v by keeping x & y same symbols

* Same $\Rightarrow r_{uv} = r_{xy}$

* opposite $\Rightarrow r_{uv} = -r_{xy}$

Regression

Avg mathematical relation, Regression analysis using line of best fit.

Types

x on y → Calculate b_{xy}
↓
x is dependent on y
↓
$x = a + by$

Calculate b_{yx} ← y on x
↓
y is dependent on x.
↓
$y = a + bx$.

Ex:-

x	y	Calculation
age	Blood pressure	y on x.
Rainfall	yield.	y on x
Dividend	profit	x on y
Sales	advertisement.	x on y

How to Calculate??

* Mean deviation method. (b_{xy}, b_{yx} = regression coefficients)

x on y → y = independent, x = dependent.

$$\boxed{x - \bar{x} = b_{xy}(y - \bar{y})}$$

b_{xy} = ① $r \cdot \dfrac{\sigma x}{\sigma y}$

② $\dfrac{\mathrm{Cov}(x,y)}{\sigma y^2}$

③ $\dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(y-\bar{y})^2}$

④ $\dfrac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n \cdot \Sigma y^2 - (\Sigma y)^2}$

x = independent, y = dependent ← y on x.

$$\boxed{y - \bar{y} = b_{yx}(x - \bar{x})}$$

b_{yx} = ① $r \cdot \dfrac{\sigma y}{\sigma x}$.

② $\dfrac{\mathrm{Cov}(x,y)}{\sigma x^2}$

③ $\dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$

④ $\dfrac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$

Properties:-

1) $b_{xy} \times b_{yx} = r^2$ [r is the gm of b_{xy} & b_{yx}]

② r, b_{xy}, b_{yx} all should have same symbol.

③ $\boxed{ax+by+c=0}$

y on x.

$b_{yx} = -\dfrac{\text{coeff of } x}{\text{coeff of } y}$

x on y.

$b_{xy} = -\dfrac{\text{coeff of } y}{\text{coeff of } x}$

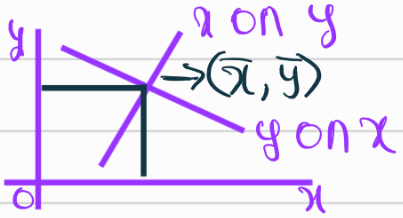
① Take an assumption
② if assumption is wrong reverse it.
③ r lies in B/w -1 to +1.

Ex:- if $2x+5y+9=0$ & $3x+4y+7=0$ identify y on x & x on y.

	$2x+5y+9=0$	$3x+4y+7=0$	$r = \sqrt{b_{xy} \times b_{yx}}$
assumption-1 (This can't be correct as r is not in B/w -1 to +1)	x on y $b_{xy} = -\dfrac{5}{2}$	y on x. $b_{yx} = -\dfrac{3}{4}$	$= \sqrt{-\dfrac{5}{2} \times -\dfrac{3}{4}} = -1.369$ (wrong assumption)
assumption-2 (Since r lies in B/w -1 to +1 this is a valid assumption)	y on x $b_{yx} = -\dfrac{2}{5}$	x on y $b_{xy} = -\dfrac{4}{3}$	$= \sqrt{-\dfrac{2}{5} \times -\dfrac{4}{3}} = -0.722$ (correct assumption)

4) $\dfrac{b_{xy} + b_{yx}}{2} \geq r$ (Am of regression coefficients must be more than r)

5) Two Regression lines always meet at their means



6) angle b/w regression lines depends on correlation
if $r = \pm 1$ angle $= 0^\circ$, they coincide.
$r = 0$ angle $= 90^\circ$, they are perpendicular.

7) Correlation is affected by scale not by origin.
Ex:- $b_{xy} = 0.51$ $b_{yx} = 0.16$ find b_{vu}, b_{uv}
$u = 5 + 3x$ → scale.
$v = 9 + 5y$

$b_{vu} = -\dfrac{\text{scale of } v}{\text{scale of } u} \times b_{yx}$.

$= -\dfrac{5}{3} \times 0.16 = -0.2666$

$b_{uv} = -\dfrac{\text{scale of } u}{\text{scale of } v} \times b_{xy}$.

