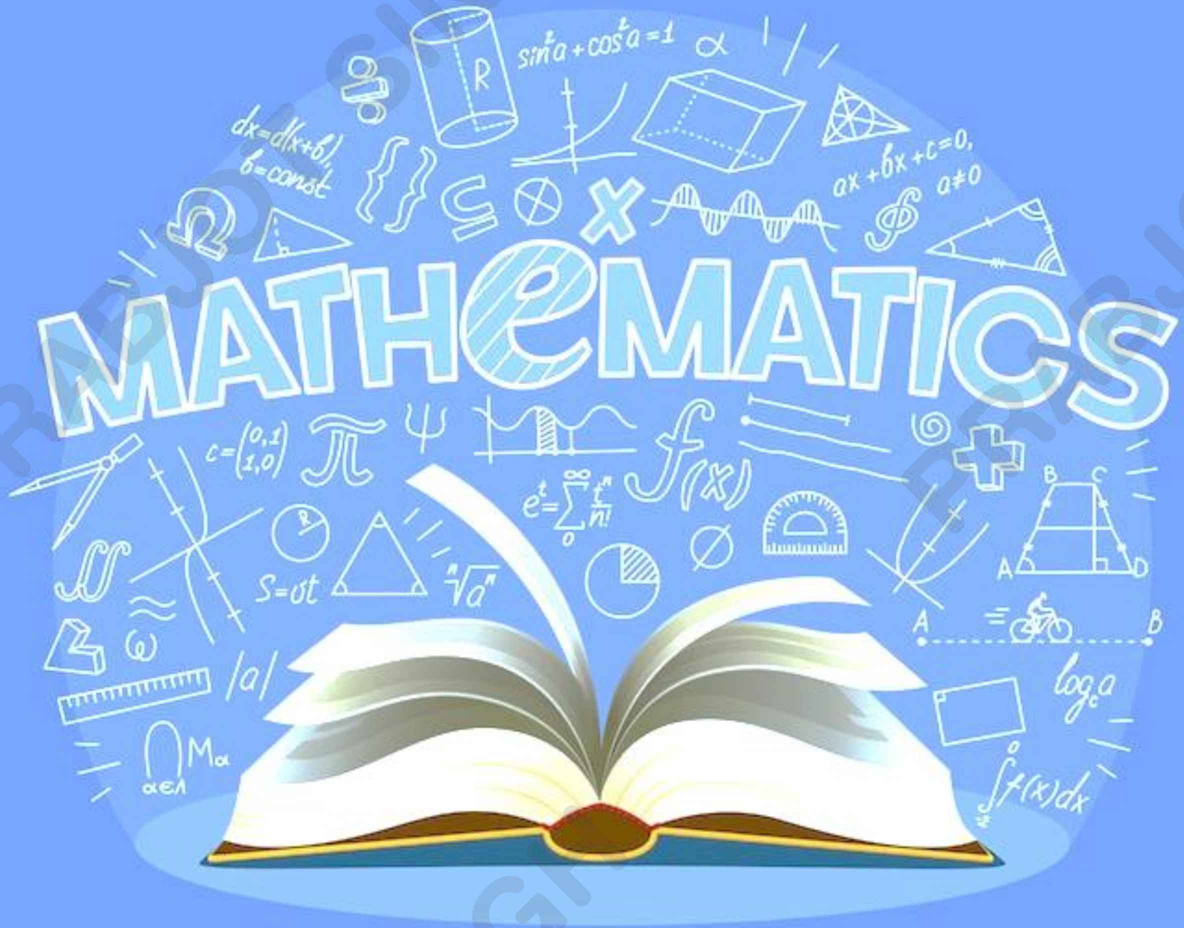
CA - FOUNDATION

MATHEMATICS 101



Concept Guide

[Covers Part A - Business Mathematics in just 24 pages!]

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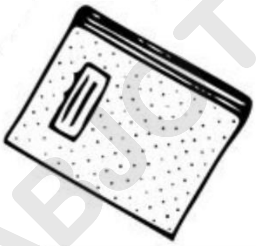
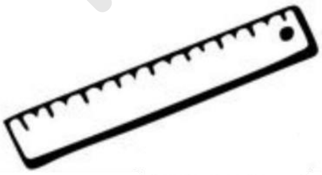
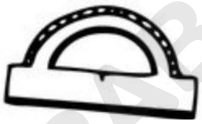






Chapter 1

Ratio & Proportion

Ratio

comparison of ≥ 2 quantities of same kind by division

antecedent → $\frac{A}{B}$ ← consequent

NOTES:

- ○ Order of the terms is important and cannot be changed
- ○ Ratios can be multiplied/divided by a common number but not added/subtracted
- ○ If number is ↑/↓ in ratio x:y, then [New quantity = y/x x old number]

Types of Ratios

Inverse

a : b => b : a

a:b:c => bc:ac:ab

Compound

Different Values	Duplicate	Triplicate	Sub duplicate	Sub Triplicate
$a : b, c : d$ $ac : bd$	$a : b$ $a^2 : b^2$	$a : b$ $a^3 : b^3$	$a : b$ $\sqrt{a} : \sqrt{b}$	$a : b$ $\sqrt[3]{a} : \sqrt[3]{b}$

Continued

a,b,c => a : b : c

Exam tips

✓ If 3,000 is to be allocated in a : b => $a = 3{,}000 \times \frac{a}{(a+b)}$ $b = 3{,}000 \times \frac{b}{(a+b)}$

✓ If x:y = 3:4, value of any equation can be obtained by taking x=3 and y=4 in equation

✓ If ratios are given to arrive at continued ratio:

2 ratios

$A : B_1$

$B_2 : C$

$AB_2 : B_2B_1 : B_1C$

3 ratios

$A : B_1$

$B_2 : C_1$

$C_2 : D$

$AB_2C_2 : C_2B_2B_1 : C_2C_1B_1 : B_1C_1D$

Proportion

Equality of 2 ratios

$$\underbrace{A : \overbrace{B :: C}^{\text{means}} : D}_{\text{extremes}}$$

$$\frac{A}{B} = \frac{C}{D}$$

$$AD = BC$$

NOTES:

- a,b,c,d are called 1st, 2nd, 3rd and 4th proportion respectively
- Continued proportion: a,b,c => a : b :: b : d.

