Vishwas CA

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CA FOUNDATION

AS PER NEW SYLLABUS !!!

QUANTITATIVE APTITUDE

COVERED :

- ICAI Study Mat. Content Simplified With All Types Of Ques.
- Summary And Short Notes.
- Tricks (Calculator/Short Cut)
- PYQs And Other Imp. Ques.

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CHAPTER-1

RATIO AND PROPORTION, INDICES, LOGARITHM

RATIO

DEFINITION OF RATIO

A ratio is a comparison between two or more quantities of the same kind, (like apple is comparable to orange but not to gold).

And they must be in same units, E.g. - Kg and gram are two different units of weights.

It is represented by a: b where a & b are two quantities of same kind and same unit.

Then, fraction of a and b is called the ratio of a and b which is written as $\frac{a}{b}$

- The quantities a and b are called terms of the ratio.

  E.g.: In 3: 4, 3 and 4 are terms of ratio.

- a is called the first term or antecedent.

  E.g.: In 3 : 4 , 3 is first term or antecedent.

- b is called the second term or consequent.

  E.g.: In 3 : 4 , 4 is second term or consequent.

Q.1 The ratio of two quantities is 3 : 4. What is antecedent and what is consequent?

(a) 3, 4 (b) 4, 3 (c) 3, 3 (d) None of these

Ans. (a)

Q. 2 The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is

(a) 16 (b) 60 (c) 22 (d) 20

Ans. (d)

Q.3 The ratio of two quantities is 5 : 8. If the consequent is 3: 40, then the antecedent is

(a) 25 (b) 40 (c) 5 (d) 8

Ans. (a)

KEY CONCEPT REGARDING RATIOS

- Both terms of a ratio can be multiplied or divided by the same (non-zero) number.
  E.g.: If we have a ratio of 2 : 3, we can multiply both terms by 2 to get 4 : 6 and do the similar with division.
- Usually, a ratio is expressed in lowest terms (or simplest form).
  E.g.: If we have a ratio of 6 : 8, we can simplify it to 3 : 4 by dividing both terms by their greatest common divisor, which is 2.
- The order of the terms in a ratio is important.
  E.g.: The ratio 2 : 3 is different from the ratio 3 : 2. They both represent different quantities.
- Ratio exists only between quantities of the same kind.
  E.g.: We can compare the ratio of apples to oranges, but not apples to minutes.
- Quantities to be compared (by division) must be in the same units.
  E.g.: If we want to compare the ratio of the lengths of two objects, one measured in inches and the other in centimeters, we need to convert them to the same unit (e.g., both in inches or both in centimeters) before dividing them.
- To compare two ratios, convert them into equivalent like fractions.
  E.g.: If we have the ratio 2 : 3 and the ratio 4 : 5, we can convert them into equivalent fractions by converting to a common denominator. The ratios become $\frac{10}{15}$ and $\frac{12}{15}$, respectively, and we can then compare them easily.

Q.4 Which ratio is greater?

1. $3\frac{1}{2} : 4\frac{1}{2}$ or 2.5 : 4.5

(a) $3\frac{1}{2} : 4\frac{1}{2}$ (b) 2.5 : 4.5 (c) Both are equal (d) Cannot be determined

Ans. (a)

Q.5 Earlier shoe company produced 2000 shoes in a day. They increased their production 5: 6. What will be the new production?

(a) 2500 (b) 2400 (c) 2000 (d) 3000

Ans. (b)

Q.6 If Rajni eats 12 chapati in a day. If she reduces her chapati by 6: 5. How many chapati does she eat now?

(a) 12 (b) 8 (c) 10 (d) 15

Ans. (c)

Q.7 Simplify the ratio $\frac{1}{2} : \frac{2}{3} : \frac{4}{5}$

(a) 15 : 20 : 24 (b) 10 : 25 : 24 (c) 2 : 3 : 5 (d) 1 : 2 : 4

Ans. (a)

Q.8 Simplify the ratio $\frac{1}{3} : \frac{1}{8} : \frac{1}{6}$

(a) 8 : 3 : 4 (b) 8 : 5 : 1 (c) 7 : 2 : 6 (d) 1 : 3 : 5

Ans. (a)

Q.9 Anand earns ₹80 in 7 hours and Promod ₹90 in 12 hours. The ratio of their earnings is

(a) 32: 21 (b) 23: 12 (c) 8: 9 (d) None of these

Ratio and Proportion, Indices, Logarithm

Ans. (a)

Q.10 The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 km in 5 hours, the speed of the first train is

(a) 10 km/hr (b) 50 km/hr (c) 70 km/hr (d) None of these

Ans. (c)

Q.11 40 feet rope is cut into 2. One piece is 18 feet longer than the other. What is the length of the shorter piece?

(a) 11 (b) 12 (c) 18 (d) 22

Ans. (c)

Q. 12 The ratio of two numbers is 7:10 and their difference is 105. The numbers are

(a) (200, 305) (b) (185, 290) (c) (245, 350) (d) None of these

Q.13 Division of ₹324 between X and Y is in the ratio 11 : 7. X & Y would get rupees

(a) (204, 120) (b) (200, 124) (c) (180, 144) (d) None of these

Ans. (d)

Q.14 If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be

(a) 5 : 8 (b) 8 : 5 (c) 5 : 3 (d) 3 : 5

Ans. (b)

Q.15 The ratio of the number of boys to the number of girls in a school of 720 students is 3 : 5. If 18 new girls are admitted in the school, then find how many new boys may be admitted so that the ratio of the number of boys to the number of girls may change to 2 : 3.

(a) 40 (b) 42 (c) 45 (d) None of these

Ans. (b)

Q.16 A bag contains 25 paise, 10 paise, 5 paise are in the ratio 3 : 2 : 1. The total value of coins is Rs. 40, then the number of 5 paise coin is:

(a) 40 (b) 80 (c) 240 (d) 480

Ans. (a)

Q.18 The ratio of boys and girls in a college is 7:5. If the number of boys increase by 40% and the number of girls increase by 20%, what will be the new ratio of boys and girls?

(a) 4:2 (b) 30:49 (c) 49:30 (d) 49:47

Ans. (c)

INVERSE RATIO

One ratio is the inverse of the other if their product is 1.

E.g.: b : a is the inverse ratio of a : b since $\frac{a}{b} \times \frac{b}{a} = 1$

Q.19 The inverse ratio of 11 : 15 is

(a) 15 : 11 (b) $\sqrt{11} : \sqrt{15}$ (c) 121 : 225 (d) None of these

Ans. (a)

Q.20 The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 25, the antecedent is

(a) 7 (b) $\sqrt{35}$ (c) 35 (d) None of these

Ans. (c)

Greater Inequality Ratio	Less Inequality Ratio
A ratio a : b will be of greater inequality if a > b	A ratio a : b will be of less inequality if a < b

The ratio 2 : 1 represents a greater inequality because the first term (2) is greater than the second term (1)

The ratio 3 : 4 represents less inequality because the first term (3) is smaller than the second term (4).

Q.21 The ratio of 8 : 15 is

(a) Greater inequality ratio (b) Less inequality ratio

(c) Equal to 1 (d) None of these

Ans. (b)

Q.22 The ratio of 20 : 15 is

(a) Greater inequality ratio (b) Less inequality ratio

(c) Equal to 1 (d) None of these

Ans. (a)

COMPOUNDED RATIO

A ratio resultant of compounding two or more ratios.

E.g.: If there are ratios 5 : 6, 7 : 8 and 9 : 10 then the compounded ratio will be:

$(5 \times 7 \times 9) : (6 \times 8 \times 10)$

$$= \frac{5 \times 7 \times 9}{6 \times 8 \times 10} = \frac{7 \times 3}{2 \times 8 \times 2} = \frac{21}{32}$$

Q.23 The ratio compounded of 5 : 6,9 : 2,4 : 3 and 1 : 5 is

(a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) None of these

Ans. (a)

Q.24 The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is

(a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) None of these

Ans. (a)

Q.25 a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7, then a : d is

(a) 2. If 4 : 35 (b) 8 : 15 (c) 16 : 35 (d) 7 : 1513

Ans. (c)

Q.26. P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is

(a) 22 : 27 (b) 27 : 22 (c) 32 : 33 (d) None of these

Ans. (b)

- When the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are considered to be **Commensurable**.
- On the other hand, if the ratio cannot be expressed as a ratio of two integers, the quantities are considered to be **Incommensurable**.

DUPLICATE RATIO

A ratio compound to itself is called a Duplicate ratio. For ratio a : b, its duplicate ratio will be $a^2 : b^2$

E.g.: For ratio 2 : 3, its duplicate ratio will be $2^2 : 3^2 = 4 : 9$

TRIPLICATE RATIO

For ratio a : b, its triplicate ratio will be $a^3 : b^3$

E.g.: For ratio 2 : 3, its triplicate ratio will be $2^3 : 3^3 = 8 : 27$

Q.27. The duplicate ratio of 5 : 9 is

(a) 81 : 25 (b) 9 : 5 (c) 25 : 81 (d) None of these

Ans. (c)

Q.28. The triplicate ratio of 5 : 7 is

(a) 25 : 49 (b) 5 : 7 (c) 125 : 343 (d) None of these

Ans. (c)

SUB DUPLICATE RATIO

For a : b, the sub duplicate ratio will be $\sqrt{a} : \sqrt{b}$

E.g.: For ratio 4 : 9, its sub duplicate ratio will be $\sqrt{4} : \sqrt{9} = 2 : 3$

SUB TRIPLICATE RATIO

For a : b, the sub triplicate ratio will be $\sqrt[3]{a} : \sqrt[3]{b}$

E.g.: For ratio 8 : 27, its sub triplicate ratio will be $\sqrt[3]{8} : \sqrt[3]{27} = 2 : 3$

Q.29. The sub-duplicate ratio of 121 : 25 is

(a) 6 : 5 (b) 36 : 25 (c) 50 : 72 (d) 11 : 5

Ans. (d)

Q.30 The sub - triplicate ratio of 64 : 27 is

(a) 16 : 9 (b) 4 : 3 (c) 27 : 64 (d) None of these

Ans. (b)

Q.31. The ratio compounded of triplicate ratio of 2 : 3 and the duplicate ratio of 3 : 4 is

(a) 1 : 4 (b) 2 : 3 (c) 1 : 6 (d) None of these

Ans. (c)

Q.32. The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 3 : 2, sub duplicate ratio 25 : 81 and sub - triplicate ratio of 1000 : 27 is

(a) 1 : 4 (b) 3 : 25 (c) 4 : 1 (d) None of these

Ans. (c)

Q.33. If A : B = 2 : 5, then (10A + 3B): (5A + 2B) is equal to:

(a) 7 : 4 (b) 7 : 3 (c) 6 : 5 (d) 7 : 9

Ans. (a)

Q.34. If 2s : 3t is the duplicate ratio of 2s - p : 3t - p, then

(a) $p^2 = 6st$ (b) $p = 6st$ (c) $2p = 3st$ (d) None of these

Ans. (a)

Q.35. If p : q is the sub-duplicate ratio of $p - x^2 : q - x^2$, then x^2 is

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{p+q}$ (d) None of these

Ans. (c)

Q.36. Three friends A, B, and C shared a total amount of ₹3,000 in the ratio of 5 : 3 : 2. How much did B receive?

(a) ₹700 (b) ₹750 (c) ₹900 (d) ₹300

Ans. (c)

Q.37. If x : y = 5 : 2 then the value of (8x + 9y) : (8x + 2y)

(a) 22 : 29 (b) 26 : 61 (c) 29 : 22 (d) 61 : 26

Ans. (c)

Q.38. Find the compounded ratio of 3 : 5 , 2 : 3, 5 :1 and 4 : 3.

(a) 4 : 1 (b) 4 : 5 (c) 3 : 8 (d) 8 : 3

Ans. (d)

Q.39. Find the compounded ratio of 7 : 4, duplicate ratio of 5 : 8, triplicate ratio of 2 : 7, sub-duplicate ratio of 16 : 25 and sub-triplicate ratio of 125 : 343.

(a) 1 : 40 (b) 4 : 45 (c) 1 : 56 (d) 7 : 58

Ans. (c)

CONTINUED RATIO

It is the comparison between the magnitudes of three or more quantities of the same kind. The continued ratio of three similar quantities a,b,c is written as a : b : c.

E.g.: Consider three similar quantities: Lengths of three sides of a triangle.

Let's say the lengths of the sides are a = 2 units, b = 4 units and c = 6 units.

The continued ratio of these side lengths is written as a : b : c, which in this case is 2 : 4 : 6.

This indicates the side lengths in comparison to each other. The ratio can be simplified to 1 : 2 : 3 by dividing all the terms by their greatest common divisor, which in this case is 2.

Q.40. The angles of a triangle are in ratio 2 : 7 : 11. The angles are

(a) (20°, 70°, 90°) (b) (30°, 70°, 80°)

(c) (18°, 63°, 99°) (d) None of these

Ans. (c)

Q.41 Find three numbers which are in the ratio 1 : 2 : 3, such that the sum of their squares is equal to 504.

(a) 6, 12, 18 (b) 3, 6, 9 (c) 4, 8, 12 (d) 5, 10, 15

Ans. (a)

PROPORTION

WHAT IS PROPORTION?

- An equality of two ratios is called a proportion. If we take four quantities a, b, c, d, they will be said to be in proportion if a/b = c/d i.e., ad = bc

- a, b, c and d are called its **terms** of the proportion.
- First and fourth terms are called **extremes.**
- Second and third terms are called **means (or middle terms).**
- Also, it can be written as **a : b :: c : d.**

E.g.: Consider four quantities: a = 2, b = 4, c = 3, and d = 6.
In this case, the terms of the proportion are 2, 4, 3 and 6.
The first and fourth terms, a = 2 and d = 6, are called the extremes.
The second and third terms, b = 4 and c = 3, are called the means or middle terms.
We can express this proportion as a : b :: c : d, which indicates the equality of the ratios a/b and c/d.
The ratio a/b is 2/4, which can be simplified to 1/2.
The ratio c/d is 3/6, which can also be simplified to 1/2.
Thus, we can say a = 2, b = 4, c = 3, and d = 6 are in proportion.19

CROSS PRODUCT RULE

Product of extremes = Product of means

Let's consider the following proportion: a, b, c and d
i.e., $\frac{a}{b} = \frac{c}{d}$
In this proportion, a and d are the extremes, while b and c are the means.
The cross-product rule states that the product of the extremes (a × d) is equal to the product of the means (b × c)

Q.42. 8, 12, *, 15 are in proportion. Then * is

(a) 20 (b) 10 (c) 15 (d) None of these

Ans. (b)

Q.43. The fourth proportional to 2, 4, 8 is

(a) 16 (b) 32 (c) 48 (d) None of these

Ans. (a)

Q.44. The fourth proportional to 2a, a^2, c is

(a) $\frac{ac}{2}$ (b) ac (c) $\frac{2}{ac}$ (d) None of these

Ans. (a)

Q.45. If four numbers $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{x}$ are proportional then x is

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{15}{2}$ (d) None of these

Q.46 The number which has the same ratio to 22 that 5 has to 11 is

(a) 11 (b) 10 (c) $\frac{15}{2}$ (d) None of these

Ans. (b)

CONTINUOUS PROPORTION

- If 3 quantities: a, b, c are of same kind (in same units), they will be in continuous proportion if a : b = b : c

  i.e., $\frac{a}{b} = \frac{b}{c}$ = or $b^2 = ac$

- The middle term b is called the mean proportional between a and c.
- a is first proportional and c is the third proportional.

E.g.: Consider three quantities: a = 2, b = 4 and c = 8

Here, $\frac{a}{b} = \frac{2}{4} = \frac{1}{2}$

$\frac{b}{c} = \frac{4}{8} = \frac{1}{2}$

Since, $\frac{a}{b} = \frac{b}{c}$ thus a, b, c are in continuous proportion.

We can say, 2 and 8 are first and third proportional respectively and 4 is the mean proportional.

Q.47 The mean proportional between 16, 25 is

(a) 18 (b) 19 (c) 20 (d) None of these

Ans. (c)

Q.48 The third proportional to 2, 6 is

(a) 12 (b) 18 (c) 24 (d) None of these

Ans. (b)

Q.48 The mean proportional between $12x^2$ and $27y^2$ is

(a) 18xy (b) 81xy (c) 8xy (d) None of these

Ans. (a)

Q.49 Find 2 numbers such that the mean proportional between them is 18 and 3rd proportional between them is 144.

(a) 9; 36 (b) 8; 32 (c) 7; 28 (d) 6; 24

Ans. (a)

Trick Go by choice

Q.50 If $A = \frac{B}{2} = \frac{C}{5}$, then A : B : C is

(a) 1 : 1 : 1 (b) 5 : 2 : 1 (c) 1 : 2 : 5 (d) None of these

Ans. (c)

Q.51 If A : B = 5 : 6 and B : C = 7 : 8, what is the value of A : B : C?

(a) 48 : 42 : 35 (b) 5 : 42 : 8
(c) 60 : 42 : 58 (d) 35 : 42 : 48

Ans. (d)

Q.52 If a : b : c = 2 : 1 : 3 and 2a – 3b + c = 8 then a + b + c is

(a) 8 (b) 6 (c) 12 (d) 15

Ans. (c)

Q.53 Find 2 numbers such that the mean proportional between them is 14 and 3rd proportional between them is 112.

(a) 7, 28 (b) 6, 36 (c) 8, 54 (d) None of these

Ans. (a)

INVERSE PROPORTION

If a ratio is equal to the reciprocal of the other, then either of them is in inverse (or reciprocal) proportion of the other.

Note: In a ratio a : b, both quantities must be of the same kind while in a proportion i.e., a : b = c : d, all the four quantities need not be of the same type.

- **Properties of Proportion:**

1. If a : b = c : d, then ad = bc **(Product of extremes = Product of means)**

E.g.: If 2 : 3 =4 : 6, then (2)(6) = (3)(4), which simplifies to 12 = 12

2. If a : b = c : d, then b : a = d : c **(Invertendo)**

E.g.: If 5 :10 = 2 : 4, then 10 : 5 = 4 : 2

3. If a : b = c : d then a : c = b : d **(Alternendo)**

E.g.: If 3 : 9 =2 : 6, then 3 : 2 = 9 : 6

4. If a : b = c : d, then (a + b) : b = (c + d) : d **(Componendo)**

E.g.: If 4 : 6 = 2 : 3 then (4 + 6) : 6 = (2 + 3) : 3, which simplifies to 10 : 6 = 5 : 3

5. If a : b = c : d, then (a - b) : b = (c - d) : d **(Dividendo)**

E.g.: If 9 : 4 = 18 : 8, then (9 - 4) : 4 = (18 -8) : 8, which simplifies to 5 : 4 = 10 : 8

6. If a : b = c : d, then (a + b) : (a - b) = (c + d) : (c - d) **(Componendo and Dividendo)**

E.g.: If 5 : 2 = 10 : 4, then (5 + 2) : (5 - 2)= (10 + 4) : (10 - 4), which simplifies to 7 : 3 = 14 : 6.

7. If a : b = c : d = e : f = ..., then each of these ratios is equal to (a + c + e +...) : (b + d + f + ----) **(Addendo)**

E.g.: If 2 : 4 = 3 : 6 = 1 : 2, then each of these ratios is equal to (2 + 3 + 1) : (4 + 6 + 2), which simplifies to 6 :12.

8. If a : b = c : d = e : f = ….. , then each of these ratios is equal to (a - c - e - ...) : (b - d - f - .....) **(Subtrahendo)**

E.g.: If 5 : 10 =3 : 6 = 1 : 2, then each of these ratios is equal to (5 - 3 - 1) : (10 – 6 - 2), which simplifies to 1 : 2

Q.54 If $\frac{X}{Y} = \frac{Z}{W}$ = , implies $\frac{Y}{X} = \frac{Z}{W}$, then the process is called

(a) Dividendo (b) Componendo (c) Alternendo (d) None of these

Ans. (d)

Q.55 If $\frac{p}{q} = \frac{r}{s} = \frac{p-r}{q-s}$, the process is called (ICAI)

(a) Subtrahendo (b) Addendo (c) Invertendo (d) None of these

Ans. (a)

Q.56 if $\frac{a}{4} = \frac{b}{5} = \frac{c}{q}$, then $\frac{a+b+c}{c}$ is

(a) 4 (b) 2 (c) 7 (d) None of these

Ans. (b)

Q.57 if $\frac{a}{4} = \frac{b}{5}$, then

(a) $\frac{a+4}{a-4} = \frac{b-5}{b+5}$ (b) $\frac{a+4}{a-4} = \frac{b+5}{b-5}$ (c) $\frac{a-4}{a+4} = \frac{b-5}{b+5}$ (d) None of these

Ans. (b)

Q.58. if $\frac{x}{2} = \frac{y}{3} = \frac{z}{7}$, then the value of x : y is

(a) $\frac{6}{23}$ (b) $\frac{23}{6}$ (c) $\frac{2}{3}$ (d) $\frac{17}{6}$

Ans. (c)

Q.59. if $\frac{5x-3y}{5y-3x} = \frac{3}{4}$, then the value of x : y is

(a) 2 : 9 (b) 7 : 2 (c) 7 : 9 (d) None of these

Ans. (d)

Q.60. $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$, then (b - c) x+ (c – a) y+ (a – b) z is equal to

(a) 1 (b) 0 (c) 5 (d) None of these

Ans. (b)

Q.61. The sum of the ages of 3 persons is 150 years. 10 years ago, their ages were in the ratio 7 : 8 : 9. Their present ages are

(a) (45, 50, 55) (b) (40, 60, 50) (c) (35, 45, 70) (d) None of these

Ans. (a)

Q.62. if a:b = 4:1, then $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ is :

(a) $\frac{5}{2}$ (b) 4 (c) 5 (d) None of these

Ans. (a)

Q.63. A person has assets worth ₹1,48,200. He wishes to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this asset, the share of his son will be:

(a) ₹24,700 (b) ₹49,400 (c) ₹74,100 (d) ₹37,050

Ans. (b)

Q.64. The students of two classes are in the ratio 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6 then the number of students in each class is:

(a) 30, 40 (b) 25, 24 (c) 40, 60 (d) 50, 70

Ans. (d)

Q.65. A dealer mixes rice costing ₹13.85 per kg with rice costing ₹15.54 and sell the mixture at ₹17.60 per kg so, he earned a profit of 14.6% on his sale price. the proportion in which he mixes the two qualities of rice is

(a) 3 : 7 (b) 5 : 7 (c) 7 : 9 (d) 9 : 11

Ans. (a)

Q.66. Two numbers are in the ratio 7 : 8. If 3 is added to each of them, their ratio becomes 8 : 9. The number are:

(a) 14, 16 (b) 24, 27 (c) 21, 24 (d) 16, 18

Ans. (c)

Q.67 Two numbers are in the ratio 2 : 3 and the difference of their squares is 320. The numbers are

(a) 12, 18 (b) 16, 24 (c) 14, 21 (d) None of these

Ans. (b)

Q.68 The monthly income of A and B are in the ratio 4 : 5 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹150 per month, find their monthly incomes.

(a) (₹40, ₹50) (b) (₹50, ₹40) (c) (₹400, ₹500) (d) None of these

Ans. (c)

INDICES

The power, also known as the **index**, tells you how many times you have to multiply the number by itself and number will be called as **base.**

In other words: If n is a positive integer, and 'a' is a real number, i.e. n ε N and a ε R (where N is the set of positive integers and R is the set of real numbers).

Then, a^n= a × a × a × a….to n factors

Q.69. The values of

(I) 2^4 (II) 3^3 (III) 18^2

(a) 8, 9, 36 (b) 16, 27, 36 (c) 16, 27, 324 (d) None of these

Ans. (c)

- Any base to the power (or index) 0 is 1 i.e., a0 = 1 for a ≠ 0.

Q.70. The value of

(I) 12^0 (II) 32^0 (III) $(\frac{1}{18})^0$

(a) 12, 32, 18 (b) 12, 32, $\frac{1}{18}$ (c) 1, 1, 1 (d) None of these

Ans. (c)

Finding the roots : If you have to find nth root of base a i.e. n a it will give value equal to 1n a .

Negative Powers: If you have power as negative integer, -n (let) for base a, then n n 1 a a − = .

Calculator trick to find n^{th} root:

1. Type the number
2. Press '√' for 12 times
3. Subtract 1 from it and then divide by n
4. Add '1'
5. Press '× =' for 12 times

E.g.: To find $(16)^{\frac{1}{4}}$

1. Type '16'
2. Press '√' for 12 times
3. Subtract 1 from it and then divide by 4
4. Add '1'
5. Press '× =' for 12 times33
We will get 2.00035 ≈ 2

Therefore, $(16)^{\frac{1}{4}}$ =2

Q.71 Find out the values of each of the indices:

(I) $\sqrt{4}$ (II) 16^{-4} (III) $(3)^{-3}$

(a) $2, \frac{1}{2}, \frac{1}{9}, \frac{1}{3^{10}}$ (b) $2, \frac{1}{2}, \frac{1}{27}, \frac{1}{3^5}$ (c) $2, \frac{1}{2}, \frac{1}{27}, \frac{1}{3^{10}}$ (d) None of these.

Ans. (c)

Q.72. The value of $(\frac{2p^2q^3}{3xy})^0$ where $p, q, x, y \neq 0$

(a) 0 (b) 1 (c) 2 (d) None of these.

Ans. (b)

LAW 1

$a^m \times a^n = a^{m+n}$

- **Case-I:** When m and n are positive integers; then $a^m \times a^n = a^{m+n}$
  E.g.: $6^7 6^2 = 6^{7+2} = 6^9$
- **Case-II:** When n = –m, m is positive integer then $a^m \times a^{-m} = a^0 = 1$
  E.g.: $2^3 2^{-3} = 2^{3-3} = 2^0 = 1$
- **Case-III:** When m and n are any integers
  Eg,: $7^{\frac{4}{3}} 7^{-2} = 7^{\frac{4}{3}-2} = 7^{\frac{4-6}{3}} = 7^{-\frac{2}{3}}$

Q.73. Find the value of $5^{\frac{3}{5}} . 5^{\frac{7}{2}}$

(a) $5^{\frac{-4}{7}}$ (b) $5^{-\frac{41}{10}}$ (c) $5^{\frac{29}{10}}$ (d) $5^{-\frac{29}{10}}$

Ans. (d)

Q.74. The value of $8^{\frac{1}{3}}$ is

(a) $3\sqrt{2}$ (b) 4 (c) 2 (d) None of these

Ans. (c)

Q.75. The value of $2 \times (32)^{\frac{1}{5}}$ is

(a) 2 (b) 10 (c) 4 (d) None of these

Ans. (c)

Q.76. The value of $2(256)^{\frac{1}{8}}$ is

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) None of these

Ans. (c)

Q.77. $4x^{-1/4}$ is expressed as

(a) $-4x^{1/4}$ (b) x^{-1} (c) $4/x^{1/4}$ (d) None of these

Q.78 Simplify $4x^{1/3} \times 2x^{-1}$ if x=4.

(a) $2^{\frac{1}{3}}$ (b) 2 (c) $2^{\frac{5}{3}}$ (d) None of these

Ans. (c)

Q.79 $x^{a-b} \times x^{b-c} \times x^{c-a}$ is equal to

(a) x (b) 1 (c) 0 (d) None of these

Ans. (b)

Q.80. The value of $4^{-3} \times 4^5$ is

(a) 4 (b) 8 (c) 16 (d) None of these

Ans. (c)

Q.81 The value of $(125)^{\frac{2}{3}} \times (625)^{\frac{3}{4}}$ is

(a) 150 (b) 625 (c) 3125 (d) None of these

Ans. (c)

LAW 2

$\frac{a^m}{a^n} = a^{m-n}$ when m and n are positive integers and m>n

Eg.: $\frac{2^3}{2^4} = 2^{3-4} = 2^{-1} = \frac{1}{2}$

Q.82 Which of the following option is correct when you simplify the following

(I) $4/(16)^{1/4}$ (II) $(27)^{2/3}/(9)^{3/2}$

(a) $2, \frac{1}{3}$ (b) $\frac{1}{2}, \frac{1}{3}$ (c) 2, -3 (d) None of these

Ans. (a)

Q.83 The value of $\frac{4}{(32)^{\frac{1}{5}}}$ *is*

(a) 8 (b) 2 (c) 4 (d) None of these

Ans. (b)

Q.84 Simplify $\frac{2a^{2-} \times a^{\frac{2}{3}} \times 6a^{\frac{7}{3}}}{9a^{a--} \times a^{a-}}$ if a =4

(a) 3 (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) 4

Ans. (b)

LAW 3

$(a^m)^n = a^{mn}$ when m and n are positive integers.