$= -\dfrac{3}{5} \times 0.51 = -0.306$

---

* data types
univariate = 1 data.
bivariate = two types of data. taken at same point of time
(bivariate is a part of multivariate)

* for $p \times q$ bivariate freq table.
no of marginal distributions $= 2$
no of conditional distributions $= p + q$.
* in a bivariate table of $p+q$ classification, no of cells $= pq$.

* Correlation is calculated for bivariate data.

INDEX NUMBERS – SUMMARY

Definition:- It is a ratio of two or more time periods, one of which is bas time period, the value at the base period serves standard point of comparison

$\boxed{\text{INDEX NUMBERS are Special type of Averages}}$

$$\text{Index number} = \frac{\text{Current year}}{\text{Base year}} \times 100$$

* Index numbers is unit free measure, so it is a relative measure
* Current year – Year prevailing one.
* Base year – year which is used for comparison, It should be a normal year & free from abnormalities, like natural calamities etc. (it should have a relative stability) [Index number of base period = 100]
* To average Index numbers we use $\boxed{\text{Geometric MEAN}}$.

☆☆☆ Difference between Index number & %age:-

$$\text{Index number} \xrightarrow{-100} \text{Percentage} \qquad \text{Percentage} \xrightarrow{+100} \text{Index number}$$

$\boxed{\text{times} \times 100 = \%\text{age}}$

Types of Index numbers

1. Price Index numbers → changes in prices
2. Quantity Index numbers → changes in Quantities
3. Value Index numbers → changes in price and quantity

$\boxed{VIN = PIN \times QIN}$

"0" (n-1) → Represents base period.
"1" (n) → Represent Current period

P_0 = Price in base Year
P_1 = Price in Current Year
Q_0 = Qty in base year.
Q_1 = Qty in Current Year

Price Index numbers:-

- Simple/unweighted Price Index numbers
  - (only price is considered)
- ☆ weighted price index numbers
  - (both price & qty is considered)

Simple Price Index numbers:-

- Simple aggregate Price Index numbers
  - $\frac{\Sigma CYP}{\Sigma BYP} \times 100$ (or) $\frac{\Sigma P_1}{\Sigma P_0} \times 100$
- Simple average of Price Relatives
  - using A.M
    - $\frac{\Sigma PR}{n} \times 100$
  - using G.M
    - $\sqrt{\text{Product of PR}} \times 100$
    - (or) $\text{Antilog}\left[\frac{\Sigma \log PR}{n}\right]$

PR = Price relative = its a ratio of current year price & base year price

$= \frac{P_1}{P_0}$ (or) $\frac{CYP}{BYP}$

weighted price Index numbers

- weighted aggregate price Index no $\left[\frac{\Sigma P_1 w}{\Sigma P_0 w} \times 100\right]$ $[w = q_1 \text{ or } q_0]$
- weighted average of Price Relatives.

1) laspeyer's (L) $= \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$
(Base year-weight)

2) Paasche's (P) $= \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100$
(current year-weight)

3) Dorbish bowley $= \frac{L+P}{2}$

4) fishers (Ideal Index) $= \sqrt{L \times P}$

5) marshall Edgeworth
$= \frac{\Sigma P_1(q_0+q_1)}{\Sigma P_0(q_0+q_1)} \times 100$

6) Walsh method.
$= \frac{\Sigma P_1\sqrt{q_0 q_1}}{\Sigma P_0\sqrt{q_0 q_1}} \times 100$

* using Am $= \frac{\Sigma PR \cdot W}{\Sigma W} \times 100$
$W = P_0 q_0$.

* using gm $= \text{Antilog}\left[\frac{\Sigma \log PR \times W}{\Sigma W}\right]$

$$\boxed{\text{group Index no} = \frac{\Sigma PR \cdot W}{\Sigma W}}$$

$$\boxed{\text{General Index no} = \frac{\sum \text{Index number} \times \text{weight}}{\sum \text{weight}}}$$

Quantity Index numbers :- They compare & measure changes in Quantity.