  Note: Continued proportion are always in Geometric progression (GP)
- Mean proportional: In continued proportion, 'b' is mean proportional. i.e., $[b^2 = ac]$

Assume: $\frac{A}{B} = \frac{C}{D}$

Properties of Properties of Proportions

Theory

Invertendo	Alternendo	Componendo	Dividendo	Comp & Divi	Addendo	Subtrahendo
$\frac{B}{A} = \frac{D}{C}$	$\frac{A}{C} = \frac{B}{D}$	$\frac{A+B}{B} = \frac{C+D}{D}$	$\frac{A-B}{B} = \frac{C-D}{D}$	$\frac{A+B}{A-B} = \frac{C+D}{C-D}$	$\frac{A+C}{B+D}$	$\frac{A-C}{B-D}$

Exam tips

✓ If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, value of any equation can be obtained by taking x=3, y=4 & z = 5 in equation

Chapter 1

Indices & logarithms

Indices representation of numbers in terms of powers to small base

a^n — base, power

Laws of indices:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{m\times n}$
- $a^0 = 1$
- $\sqrt[n]{a} = a^{1/n} = a^{-n}$
- If $a^x = a^y$, then $x = y$
- If $a = b$, then $a^b = b^a$
- $\sqrt{a\sqrt{a\sqrt{a\ldots\infty}}} = a$
- $\sqrt{a\sqrt{a\sqrt{a\ldots n \text{ times}}}} = a^{\frac{2^n-1}{2^n}}$
- If $x = a^{1/2} + a^{-1/2}$, then $x^2 - 2 = a + 1/a$
- If $x = a^{1/2} - a^{-1/2}$, then $x^2 + 2 = a + 1/a$
- If $x = a^{1/3} - a^{-1/3}$, then $x^3 + 3x = a - 1/a$
- If $x = a^{1/3} + a^{-1/3}$, then $x^3 - 3x = a + 1/a$

Algebraic equations:

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)(a-b) = a^2 - b^2$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
- $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $a^3 + b^3 + c^3 = (a+b+c)(a^2+b^2+c^2-ab-bc-ac) + 3abc$
- If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Exam tips

- ✓ Questions with complex 'n' powers can be simply solved by substituting 'n' with 1 or 0
- ✓ If powers are given in cyclic order, the resultant answer will always be a^0 i.e., 1
- ✓ If $a^x = b^y = c^z$, any equation containing x, y, z can be solved by equating base to 'k'. i.e., $a = K^{1/x}$, $b = k^{1/y}$, $c = k^{1/z}$ and solving for $a \times b = c$

logarithms

power to which base must be raised to produce the number

$a^n = x$

$\boxed{\log_a x = n}$

NOTES:

- If base to the log is not mentioned, assume the same as 10 [common log]
- In a common log, 'n' can be derived as logarithm table as follow:

$$\log_{10}4594 = \underbrace{3}_{\text{Characteristic}} + \underbrace{0.6623}_{\text{Mantissa}}$$

Characteristic ↓ Power required to base 10 to reach 'n' value

Mantissa ↓ Derived from log table

Digits	Power
4	3
3	2
1	0
1 decimal	$\bar{1}$
2 decimals	$\bar{2}$

Laws of logarithms:

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a m^n = n \log_a m$
- $\log_{a^b} x = 1/b \log_a x$
- $\log_a mn = \log_a m + \log_a n$
- $\log_a m/n = \log_a m - \log_a n$
- $\log_a x = 1 \div \log_x a$
- $\log_a m = \log_b m \div \log_b a$
- $a^{\log_a x} = x$
- If $\log x = \log y$, then $x = y$
- If $\log_a x = n$, then $x = \text{antilog } n$

Exam tips

- ✓ In case of multiple logs, i.e., log(log(log m)), compute value from innermost
- ✓ Value of log a on calculator => $\sqrt{a}$ (13 times) → - 1 → multiply 3558
- ✓ Value of a^n on calculator
  - If n is whole number => Type a → Press x → Press = (n times)
  - If n has decimals => $\sqrt{a}$ (12 times) → - 1 → multiply n → + 1 → Press x = (12 times)
- ✓ Value of $a^{1/n}$ on calculator => $\sqrt{a}$ (12 times) → - 1 → ÷ n → + 1 → Press x = (12 times)

Chapter 2

Equations

Equations mathematical statement of equality. The derivation (solution) of the unknown in the equation is known as roots.

