

Chapter 1 :-
Ratio and Proportion, Indices, Logarithms :-

Q1:- Ratio, Unit :- Test

To Compare two quantities in some units.

5 kg Rice with 2 kg flour

Expressed as a fraction $\frac{R}{F} = \frac{5}{2} = 5:2$ (5 is to 2)
(5 ratio 2)

In simplest fraction $\frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2} = (3:2)$

Expressed as $a:b$ or $\frac{a}{b}$
 $a \rightarrow$ Numerator
 $b \rightarrow$ Denominator
 a - antecedent (1st term)
 b - Consequent (2nd term)

Ratio is always unit less

Q:-1 Ratio of 150 gm and 2 kg is

a) 3:40

b) 15:1

c) 5:40

d) Can't be determined

$$\begin{aligned} & \frac{150 \text{ gm}}{2 \text{ kg}} \\ &= \frac{3 \times 150 \text{ gm}}{2 \times 1000 \text{ gm}} \\ &= \frac{3}{40} = (3:40) \end{aligned}$$

Important

$$\begin{aligned} 1 \text{ kg} &= 1000 \text{ gm} \\ \frac{1}{2} \text{ kg} &= 500 \text{ gm} \\ 1 \text{ gm} &= \frac{1}{1000} \text{ kg} \\ \text{kg} &\xrightarrow{\times 1000} \text{ gm} \\ &\xleftarrow{\times \frac{1}{1000}} \end{aligned}$$

2. The ratio of two quantities 3:4. If the antecedent is, the consequent is.

a) 16

b) 60

c) 22

d) 20

$$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

Topic:- Transformation of Ratios

$$\text{Ratio} = a:b$$

1.) Duplicate Ratio of $a:b = a^2:b^2$

Example

$$= 2:3 \text{ Duplicate } (2)^2:(3)^2$$

$$\text{ratio} = 4:9$$

2.) Triplicate Ratio of $a:b = a^3:b^3$

Example

$$= 2:3 \text{ Triplicate } (2)^3:(3)^3$$

$$\text{ratio} = 8:27$$

3.) Sub-duplicate Ratio of $a:b = \sqrt{a}:\sqrt{b}$

Example

$$= 16:25 \text{ Sub } \sqrt{16}:\sqrt{25}$$

$$\text{D.R} = 4:5$$

4.) Sub-Triplicate Ratio of $a:b = \sqrt[3]{a}:\sqrt[3]{b}$ or $a^{\frac{1}{3}}:b^{\frac{1}{3}}$

Example

$$= 27:64 \text{ Sub } (27)^{\frac{1}{3}}:(64)^{\frac{1}{3}}$$

$$\text{T.R} = 3:4$$

5.) Inverse ratio of $a:b = b:a$

Example

$a:b \rightarrow a \& b \rightarrow$ integers
 $2:3 \xrightarrow{\text{Inv ratio}} 3:2$
 Commensurable ratio

$a \& b \rightarrow$ non-integers
 Irrational
 Incommensurable ratio
 $x:y \rightarrow \begin{cases} x > y & \text{(greater inequality ratio)} \\ x < y & \text{(lesser inequality)} \\ x = y & \text{(equal ratio } \rightarrow 1:1) \end{cases}$

1. The inverse ratio of $11:15$ is

- a.) $15:11$ ✓ b.) $\sqrt{11}:\sqrt{15}$ c.) $121:225$ d.) None of these

$11:15$ Inverse ratio $15:11$

2. The ratio of the quantities is $5:7$. If the consequent of its inverse ratio is 5 , the antecedent is

- a.) 5 ✓ b.) $\sqrt{5}$ c.) 7 ✓ d.) None of these

$\frac{5}{7}$ IR $\frac{7}{5} \rightarrow A$
 $5 \rightarrow C$

3. The duplicate ratio of $3:4$ is

- a.) $\sqrt{3}:2$ b.) $4:3$ c.) $9:16$ ✓ d.) None

$(3)^2 : (4)^2$
 $= 9:16$

4. The sub-duplicate ratio of 25:36 is

- a.) 6:5 b.) 36:25 c.) 50:72 d.) 5:6 ✓

$$\sqrt{25} : \sqrt{36} \\ = 5 : 6$$

5. The triplicate ratio of 2:3 is

- a.) 8:27 ✓ b.) 6:9 c.) 3:2 d.) None

$$(2)^3 : (3)^3 \\ = 8 : 27$$

6. The sub-triplicate ratio of 8:27 is

- a.) 27:8 b.) 24:81 c.) 2:3 ✓ d.) None

$$(8)^{1/3} : (27)^{1/3} \\ = 2 : 3$$

Topic: Compound Ratio

Compound Ratio of a:b and c:d

(Combined)

