MATHS MARATHON

CONCEPT SECTION

JK SHAH CLASSES

Day 1 video

Day 2 video

Link to documents

Remember

Easy Chapter makes you win!

Chapter sequence

1) Time Value of Money (SI-CI) ***
2) Ratio and Proportion*
3) Laws of indices *
4) Logarithm *
5) Equations***
6) Permutation Combination**
7)Sequence and Series**
8) Time Value of Money (Annuity, CAGR, Bond, inflation, Growth rate, discount rate, Effective and nominal rate of interest …)***
9)Set Relations and Functions
10) Linear inequality
11)Derivatives and Integration

Let's start

18 Chapters

100 Questions

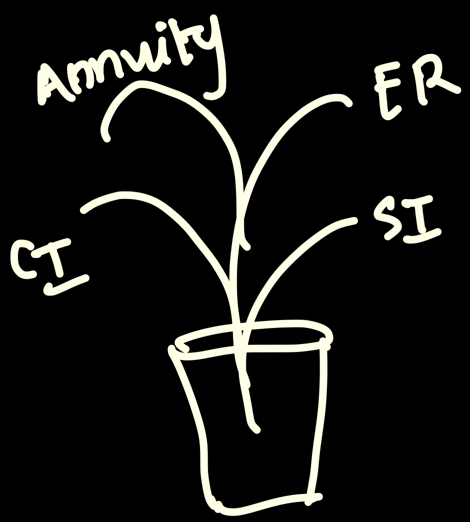
120 minutes

1 min 12 sec per question

OMR marking using pencil

make sure battery is ok (No display fading)

Time Value of Money (SI - CI)



SI → AP

CI → GP

SI Simple interest

P = 100Cr   R = 10%   N = 3

N	Int
1	10
2	10
3	10

$SI_1 = CI_1$

$\frac{R}{100} \times P$

$A = 100 + 30 = 130$

$$SI = 3 \times \frac{10}{100} \times 100$$

$$SI = \frac{P \cdot N \cdot R}{100}$$

$$N_{years} = \frac{N_{months}}{12} = \frac{N_{day}}{365}$$

$SA = A = P + SI$

$= P + \frac{PNR}{100}$

$$\boxed{A = P\left(1 + \frac{NR}{100}\right)}$$

SI:- Money becomes K times

$P = x \qquad A = Kx$

$$A = P\left(1 + \frac{NR}{100}\right)$$

$$K\cancel{x} = \cancel{x}\left(1 + \frac{NR}{100}\right)$$

$$(K - 1) = \frac{NR}{100}$$

$$\boxed{\therefore \quad NR = 100\,(K - 1)}$$

$K = 2$ Doubles

$= 3$ Triples ...

$$\begin{tikzcd}
x \arrow[r, "5y", "+x"'] & 2x \arrow[d, "+x"', "5\text{ years}"] \\
4x \arrow[r, dash, "5y"'] & 3x
\end{tikzcd}$$

Total 15 y for money to quadraple

Compound Interest

$P = 100 \qquad N = 3 \qquad R = 10\%$

N	Int	P = 100
1	10	P = 110
2	11	P = 121
3	12.1	P = 133.1

$$\begin{array}{r} 133.1 \\ -\ 100 \\ \hline CI \quad 33.1 \end{array}$$

$A_1 = 100 + 10$

$= P + \frac{R}{100} \cdot P$

$A_1 = P\left(1 + \frac{R}{100}\right)$ ✓

$A_2 = 110 + 11$

$= A_1 + \frac{R}{100} A_1$

$= A_1\left(1 + \frac{R}{100}\right)$

$A_2 = P\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)$

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$CI = A - P$$

$$= P\left(1 + \frac{R}{100}\right)^n - P$$

$$= P\left[\left(1 + \frac{R}{100}\right)^n - 1\right]$$

$$R \longrightarrow \frac{R}{2} \qquad 1 \longrightarrow 2$$

$$n \longrightarrow 2n$$

$$R \longrightarrow \frac{R}{m} \qquad n \longrightarrow nm$$

$$\left.\begin{aligned} A &= P\left(1+\frac{R}{m100}\right)^{nm} \\ CI &= P\left[\left(1+\frac{R}{m100}\right)^{nm}-1\right] \end{aligned}\right\}$$

m = no of conversions

m = 2 Half yearly

= 4 Quarterly

= 12 Monthly

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\cdots$$

$$A = P\left(1+\frac{R_1}{m_1 100}\right)^{m_1 n_1}\left(1+\frac{R_2}{m_2 100}\right)^{m_2 n_2}$$

Papa Method

$P = 100 \quad N = 3 \quad R = 10\%$

$A = 100 + 10\% + 10\% + 10\%$

$\underline{\underline{133.1}}$

Scrap value = Original value − R% − R% − R%...

$$SV = OV\left(1 - \frac{R}{100}\right)^n$$

$$\text{New Popu} = \text{Old Popu}\left(1 + \frac{R}{100}\right)^n$$

Money becomes <u>K times</u>

Double	Triple
Rule 72	Rule 114
NR $\doteq$ 72	NR $\doteq$ 114
✓ ?	
? ✓	

When option are not too close

Population double @ 5% pa

Find no of years. 14.4

a) 14 y 2 m   b) 14 y 3 m

$P = x \qquad A = 2x$

$$2x = x\left(1 + \frac{r}{100}\right)^n$$

$$2 = \left(1 + \frac{5}{100}\right)^n$$

$$1.05^{n} = 2$$

Power = Stepno − 1

$1.05 \times$ =

=

=

= 1.978 Step 15

= 2

= 2.078 Step 16

$$1.98 \xrightarrow{0.02} 2 \xrightarrow{0.08} 2.08$$

2 : 8

Step 15 —— Step 16

Step no = 15.2

Power = 15.2 − 1

= 14.2

14 y 0.2 of year

$\frac{0.25}{3}$

14 y 3 months

= 0.8 × 2

= 1.6

Popn doubles @ 2%

$$2x = x\left(1 + \frac{2}{100}\right)^n$$

$$1.02^n = 2$$

$1.02 \times \vdots$ 1.99 Step 36

Power $= 36 - 1 = \underline{\underline{35}}$

$CI_1 - SI_1 = 0$

$CI_2 - SI_2 = Pi^2$

$i = \frac{R}{100}$

$CI_3 - SI_3 = Pi^2(i+3)$

$CI_4 - SI_4$

Seperately

find Seperately.

Ratio :- Compare 2 Quantities
(same type/unit)

Qty → Fully Quantifiable
(rational)

Qty → commensurable
Rational no's

① $\frac{a}{b} = \frac{2}{3}$

$a = 2$
$b = 3$
✗

$a = 2x$
$b = 3x$
✓

② 500000 ÷ in ratio 3 : 7 amongst A & B

A's share = $\frac{3}{10} \times 500000 = 150000$

B's share = $\frac{7}{10} \times 5L$ or

$$\begin{array}{r} 500000 \\ -150000 \\ \hline 350000 \end{array}$$

(3) 260060 Re is ÷ amongst A, B, C such that

$$\frac{A}{B} = \frac{2}{3} \qquad \frac{B}{C} = \frac{4}{5}$$

$$\begin{array}{ccccc} A & : & B & : & C \\ 2 & & 3 & & \\ & & 4 & & 5 \\ \hline 8 & : & 12 & : & 15 \end{array}$$

$$A\text{'s share} = \frac{8}{35} \times 260000$$

$$B\text{'s share} = \frac{12}{35} \times 260000$$

$$C\text{'s share} = \frac{15}{35} \times 260000$$

Compound ratio a/b , c/d , e/f

$$CR = \frac{a\,c\,e}{b\,d\,f}$$

$a/b \longrightarrow$ Duplicate a^2/b^2

Triplicate a^3/b^3

Sub duplicate $\sqrt{a}/\sqrt{b}$

Sub Triplicate $\sqrt[3]{a}/\sqrt[3]{b}$

Inverse ratio ..

$a : b \longrightarrow b : a$

$a : b : c \longrightarrow bc : ac : ab$

$\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$

$a : b : c : d \longrightarrow bcd : acd : abd : abc$

Proportion : Equality of Ratio

$$\frac{a}{b} = \frac{c}{d} \qquad a : b :: c : d$$

3rd proportional

extremes mean

4th prop

Continued proportion

$$\frac{a}{b} = \frac{b}{c} \Rightarrow a : b : c$$

mean proportion?

3rd prop ?

$$b^2 = ac$$

$$\boxed{b = \sqrt{ac}}$$

mean prop x & y

$$\sqrt{xy}$$

$a/b = c/d$

$$\frac{a+b}{b} = \frac{c+d}{d}$$ Componendo

$$\frac{a-b}{b} = \frac{c-d}{d}$$ Dividendo.

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$ Componendo Dividendo

$$\frac{a}{c} = \frac{b}{d}$$ alternendo

$$\frac{b}{a} = \frac{d}{c}$$ invertendo

$$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$$ Addendo

$$= \frac{a-c}{b-d}$$ Subtrahendo

$$
\begin{tikzcd}
x \arrow[dr] & & y \arrow[dl] \\
& \text{Average} \arrow[dl] \arrow[dr] & \\
|y - \text{Avg}| & & |x - \text{Avg}|
\end{tikzcd}
$$

Laws of Indices

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$m \rightarrow$ power

$n \rightarrow$ root

$a \downarrow$ base

$$\underbrace{a^m \cdot a^n}_{\text{same base}} = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$$

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

$$(a \cdot b)^c = a^c b^c$$

$$6^2 = 3^2 \cdot 2^2$$

$$(a+b)^c \neq a^c + b^c$$

expansion formula

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$= a^3 + 3ab(a+b) + b^3$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$a^3 = 8$ root

power

$a = 8^{1/3}$

$a^{1/3} = 2$ power

root

$a = 2^3$

$$\frac{1}{\sqrt{a}-\sqrt{b}}$$

Multiply Num & Deno by its conjugate

$$\frac{\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}$$

$$= \frac{\sqrt{a}+\sqrt{b}}{a-b}$$ //

$$x^{3} = 2^{3}$$

$$\therefore \boxed{x = 2}$$

$$x^{3} = x^{K}$$

$$\boxed{K = 3}$$

$$a^{m} = \frac{1}{a^{-m}} \qquad \frac{1}{a^{-m}} = a^{m}$$

$$\left(a^{b}\right)^{c} = a^{b \times c}$$

Logarithm : $\times \div \; + \; -$

1) $\log(ab) = \log a + \log b$

2) $\log(a/b) = \log a - \log b$

3) $\log a^m = m \log a$

4) If $\log_b a = x$

then $a = b^x$

5) $\log_b a = \dfrac{\log_{\#} a}{\log_{\#} b}$

6) $\log_a a = 1$

7) $\log_e e = 1$

8) $\log_{\#} 1 = 0$

9) $\log 0 = \text{N.D}$

$\log(-ve) = \text{ND}$

10) $a^{\log_a b} = b$

$e^{\log_e f(x)} = f(x)$

11) $\dfrac{1}{\log_b a} = \dfrac{1}{\frac{\log a}{\log b}} = \dfrac{\log b}{\log a}$