Types of Equations:

- Simple Equation → $ax + b = 0$, where 'x' is unknown
- Linear Equation → $ax + by + c = 0$, where 'x' and 'y' are unknown
- Quadratic Equation → $ax^2 + bx + c = 0$. "x" in this case would have upto 2 roots
- Cubic Equation → $ax^3 + bx^2 + cx + d = 0$, "x" in this case would have upto 3 roots

Solving Linear Equation

Two equations "$a_1 x + b_1 y + c_1 = 0$" & "$a_2 x + b_2 y + c_2 = 0$" can be solved by any of the ways:

Elimination method

Step 1: Multiply Eq 1 with a_2

Step 2: Multiply Eq 2 with a_1

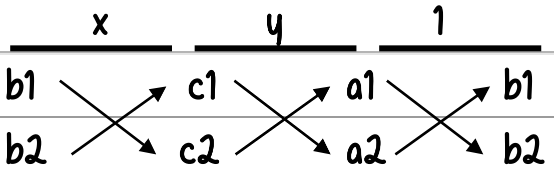
Step 3: Step 1 – Step 2 to arrive at "y"

Step 4: Solve for "x" by substituting "y" in Eq 1 or Eq 2

Substitution method

Ditch everything and simply substitute options in equation to check matches!

Cross multiplication method



i.e., $\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1}$

or, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ & $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Solving Quadratic Equation

The roots 'α' & 'β' of equation "$ax^2 + bx + c = 0$" can be identified by any of the ways:

Factorisation method

The LCM of [a x c] should be considered such that sum of LCM results in b simplifying to the equation $(x - \alpha)(x - \beta) = 0$

Formula method

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

NOTES:

- **Nature of Roots:** In a quadratic equation “$b^2 - 4ac$” is known as Discriminant.

If Discriminant	*Nature of Roots*
$= 0$	Real & Equal
Perfect Square	Real, Unequal & Rational
Not Perfect Square	Real, Unequal & Irrational*
< 0	Imaginary (Does not exist)

*Irrational roots occur in conjugate pairs i.e., if $\alpha = p + \sqrt{q}$, then $\beta = p - \sqrt{q}$

- **Properties of Roots:**

$$[\alpha + \beta] = -b/a \qquad \alpha\beta = c/a$$

- **Constructing Quadratic equation:** Equation of roots ‘α’ & ‘β’ can be formed by:

$$x^2 - [\alpha + \beta]\,x + \alpha\beta = 0$$

Solving Cubic Equation

The roots ‘α’,‘β’ & ‘γ’ of equation “$ax^3 + bx^2 + cx + d = 0$” can be identified by any of the ways:

Trial & Error method	Properties of roots
Identify first root by substituting x with values	$[\alpha + \beta + \gamma] = -b/a$
that satisfy the equation. With $(x - \alpha)$ as a	$\alpha\beta\gamma = -d/a$
factor, solve for the remaining quadratic	Identify the option which matches
equation to obtain β & γ	the properties with equation

Algebraic equations:

- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$
- $\alpha^3 - \beta^3 = (\alpha - \beta)[(\alpha + \beta)^2 - \alpha\beta)$
- $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

Infinite series

In case of infinite series, identify the roots such that $\beta - \alpha = 1$

- $\sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$ = Higher root (β)
- $\sqrt{x - \sqrt{x - \sqrt{x - \ldots\infty}}}$ = Smaller root (α)

Chapter 3

Linear Inequalities

Inequalities statements where 2 quantities are unequal but a relationship exists between them.

Eg: $ax + by + c > 0$ $ab + by + c < 0$ $ax + by + c \geq 0$ $ax + by + c \leq 0$

NOTES:

- In an inequality, the sign is very important and cannot be reversed unless both sides are multiplied by -ve sign.
- Range of a variable are expressed by [lower value, upper value]. If a circle bracket "(" is used, it means the value is not included. However, if a square bracket "[" is used, it means, value is included. For eg, [-2, 3) means variable x falls between $-2 \leq x < 3$
- Range of sin x and cos x is [-1 , 1]

Solving Linear inequalities in two vairables

Step 1: Replace inequality with an equal sign

Step 2: Find co-ordinates of line by taking x = 0 to get y and y = 0 to get x

Step 3: Plot the co-ordinates received to form straight line in graph

Step 4: Shading in graph takes place by following rule:

Scenario	*Shading side*	
X >	Right	Consider the same only after making co-efficients of vairables as + ve.
X <	Left	
Y >	Up	
Y <	Down	

Exam tips

✓ In a scenario based question involving, resources, people, work, time etc., an inherent basic condition is also $x \geq 0, y \geq 0$

Chapter 4

Mathematics of finance

Simple Interest

interest on principal for entire period of borrowing.

No interest is paid on interest

$$SI = \overset{\text{principal}}{P} \times \underset{\text{No. of years}}{N} \times \overset{\text{Interest rate p.a.}}{R}$$

$$\underset{\text{Amount (Final value)}}{A_n} = P + SI \quad \text{OR} \quad \underset{\text{Amount (Final value)}}{A_n} = P[1 + NR]$$

NOTES:

- R is expressed in absolute numbers (decimals). Not in %
- N should always be taken in years. Months can be converted to years by dividing by 12
- Amounts in SI are always in Arithmetical Progression (AP). i.e., $A_1 \underbrace{\,,\,}_{(+)\,SI} A_2 \underbrace{\,,\,}_{(+)\,SI} A_3 \ldots$

Compound Interest

interest on progressing principal for a defined period of borrowing. Interest is also paid on interest

$$CI = P[(1+r)^n - 1] \qquad A_n = P[1+r]^n \qquad \underset{\text{Effective rate of interest}}{E} = (1+r)^n - 1$$

NOTES:

- r and n should be expressed in same conversion period as interest is compounded. Eg: if interest is compounded quarterly, "r" would be taken as Interest rate p.a. / 4 and 1 year would be converted to 4 periods
- Amounts in CI are always in Geometric Progression (GP). i.e., $A_1 \underbrace{\,,\,}_{\times(1+r)} A_2 \underbrace{\,,\,}_{\times(1+r)} A_3 \ldots$

Other Formulas:

- WDV of asset = Original Cost $[1 - \text{Dep\%}]^n$
- $CI_2 - SI_2 = PR^2$
- $CI_3 - SI_3 = PR^2\,[R + 3]$

Annuity series of constant payments/receipts made over constant period of time.