For price Index numbers → Interchange P & Q.

particulars	price Index numbers	Quantity Index number
1) Simple aggregate	$\frac{\sum P_1}{\sum P_0} \times 100$	$\frac{\sum q_1}{\sum q_0} \times 100$
2) Simple average of relatives		
using AM	$\frac{\sum P_1/P_0}{n} \times 100$	$\frac{\sum q_1/q_0}{n} \times 100$
using GM	$\sqrt[n]{P_1/P_0} \times 100$	$\sqrt[n]{q_1/q_0} \times 100$
3) laspeyre	$\frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100$	$\frac{\sum q_1 P_0}{\sum q_0 P_0} \times 100$
4) paasche	$\frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100$	$\frac{\sum q_1 P_1}{\sum q_0 P_1} \times 100$
5) Dorbish bowley	$(L+P)/2$	$L+P/2$
6) Fishers Ideal Index no.	$\sqrt{L \times P}$	$\sqrt{L \times P}$
7) weighted average of relatives	$\frac{\sum PR \times W}{\sum W}$	$\frac{\sum QR \times W}{\sum W}$

3) Value Index numbers :- $\boxed{\text{Value} = \text{Price} \times \text{Qty}}$ (or) PIN × QIN

Value ratio $= \frac{\sum P_1 q_1}{\sum P_0 q_0}$ (or) $\frac{\sum P_1 q_1}{\sum P_0 q_0} \times 100$

* Purchase power ⇒ Reciprocal of IN
* Cost of living index:

using Aggregate method $= \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100$ (laspeyre method)

family budget method $= \frac{\sum P.R \times W}{\sum W}$ [mostly used method]

Tests of Adequacy :- 4 tests

unit test
↓
given Index number should be unit free.
(Simple aggregate Index no. doesn't satisfy this test)

time Reversal test
(given by fisher)
↓
$\boxed{P_{01} \times P_{10} = 1}$
↓
P_{01} = CY upon BY.
P_{10} = BY upon CY.

factor reversal test
(given by fisher)
↓
$\boxed{P_{01} \times Q_{01} = V_{01}}$
(only fishers satisfy this test)

Circular test (Extension of TRT)
↓
$P_{01} \times P_{12} \times P_{20} = 1$

Index number	UT	TRT	FRT	CT
Simple aggregate Index no	✗	✓	✗	✓
Simple avg of Price relatives - using AM	✓	✗	✗	✗
- using GM	✓	✓	✗	✓
weighted avg of price relatives - using AM	✓	✗	✗	✗
- using GM	✓	✓	✗	✗
laspeyer's method	✓	✗	✗	✗
Paasche's method	✓	✗	✗	✗
Dorbish bowley	✓	✗	✗	✗
fishers method (ideal Index number)	✓	✓	✓	✗
marshall edgeworth	✓	✓	✗	✗
walsch	✓	✓	✗	✗

Summary - Probability

1) Probability/chance/odds

$P(E) = \frac{\text{favourable Cases}}{\text{total Cases}} = \frac{FC}{TC}$

2) earlier branch of mathematics now branch of Statistics

3) two Broad divisons

Subjective P(E) - applied in decison making, used in management.

Objective/classical/Priori P(E) - Regularly used Probability where outcomes are known in advance.

4) Random Experiment = Any activity performed Randomly.

5) outcomes of Random Experiment = events

Simple/elementary → Cannot be decomposed further.

Composite/compound → Can be decomposed into two or more events.

6) mutually Exclusive Events/disjoint events:- if one event happens other event doesn't happen.

$\boxed{P(A \cap B) = 0, \quad P(A \cap B \cap C) = 0}$

7) mutually exhaustive events:- happening of Events lead to Sample Space.

$\boxed{P(A \cup B) = 1, \quad P(A \cup B \cup C) = 1}$

8) equilikely/Equi probable events:-

$P(A) = P(B)$ (or) $P(A) = P(B) = P(C)$

9) Mutually Independent events:- Events are not dependent on each other.

$$\boxed{\begin{aligned} P(A \cap B) &= P(A) \times P(B) \\ P(A \cap B \cap C) &= P(A) \times P(B) \times P(C) \end{aligned}}$$

if A & B are independent.

$\left.\begin{array}{l} A, B^c \\ A^c, B \\ A^c, B^c \end{array}\right\}$ All are independent.

* $P(A \cup B) = 1 - P(A')(B')$ (for Independent only)

10) Favourable cases + unfavourable cases (FC + UFC) = total Cases/Sample space.

$\therefore P(FC) + P(UFC) = P(TC)$

$P(TC) = P(SS) = 1$.