E.g.: $((16)^{1/4})^5 = (16)^{\frac{1}{4}\times 5} = 16^{\frac{5}{4}} = (2^4)^{\frac{5}{4}} = 2^{4\times\frac{5}{4}} = 2^5$

Q.85. Find the value of $[(27)^{\frac{2}{3}}(9)^{\frac{3}{2}}]^{\frac{1}{5}}$

(a) 27 (b) 3 (c) 3^5 (d) None of these

Ans. (b)

Q.86. The value of $(\frac{8}{27})^{\frac{1}{3}}$ *is*

(a) 2/3 (b) 3/2 (c) 2/9 (d) None of these

Ans. (a)

Q.87.Q $[(x^n)^{n-\frac{1}{n}}]^{\frac{1}{n+1}}$

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(a) x^n (b) x^{n+1} (c) x^{n-1} (d) None of these

Ans. (c)

Q.88. The value of $[\frac{x^a}{x^b}]^{ab} \times [\frac{x^b}{x^c}]^{bc} \times [\frac{x^c}{x^a}]^{c+a}$

(a) 1 (b) 0 (c) 2 (d) None of these

Ans. (a)

Q.89 if $a^3 - b^3 = (a-b)(a^2+ab+b^2)$, then simplified form of $[\frac{x^l}{x^m}]^{l^2+lm+m^2} \times [\frac{x^m}{x^n}]^{m^2+mn+n^2} \times [\frac{x^n}{x^l}]^{l^2+ln+n^2}$

(a) 0 (b) 1 (c) x (d) None of these.

Ans. (b)

Q.90 If $(25)^{150} = (25x)^{50}$, then the value of x will be

(a) 5^3 (b) 5^4 (c) 5^2 (d) None of these

Ans. (b)

Q.91. The value of $\frac{64(b^4a^3)^6}{[4(a^3b)^2 \times (ab)^2]}$ is

(a) $16a^{10}b^{20}$ (b) $4a^{20}b^{10}$ (c) $8a^{10}b^{20}$ (d) $4a^{10}b^{20}$

Ans. (a)

Q.92 On simplification, $\frac{1}{1+a^{m-n}+a^{m-p}} + \frac{1}{1+a^{n-m}+a^{n-p}} + \frac{1}{1+a^{p-m}+a^{p-n}}$ is equal to

(a) 0 (b) a (c) 1 (d) $\frac{1}{a}$

Ans. (c)

Q.93. $[\{(2)^{1/2}.(4)^{3/4}.(8)^{5/6}.(16)^{7/8}.(32)^{9/10}\}^{4}]^{3/25}$ is

(a) A fraction (b) an integer (c) 1 (d) None of these

Ans. (b)

LAW 4 $(ab)^n = a^n b^n$ when n can take all of the values.

E.g.: 6^3 can be written as $(2 \times 3)^3 = 2^3 \times 3^3$

Q.94. The value of $(8)^3(27)^3$ is

(a) 6 (b) 6^{18} (c) 6^9 (d) None of these

Ans. (b)

Q.95. Simplify $6a^2b^3c^2 \times 4b^{-3}c^{-2}$d.

(a) 0 (b) $24a^2$d (c) $24a^2$bcd (d) None of these

Ans. (b)

Q.96. The simplified value of $16 \times^{-3} y^2 \times 8^{-1} \times x^3y^{-2}$ is

(a) 2xy (b) $\frac{xy}{2}$ (c) 2 (d) None of these

Ans. (c)

Q.97. If $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}}$ and $xyz = 1$, then the value of $p + q + r$ is

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) None of these.

Ans. (b)

Q.98. If $3^{x+2} = 27$, what is the value of x?

(a) 1 (b) 0 (c) 2 (d) None of these

Ans. (a)

Q.99. If $2^x = 3^y = 6^{-z}$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is

(a) 1 (b) 0 (c) 2 (d) None of these

Ans. (b)

Q.100. The value of $\{(x+y)^{2/3}(x-y)^{3/2} / \sqrt{x+y} \times \sqrt[3]{(x-y)^2}\}^6$ is

(a) $(x+y)^2$ (b) $(x-y)$ (c) $(x+y)$ (d) None of these.

Ans. (c)

LOGARITHM

WHAT IS LOGARITHM?

- For all real numbers n and all positive numbers a and x where $a \neq 1$, $x = \log_a n$ if and only if $n = a^x$.

E.g.: For n = 1000 and a = 10.

Here, $n = 1000 = (10)^3$ which is of the form n = ax

- The expression **$\log_a n$** represents the base or index to which the base "a" must be raised in order to obtain x.

Q.101. Log_2 8 is equal to

(a) 2 (b) 8 (c) 3 (d) None of these.

Ans. (c)

Q.102. $log_{\sqrt{2}}$ 64 is equal to

(a) 12 (b) 6 (c) 1 (d) None of these

Ans. (a)

Q.103. $log_{2\sqrt{3}}1728$ *is equal to*

(a) $2\sqrt{3}$ (b) 2 (c) 6 (d) None of these

Ans. (c)

Q.104. Find the value of $log_3(\frac{1}{9})$

(a) 2 (b) -2 (c) 3 (d) None of these

Ans. (b)

Q.105. The value of $log_{2\sqrt{3}}5832$ is

(a) $2\sqrt{3}$ (b) 6 (c) 12 (d) cannot be determined.

Ans. (c)

- **Calculator Trick to find out Logarithm:**

1. Type number and press root for 15 times.
2. Subtract 1 after step 1.
3. Divide number by 0.00007027 or multiply by 14230.

Q.106. Find

(I) log 4.221 (II) log 0.2312 (III) log 0.001294

FUNDAMENTAL LAW LOGARITHMS

LAW 1

Logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers to the same base, i.e. $\log_a mn = \log_a m + \log_a n$

E.g.: $\log_a(10) = \log_a(2 \times 5) = \log_a 2 + \log_a 5$

Q.107 log 20 can be expressed as

(a) log 2 + log 5 (b) log 5 + log 8 (c) log 5 + log 4 (d) None of these

Ans. (c)

Q.108. log 6 + log 5 is expressed as

(a) log 11 (b) log 30 (c) log 5/6 (d) None of these

Ans. (b)

Q.109. log (1 × 2 × 3) is equal to

(a) log 1 + log 2 + log 3 (b) log 3
(c) log 2 (d) None of these

Ans. (a)

Q.110. If log x + log y = log (x + y), y can be expressed as

(a) x - 1 (b) x (c) $\frac{x}{x-1}$ (d) None of these

Ans. (c)

Q.111. Given log 2 = 0.3010 and log 3 = 0.4771, the value of log 6 is

(a) 0.9030 (b) 0.9542 (c) 0.7781 (d) None of these

Ans. (c)

Q.112. Which of the following statements does not hold?

(a) $\log_{100} 1 = 0$ (b) $\log_5 5 = 1$

(c) $\log(2 + 3) = \log(2 \times 3)$ (d) $\log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$

Ans. (c)

Q.113. Given that $\log_{10} 2 = x$, $\log_{10} 3 = y$, then $\log_{10} 60$ is expressed in terms of x and y

(a) x – y + 1 (b) x + y + 1 (c) x – y – 1 (d) None of these

Ans. (b)

LAW 2

The logarithm of the quotient of two numbers is equal to the difference of their logarithms to the same base, i.e., $log_a (\frac{m}{n}) = log_a m - log_a n$

E.g.: $log_4 (\frac{27}{5}) = log_4(27) - log_4(5)$

Q.114. $\log (\frac{32}{4})$ is equal to

(a) $\frac{\log 32}{\log 4}$ (b) $\log 32 - \log 4$ (c) 23 (d) None of these

Q.115. log 20 – log 5 is equal to

(a) log 6 (b) log 16 (c) log 4 (d) None of these

Ans. (c)

Q.116. The simplified value of $\log_{10} 5 + \log_{10} 8 - \log_{10} 4$ is

(a) $\frac{1}{2}$ (b) 4 (c) 1 (d) None of these

Ans. (c)

Q.117. Given that $\log_{10} 2 = x$, $\log_{10} 3 = y$, then $\log_{10} 1.2$ is expressed in terms of x and y as

(a) x + 2y – 1 (b) x + y – 1 (c) 2x + y – 1 (d) None of these

Ans. (c)

Q.118. If $\log_{10}2 = y$ and $\log_{10}3 = x$, then the value of $\log_{10}15$ is

(a) x – y + 1 (b) x + y + 1 (c) x – y - 1 (d) y – x + 1

Ans. (a)

Q.119. If $\log_3 x = 2$ and $\log_3 y = 3$, what is the value of $log_3 (\frac{x}{y})$?

(a) 1 (b) –1 (c) 0 (d) 5

Ans. (b)

Q.120 Given that log x = m + n and log y = m - n, the value of log $log \frac{10x}{y^2}$ is expressed in terms of m and n as

(a) 1 – m + 3n (b) m – 1 + 3n (c) m + 3n + 1 (d) None of these

Ans. (a)

LAW 3

Logarithm of the number raised to the power is equal to the index of the power multiplied by the logarithm of the number to the same base i.e. $\log_a m^n = n\log_a m$

E.g.: $\log_2 2^4 = 4 \log_2 2$

Q.121. $\log_3 81$ is equal to

(a) 2 (b) 8 (c) 3 (d) None of these

Ans. (d)

Q.122. The value of log 0.0001 to the base 0.1 is

(a) –4 (b) 4 (c) (d) None of these

Ans. (b)

Q.123. $\log (\frac{1}{81})$ to the base 9 is equal to

(a) 2 (b) $\frac{1}{2}$ (c) –2 (d) None of these

Ans. (c)

Q.124. The value of $\log_2 [\log_2 \{\log_3 (\log_3 27^3)\}]$ is equal to

(a) 1 (b) 2 (c) 0 (d) None of these

Ans. (c)

Q.125 The value of $\log(1^3 + 2^3 + 3^3 + ... + n^3)$ is equal to

(a) 3 log 1 + 3 log 2 + ... + 3 log n

(b) 2 log n + 2 log(n + 1) – 2 log 2

(c) log n + log(n + 1) + log(2n + 1) – log 6

(d) 1

Ans. (b)

Q.126. The value of $\log_{60} 3 + \log_{60} 4 + \log_{60} 5$ is

(a) $\log_{60} 12$ (b) $\log_{60} 30$ (c) 1 (d) 0

Ans. (c)

Q.127. The simplified value of 2 log105 + log10 8 – 12 log10 4 is

(a) 12 (b) 4 (c) 2 (d) None of these

Ans. (c)

Q.128 $\log [1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$ can be written as

(a) $\log x^2$ (b) log x (c) $\log \frac{1}{x}$ (d) None of these

Ans. (b)

CHANGE OF BASE

$\log_6 m = \frac{\log_a m}{\log_a b}$

E.g.: $\log_3 27 = \frac{\log_{10} 27}{\log_{10} 3} = \frac{\log_{10} 3^3}{\log_{10} 3} = \frac{3\log_{10} 3}{\log_{10} 3} = 3$

Q.129. If $log_2 x + log_4 x + log_{16} x = \frac{21}{4}$ these x is equal to

(a) 8 (b) 4 (c) 16 (d) None of these

Ans. (a)

Q.130. The value of $(\log_b a \times \log_c b \times \log_a c)^3$ is equal to

(a) 3 (b) 0 (c) 1 (d) None of these

Ans. (c)

ANTILOGARITHM

- If x is the logarithm of a given number n with a given base, then n is called the antilogarithm (antilog) of x to that base. This can be expressed as follows:
- If $\log_a n = x$ then n = antilog x

To find Antilogarithm:

1. Multiply number by 0.000070274 or divide by 14230

2. Add 1 after step 1

3. Press "multiply & =" button for 15 times

Q.131. Find the antilog of

(i) 2.4523 (ii) -1.0451

Q.132 The value of $(\frac{8}{27})^{\frac{-1}{3}} \times (\frac{32}{243})^{\frac{-1}{5}}$ is

(a) 9/4 (b) 4/9 (c) 80 (d) 480

Ans. (a)

Q.133. $[(x^n)^{\frac{n^2-1}{n}\frac{1}{n+1}}]$ is equal to

(a) x^n (b) x^{n+1} (c) x^{n-1} (d) None of these

Ans. (c)

Q.134. $log_{\sqrt{2}} 64$ *is equal to*

(a) 12 (b) 6 (c) 1 (d) None of these

Ans. (a)

Q.135. If $log_4 x + log_{16} x + log_{64} x + log_{256} x = \frac{25}{6}$ then the value of x is

(a) 64 (b) 4 (c) 16 (d) 2

Ans. (c)

Q.136 The simplified value of $log \sqrt[*]{729\sqrt[3]{9^{-1} \cdot 27^{-4/3}}}$ is

(a) log 3 (b) log 2 (c) $\log^{1/2}$ (d) None of these

Ans. (a)

Q.137 Simplify $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) ½ (b) 3/2 (c) 2/3 (d) 1/3

Ans. (b)

Q.138 If $\log_3[\log_4(\log_2 x)] = 0$, then the value of 'x' will be

(a) 4 (b) 8 (c) 16 (d) 32

Ans. (c)

Q.139 The ratio of the number of ₹5 coins and ₹10 coins is 8 : 15. If the value of ₹5 coins is ₹360, then the number of ₹10 coins will be

(a) 72 (b) 120 (c) 135 (d) 185

Ans. (c)

Q.140 The number which when subtracted from each of the terms of the ratio 19 : 31 reducing it to 1 : 4 is

(a) 15 (b) 5 (c) 1 (d) None of these

Ans. (a)

Q.141 $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$ equal to

(a) 3 (b) 2 (c) 1 (d) 0

Ans. (b)

Q.142 If $log_a(\sqrt{3}) = \frac{1}{6}$ find the value of 'a'

(a) 81 (b) 9 (c) 27 (d) 3

Ans. (c)

Q.143 The value of $\frac{1}{1+a^{y-x}} + \frac{1}{1+a^{x-y}}$ is given by

(a) –1 (b) 0 (c) 1 (d) None of these

Ans. (c)

Q.144 log(1 + 2 + 3) is equal to

(a) log1 + log2 + log3 (b) log (1 × 2 × 3)

(c) Both (d) None of these

Ans. (c)

Q.145 $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$ then find the value of x

(a) 6 (b) 2 (c) 5 (d) 9

Ans. (a)

Q.146 Find the value of $\frac{3t^{-1}}{t^{-\frac{1}{3}}}$

(a) $t^{\frac{3}{3}}$ (b) $t^{\frac{3}{2}}$ (c) $\frac{3}{t^{\frac{1}{3}}}$ (d) $\frac{3}{t^2}$

Ans. (a)

Q.14 74, *, 9, $13\frac{1}{2}$ are in proportion. Then * is

(a) 6 (b) 8 (c) 9 (d) None of these

Ans. (a)

PREVIOUS YEAR QUESTIONS

RATIO, PROPORTION, INDICES AND LOGARITHM

Q.1 Divide 27 into two parts, so that 5 times the first and 11 times the second together equal to 195, then the ratio of first and second part is: [Dec. 2023]

(a) 17 : 10
(b) 15 : 12
(c) 14 : 13
(d) 16 : 11

Ans. (a) 17 : 10

Q.2 If $\frac{9^n \times 3^5 \times 27^5}{3 \times 81^4} = 27$, then the value of n is [Dec. 2023]

(a) 2
(b) 0
(c) 3
(d) 4

Ans. (b) 0

Q.3 Given that $log_{10} x = m + n - 1$ and $log_{10} y = m - n$, the value of $log_{10}\left(\frac{100x}{y^2}\right)$ expressed in terms of m and n is [June 2023]

(a) 1 – m + 3n
(b) m – 1 + 3n
(c) m + 1 + 3n
(d) $m^2 - n^2$

Ans. (a) 1 – m + 3n

Q.4 The value of $[log_{10}(5log_{10} 100)]^2$ is [June 2023]

(a) 1
(b) 2
(c) 10
(d) 25

Ans. (a) 1

Q.5 If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then find the value of $\left(\frac{a+b+c}{3}\right)^3$ [June 2023]

(a) abc
(b) 9abc
(c) 1/abc
(d) 1/9abc

Ans. (a) abc

Q.6 If $x = y^a, y = z^b, z = x^c$, then the value of abc is [June 2023]

(a)1
(b) 2
(c) 3
(d) 4

Ans. (a)1

Q.7 A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1,000 more than D, what is B's share? [Dec 2022]

(a) 2000
(b) 1500
(c) 2500
(d) 1000

Ans. (a) 2000

Q.8 A group of 400 soldiers posted at border area had a provision for 31 days. After 28 days 280 from this group were called back. Find the number of days for which the remaining rations will be sufficient? [Dec 2022]

(a)3
(b) 6
(c) 8
(d) 10

Ans. (d) 10

Q.9 By simplifying $(2a^3 b^4)^6 / (4a^3b)^2 \times (a^2b^2)$, the answer will be [Dec 2022]

(a) $4a^2b^3$
(b) $4a^6b$
(c) $4a^{10}b^{10}$
(d) $4a^{10}b^{20}$

Ans. (d) $4a^{10}b^{20}$

Q.10 $\log_3 4 . \log_4 5 \log_5 6 . \log_6 7 . \log_7 8 . \log_8 9$ equal to : [Dec 2022]

(a)3
(b) 2
(c) 1
(d) 0

Ans. (b) 2

Q.11 If $\log_{10} 2 = y$ and $\log_{10} 3 = x$, then the value of $\log_{10} 15$ is: [Dec 2022]

(a) x – y + 1
(b) x – y - 1
(c) x + y + 1
(d) y – x + 1

Ans. (a) x – y + 1

Q.12 $\log \frac{p^2}{qr} + \log \frac{q^2}{pr} + \log \frac{r^2}{pq} =$ ____ [June 2022]

(a) pqr

(b) 1/pqr
(c) 1
(d) 0

Ans. (d) 0

Q.13 If $\log_a \sqrt{3} = \frac{1}{6}$ find the value of a
[June 2022]
(a) 9
(b) 81
(c) 27
(d) 3
Ans. (c) 27

Q.14 Find the value of $\frac{3t^{-1}}{t^{\frac{-1}{3}}}$
[June 2022]
(a) $\frac{3}{t^{\frac{2}{3}}}$
(b) $\frac{3}{t^{\frac{3}{2}}}$
(c) $\frac{3}{t^{\frac{1}{3}}}$
(d) $\frac{3}{t^2}$
Ans. (a) $\frac{3}{t^{\frac{2}{3}}}$

Q.15 Find the value of z from $(\sqrt{9})^{-8} \times (\sqrt{3})^{-5} = 3^z$
[June 2022]
(a) 2/21
(b) -21
(c) 21
(d) -2/21
Ans. (d) -2/21

Q.16 A bag has 105 coins containing some 50 paise, and 25 paise coins. The ratio of the number of these coins is 4: 3. The total value (in ₹) in the bag is
[Dec 2021]
(a) 43.25
(b) 41.25
(c) 39.25
(d) 35.25
Ans. (b) 41.25

Q.17 If $\log_{10} 3 = x$ and $\log_{10} 4 = y$, then the value of $\log_{10} 120$ can be expressed as
[Dec 2021]
(a) x - y + 1
(b) x + y + 1
(c) x - y - 1
(d) 2x + y - 1

Ans. (b) x + y + 1

Q.18 Find the value of log (x^6) if log (x) + 2 log(x^2) + 3 log (x^3) = 14.
[Dec 2021]
(a) 3
(b) 4
(c) 5
(d) 6
Ans. (d) 6

Q.19 Incomes of *R* and *S* are in the ratio 7: 9 and their expenditures are in the ratio 4: 5. Their total expenditure is equal to income of *R*. What is the ratio of their savings?
[Dec 2021]
(a) 23:36
(b) 28:41
(c) 31:43
(d) 35:46
Ans. (d) 35:46

Q.20 The value of $\frac{6^{n+4}+3^{n+3}\times 2^{n+3}}{5\times 6^n+6^n}$ is :
[Dec 2021]
(a) 232
(b) 242
(c) 252
(d) 262
Ans. (c) 252

Q.21 In a department, the number of males and females are in the ratio 3: 2. If two males and 5 females join department, then the ratio becomes 1: 1, initially the number of female in the department is [Dec 2021]
(a) 9
(b) 6
(c) 3
(d) 8
Ans. (b) 6

Q.22 If $(\frac{3a}{2b})^{2x-4} = (\frac{b}{3a})^{2x-4}$, for some a and b, then the value of x is
[Dec 2021]
(a) 8
(b) 6
(c) 4
(d) 2
Ans. (d) 2

Q.23 The value of $(1 - \sqrt[3]{0.027}\,(\frac{5}{6})\,(\frac{1}{2})^2)$ is :

[Dec 2021]
(a) 11/16
(b) 13/16
(c) 15/16
(d) 1
Ans. (c) 15/16

Q.24 The salaries of A,B, and C are of ratio 2:3:5. If the increments of 15%, 10% and 20% are done to their respective salaries, then find the new ratio of the
[July 2021]
(a) 3 : 3 : 10
(b) 10 : 11 : 20
(c) 23 : 33 : 60
(d) 33: 60: 23
Ans. (c) 23 : 33 : 60

Q.25 If $\log_4 x+\log_{16} x+\log_{64} x+\log_{256} x=25/6$ then the value of x is.
[July 2021]
(a) 64
(b) 4
(c) 16
(d) 2
Ans. (c) 16

Q.26 If A : B = 5 : 3 , B : C = 6 : 7 and C : D = 14 : 9, then the value of A : B : C : D
[July 2021]
(a) 20 : 14 : 12 : 9
(b) 20 : 9 : 12 : 14
(c) 20 : 9 : 14 : 12
(d) 20: 12: 14 : 9
Ans. (d) 20: 12: 14 : 9

Q.27 Find the value of $\frac{3t^{-1}}{t^{-1/3}}$

(a) $\frac{3}{t^{2/3}}$
(b) $\frac{3}{t^{3/2}}$
(c) $\frac{3}{t^{1/3}}$
(d) $\frac{3}{t^2}$
Ans. (a) $\frac{3}{t^{2/3}}$

Q.28 In a certain business, A and B received Profit in a certain ratio; B and C received profits in the same ratio. If A gets ₹1,600 and C gets ₹2,500, then how much does B get?
[Jan. 2021]
(a) ₹2,000
(b) ₹2,500

(c) ₹1,000
(d) ₹1,500
Ans. (a) ₹2,000

Q.29If xy+yz+zx= - 1then the value of is $\left(\frac{x+y}{1+xy}\ \frac{z+y}{1+zy}\ \frac{x+z}{1+zy}\right)$ July 21

(a) xyz
(b) $\frac{-1}{yz}$
(c) $\frac{1}{xyz}$
(d) $\frac{1}{x+y+z}$
Ans.(c)$\frac{1}{xyz}$

Q.30 The ratio of two quantities is 15 : 17. If the consequent of its inverse ratio is 15, then the antecedent is.
[Jan. 2021]
(a) 15
(b) √15
(c) 17
(d) 14
Ans. (c) 17

Q.31 The salaries of A, B and C are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salaries, then what will be the new ratio of their salaries?
[Jan. 2021]
(a) 3 : 3 : 10
(b) 10 : 11: 20
(c) 23 : 33 : 60
(d) Cannot be determined
Ans. (c) 23 : 33 : 60

Q.32 If a : b = 3: 7 then 3a + 2b : 4a + 5b = ?
[Dec. 20]
(a) 27 : 43
(b) 23 : 47
(c) 24 : 51
(d) 29 : 53
Ans. (b) 23 : 47

Q.33 log 9 + log 5 is expressed as
[Dec. 2020]
(a) log (9/5)

(b) log (5/9)
(c) log 4
(d) Log 45
Ans. (d) Log 45

Q.34 Find the value 'a' from the following.

$(\sqrt{9})^{-5} \times (\sqrt{3})^{-7} = (\sqrt{3})^{-a}$

(a) 13
(b) 11
(c) 15
(d) 17
Ans.(d) 17

Q.35 The ratio of no. of boys and the no. of girls in a school is found to be 15:32. How many boys and equal no. of girls should be added to bring the ratio to 2/3?
[Dec. 20]
(a) 20
(b) 19
(c) 23
(d) 27
Ans. (b) 19

Q.36 If $\log_a (\sqrt{9}) = \frac{1}{6}$ find the value of 'a'
[Dec. 20]
(a) 81
(b) 9
(c) 27
(d) 3
Ans. (c) 27

SUMMARY

RATIO

- If a and b are two quantities in the same units, then the fraction ab is called the ratio of a to b. It is written as a : b.

- The quantities a and b are called the terms of the ratio, a is called the first term or antecedent and b is called the second term or consequent.
- The ratio compounded of the two ratios a : b and c : d is ac : bd.
- A ratio compounded of itself is called its duplicate ratio given by $a^2 : b^2$ and the triplicate ratio of a : b is $a^3 : b^3$.

Similarly, Sub- duplicate ratio of a : b is $a^{1/2} : b^{1/2}$ and the sub-triplicate ratio of a : b is $a^{1/3} : b^{1/3}$.
Inverse ratio for any ratio a : b is b : a

- Continued Ratio is the relation between three or more Quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as a : b : c.

PROPORTION

- The quantities a, b, c, d are called terms of the proportion; a, b, c and d are called its first, second, third and fourth terms respectively and a : b = c : d.
  First and fourth terms are called extreme terms, whereas Second and third terms are called means (or middle terms).
- If a : b = c : d are in proportion then $\frac{a}{b} = \frac{c}{d} \Rightarrow$ ad = bc i.e.
  Product of extremes = Product of means. This is called the cross product rule.
- Three quantities a, b, c of the same units are said to be in continuous proportion, then the middle term b is called the mean proportional between a and c, a is the first proportional and c is the third proportional.
- Few properties of proportions :

  - If a : b = b : c i.e., $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$
  - Invertendo: p : q = r : s ⇒ q : p = s : r
  - Alternendo: a : b = c : d ⇒ a : c = b : d
  - Componendo: a : b = c : d ⇒ a + b : b = c + d : d
  - Dividendo: a : b = c : d ⇒ a – b : b = c – d : d
  - Componendo & Dividendo: a : b = c : d ⇒ a + b : a – b = c + d : c – d
  - Addendo: a : b = c : d = a + c : b + d
  - Subtrahendo: a : b = c : d = a – c : b – d
  - Generalising: If a : b = c : d = e : f = ... each of these ratios = (a – c – e – ...) : (b – d – f – ...)

INDICES: (IF BASE IS SAME)

- $a^m \times a^n = a^{m+n}$
- $a^m / a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{-m} = \frac{1}{a^m}$ & $\frac{1}{a^{-m}} = a^m$
- $if\ a^x = a^y, then\ x = y$
- $if\ x^a = y^a, then\ x = y$
- $\sqrt[n]{a} = a^{1/n}$
- $if\ a^x = n\ then\ x = log_a n$
- $log_a a = 1$

LOGARITHMS

- If base of log is *e*, then it is a natural logarithm and if base is 10 then it is called common logarithm.
  $loga\, mn = loga\, m + loga\, n$
  $loga\, (m/n) = loga\, m - loga\, n$
  $log_a\, m^n = nlog_a\, m$
  $log_a\, 1 = 0$
- $log_b a \times log_a\, b = 1$
- $log \quad a \times \frac{\log a}{\log b}$
- $log_b\, a \times log_c b = log_c a$
- $log_b\, a = 1/log_a b$
- $a\, log_a^x = x\ (Invrse\ logarithm)$

CHAPTER-2

EQUATION

If I ask you to buy some bananas and few apples such that their sum is equal to 15. How will you write mathematically?

We can represent it as:

Let the number of bananas be x and the number of apples be y having prices as a and b respectively.

$Then, ax + by = 15$

WHAT IS EQUATION?