Present value of Annuity received in Perpetuity (indefinitely)

If no growth $\frac{A}{r}$ Annuity If growth $\frac{A}{r-g}$

Present Value & Future Value

Single payment [Invest once]	Annuity Regular [Invest at end of each year]	Annuity Due [Invest at start of each year]
$PV = \frac{FV}{(1+r)^n}$	$PV = A \times \sum_{1}^{n} \frac{1}{(1+r)^n}$	$PV = \left[A \times \sum_{1}^{n-1} \frac{1}{(1+r)^{n-1}} \right] + A$
$FV = PV(1+r)^n$	$FV = \left[A \times \sum_{1}^{n-1} (1+r)^{n-1} \right] + A$	$FV = A \times \sum_{1}^{n} (1+r)^n$

Exam tips

- ✓ Value of $\sum_{1}^{n} (1+r)^n$ on calculator => (r + 1) x 1 → Press = (n times) → Press GT
- ✓ Value of $\sum_{1}^{n} \frac{1}{(1+r)^n}$ on calculator => (r + 1) → Press ÷ → Press = (n times) → Press GT

Applications of Time Value of Money

○ Sinking Fund → Fund created by crediting series of periodic payments in annuity which is compounded annually.

$$\boxed{\text{i.e., Accumulation in Sinking Fund} = \left[A \times \sum_{1}^{n-1} (1+r)^{n-1} \right] + A}$$

○ Leasing → Decision to lease an asset or purchase the asset

	Lessor point of view (Income perspective)	Lessee point of view (Expense perspective)
Lease if:	PV of Lease rent > Purchase price	PV of Lease rent < Purchase price

- <u>Investment decision</u> → Decision to invest in an asset/project etc. based on **Net Present Value** (NPV)

$$\boxed{NPV = PV \text{ of Inflows} - PV \text{ of Outflows}}$$

Scenario	*Decision*
If NPV is +ve	Accept the Project/Asset
If NPV is 0	Indifference point
If NPV is -ve	Reject the Project/Asset

Note: Where 2 projects are given to compare, accept the project with higher NPV

- <u>Valuation of Bond</u> → Price at which bond can be purchased (IP) considering the interest received p.a. (I), Redemption price (RP) and expected rate of return (r)

Int. (value) as per Bond rate

$$\boxed{IP = \frac{I}{\sum_{1}^{n}(1+r)^n} + \frac{RP}{(1+r)^n}}$$

ROI (rate) as per investor expectation

Scenario	*Decision*
If IP < Actual price	Purchase the Bond
If IP = Actual price	Indifference point
If IP > Actual price	Reject the Bond

- <u>Compound Annual Growth Rate (CAGR)</u> → Growth over period of time of a particular element (revenue, units, users etc.)

$$\boxed{CAGR = \left[\left(\frac{\text{Current value}}{\text{Initial Value}}\right)^{1/n} - 1\right] \times 100}$$

Chapter 5

Permutation & Combination

Permutation

ways of selecting things with due due attention to arrangement

items available → ^nP_r ← selection

NOTES:

- **Factorial:** multiplication of all integers from 1 to n. For eg 0! = 1, 3! = 1 x 2 x 3 = 6

Permutation Formulas

- $^nP_r = \frac{n!}{(n-r)!}$
- $^nP_n = n!$
- $^nP_0 = 1$

Combination

ways of selecting things where arrangement is not important

items available → ^nC_r ← selection

NOTES:

- In combination there is no regard to the order of arrangement. These are mere cases cases of selection.

Combination Formulas

- $^nC_r = \frac{n!}{(n-r)!\,r!}$
- $^nC_n = 1$
- $^nC_0 = 1$
- $^nC_r = {}^nC_{(n-r)}$

- **Relationship:** Combination (selection) x Arrangement = Permutation

$$\boxed{^nC_r \times r! = {}^nP_r}$$

SPECIAL SCENARIOS

Arrangement of words

- Simple Arrangement → Assume formed words need not have a meaning.

  Eg: Arrange "RAHUL" => $^5P_5$

- Rearrangement → Question does not require the original word

  Eg: Rearrange "RAHUL" => $^5P_5 - 1$

- Letters come together → Consider letters as one unit and provide Permutation. Separate permutation for letters inside unit unless restricted

Eg: Arrange "RAHUL" such that Vowels come together

=> RHL $\boxed{AU}$ $\underbrace{^4P_4}_{\text{Letter arrangement}} \times \underbrace{^2P_2}_{\text{Vowel arrangement}}$

- Letters don't come together → If particular way is restricted consider
  Total ways (-) restricted ways

Eg: Arrange "RAHUL" such that Vowels don't come together => $^5P_5 - [^4P_4 \times {}^2P_2]$

- Restricted places → Conditional Permutation. Draw places and identify letters allowed in restricted place. Remaining places = (No. of letters - 1) in reducing sequence.

Total ways = Multiply all selection

Eg: Arrange "RAHUL" such that R does not appear in the last place

=> _ _ _ _ _
4 x 3 x 2 x 1 x 4
(A,H,U,L)

Note: Permutation only works when letters to be placed are ≤ the spaces available. Where spaces available are more than letters, switch to the reverse condition. Eg: If vowels in word "DAUGHTER" need to occupy odd places, Permutation cannot be done since there are 3 vowels and 4 odd spaces. Instead, reverse condition for 5 constants to occupy 4 even spaces and 1 odd space.