$= \log_a b$

$\dfrac{1}{\log_b a} = \log_a b$

$\log(a \cdot b) = \log a + \log b$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

$\log a^m = m \log a$

If $\log_b a = x \quad a = b^x$

$\log_b a = \dfrac{\log a}{\log b}$

$\log_b a = \dfrac{1}{\log_a b}$

$\log_a a = 1$

$\log e = 1$

$\log 1 = 0$

$\log 0 = \text{ND}$

$\log(-k) = \text{ND}$

No of digits in 2^{64} $\log 2 = 0.3010$

$$\text{Power} \times \log(\text{base}) = \text{value}$$

(round off to greater integer)

$2^{10} = 1024$

$10 \times \log 2$

$= 10 \times 0.3010$

$= 3.010$

$= \underline{\underline{4}}$ digits

2^{20}

$20 \times \log 2$

20×0.3010

$= 6.020$

$= 7$ digits.

log (No)

Char

4dig $\to$ 3

3dig $\to$ 2

0 dig $\to$ $\bar{1}$

Mantissa

Log table

Antilog

Char dig

3 $\longrightarrow$ 4

2 $\longrightarrow$ 3

5 $\longrightarrow$ 6

Antilog table

Equations $\longrightarrow$ Roots ✓ ✓

Satisfy LHS = RHS

CAFC $\quad$ Q $\rightleftarrows$ Opt $\quad$ put roots into equation

From options

② Quadratic Equation

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \alpha, \beta$$

$$\boxed{\alpha + \beta = -\frac{b}{a}} \qquad \boxed{\alpha\beta = \frac{c}{a}}$$

$\Delta = b^2 - 4ac$	Nature of roots
$\Delta < 0$	Imaginary & conjugates $(a+bi) \to (a-bi)$
$\Delta = 0$	Real & $=$
$\Delta > 0$	Real & $\neq$
$\Delta > 0$ & perfect square	Real, $\neq$, rational
$\Delta > 0$ & not a perfect square	Real, $\neq$, irrational & conjugates $p+\sqrt{q} \rightsquigarrow p-\sqrt{q}$
$\Delta \geq 0$	Real

Eqn $\begin{cases} \alpha+\beta \\ \alpha\beta \end{cases}$

$$\boxed{\begin{aligned} \alpha^3 + \beta^3 &= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) \\ \alpha^2 + \beta^2 &= (\alpha+\beta)^2 - 2\alpha\beta \end{aligned}}$$

$$(\alpha - \beta) = ?$$

$$\begin{aligned} (\alpha-\beta)^2 &= \alpha^2 - 2\alpha\beta + \beta^2 \\ &= \underbrace{\alpha^2 + 2\alpha\beta + \beta^2} - 4\alpha\beta \\ &= (\alpha+\beta)^2 - 4\alpha\beta \end{aligned}$$

$$\boxed{(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}}$$

$$\alpha^2 - \beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$$\alpha^3 - \beta^3 = (\alpha-\beta)\left(\underline{\alpha^2 + \alpha\beta + \beta^2}\right)$$

$$\boxed{\alpha^3 - \beta^3 = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\left[(\alpha+\beta)^2 - \alpha\beta\right]}$$

	Sum	Prod
Quad	$-b/a$	$\frac{c}{a}$
$\boxed{\text{Cubic}}$	$-b/a$	$\boxed{-}\frac{d}{a}$
Bi-Quad	$-b/a$	$\frac{e}{a}$

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$
$$\Downarrow$$
$$\frac{a_1}{a_2} \; ? \; \frac{b_1}{b_2}$$

- $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ — Unique Sol^n — Consistent Equ
- $\frac{a_1}{a_2} = \frac{b_1}{b_2}$
  - $= \frac{c_1}{c_2}$ — Infinite Solution
  - $\neq \frac{c_1}{c_2}$ — No Solution — Inconsistent equation.

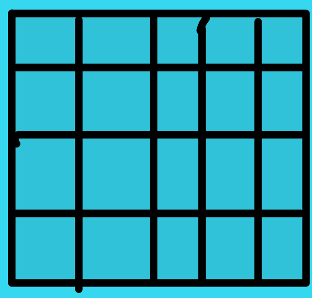
When Sum & Prod of roots are Known then eqn is given by

$$x^2 - (\text{Sum})x + \text{Prod} = 0$$

$$x^2 - Sx + P = 0$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

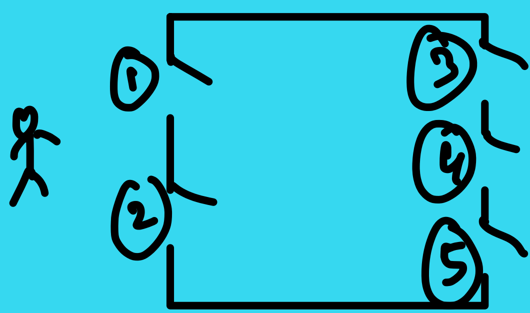
Permutation & Combination



$1+2+3+\cdots\cdots = 20$

$5+5+5+5 = 20$

$4 \times 5 = 20$



1 2

$3 + 3 = 6$

(1 3) (2 3)
(1 4) (2 4)
(1 5) (2 5) } 6 ways

Enter Exit
$2 \times 3 = 6$ ways

$E_1 \rightarrow E_2$

$n(E_1) \times n(E_2)$ 'and'

E_1 ✓ E_2 ✓
E_2 ✗ E_1 ✗

$n(E_1) + n(E_2)$ 'or'

$$A \;\substack{\longrightarrow\\\longrightarrow\\\longrightarrow}\; B \;\substack{\longrightarrow\\\longrightarrow}\; C$$

Round Trip A to C via B

$3 \times 2 \times 2 \times 3$

$= 36$ ways

Can't use same root

$3 \times 2 \times 1 \times 2$

$= 12$ ways

<u>Factorial</u> $n!$ or $\underline{|n}$

$$n! = 1 \times 2 \times 3 \times \cdots \times n$$

$$n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$$

$$\mathbf{n! = n(n-1)!}$$

$$5! = 5 \cdot \underline{4 \cdot 3 \cdot 2 \cdot 1}$$

$$4! = \underline{4 \cdot 3 \cdot 2 \cdot 1}$$

$$\boxed{5! = 5 \cdot 4!}$$

$1! = 1$

$2! = 2$

$3! = 6$

$4! = 24$

$5! = 120$

$6! = 720$

$7! = 5040$

$8! = 40320$

$9! = 362880$

<u>Permutation</u> :- Arrangements

$\Downarrow$

$AB \neq BA$

$\Downarrow$

Order matters

$P(n,r)$ or ${}^{n}P_{r}$

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$${}^{7}P_{3} = \frac{7!}{4!} = \frac{7\cdot6\cdot5\cdot\cancel{4!}}{\cancel{4!}} = 7\cdot6\cdot5$$

$$^{7}P_{3} = 7 \cdot 6 \cdot 5$$

$$^{8}P_{2} = 8 \cdot 7$$

$$\boxed{^{n}P_{2} = n(n-1)}$$

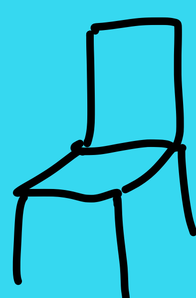
$$^{n}P_{3} = n(n-1)(n-2)$$

$$^{n}P_{r} = n(n-1)(n-2)\cdots[n-(r-1)]$$

$$^{n}P_{r} = n(n-1)(n-2)\cdots[n-r+1]$$

$^{n}P_{r}$ will have 'r' factors

ABC	ABC	ABC
hhh	hh	h
ABC	AB	A
ACB	AC	B
BAC	BA	C
BCA	BC	
CAB	CA	
CBA	CB	



$^{3}P_{1} = 3$

$^{3}P_{3} = 6$ ✓ $\quad$ $^{3}P_{2} = 6$ ✓

$$\boxed{^{n}P_{1} = n}$$

$^3P_3 = 3!$ $\boxed{^nP_n = {}^nP_{n-1}}$

$\boxed{^nP_n = n!}$

$^3P_2 = 3 \cdot 2 = 6$

$^3P_3 = 3 \cdot 2 \cdot 1 = 6$

$^nP_r = \dfrac{n!}{(n-r)!}$ $^7P_3 = 7 \cdot 6 \cdot 5$

$^nP_1 = n$ $^nP_n = n!$

$^nP_{n-1} = n!$ eg: $^5P_4 = {}^5P_5 = \underline{\underline{5!}}$

Permutation of alike objects

ABHISHEK MEHTA TWS = $\dfrac{13!}{2!\,3!\,2!}$

3obj

ABC	AAC	ABCD	AAAD
ACB	ACA		AADA
BAC	CAA	4!	ADAA
BCA		24	DAAA
CAB			4 ways
CBA			

6 ways $\xrightarrow[\div 2]{}$ 3 ways

24 → 4 ways

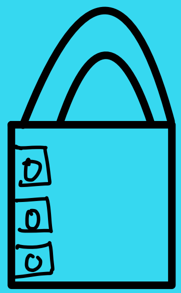
$\div 6$

$\div 3!$

Triplet

$$\boxed{\text{Alike Obj} \rightarrow \text{Total ways} = \frac{n!}{p!\,q!\,r!}}$$