- In simple language, equation is a mathematical statement of equality.
- When we equate '=' variable/variables with constant and find out the value of variable. That will be called an equation.

  E.g.: 2x + 3 = 0, where x is a variable and 3 is a constant.

In this chapter, you have to focus on understanding the questions and forming the equation, if done properly you have solved 50% of the question and 50% is just the steps which you will learn.

CONDITIONAL EQUATION

- In simple language, an equation is a mathematical statement of equality.

E.g.: $\frac{x+2}{2} = 5$

IDENTITY

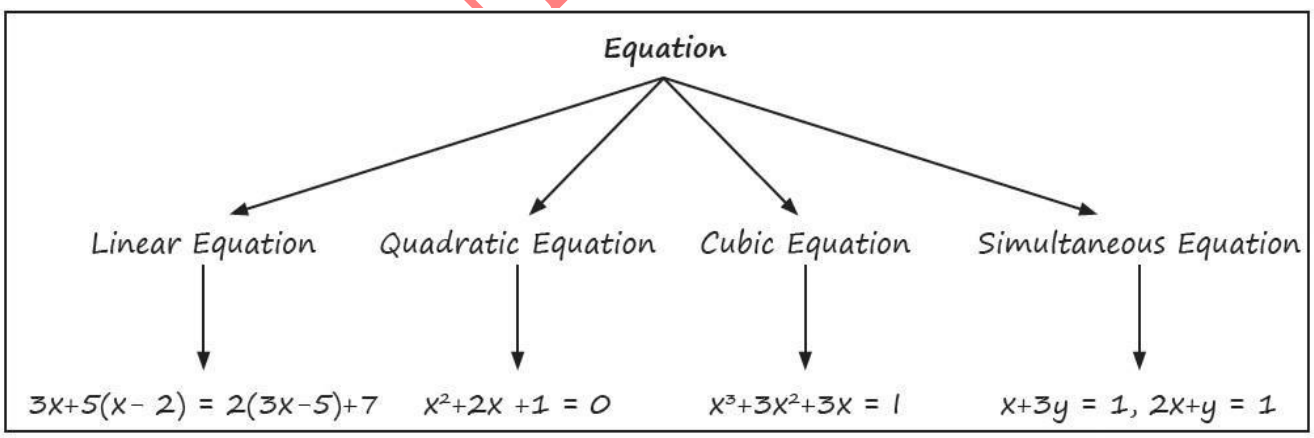
- If the equality is true for all values of the variable involved, the equation is called an identity.

E.g.: $sin^2x + cos^2x = 1$

Solution of the Equation or Root of the Equation

The value of the variable which satisfies the equation will be the solution or root of the equation.

Equation			
Linear Equation	Quadratic Equation	Cubic Equation	Simultaneous Equation
3x+5(x- 2) = 2(3x-5)+7	$x^2+2x+1 = 0$	$x^3+3x^2+3x = I$	x+3y = 1, 2x+y = 1



LINEAR EQUATION (SIMPLE EQUATION)

- An equation in the form of ax + b = 0 will be called a simple equation or linear equation.

E.g.: 2x + 3 = 0 where a = 2, b = 3.

Q.1 The equation $2x - 1 = x + 3$will be satisfied for x equal to

(a) –1 (b) 2 (c) 4 (d) –2

Ans. (c)

Q.2 The solution of an equation $\frac{x+7}{3} = 4 + \frac{x}{2}$ is

(a) –10 (b) 20 (c) 20 (d) 4

Ans. (a)

Q.3 The sum of two numbers is 20 and their difference is 4. The numbers are

(a) (12, 15) (b) (12, 8) (c) (2, 8) (d) None of these

Ans. (b)

Q.4 8 is the solution of the equation

(a) $\frac{x+4}{4}+\frac{x-5}{3}=11$ (b) $\frac{x+4}{2}+\frac{x+10}{9}=8$

(c) $\frac{x+24}{5}=4+\frac{x}{4}$ (d) $\frac{x-15}{10}+\frac{x+5}{5}=4$

Ans. (b)

Q.5 If we subtract twice Ram's present age from his age 15 years from the present, the result would be equal to his age after 3 years. Find Ram's present age.

(a) 10 (b) 12 (c) 15 (d) 6

Ans. (d)

Q.6 Pick up the correct value x for which $\frac{x}{0.5}-\frac{1}{0.05}+\frac{x}{0.005}-\frac{1}{0.0005}=0$

(a) x = 0 (b) x = 1 (c) x = 10 (d) None of these

Ans. (c)

Trick: Go by choices:

Q.7 One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number was

(a) 320 (b) 400 (c) 480 (d) None of these

Ans. (c)

Q.8 For a certain commodity, the demand equation giving demand 'd' in kg, for a price 'p' in rupees per kg is d = 50(20 – p). The supply equation gives the supply 's' in kg, for a price 'p' in rupees per kg is s = 75(p – 3). The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.

(a) (9.8, 150) (b) (20, 405) (c) (15, 150) (d) None of these

Ans. (a)

Q.9 The equation $-7x+1=5-3x$ will be satisfied for x equal to

(a) 2 (b) – 1 (c) 1 (d) None of these

Ans. (b)

Q.10 What will be the correct value x for which $\frac{x}{0.1}-\frac{1}{0.01}+\frac{x}{0.001}-\frac{x}{0.1}=0$

(a) –10 (b) 2 (c) 10 (d) –5

Ans. (c)

Q.11 Divide 65 into two parts such that two times the first part exceeds one fourth of the second by 40. The parts are.

(a) (30, 35) (b) (20, 45) (c) (15, 50) (d) None of these

Ans. (d)

Q.12 The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator the fraction increases by unity. The fraction is

(a) 3/5 (b) 1/3 (c) 7/9 (d) None of these

Ans. (a)

Q.13 If a number of which the half is greater than $\frac{1}{5}th$ of the number by 15 then the number is

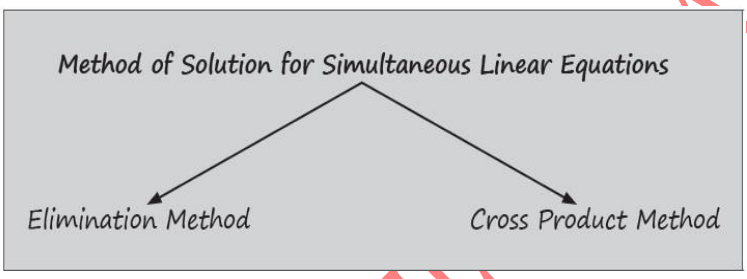
(a) 50 (b) 40 (c) 80 (d) None of these

Ans. (a)

SIMULTANEOUS LINEAR EQUATIONS

What will we study here?

- Simultaneous linear equation is basically a method of solving two equations given simultaneously at a time, where we need to find solution for two different variable
- The general form of that type of equation will be: ax + by + c = 0, where, x and y are two variables.
- So, let's say two equations possibly are:
  $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
  In above equation, we need to find the value (solution or roots) of two linear equation which has two variables x and y.



ELIMINATION METHOD

- In this method two given linear equations are reduced to a linear equation in one variable by eliminating one of the variables and then solving for the other variable.

Q.14 Solve for x and y:

2x + y = 9 and 2x – 3y = 5

Q.15 Solve for x and y:

2x – y = –2 and 4x – 3y = –12

(a) (1, 2) (b) (3, 8) (c) (4, 4) (d) (8, 3)

Ans. (b)

Q.16 The solution of the set of equations 3x + 4y = 7, 4x – y = 3 is

(a) (1, –1) (b) (1, 1) (c) (2, 1) (d) (1, –2)

Ans. (b)

Q.17 The values of x and y satisfying the equations 3x – y = 3, x + 2y = 8 are given by the pair:

(a) (3, 2) (b) (–2, –3) (c) (2, 3) (d) None of these

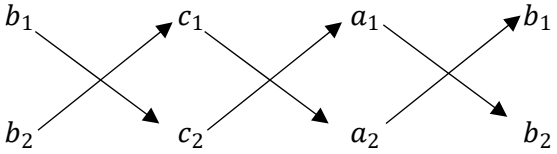
Ans. (c)

CROSS MULTIPLICATION METHOD

- Let two equation be:

$a_1x + b_1y + c1 = 0 \; and \; a_2x + b_{2y} + c_2 = 0$

Write coefficients of $x \; and \; y$, also constants in both the equations, likes:



$\frac{x}{b_1c_2-b_2c_1} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{a_1b_2-a_2b_1}$

i.e., $x = \frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$ *and* $y = \frac{c_1a_2-c_2a_1}{a_1b_{a2}-a_2b_1}$

Q.18. The simultaneous equations 7x-3y=31,9x-5y =41 have solutions given by

(a) (-4, -1) (b) (-1, 4) (c) (4, -1) (d) (3, 7)

Ans. (c)

Trick: Put all the options in given equations, you will see only option (c) will be satisfying both equations.

Q.19 1.5x+ 3.6y=2.1,2.5(x+1)=6y have solutions as

(a) (0.2,0.5) (b) (0.5,0.2) (c) (2,5) (d) (-2,-5)

Ans. (a)

Q.20 A number consists of two digits. The digits in the ten's place is 3 time the digit in the unit's place. If 54 is subtracted from the number, the digits are reversed. The number is

(a) 39 (b) 92 (c) 93 (d) 94

Ans. (c)

Trick: Only for option (c): 93

If 54 is subtracted from the number, the digits are reversed 93-54=39

Hence, the correct option is (c)

Q.21 1.5x+2.4y=1.8, 2.5(x+1)=7y have solutions as

(a) (0.5, 0.4) (b)(0.4, 0.5) (c) $(\frac{1}{2}, \frac{2}{5})$ (d) (2, 5)

Ans. (b)

Q.22 Solve for x and y: $\frac{x}{2} + \frac{2y}{3} = -1$ *and* $x - \frac{y}{3} = 3$

(a) x = 2, y = -2 (b) x = 2, y = 3 (c) x = 1, y = 2 (d) x = 2, y = -3

Ans. (d)

Q.23 The solution to the given set of equations $\frac{x}{5} + \frac{y}{6} + 1 = \frac{x}{6} + \frac{y}{5} = 28$

(a) (6, 9) (b) (9, 6) (c) (60, 90) (d) (90, 60)

Ans. (c)

Q.24 The solution for the pair of equation $\frac{1}{16x} + \frac{1}{15y} = \frac{9}{20}, \frac{1}{20x} - \frac{1}{27y} = \frac{4}{45}$ is given by,

(a) $(\frac{1}{2}, \frac{1}{3})$ (b) $(\frac{1}{2}, \frac{1}{3})$ (c) $(\frac{1}{4}, \frac{1}{3})$ (d) $(\frac{1}{2}, \frac{1}{4})$

Ans. (c)

Q.25 The cost prices of 3 pens and 4 bags is ₹324 and 4 pens and 3 bags is ₹257, then the cost of 1 pen is equal to

(a) ₹16 (b) ₹8 (c) ₹50 (d) ₹75

METHOD OF SOLVING SIMULTANEOUS LINEAR EQUATION WITH THREE VARIABLES

ax + by + cz = d, where x, y, z are variable, d is a constant and a, b, c are coefficients.

Q.26 Solve for x, y and z, 2x + 3y + 4z = 0, x + 2y – 5z = 0, 10x + 16y – 6z = 0

(a) (0, 0, 0) (b) (1, –1, 1) (c) (3, 2, –1) (d) (1, 0, 2)

Ans. (a)

Trick:

Put all the options in given equations, you will see only option (a) will be satisfying all three equations.

$2 \times 0 + 3 \times 0 + 4 \times 0 = 0$

$0 + 2 \times 0 - 5 \times 0 = 0$
$10 \times 0 + 16 \times 0 - 6 \times 0 = 0$
Therefore, the required solution is (0, 0, 0).
Hence, the correct option is (a)

Q.27 The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.
(a) 60 years (b) 52 years (c) 51 years (d) 50 years
Ans. (d)

Q.28 A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed then find the number.
(a) 54 (b) 53 (c) 45 (d) 55
Ans. (c)

Q.29 $\frac{xy}{x+y} = 20, \frac{yz}{y+z} = 40, \frac{zx}{z+x} = 24$
(a) (120, 60, 30) (b) (60, 30, 120) (c) (30, 120, 60) (d) (30, 60, 120)
Ans. (d)

Q.30 The wages of 8 men and 6 boys amount to ₹33. If 4 men earn ₹4.50 more than 5 boys, determine the wages of each man and boy.
(a) (₹0.50, ₹3) (b) (₹3, ₹1.50) (c) (₹2.50, ₹2) (d) (₹2, ₹2.50)
Ans. (b)

Q.31 If $\frac{10}{x+y} + \frac{2}{x-y} = 4$ *and* $\frac{15}{x+y} - \frac{5}{x-y} = -2$, then (x, y) is
(a) (3, 2) (b) (–3, 2) (c) (3, –2) (d) (–3, –2)
Ans. (a)

Q.32 $\frac{4}{x} - \frac{5}{y} = \frac{x+y}{xy} + \frac{3}{10}, 3xy = 10(y - x)$
(a) (2, 5) (b) (5, 2) (c) (2, 7) (d) (3, 4)
Ans. (d)

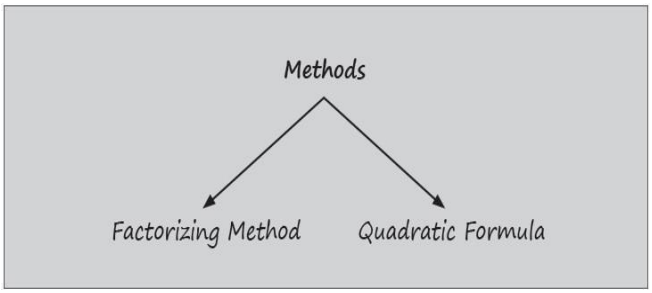
Q.33 The solution for the pair of $\frac{xy}{y-x} = 110, \frac{xy}{z-y} = 132, \frac{zx}{z+x} = 110$ is given by
(a) (12, 11, 10) (b) (10, 11, 12) (c) (11, 10, 12) (d) (12, 10, 11)
Ans. (b)

Q.34 The demand and supply equations for a certain commodity are 4q + 7p = 17 and $p = \frac{9}{3} + \frac{7}{4}$ respectively where p is the market price and q is the quantity then the equilibrium price and quantity are
(a) $2, \frac{3}{4}$ (b) $2, \frac{3}{4}$ (c) $5, \frac{3}{4}$ (d) None of these
Ans. (a)

QUADRATIC EQUATION

- An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called
- quadratic equation or equation of the second degree.
- When b = 0, the equation is called a pure quadratic equation i.e., $ax^2 + c = 0$
- When $b \neq 0$, the equation is called an affected Quadratic.

E.g.: (i) $3x^2 + 2x + 1 = 0$ (ii) $x^2 - x = 0$
How to find out the roots of a quadratic equation: $ax^2 + bx + c = 0$ $(a \neq 0)$



Q.35 Solve $x^2 - 8x + 16 = 0$ and find the roots.

(a) 4, –4 (b) 4, 0 (c) 4, 4 (d) 1, 4

Ans. (c)

Q.36 The positive root of the equation $2x^2 - 5x = 3$ is

(a) 5 (b) 3 (c) 4 (d) 1

Ans. (b)

SUM AND PRODUCT OF ROOTS

Let α and β be the roots of equation $ax^2 + bx + c = 0$.

Sum of roots: $\alpha + \beta = -\frac{b}{a}$

Product of roots: $\alpha\beta = \frac{c}{a}$

Q.37 if the roots of the quadratic equation $2x^2 + 5x - 3 = 0$ are α and β, what is the value of $|\alpha - \beta|$?

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{7}{2}$ (d) $\frac{5}{2}$

Ans. (c)

FOR QUADRATIC EQUATION

$ax^2 + bx + c = 0$

Nature of Roots for $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. If $b^2 - 4ac = 0$, then the roots are real and equal.

2. If $b^2 - 4ac > 0$, then the roots are real and unequal (or distinct).

3. If $b^2 - 4ac < 0$, then the roots are imaginary.

4. If $b^2 - 4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct).

5. If $b^2 - 4ac > 0$ but not a perfect square, the roots are real, irrational and unequal.

Q.37 If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal, then the value of m is

(a) –3 (b) –1 (c) 1 (d) –2

Ans. (d)

Q.38 For what value of k the given equation has real roots: $x^2 - 10x + k = 0$

(a) $k \leq 25$ (b) $k \geq 25$ (c) $k \leq 100$ (d) None of these

Ans. (a)

Q.39 Five times of a positive whole number is 3 less than twice the square of the number. The number is

(a) 3 (b) 4 (c) –3 (d) 2

Ans. (a)

Q.40 If α and β are the roots of the equation $2x^2 - 7x + 3 = 0$, then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is equal to

(a) $\frac{15}{4}$ (b) $\frac{37}{6}$ (c) $\frac{28}{5}$ (d) None of these

Ans. (b)

Q.41 A solution of the quadratic equation $(a + b - 2c)x^2 + (2a - b - c)x + (c + a - 2b) = 0$ is

(a) x = 1 (b) x = –1 (c) x = 2 (d) x = –2

Ans. (b)

Q.42 The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x, form an equation in x and hence find the numbers.

(a) (7, 10) (b) (4, 4) (c) (3, 5) (d) (2, 6)

Ans. (c)

Q.43 If L + M + N = 0 and L, M, N are rationals then the roots of the equation: $(M + N - L)x^2 + (N + L - M)x + (L + M - N) = 0$ are

(a) real and irrational (b) real and rational

(c) imaginary and equal (d) real and equal

Ans. (b)

Note:

(a) Irrational roots in conjugate pairs that is if $(m + \sqrt{n})$ is a root then $(m - \sqrt{n})$is the other root of the same equation.

(b) If one root is reciprocal to the other root then their product is 1 and so c/a =1 i.e. c = a

(c) If one root is equal to another root but opposite in sign then their sum = 0 and so b/a = 0. i.e. b = 0.

Q.44 The roots of the equation $x^2 + (2p - 1)x + p^2 = 0$ are real if.

(a) $p \geq 1$ (b) $p \leq 1$ (c) $p \geq \frac{1}{4}$ (d) $p \leq \frac{1}{4}$

Ans. (d)

Q.45 What will be the value of k, if the roots of the equation $(k - 4)x^2 - 2kx + (k + 5) = 0$ are equal?

(a) 18 (b) 20 (c) 19 (d) 21

Ans. (c)

Q.46 If the roots of the equation $x^2 - 8x + m = 0$ exceeds the other by 4, then the value of m is

(a) m = 10 (b) m = 11 (c) m = 9 (d) m = 12

Ans. (d)

Q.46 If the sum of two numbers is 11 and the sum of their squares be 85, then the numbers are

(a) 5 and 6 (b) 4 and 7 (c) 2 and 9 (d) None of these

Ans. (c)

Q.47 If α and β are the roots of $x^2 = x + 1$ then value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is

(a) $2\sqrt{5}$ (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) $-2\sqrt{5}$

Ans. (d)

Q.48 Solve the equation if $2^{2x+3} - 3^2 \cdot 2^x + 1 = 0$, then the values of x are

(a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, –3

Ans. (d)

Q.7 The Value of $2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\ddots}}}}$

(a) $1 \pm \sqrt{2}$ (b) $2 \pm \sqrt{5}$ (c) $1 \pm \sqrt{2}$ (d) None of these

Ans. (c)

Q.49 If $p \neq q$ *and* $p^2 = 5p - 3$ and $q^2 = 5q - 3$ the equation having roots as p and q is

(a) $x^2 - 19x + 3 = 0$ (b) $3x^2 - 19x - 3 = 0$
(c) $3x^2 - 19x + 3 = 0$ (d) None of these

Ans. (c)

Q.50 The solutions of the equation $\frac{6x}{x+1} + \frac{6(x+1)}{x} = 13$ are

(a) (2, 3) (b) (3, –2) (c) (–2, –3) (d) (2, –3)

Ans. (d)

Q.51 The satisfying values of x for the equation, $\frac{1}{x+p+q} + \frac{1}{x} + \frac{1}{p} + \frac{1}{q}$ are

(a) (p, q) (b) (–p, –q) (c) (p, –p) (d) (–p, q)

Ans. (b)

Q.52 The equation $(\frac{l-m}{2})x^2 - (\frac{l+m}{2})x + m = 0$ has got two values of x to satisfy the equation given as

(a) $(1, \frac{2m}{l-m})$ (b) $(1, \frac{m^2}{l-m})$ (c) $(1, \frac{2l}{l-m})$ (d) $(1, \frac{l}{l-m})$

Ans. (a)

Q.53 There are two consecutive numbers such that the difference of their reciprocals is $\frac{1}{240}$. The numbers are

(a) (15, 16) (b) (17, 18) (c) (13, 14) (d) (12, 13)

Ans. (a)

Q.54 A distributor of apple Juice has 5000 bottles in the store that it wishes to distribute in a month. From experience it is known that demand D (in number of bottles) is given by $D = -2000p^2 + 2000p + 17000$. The price per bottle that will result zero inventory is

(a) ₹3 (b) ₹5 (c) ₹2 (d) None of these

Ans. (a)

CUBIC EQUATION

An equation of 3rd degree or where the maximum power of x is 3 is called as cubic equation. Cubic Equation will look like: $ax^3 + bx^2 + cx + d = 0$ where $a \neq 0$ *will be called as cubic equation.*

E.g.: $x^3 - 6x^2 + 11x - 6 = 0$

For cubic equation, $ax^3 + bx^2 + c + d = 0$

Sum of roots $= -\frac{b}{a}$

Sum of roots taken two at a time $= \frac{c}{a}$

Product of roots $= -\frac{d}{a}$

Q.55 The roots of $x^3 + x^2 - x - 1 = 0$ are

(a) (–1, –1, 1) (b) (1, 1, –1) (c) (–1, –1, –1) (d) (1, 1, 1)

Ans. (a)

Q.56 Solve $x^3 - 6x^2 + 5x + 12 = 0$

(a) 1, 3, 4 (b) –1, 3, 4 (c) 1, 6, 2 (d) 1, –6, –2

Ans. (b)

Q.57 The age of a person is 8 years more than thrice the age of the sum of his two grandsons who were twins. After 8 years his age will be 10 years more than twice the sum of the ages of his grandsons. Then the age of the person when the twins were born is

(a) 86 years (b) 73 years (c) 68 years (d) None of these

Ans. (b)

Q.58 If α and β are the roots of the equation $x^2 + x + 5 = 0$, then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is equal to

(a) 16/5 (b) 2 (c) 3 (d) 14/5

Ans. (d)

Q.59 A number consists of two digits such that the digit in one's place is thrice the digit at ten's place. If 36 be added then the digits are reversed. Find the number

(a) 62 (b) 26 (c) 39 (d) None

Ans. (b)

Q.60 The value of p for which the difference between the root of equation $x^2 + px + 8 = 0$ is 2 is

(a) +2 (b) +4 (c) +6 (d) +8

Ans. (c)

Q.62 If the quadratic equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root then p + q = ?

(a) 0 (b) 1 (c) –1 (d) 2

Ans. (c)

Q.63 If a and b are the roots of the equation $2x^2 + 5x + k = 0$, and $4(a^2 + b^2 + ab) = 23$, then which of the following is true?

(a) $k^2 - 3k + 2 = 0$ (b) $k^2 - 2k - 2 = 0$

(c) $k^2 - 2k - 3 = 0$ (d) $k^2 + 3k - 2 = 0$

Ans. (a)

Q.64 The cost of 2 oranges and 3 apples is ₹28. If the cost of an apple is doubled then the cost of 3 oranges and 5 apples is ₹75. The original cost of 7 oranges and 4 apples (in ₹) is

(a) 59 (b) 47 (c) 71 (d) 63

Ans. (a)

Q. 65 The value of 'k' is ... If 2 is a root of the following cubic equation: $x^3 - (k + 1) x + k = 0$

(a) 2 (b) 6 (c) 1 (d) 4

Ans. (b)

Q.66 If the quadratic equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root then p + q =?

(a) 0 (b) 1 (c) –1 (d) 2

Ans. (c)

PREVIOUS YEAR QUESTIONS

EQUATION

Q.1 The solution of cubic equation $x^3 - 23x^2 + 142x - 120 = 0$ is given by the triplet:
[Dec. 2023]
(a) (1, 10, 12)
(b) (1, –10, 12)
(c) (–1, –10, –12)
(d) (1, 10, –12)
Ans. (a) (1, 10, 12)

Q.2 If 2x = 4y = 8z and 1/2x + 1/4y + 1/6z = 24/7, then the value of z is:
[Dec. 2023]
(a) 7/16
(b) 7/32
(c) 7/48
(d) 7/64
Ans. (c) 7/48

Q.3 The solution of the linear simultaneous equations 2x – y = 4 and 3x + 4 y = 17 is
[Dec. 2023]
(a) x = 3; y = 2
(b) x = 2; y = 3
(c) x - –3; y = –2
(d) x = –2; y = –3
Ans. (a) x = 3; y = 2

Q.4 The roots of the equation $x^3 + x^2 - x - 1 = 0$ are
[Dec. 2023]
(a) x = 1, x = –1 , x = –1
(b) x = 1, x = 1, x = –1
(c) x = –1, x = –1, x = –1
(d) x = 1, x = 1, x = 1
Ans. (a) x = 1, x = –1 , x = –1

Q.5 Given $x = \frac{\sqrt{5}+\sqrt{5}}{\sqrt{5}-\sqrt{5}}$ and $y = \frac{\sqrt{5}-\sqrt{5}}{\sqrt{5}+\sqrt{3}}$, Then find the value of $\frac{1}{x^2} + \frac{1}{y^2}$
[Dec. 2023]
(a) 63
(b) 61
(c) 62
(d) 60
Ans. (c) 62

Q.6 If α and β are the roots of the equation$x^2 - 4x + 1 = 0$, then value of $\alpha^3 + \beta^3$ will be
[Dec. 2023]
(a) –76
(b) 76
(c) –52
(d) 52
Ans. (d) 52

Q.7 The age of a person is four times the sum of the ages of his two sons. After 10 years his age will be doubled the sum of their ages. Then the present age of the man is
[June 2023]
(a) 56 years
(b) 45 years
(c) 60 years
(d) 64 years
Ans. (c) 60 years

Q.8 If α and β are the roots of the equation $x^2 - (n^2 + 1)x + \frac{1}{2}(n^4 + n^2 + 1) = 0$, then the value of $\alpha^2 + \beta^2$ is
[June 2023]
(a) 2n
(b) n^2
(c) $2n^2$
(d) n^3
Ans. (b) n^2

Q.9 If α and β are the roots of the equation $x^2 - 2x - 3 = 0$, then the equation whose roots are $\alpha + \beta$ and $\alpha - \beta$ is
[June 2023]
(a) $x^2 - 6x - 8 = 0$
(b) $x^2 - 6x + 8 = 0$
(c) $x^2 + 6x + 8 = 0$
(d) $x^2 + 6x - 8 = 0$
Ans. (b) $x^2 - 6x + 8 = 0$

Q.10 What will be the value of k, if the roots of the equation $(k-4)x^2 - 2kx + (k+5) = 0$ are equal
[Dec 2022]
(a) 18
(b) 20
(c) 19
(d) 21
Ans. (b) 20

Q.11 If the cost of 3 bags and 4 pens is Rs. 267 where as the cost of 4 bags and 3 pens is Rs. 324, then the cost of one bag is:
[Dec 2022]
(a) 8
(b) 24
(c) 32
(d) 75
Ans.