Arrangement of Digits

- Simple Arrangement → Assume Digits are not repeating

Eg: Form 4 Digit number using 1 to 5 => $^5P_4$

- Single Place restriction → Conditional Permutation. Draw places and identify Digits allowed in restricted place. Remaining places = (No. of Digits - 1) in reducing sequence.

Eg: Form 4 Digit number > 5,000 using 3 to 7 => _ _ _ _
3 x 4 x 3 x 2
(5,6,7)

- Multiple Place restrictions → Split restrictions. Fix primary restrictions and apply conditional permutation. Total ways = Sum of all ways

Eg: Form 4 Digit number > 2,300 using 1 to 5

=> Between 2000 - 2999 (+) => Above 2999

$2\ \_\ \_\ \_$ $\qquad$ $\_\ \_\ \_\ \_$

$1 \times \underbrace{3}_{(3,4,5)} \times 3 \times 2$ $\qquad$ $\underbrace{3}_{(3,4,5)} \times 4 \times 3 \times 2$

Note: If 0 is given as a Digit, it would be considered as a restriction for the first place as that would reduce the Digit formation. Consider Conditional Permutation accordingly

Eg: 4 Digit number using 0, 1, 2, 3, 4, 5 => $\_\ \_\ \_\ \_$

$\underbrace{4}_{(1,2,3,4,5)} \times 4 \times 3 \times 2$

Sum of Numbers Formed with Digits

- **No repetitions** → Shortcut is $[\text{Sum of Digits}](\underbrace{n}_{\text{No. of digits}} - 1)! \times \underbrace{1111}_{\text{No. of 1 = No. of digits}}$

Eg: Sum of all 4-digit number with 1, 3, 5, 7 => $[1+3+5+7] \times 3! \times 1111$

- **Digits repeating** → Divide Sum with (No. of Digits repeating)!

Eg: Sum of all 4-digit number with 1, 3, 3, 5 => $\frac{[1+3+3+5] \times 3! \times 1111}{2!}$

- **Digits contain 0** → Since 0 cannot appear in the first place, Compute Sum of all Digits (-) Sum of Digits if 0 occurs in first place

Eg: Sum of all 4-digit numbers with 0, 1, 3, 5

=> $[0+1+3+5] \times 3! \times 1111$ (-) $[1+3+5] \times 2! \times 111$

Factorisation

- **Total Factors** → To find the total number of factors of a number, identify the prime factors in the number and report them in indices i.e., $(p)^a (q)^b (r)^c$

Total Ways = $(a + 1)(b + 1)(c + 1)$

- **Different Factors** → Since the number is a factor of itself, too,

Different Ways = $[(a + 1)(b + 1)(c + 1)] - 1$

Repetition

- Undefined repetitions If same object can be repeated any number of times in the placements, Ways = n^r

  Eg: Form 4 digit number using any number between 1 to 9. Same number can be used again

  => $\underset{9 \times 9 \times 9 \times 9}{\_\ \_\ \_\ \_}$ (or) 9^4

- Identified repetitions If certain objects are repeated in a given placement, impact of repetitions would be removed by dividing the No. of ways by (No. of repetitions used)!

  Eg: Arrange "AGARWAL" using all letters => $\dfrac{^7P_7}{3!}$ ← Repetitions of A

Note: The above only works since all objects given are used in the placement. Where placements given are less than a objects available, given objects will need to be grouped and identified for various possibilities where repetitions may occur/not occur

Eg: Form 4 letter word using "AGARWAL" => 4 letter word can be formed by using the following unique letters: $\boxed{AAA}$ G R W L

Possibilities: No repetitions + 2 letters repeating + 3 letters repeating

$$^5C_4 \times 4! + \frac{^3C_2 \times {^4C_2} \times 4!}{2!} + \frac{^3C_3 \times {^4C_1} \times 4!}{3!}$$

Arrangement (4!) ; 2! ← Repetitions of A → 3!

Occurrence

- Item always occurs → Use Combination to fill other available places.

  Arrangement of places = (Total places)!

  Eg: Arrange 10 items in 4 places such that 1 item always occurs => $^9C_3 \times 4!$

- Item never occurs → Ignore the item as if it was never part of the list. Can use Permutation directly.

  Eg: Arrange 10 items in 4 places such that 1 item never occurs => $^9P_4$

Circular Permutations

- **Arrangement of persons** → If directions of seating is also relevant, Ways = (n-1)!

  If directions of seating is not relevant (should not sit with same neighbour twice),

$$\text{Ways} = \frac{1}{2} \times (n-1)!$$

- **Arrangement of beads in necklace** → Directions does not matter. Ways = $\frac{1}{2} \times (n-1)!$

Seating arrangement

- **Alternate seating** → Place the persons with higher quantity first. The remaining spots if fixed will be used by the persons with lower quantity.

  Eg: Place 6 boys and 5 girls in row such that no 2 girls or 2 boys sit together

$$\Rightarrow \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}$$

$$\underbrace{{}^6P_6}_{\text{Boys arrangement}} \times \underbrace{{}^5P_5}_{\text{Girls arrangement}}$$

  Note: If spaces for lower quantity is not fixed, consider spaces before and after higher quantity as options available as well.

  Eg: Place 5 girls and 6 boys such that no 2 girls sit together

  => No restriction on placement of boys

$$\underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G} \quad \Rightarrow \underbrace{{}^6P_6}_{\text{Boys arrangement}} \times \underbrace{{}^7P_5}_{\text{Girls arrangement}}$$

Distribution

- **Identical items** → Placement of identical items do not hold relevance as there is no differentiation. Hence, No. of Ways will be divided by (Identical items)!