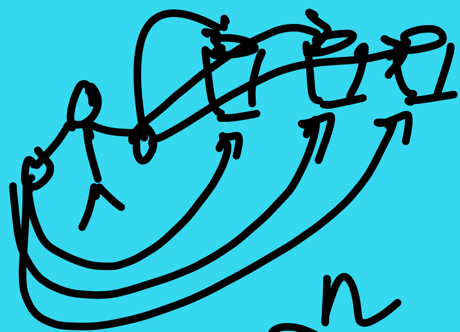
Permutation with repetition



0 1 2 3 . . . 9

$10 \cdot 10 \cdot 10 = 10^3$

$= 1000$



$3 \cdot 3 = 3^2$

(possibilities) → (Stable) → (Fixed)

n, moving, variable

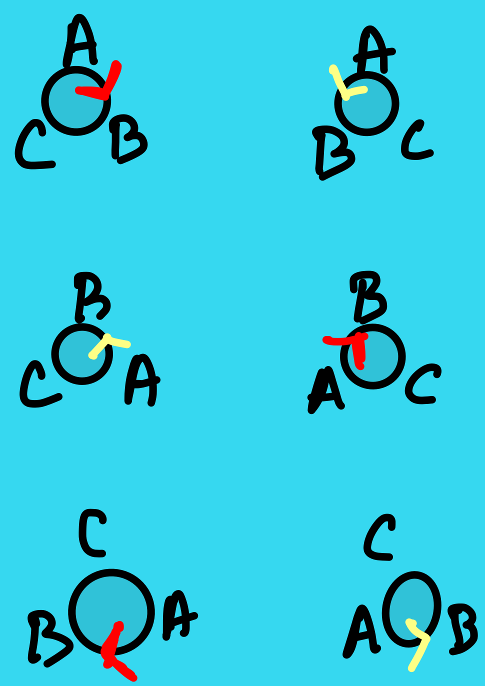
3 coin tossed → $2 \cdot 2 \cdot 2 = 2^3$

3 Friends ⇝ 4 Hotels

$4 \cdot 4 \cdot 4 = 4^3$

Permutation of Circular arrangements

ABC
ACB
BAC
BCA
CAB
CBA



6 ways $\xrightarrow[\div 3]{}$ 2 ways

$\div$ (no of objects)

Linear $\xrightarrow[\div n]{}$ Circular

$$n! \xrightarrow[\div n]{} \frac{n!}{n}$$

$\therefore$ No of ways to arrange obj in a circle

$$= \frac{n!}{n} = \frac{\cancel{n}(n-1)!}{\cancel{n}} = (n-1)!$$

Circular arrangements $= (n-1)!$

In case of MALA $= \dfrac{(n-1)!}{2}$

arr of stones, beads, diamond, flower .... Necklace

COMBINATION :- Selection → Order doesn't matter (making choices)

AB = BA

^nC_r or $C(n, r) = \dfrac{n!}{(n-r)!\,r!}$

Permutation

ABC
↓
A
B
C
$^3P_1 = 3$

ABC
↓↓
AB
AC
BA
BC
CA
CB
= 6 ways

ABC
↓↓↓
ABC
ACB
BAC
BCA
CAB
CBA
$^3P_3 = 3!$
6 ways

Combination

ABC
↓
A
B
C
$^3C_1 = 3$ ✓

ABC
↓↓
AB
AC
BC
✓ $^3C_2 = 3$

ABC
↓↓↓
ABC 1 way
$^3C_3 = 1$

$^nP_1 = {}^nC_1 = n$

	P		C
$r=3$	6	$\xrightarrow{\;3!\;}$	1
$r=2$	6	$\xrightarrow{\;2!\;}$	3
$r=1$	3	$\xrightarrow{\;1!\;}$	3

$$^{n}C_{r} = \frac{^{n}P_{r}}{r!}$$

$$^{n}P_{r} = {^{n}C_{r}} \cdot r! \quad ✓ \quad ✓$$

$$\boxed{^{n}C_{r} = \frac{n!}{(n-r)!\, r!}}$$

Arrangement cannot be done without Selection

but Selection is independent of arrangement.

(a) 3 chairs 7 ppl

$^7P_3 \leftarrow {}^7C_3 \cdot 3!$

(b) 7 chairs 3 ppl

$^7P_3 = {}^7C_3 \cdot 3!$

$$^5C_2 = \frac{5!}{3!\ 2!} = \frac{5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}\ 2!}$$

$$^5C_2 = \frac{5 \cdot 4}{2!} = 10 \qquad \boxed{^5C_2 = 10}$$

$$^5C_3 = \frac{5 \cdot 4 \cdot 3}{3!} = \frac{60}{6} = 10 \cdot \qquad \boxed{^5C_3 = 10}$$

$$^{10}C_6 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{720} = 210$$

$${}^{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{24} = 210$$

$${}^{10}C_8 = {}^{10}C_2$$

45 45

$${}^{n}C_r = {}^{n}C_{n-r}$$

no of Selection = no of Rejections

(2) If ${}^{n}C_x = {}^{n}C_y$

i) $x = y$ or

ii) $n = x + y$

iii) Both

③ $^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Homemade → A.M

$$\boxed{^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}}$$

$^{13}C_{3} + {}^{13}C_{4} = {}^{14}C_{4}$

④ Selection of 'n' different items.

3 friends ... Invitation

$$ {}^{3}C_0 + {}^{3}C_1 + {}^{3}C_2 + {}^{3}C_3 $$

$$ = 1 + 3 + 3 + 1 $$

$$ = 8 $$

$$ = 2^{3} $$

$$ {}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + \dots + {}^{n}C_n = 2^{n} $$

Selection of atleast 1 of 'n' different items

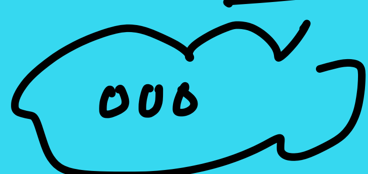
$$ {}^{n}C_1 + {}^{n}C_2 + \dots + {}^{n}C_n = 2^{n} - 1 $$

Selection of alike objects

4 Ip 3 SS 2 Jio

$$ 5 \text{ ways} \left\{ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \right. \quad 4 \text{ ways} \left\{ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right. \quad 3 \text{ ways} \left\{ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \right. $$

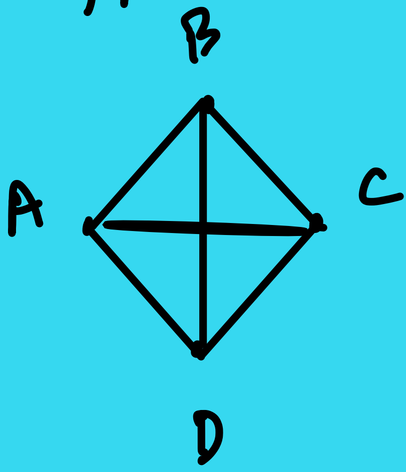
$$ (4+1)(3+1)(2+1) - 1 $$



$$ = 5 \cdot 4 \cdot 3 - 1 = \underline{59} $$

$(p+1)(q+1)(r+1) - 1$

Application of Combination



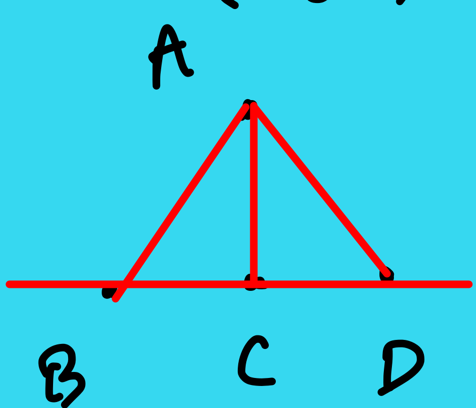
$^4C_2 = \frac{4\cdot 3}{2} = \underline{\underline{6}}$

no (lines) $= {}^nC_2$

no (Δ) $= {}^nC_3$

no (O) $= n(\Delta) =$

'n' pts non-collinear



$^6C_2 - {}^3C_2 + 1$

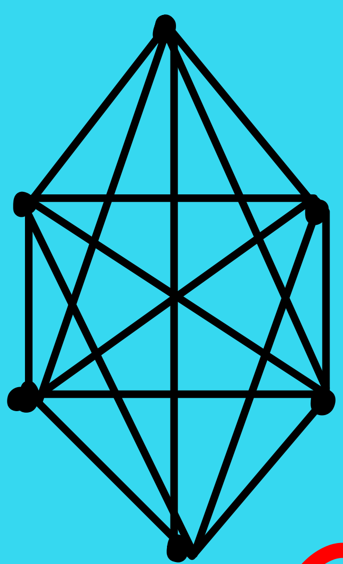
$n(\text{lines}) = {}^nC_2 - {}^xC_2 + 1$