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} = ac:bd$$



product of all ratios given

1. The ratio compounded of 2:3, 3:4, 5:6 and 8:10 is

a) 1:1

b) 1:5

c) 3:8

d) None

$$\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6} \times \frac{8}{10} = \frac{720}{720} = 1:1$$

2. The ratio compounded of 4:9 and the duplicate ratio of 3:4 is

a) 1:4

b) 1:3

c) 3:1

d) None

$$\frac{4}{9} \times \frac{(3)^2}{(4)^2} = \frac{4 \times 9}{9 \times 16} = \frac{1}{4} = 1:4$$

3. The ratio compounded of 4:5, 9:7, the triplicate ratio of 3:4 and the triplicate ratio of 2:3 is

a) 4:512

b) 3:32

c) 1:12

d) None

$$\frac{4}{5} \times \frac{9}{7} \times \frac{(3)^3}{(4)^3} \times \frac{(2)^3}{(3)^3} = \frac{4}{5} \times \frac{9}{7} \times \frac{8}{64} = \frac{9}{70} = 9:70$$

4. If $a:b = 2:3$, $b:c = 5:7$, then find $a:b:c$

- a) 10:15:21 b) 6:9:14 c) 4:6:21 d) None

$$\frac{a}{b} = \frac{2}{3}, \quad \frac{b}{c} = \frac{5}{7}$$

$$\frac{a}{b} = \frac{2 \times 5}{3 \times 5}; \quad \frac{b}{c} = \frac{5 \times 3}{7 \times 3}$$

$$\frac{a}{b} = \frac{10}{15}; \quad \frac{b}{c} = \frac{15}{21}$$

$$a:b:c = 10:15:21$$

5. If $a:b = 3:4$, the value of $(2a+3b):(3a+4b)$ is

- a) 54:25 b) 8:25 c) 17:24 d) 18:25

$$a:b = 3:4, \quad \frac{2a+3b}{3a+4b} = ?$$

$$\frac{a}{b} = \frac{3}{4}$$

$$\frac{\frac{2a+3b}{b}}{\frac{3a+4b}{b}} = \frac{2\left(\frac{3}{4}\right)+3}{3\left(\frac{3}{4}\right)+4}$$

$$= \frac{\frac{6+12}{4}}{\frac{9+16}{4}} = \frac{18}{25}$$

$$= 18:25$$

6. Two numbers are in the ratio 2:3. If 4 be subtracted from each, they are in the ratio 3:5. The numbers are.

- a.) [6, 24] b.) [4, 6] c.) [2, 3] d.) None

$$i.) \frac{16}{24} = \frac{2}{3}$$

$$ii.) \frac{16-4}{24-4} = \frac{12}{20} = \frac{3}{5}$$

7. The angles of a triangle are in ratio 2:7:11. The angles are

- a.) (20°, 70°, 90°) b.) (30°, 70°, 80°) c.) (18°, 63°, 99°) d.) None *

The angles of a triangle sum = 180°

$$2x + 7x + 11 = 180^\circ$$

$$9x = 180^\circ$$

$$x = \frac{180^\circ}{9}$$

$$2(9), 7(9), 11(9)$$

$$18, 63, 99$$

8. The ratio of two numbers is 7:10 and their difference is 45. The numbers are.

- a.) (200, 305) b.) (185, 290) c.) (245, 350) d.) None

$$\frac{200}{305} = \frac{40}{61}$$

$$\frac{185}{290} = \frac{37}{58}$$

$$\frac{245}{350} = \frac{49}{70} = \frac{7}{10} = 7:10$$

9. P, Q and R are three cities. The ratio of average temperature between P and Q is 11:12 and that between P and R is 9:8. The ratio between the average temperature of Q and R is.