  Eg: Provide 5 oranges & 3 mangoes to 8 students => $\frac{8!}{5!\,3!}$

- **Grouping** → Distributing items in group will follow same principle as above with the exception that if there are identical groups, then ways would be divided by (No. of Identical group)!

Eg: Split 12 students into 3 groups => $\frac{12!}{4!\,4!\,4!\,3!}$

Open Selection

Different things	Alike things
Zero or more selections → If zero or more different things can be selected, Ways = 2^n	**Zero or more selections** → If zero or more alike things can be selected, Ways = $(n+1)$
One or more selections → If atleast one different things needs to be selected, Ways = $2^n - 1$	**One or more selections** → If atleast one alike things needs to be selected, Ways = n
Multiple different things → Multiplication of all ways i.e., Zero or more = $2^{n_1} \times 2^{n_2}$ Atleast one = $(2^{n_1} - 1)(2^{n_2} - 1)$	**Multiple alike things** → Multiplication of all alike ways i.e., Zero or more = $(n_1 + 1)(n_2 + 1)$ Atleast one = $[(n_1 + 1)(n_2 + 1) - 1]$

Note: 2 is considered here on the assumption that there is only 2 choices: Select or leave. If there are more choices, increase 2 by such additional choices.

Geometry

- **Straight Line** → A line can be created by joining any 2 points. Ways = $^{\text{Total Points}}C_2$

- **Triangle** → A triangle can be formed by joining any 3 points. Ways = $^{\text{Total Points}}C_3$

  Note: A triangle cannot be formed if all 3 points are collinear (in straight line). In such a case, remove the ways in which such collinear points can be selected for triangle.

  Eg: Form triangle using 15 points where 8 of them are collinear => $^{15}C_3 - {}^{8}C_3$

- **Diagonals** → While selecting points in a shape to form a line, the sides of the shape also get selected. Hence, Ways = $^{\text{Total Points}}C_2$ (-) no. of sides in the shape

- **Parallelogram** → To form a llgm, one needs 2 lines on x axis and 2 lines on y axis

  Ways = $^{\text{lines on x axis}}C_2 \times {}^{\text{lines on y axis}}C_2$

Sequence & Series

Ordered Collection of numbers formed by definitive rule is called as Sequence

Eg: 2 , 4 , 6 , 8 .... n

Sum of elements of a sequence to n terms is called Series

Eg: 2 + 4 + 6 + 8 +......... + n

Arithmetic Progression

Sequence in which next term is obtained by adding a common difference "d"

Eg: 1, 2, 3, 4, 5, ...

To find n^{th} term in AP

$$a_n = a + (n-1)\,d$$

First term common difference

To find Sum of n terms in AP

$$S_n = n\,(a + l)\,/\,2 \quad \text{OR} \quad S_n = \frac{n}{2}\left[2a + (n-1)\,d\right]$$

n^{th} term

Sum of series (formulas):

- 1^{st} n natural numbers $(1 + 2 + 3 + 4 + \ldots\ldots + n) = n(n + 1)/2$
- 1^{st} n odd number $(1 + 3 + 5 + 7 + \ldots\ldots + (2n - 1) = n^2$
- Squares of 1^{st} n natural number $(1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + n^2) = n(n + 1)(2n + 1)/6$
- Cubes of 1^{st} n natural number $(1^3 + 2^3 + 3^3 + 4^3 + \ldots\ldots + n^3) = [n(n + 1)/2]^2$

Geometric Progression

Sequence in which next term is obtained by multiplying by a constant multiplier "r"

Eg: 2, 4, 6, 8, 10, ...

To find constant multiplier

$$r = a_n / a_{(n-1)}$$

To find Sum of n terms in GP

If r<1 $$S_n = \frac{a\,(1 - r^n)}{(1 - r)}$$ OR If r>1 $$S_n = \frac{a\,(r^n - 1)}{(r - 1)}$$

To find n^{th} term in GP

$$a_n = a\,r^{(n-1)}$$

First term constant multiplier

To find Sum of ∞ terms in GP

$$S_\infty = \frac{a}{(1 - r)}$$

Arithmetic Mean	Geometric Mean
Value in between 2 terms of an AP.	Value in between 2 terms of an GP.
i.e., ‘b’ is the AM between ‘a’ and ‘c’ in an AP of a, b, c where	i.e., ‘b’ is the GM between ‘a’ and ‘c’ in a GP of a, b, c where
$b = \frac{a+c}{2}$	$b^2 = ac$

Solving Sequence & Series

Sequence is given. Find		Find sequence by using	
n^{th} formula	Sum formula	AP/GP formula	Sum formula
Substitute n = 1, 2, 3 in options. Find option matching sequence	Find S_1, S_2, S_3 from sequence. Substitute n = 1, 2, 3 in options. Find option matching with the sums	Substitute n = 1, 2, 3 in formula and arrive at sequence.	Substitute n = 1, 2, 3 in formula and arrive at S_1, S_2, S_3 $a_1 = S_1$, $a_2 = S_2 - S_1$ $a_3 = S_3 - S_2$

Note: If AP/GP formula is asked and sum formula is requested or vice versa, arrive at sequence using first formula and substitute in second formula to verify.