$$n(\Delta) = {}^{n}C_3 - {}^{x}C_3$$

No of diagonals

Hexagon 'n' = 6

6 sides $\Leftrightarrow$ 6 vertices



$$ {}^{n}C_2 - n $$

$$ {}^{6}C_2 - 6 $$

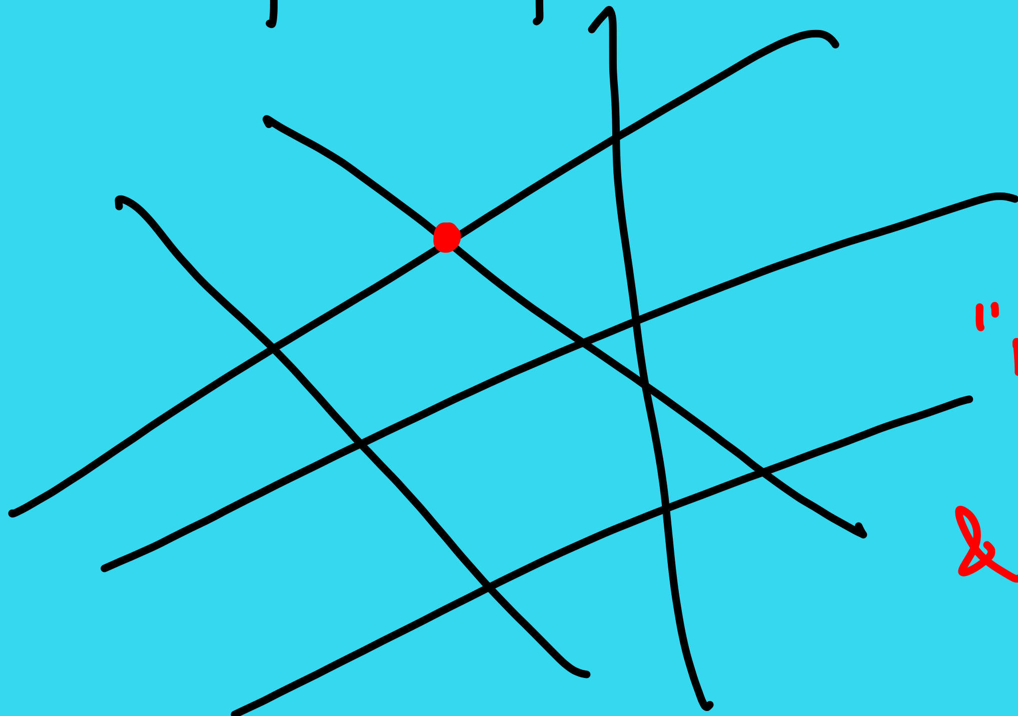
$$ = \frac{6 \cdot 5}{2} - 6 $$

$$ = 15 - 6 $$

$$ = 9 $$

$$ \frac{n(n-3)}{2} $$

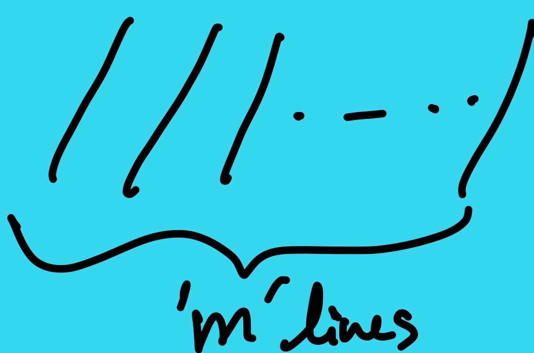
No of pts of intersection

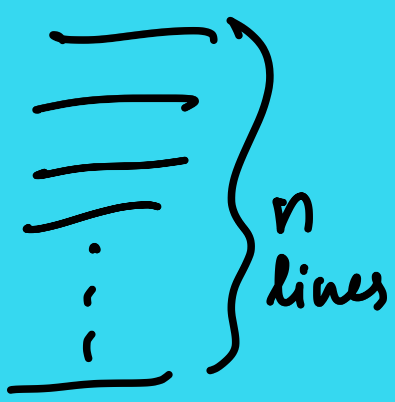


$^{n}C_2$

"No 2 lines are parallel
& No 3 lines are concurrent"

No of Parallelogram





$^{m}C_2 \cdot {^{n}C_2}$

Sequence & Series

$1, 2, 3 \ldots$ A.P

$1, 2, 4 \ldots$ G.P

$1, \frac{1}{2}, \frac{1}{3} \ldots$ HP

AP (difference constant)

t_1	t_2	t_3	t_4
2	4	6	8

$$d = t_2 - t_1$$
$$= t_3 - t_2$$
$$= t_4 - t_3$$

$$\boxed{d = t_n - t_{n-1}}$$

$$t_1 = a$$
$$t_2 = a + d$$
$$t_3 = a + 2d$$
$$t_4 = a + 3d$$

$$\boxed{t_n = a + (n-1)d}$$

d

$$t_n - a = (n-1)d$$

$$\boxed{d = \frac{t_n - a}{n-1}}$$

n

$$t_n - a = (n-1)d$$

$$\frac{t_n - a}{d} = n - 1$$

$$\boxed{d = \frac{t_p - t_q}{p - q}}$$

$$\boxed{n = \frac{t_n - a}{d} + 1}$$

$S_n = ?$

$S_n = t_1 + t_2 + \cdots + t_n$

$S_n = a + (a+d) + \cdots + (l-d) + l$ — (1)

$S_n = l + (l-d) + \cdots + (a+d) + a$ — (2)

(1) + (2)

$$2S_n = \underbrace{(a+l) + (a+l) + \cdots + (a+l)}_{n \text{ terms}}$$

$2S_n = n(a+l)$

$$\boxed{S_n = \frac{n}{2}(a + t_n)}$$

$$S_n = \frac{n}{2}\left[a + a + (n-1)d\right]$$

$$\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

AP (d)

$$d = t_n - t_{n-1} = \frac{t_n - a}{n-1} = \frac{t_p - t_q}{p-q}$$

$$t_n = a + (n-1)d$$

$$= S_n - S_{n-1}$$

$1 \; 2 \; 3 \; 4 = 10 \quad S_4$

$1 \; 2 \; 3 \quad = -6 \quad S_3$

$4 \quad t_4$

$t_4 = S_4 - S_3$

$$n = \frac{t_n - a}{d} + 1$$

$$S_n = n/2\,(a + t_n)$$

$$= n/2\,\left[2a + (n-1)d\right]$$

$$\sum x = n\bar{x}$$

$$S_n = n\left(\frac{a+t_n}{2}\right)$$

G.P

GP (ratio)

t_1	t_2	t_3	t_4
2	4	8	16

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3}$$

$$\boxed{r = \frac{t_n}{t_{n-1}}}$$

$t_n = ?$

$t_1 = a$

$t_2 = ar$

$t_3 = ar^2$

$t_4 = ar^3$

$$\boxed{t_n = a r^{n-1}}$$

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1} \quad — ①$$

$$rS_n = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n \quad — ②$$

② − ①

$$ar^{n-1} \cdot r^1 = ar^{n\cancel{-1+1}}$$

$$rS_n - S_n = ar^n - a$$

$$S_n(r-1) = a(r^n - 1)$$

$$\therefore \boxed{S_n = \frac{a(r^n - 1)}{r-1}} \qquad r > 1$$

$$\boxed{S_n = \frac{a(1 - r^n)}{1-r}} \qquad r < 1$$

$$S_n = 5 + 5 + 5 + 5$$
$$= 4 \times 5$$

$$\boxed{S_n = n.a} \qquad r = 1$$

$n \to \infty, \quad r < 1$

$$S_n = a\left[\frac{1 - (<1)^{\infty}}{1 - r}\right]$$

$$= a\left[\frac{1 - 0}{1 - r}\right]$$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

$n \to \infty$

$r < 1$

$$\boxed{t_n = S_n - S_{n-1}}$$

$2+4+8+16 = 30 \quad S_4$

$2+4+8 = 14 \quad S_3$

$\overline{\quad 16 \quad} \; t_4$

$$t_4 = S_4 - S_3$$

GP $r = \frac{t_n}{t_{n-1}}$

$t_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r - 1}$ $r > 1$

$= \frac{a(1 - r^n)}{1 - r}$ $r < 1$

$= \frac{a}{1 - r} = \frac{FT}{1 - CR}$ $n \to \infty$, $r < 1$

To Show a b c ... AP

$b - a = c - b$

$\boxed{2b = a + c}$ $b = \frac{a+c}{2}$

To Show a b c ... GP

$\frac{b}{a} = \frac{c}{b}$

$$\boxed{b^2 = ac}$$

$$b = \sqrt{ac}$$

<u>HP</u> :- Seq is in HP if their reciprocals are in AP

$1, \; 1/2, \; 1/3 \cdots$ HP

$1, \; 2, \; 3 \cdots$ AP

$$t_n = \frac{1}{a+(n-1)d} \longrightarrow AP$$

Rel^n bet^n AM, GM, HM

$$GM^2 = AM \times HM$$

AM > GM > h.m

2, 8

$\to$ 5 AM

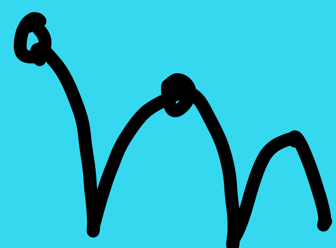
$\to$ 4

$\to \dfrac{2ab}{a+b} = \dfrac{32}{10} = 3.2$ HM

Distance Travelled by bouncing ball

$$d_{\infty} = h\,\frac{(1+r)}{(1-r)}$$

bounce



$r<1$

$$\sum_{r=1}^{n} r = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{r=1}^{n} r^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{r=1}^{n} r^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

r starts from 1 to n

$$AP \xrightarrow{\times \ \div \ + \ -} AP$$

$2 \quad 4 \quad 6 \quad 8 \quad \ldots$ AP

$\div 2$

$1 \quad 2 \quad 3 \quad 4 \quad \ldots$ AP

$\times 3$

$3 \quad 6 \quad 9 \quad 12 \quad \ldots$ AP

$+1 \quad 4 \quad 7 \quad 10 \quad 13 \quad \ldots$ AP

$-3 \quad 1 \quad 4 \quad 7 \quad 10 \quad \ldots$ AP

Time Value of Money

- Annuity
- ER & NR
- CAGR ✓
- BOND ✓

Annuity [FIX installment investment interval of time]

- Annuity regular / ordinary (end)
- Annuity immediate/ Due (start)

I : <u>FUTURE VALUE</u>

a) FV of a single investment

$$\boxed{A = P(1+i)^N}$$

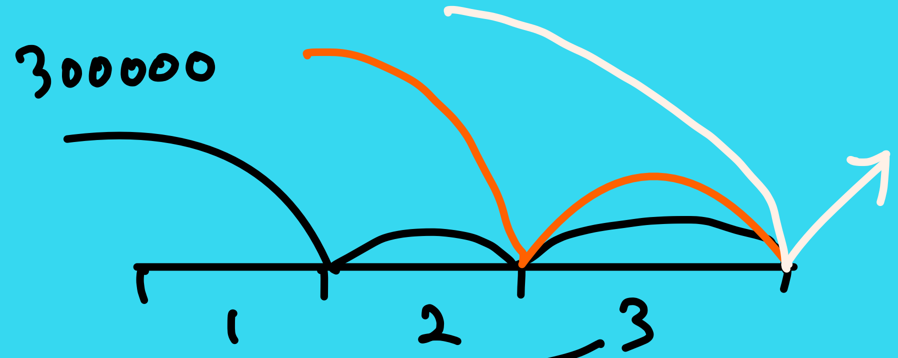
$P = 1000000$ $\quad R = 10\%$ $\quad N = 10$