Q.12 If the roots of the equation $x^2 - px + q = 0$ are in the ratio 2 : 3 then
[Dec 2022]
(a) $p^2 = 25q$
(b) $p^2 = 5q$
(c) $6p^3 = 5q$

(d) $6p^2 = 25q$
Ans. (d) $6p^2 = 25q$

Q.13 The solution of the following system of linear equation 2x – 5y + 4 = 0 and 2x + y – 8 = 0 will be
[Dec 2022]
(a) (-2,2)
(b) (2,-3)
(c) (1,-4)
(d) (3,2)
Ans. (d) (3,2)

Q.14 If one root of $5z^2 + 13z + y = 0$ be reciprocal of the other then the value of y is
[June 2022]
(a) 1/5
(b) -1/5
(c) 5
(d) -5
Ans. (c) 5

Q.15 A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as the shortest. What is the possible length for the shortest piece?
[June 2022]
(a) 22
(b) 20
(c) 15
(d) 18
Ans. (a) 22

Q.16 If the square of a number exceeds twice of the number by 15 , then number that satisfies the condition is
[Dec 2021]
(a) -5
(b) 3
(c) 5
(d) 15
Ans. (c) 5

Q.17 In a multiple choice question paper consisting of 100 questions of $1 mark$ each, a candidate get 60% marks. If the candidate attempted all questions and there was a penalty of 0.25 marks for wrong answers, the difference between number of right answer and wrong answers is :
[Dec 2021]
(a) 32
(b) 36
(c) 40
(d) 38
Ans. (b) 36

Q.18 If one root is half of the other of a quadratic equation and the difference in roots is a then the equation is
[Dec 2021]
(a) $x^2 + ax + 2a^2 = 0$

(b) $x^2 - 3ax - 2a^2 = 0$
(c) $x^2 - 3ax + 2a^2 = 0$
(d) $x^2 + ax - 2a^2 = 0$
Ans. (c) $x^2 - 3ax + 2a^2 = 0$

Q.19 The cost of 2 oranges and 3 apples is ₹28. If the cost of an apple is doubled then the cost of 3 oranges and 5 apples is ₹75 The original cost of 7 oranges and 4 apples (in ₹) is
[July 2021]
(a) 59
(b) 47
(c) 71
(d) 63
Ans. (a) 59

Q.20 If a and B are the roots of the equation $2x^2+5x+k=0$, and $4(a^2 + b^2 + ab) = 23$, then which of the following is true? [July 2021]
(a) $k^2 + 3k - 2 = 0$
(b) $k^2 - 2k + 3 = 0$
(c) $k^2 - 2k - 3 = 0$
(d) $k^2 - 3k + 2 = 0$
Ans. (d) $k^2 - 3k + 2 = 0$

Q.21 The value of 'k' is ..... If 2 is a root of the following cubic equation: $x^3 - (k + 1) x + k = 0$
[July 2021]
(a) 2
(b) 6
(c) 1
(d) 4
Ans. (b) 6

Q.22 The rational roots of the equation $0 = 2p^3 - p^2 - 4p + 2$ is
[Dec. 2020]
(a) 2
(b) -2
(c) 1/2
(d)-1/2
Ans. (c) ½

Q.23 Solving equation $m + \sqrt{m} = \frac{6}{25}$ the value of 'm' works out to
[Dec. 2020]
(a)2/25
(b) 1/25
(c)3/25
(d) 1
Ans. (b) 1/25

Q.24 Solving equation $3g^2 - 14g + 16 = 0$ we get roots as
[Dec. 2020]
(a) 0
(b) ± 5

(c)8 and 2/3
(d) 2 and 8/3
Ans. (d) 2 and 8/3

Q.25 If $2x^2 - (a+6)2x + 12a = 0$ then roots are
[Dec. 2020]
(a) 4 & a^2
(b) 6 & a
(c)3 & 2a
(d) 6 & 3a
Ans. (b) 6 & a

Q.26 The value of P for difference between the root of equation$x^2 + px + 8 = 0$ is 2, is
[Dec. 2020]
(a) ± 2
(b) ± 4
(c) ±6
(d) ±8
Ans. (c) ±6

Q.27 The 3 roots of the equation $x^3 + 9x^2 - x - 9 = 0$ is
[Nov. 2019]
(a) 1,-1,-9
(b) -1,-9,-1
(c) 1,1,-9
(d)1,-1,9
Ans. (a) 1,-1,-9

SUMMARY

- A simple equation in one variable x is in the form ax + b = 0, where a, b are known constants and a ≠ 0.
- The general form of a linear equations in two variables, x and y is ax + by + c = 0, where a, b are non- zero coefficients and c is a constant.
- Two such equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ form a pair of simultaneous equations in x and y.
- A value for each unknown which satisfies simultaneously both the equation will give the roots of the equations.
- **Elimination Method**: In this method two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknowns and then solving for the other unknown.
- **Cross Multiplication Method**: For two equations:

  $a_1x + b_1y + c_1 = 0$

  $a_2x + b_2y + c_2 = 0$

  The solution of the equation are:

  $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $\quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

QUADRATIC EQUATION

- An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with a≠ 0 is called a quadratic equation or equation of the second degree.
- When b=0, the equation is called a pure quadratic equation; and when b ≠ 0 the equation is called an affected quadratic.

  The value of x that satisfies the given quadratic equation is called as the roots of the equation. The roots of a quadratic equation $ax^2 + bx + c = 0$ are

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- Sum and Product of the roots of quadratic equation: $ax^2 + bx + c = 0$

  $$Sum\ of\ roots = -\frac{b}{a} = -\frac{-coefficient\ of\ x}{coefficients\ of\ x^2}$$

  $$Produt\ of\ the\ roots = \frac{c}{a} = \frac{constant\ term}{coefficient\ of\ x^2}$$
- To construct a quadratic equation for the equation $ax^2 + bx + c = 0$ we have x^2 + (Sum of the roots)x + Product of the roots = 0

  Nature of the roots : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

  (i) If $b^2 - 4ac = 0$, then the roots are real and equal;

  (ii) If $b^2 - 4ac > 0$, then the roots are real and unequal (or distinct);

  (iii) If $b^2 - 4ac < 0$, then the roots are imaginary;

  (iv) If $b^2 - 4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct);

  (v) If $b^2 - 4ac > 0$, but not a perfect square the roots are real, irrational and unequal.

  Cubic Equation: An equation of 3rd degree is called as a cubic equation.

  i.e., $ax^3 + bx^2 + cx + d = 0$

  For cubic equation, $ax^3 + bx^2 + cx + d = 0$

  Sum of roots $= -\frac{b}{a}$

  Sum, of roots taken two at a time $= \frac{c}{a}$

  Product of roots $= -\frac{d}{a}$

LINEAR INEQUATION

- If the equality symbol ' = ' in a linear equation is replaced by an inequality symbol ($<, >, \leq,$ *or* $\geq$), then the statement is called linear inequality.

  E.g.: $x \leq 3, x + y > 7$ etc.

- Thing to remember for the sign of the inequality:

1. **Addition/Subtraction:** The sign of inequality doesn't change on adding or subtracting the same quantity to both sides.

   E.g.: Since, $5 < 2$

   $\Rightarrow 5 + 1 < 2 + 1 \Rightarrow 6 < 3$, which is true

   Also, $5 < 2$

   $\Rightarrow 5 - 1 < 2 - 1$

   $\Rightarrow 4 < 1$, which is true

2. **Multiplication/Division:** The sign of inequality doesn't change while multiplying and dividing with positive integers on both sides. However, the sign changes when both sides get multiply or divide by a negative integer.

   E.g.:

   On multiplying and dividing with positive integers.

   Since, $5 < 2$

   On multiplying both the sides by 2, then

   $5 \times 2 < 2 \times 2 \Rightarrow 10 < 4$

   Also, $12 > 4$

   - On dividing both the sides by 4, then

     $12 \div 4 > 4 \div 4$

     $3 > 1$

   - On multiplying and dividing with negative integers.

     Since, $15 > 3$

     On multiplying both the sides by –2, then

     $15 \times (-2) < 3 \times (-2)$

     $-30 < -6$

     Also, $10 > 6$

     On dividing both the sides by –2, then

     $\Rightarrow 10 \div (-2) > 6 \div (-2)$

     $\Rightarrow -5 < -3$

WHY TO STUDY?

- The weight of all the students in your class is less than 90 kgs.
- Conference room can occupy at most 40 tables or chairs.
- Rajesh has ₹100 and wants to buy some notebooks and pens. The cost of one notebook is ₹40 and that of a pen is ₹20.

  So, whenever there is a limit to something and not an equality for sure then we need to study the inequalities.
- These Type of inequalities occur in business whenever there is a limit on supply, demand, sales etc.

  E.g.: Raju wants to produce sofas. But the material needed can be only get on some conditions such that he has to buy cloth of sofa at least for 40 sets of sofas and he can get wood for maximum for 100 sets.

  So, in such case he needs to adjust his demands and sales accordingly and for this we need to understand the concept of inequalities.

LINEAR INEQUATION IN ONE VARIABLE

- As the heading says, when there is an inequality on one variable then it will be called as linear inequation in one variable.

  E.g.: $x \geq 5, y < 3$

  And the range where the equation satisfies is called as intervals solution space also abbreviated as S.S.

  Intervals (Solution space) of an inequality:
- If $a < b$, and $a < x < b$ means that $a < x$, and $x < b$. That is, x is between a and b.

  The following table will be making you understand it properly:

Interval	Inequality
[a, b]	$a \leq x \leq b$
[a, b]	$a \leq x < b$
(a, b]	$a < x \leq b$
(a, b)	$a < x < b$

E.g.: $(-2, 5)$ is an open interval that includes all the real numbers greater than -2 but less than 5.

$[-2, 5)$ is an interval that includes all the real numbers greater than and equal to -2 but less than 5.

$(-2, 5]$ is an open interval that includes all the real numbers greater than -2 but less than and equal to 5.

$[-2, 5]$ is closed interval that includes all the real numbers from -2 to 5 including the endpoints.

Q.1 The solution of the inequality $8x + 6 < 12x + 14$ is

(a) (-2, 2) (b) (0, 2) (c) $(2, \infty)$ (d) $(-2, \infty)$

Ans. (d)

Q.2. The solution set of the equations x + 2 > o and 2x – 6 > 0 is

(a) $(-2, \infty)$ (b) $(3, \infty)$ (c) $(-\infty, -2)$ (d) $(-\infty, -3)$

Ans. (b)

Q.3 Find the range of real of x satisfying the inequalities $3x - 2 > 7$ *and* $4x - 13 > 15$.

(a) $x > 3$ (b) $x > 7$ (c) $x < 7$ (d) $x < 3$

Ans. (b)

Q.4 Solve the inequality: $\frac{(3x-1)}{2} \leq \frac{(x+2)}{4}$.

(a) $x \leq 2$ (b) $x \leq 0.8$ (c) $x \geq 1.5$ (d) $x \geq 2$

Ans. (b)

Q.5 if $3x + 2 < 2x + 5$ *and* $4x - 5 \geq 2x - 3$, then x can take the following value

(a) 3 (b) -1 (c) 2 (d) -3

Ans. (c)

LINEAR INEQUATION IN TWO VARIABLES

- Again, as the heading says, when there is an inequality in two variables then it will be called as linear inequation in two variables. For example: $ax + by \leq c$, $x + 2y < 6$, $x + y \geq 5$, $3x + y > 9$ are some examples of the linear inequality of two variables.

E.g.: Let's plot a graph of $x + 2y \leq 10$

The line of equation corresponding to above inequality is $x + 2y = 10$

When $x = 0$ then $y = 5$

When $y = 0$ then $x = 10$

Thus, the points are (0, 5) and (10, 0).

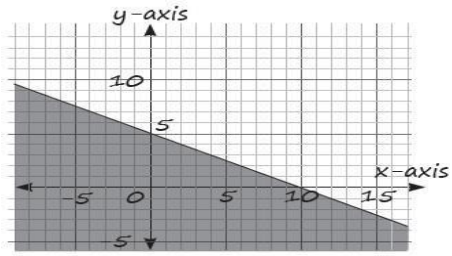
Now, put x = 0 and y = 0 in given inequality, we get

$0 + 0 \leq 10$

$0 \leq 10$, which is true

Thus, the shaded region will be towards the origin.

Therefore, the required graph is:



Q.6 Rajesh has ₹100 and wants to buy some notebooks and pens. The cost of one notebook is ₹40 and that of a pen is ₹20. Draw the linear inequality

(a) $40x + 20y < 100$ (b) $40x + 20y \leq 100$

(c) $40x - 20y > 100$ (d) $40x + 20y > 100$

Ans. (b)

Q.7 There are 150 students in a class. If the number of boys is x and number of girls are y, then which of the following inequalities represents the situation?

(a) $x + y \leq 150$ (b) $x + y \geq 150$

(c) $x + y \neq 150$ (d) None of these

Ans. (a)

Q.8 Fisherman catches x fishes and y crabs for his family to eat on daily basis till the time it reaches a number of minimum 10. Now tell, x and y can be related by the inequality

(a) x + y ≠ 10 (b) x + y ≤ 10, x ≥ 0, y ≥ 0

(c) x + y ≥ 10, x ≥ 0, y ≥ 0 (d) None of these

Ans. (c)

Q.9 An average size fish can feed 4 people while one crab can feed only one people but the employer has to maintain an output of at least 15 people per done. This situation can be expressed as

(a) $4x + y \leq 15$ (b) $4x + y > 15$

(c) $4x + y \geq 15, x \geq 0, y \geq 0$ (d) None of these

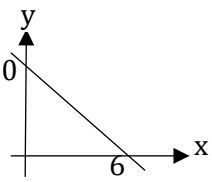
Ans. (c)

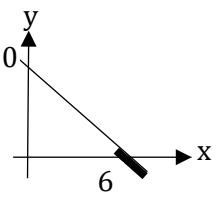
Q. 10 But he makes sure that he takes home more than 2 fishes to each crab. This situation can be expressed as

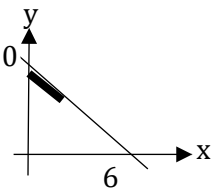
(a) $x \geq y/2$ (b) $y \leq x/2$ (c) $y \geq x/2$ (d) $2x > y$

Ans. (b)

Q.11 The graph to express the inequality $5x + 3y \geq 30$ is

(a) 

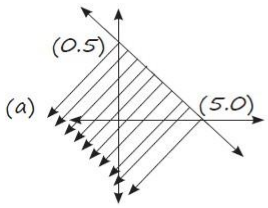
(b) 

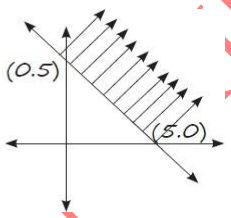
(c) 

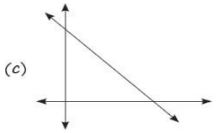
(d) None of these

Ans. (c)

Q.12 The graph to express the inequality $x + y \leq 5$ is

(a) 

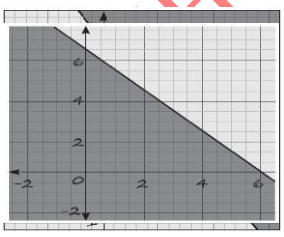
(b) 

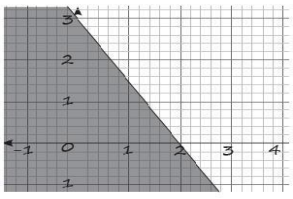
(c) 

(d) None of these

Ans. (a)

Q. 13 The graph to express the inequality $3x + 2y \leq 6$ is

(c) 

(b) 

(d) None of these

Ans. (b)

Q. 14 On solving the inequalities $2x + 5y + 20, 3x + 2y \leq 12, x \geq 0, y \geq 0$, we get the following situation

(a) $(0,0), (0,4), (4,0)$ *and* $(\frac{20}{11},\frac{36}{11})$
(b) $(0,0), (10,0), (0,6)$ *and* $(\frac{20}{11},\frac{36}{11})$
(c) $(0,0), (0,4), (4,0)$ *and* $(2,3)$
(d) $(0,0), (10,0), (0,6)$ *and* $(2,3)$

Ans. (a)

Q. 15 A firm makes two types of products: Type A and Type B. The profit on product A is ₹20 each and that on product B is ₹30 each. Both types are processed on three machines M1, M2 and M3. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Product A	Product B	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

The constraints can be formulated taking x_1 = number of units A and x_2 = number of unit of B as

(a) $x_1 + x_2 \le 12$
$5x_1 + 2x_2 \le 50$
$2x_1 + 6x_2 \le 60$
$3x_1 + 3x_2 \le 36$

(b) $3x_1 + 3x_2 \le 36$
$5x_1 + 2x_2 \le 50$
$2x_1 + 6x_2 \le 60, x_1 \ge 0, x_2 \ge 0$

(c) $2x_1 + 6x_2 \le 60$
$5x_1 + 2x_2 \le 50, x_1 \ge 0, x_2 \ge 0$

(d) None of these

Ans. (c)

Q.16 The union forbids employer to employ less than two experienced people (x) to each fresh person (y). This situation can be expressed as

(a) $x \le y/2$ (b) $y \le x/2$ (c) $y \ge x/2$ (d) None of these

Ans. (b)

Q.17 On the average an experienced person does 7 units of work while a fresh one works 5 units of work daily but the employer has to maintain an output of at least 35 units of work per day. The situation can be expressed as:

(a) $7x + 5y < 35$ (b) $7x + 5y \le 35$
(c) $7x + 5y > 35$ (d) $7x + 5y \ge 35$

Ans. (d)

Q.18 Solution space of the inequalities $2x + y \le 10$ and $x - y \le 5$:

(i) Include the origin.
(ii) Includes the point (4, 3)
Which one is correct?

(a) Only (i) (b) Only (ii) (c) Both (i) and (ii) (d) None of these

Ans. (a)

Q.19 The linear relationship between two variables in an inequality:

(a) $ax + by \le c$ (b) $axby \le c$
(c) $axy + by \le c$ (d) $ax + bxy \le c$

Ans. (a)

Q.20 The rules and regulations demand that the employer should empty not more than 5 experienced hands to 1 fresh one and this fact is represented by: (Taking experienced person as x and fresh person as y)

(a) $y \ge \frac{x}{5}$ (b) $5y \le x$ (c) $5y \ge x$ (d) None of these

Ans. (a)

Q.21 if $a > 0$ *and* $b < 0$, it follows that

(a) $\frac{1}{a} > \frac{1}{b}$ (b) $\frac{1}{a} < \frac{1}{b}$ (c) $\frac{1}{a} = \frac{1}{b}$ (d) None of these

Ans. (a)

Q.22 A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per kg of each food is shown below:

	A	B	C	D
Food I:	2	1	1	2
Food II:	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II, the situation can be expressed as

(a) $2x + y \leq 9, x + y \leq 7, x + 2y \leq 10, 1x + 2y \leq 12, x > 0, y > 0$

(b) $2x + y \geq 30, x + y \leq 7, x + 2y \leq 10, x + 3y \geq 12$

(c) $2x + y \geq 9, x + y \geq 7, x + y \leq 10, x + 3y \geq 12$

(d) $2x + y \geq 9, x + y \geq 7, x + 2y \geq 10, 2x + 3y \geq 12, x \geq 0, y \geq 0$

Ans. (d) Given, Quantity of food $I = x$ units

Quantity of food $II = y$ *units*

Since, the content should be atleast 9 units of vitamin A, 7 units of Vitamin B, 10 units of Vitamin C and 12 units of vitamin D

$\Rightarrow 2x + y \geq 9, x + y \geq 7, x + 2y \geq 10, 2x + 3y \geq 12,$

Also, the quantity of food cannot be negative.

$\Rightarrow x \geq 0, y \geq 0$

Hence, the final correct answer is option (d)

i.e., $2x + y \geq 9, x + y \geq 7, x + 2y \geq 10, 2x + 3y \geq 12, x \geq 0, y \geq 0$

Q.23 On solving the inequalities $5x + y \leq 100$, $x + y \leq 60$, $x \geq 0$, $y \geq 0$, we get the following situation:

(a) (0, 0), (20, 0), (10, 50) & (0, 60)

(b) (0, 0), (60, 0), (10, 50) & (0, 60)

(c) (0, 0), (20, 0), (0, 100) & (10, 50)

(d) None of these

Ans. (a) The inequalities are:

$5x + y \leq 100$

$x + y \leq 60$

The line of equation for inequality: $5x + y \leq 100$

$\Rightarrow 5x + y = 100$

Reference of points:

x	0	20	10
y	100	0	50

The line of equation for inequality: $x + y \leq 60$

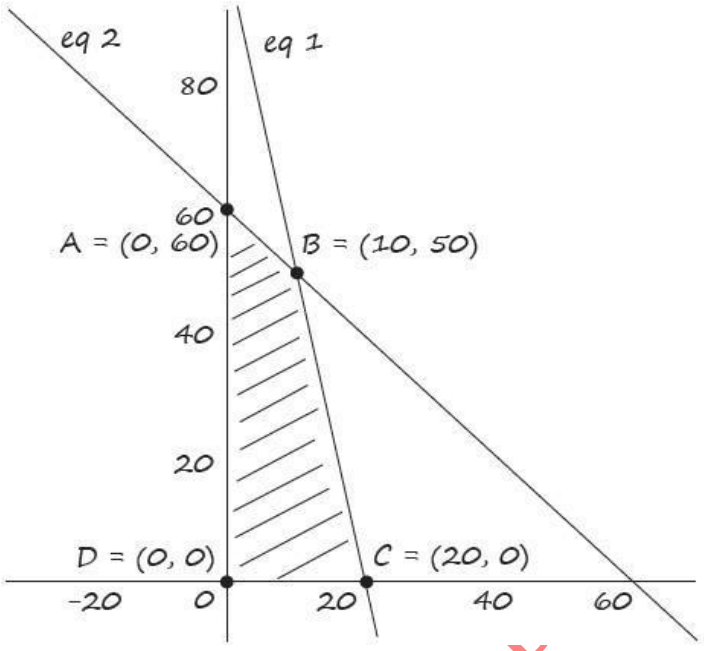
$\Rightarrow x + y = 60$

Reference of points:

x	0	60	10
y	60	0	50

For the inequalities, the value of x and y will lie either on the lines $x + y = 60$ and $5x + y = 100$ or below them.

It is given $x \geq 0, y \geq 0$.
Thus the value of x and y belongs to 1st quadrant
The graph for the given set of inequalities is as follows:



Thus, we get the four points
$A = (0,60); B = (10,50); C = (20,0); d = (0,0)$
Hence the correct option is (a)

Q.24 An employer recruits experienced (x) and fresh workmen (y) under the condition that he cannot employ more than 11 people. x and y can be related by the inequality
(a) $x + y = 11$ (b) $x + y \leq 11, x \geq 0, y \geq 0$
(c) $x + y > 11, x \geq 0, y \geq 0$ (d) None of these
Ans. (b)

Q.25 The solution of the inequality $\frac{(5-2x)}{3} \leq \frac{x}{6} - 5$ is
(a) $x \geq 8$ (b) $z \leq 8$ (c) $x = 8$ (d) None of these
Ans. (a)

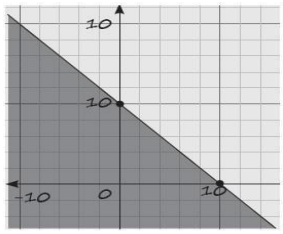
Q.26 Which of the following situation do we get on solving the inequalities: x – y ≤ 2, x + y ≤ 4 and x ≥ 1 ?
(a) (1, 3), (1, 0), (2, 0), (3, 1)
(b) (1, 1), (2, 0), (3, 1), (6, 4)
(c) (1, 0), (2, 0), (3, 1), (7, 5)
(d) None of these
Ans. (a)

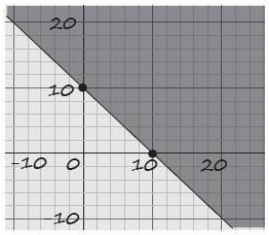
Q.27 The solution of the set of inequalities $2x + y \geq 12, 5x + 8y \geq 74, x + 6y \geq 24, x \geq 0, y \geq 0$ are
(a) $(24,0), (\frac{126}{11}, \frac{23}{11}), (2,8), (0,12)$
(b) $(0,24), (2,8), (0,12), (\frac{126}{11}, \frac{23}{11})$

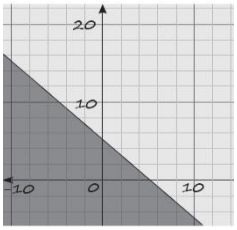
(c) (8,4), (2,8), (0,12), (0,24)
(d) (8,4), (0,0), (0,6), (2,0)

Ans. (a)

Q.28 Which of the following graph represents the inequality $x + y \leq 10$?

(a) 

(b) 

(c) 

(d) None of these

Ans. (a)

Q.29 Vitamin A and B are found in food F_1 and F_2. One unit of F_1 contains 20 units of vitamin A and 30 units of vitamin B. One unit of F_2 contains 60 units of vitamin A and 40 units of vitamin B. Cost per unit of F_1 and F_2 are ₹3 and ₹4 respectively. The minimum daily requirement of vitamin A and B is 80 and 100 units respectively.

Problem is to determine the mixture of food F_1 and F_2, which meets the requirement at minimum cost by assuming that x_1 units of food F_1 and x_2 units of food F_2 are required to fulfill the need of vitamins. The constraints are

(a) $20x_1 + 60x_2 \leq 80,\ 30x_1 + 40x_2 \leq 40,\ x_1 \leq 0;\ x_2 \leq 0$
(b) $20x_1 + 60x_2 \geq 80,\ 30x_1 + 40x_2 \leq 40,\ x_1 \geq 0;\ x_2 \leq 0$
(c) $20x_1 + 60x_2 \geq 80,\ 30x_1 + 40x_2 \geq 40,\ x_1 \geq 0;\ x_2 \geq 0$
(d) $20x_1 + 60x_2 \leq 80, 30x_1 + 40x_2 \geq 40, x_1 \leq 0; x_2 \geq 0$

Ans. (c)

Q. 30 A car manufacturing company manufacturers cars of two types A and B. Model A requires 150 man hours for assembling, 50 man-hours for painting and 10 man-hours for checking and testing. Model B requires 60 man-hours for assembling, 40 man-hours for painting and 20 man-hours for checking and testing. There are available 30 thousand man-hours for assembling,13 thousand man-hours for painting and 5 thousand man-hours for checking and testing. Express the above situation using linear inequalities. Let the company manufacture x units of type A model of car and y units of type B model of car.