Chapter 7

Sets, Relations & Functions

Sets collection of well-defined distinct objects

Roster form	Set builder form
A = {a, e, i, o, u}	A = set of vowels

Properties of sets

- Null set: Empty set containing no elements. Represented by { } or ϕ
- Singleton set: Set containing only 1 element. Eg: { 1 }
- Equal set: Set containing only 1 element. Eg: If A = set of natural numbers, B = set of +ve integers, then A = B
- Subsets: The number of sub-sets of a set is 2^n. If A = { 1 , 2 , 3 }, subset = $2^3 = 8$

  ϕ, { 1 }, { 2 }, { 3 }, { 1 , 2 }, { 1 , 3 }, { 2, 3 }, { 1, 2, 3 }

  Subset are denoted by ⊂ or ⊆. i.e., { 1 , 2 } ⊂ { 1 , 2 , 3 }
- Proper Subsets: Sub-sets does not include the main set. => $2^n - 1$, i.e., $2^3 - 1 = 7$

  ϕ, { 1 }, { 2 }, { 3 }, { 1 , 2 }, { 1 , 3 }, { 2, 3 }
- Power Set: Collection of all subset

  P(A) = { ϕ, { 1 }, { 2 }, { 3 }, { 1 , 2 }, { 1 , 3 }, { 2, 3 }, { 1, 2, 3 } }

Relation between sets

For the given sets, A = {1, 2, 3}, B = {1, 3, 5, 7, 9} and S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

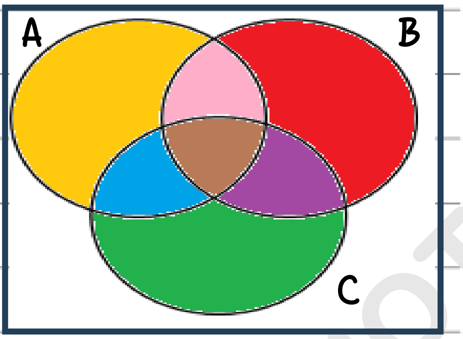
- S => S is the universal set since it contains all elements of A & B
- A ∩ B => A intersection B contains all common elements in both sets i.e., {1, 3}
- A ∪ B => A union B contains all unique elements in both sets i.e., {1, 2, 3, 5, 7, 9}
- A - B => Contains all unique elements in A which are not present in B i.e., {2}
- B - A => Contains all unique elements in B which are not present in A i.e., {5, 7, 9}
- A′ => A complement refers to all elements of S which is not present in A i.e., {4, 5, 6, 7, 8, 9, 10}

Rules of Set

- A ∩ S = A
- A ∪ S = S
- (A ∪ B)′ = A′ ∩ B′
- (A ∩ B)′ = A′ ∪ B′

- A x B => **Cartesian Product** of A & B refers to all pairs of (a, b) where "a" belong to Set A and "b" belongs to set B i.e., { (1,1), (1,3), (1,5), (1,7), (1,9), (2,1), (2,3), (2,5), (2,7), (2,9), (3,1), (3,3), (3,5), (3,7), (3,9) }

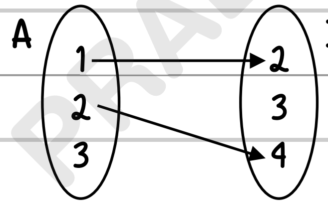
Euler - Venn Diagram



- $n(A \cup B) \Rightarrow n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) \Rightarrow n(A) + n(B) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- n(Only B & C) => $n(B \cap C) - n(A \cap B \cap C)$
- n(Only C) => $n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Relations

instance where **atleast 1 element** of Set A has a mapping to Set B



Types of Relations

Reflexive	Symmetric	Transitive	Equivalence
If Element a is related to itself Eg: a is equal to a	If a is related to b, then b is related to a Eg: If a knows b, then b knows a	If a is related to b & b is related to c, then a is related to c Eg: If a ∥ b & b ∥ c then, a ∥ c	If a relation is reflexive, symmetric & transitive Eg: x = y

functions

a **relation** where **all elements** of Set A are mapped to **any 1 element** of Set B.

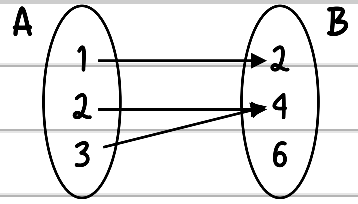
domain co-domain

F : A → B

NOTES:

- It is not necessary for elements in Set A to have a unique element in Set B.

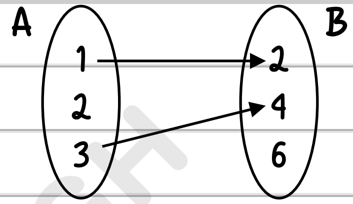
Case A



Function

Set A has 1 image each in B

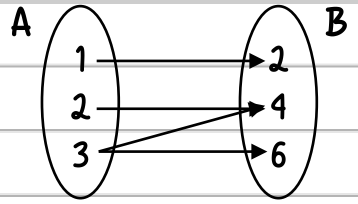
Case B



NOT a function.

2 is not mapped

Case C



NOT a function.

3 is mapped twice

- Domain: All Elements of Set A
- Co-Domain: All Elements of Set B (whether mapped or not)
- Range: Elements of Set B which are mapped to Set A

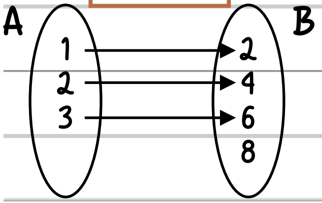
Eg: In Case A, Domain = {1, 2, 3}, Co-domain = {2, 4, 6}, Range = {2, 4}

Note: If the elements are not finite, express them using circle or square bracket [Refer Linear Inequality Page 7]

- In graphical terms, a relation becomes a function if a parallel line drawn to y axis intersects with the equation at only one point.