$i = 0.1$

$$A = P(1+i)^N = 1000000(1.1)^{10}$$

$$= \underline{25.93 \text{ Lakhs}}$$

b) **F·V of Annuity Regular**



$$3(1\cdot 1)^2 + 3(1\cdot 1)^1 + 3(1\cdot 1)^0$$

$$A(N,i) = A(1+i)^0 + A(1+i)^1 + A(1+i)^2 + \cdots A(1+i)^{N-1}$$

$$= A\left[1 + (1+i)^1 + (1+i)^2 + \cdots + (1+i)^{N-1}\right]$$

$$a = 1 \quad r = (1+i) \quad n = N$$

$$= A \cdot 1 \left[\frac{(1+i)^N - 1}{1+i-1}\right] \qquad \frac{a(r^n-1)}{r-1}$$

$$\boxed{A(N,i) = A\left[\frac{(1+i)^N - 1}{i}\right]}$$

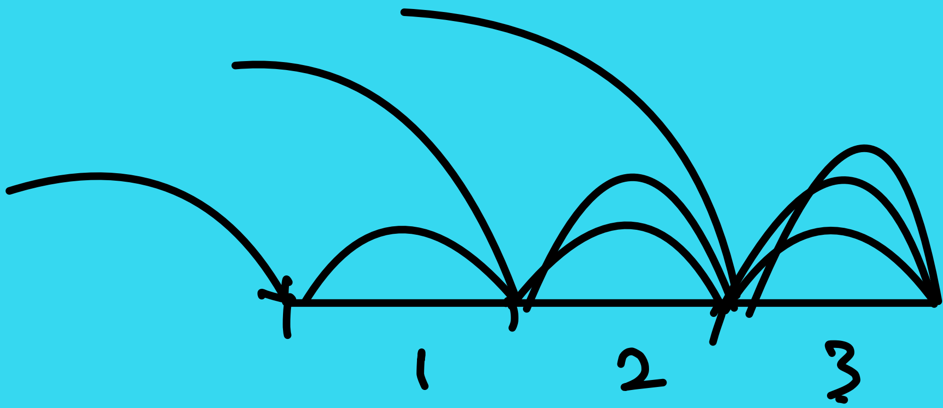
$$N = nm$$

$$i = \frac{R}{m100}$$

(c) **F·V of Annuity immediate / Due**

$$\boxed{A_I = (1+i) * A(N,i)}$$

When inv starts from today



$$3(1.1)^3 + 3(1.1)^2 + 3(1.1)^1$$

$$= (1.1)\left[\,3(1.1)^2 + 3(1.1)^1 + 3(1.1)^0\,\right]$$

$$= (1+i) * A(N, i)$$

PRESENT VALUE

a) PV of single investment

$$V = \frac{A}{(1+i)^N}$$

$$A = P(1+i)^N$$

$$P = \frac{A}{(1+i)^N}$$

$A = 2500000 \quad N = 10 \quad R = 10\%$

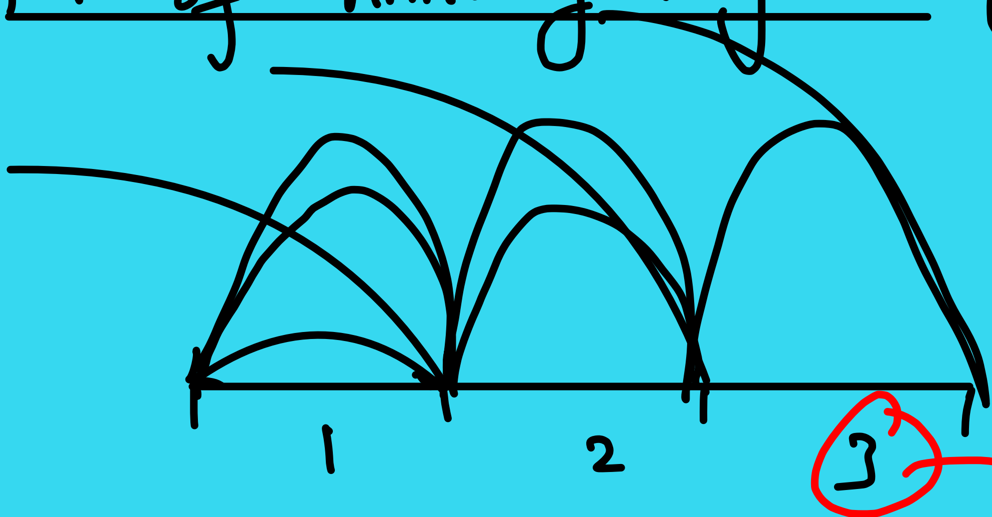
$$V = P = \frac{2500000}{(1.1)^{10}}$$

$2500000 \div 1.1$ = = = = = = = = = =

$= 9.63$ Lak

b) P.V of Annuity Regular (LOAN)

1L



$$\frac{1L}{1.1} + \frac{1L}{(1.1)^2} + \frac{1L}{(1.1)^3}$$

$$V(N,i) = \boxed{\frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \cdots \frac{A}{(1+i)^N}}$$

$a = \dfrac{A}{1+i}$, $r = \dfrac{1}{1+i}$, $n = N$

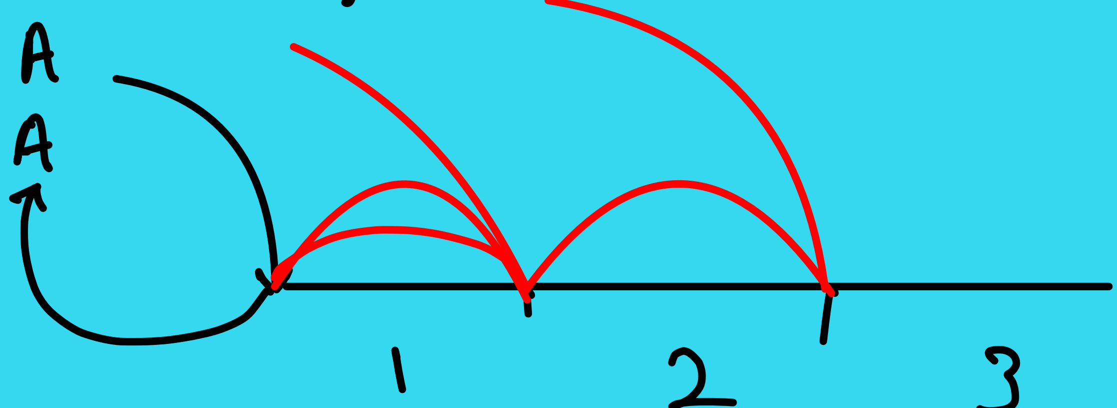
$$= \frac{A}{1+i}\left[\frac{1-\left(\frac{1}{1+i}\right)^N}{1-\frac{1}{1+i}}\right]$$

$$= \frac{A}{\cancel{1+i}}\left[\frac{1-(1+i)^{-N}}{\dfrac{\cancel{1}+i-\cancel{1}}{\cancel{1+i}}}\right]$$

$$\boxed{V(N,i) = A\left[\frac{1-(1+i)^{-N}}{i}\right]}$$

Loan ...

c) P.V of Annuity Imm/Due :-



$$\boxed{V_I = \underline{A} + V(N, i)}$$

$N-1$

No of installments reduces by 1

$$V(N,i) = A\left[\frac{1-\left(\frac{1}{1+i}\right)^{N\to\infty}}{i}\right]$$

$$= A\left[\frac{1-0}{i}\right]$$

$$\boxed{V = \frac{A}{i}}$$

Perpetuity

Charity
forever
$n \to \infty$

'n' is neither asked
nor 'n' is given

A : Recurring investment / installment

$i = \frac{R}{m100}$ $N = nm$

Annuity

Annuity Regular (end)

Annuity Due / Imm (start)

$$A(N,i) = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$V(N,i) = A\left[\frac{1-(1+i)^N}{i}\right]$$

$$A_I = (1+i)\,A(N,i)$$

$$V_I = A + V(N,i)$$

N reduces by 1

$$V = \frac{A}{i}$$ Perpetuity

$$V(N,i) = A\left[\frac{1-(1+i)^{-N}}{i}\right]$$

$$= A\left[\frac{1-\frac{1}{(1+i)^N}}{i}\right]$$

$$= A\left[\frac{(1+i)^N - 1}{i\,(1+i)^N}\right]$$

$$\boxed{V(N,i) = \frac{A(N,i)}{(1+i)^N}}$$

$$P = \frac{A}{(1+i)^N}$$

A	10%		
B	10%		$A = B = C$
C	10%		

A	10%	$m=2$	
B	10%	$m=4$	$C > B > A$
C	10%	$m=12$	

A	10.25%	
B	10%	$A > B > C$
C	9.75%	

Bank	R	Conversions
A	10.25%	$m=1$
B	10%	$m=2$
C	9.75%	$m=4$

Nominal Rate $\longrightarrow$ Effective Rate

$$SI_1 = CI_1$$

$$\frac{\cancel{P}\cdot 1\cdot R}{100} = \cancel{P}\left[\left(1+\frac{R}{m100}\right)^{1.m} - 1\right]$$