Then the inequalities are:

(a) $5x + 2y \geq 1000,\ 5x + 4y \geq 1300,\ x + 2y \leq 500,\ x \geq 0,\ y \geq 0$
(b) $5x + 2y \leq 1000,\ 5x + 4y \leq 1300,\ x + 2y \leq 500,\ x \geq 0,\ y \geq 0$
(c) $5x + 2y \leq 1000,\ 5x + 4y \leq 1300,\ x + 2y \leq 500,\ x \geq 0,\ y \geq 0$
(d) $5x + 2y = 1000, 5x + 4y \geq 1300, x + 2y = 500, x \geq 0, y \geq 0$

Ans. (b)

PREVIOUS YEAR QUESTIONS

LINEAR INEQUALITIES

Q.1 The solution of the inequality $\frac{5-2x}{3} \leq \frac{x}{6} - 5$ is:
[Dec. 2023]
(a) $x \geq 8$
(b) $x \geq 7$
(c) $x \leq \frac{80}{3}$
(d) $x \geq \frac{40}{3}$
Ans. (a) $x \geq 8$

Q.2 A software company should recruit more than or equal to 10 employees at a time in their recruitment drive. Under these conditions, company recruits experienced (x) and freshers (y) employees. The value of x and y can be related by the following inequality. [Dec. 2023]
(a) $x + y \leq 10, x \geq 0, y \geq 0$
(b) $x + y \geq 10, x \geq 0, y \geq 0$
(c) $x + y \neq 10, x \geq 0, y \geq 0$
(d) $x + y \geq 10, x \leq 0, y \leq 0$
Ans. (b) $x + y \geq 10, x \geq 0, y \geq 0$

Q.3 The largest side of a triangle is 3 times the shortest side and third side is 4 *cm* shorter than largest side. If the perimeter of the triangle is at least 59 *cm*, what is the length of the shortest side?
[June 2023]
(a) Less than 7cm
(b) Greater than or equal to 7cm
(c) Less than 9cm
(d) Greater than or equal to 9cm
Ans. (d) Greater than or equal to 9cm

Q.4 In a garment factory an average experienced person does 5 units of work while a fresh one does 3 units of work daily but the employer has to maintain output of at least 30 units of work per day. This situation can be expressed as
[June 2023]]
(a) $5x + 3y \leq 30$
(b) $5x + 3y \geq 30$
(c) $5x + 3y \geq 30\,, x \geq 0\,, y \geq 0$
(d) $5x + 3y \leq 30\,, x \geq 0\,, y \geq 0$
Ans. (c) $5x + 3y \geq 30\,, x \geq 0\,, y \geq 0$

Q.5 A fertilizer company produces two types of fertilizers called grade I and grade II. Each of these types is processed through two critical chemical plant units. The plant has maximum of 180 hours available in a week. Manufacturing one bag of grade I fertilizer requires 4 hours in the plant. Manufacturing one bag of grade II fertilizer requires 10 hours in the plant. Express this using linear

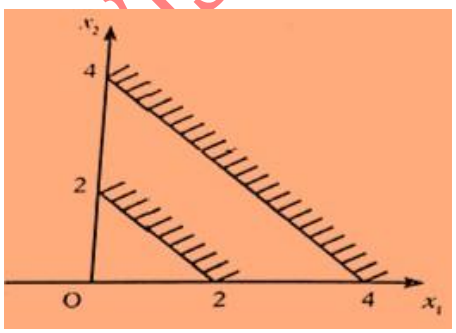
inequality.
[June 2023]
(a) $5x_1 + 3x_2 \leq 180$
(b) $4x_1 + 5x_2 > 180$
(c) $2x_1 + 5x_2 > 180$
(d) $4x_1 + 10x_2 \leq 180$
Ans. (d) $4x_1 + 10x_2 \leq 180$

Q.6 If 2x +5 > 3x + 2 and 2x – 3 <= 4x – 5, the 'x' can take which of the following value?
[Dec 2022]
(a) 4
(b) -4
(c) 2
(d) -2
Ans. (c) 2

Q.7 XYZ Company has a policy for its recruitment as : it should not recruit more than eight men (x) to three women (y). How can this fact be expressed in inequality?
[Dec 2021]
(a) $3y \geq 8x$
(b) $3y \leq \frac{x}{8}$
(c) $8y \geq 3x$
(d) $8y \leq 3x$
Ans. (c) $8y \geq 3x$

Q.8 If y = 4 + 9 sin 5x then which hold good
[July. 2021]
(a) $-5 \leq y \leq 13$
(b) $-4 \leq y \leq 8$
(c) $0 < y < 0$
(d) $-5 < y < 5$
Ans. (a) $-5 \leq y \leq 13$

Q.9 The region indicated by the shading in the graph is expressed by the inequalities
[Dec 2021]



(a) $x_1 + x_2 \leq 2$, $x_1 + x_2 \geq 4$, $x_1 \geq 0$, $x_2 \geq 0$
(b) $x_1 + x_2 \leq 2$, $x_1x_2 + x_2 \leq 4$, $x_1 \geq 0$, $x_2 \geq 0$
(c) $x_1 + x_2 \geq 2$, $x_1 + x_2 \geq 4$, $x_1 \geq 0$, $x_2 \geq 0$
(d) $x_1 + x_2 \leq 2$, $x_1 + x_2 > 4$, $x_1 \geq 0$, $x_2 \geq 0$
Ans. (c) $x_1 + x_2 \geq 2$, $x_1 + x_2 \geq 4$, $x_1 \geq 0$, $x_2 \geq 0$

SUMMARY

- The chapter on inequalities explores mathematical relationships between expressions or numbers that are not equal. Inequalities are represented using symbols such as " < " (less than), " >" (greater than), "≤" (less than or equal to), and "≥" (greater than or equal to).
- **Inequality Symbols:** Inequalities are expressed using symbols such as " < ", " > ", "≤" and "≥" to compare two quantities or expressions.
- **Solving Linear Inequalities:** Solving linear inequalities involves finding the range of values for two variables that satisfy the given inequality. This often includes algebraic manipulation, such as adding, subtracting, multiplying, or dividing both sides of the inequality by constants.
- **Graphical Representation:** One of the fundamental aspects of this chapter is the graphical representation of linear inequalities. These inequalities can be visualized on a coordinate plane as shaded regions. The region below or above a line (depending on the inequality symbol) represents the set of points that satisfy the inequality.
- **Word Problems:** Inequality concepts are applied to real-world situations in word problems. These problems require translating verbal descriptions into mathematical inequalities and solving for the unknown quantities.
- Overall, understanding linear inequalities and their graphical representation is crucial in mathematics and real-life applications. It helps describe relationships where quantities are not necessarily equal but can fall within a range of values. Solving linear inequalities is a fundamental skill in algebra and has numerous applications in fields such as economics, science, and engineering.

WHAT IS TIME VALUE OF MONEY AND WHY IT IS SO IMPORTANT?

Time Value of Money (TVM) is an important concept that money in hand today is worth more than a money promised in the future. Basically, it is a fundamental financial concept that recognizes the worth of money over time. It emphasizes the principle that a sum of money available today is worth more than the same amount of money in the future. This is primarily because money has the potential to earn returns or interest when invested or utilized immediately.

WHY IS INTEREST PAID?

Interest compensates for the time value of money, recognizing that receiving money earlier is generally more beneficial than receiving it later. By paying interest, the borrower compensates the lender for the temporary use of their funds.

- **Opportunity Cost:** When individuals or organizations lend money, they forgo potential alternative uses of those funds. The interest paid serves as compensation for the opportunity cost of not investing or using the money elsewhere.
- **Inflation:** Inflation erodes the purchasing power of money over time. Lenders charge interest to account for the expected decrease in the value of money due to inflation. This ensures that the lender's real purchasing power is maintained or increased.
- **Liquidity Preference:** Lenders may require interest as compensation for the lack of immediate access to their funds. By lending money, they forgo the ability to use that money for their own immediate needs or expenses.
- **Risk Factor:** Interest also accounts for the risk associated with lending money. Lenders assess the creditworthiness of borrowers and charge interest rates based on the perceived risk of default. Higher-risk borrowers are charged higher interest rates to compensate for the increased probability of non-repayment.

These factors collectively determine the interest rate charged on loans, bonds, or other financial instruments, allowing lenders to earn a return on their capital while borrowers gain access to funds for various purposes.

DEFINITIONS

Interest: Fee paid for the use of another party's money. To the borrower it is the cost of renting money, to the lender the income from lending it.

E.g.: The principal amount invested by Anita is ₹10,000. After 1 year, she received the accumulated amount or balance, including both the principal and the interest, as ₹10,500.

Thus, the difference between ₹10,500 and ₹10,000 which is ₹500 is the interest she earned in a year.

Principal: The total amount of money borrowed (or invested), not including any interest.

E.g.: Sarah wants to buy a new laptop and decides to take a loan of ₹30,000 from a finance company. The ₹30,000 is the principal amount she borrows, not including any interest.

Rate of Interest: The interest rate (rate of interest) is the rate at which interest is charged on the principal amount which is landed or borrowed. Rate of interest is usually expressed as percentages.

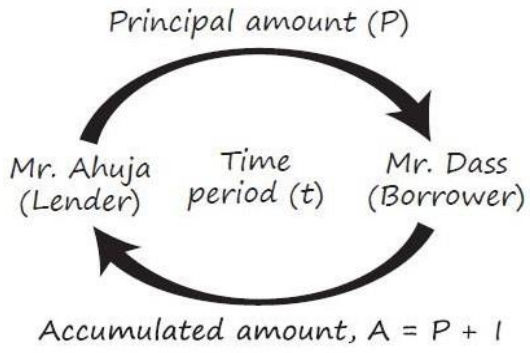
E.g.: Mark wants to invest his savings in a fixed deposit with a bank. The bank offers an annual interest rate of 6% on fixed deposits. The 6% is the rate of interest at which Mark's money will earn interest over time.

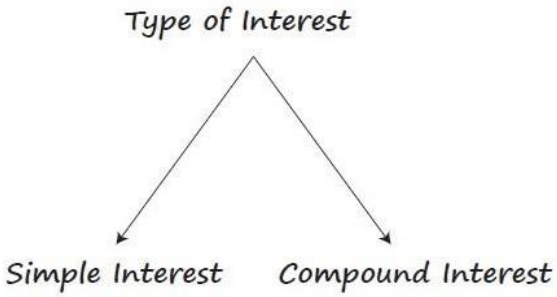
Time Period: The time period refers to the duration for which the principal amount is borrowed or invested, and during which interest is charged or earned. It is the length of time over which the financial transaction takes place.

E.g.: If John borrows ₹50,000 from a bank at an annual interest rate of 8% for a time period of 2 years, then the time period in this case is 2 years.

Accumulated amount (or Balance): The amount which you receive after the due period including the principal and the interest in called as accumulated amount.

E.g.: Ankita invested ₹20,000 in a savings account that earns an annual interest rate of 4%. After three years, she checked her account balance and found it to be ₹22,408. The ₹22,408 is the accumulated amount or balance, including the principal and the interest earned over the three-year period.





SIMPLE INTEREST

- Simple interest is the way when we take interest at a simple rate on the principal amount over a period of time.
- Simple interest increases (or decreases) if Principal amount increases (or decreases) or if interest rate increases (or decreases) or if the time period increases (or decreases).
- It means Simple interest(I) is directly proportional to Principal Amount (P), Interest rate (R%), Time period (T), Therefore,

$$\text{S.I} = \frac{P \times \text{R} \times \text{T}}{100}$$

Q.1. S.I. on ₹3,500 for 3 years at 12% per annum is

(a) ₹1,200 (b) ₹1,260 (c) ₹2,260 (d) None of these

Ans. (b)

Q.2 Sania deposited ₹50,000 in a bank for two years with the interest rate of 5.5% p.a. How much interest would she earn?

(a) ₹2,750 (b) ₹5,500 (c) ₹55,500 (d) None of these

Ans. (b)

Calculator trick: I = 50,000 × 2 × 5.5% = 5500

Therefore, she earned ₹5,500 as interest.

Hence, the correct option is (b).

Q.3 If P = ₹5,000, T = 1 year, I = ₹300, R will be

(a) 5% (b) 4% (c) 6% (d) None of these

Ans. (c)

Q.4 How much interest will be earned on ₹500 at 10% simple interest for 5 years and what will be the accumulated amount?

(a) ₹125, ₹625 (b) ₹250, ₹500

(c) ₹250, ₹750 (d) None of these

Ans. (c)

Calculator trick: I = 500 × 5 × 10% = 250
A = P + I = 500 + 250 = 750

Hence, the correct option is (c).

Q.5 P = ₹12,000, A = ₹16,500, T = $2\frac{1}{2}$ years. Rate percent per annum simple interest will be

(a) 15% (b) 12% (c) 10% (d) None of these

Ans (a)

Calculator trick: Steps to follow:
1. 16500 ÷ 12000
2. –1
3. ×100
4. ÷2.5 which gives the required result i.e., 15%.

Hence, the correct option is (a)

Q.6. P=₹8,500, A= ₹10,200, R=$12\frac{1}{2}$% SI, T will be

(a) 1 year 7 months (b) 2 years

(c) $1\frac{1}{2}$ years (d) None of these

Ans. (a)

Q.7 A sum of money amounts to ₹6,200 in 2 years and ₹7,400 in 3 years. The principal and rate of interest are

(a) (a) ₹3,000, 20% (b) ₹3,000, 20%

(c) ₹3,500, 15% (d) None of these

Ans. (a)

Trick:
If simple interest for 1 year be S.I then
Amount in 2 years = A_2 =P + 2 (S.I)
Amount in 3 years = A_3 =P + 3 (S.I)
Thus $A_3 - A_2$ = S.I
S.I. = 7400 – 6200 = 1200
=> $\frac{P\times R\times T}{100}$ =1200
=> 12,000

Go by choices:
Option (a): ₹3,800, 31.58%
PR = 3800 × 31.58 = 120004 ≈ 120000
Option (b): ₹3,000, 20%
PR = 3000 × 20 = 60000 ≠ 120000
Option (c): ₹3,500, 15%
PR = 3500 × 15 = 52500 ≠ 120000

Hence, the correct option is (a).

Q.8 A sum of money doubles itself in 10 years. The number of years it would triple itself is

(a) 25 years (b) 15 years (c) 20 years (d) None of these

Ans. (c)

Q.9 Find the rate of interest if the amount owned after 3 months is ₹525, borrowed amount being ₹500

(a) 25% (b) 20% (c) 30% (d) None of these

Ans. (b)

Q.10 Rohit invested ₹50,000 in a bank at the rate of 5.5% p.a. simple interest rate. He received ₹69,250 after the end of term. Find the period for which the sum was invested by Rohit.

(a) 7 Years (b) 5 years (c) 10 years (d) None of these

Ans. (a)

Q.11 Kiran took some amount from Suraj for 6 and $\frac{1}{2}$years at the rate of 4% p.a. simple interest. And Kiran after that time has to pay ₹31,500 at the end of the term. What amount did Kiran took from Suraj?

(a) ₹25,240.40 (b) ₹30,000 (c) ₹25,000 (d) ₹21,550

Ans. (c)

Q.12 A farmer borrowed ₹3600 at the rate of 15% simple interest per annum. At the end of 4 years, he cleared this account by paying ₹4000 and a cow. The cost of the cow is

(a) ₹1,000 (b) ₹1,200 (c) ₹1550 (d) ₹1,760

Ans. (d)

Q.13 If P = ₹3,600, A = ₹6,400, then Simple interest i.e. I will be

(a) ₹1,800 (b) ₹2800 (c) ₹1600 (d) ₹9000

Ans. (b)

Q.14 ₹80,000 is invested to earn a monthly interest of ₹1200 at the rate of_____p.a. simple interest.

(a) 12% (b) 14% (c) 16% (d) 18%

Ans. (d)

Q.15 How long will it take for a sum to quadruple itself at 10% per year simple interest?

(a) 13% (b) 15 years (c) 25 years (d) 30 years

Ans. (d)

COMPOUND INTEREST

The interest calculated on the initial principal, which also includes all of the accumulated interest of previous periods of a deposit or loan.

SYSTEM USED BY BANKS

For example: Suppose you deposit ₹1,000,000 in CITI bank for 3 years at 5% p.a. compounded annually. To calculate the Interest, the concept of compound interest is used.

Now, what will be the formula for it?

C. I $= P\left(1 + \frac{r}{100}\right)^t - P$

$A = P\left(1 + \frac{r}{100}\right)^t$

Q.16 Raju took a loan of ₹10,000 from bank at the compound interest rate of 7% annually for 3 years. How much Raju needs to pay bank now?

(a) ₹12,250.43 (b) ₹22,250.43 (c) ₹2,250.43 (d) None of these

Ans. (a)

Calculation trick:

(i) 1.07 × = =

(ii) Then × 10000

Q.17 If P=₹1,000 R=5% p.a. n=4; What is Amount and C.I.?

(a) ₹1,215.50, ₹215.50 (b) ₹1,125.50, ₹125

(c) ₹2,115, ₹115 (c) None of these

Ans. (a)

Calculation trick: 1.05 × = = =
Then × 1000 which gives 1215.50 as amount
Subtract 1000 from it to get the value of C.I.

Q.18 ₹100 will become after 20 years at 5% p.a. compound interest of

(a) ₹250 (b) ₹205 (c) ₹165.33 (d) None of these

Ans. (c)

Q.19 Mr. X borrowed ₹5,120 at 12.5% p.a. C.I. At the end 3 years, the money was repaid along with the interest accrued. The amount of interest paid by him is.

(a) ₹2100 (b) ₹2170 (c) ₹2000 (d) None of these

Ans. (b)

Q.20 If ₹64 amount to ₹83.20 in 2 years, what will ₹86 amount to in 4 years at the same rate percent per annum?

(a) 127.60 (b) 147.60 (c) 145.34 (d) 117.60

Ans. (c)

Q.21 10 years ago the earning per share (EPS) of ABC Ltd. was ₹5 share. Its EPS for this year is ₹22. Compute at what rate, EPS of the company grows annually?

(a) 15.97% (b) 16.77% (c) 18.64% (d) 14.79%

Ans. (a)

CONVERSION PERIOD

The time period according to which it is going to be compounded.

E.g.: we can take a compounding period–annually, semi-annually, quarterly, monthly or daily.

TYPICAL CONVERSION PERIOD

Conversion Period	Description	Number of Conversion period in a year (c)
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded Semi Annually	2
12 months	Compounded Annually	1

Q.22 ₹5,000 is invested at annual rate of interest of 5%. What is the amount after two years if compounding is done]

(I) Annually (II) Semi-annually (III) Quarterly (IV) monthly

FORMULAS FOR COMPOUND INTEREST

- $A_n = P(1 + i)^n$
- Compound Interest (C. I) $= P[(1 + i)^n - 1]$

where, A_n = Amount accumulated after n^{th} period

i = Annual rate of interest

P = Principal amount

n is total conversion i.e. time period * total no. of conversions

Q.23 The C.I on ₹16000 for $1\frac{1}{2}$ years at 10% p.a. payable half-yearly is

(a) ₹2,222 (b) ₹2,522 (c) ₹2,500 (d) None of these

Ans. (b)

Q.24 In what time will ₹4,000 amount to ₹4,410 at 10% per annum interest compounded half-yearly?

(a) 1 year (b) 2 years (c) 2.5 years (d) Cannot be determined

Ans. (a)

Q.25 The difference between the S.I and C.I on ₹2,400 for 2 years at 5% p.a. is

(a) ₹5 (b) ₹10 (c) ₹16 (d) ₹6

Ans. (d)

Q.26 The difference between C.I and S.I. on a certain sum of money invested for 3 years at 6% p.a. is ₹110.16. The principal is

Q.27 In how many years a sum of money triples at 5% pa. compound interest payable on a half- yearly basis?

(a) 15 years 7 months (b) 18 years 6 months

(c) 18 years 8 months (d) 22 years 3 months

Ans. (d)

Q.28 Find the compound interest of ₹700 invested for 15 years at 8% compounded semiannually.

(a) ₹1570.37 (b) ₹2270.37 (c) ₹1800.75 (d) None of these

Ans. (a)

Q.29 How long will it take for ₹5000 amount to ₹7000 if it is invested at 8% compounded quarterly?

(a) $4\frac{1}{4}$ years (b) $3\frac{1}{2}$ years (c) $5\frac{1}{4}$ years (d) None of these

Ans. (a)

Q.30 The difference between the simple and compound interest on a certain sum for 2 years at 4% p.a. is ₹600. The sum is

(a) ₹3,50,000 (b) ₹3,75,000 (c) ₹3,27,500 (d) ₹2,97,550

Ans. (b)

Q.31 A person opened an account on Jan 2019 with a deposit of ₹500. The account paid 5% interest compounded quarterly. In July 2019 he closed the account and added enough additional money to invest in a 6-months' time deposit for ₹800, earning 5% compounded monthly.

(I) How much additional amount did the person invest on July 1?

(II) What was the maturity value of his time deposit on Jan 1, 2020?

(III) How much total interest was earned?

(a) ₹512.58, ₹820.21, ₹32.79 (b) ₹512.58, ₹820.21, ₹12.58

(c) ₹237.42, ₹820.21, ₹32.79 (d) ₹1037.42, ₹820.21, ₹32.79

Ans. (c)

Q.32 The annual birth and death rates per 1,000 are 30 and 10 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

(a) 35 years (b) 30 years (c) 25 years (d) none of these

Ans. (a)

EFFECTIVE RATE OF INTEREST

- If interest is compounded more than once a year, the effective interest rate for a year exceeds the per annum interest rate.
- For example: If we invest ₹1000 for a year at the compound interest rate of 5% quarterly. The effective rate of interest will be more when compounded in comparison to 5%.

So, to find out effective interest rate:

Effective Rate of Interest (E) *is given as*

$$E = ((1 + \frac{r}{100c})^{c} - 1) \times 100\%$$

Q.33 The effective rate of interest corresponding to a nominal rate 4% p.a. payable half yearly is

(a) 4% (b) 4.04% (c) 4.45% (d) None of these

Ans. (b)

Q.34 Which is a better investment: 7% per year compounded monthly or 7.5% per year compound interest?

(a) 7% compound interest (b) 7.5% compound interest

(c) Both are same (d) Cannot be determined

Ans. (b)

Q.35 A machine is depreciated at the rate of 10% on reducing balance. The original cost of the machine was ₹50,000 and its ultimate scrap value was ₹15,000. The effective life of the machine is

(a) 10 years (b) 11.4 years (c) 12.5 years (d) None

Ans. (b)

Q.36 The effective rate of interest corresponding to a nominal rate 3% p.a. payable half yearly is

(a) 3.2% p.a. (b) 3.25% p.a. (c) 3.0225% p.a. (d) None of these

Ans. (c)

Q.37 The effective annual rate of interest corresponding to a nominal rate of 6.5% p.a. compounded quarterly is

(a) 6.0% (b) 6.25% (c) 6.50% (d) 6.66%

Ans. (d)

Q.38 An investment earns 5% interest per annum, compounded half yearly. What is the effective annual rate of interest?

(a) 5.0650% (b) 5.0925% (c) 5.0625% (d) 5.1525%

Ans. (c)

Q.39 A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹1,00,000 and its ultimate scrap value was ₹30,000. The effective life of the machine is

(a) 4.5 years (appx.) (b) 5.4 years (appx.)

(c) 5 years (appx.) (d) None of these

Ans. (b)

ANNUITY

In many cases, individuals have regular financial obligations where they make equal payments at fixed intervals, such as monthly or yearly. Examples of such payments include life insurance premiums, rent for a rented house, housing loan payments, vehicle loan installments, etc. This consistent payment of a fixed amount over a specified time period is known as an annuity.

An annuity can be defined as a sequence of periodic payments or receipts made regularly over a specific duration. It applies to situations where individuals receive a fixed amount regularly, like pension payments or rental income from a property.

TAKING THREE CASES

Case 1: When the payment or receipt amount is not the same.

Year end	Payment (₹)
2014	1,000
2015	2,000
2016	3,000
2017	4,000

Case 2: When Payment (or received) amount is same, but the period is not constant.

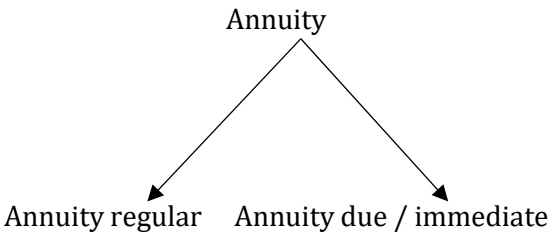
Year end	Payment (₹)
2014	1,000
2015	1,000
2018	1,000
2020	1,000

Case 3: When payment and periodic period are same.

Year end	Payment (₹)
2014	2,000
2015	2,000
2016	2,000
2017	2,000

Here, case 1 and 2 cannot be an annuity whereas case 3 can be an annuity since the payment and the time interval between the consecutive payments are same.

There are two types of Annuity:



REGULAR ANNUITY

In a regular annuity, the first payment or receipt occurs at the end of the first period. For example, consider the following table:

Year end	Payment (₹)
2014	2,000
2015	2,000
2016	2,000
2017	2,000

In this case, the first payment or receipt takes place at the end of the first year, making it a regular annuity.

ANNUITY DUE OR ANNUITY IMMEDIATE

In an annuity due or annuity immediate, the first payment or receipt is made today, at the beginning of the annuity.

For example, consider the following table:

the beginning of	Payment (₹)
2014	2,000
2015	2,000
2016	2,000
2017	2,000

In this case, the first payment or receipt is made at the beginning of the first year. This type of annuity is referred to as annuity due or annuity immediate.

FUTURE VALUE

Future value refers to the monetary worth of an investment or sum of money at a specific point in the future. It represents the value that today's money will grow to over time when compounded at a certain interest rate.

We know that,

$$An = P(1 + i)^n$$

Future value of a single cash flow can be computed by the above formula. Replace A by future value (F) and P by single cash flow (C.F.), therefore

$$F = C.F.(1 + i)^n$$

Let's consider an example to illustrate the concept of future value. Suppose you invest ₹10,000 in a fixed deposit that offers an annual interest rate of 5%. At the end of the first year, your investment will grow to ₹10,500. This amount includes the original principal of ₹10,000 and the interest earned of ₹500. Therefore, we can say that ₹10,000 invested today is expected to be worth ₹10,500 in one year's time, assuming an interest rate of 5%.

FUTURE VALUE OF AN ANNUITY REGULAR

Q.40 You invest ₹1000 in a three year investment that pays you 6 % per annum. Calculate the future value of the investment.

(a) ₹1191.1 (b) ₹1200.5 (c) ₹900.8 (d) ₹1000

Ans. (a)

CALCULATING THE FORMULA FOR FUTURE VALUE OF ANNUITY

If A be the periodic payments, the future value $A(n, i)$ of the annuity is given by

$$A(n, i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Q.41 Find the future value of an annuity of ₹10000 made annually for 5 years at an interest rate of 12% compounded annually.

Given that $(1.12)^5 = 1.7623$.

(a) ₹52500.64 (b) ₹60000.50 (c) ₹63528.47 (d) None of these

Ans. (c)

Calculator trick:

1. 1.12 × = = = =

2. −1

3. × 10000

4. ÷0.12

Q.42 ₹500 is invested at the end of each month in an account paying interest 12% per year compounded monthly. What is the future value of this annuity after the 9th payment?

(a) ₹4000 (b) ₹4684.36 (c) ₹5526.64 (d) None of these

Ans. (b)

Calculator trick:

1. 1.01 × = = = = = = = = =

2. −1

3. ÷0.01

4. ×500

FUTURE VALUE OF ANNUITY DUE OR ANNUITY IMMEDIATE

To calculate the future value of an Annuity due/Annuity immediate:

Step 1: Calculate the future value as though it is an ordinary annuity.

Step 2: Multiply the result by $(1 + i)$

Q.43 Rahul invests ₹10,000 every year starting from today for next 10 years. Suppose interest rate is 8% per annum compounded annually. Calculate future value of the annuity.

Given that $(1 + 0.08)^{10} = 2.15892500$

(a) ₹1,56,455 (b) ₹1,60,855 (c) ₹1,90,865 (d) ₹2,00,505

Ans. (a)

Q.44 The amount of an annuity certain of ₹150 for 12 years at 3.5% p.a C.I is

(a) ₹2,190.28 (b) ₹1,290.28 (c) ₹2,180.28 (d) None of these

Ans. (a)

Q.45 If the amount of an annuity after 25 years at 5% p.a. C.I is ₹50,000, the annuity will be

(a) ₹1,406.90 (b) ₹1,047.62 (c) ₹1,146.90 (d) None of these

Ans. (b)

Q.46 A person invests ₹500 at the end of each year with a bank which pays interest at 10% p.a. C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is

(a) ₹11,761.36 (b) ₹10,000 (c) ₹12,000 (d) none of these

Ans. (a)

Q.47 At an interest rate of 14% compounded annually, how much should be invested at the end of each year for a period of 7 years to accumulate a total amount of ₹11,000 by the end of the 7th year?

(a) ₹950 (b) ₹1400 (c) ₹1030 (d) ₹2000

Ans. (c)

Q.48 Sahil decided to invest ₹50,000 every year starting from today for next 12 years. If the interest rate is 7.5% per annum compounded annually, then calculate the future value of the annuity.

[Given that $(1 + 0.075)^{12} = 2.3817796$]

(a) ₹990,275 (b) ₹900,298 (c) ₹890,275 (d) None of these

Ans. (a)

Q.49 Raju invests ₹20,000 every year in a deposit scheme starting from today for next 12 years. Assuming that interest rate on this deposit is 7% per annum compounded annually. What will be the future value of this annuity? Given that $(1 + 0.07)^{12} = 2.25219150$

(a) ₹540,576 (b) ₹382,813 (c) ₹643,483 (d) ₹357,769

Ans. (b)

PRESENT VALUE

WE UNDERSTOOD WHAT THE FUTURE VALUE IS!

- The value of today's money in the future, right!!
- Now, present value means: Value of future money in the present.

Present Value refers to the current value of a future sum of money, considering the time value of money and the expected rate of return. It represents the amount of money that would need to be invested today to equal the future amount at a given interest rate.

So, formula for Present value for amount An, time period n period at the rate of interest per period i can easily be obtained from the below equation:

$$An = P(1+i)^n$$

$$\text{i.e., } P = \frac{A_n}{(1+i)^n}$$

PRESENT VALUE OF ANNUITY REGULAR

The present value of an annuity regular refers to the current worth of a series of periodic payments or receipts over a specified time period. It helps determine the value of the annuity in today's terms.