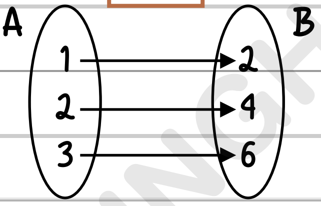
Types of functions

One-One



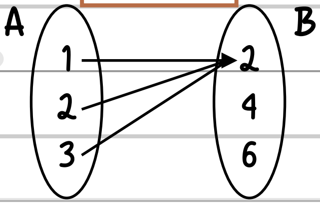
Elements in Set B is mapped only once

Onto



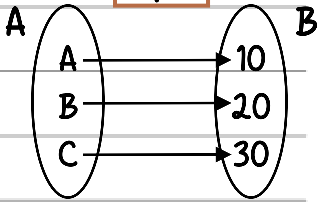
Co-domain = Range

Constant



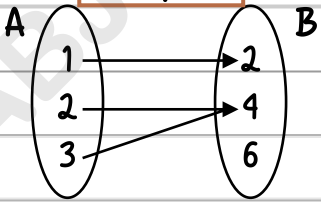
Domain is mapped to one element in Set B

Equal



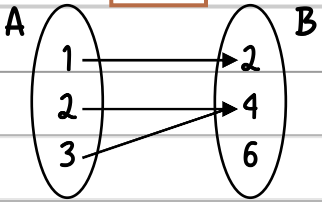
Mapping in Set A & Set B are unique

Many-one



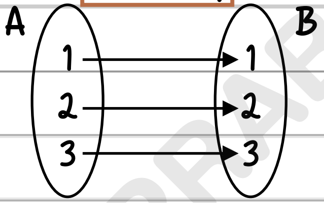
Elements in Set B mapped more than once

Into



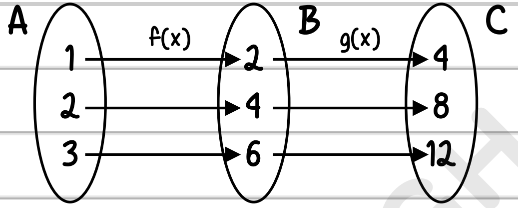
Co-domain ≠ Range

Identity



Domain = Range

Composite function Where 2 functions are involved such that $f(A) = B$ and $g(B) = C$, then it can be said that $g[f(A)] = C$. This can be written as $gof(A)$



Here, $f(1) = 2, g(2) = 4$

Therefore, $gof(1)$ or $g[f(1)] = 4$

and $fog(4)$ or $f[g(4)] = 1$

Inverse function If f is a one-one and Onto function, such that $f(x) = y$, then inverse function $f^{-1}(y) = x$. For eg: if $f(1) = 2$, then $f^{-1}(2) = 1$.

<u>Steps to find inverse function of f(x)</u> [say $f(x) = 2x$]

Step 1: Write the given equation in terms of y i.e., $y = 2x$

Step 2: Derive the equation of x in terms of y i.e., $x = y/2$

Step 3: Replace x with y and y with x in all places i.e., $y = x/2$

$$\Rightarrow f^{-1} = x/2$$

chapter 7

Limits & Continuity

limit value that a function approaches the output for the given input values

value x tends to $\lim_{x \to a} f(x)$ function

Steps to solving limits

Step 1: Determine whether the limit exists.

A limit for a function only exists when Left hand limit = Right hand limit

$$\boxed{\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)}$$

Step 2: Substitute "a" in f(x) to check if the value is determinate or undefined

Undefined	Defined
$\frac{a}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, 1^∞, $\sqrt{-a}$, 0^∞, 0^0, ∞^0	$\frac{0}{a} = 0$, $\frac{a}{\infty} = 0$, $\infty + \infty = \infty$

Step 3: If f(x) is determinate, compute value of limit substituting a in f(x)

Step 4: If value is undefined, using any of the following methods, to make f(x) determinate.

Factorisation method	Differentiate method	Rationalisation method
(applicable in case of fraction with quadratic equations)	*(applicable in case value of f(x) is 0/0 or ∞/∞)*	*(applicable if f(x) has irrational equations)*
Factorise equations to eliminate common factor in Nr and Dr to remove the undefined form	Differentiate Nr & Dr until f(x) becomes determinate Note: $\frac{dy}{dx} x^n = n(x)^{n-1}$, $\frac{dy}{dx} a = 0$	Multiple Nr and Dr with conjugate pairs. Eliminate undefined form

Calculator trick

(applicable only in case of above four and not when standard functions are used)

If x → a, substitute the x in the equation with marginally high number, eg: 0 becomes 0.01

If x → ∞, substitute the x in the equation with 100

The nearest option resembling with value arrived in the calculator is the answer

Standard functions of limits:

- $\lim_{x \to 0} \frac{e^{f(x)} - 1}{f(x)} = 1$
- $\lim_{x \to 0} \frac{a^{f(x)} - 1}{f(x)} = \log a$
- $\lim_{x \to 0} \frac{\log[1+f(x)]}{f(x)} = 1$
- $\lim_{x \to a} \frac{f(x)^n - a^n}{f(x) - a} = n(a)^{n-1}$
- $\lim_{x \to \infty} f(x)^x = e^{x[f(x)-1]}$

continuity

a function which at a given interval has no break in the graph of the function in the entire interval range

A function is said to be continuous only if the following condition is satisfied:

$$\boxed{\lim_{x \to a^-} f(x) = \lim_{x \to a} f(x) = \lim_{x \to a^+} f(x)} \quad \text{OR} \quad \boxed{f(a^-) = f(a) = f(a^+)}$$

Modulus function

Modulus function (or absolute value function) means reporting only +ve value of the function as the output. For eg, |2| = 2 and |-2| is also 2

In case of a modulus function i.e., |f(x)|, the limit of the functions can be arrived as follows:

$$f(x) = \begin{cases} f(x) & \text{for } f(x) \geq 0 \\ -f(x) & \text{for } f(x) < 0 \end{cases}$$