$P(FC) = 1 - P(UFC)$

$P(\text{atleast } 1) = 1 - P(\text{none})$

11) odds in favour = FC : UFC

odds in Against = UFC : FC.

12) Range of P(E) = 0 to 1

0 = impossible event

1 = certain/Sure event.

13) Conditional Probability:- happening of an event is based on condition.

* if condition is impossible then main event becomes impossible.

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ A - main event, B - condition.

$P(A'|B') = 1 - P(A \cup B) / 1 - P(B)$

* If events are conditional and independent $P(A|B) = P(A)$, $P(B|A) = P(B)$

14) Some useful Conclusions:

* $P(A \cup B) = P(A) + P(B)$
  $P(A \cup B \cup C) = P(A) + P(B) + P(C)$ } exclusive events
* $P(A) + P(B) = 1$
  $P(A) + P(B) + P(C) = 1$ } exclusive & exhaustive events
* $P(A^c) = 1 - P(A)$
  $P(A) + P(A^c) = 1$
* A & B write Exam - Independent events.
* A & B participate in race - Exclusive events
* known that / given that - Conditional Probability.
* or, atleast - union
* and, both - intersection
* but not = difference $(A-B)$

15) Some useful Probabilities:-

Coins

Single toss :- $TC = \{H, T\}$

$P(H) = 1/2$, $P(T) = 1/2$.

in a single toss, head & tail are exclusive, exhaustive, independent.

two toss

$TC = \{HH, HT, TH, TT\}$

$P(0H) = P(0T) = 1/4$

$P(1H) = P(1T) = 2/4$

$P(2H) = P(2T) = 1/4$

$$\boxed{\text{Sum of } P(E) = 1}$$

three toss:-

$TC = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

$P(0H) = P(0T) = 1/8$

$P(1H) = P(1T) = 3/8$

$P(2H) = P(2T) = 3/8$

$P(3H) = P(3T) = 1/8$

* when n coins are tossed the probability of getting exactly r heads/tails $= {}^nC_r / 2^n$

dice:-

Single throw

$TC = \{1, 2, 3, 4, 5, 6\}$

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$

two dice throw:-

outcomes	P(E)	outcomes	P(E)
2	1/36	8	5/36
3	2/36	9	4/36
4	3/36	10	3/36
5	4/36	11	2/36
6	5/36	12	1/36
7	6/36		

P(same no on both dice) $= 6/36 = 1/6$

Cards:-

52 Cards + 2 Jokers

4 Suits → 13 heart, 13 diamond, 13 club, 13 spade

heart, diamond: (26) Red cards

club, spade: Black Cards (26)

each Suit has 13 cards = A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K.

A, J, Q, K = Picture cards.

J, Q, K = honor cards, 5 cards - Poker hand.

* P(Red card) = P(black card) = $\frac{26}{52} = \frac{1}{2}$

* P(1 heart) = P(1 diamond) = P(1 club) = P(1 spade) = 13/52 = 1/4.

* P(Jack) = P(queen) = P(king) = P(ace) = 4/52 = 1/13

* take Jokers only if question specifies to take.

* Random variables / Stochastic variables

1) if Random variables (RV) & probabilities are written together we call it Probability distribution.

2) mathematical Expectation / mean / μ

$E(x) = \sum RV \times P(E)$

$= \sum_{i=1}^{n} x_i P_i = x_1 P_1 \times x_2 P_2 \ldots x_n P_n$

3) Variance (σ^2) $= \sum RV^2 \times P(E) - E(x)^2$

$= \sum$ Random variable^2 × Probability − Mathematical Expectation^2

4) Variance notations

$\Rightarrow E(x-\mu)^2 = E(x^2) - \mu^2$.

$= E(x^2) - (E(x))^2$

5) In case of a game

mathematical Expectation < participation amount

(the game is unfair)

6) mathematical Expection of

- Rolling 2 dice = 7
- tossing 2 coins = 1
- tossing 3 coins = 1.5

7) properties of mathematical Expectation

* $E(K) = K$ (K = constant)
* $E(K \cdot x) = K \cdot E(x)$
* $E(x+y) = E(x) + E(y)$
* $E(x-y) = E(x) - E(y)$
* $E(x \times y) = E(x) \times E(y)$ where x & y are independent.