To calculate the present value of an annuity regular, we can use the following formula:

$$P.V = A \times \frac{(1+i)^n - 1}{i \times (1+i)^n} = A.P(n,i)$$

Where, $P(n, i) = \frac{(1+i)^n - 1}{i \times (1+i)^n}$

One more formula is $A = \frac{V}{P(n,i)}$ and it uses amortization, where amortization refers to the process of gradually paying off a debt over a specific period of time through regular instalments. It involves the systematic reduction of the principal amount owed, along with the payment of interest, until the debt is fully repaid.

In case of loan if you pay your annuity properly i.e. principal amount + interest, instalment are same and paid over equal period of time then it is said that loan is amortized.

For example, let's consider an annuity regular where you receive ₹5,000 at the end of each year for five years, and the interest rate is 6% per annum. Using the formula, we can calculate the present value of this annuity regular.

Q.50 The present value of an annuity of ₹80 a year for 20 years at 5% p.a is

(a) ₹997 (b) ₹900 (c) ₹1,000 (d) none of these

Ans. (a)

Calculator trick:

1. Start from MRC

2. Find (1.05)20 i.e., 1.05 × = = = = = ... (21 steps)

3. Click M+ to store the value

4. Now, subtract 1

5. ÷0.05

6. ÷MRC

7. ×80

In short find (1.05)20 - 1 ÷ 0.05 ÷ MRC = then × 80

Q.51 A person bought a house paying ₹20,000 cash down and ₹4,000 at the end of each year for 25 years at 5% p.a. C.l. The cash down price is

(a) ₹75,000 (b) ₹76,000 (c) ₹76,370.80 (d) None of these

Ans. (c)

Calculator trick:

Find $(0.05)^{25}$ and store it then –1 ÷ 0.05 ÷ MRC = ×4000

PRESENT VALUE OF ANNUITY DUE OR ANNUITY IMMEDIATE

Steps to calculate Present Value for Annuity Immediate/Due

1. Calculate the present value for (n – 1) period as annuity regular
2. Add the initial payment (receipt) in the above.

Q.52 Your Papa wants to give you ₹10000 every year starting from today for next 5 years as a gift. So, being a smart father, he invested at the interest rate of 15% in mutual funds today morning. What should be the exact amount to be invested such that it exactly gives the money to be gifted?

(a) ₹50,000.0 (b) ₹28,549.8 (c) ₹38,549.8 (d) None of these

Ans. (c)

Q.53 Suppose you bought a flat worth ₹40,00,000 at the interest rate of 5% where you need to pay an equal 10 instalments at the end of every year where initially you gave to the bank ₹10,00,000. So, what will be the amount of every instalment?

(a) ₹97,128.25 (b) ₹2,85,900.800

(c) ₹9,71,000.0 (d) ₹3,88,513.725

Ans. (d)

Q.54 Find the present value of ₹1000 due at the end of 4 years if money is 7% effective.

(a) ₹616.56 (b) ₹812.75 (c) ₹762.89 (d) None of these

Ans. (c)

Q.55 Calculate the present value of an annuity of ₹2,000 payable at the end of each year for a period of 5 years. The rate of interest is 8% compounded annually.

(a) ₹7,985 (approx) (b) ₹8,360 (approx)

(c) ₹12,000 (approx) (d) None of these

Ans. (a)

Q.56 A loan of ₹1,02,000 is to be paid back in two equal annual instalments. If the rate of interest is 4% p.a. compounded annually, then the total interest charged under this instalment plan is

(a) ₹6,160 (b) ₹8,120 (c) ₹5,980 (d) ₹7,560

Ans. (a)

FEW MORE USE OF TIME VALUE OF MONEY

- Sinking funds
- Leasing
- Capital expenditure
- Valuation of bond
- Perpetuity
- Net present value technique (NPV)
- Compound Annual Growth Rate (CAGR)

SINKING FUNDS

Sinking funds refer to setting aside money over a period of time to accumulate a specific amount needed to repay a debt or fund a future obligation like debentures, renew items etc.

It involves making regular payments into an account or investment to ensure sufficient funds are available when needed.

Example: A homeowner setting aside a certain amount of money each month in a savings account to accumulate funds for a future major home repair, such as replacing the roof or renovating the kitchen.

Thus, using formula $A = P.A\ (n, i)$; where $A(n, i) = [\frac{(1+i)^n - 1}{i}]$

Q.57 Company Alpha needs to pay ₹10,00,000 to banks after 5 years for which they started investing every year at the end with equal instalment with an interest rate of 10%.

(a) ₹1,00,000 (b) ₹2,63,785.64 (c) ₹1,63,797.48 (d) None of these

Ans. (c)

Q.58 A company creates a sinking fund of ₹2,00,000 in a bank account for 15 years for which the bank offers interest rate 6% per annum. The yearly payment to be paid by company is approximately

(a) ₹8,945 (b) ₹8,145 (c) ₹8,593 (d) ₹9,645

Ans. (c)

Q.59 A machine costs ₹5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹25000. What amount should be set aside every year if the sinking fund investment accumulate at 3.5% compound interest p.a.?

(a) ₹16,000 (b) ₹16,500 (c) ₹16,050 (d) ₹16,005

Ans. (c)

Q.60 A sinking fund is created for redeeming debentures worth ₹5 lakhs at the end of 25 year. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?

(a) ₹12,006 (b) ₹12,040 (c) ₹12,039 (d) ₹12,035

Ans. (a)

Q.61 Sinking fund factor is the reciprocal of:

(a) Present value interest factor of a single cash flow

(b) Present value interest factor of an annuity

(c) Future value interest factor of an annuity

(d) Future value interest factor of a single cash flow

Ans. (c)

LEASING

Leasing is a financial arrangement in which the owner of an asset, known as the lessor, grants the right to use the asset to another party, known as the lessee, for a specified period of time. The lessee pays a periodic fee, known as the lease rental, to the lessor for the use of the asset.

In a lease agreement, the lessor retains ownership of the asset while providing the lessee with the right to use it. The asset can be tangible, such as equipment, machinery, vehicles, or real estate, or intangible, such as intellectual property or software.

Q.62 ABC Ltd. wants to lease out an asset costing ₹3,60,000 for a five year period. It has fixed a rental of ₹1,05,000 per annum payable annually starting from the end of first year. Suppose rate of interest is

14% per annum compounded annually on which money can be invested by the company. Is this agreement favorable to the company?

(a) The agreement is favorable (b) The agreement is not favorable

(c) No difference (d) Cannot be determined

Ans. (a)

Q.63 A company is considering of buying two electronics either by making full payment of ₹17,000 or by leasing it for five years at an annual rate of ₹5000. Which course of action is preferable if the company can borrow money at 14% compounded annually?

(a) Buying the electronics by making a full payment of ₹17,000 is preferable.

(b) Leasing the electronics for five years at an annual rate of ₹5000 is preferable.

(c) Both options are equally preferable.

(d) It cannot be determined based on the given information.

Ans. (a)

Q.64 A company is considering buying a piece of equipment for ₹75,000 or leasing it for 5 years at an annual rent of ₹18,000. If the company's cost of capital is 12%, which option is more financially attractive?"

(a) Leasing the equipment is more financially attractive

(b) Buying the equipment is more financially attractive

(c) Both options have the same cost

(d) Cannot be determine

Ans. (a)

CAPITAL EXPENDITURE

Capital expenditure refers to the funds spent by a company or organization to acquire or improve long-term assets that are expected to generate benefits over an extended period. It involves investing in assets that have a useful life beyond the current accounting period and are not intended for immediate resale.

The purpose of capital expenditure is to enhance the productive capacity, efficiency, or competitiveness of a business. These expenditures are aimed at acquiring or upgrading assets such as property, plant, equipment, machinery, technology, or infrastructure that will support the company's operations and contribute to future revenue generation.

E.g.: A manufacturing company may invest in new machinery to increase production capacity and improve efficiency. A technology company may invest in research and development to develop innovative products or acquire intellectual property rights. A real estate developer may invest in acquiring land and constructing new properties.

Q.65 A cloth making machine can be purchased for ₹200000. Machine will contribute ₹50000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether the machine should be purchased or not.

(a) Leasing is preferable (b) Should be purchased

(c) No difference (d) Cannot be determined

Ans. (a)

Q.66 Juicer A with a useful life of seven years costs ₹15,000 while another Juicer B with a useful life of five years costs ₹12,000. The first juicer saves labour expenses of ₹3,000 annually and the second one saves labour expenses of ₹3,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

(a) Buying Juicer A is the preferred course of action.

(b) Buying Juicer B is the preferred course of action.

(c) Both options are equally preferred.

(d) It cannot be determined based on the given information.

VALUATION OF BOND

Valuation of a bond refers to the process of determining the fair value or price at which a bond should be bought or sold in the market. Bonds are debt securities issued by governments, municipalities, or corporations to raise capital. When an investor buys a bond, they are essentially lending money to the issuer in exchange for periodic interest payments and the return of the principal amount at maturity.

When we say that bonds are "issued," it means that the bond issuer creates and offers the bonds to the market for purchase by investors. The issuer, which can be a government entity, a local municipality, or a corporate entity, issues the bonds as a means of borrowing money from investors. By selling bonds, the issuer receives an upfront sum of money from investors, which it agrees to repay at a future date, along with periodic interest payments.

Q.67 Rajesh intends purchasing a three year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and Rajesh requires a rate of return of 20%

(a) ₹900.35 (b) ₹789.35 (c) ₹1000 (d) None of these

Ans. (b)

Alternative Solution:

Valuation of bond = Present value of A for n periods + Present value of B for n periods

$$= \frac{A((1+i)^n - 1)}{i(1+i)^n} + \frac{B}{(1+i)^n}$$

Q.68 An investor is considering purchasing a four-year bond with a face value of ₹10,000 and a nominal interest rate of 6%. The investor requires a rate of return of 8%. What is the maximum price the investor should be willing to pay for the bond?

(a) ₹9000 (b) ₹9500 (c) ₹10000 (d) ₹10500

Ans. (b)

Q69. An investor intends purchasing a three year ₹1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

(a) ₹1000 (b) ₹887.152 (c) ₹907.125 (d) None of these

Ans. (c)

PERPETUITY

When you get an annuity for unlimited time, it becomes Perpetuity.

We need to understand two basic things:

1. When we talk about money far in the future, the present value of it becomes today extremely low.
2. And principal is never repaid, that means we do not have to think about the principal.

Then, $PVA_{\infty} = \frac{R}{i}$

Where R = the payment or receipt each period

i = the interest rate

Q.70 Hari wants to retire and receive ₹15,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 7% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

(a) ₹25,71,428 (b) ₹2,14,285 (c) ₹1,80,000 (d) None of these

Ans. (a) Given, R=₹15,000, c = 12 and r = 7%

$$=> i = \frac{7\%}{12} = \frac{7}{1200}$$

We know that,

$$PVA_{\infty} = \frac{R}{i} = \frac{15000}{\frac{7}{1200}} = \frac{15000}{7} \times 1200 = ₹25,71,428 \text{ (approx)}$$

Therefore, Hari need ₹25,71,428 to set aside to achieve his perpetuity goal.

Hence, the correct option is (a)

CALCULATING GROWING PERPETUITY

Growing perpetuity means the periodic installment is increasing with fixed interest rate

$$\text{PVA} = \frac{\text{A}}{\text{i} - \text{g}}$$

Q.71 Assuming that the discount rate is 8% per annum, how much would you pay to receive ₹1000, growing at 4%, annually, forever?

(a) ₹25,000 (b) ₹2,500 (c) ₹1,00,000 (d) None of these

Q.72 A person wants to save for his retirement and plans to make annual deposits of ₹50,000 at the end of each year for perpetuity. If the rate of interest is 12%. What is the present value of his retirement fund?

(a) ₹500,000 (b) ₹416,667 (c) ₹600,000 (d) ₹450,000

Ans. (b)

Q.73 If the discount rate is 16% per annum, then how much a company has to pay to receive ₹360 growing at 10% annually forever?

(a) ₹6,000 (b) ₹5,800 (c) ₹5,400 (d) ₹5,200

Ans. (a)

Q74. At a discount rate of 8% per year, what would be the present value of receiving ₹300 growing at an annual rate of 6% indefinitely?

(a) ₹11,500 (b) ₹12,000 (c) ₹15,000 (d) ₹10,000

Ans. (c)

RATE OF RETURN

Net Present Value Technique (NPV)

Net present value = Present value of cash inflow – Present value of cash outflow

Q.75 Compute the net present value for a project with a net investment of ₹1,00,000 and net cash flows year one is ₹55,000 for year two is ₹80,000 and for year three is ₹15,000. Further, the company's cost of capital is 10%.

(a) ₹50,005 (b) ₹49,995 (c) ₹27,340 (d) ₹1,27,340

Ans. (c)

DECISION RULE

If NPV > 0, Accept the Proposal

If NPV < 0, Reject the Proposal

COMPOUND ANNUAL GROWTH RATE (CAGR)

It is the method to find the smoothed annualized gain of an investment over a given time period.

$$CAGR(t_o, t_n) = \left(\frac{V(t_n)}{V(t_o)}\right)^{\frac{1}{t_n - t_o}} - 1$$

Where $V(t_o)$= Beginning Period; $V(t_n)$= End Period

Trick:

Steps to find nth root of any number:

1. Write the number.

2. Press √ ..... 12 times

3. –1

4. ÷n

5. +1

6. ×= .... 12 times

Q.76 A company recorded its annual revenues over a five-year period as follows:

Year	2001	2002	2003	2004	2005
Revenue (in thousand ₹)	100	110	121	133	146.41

What is the compound annual growth rate (CAGR) of the company's revenue over this five-year period?

(a) 10% (b) 20% (c) 30% (d) 40%

Ans. (a)

Calculation trick:

To find 4th root

1. 146410 ÷ 100000
2. Press root 12 times
3. –1
4. ÷4 (because of fourth root)
5. +1
6. ×= ..... 12 times

NOMINAL RATE OF INTEREST

Nominal Interest Rate = Real Interest Rate + Inflation

Q.77 If the nominal rate of growth is 17% and inflation is 9% for five years. Let P be the Gross Domestic Product (GDP) amount at the present year then the projected real GDP after 6 years is

(a) 1.587 P (b) 1.921 P (c) 1.403 P (d) 2.51 P

Ans. (a)

Q.78 If the discount rate is 14% per annum, then how much a company has to pay to receive ₹280 growing at 9% annually forever?

(a) ₹5600 (b) ₹2,800 (c) ₹1,400 (d) ₹4,200

Ans. (a)

Q.79 Let the operating profit of a manufacturer for five years is given as:

Year	1	2	3	4	5	6
Operating profit (in lakh ₹)	90	100	106.4	107.14	120.24	157.35

Then operating profit of compound annual growth rate (CAGR) for year 6 with respect to year 2 is given at

(a) 9% (b) 12% (c) 11% (d) 13%

Ans. (b)

Q.80 The profit (in lakh ₹) of a company for the past five years are as follows:

Year	1	2	3	4	5
Profit	20	25	30	35	40

Find the compound annual growth rate (CAGR) of the profit for the period from year 1 to year

(a) 15% (b) 19% (c) 25% (d) 35%

Ans. (b)

PREVIOUS YEAR QUESTIONS

MATHEMATICS OF FINANCE

Q.2. How much amount is required to be invested every year so as to accumulated 30,000 at the end of 10 years if the interest compounded annually at 10%. Given A (10,0.1) = 15.9374
[Dec. 2023]
(a) ₹1882.36
(b) ₹1828.30
(c) ₹1832.65
(d) ₹1853.65
Ans. (a) ₹1882.36

Q.3 Suppose Mr. X invested ₹5,000 every year starting from today in mutual fund for next 10 years. Assuming that average return compounded annually is at 18% per annum. What is future value?
[Dec. 2023]
(a) ₹183,677.68
(b) ₹1,38,678.85
(c) ₹1,83,776.53
(d) ₹1,38,774.55
Ans. (d) ₹1,38,774.55

Q.4 A person wants to open a shop have two options to acquired a commercial space either by leasing for 10 years at annual rent of ₹2,00,000 or by purchasing the space for ₹12,00,000. If person can borrow money at 14% compounded per annum. Which alternate is most suitable? Given P(10,0.14) = 5.21611 [Dec. 2023]
(a) Leasing
(b) Purchase
(c) Can't say
(d) Data insufficient
Ans. (a) Leasing

Q.5 What is the effective rate of interest when principal amount of ₹50,000 deposited in a nationalized bank for one year, corresponding to a nominal rate interest 8% per annum compounded quarterly, given $(1.02)^4 = 1.0824$
[Dec. 2023]
(a) 10.38%
(b) 8.08%
(c) 8.16%
(d) 8.24%
Ans. (d) 8.24%

Q.6 Manoj invests ₹12,000 at 6% per annum simple interest to obtain a total amount of ₹14,880. What is the time for which the amount was invested?
[Dec. 2023]
(a) 3 years

(b) 4 years
(c) 2 years
(d) 5 years
Ans. (b) 4 years

Q.7 Mr.X makes a deposit of ₹50,000in the bank for a period of 21/2 years. If the rate of interest is 12% per annum compounded half yearly, then the maturity value of the money deposited by Mr. X is: [Where (1.06)5 = 1.3382] [Dec. 2023]
(a) ₹66,910
(b) ₹66,123
(c) ₹67,925
(d) ₹65,550
Ans. (a) ₹66,910

Q.8 What will be the future value of an annuity of ₹2,500 made annually for 12 years at interest rate of 5% compounded annually if (1.05)12 = 1.7958 [Dec. 2023]
(a) ₹37,588.58
(b) ₹39,790.00
(c) ₹40,873.13
(d) ₹42,603.68
Ans. (b) ₹39,790.00

Q.9 If the initial investment of ₹4,00,000 becomes ₹6,00,000 in 24 months, then the Compound Annual Growth Rate (CAGR) is: [Dec. 2023]

(a) 30.33%
(b) 22.4%
(c) 19.46%
(d) 14.47%
Ans. (b) 22.4%

Q.10 Mrs. X invests in an annuity immediately that promises annual payments of ₹50,000 for the next 16 years. If the interest rate is 6% compounded annually then the approximate present value of this annuity is, where (1.06)15 = 2.3935. [Dec. 2023]
(a) ₹5,51,217.75
(b) ₹5,75,900.00
(c) ₹5,05,288.08
(d) ₹5,35,612.45
Ans. (d) ₹5,35,612.45

Q.11 A machine costing ₹1,00,000 has useful of 10 years. If the rate of depreciation is 12%, what is scrap value of the machine at the end of life? Given (0.88)10 = 0.27850 [Dec. 2023]
(a) ₹25,850
(b) ₹26,850
(c) ₹27,850

(d) ₹28,850
Ans. (c) ₹27,850

Q.12 Compute the compound interest on ₹6,000 for 11/4 years at 8% per annum. Interest will be compounded quarterly. [Dec. 2023]
(a) 642
(b) 630.78
(c) 634.68
(d) 624.48
Ans. (d) 624.48

Q.13 The population of a city increases at the rate of 5% every year. What will be the population of the city in the year2023, if its population in 2021 was 1,00,000? [Dec. 2023]
(a) 1,05,500
(b) 1,10,250
(c) 1,15,2400
(d) 1,20,550
Ans. (b) 1,10,250

Q.14 Mr. XYZ invested ₹60,000 in a nationalized bank in the form of fixed deposit at the rate of 7.5% per annum simple interest rate. He received ₹73,500 after the end of the term of fixed deposit. Calculate the period for which ₹60,000 was invested in fixed deposit. [Dec. 2023]
(a) 3 Years
(b) 3.5 Years
(c) 4 Years
(d) 4.5 Year
Ans. (a) 3 Years

Q.15 Calculate the present value of ₹2,000 to be required after 10 years Compounded annually at 5% per annum given (1.05)10 = 1.62889 [Dec. 2023]
(a) 1,227.82
(b) 1,282.48
(c) 1,328.35
(d) 1,822.65
Ans. (a) 1,227.82

Q.16 Mrs. Paul invested ₹1,00,000 in a mutual fund scheme. She got a dividend of ₹10,000 for first year, ₹ 12,000 for second year, ₹ 16,000 for third year, ₹ 18,000 for fourth year and ₹ 21,000 for fifth year. What is Compounded Annual Growth Rate (CAGR) on dividend return? [June 2023]
(a) 20.38%
(b) 18.59%
(c) 16.36%
(d) 15.89%
Ans. (a) 20.38%

Q.17 Govinda's mother decides to gift him ₹50,000 every year starting from today for the next five years. He deposits this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the present value of this annuity?
Given $P(4,0.10) = 3.16987$ [June 2023]

(a) 280403.5
(b) 208493.5
(c) 208943.5
(d) 258493.5
Ans. (b) 208493.5

Q.18 The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is [June 2023]

(a) 15 years
(b) 17 years
(c) 19 years
(d) 20 years
Ans. (b) 17 years

Q.19 The difference between C.I and S.I on a certain sum of money invested for 3 years at 6% pa is 110.16 . The principle is [June 2023]
(a) 3000
(b) 3700
(c) 10000
(d) 12000
Ans. (c) 10000

Q.20 A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being ₹23,240 and ₹9,000 respectively. For how many years the machine was put to use? [June 2023]
(a) 7
(b) 8
(c) 9
(d) 10
Ans. (c) 9

Q.21 A company wants to replace its existing machine at the end of 10 years. The expected cost of machine would be ₹10,00,000. If the management creates a sinking fund, how much provision needs to be made at the end of each year which can earn at the interest rate of 10% compounded annually.
Given $A(10,0.1) = 15.937425$ [June 2023]
(a) 74625
(b) 72514
(c) 62745

(d) 67245
Ans. (c) 62745

Q.22 Mr. Ram invested a total of ₹1,00,000 in two different banks for a fixed period. The first bank yields an interest of 9% per annum and the second at 11% per annum. If the total interest at the end of one year is 9.75% per annum, then the amount invested in these banks are respectively

[June 2023]

(a) 52500, 47500

(b) 62500, 37500

(c) 57500, 42500
(d) 67500, 32500
Ans. (b) 62500, 37500

Q.23 Suppose you have decided to make a Systematic Investment Plan (SIP) in a mutual fund with ₹1,00,000 every year from today for next 10 years at the rate of 10% per annum compounded annually. What is the future value of this annuity? Given $1.1^{10} = 2.59374$ [June 2023]

(a) 1735114

(b) 1753411

(c) 1735411
(d) 1753114
Ans. (d) 1753114

Q.24 Jonny wants to have ₹2,00,000 in his saving account after three years. The rate of interest offered by bank is 8% per annum compounded annually. How much should he invest today to achieve his target amount? [June 2023]

(a) 147489.10

(b) 158766.44

(c) 171035.59
(d) 184417.96
Ans. (b) 158766.44

Q.25 Mr. Sharad got his retirement benefits amounting to ₹50,00,000. He wants to receive a fixed monthly sum of amount for his rest of life, starting after one month and thereafter he wants to pass on the same to future generation. He expects to earn an interest of 9% compounded annually. Determine how much perpetuity amount he will receive every month? [June 2023]

(a) 39500

(b) 38500

(c) 37500
(d) 39500
Ans. (c) 37500

Q.26 If the discount rate is 10% p.a. how much would you pay to receive ₹2500 growing at 8% annually forever?

[June 2023]

(a) 125000

(b) 250000

(c) 150000
(d) 200000
Ans. (a) 125000

Q.27 The compound interest on ₹15,625 for 9 months at 16% per annum compounded quarterly is

[June 2023]

(a) 1851

(b) 1941

(c) 1951
(d) 1961
Ans. (c) 1951

Q.28 A car is available for ₹4,98,200 cash payment or ₹60,000 cash down payment followed by three equal annual instalments. If the rate of interest charged is 14% per annum compounded yearly, then total interest charged in the instalment plan is (Given $P(3,0.14) = 2.32163$)
[June 2023]
(a) 146314
(b) 146137
(c) 128040
(d) 158040
Ans. (c) 128040

Q.29 The Nominal rate of interest is 10% per annum. The interest is compounded quarterly. The effective rate of interest per annum will be
[June 2023]
(a) 10%
(b) 10.40%
(c) 10.25%
(d) 10.38%
Ans. (d) 10.38%

Q.30 Sinking fund factor is the reciprocal of :
[Dec 2022]
(a) Present value interest factor of a single cash flow
(b) Present value interest factor of an annuity
(c) Future value interest factor of an annuity
(d) Future value interest factor of a single cash flow
Ans. (c) Future value interest factor of an annuity

Q.31 A sum of money doubles itself in 4 years at certain compound interest rate. In how many years this sum will become 8 times at the same compound interest rate
[Dec 2022]
(a) 12 years
(b) 14 years
(c) 16 years
(d) 18 years
Ans. (a) 12 years

Q.32 Rs. 5,000 is invested every month and in an account paying interest @ 12% per annum compounded monthly. What is the future value of this annuity just after making 11th payment" (Given that $(1.01)^{11}$= 1.1156)
[Dec 2022]
(a) 57800
(b) 56100
(c) 56800
(d) 57100
Ans. (a) 57800

Q.33 The difference between compound interest and simple interest on an amount of Rs. 15,000 for 2 years is Rs. 96. What is the rate of interest per Annum ?
[Dec 2022]
(a) 9%
(b) 8%
(c) 11%
(d) 10%
Ans. (b) 8%

Q.34 A sum of money invested of compound interest double itself in four years. In how many years it become 32 times of itself at the same rate of compound interest
[Dec 2022]
(a) 12 years
(b) 16 years
(c) 20 years
(d) 24 years
Ans. (c) 20 years

Q.35 Mr. Prakash invested money in two schemes 'A' and 'B' offering compound interest at the rate of 8% and 9% per annum respectively. It the total amount of interest accrued through these two schemes together in two years was Rs. 4818.30 and total amount invested was Rs. 27,000. What was the amount invested in schemes 'A'?
[Dec 2022]
(a) 12000
(b) 12500
(c) 13000
(d) 13500
Ans. (a)12000

Q.36 Mr. A invested Rs. 10,000 every year for next 3 years at the interest rate of 8 percent per annum compounded annually. What is future value of the annuity?
[Dec 2022]
(a) 32644
(b) 32464
(c) 34264
(d) 36442
Ans. (b) 32464

Q.37 10 years ago the earning per share (EPS) of ABC Ltd. was Rs. 5 share Its EPS for this year is Rs. 22. Compute at what rate, EPS of the company grow annually ?
[Dec 2022]
(a) 15.97%
(b) 16.77%
(c) 18.64%
(d) 14.79%
Ans. (a) 15.97%

Q.38 The effective annual rate of interest corresponding to a normal rate of 6% per annum payable half yearly is:
[Dec 2022]
(a) 6.06%
(b) 6.07%
(c) 6.08%
(d) 6.09%
Ans. (d) 6.09%

Q.39 How much amount is required to be invested every year so as to accumulate Rs. 5,00,000 at the end of 12 years if interest is compounded annually at 10% {Where A (12,0.1) = 21.384284}
[Dec 2022]

(a) 23381.65
(b) 24385.85
(c) 26381.65
(d) 28362.75
Ans. (a) 23381.65

Q.40 A farmer borrowed Rs. 3600 at the rate of 15% simple interest per Annum. At the end of 4 years, he cleared this account by paying Rs. 4000 and a cow. The cost of the cow is:
[Dec 2022]
(a) 1000
(b) 1200
(c) 1550
(d) 1760
Ans. (d) 1760

Q.41 If Rs 64 Amount to Rs. 83.20 in 2 years, what will Rs 86 Amount to in 4 years at the same Rate percent per annum ?
[Dec 2022]
(a) 127.60
(b) 147.60
(c) 145.34
(d) 117.60
Ans. (c) 145.34

Q.42 A machine worth Rs 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to Rs. 2,00,750
[Dec 2022]
(a) 5 years 5 month
(b) 5 years 6 months
(c) 5 years 7 months
(d) 5 years 8 months
Ans. (b) 5 years 6 months

Q.43 A company creates a sinking fund of Rs. 2,00,000 in a bank account for 15 years bank offers interest rate 6% per annum, the yearly payment to be paid by company is approximately (if need, use: $1.06^{14} = 2.2609$)
[June 2022]
(a) 8592
(b) 8145
(c) 9345
(d) 9645
Ans. (a) 8592