$$R_{eff} = 100\left[\left(1+\frac{R}{m100}\right)^{m} - 1\right]$$

Formula is same like CI with $P=100$, $n=1$

$$R_C = 100\left[\left(1+\frac{9.75}{400}\right)^{4} - 1\right] = 10.11\%$$

$$R_B = 100\left[\left(1+\frac{10}{200}\right)^{2} - 1\right] = 10.25$$

$R_A = 10.25$

$$\boxed{A = B > C}$$

Set
Relations
Functions

Set :- Coll^n of well defined unique elements

A B H I S H E K

ROSTER FORM

$A = \{A, B, H, I, S, E, K\}$

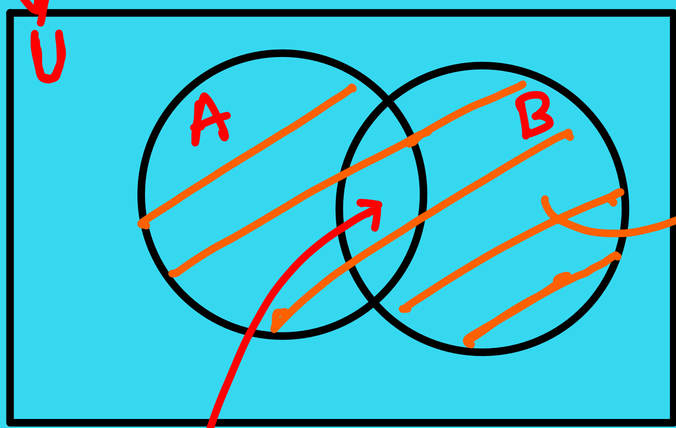
Set of first 100 Natural no's

SET BUILDER FORM

$B = \{x / x \in N, \ x < 101\}$

VENN DIAGRAM

Universal



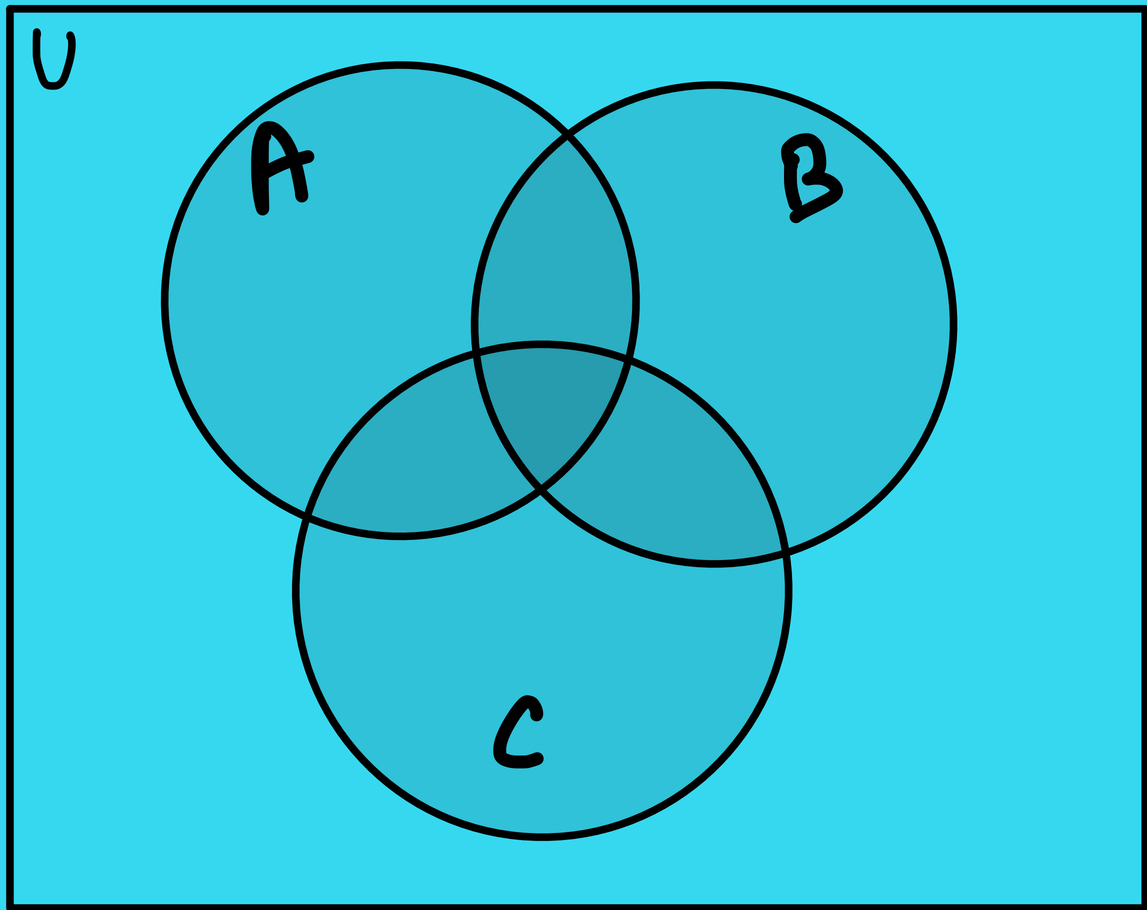
$A \cup B$

$A \cap B$ ✓ Joint set

$n(A \cap B) = 0$ Disjoint Set

$$A \cup B = A + B - A \cap B$$

$$\boxed{A \cup B = A + B - AB}$$



$$A \cup B \cup C = A + B + C - AB - BC - AC + ABC$$

<u>Types</u>

$\{\ \}$, ϕ Null / void / Empty Set

$\{1\}$ Singleton $\{0\}$

$\{1, 2, 3\} \rightarrow$ finite Set

$\{1, 2, 3, \ldots\} \rightarrow$ Infinite Set

$A\{a, b\}$

$B\{b, a\}$

$A = B$ Equal Set

$A = \{a, b\}$

$B = \{1, 2\}$

$A \neq B$

$n(A) = n(B)$

Equivalent Set

Subset :- If all A's belongs to B's

$$A \subseteq B$$

eg:- Isoceles Δ , Equilateral Δ

$$E\Delta \subseteq I\Delta$$

$$\square \subset \square$$

Proper Subset :-

If A's belongs to B's but $\exists$ in B which cannot belong to A.

eg $\quad N \subset W \subset I \subset R \subset Z$

$A = \{1, 2, 3\}$

$P(A)$ = Power Set

= Set of all Subsets

$= \{$

$\{\ \}$, 1

$\{1\}\ \{2\}\ \{3\}$ 3

$\{1,2\}\ \{2,3\}\ \{1,3\}$ 3

$\{1,2,3\}$ 1

$\}$

$1 + 3 + 3 + 1 = 8$

$^3C_0 + {^3C_1} + {^3C_2} + {^3C_3} = \underline{\underline{2^3}}$

No of Subsets $= 2^n$

No of non-empty SS $= 2^n - 1$

$\{\}$

No of Proper SS $= 2^n - 1$

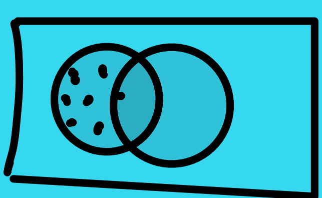
$\{1, 2, 3\}$

No of non-empty proper SS $= 2^n - 2$

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A - B = \{1, 2\}$



$B - A = \{5, 6\}$

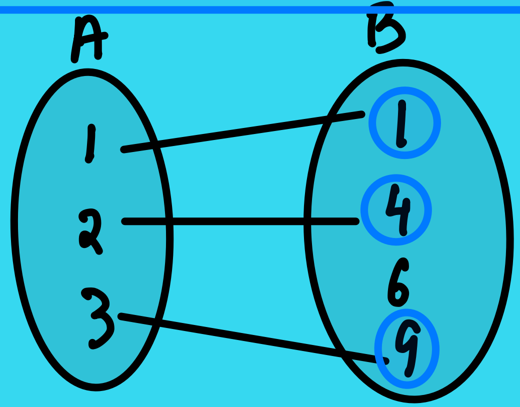
$A = \{1, 2, 3\}$ 3 $B = \{1, 4, 6, 9\}$ 4

$$A \times B = \{(1,1), (1,4), (1,6), (1,9), (2,1), (2,4), (2,6), (2,9), (3,1), (3,4), (3,6), (3,9)\}$$

12

$$\boxed{R_1 = \{(1,1)\ (2,4)\ (3,9)\}}$$

Domain (input)



Co-domain

Right
Related (output)
RANGE $= \{1, 4, 9\}$

$$R^{-1} = \{(1,1), (4,2), (9,3)\}$$

$n(A) = m$

$n(B) = n$

$n(A \times B) = mn$

$SS(A \times B) = 2^{mn}$

no of Relation = No of Subsets of $A \times B$

$= 2^{mn}$

TYPES OF RELATION

$A = \{a, b, c\}$

Relation on A = $A \times A$

$$= \{ \begin{matrix} aa & ab & ac \\ ba & bb & bc \\ ca & cb & cc \end{matrix} \}$$

$R_1 = \{ (aa)\ (bb)\ (cc) \}$

$\boxed{\text{Reflexive Relation}}$

$\forall\, a \in A$, (a,a)

For every element in A

same same element must be related

$R_2 = \{ (ab), (b,a)\ (a,c)\ (c,a) \}$

if (a,b) then (b,a) must exist

Symmetric Relation

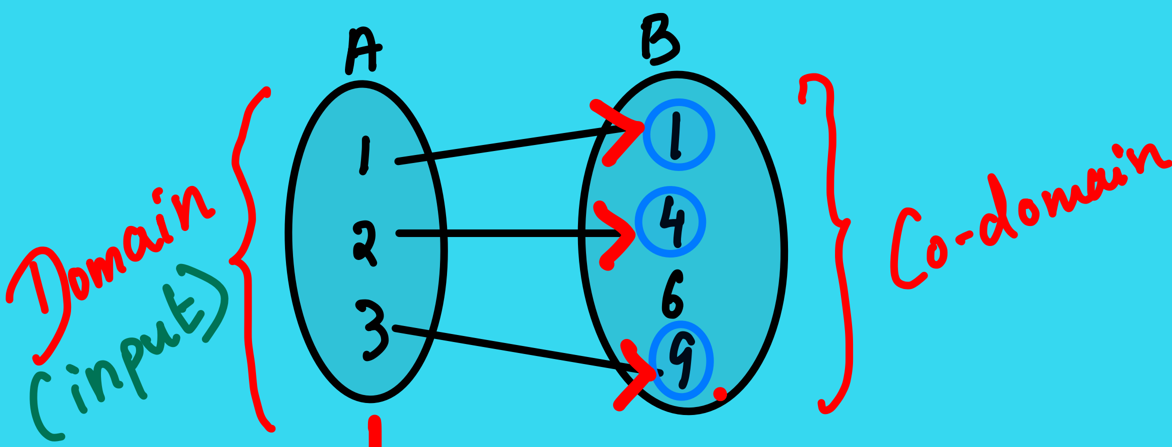
$R_3 = \{ (a\,b)\ (b, c)\ (a, c) \}$

$p\ m$	$m\ u$	$p\ u$	
✓	✓	✓	T
✓	✓	✗	N.T
✓	✗	—	T
✗	—	—	T

TRANSITIVE RELATION

$(a\,b)\ (b, c)\ (a, c)$ ✓

If all three relation exist then Rel is called equivalence relation.



All elements are related → to unique elements

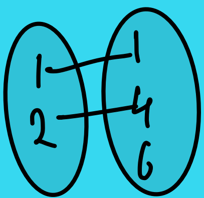
F : valid for all domain (input)
and
must give single value in its range (output)

Types

One to one — into, onto

many to one — into, onto

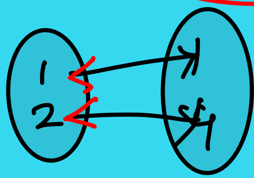
into



$R \subset$ Co-domain

one-one into f^n

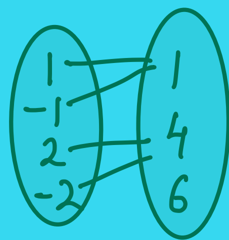
onto



$R =$ Co-domain

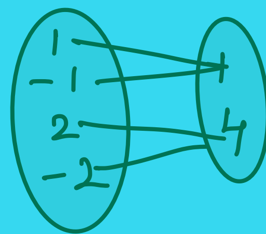
Inverse exist

into (many to one)



$R \subset$ Co-do

onto (many to one)



$R =$ Co-domain

$$f(x) = x^2 \qquad g(x) = x + 2$$

$$f \circ g = f[g(x)]$$
$$= [g(x)]^2$$
$$= (x+2)^2$$

$$g \circ f = g[f(x)]$$
$$= f(x) + 2$$
$$= x^2 + 2$$

Inverse function : When function is one to one & onto

$$f(x) = \frac{3x-5}{2}$$

ISOLATE 'x'