8) if $y = ax + b$ & $Var(x) = K$ then $var(y) = a^2 \cdot Var(x)$

Theoretical Distribution - Summary

Types

- Discrete distributions
  - * Binomial distribution
  - * Poisson distribution
- Continuous distributions
  - normal Distribution

1) Binomial distribution:-

* Probability mass function (PMF) $= {}^nC_x\, p^x q^{n-x}$.

n = no of trials

$\boxed{p+q=1}$

p = Probability of success

q = Probability of failure

x = no of successes

p, q = they are exclusive, exhaustive, independent.

$p \neq$ certain or impossible.

n = finite trials.

* Properties:-

a) mean $= np$, variance $= npq$, SD $= \sqrt{npq}$

b) mean > variance

c) maximum value of variance $= n/4$.

d) $p = 0.5 \rightarrow$ Symmetric

$p < 0.5 \rightarrow$ Positively Skewed.

$p > 0.5 \rightarrow$ negatively skewed.

e) mode depends on $(n+1)p$

- decimal → Unimodal (Remove decimal)
- Integer → Bi modal. $(n+1)p$, $(n+1)p - 1$.

f) Biparametric distribution with parameters n, p

* Every Experiment where the outcomes can be divided into two parts these Binomial distribution can be applied.

Ex:- coins, true or false questions, cards, dice, ODI matches etc.

2) Poisson distribution:-

* if $n \rightarrow \infty$, $p \rightarrow 0$ } then Binomial distribution Becomes Poisson distribution

Such that n, p can be multiplied to get finite value m

* PMF $= e^{-m} \cdot m^x / x!$

$e = 2.718281828$.

$m = n \cdot p$, x = no of successes.

Examples:-

1) no of printing mistakes in large book.

2) no of road accidents in a busy Road.

3) no of Radio active elements per minute in fusion process

4) no of demands per minute in health centre etc.

* Properties:-

a) mean = variance $= m = np$

b) Standard deviation $= \sqrt{np}$

c) coefficient of variance $= 100/\sqrt{m}$

d) mode depends on value of m

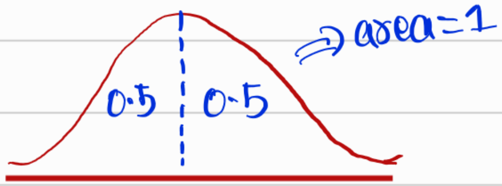
if m is decimal - unimodal.
m is integer - Bimodal $(m, m-1)$
e) it is always positively skewed.
f) uniparametric distribution - m

3) Normal distribution:-

* Continuous Probability distribution with parameters μ & σ^2 (Biparametric distribution)
* Bell shaped curve & mean = median = mode.
* Probability density function (PDF) $= \dfrac{e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}}{\sigma\cdot\sqrt{2\pi}}$

properties:-

a) area under normal curve $= 1$
b) area under either halves $= 0.5$
c) the two tails of curve extend indefinitely & never touch axis



d) points of inflexion $\Rightarrow \mu-\sigma, \mu+\sigma$ (μ = mean)
e) $6QD = 5MD = 4SD$

f) $Q_1 = \mu - 0.675\sigma$; $Q_3 = \mu + 0.675\sigma$
g) it is unimodal distribution as it has only one peak.
h) area coverage.
$\mu-\sigma, \mu+\sigma$ = 68.27% of values.
$\mu-2\sigma, \mu+2\sigma$ = 95.45% of values
$\mu-3\sigma, \mu+3\sigma$ = 99.73% or 100%.
except 0.27% of values.
for any other area coverage we use z table with $z\left|\frac{x-\mu}{\sigma}\right|$ values
i) $z + 0.5 = \phi$, $\phi - 0.5 = z$ values
j) Skewness $= 0$

Additive property:-

Binomial distribution	Poisson distribution	normal distribution
if $x \sim (n_1, P)$ & $y \sim (n_2, P)$ then $x+y \sim (n_1+n_2, P)$	$x \sim (m_1)$ & $y \sim (m_2)$ then $x+y \sim (m_1+m_2)$	$x \sim (\mu_1, \sigma_1^2)$ & $y \sim (\mu_2, \sigma_2^2)$ then $x+y \sim (\mu_1+\mu_2, \sqrt{\sigma_1^2+\sigma_2^2})$