Q.44 The annual rate of simple interest is 12.5%. In how many years does the principal double?
[June 2022]
(a) 11 years
(b) 9 years
(c) 8 years

(d) 7 years
Ans. (c) 8 years

Q.45 The present value of Rs. 2,000, after 8 years at the rate of 6% per annum, is____.(1.06^8 = 1.59385)
[June 2022]
(a) 1054
(b) 1254
(c) 3054
(d) 2054
Ans. (b) 1254

Q.46 There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of Rs. 12,000 after 3 years at the same rate?
[June 2022]
(a) 3972
(b) 2160
(c) 3120
(d) 3742
Ans. (a) 3972

Q.47 An investor is saving to pay off an obligation of Rs. 15,250 which will be due in seven years, if the investor is earning 7.5% simple interest rate per annum, he must deposit Rs._____to meet the obligation.
[June 2022]
(a) 8000
(b) 9000
(c) 10000
(d) 11000
Ans. (c) 10000

Q.48 An investment is earning compound interest, Rs. 100 invested in the year 2 accumulated to Rs. 105 by year 4. If Rs. 500 invested in the year 5, will become Rs… ….by year 10.
[June 2022]
(a) 364.80
(b) 564.80
(c) 464.80
(d) 664.80
Ans. (b) 564.80

Q.49 Ramesh invests Rs. 20,000 per year in a stock index fund, which earns 9% per year, for the next ten years. What would be the closest value of the accumulated value of the investment upon payment of the last instalment? (1.09^{10} = 2.36736)
[June 2022]
(a) 388764.968
(b) 303858.594
(c) 268728.484

(d) 408718.364
Ans. (b) 303858.594

Q.50 Madhu takes a loan of Rs. 50,000 from XYZ Bank. The rate of interest is 10% per annum. The first instalment will be paid at the end of year 5. Determine the amount (in Rs.) of equal instalments, if Madhu wishes to repay the amount in five instalments?
[June 2022]
(a) 19510
(b) 19430
(c) 19310
(d) 19630
Ans. (c) 19310

Q.51 Raj made an investment of Rs. 15,000 in a scheme and at the time of maturity the amount was Rs. 25,000. If Compound Annual Growth Rate (CAGR) for this investment is 8.88%. Calculate the approximate number of years for which he has invested the amount.
[June 2022]
(a) 6
(b) 7.7
(c) 5.5
(d) 7
Ans. (a) 6

Q.52 The present value of ₹25,000 to be received after 10 years at 6% per annum compounded annually is ₹ ($1.06^5 = 1.33823$)
[Dec 2021]
(a) 15960
(b) 13960
(c) 11960
(d) 17960
Ans. (b) 13960

Q.53 A sum of money is put at 20% compound interest rate p.a. At which year the aggregated amount just exceeds the double of the original sum?
[Dec 2021]
(a) 5
(b) 6
(c) 4
(d) 3
Ans. (c) 4

Q.54 An amount is lent at R% simple interest for R years and the simple interest amount was one fourth of the principal amount. Then R is…….
[Dec 2021]
(a) 5
(b) 6
(c) 5.5

(d) 61.5
Ans. (a) 5

Q.55 It needs to pay ₹5,00,000 after 10 years. He invested a sum in a scheme at 9% rate of interest compounded half-yearly. How much amount (in ₹) he invested? $(1.045^{20} = 2.41171)$
[Dec 2021]
(a) 397321
(b) 207321
(c) 297321
(d)340321
Ans. (b) 207321

Q.56 A company needs ₹10,000 in five years to replace as equipment. How much (in ₹) must be invested now at the interest rate of 8% p.a. is order to provide for the equipment?
[Dec 2021]
(a) 6606
(b) 6806
(c) 10500
(d) 11500
Ans. (b) 6806

Q.57 If the compound interest earned at i% p.a. in n years is to be earned at s% simple interest rate for n years, the $s =$
[Dec 2021]
(a) i
(b) $i\frac{1}{n}$
(c) $\frac{(1+i)^n-1}{n}$
(d) $\frac{1-(1+i)^n}{n}$
Ans. (c) $\frac{(1+i)^n-1}{n}$

Q.58 Simple interest on a sum of money is amount to ₹59,000 in 3 years and ₹62,000 in 4 years at same rate of interest. What are the principal amount and rate of interest?
[Dec 2021]
(a) 50000, 6%
(b) 45000, 5.5%
(c) 55000, 5%
(d) 52000, 7%
Ans. (a) 50000, 6%

Q.59 Cost of a laptop is ₹1,10,000 and its value depreciate 12% annually its life is 6 years its scrap value. …. times its cost
[Dec 2021]
(a) 0.464
(b) 0.42
(c) 0.4

(d) 0.48
Ans. (a) 0.464

Q.60 A sum of money in simple interest doubles itself in 7 years. How many years will it take to triple itself?
[Dec 2021]
(a) End of 12 years
(b) End of 14 years
(c) End of 18 years
(d) End of 16 years
Ans. (b) End of 14 years

Q.61 S deposits an amount in bank which gives 10% compound interest, compounded annually for 5 years. What is effective rate of simple interest?
[Dec 2021]
(a) 12.21
(b) 11.11
(c) 13.21
(d) 12.81
Ans. (a) 12.21

Q.62 Mr. X wants to accumulate Rs. 50,00,000 at the end of 10 years. Then how much amount is required to be invested every year if interest is compounded annually at 10% (Given that $P(10,0.10) = 15.9374298$)
[Dec 2021]
(a) 313726.87
(b) 413726.87
(c) 353726.87
(d) 453726.87
Ans. (a) 313726.87

Q.63

A certain sum amounts to ₹15748 in 3 years at simple interest at r% p.a. The same sum amounts to ₹16,510 at (r + 2)% p.a. simple interest in the same time. What is the value of r?

[June 2021]
(a) 10%
(b) 8%
(c) 12%
(d) 6%
Ans. (b) 8%

Q.64 The effective rate of return for 24% per annum convertible monthly is given as
[June 2021]
(a) 24%
(b) 26.82%

(c) 18%
(d) 24.24 %
Ans. (b) 26.82%

Q.65 What is the difference (in Rs.) between the simple interest and the compound interest on a sum of Rs. 8,000 for 2 25 years at the rate of 10% p.a., when the interest is compounded yearly?
[June 2021]
(a) 135.75
(b) 129.50
(c) 151.75
(d) 147.20
Ans. (d) 147.20

Q.66 A sum of Rs. X amounts to Rs. 27,900 in 3 years and to Rs. 41,850 in 6 years at a certain rate percent per annum, when the interest is compounded yearly. The value of x is
[June 2021]

(a) 16,080
(b) 18,600
(c) 18060
(d) 16800
Ans. (b) 18,600

Q.67 If the cost of capital be 12% per annum., then the net present value (in nearest Rs.) from the given cash flow is given as
[June 2021]

Year	0	1	2	3
Operating profit (in thousand Rs.)	(100)	60	40	50

(a) 3,1048
(b) 34185
(c) 51048
(d) 24187
Ans. (d) 24187

Q.68 The future value of annuity of Rs. 2,000 for 5 years at 5% compounded annually is given (in nearest Rs.)as [June 2021]
(a) 51,051
(b) 21021
(c) 15624 (11051)
(d) 61254
Ans. (c) 15624 (11051)

Q.69 A loan of Rs. 1,02,000 is to be paid back in two equal annual instalments. If the rate of interest is 4% p.a., compounded annually, then the total interest charged (in Rs.) under this instalment plan is

[June 2021]
(a) 6,160
(b) 8120
(c) 5980
(d) 7560
Ans. (a) 6,160

Q.70 If the nominal rate of growth is 17% and inflation is 9% for the five years. Let P be the Gross Domestic Product (GDP) amount at the present year then the projected real GDP after 6 years is [June 2021]
(a) 1.587 P
(b) 1.921 P
(c) 1.403 P
(d) 2.51 P
Ans. (a) 1.587 P

Q.71 If a person bought a house by paying Rs. 45,00,000 down payment and Rs. 80,000 at the end of each year till the perpetuity assuming the rate of interest as 16% , the present value of house (in Rs.) is given as [June 2021]
(a) 47,00,000
(b) 45,00,000
(c) 57,80,000
(d) 50,00,000
Ans. (d) 50,00,000

Q.72 Find the future value of annuity of Rs. 1,000 made annually for 7 years at interest rate of 14% compounded annually [Given that 1.147 = 2.5023]
[Jan. 2021]
(a) Rs. 10,730.7
(b) Rs. 5,365.35
(c) Rs. 8,756
(d) RS. 9,892.34
Ans. (a) Rs. 10,730.7

Q.73 Assuming that the discount rate is 7% p.a. how much would you pay to receive Rs. 200, growing at 5% annually, for ever?
[Jan. 2021]
(a) Rs. 2, 500
(b) Rs. 7,500
(c) Rs. 5,000
(d) Rs. 10,000
Ans. (d) Rs. 10,000

Q.74 Rs. 800 is invested at the end of each month in an amount paying interest 5% per year compounded monthly. What is the future value of this annuity after tenth payment? (Given that (1.005)10 = 1.0511) [Jan. 2021]

(a) Rs. 4,444
(b) Rs. 3,491
(c) Rs. 8,756
(d) Rs. 8,176
Ans. (d) Rs. 8,176

Q.75 Rs. 2,500 is paid every year for 10 years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually?
[Dec. 2021]

(a) 13,040.27
(b) 14,674.21
(c) 15,847.90
(d) 16,345.11
Ans. (a) 13,040.27

Q.76 Find the present value of Rs. 1,00,000 be required after 5 years if the rate of interest is 9% given that (1.09)5 = 1.5386
[Dec. 2021]
(a) 78,995.98
(b) 88,992.43
(c) 64,994.20
(d) 93,902.12
Ans. (c) 64,994.20

Q.77 Determine the present value of perpetuity Rs. 10 per month for infinite period at an effective rate of interest of 14% p.a.?
[Dec. 2021]
(a) Rs. 657
(b) Rs. 757
(c) Rs. 857
(d) Rs. 957
Ans. (c) Rs. 857

Q.78 Find the future value of annuity of Rs. 1,000 made annually for 7 years at interest rate of 14% compounded annually. Given that (1.14)7 = 2.5023
[Dec. 2021]
(a) 10,730.7
(b) 5,365.36
(c) 8,756
(d) 9,892.34
Ans. (a) 10,730.7

Q.79 A five year annuity due has periodic cash flow of Rs. 100 each year, If the interest rate is 8% the future value of this annuity is given by
[Dec. 2021]

(a) (Rs. 100) × (future value at rate 8% for 5 years) × (0.08)
(b) (Rs. 100) × (future value at rate 8% for 5 years) × (1–.08)
(c) (Rs. 100) × (future value at rate 8% for 5 years) × (1+.08)
(d) (Rs. 100) × (future value at rate 8% for 5 years) × (1/0.08)
Ans. (c) (Rs. 100) × (future value at rate 8% for 5 years) × (1+.08)

SUMMARY

- The sum of money received in the future is less valuable than it is today.
- **Interest:** Price paid by the borrower for the use of lender's money, Or we say if we borrow some money from a person for a certain time period then the extra amount paid than the initial amount is called interest.
- **Simple Interest:** It is interest computed on principal for entire period of borrowing.

  Formulae: $\text{S. I.} = \frac{\text{P}\times\text{R}\times\text{T}}{100}$

  $\text{A} = \text{P} + \text{S. I.}$

  Where,

  A = Final value of an investment (accumulated amount)

  P= Initial value of an investment (Principal amount)

  R=rate of interest in decimal

  S.I. = amount of interest.

  T = time in years
- **Compound interest:** Interest that occurs when earning for each specified time period is added to the principal amount.

  Formula: $\text{C. I.} = \text{P}\left(1 + \frac{\text{r}}{100}\right)^{\text{n}} - \text{P}$

  $A = P\left(1 + \frac{r}{100}\right)^{n}$

  Where,

  P = Initial value of an investment (Principal Amount)

  r = Annual rate of interest

  n = $t \times no.$ Of conversions per year (c)
- **Effective rate of interest:** Effective interest rate is calculated by formula

  $\text{E} = \left(\left(1 + \frac{r}{100c}\right)^{c} - 1\right) \times 100\%$
- **Annuity** can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.
  - Annuity may be of two types:

  (i) Annuity Regular: In annuity regular first payment/receipt takes place at the end of first period.

(ii) Annuity Due or Annuity Immediate: When the first receipt or payment is made today (at the beginning of the annuity) it is called annuity due or annuity immediate.

- **Future Value of Annuity Regular:** If A be the periodic payments, the future value A (n,i) of the annuity is given by

$$A(n,i) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

- **Future value of an Annuity Due/Annuity Immediate** = Future value of annuity regular × (1 + i) where i is the interest rate in decimal
- **Present value of an Annuity Regular:** The present value P of the Annuity A due at the end of n period at the rate of r per interest period may be obtained by

$$P.V = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

- **Present Value of an Annuity Due or Annuity Immediate:**

(i) Calculate the present value for (n – 1) period as annuity regular.

(ii) Add the initial payment (receipt) in the above.

- **Capital Expenditure (investment decision):** Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment.
- **Valuation of Bond:** A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.
- **Perpetuity:** When you get an annuity for unlimited time, it becomes Perpetuity.

$$PVA_{\infty} = \frac{R}{i}$$

- **Calculating Growing Perpetuity:**

Growing perpetuity means the periodic installment is increasing with fixed interest rate.

$$\text{PVA} = \frac{\text{A}}{\text{i}-\text{g}}$$

- **Rate of Return:**

Net present value = Present value of cash inflow – Present value of cash outflow

- **Compound Annual Growth Rate (CAGR):**

It is the method to find the smoothed annualized gain of an investment over a given time period

$$CAGR(t_o, t_n) = \left(\frac{V(t_n)}{V(t_o)}\right)^{\frac{1}{t_n - t_o}} - 1$$

Where, $V(t_o)$=Beginning Period and $V(t_n)$=End Period

CHAPTER-5

BASIC CONCEPT OF PERMUTATION AND COMBINATION

Permutation and combination are essential concepts that help solve complex counting problems. For instance, let's consider the scenario of arranging members of the Lok Sabha in different seating arrangements.

FUNDAMENTAL PRINCIPLE OF COUNTING

- **Multiplication Rule**: If one task can be done in m ways and another task which is independent of the first task can be done in n ways, after the first task has been performed, then the number of possible ways in which both the tasks can be done simultaneously is m × n.

E.g.: Let's say Task A is choosing a shirt from a collection of 5 different shirts, and Task B is selecting a pair of pants from a collection of 3 different pants. Using the Multiplication Rule:

Number of ways to choose a shirt = 5

Number of ways to select a pair of pants = 3

Total number of ways to choose a shirt and pants simultaneously = 5 × 3 = 15

Q.1 Raghav has 3 different types of shirts and 2 different types of trousers. Whenever he goes out, he likes to wear a shirt and a trouser. In how many ways can he decide what to wear?

(a) 5 (b) 4 (c) 6 (d) 8

Ans. (c)

Addition Rule: If one task can be done in m ways and another task which is independent of the first task can be done in n ways, then the total number of ways either of them can perform is m + n.

E.g.: Task A: Choosing a dessert from a menu with 4 options.

Task B: Selecting a drink from a menu with 3 options.

Number of ways to choose a dessert = 4

Number of ways to select a drink = 3

Total number of ways to either choose a dessert or select a drink = 4 + 3 = 7

Q.2 Rani has 3 different types of shoes and 2 different types of sandals. Whenever she goes out, she likes to wear either a shoe or a sandal. In how many ways can she decide what to wear?

(a) 3 (b) 2 (c) 5 (d) 6

Ans. (c)

What we understand is that, if "And" comes in a statement use MULTIPLICATION RULE, when "Or" comes in a statement use ADDITION RULE.

Q.3 A room has 6 doors. In how many ways can a man enter the room through one door and come out through a different door?

(a) 30 (b) 36 (c) 64 (d) 80

Ans. (a)

Q.4 In a class there are 27 boys and 14 girls. The teacher wants to select 1 boy or 1 girl to represent the class in a function. In how many ways can the teacher make this selection?

(a) 41 (b) 224 (c) 278 (d) 378

Ans. (a)

Q.5 Given 4 flags of different colors, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

(a) 5 (b) 4 (c) 12 (d) 8

Ans. (c)

Q.6 In a monthly test, the teacher decides that there will be three questions, one from each of Exercise 7, 8 and 9 of the textbooks. If there are 12 questions in Exercise 7, 18 in Exercise 8 and 9 in Exercise 9, in how many ways can three questions be selected?

(a) 1214 (b) 1234 (c) 6000 (d) 1944

Ans. (d)

Q.7 In how many ways can 5 letters be posted in 4 letter boxes?

(a) 120 (b) 1024 (c) 625 (d) None of these

Ans. (b)

Q.8 Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, when

(I) the repetition of the letters is not allowed.

(II) the repetition of the letters is allowed.

(a) 16, 186 (b) 24, 256 (c) 24, 172 (d) None of these

Ans. (b)

Q.9 There are 10 trains running between Calcutta and Delhi. The number of ways in which a person can go from Calcutta to Delhi and return by a different train is

(a) 99 (b) 90 (c) 80 (d) None of these

Ans. (b)

Q.10 A person can go from place 'A' to 'B' by 11 different modes of transport but is allowed to return back to 'A' by any mode other than the one earlier. The number of different ways, the entire journey can be complete is

(a) 110 (b) 10^{10} (c) 9^5 (d) 10^9

Ans. (a)

Q.11 Consider the word "TRAIN". How many distinct 5-letter words can be formed from its letters without repeating any letter?

(a) 120 (b) 24 (c) 360 (d) 30

Ans. (a)

Q.12 How many 3-digit odd numbers can be formed using the digits 5, 6, 7, 8, 9, if the digits can be repeated?

(a) 55 (b) 75 (c) 65 (d) 36

Ans. (b)

Q.13 In how many ways can 3 letters be posted in 4 letter boxes?

(a) 24 (b) 27 (c) 64 (d) None of these

Ans. (c)

THE FACTORIAL

If we take a number 'n', then n! or ∠n is called the factorial of 'n' and the value of n! is equal to multiplication of 1 to n i.e. $n! = 1 \times 2 \times 3 \times 4 \times \ldots \times n$ or we can write, $n! = n(n-1)(n-2)\ldots 3 \times 2 \times 1$ Note: $0! = 1$

Q.14 Find the value of

(I) 8! (II) 5! (III) $\frac{10!}{4!}$

(a) 5040, 720, 75600 (b) 20,160, 120, 151200

(c) 40,320, 120, 151200 (d) 40,320, 520, 151100

Ans. (c)

Q.15 Find X if $\frac{1}{6!} + \frac{1}{5!} + \frac{X}{7!}$

(a) 49 (b) 81 (c) 88 (d) 91

Ans. (a)

Q.16 Find the value of n if $(n+1)! = 30(n-1)!$

(a) 6 (b) 5 (c) 7 (d) None of these

Ans. (b)

Q.17 The value of $\frac{9!}{5!}$ is

(a) 362,880 (b) 15,120 (c) 3024 (d) None of these

Ans. (c)

Q.18 Find n if $(n+2)! = 12(n)!$

(a) 4 (b) 3 (c) 6 (d) None of these

Ans. (a)

Q.19 Find the value of X if $\frac{1}{9!} + \frac{1}{10!} + \frac{X}{11!}$

(a) 11 (b) 121 (c) 150 (d) None of these

Ans. (b)

LET US UNDERSTAND THE TWO WORDS

1. **PERMUTATION:** In P**ermutation**, we consider the arrangement of objects in a specific order. The order of the arrangement matters.

   For example, let's consider the football team selection scenario from your class of 30 students. The coach wants to select 11 students for the football team and arrange them in specific positions: forward player, midfielder, backward player, and goalkeeper.

   In this case, the order of selection and arrangement of the players is important.

2. **COMBINATION:** In C**ombination**, we consider the selection of objects without any specific order. The order of the selection does not matter.

   Continuing with the football team example, if we only want to select 11 students for the team without considering their specific positions, we are dealing with combinations. Here, the order in which the students are selected does not matter.

In case of Permutation: We do arrangements, and we say order matters.

In case of Combination: We do selection, and we say order doesn't matter.

PERMUTATIONS

A permutation determines the number of possible arrangements in a set when the order of the arrangements matters. It can be calculated using the formula:

$$nP_r = \frac{n!}{(n-r)!}$$

where n is the total number of objects and r is the number of objects to be arranged.

E.g., if we have three different boxes: one Yellow, one Green, and one Red and we want to arrange them on a table, the number of possible arrangements (permutations) would be:

$${}^3P_3 = \frac{3!}{(3-3)!} + \frac{3}{0!} + \frac{3!}{1} = 6$$

In the scenario where all n students are winners, the permutation formula becomes:

$${}^nP_n = \frac{n!}{(n-n)!} + \frac{n!}{0!} + n!$$

Q.20 Evaluate each of the following:

(I) ${}^4P_2$ (II) ${}^7P_3$ (III) ${}^{10}P_6$

Q.21 In ${}^nP_r$, the restriction is

(a) n > r (b) n ≥ r (c) n ≤ r (d) None of these

Ans. (b)

Q.22 If ${}^nP_4 = = 12 \times {}^nP_2$, then n is equal to

(a) –1 (b) 6 (c) 5 (d) None of these

Ans. (b)

Hint : Go by choices:

Q.23 If ${}^5P_r$ =60, then the value of r is

(a) 3 (b) 2 (c) 4 (d) None of these

Ans. (a)

Q.24 If $n_1 + n_{2P_2} = 132 \; and \; n_1 - n_{2P_2} = 30, then$

(a) $n_1 = 6, n_2 = 6$ (b) $n_1 = 10, n_2 = 2$

(c) $n_1 = 9, n_2 = 3$ (d) None of these

Q.25 How many 4 letter words can be formed from 'COMPUTER'?

(a) 1223 (b) 1680 (c) 7880 (d) 7200

Ans. (b)

Q.26 The number of arrangements of the letters in the word ₹FAILURE', so that vowels are always coming together is

(a) 576 (b) 575 (c) 570 (d) None of these

Ans. (a)

Q.27 10 examination papers are arranged in such a way that the best and worst papers never come together. The number of arrangements is

(a) 9 × 8! (b) 10! (c) 8 × 9! (d) None of these

Ans. (c)

Q.28 3-digit numbers to be formed out of the figures 0, 1, 2, 3, 4, 7, 8, 9 (no digit is repeated) then number of such numbers is

(a) 336 (b) 294 (c) 1050 (d) None of these

Ans. (b)

Q.29 If 19 states teams are participating in a national singing contest then the number of ways the first, second and third positions may be won is

(a) 5814 (b) 93024 (c) 342 (d) None of these

Ans. (a)

Q.30 In how many ways among 8 students – (5 boys and 3 girls), can school select School Prefect, Head boy and Head Girl, if no member can hold two positions and each boy and girl is eligible for School Prefect too?

(a) 90 (b) 150 (c) 60 (d) None of these

Ans. (a)

Q.31 The number of ways the letters of the word ₹TRIANGLE' to be arranged so that the word 'ANGLE' will be always present is

(a) 20 (b) 60 (c) 24 (d) 32

Ans. (c)

Q.32 How many numbers divisible by 5 of 6 digits can be made from the digit 2, 3, 4, 5, 6, 7?

(a) 120 (b) 600 (c) 240 (d) None of these

Ans. (a)

Q.33 Find the value of n if : ${}^{n-1}P3 : {}^{n}P4 = 1:9$

(a) 3 (b) 9 (c) 10 (d) None of these

Ans. (b)

Q.34 If there are 6 books on Accounts, 3 on Business Mathematics and 2 on Economics. In how many ways can we place them if the books on the same subject are to be together ?

(a) 5184 (b) 8,640 (c) 25,920 (d) 51,840

Ans. (d)

Q.35 How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

(a) 40320 (b) 5040 (c) 6720 (d) None of these

Ans. (a)

Q.36 The total number of ways in which six '+ ' and four '- ' sign can be arranged in a line such that no two '- ' sign occur together is

(a) $\frac{7!}{3!}$ (b) $\frac{6!\times 7!}{3!}$ (c) 35 (d) None of these

Ans. (c)

Q.37 How many numbers of seven-digit numbers which can be formed from the digits 3, 4, 5, 6, 7, 8, 9 no digits being repeated are not divisible by 5?

(a) 4320 (b) 4690 (c) 3900 (d) 3890

Ans. (a)

Q.38 In how many ways can the letters of the word 'STRANGE' be arranged so that the vowels come together?

(a) 120 (b) 720 (c) 1440 (d) None of these

Ans. (c)

CIRCULAR PERMUTATION

Circular Permutation refers to arrangements where the objects or individuals are arranged in a circular manner. Unlike linear permutation, where the arrangement is in a straight line, circular permutation involves arranging objects in a circular form.

DIFFERENCE BETWEEN LINEAR PERMUTATION AND CIRCULAR PERMUTATION

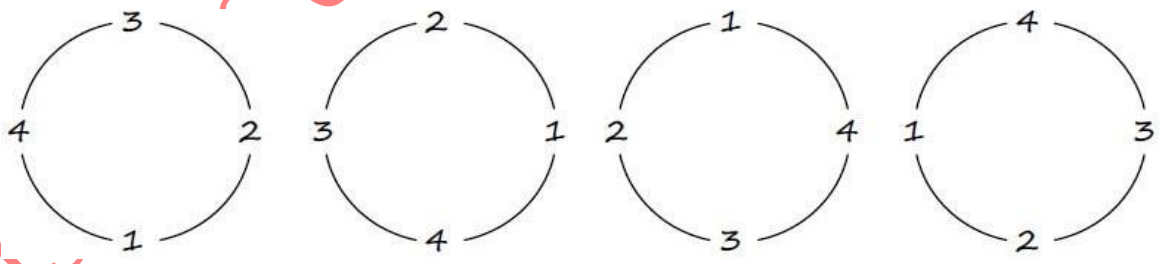
Let's consider two scenarios: the sitting arrangement of a langar (community meal) and the sitting arrangement of a circular table party.

In the case of a langar, where people sit in a line, it follows the concept of linear permutation. Each person has a distinct seat, and the order in which they sit matters. For example, if there are 4 people, the number of linear permutations would be 4!.

On the other hand, in the case of a circular table party, the arrangement is circular. The seats are arranged in a circle, and the order in which individuals sit becomes significant. Circular permutation takes into account that the arrangement repeats after one full revolution. To calculate the number of circular permutations, we use the formula (n – 1)!, where n represents the number of objects or individuals.

For example, if there are 4 people at the circular table, the number of circular permutations would be 3!.

Let's say there are 4 people sitting in circular arrangement as shown in the below figure:



Clearly, all the four arrangements are similar to each other.

i.e., these 4-people permutations equal to one in circular.

Thus, n ordinary permutations are equal to one permutation.

Hence, there are ${}^{n}P_{n}$ / n ways in which n things can be arranged in circular permutations which is equal to (n – 1)!.

Q.39 The number of ways in which 6 boys form a ring is

(a) 100 (b) 110 (c) 120 (d) None of these

Ans. (c)

Q.40 The number of ways in which 7 boys sit around a table so that two particular boys may sit together is

(a) 240 (b) 200 (c) 120 (d) None of these

Ans. (a)

Q.41 3 ladies and 3 gents can be seated at a round table so that any two and only two of the ladies sit together. The number of ways is

(a) 70 (b) 27 (c) 72 (d) None of these

Ans. (c)

Q.42 5 persons are sitting in a round table in such way that tallest person is always on the right–side of the shortest person; the number of such arrangements is

(a) 6 (b) 8 (c) 24 (d) None of these

Ans. (a)

Q.43 If 50 different jewels can be set to form a necklace then the number of ways is

(a) $\frac{50!}{2}$ (b) $\frac{49!}{2}$ (c) 49! (d) None of these

Ans. (b)

PERMUTATION WITH RESTRICTIONS

Theorem 1: Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is ${}^{n-1}P_r$.

Q.44 How many ways 5 glasses of Coca – cola can be served to 10 people if one says that he does not drink coca cola?