Find $f^{-1} = ?$

$$2f(x) = 3x - 5$$

$$2f(x) + 5 = 3x$$

$$\therefore x = \frac{2f(x)+5}{3}$$

isolated

replace f(x) by x

$$\boxed{\frac{2x+5}{3}}$$

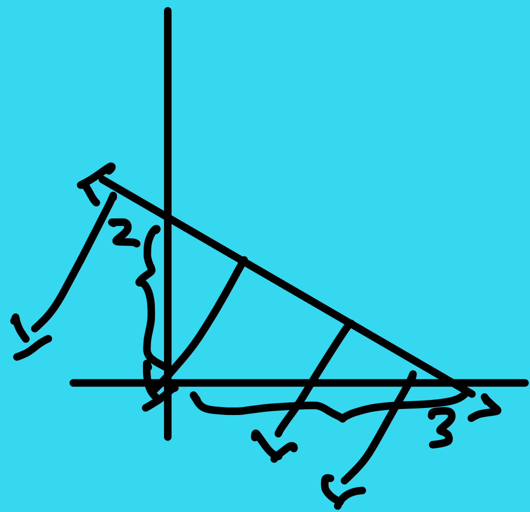
Linear Inequalities $\leq$ $\geq$

$$2x + 3y \leq 6$$

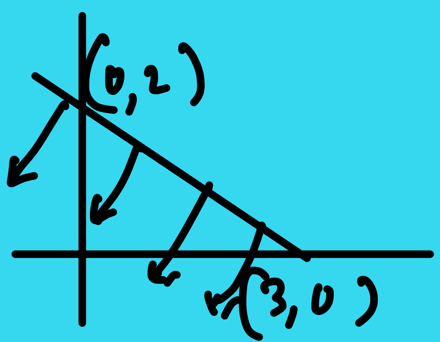
$x=0 \quad y=2$

$x=3 \quad y=0$

$$\frac{x}{3} + \frac{y}{2} \leq 1$$

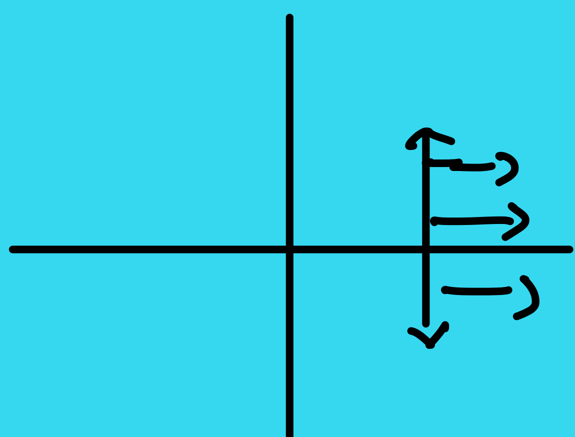


$(0,2) \; (3,0)$

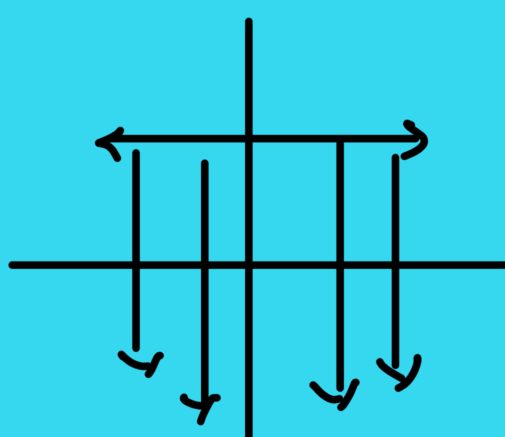


$x = \text{const} \rightarrow \parallel^{el}$ Y axis

$y = \text{const} \rightarrow \parallel^{el}$ X axis



$x \geq 2$



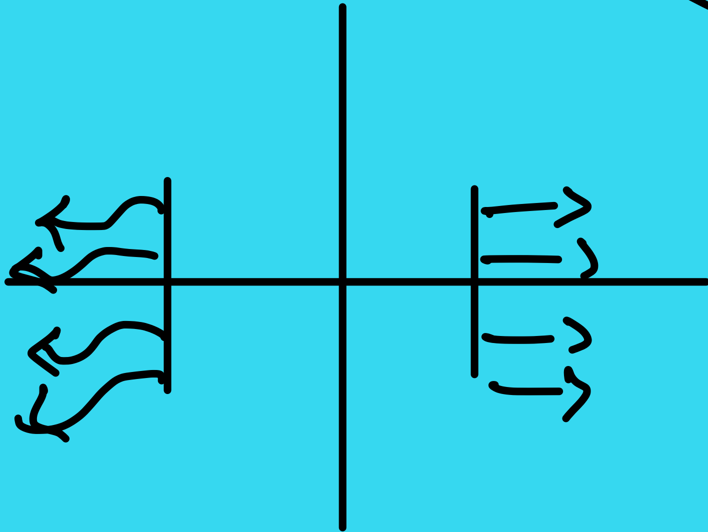
$y \leq 2$

$|x| \geq 2$

$x \geq 2$

or

$x \leq -2$

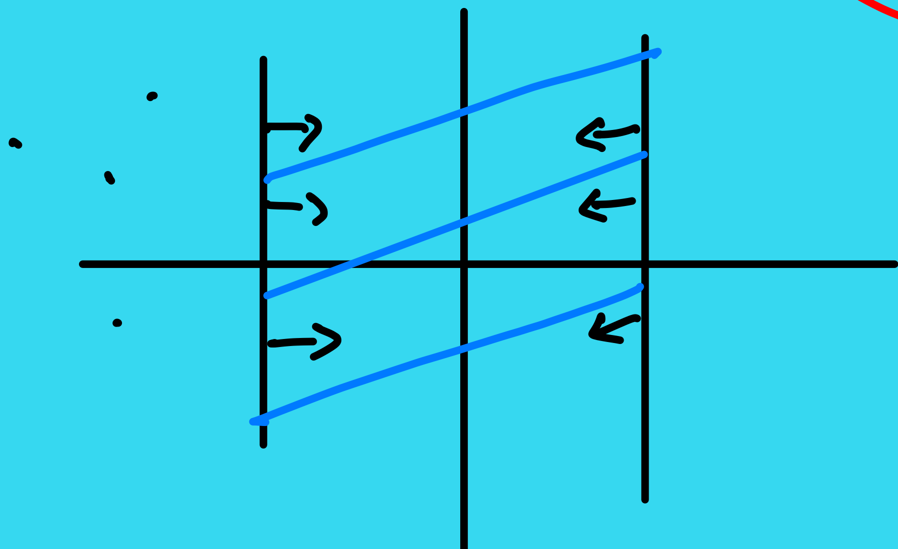


$|x| \leq 2$

$x \leq 2$

and

$x \geq -2$



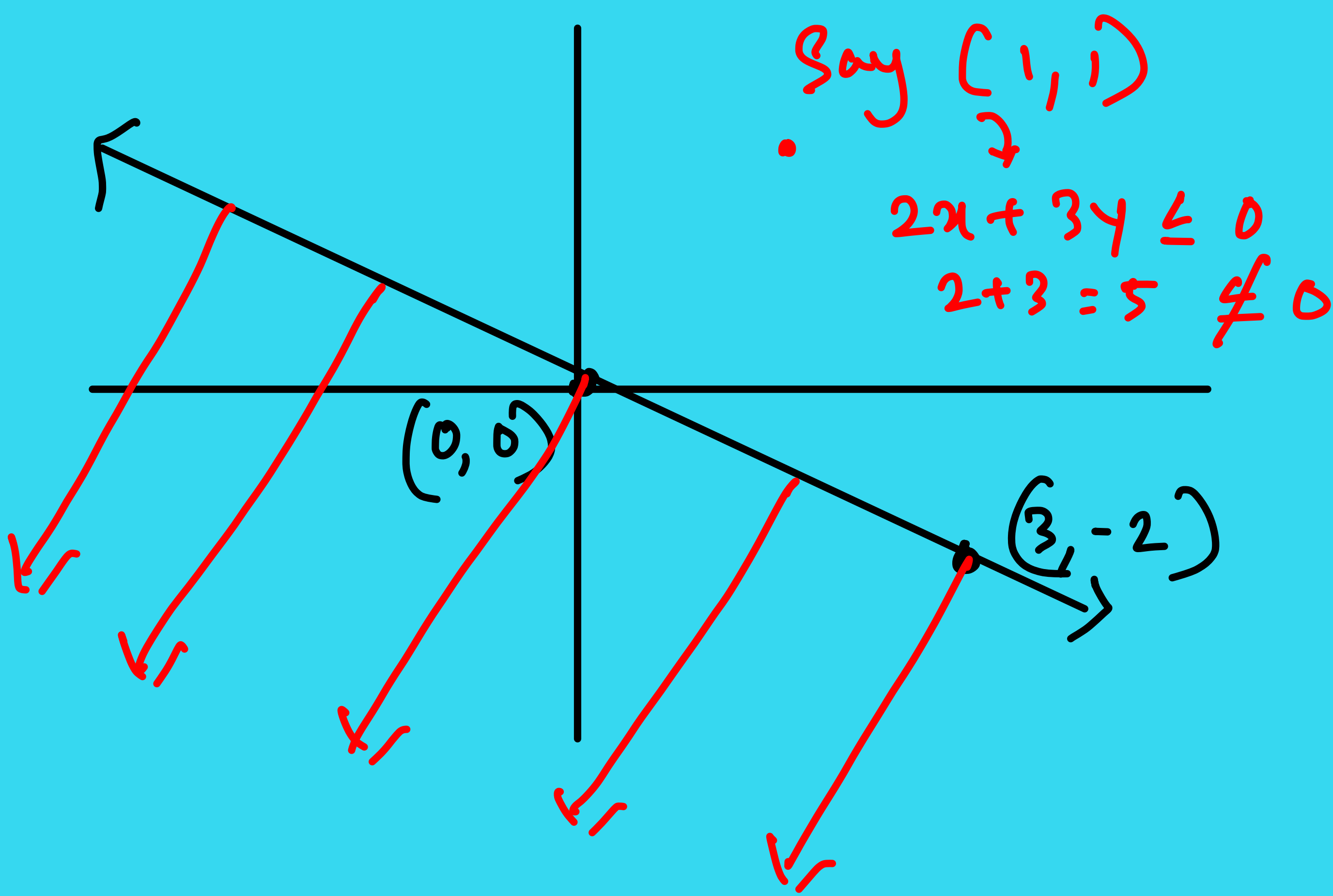
$2x + 3y \leq 0$ → $(0, 0)$

→ 2nd point

→ put x = coeffient of y

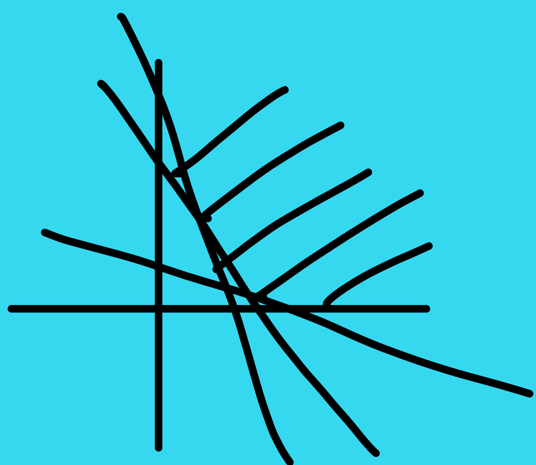
$x = 3, \ y = -2$

$(0, 0) \quad (3, -2)$

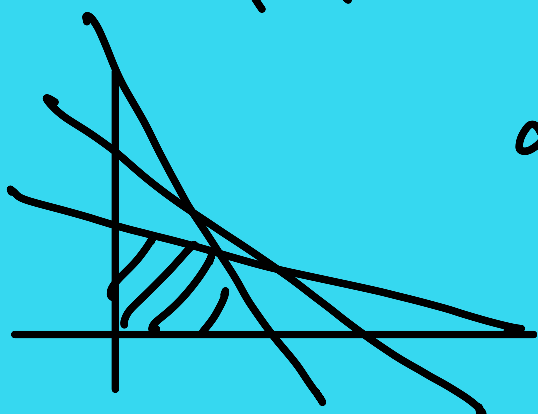


Product Types
Column wise

	A	B	Min Req $\geq$
Content/ Process			Max Available $\leq$



all eqn will have $\geq$



all eqn will have $\leq$

& $\underbrace{x, y \geq 0}_{\text{no of item can't be -ve}}$

Employer cannot hire more than

5 exp person (x) to $\underline{1}$ fresh one (y)

ratio

$$\frac{x}{y} \leq \frac{5}{1}$$

$$x \leq 5y$$

$$\frac{x}{5} \leq y$$

$$\therefore \; y \geq \frac{x}{5}$$

Derivatives $\rightarrow$ dy/dx $\rightarrow$ differential coefficient

$\downarrow$

Gradient $\leftarrow$ Slope of tangent

$\parallel$

$\dfrac{dy}{dx}$

Second order

log

implicit

u/v

uv

Chain rule

Basic

$y \rightarrow \dfrac{dy}{dx},\ y_1,\ y'$

$f(x) \rightarrow f'(x)$

$K \rightarrow 0$

$K f(x) \rightarrow K \cdot f'(x)$

$x^n \rightarrow n\, x^{n-1}$

$x \rightarrow 1$

$\dfrac{1}{x} \rightarrow \dfrac{-1}{x^2}$

$\log x \rightarrow 1/x$

$a^x \rightarrow a^x . \log a$

$e^x \rightarrow e^x$

$\sqrt{x} \rightarrow \dfrac{1}{2\sqrt{x}}$

$$y = a^x + x^a + e^x + x^e + 2^x + x^2 + a^a$$

$$\frac{dy}{dx} = a^x la + a \cdot x^{a-1} + e^x + e x^{e-1} + 2^x l2 + 2x + 0$$

<u>Chain rule</u> :-

$$y = \log(\cancel{\log x})$$

$$\frac{dy}{dx} = \frac{1}{\log x} \cdot (\log x)' = \frac{1}{\log x} \cdot \frac{1}{x}$$

$$y = \sqrt{x^2 + 5}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^2+5}} (x^2 + 5)'$$

$$= \frac{1}{\cancel{2}\sqrt{x^2+5}} (\cancel{2}x) = \frac{x}{\sqrt{x^2+5}}$$

$$y = \log\left(x + \sqrt{x^2+a^2}\right)$$

$$\frac{dy}{dx} = \frac{1}{\left(x+\sqrt{x^2+a^2}\right)}\left(1 + \frac{1}{\cancel{2}\sqrt{x^2+a^2}}\,\cancel{2}x\right)$$

$$= \frac{1}{\cancel{\left(x+\sqrt{x^2+a^2}\right)}}\left(\frac{\cancel{\sqrt{x^2+a^2}+x}}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{1}{\sqrt{x^2+a^2}}$$

$$\frac{1}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$y = uv$$

$$\frac{dy}{dx} = uv' + vu'$$

eg $y = x \log x$

$$\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \, (1)$$

$$= 1 + \log x$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$y = \frac{x}{\log x}$$

$$\frac{dy}{dx} = \frac{\log x\,(1) - x\left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{\log x - 1}{(\log x)^2}$$

Implicit f^n

$$x^2 + 2hxy + y^2 = 0$$

$$\frac{dy}{dx} = -\left[\frac{\text{diff wrt } x,\ y = \text{constant}}{\text{diff wrt } y,\ x = \text{constant}}\right]$$

$$\frac{dy}{dx} = -\left[\frac{2x + 2hy}{2hx + 2y}\right]$$

$$= -\left[\frac{x + hy}{hx + y}\right]$$

If options are in one variable form we need to avoid shortcut & seperate x & y properly

<u>Logarithmic</u> (complex / $f(x)^{g(x)}$ ✓ Short cut $\underbrace{f(x)^{g(y)} / g(y)^{f(x)}}_{\text{log}}$ → implicit shortcut