- a) 22:27 b) 27:22 c) 32:33 d) End of 16 years

$$P:Q = 11:12 \Rightarrow \frac{Q}{P} = \frac{12}{11} \times \frac{9}{9} = \frac{108}{99}$$

$$P:R = 9:8 \Rightarrow \frac{P}{R} = \frac{9}{8} \times \frac{11}{11} = \frac{99}{88}$$

$$\frac{Q}{P} \times \frac{P}{R} = \frac{108}{99} \times \frac{99}{88}$$

$$\frac{Q}{R} = \frac{108}{88} = \frac{27}{22} = 27:22$$

10. If $x:y = 3:4$, the value of $x^2y + xy^2 : x^3 + y^3$ is

- a) 13:12 b) 12:13 c) 11:31 d) None

$$\frac{x}{y} = \frac{3}{4}, \text{ The value } \frac{x^2y + xy^2}{x^3 + y^3}$$

$$= \frac{\frac{x^2y}{y^3} + \frac{xy^2}{y^3}}{\frac{x^3}{y^3} + \frac{y^3}{y^3}} = \frac{\frac{9}{16} + \frac{3}{4}}{\frac{27}{64} + 1}$$

$$= \frac{\frac{x^2}{y^2} + \frac{x}{y}}{\frac{x^3}{y^3} + 1} = \frac{\frac{9+12}{16}}{\frac{27+64}{64}} = \frac{21}{16} \times \frac{64}{91} = \frac{12}{13} = 12:13$$

11. If $2x:3y$ is duplicate ratio of $2x-k:3y-k$, then

- a.) $k^2 = 6xy$ b.) $k = 6xy$ c.) $2k = 3xy$ d.) None

$$\frac{2x}{3y} = \frac{(2x-k)^2}{(3y-k)^2}$$

GBT Put $x=1, y=1$

$$= \frac{2}{3} = \frac{(2-k)^2}{(3-k)^2}$$

$$= \frac{2}{3} = \frac{4+k^2-4k}{9+k^2-6k}$$

$$= \frac{18+2k^2-12k}{9+k^2-6k} = \frac{12+3k^2-12k}{9+k^2-6k}$$

$$= \frac{18-12}{9+k^2-6k} = \frac{3k^2-2k^2}{9+k^2-6k}$$

$$= 6 = k^2 \quad / \quad k^2 = 6(1)(1) \rightarrow k^2 = 6$$

12. If $P:q$ is the sub-duplicate ratio of $p-z^2$:
 $q-z^2$ then z^2 is

- a.) $\frac{Pq}{P+q}$ b.) $\frac{P}{P+q}$ c.) $\frac{Pq}{P-q}$ d.) None

$$\frac{P}{q} = \frac{\sqrt{P-z^2}}{\sqrt{q-z^2}}$$

GBT Put $P=1, q=2$

$$\frac{1}{2} = \frac{\sqrt{1-z^2}}{\sqrt{2-z^2}}$$

sq. both sides

$$= \frac{1}{4} = \frac{1-z^2}{2-z^2} = \frac{2-z^2}{2-z^2} = \frac{4-4z^2}{2-z^2}$$

$$= 2-z^2 = 4-4z^2$$

$$= 4z^2 - z^2 = 4-2$$

$$= 3z^2 = 2$$

$$= z^2 = \frac{2}{3}$$

13. Divide 27 into two parts, so that 5 times the first and 11 times the second together equal to 195 then the ratio of first and second part is.

- a) 17:10 b) 15:12 c) 14:13 d) 16:11

$$\begin{array}{l} 17 \rightarrow 17 \times 5 = 85 \\ 10 \rightarrow 10 \times 11 = 110 \\ \hline 195 \end{array}$$

14. In a department the number of males and females are in the ratio 3:2. If two males and 5 females join department, the ratio becomes 1:1, initially the number of female in the department is.

- a) 9 b) 6 c) 3 d) 8

$$\frac{M}{f} = \frac{3}{2} \Rightarrow M = \frac{3f}{2}$$

$$\frac{M+2}{f+5} = \frac{1}{1}$$

$$M+2 = f+5$$

$$\frac{3f}{2} + 2 = f + 5$$

$$\frac{3f}{2} - f = 5 - 2$$

$$\frac{f}{2} = 3$$

$$f = 6$$