$$y = (x+5)^{10}(9-x)^5$$

$$\log y = 10\log(x+5) + 5\log(9-x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{10}{x+5} + \frac{5}{9-x}(-1)$$

$$\frac{dy}{dx} = y\left(\frac{10}{x+5} - \frac{5}{9-x}\right)$$

2) $y = x^x$

$$\frac{dy}{dx} = y\left[\text{power} \times \log \text{base}\right]$$

$$y = u^v$$

$$\frac{dy}{dx} = u^v\left[\underbrace{v \times \log u}_{\text{prod rule}}\right]$$

$$y = x^x \qquad x \times \log x$$

$$\frac{dy}{dx} = x^x\left[x \cdot \frac{1}{x} + \log x\,(1)\right]$$

$$\boxed{\frac{dy}{dx} = x^x\left(1 + \log x\right)}$$

Dont apply if power is function of y

esa hua toh

Take log

& implicit shortcut

Parametric

$y = at^2 \qquad x = 2at$

$$\frac{2at}{2a}$$

$$\frac{dy}{dx} = t$$

Integration

$$\int f'(x)\,dx = f(x) + c$$

diff option & get Question

$$\int \frac{1}{x}\,dx = \log x + c$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + c$$

$$\int a^x\,dx = \frac{a^x}{\log a} + c$$

$$\int e^x\,dx = e^x + c$$

$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c$$

$$\int \frac{1}{ax+b}\,dx = \frac{\log|ax+b|}{a} + c$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int f'(ax+b)\,dx = \frac{f(ax+b)}{a} + c$$

Method of Substitution

$f(x) \longrightarrow f'(x)$ ek hi Q mein dikhe toh .... given method

eg) $\int \frac{1}{x}\,dx = \log|x| + c$

$\hookrightarrow \int \frac{f'(x)\,dx}{f(x)} = \log|f(x)| + c$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$\hookrightarrow \int f(x)^n f'(x)\,dx = \frac{f(x)^{n+1}}{n+1} + c$

$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + C$$

$$\int \frac{f'(x)\,dx}{\sqrt{f(x)}} = 2\sqrt{f(x)} + C$$

$$\int e^{x}\,dx = e^{x} + C$$

$$\int e^{f(x)} \cdot f'(x)\,dx = e^{f(x)} + C$$

Integration by parts

$$\int uv\,dx = u\int v\,dx - \int\left[\int v\,dx \times \frac{du}{dx}\right]dx$$

same

$$\int a\,e\,dx = a\,e_1 - a'\,e_2 + a''\,e_3 + c$$

$$\int x^2 e^x\,dx = x^2 e^x - 2x\,e^x + 2\,e^x + c$$

$$\int x^3 a^x\,dx = x^3\frac{a^x}{\ln a} - 3x^2\frac{a^x}{(\ln a)^2} + 6x\frac{a^x}{(\ln a)^3}$$

$$-\frac{6a^x}{(\ln a)^4} + c$$

$$\int x^n \log x \, dx = \frac{x^{n+1}}{n+1}\left(\log x - \frac{1}{n+1}\right) + c$$

$$\int x \log x \, dx = \frac{x^2}{2}\left(\log x - \frac{1}{2}\right) + C$$

$$\int x^5 \log x \, dx = \frac{x^6}{6}\left(\log x - \frac{1}{6}\right) + c$$

$$\int \log x \, dx = \int x^0 \log x \, dx$$

$$= x\left(\log x - 1\right) + C$$

$$\boxed{\int \log x \, dx = x \log x - x + C}$$

Partial fraction

$$\int \frac{1\,dx}{(x-2)(x-3)} = A\log|x-2| + B\log|x-3| + C$$

put $x=2$ in $\frac{1}{x-3} = \frac{1}{-1} = -1$

put $x=3$ $\frac{1}{x-2} = \frac{1}{1} = 1$

Definite integral

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\int_{-a}^{a} f(x) \begin{cases} \text{even} = 2\int_{0}^{a} f(x)\,dx \\ \text{odd} = 0 \end{cases}$$

$$\int e^{x}\left[f(x) + f'(x)\right]dx =$$

$$= e^{x} f(x) + c$$

$$f(x) \longrightarrow f'(x) \longrightarrow f''(x)$$

$$f'(x) = 0 \qquad x \left\langle \begin{matrix} a \\ \\ b \end{matrix} \right.$$

$f''(a) < 0$ maxima $\rightarrow$ $\underset{\text{max}}{x = a}$

$f''(b) > 0$ minima $\rightarrow$ $\underset{\text{min}}{x = b}$

Max value $\quad f(x)\Big|_{x=a}$

Min value $\quad f(x)\Big|_{x=b}$

App of derivatives

$$C(x) = \text{Fix cost} + \text{Variable Cost}$$

$$MC(x) = \frac{d}{dx} C(x)$$

$$AC(x) = \frac{C(x)}{x}$$

$$R(x) = px$$

$$AR(x) = \frac{R(x)}{x} = p$$

$$MR(x) = \frac{d}{dx} R(x)$$

$$P(x) = R(x) - C(x)$$

$$MP(x) = \frac{d}{dx} P(x)$$

$$AP(x) = \frac{P(x)}{x}$$

$$C = f(i)$$

marginal propensity of consumption

$$MPC = \frac{d}{di}(C)$$

$$MPS = 1 - MPC$$

$\hookrightarrow$ Saving.