(a) 10000 (b) 27506 (c) 6290 (d) 15120

Ans. (d)

Theorem 2: Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is: $r \cdot {}^{n-1}P_{r-1}$

Q.45 The number of arrangements of 6 different things taken 3 at a time in which one particular thing always occurs is :

(a) 60 (b) 25 (c) 30 (d) None of these

Ans. (a)

Q.46 How many four digit numbers greater than 5000 can be formed out of the digits 0, 1, 2, 3, 5, 7, 8, 9 if no digit is repeated in any number?

(a) 330 (b) 840 (c) 460 (d) None of these

Ans. (b)

Q.47 The sum of all 4-digit number containing the digits 2, 4, 6, 8, without repetitions is

(a) 1,33,330 (b) 1,22,220 (c) 2,13,330 (d) 1,33,320

Ans. (d)

(Hint Formula: $(n-1)! \times sum\ of\ digits \times (11111 \ldots \ldots n\ times)$)

Q.48 The number of ways in which 7 girls form a ring is

(a) 120 (b) 710 (c) 720 (d) None of these

Ans. (a)

Q.49 How can we arrange 10 people on a circular table such that two people do not want to sit together?

(a) 7 × 8! (b) 10 × 9! (c) 10! (d) None of these

Ans. (a)

Q.50 There are 6 varieties of Rice, 3 of Breads and 2 of sweets. In how many ways can these be placed on the table if the same types of dishes to be are to be together?

(a) 15,000 (b) 20,450 (c) 51,840 (d) None of these

Ans. (c)

Q.51 The number of numbers lying between 100 and 1000 can be formed with the digits 1, 2, 3, 4, 5, 6, 7 is

(a) 210 (b) 200 (c) 110 (d) None of these

Ans. (a)

Q.52 The value of $\sum_{r=1}^{r} r$, ${}^{r}P_{r}$ is

(a) 10! (b) 10! - 1 (c) 11! (d) 9! – 1

Ans. (b)

Q.53 The number of 4 digit numbers greater than 5,000 can be formed out of the digit 3, 4, 5, 6 and 7 (No. digit is repeated).The number of such is

(a) 72 (b) 27 (c) 70 (d) None of these

Ans. (a)

COMBINATION

The number of ways in which selection is done where order does not matter can be calculated as ${}^{n}C_{r}$ where

${}^{n}C_{r} = \frac{n!}{r! \times (n-r)!}$

Q.54 ${}^{12}C_{8}$ =

(a) 215 (b) 495 (c) 745 (d) None of these

Ans. (b)

Q.55 In how many ways can I select 5 cards from a pack of 52?

(a) 2598960 (b) 2598830 (c) 2600480 (d) None of these

Ans. (a)

Q.56 The number of straight lines obtained by joining 16 points on a plane, no three of them being on the same line is

(a) 120 (b) 110 (c) 210 (d) None of these

Ans. (a)

Q.57 Out of 7 boys and 4 girls, a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is

(a) 429 (b) 439 (c) 419 (d) 441

Ans. (d)

Q.58 If there are 40 guests in a party. If each guest takes a shake hand with all the remaining guests. Then the total number of hands shake is

(a) 780 (b) 840 (c) 1560 (d) 1600

Ans. (a)

Q.59 The number of diagonals in a decagon is

(a) 30 (b) 35 (c) 45 (d) None of these

Ans. (b)

Q.60 A CA needs three accountants and ten men apply. In how many ways can these selections take place?
(a) 6,04,800 (b) 36,28,800 (c) 6 (d) 120

Ans. (a)

Q.61 Army General wishes to simultaneously promote 3 of its 5 Captains to Majors. In how many ways can these promotions take place?
(a) 10 (b) 20 (b) 40 (d) None of these

Ans. (a)

Q.62 Find the value of n in 4. ${}^{n}C_2 = {}^{n+2}C_3$
(a) 2 (b) 7 (c) Both (a) and (b) (d) None of these

Ans. (c)

Q.63 If ${}^{8}P_r = 6720$ and ${}^{8}C_r = 56$; find the value or r
(a) 2 (b) 3 (c) 4 (d) None of these

Ans. (b)

Q.64 A committee of 7 members is to be chosen from 6 CA, 4 CS and 5 CS. In how many ways can this be done if in the committee, there must be at least one member from each group and at least 3 Chartered Accountants?
(a) 2570 (b) 1200 (c) 3570 (d) None of these

Ans. (c)

Q.65 Out of 6 boys & 4 girls, find the number of ways for selecting 5 members for a committee in which there are exactly two girls?
(a) 120 (b) 1440 (c) 720 (d) 71

Ans. (a)

SOME RESULTS OF COMBINATION TO TAKEN CARE

- Formula: ${}^{n}C_r = \frac{n!}{r! \times (n-r)!}$
- If r = n then, ${}^{n}C_r = {}^{n}C_n = 1$
- Similarly, if r = 0, then ${}^{n}C_r = {}^{n}C_0 = 1$
- Also, note that $0 \leq r \leq n$, then only ${}^{n}C_r$ exist. Similarly, ${}^{n}C_{n-r}$ exist only when $0 \leq n - r \leq n$
- ${}^{n}C_r = {}^{n}C_{n-r}$

Two more important results:

1. ${}^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$
2. ${}^{n}P_r = {}^{1}P_r + r.\ {}^{n-1}P_{r-1}$

Q.66 The value of ${}^{11}C_2 + {}^{11}C_3$ is
(a) 210 (b) 200 (c) 220 (d) None of these

Ans. (c)

PERMUTATIONS WHEN SOME OF THE THINGS ARE ALIKE, TAKEN ALL AT A TIME

Let us take the case, where there are n things in which n1 things are alike of one kind, n2 things alike of second kind and n3 things are alike of third kind. Then the number of possible permutations when all n things taken at a time is: $P = \frac{n!}{n_1! n_2! n_3!}$

PERMUTATIONS WHEN EACH THING MAY BE REPEATED ONCE, TWICE, ... UPTO TIMES IN ANY ARRANGEMENT

The number of permutations of n things taken r at time when each thing may be repeated r times in any arrangement = n^r

- Combinations of n different things taking some or all of n things at a time = $2^n - 1$.
- Combination of n things taken some or all at a time when n_1 things are alike of one kind, n_2 things are alike of second kind & n_3 things are alike of third kind:

$= (n_1 + 1) \times (n_2 + 1) \times (n_3 + 1) - 1$

- If we have to select the combination such that r1 things to be selected from n_1 and r_2 things to be selected from n_2 then, total selections are: ${}^{n1}C_{r1} + {}^{n2}C_{r2}$

Q.67 How many different permutations are possible from the letters of the word 'MATHEMATICS'?

(a) 11! (b) $\frac{11!}{2!\times2!\times2!}$ (c) $\frac{11!}{2!\times2!}$ (d) None of these

Ans. (b)

Q.68 Rajesh is planning Christmas party, in how many ways he can invite his 11 friends for the party?

(a) 11! (b) 2048 (c) 2047 (d) None of these

Ans. (c)

Q.69 By how many different ways you can take 10 Donuts, 6 Wafes and 8 pastries from your pantry for the picnic?

(a) 692 (b) 693 (c) 480 (d) None of these

Ans. (a)

Q.70 The number of ways in which 9 things can be divided into twice groups containing 2, 3, and 4 things respectively is

(a) 1250 (b) 1260 (c) 1200 (d) None of these

Ans. (b)

Q.71 The number of ways a person can contribute to a fund out of 1 ten–rupee note, 1 five–rupee note, 1 two–rupee and 1 one-rupee note is

(a) 15 (b) 25 (c) 10 (d) None of these

Ans. (a)

Q.72 ${}^{(n-1)}P_r + r.{}^{(n-1)}P_{(r-1)}$ is equal to

(a) ${}^nC_r$ (b) $\frac{n}{r!(n-r)!}$ (c) ${}^nP_r$ (d) None of these

Q.73 ${}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \ldots + {}^nC_n$

(a) $2^n - 1$ (b) 2^n (c) 2n + 1 (d) None of these

Ans. (a)

Q.74 The number of ways in which 12 students can be equally divided into three groups is

(a) 5775 (b) 7575 (c) 7755 (d) None of these

Ans. (a)

Q.75 The numbers of ways in which 15 mangoes can be equally divided among 2 students is

(a) $\frac{15}{(5!)^4}$ (b) $\frac{15}{(5!)^3}$ (c) $\frac{15}{(5!)}$ (d) None of these

Ans. (b)

Q.76 The letters of the word 'BOOKKEEPER' are rearranged to form different arrangements. How many different arrangements can be formed considering the repetitions of the letters?

(a) 10! (b) $\frac{10!}{2!\times2!\times2!\times3!}$

(c) $\frac{10!}{2!\times3!}$ (d) None of these

Ans. (b)

Q.77 If 3 books on computer, 3 books on commerce and 5 books on economics are arranged in such away that the books of same subject are kept together, then the number of ways in which this can be done are

(a) 4320 (b) 35820 (c) 35920 (d) 25920

Ans. (d)

Q.78 The number of triangles that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line is

(a) 185 (b) 175 (c) 105 (d) 115

Ans. (a)

Q.79 The number of numbers between 1000 and 10000, which can be formed by the digits 1, 2, 3, 4, 5, 6 without repetition is

(a) 720 (b) 180 (c) 360 (d) 540

Ans. (c)

Q.80 The number of ways in which 4 people can occupy 9 vacant seats is

(a) 6048 (b) 3024 (c) 1512 (d) 4536

Ans. (b)

Q.81 How many different permutations are possible from the letters of the word 'CALCULUS'?

(a) 40,320 (b) 20,160 (c) 10,080 (d) 5,040

Ans. (d)

Q.82 If ${}^{10}C_3 + 2.\ {}^{10}C_4 + {}^{10}C_4 = {}^{n}C_5$ then value of n is

(a) 10 (b) 11 (c) 12 (d) 13

Ans. (c)

Q.83 The maximum number of points of intersection of 10 circles of will be

(a) 2 (b) 20 (c) 90 (d) 180

Ans. (b)

Q.84 If ${}^{n+1}C_{r+1} : {}^{n}C_r : {}^{n-1}C_{r-1} = 8:3:1$, then n is equal to

(a) 22 (b) 16 (c) 10 (d) 15

Ans. (d)

Q.85 There are 6 men and women in a group , then the number of ways in which a committee of 5 persons can be formed of them , if the committee is to include at least 2 women are

(a) 180 (b) 186 (c) 120 (d) 105

Ans. (b)

Q.86 A student has three books on computer, three books on economics and five books on commerce. If these books are to be arranged subject wise , then these can be placed on shelf in the number of ways

(a) 25290 (b) 25920 (c) 4230 (d) 4320

Ans. (b)

Q.87 If six times the number of permutations of 'n' items taken 3 at a time is equal to seven times the number of permutations of (n – 1) items taken at a time, then the value of 'n' will be

(a) 7 (b) 9 (c) 13 (d) 21

Ans. (d)

Q.88 If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{98}$, then value of x will be

(a) 999 (b) 998 (c) 997 (d) None of these

Ans. (a)

Q.89 Number of ways of shaking hands in a group of 10 persons shaking hands to each other are

(a) 45 (b) 54 (c) 90 (d) 10

Ans. (a)

Q.90 If ${}^{n}P_r = 3024$ and ${}^{n}C_r = 126$, then find n and r

(a) 9, 4 (b) 10, 3 (c) 12, 4 (d) 11, 4

Ans. (a)

Q.91 The number of permutations of the word 'ACCOUNTANT' is

(a) $10! \div (2!)^4$ (b) $10! \div (2!)^3$ (c) $10!$ (d) None of these

Ans. (a)

Q.92 In how many ways that the crew of an eight be arranged so that if 3 of crew can row only on a stroke side and 2 row on the other side is

(a) 1728 (b) 256 (c) 164 (d) 126

Ans. (a)

Q.93 5 boys and 3 girls are to be seated together such that no two girls are together

(a) 14, 4000 (b) 2400 (c) 720 (d) None of these

Ans. (a)

PREVIOUS YEAR QUESTIONS

PERMUTATION & COMBINATION

Q.1 In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?
[Dec. 2023]
(a) 810
(b) 1440
(c) 25200
(d) 50400
Ans. (d) 50400

Q.2 If $^{15}C_{3r} = {}^{15}C_{r+3}$ then r is equal to:
[Dec. 2023]
(a) 5
(b) 4
(c) 3
(d) 2
Ans. ((c) 3

Q.3 Find 'n' if $^{n}P_2 = 72$
[Dec. 2023]
(a) 12
(b) 36
(c) 24
(d) 9
Ans. (d) 9

Q.4 A committee of 3 women and 4 men is to be formed out of 8 women and 7 men. Mrs. Kajal refuses to serve in a committee in which Mr. Yash is a member. The number of such committees can be
[June 2023]
(a) 1530
(b) 1500
(c) 1520
(d) 1540
Ans. (d) 1540

Q.5 In the next world cup of cricket there will be 12 teams, divided equally in two groups. Teams of each group will play a match against other. From each group 3 top teams will qualify for the next round. In this round each team will play against others once. Four top teams of this round will qualify for the semifinal round, when each team will play against the others once. Two top teams of this round will go to the final round, where they will play the best of three matches. The minimum number of matches in the next world cup will be
[June 2023]
(a) 50
(b) 53
(c) 49
(d) 43

Ans. (b) 53

Q.6 Find the number of ways in which the letters of the word SOFTWARE be arranged such that all the vowels are always together?
[June 2023]
(a) 720
(b) 1440
(c) 2880
(d) 4320
Ans. (d) 4320

Q.7 If ${}^{6}P_{2r} = 12 \times {}^{6}P_{r}$, then r is equal to
[June 2023]
(a) 1
(b) 2
(c) 3
(d) 4
Ans. (b) 2

Q.8 How many 3-digit odd numbers can be formed using the digits 5, 6, 7, 8, 9. If the digits can be repeated ?
[Dec. 2022]
(a) 55
(b) 75
(c) 65
(d) 85
Ans. (b) 75

Q.9 If ${}^{n}P_{r} = 3024$ and ${}^{n}C_{r} = 126$, then find n and r.
[Dec. 2022]
(a) 9,4
(b) 10,3
(c) 12,4
(d) 11,4
Ans. (b) 10,3

Q.10 The number of ways 4 boys and 3 girls can be seated in a row so that they are alternates :
[Dec. 2022]
(a) 12
(b) 288
(c) 144
(d) 256
Ans. (c) 144

Q.11 There are 20 points in a plane area. How many triangles can be formed by these points if 5 points are collinear?
[Dec 2022]
(a) 550
(b) 560
(c) 1130

(d) 1140
Ans. (c) 1130

Q.12 A multiple-choice test contains five questions and each has four possible options. How many different answer keys are possible?
[June 2022]
(a) 20
(b) 120
(c) 256
(d) 1024
Ans. (d) 1024

Q.13 The solution for $\frac{n!}{10}$ = ${}^{n-1}P_{n-3}$ then n =
[June 2022]
(a) 5
(b) 6
(c) 7
(d) 8
Ans. (a) 5

Q.14 The total number of sitting arrangements of 8 members of a board on a row with the president and the vice-president occupying central places is.
[Dec. 2021
(a) 7! × 2!
(b) 6! × 2!
(c) 6!
(d) 7!
Ans. (b) 6! × 2!

Q.15 Out of 7 boys and 4 girls, a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is
[Dec. 2021]
(a) 439
(b) 429
(c) 419
(d) 441
Ans. (d) 441

Q.16 There are 10 flights operating between City-A to City-B. The number of ways in which a person can travel from City-A to City-B and return by a different flight is
[June 2021]
(a) 90
(b) 95
(c) 80
(d) 78
Ans. (a) 90

Q.17 How many 4 letter words with or without meaning, can be formed out of the letters of the word. 'LOGARITHMS'. If repetition of letters is not allowed?
[June 2021]
(a) 7020
(b) 5040
(c) 1480
(d) 2520
Ans. (b) 5040

Q.18 Six points are marked on a straight line and five points are marked on another line which is parallel to the first line. How many straight lines, including the first two can be formed with these points?
[June 2022]
(a) 28
(b) 30
(c) 32
(d) 34
Ans. (c) 32

Q. 19The number of four letter words can be formed using the letters of the word

5040
(b) 720
(c) 90
(d) 30240 3360
Ans.(d) 30240

Q. 20 The number of word that can be formed using the letters of "PETROL" such that the word do not have "P" in the first position, is

(a)720
(b) 120
(c) 600
(d) 540

Q. 21

Q. 21 If $^{n}P_6 = 20\ ^{n}P_4$ then the value of n is given by

March 2021
(a) n=5
(b) n=3
(c) n=9

Q.22 =9

Q. 22 How many numbers of seven digit numbers which can be formed from the digits 3, 4, 5, 6, 7, 8, 9 no digits begin repeated are not divisible by 5?
(a) 4320
(b) 4690
(c) 3900
(d) 3890
Ans. (a) 4320

Q. 23 A person can go from place 'A' to 'B' by 11 different modes of transport but is allowed to return back to ''A'' by any mode other than the one earlier. The number of different ways, the entire journey can be complete is

July 2021

(a) 110
(b) 10
(c) 9^5
(d) 10^9

Ans. (a) 110

Q.24 A business house wishes to simultaneously elevate two of its six branch heads. In how many ways these elevations can take place?
[Jan. 2021]
(a) 12
(b) 3
(c) 6
(d) 15
Ans. (d) 15

Q.25 There are ten flights operating between city A and city B. The number of ways in which a person can travel from city A to city B and return by different flight, is
[Jan. 2021]
(a) 90
(b) 95
(c) 80
(d) 78
Ans. (a) 90

Q.26 $^nC_p + 2^nC_{p-1} + {}^nC_{p-2} = ?$
[Jan. 2021]
(a) ^nC_p
(b) $^{n+2}C_p$
(c) $^{n+1}C_{p+1}$
(d) $^{n+2}C_{p-1}$
Ans. (b) $^{n+2}C_p$

Q.27 Out of 7 boys and 4 girls a team of a debate club of 5 is to be chosen. The number of teams such that each team includes at least one girl is _____
[Dec. 2020]
(a) 429
(b) 439
(c) 419
(d) 441
Ans. (d) 441

Q.28 From a group of 8 men and 4 women, 4 persons are to be selected to form a committee so that at least 2

women are there on the committee. In how many ways can it be done?
[Dec. 2020]
(a) 201
(b) 168
(c) 202
(d) 220
Ans. (a) 201

Q.29 A fruit basket contains 7 apples, 6 bananas and 4 mangoes. How many selections of 3 fruits can be made so that all 3 are apples?
[Dec. 2020]
(a) 120 ways
(b) 35 ways
(c) 168 ways
(d) 70 ways
Ans. (b) 35 ways

SUMMARY

- **Multiplication rule:** If a certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n' different ways, then total number of ways of doing both things simultaneously $= m \times n$.
- **Addition rule:** If there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectively then either of two jobs can be done in $(m + n)$ ways.
- **Factorial: The factorial n, written as n! or $\lfloor n$,** represents the product of all integers from 1 to n both inclusive i.e. n! = $n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$ and note that $0! = 1$.
- **Permutation:** The ways of arranging or selecting a smaller or equal number of objects from a collection of objects with due regard being paid to the order of arrangement or selection, are called permutations.
- The number of permutations of n things chosen r at a time is given by: ${}^{n}P_{r} = \frac{n!}{(n-r)!}$
- Circular permutation:
  (i) Arranging n things in circular arrangement is given by $\frac{{}^{n}P_{n}}{n} = (n-1)!$ Ways in which all the n things can be arranged in a circle.
  (ii) Number of permutations of n distinct objects taken r at a time when a particular object is not taken I any arrangements is ${}^{n-1}P_{r}$
  (iii) Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is r. ${}^{n-1}P_{r-1}$
- **Combinations:** The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations given by: ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$
- Permutations when some of the things are alike, taken all at a time is $\frac{n!}{n_1!.n_2!.n_3!....}$
- Permutations or r things out of n when each thing may be repeated once, twice,...upto r times in any arrangement n^r.
- The total number of ways in which it is possible to form groups by taking some or all of n things $2^n - 1$. The total, number of ways in which it is possible to make groups by taking some or all out of $n\ (= n_1 + n_2 + n_3 + \ldots)$ things, where n1 things are alike of one kind and so on, is given by $\{(n_1 + 1)(n_2 + 1)(n_3 + 1) \ldots\} - 1$.

- The combinations of selecting r_1 things from a set having n_1 objects and r_2 things from a set having n_2 objects where combination of r_1 things, r_2 things are independent is given by : ${}^{n1}C_{r_1} \times {}^{n2}C_{r_2}$

CHAPTER-6
SEQUENCE AND SERIES ARITHMETIC AND GEOMETRIC PROGRESSIONS

Why to study sequence and series?

- When you start doing a job, then you will be working on a lot of data files and then you need to identify a particular pattern in between to understand or analyse it.

WHAT IS SEQUENCE?

When numbers are separated by "," and all numbers follow a particular law or some definite rule then numbers are said to be in sequence and the number is called an element. Generally, the sequence is written as $a_1, a_2, a_3, a_4, a_n, \ldots\ldots\ldots$ where $a_1, a_2, a_3, a_4, \ldots, a_n$ are its terms and n is the natural number and represents the number of terms or we can say nth term is the number at the nth position of the particular sequence and is denoted by a_n.

E.g.: Tell which one is in the sequence?

(i) 1, 14, 3, 5, 8 (ii) 1, 2, 5, 15, 20, 30

(iii) 1, 2, 3, 4, 5, 6, 7 (iv) 50, 45, 30, 15, 3, 1

We can see that,

In (i), the numbers do not obey any rule or law, thus it is not a sequence.

In (ii), the numbers are in ascending order but it does not obey any rule.

In (iii), the difference between the consecutive numbers is 1 or by adding 1 to each number, we get the next number, thus it forms a sequence.

In (iv), the numbers are in descending order, but it does not obey any rule. Thus, the numbers in (iii) only form a sequence.

Some of the sequences:

1. Sequence of natural numbers (N): 1, 2, 3, 4, 5, ...
2. Sequence of square numbers (n^2): 1, 4, 9, 16, 25, ...
3. Sequence of $\frac{1}{n}$: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\ldots\ldots$
4. Sequence of odd positive numbers: 1, 3, 5, 7, 9, ...
5. Sequence of even positive numbers: 2, 4, 6, 8, 10, ...
6. Sequence of $\frac{1}{(n+2)}$: $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots\ldots\ldots$
7. Sequence of $\frac{n}{(n+2)}$: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\ldots\ldots$

Q.1 Complete the sequence: 1, 4, 7, 10, 13, ?

(a) 21 (b) 17 (c) 16 (d) None of these

Ans. (c)

Q.2 Tell the next number in the sequence 1, 8, 27, 64,____.

(a) 125 (b) 256 (c) 111 (d) None of these

Ans. (a)

Q.3 Prime numbers less than 20 are

(a) 1, 2, 3, 11, 13, 19

(b) 2, 3, 5, 7, 11, 13, 17, 19

(c) 2, 5, 11, 13, 19

(d) None of these

Ans. (b)

Q.4 The first three terms of sequence when n^{th} term t_n is $n^2 - 2n$ are

(a) –1, 0, 3 (b) 1, 0, 2 (c) –1, 0, –3 (d) None of these

Ans. (a)

SERIES

When we add the terms of the sequence, then it is called a series.

Let $a_1, a_2, a_3, ..., a_n$, be the sequence then the series can be written as $a_1 + a_2 + a_3 + ... + a_n + ...$

E.g.: If the sequence is 1, 2, 3, 4, 5, 6, ... then the series is 1 + 2 + 3 + 4 + ...

TYPE OF SERIES

- **Finite series:** A finite series is a series that has a specific number of terms. The condition of the terms in the series stops after stops after a certain point

  E.g.: the series 1 + 2 + 3 + 4 + 5 is a finite series as it has a definite end.
- **Infinite series:** An infinite series is a series that continues indefinitely, without an end. The addition of terms in the series goes on indefinitely.

E.g.: the series 1 + 2 + 3 + 4 + 5 + ... is an infinite series as there is no specific endpoint.

Q.5 If $u_1 = 2, u_2 = 4, u_3 = 6, u_4 = 8$, then S_4 is:

(a) 20 (b) 15 (c) 5 (d) None of these

Ans. (a)

Q.6 The n^{th} element of the sequence –1, 2, –4, 8,....is

(a) $(-1)^n(2)^{n-1}$ (b) $(2)^{n+1}$ (c) 2^n (d) 2^{n-1}

Ans. (a)

Q.7 The sum to ∞ of the series $-5, 25, -125, 625$ ... can be written as

(a) $\sum_{k=1}^{\infty}(-5)^k$ (b) $\sum_{k=1}^{\infty}5^k$ (c) $\sum_{k=1}^{\infty}-5^k$ (d) None of these

Ans. (a)

Q.8 $\sum_{i=4}^{7}\sqrt{2i-1}$ can be written as

(a) $\sqrt{7}+\sqrt{9}+\sqrt{11}+\sqrt{13}$ (b) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$

(c) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$ (d) None of these

Ans. (a)

ARITHMETIC PROGRESSION (A.P.)

- If the common difference (d) between consecutive terms is same then the sequence is called Arithmetic Progression (A.P.).
- We can also say that a_1, a_2, a_3, a_4, a_n is in Arithmetic Progression (A.P.) when

  $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... = a_n - a_{n-1}$

E.g.: 1, 4, 7, 10, 13, ...

In the given sequence, the common difference between consecutive terms is 3. Therefore, it is an Arithmetic Progression (A.P.) with a common difference (d) of 3.

In an Arithmetic Progression (A.P.), each term is obtained by adding a constant value (d) to the previous term.

Thus, if the first term is 'a' and the common difference is 'd', then the nth term will be: $a_n = a + (n-1)d$

Now in general, an A.P series can be written as: $a, a+d, 2d, a+3d, ... a+(n-1)d$

Sequence and Series-Arithmetic and Geometric Progressions

Q.9 If the sequence 20, 15, 10, 5, 0, –5, –10 is in A.P., then the common difference is

(a) 5 (b) – 5 (c) 10 (d) None of these

Ans. (b)

Q.10 If the first term of an A.P is 15 and the common difference is 4, then the required A.P. will be

(a) 15, 19, 21, 23, ... (b) 15, 19, 23, 27, ...

(c) 11, 15, 19, 25, ... (d) None of these

Ans. (b)

Q.11 The 6^{th} term of the progression 1, 2, 5, 7 ... is

(a) 5 (b) 6 (c) 11 (d) None of these

Ans. (c)

Q.12 Which term of the progression –1, –3, –5,.... is –39?

(a) 21st (b) 20th (c) 19th (d) None of these

Ans. (b)

Q.13 The first term of an A.P. is 5, the common difference is 3, and the last term is 80; find the number of terms.

(a) 78 (b) 26 (c) 15 (d) None of these

Ans. (b)

Q.14 $(x+1), 3x, (4x+2)$ are in A.P., then the value of X is

(a) 2 (b) 3 (c) 4 (d) 5

Ans. (b)

Q.15 Which term of the progression $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, ... $\frac{19}{2}$?

(a) 5 (b) 10 (c) 15 (d) None of these

Ans. (b)

Q.16 The 20th term of arithmetic progression whose 6th term is 38 and 10th term is 66 is

(a) 136 (b) 118 (c) 178 (d) 210

Ans. (a)

Q.17 If the pth term of an A.P. is q and the qth term is q then what will be the value of the term?

(a) 0 (b) 1 (c) p + q – 1 (d) 2(p + q – 1)

Ans. (a)

Q.18 If 2^{nd} term = 7 and 7^{th} term = 27. Then what is the required A.P.?

(a) 3,7,11,15, ... (b) 3, 5,7,9, ... (c) 5,6,7,8, ... (d) None of these

Ans. (a)

Q.19 The first three terms of a sequence when nth term (t_n) is $n^3 - n$ are:

(a) 6, 24, 25 (b) 0, 6, 24 (c) 1, 2, 5 (d) None of these

Ans. (a)

Q.20 The first term and the common difference of the A.P.: 3, 1, –1, –3, ... is

(a) 3, –1 (b) 3, –2 (c) 1, –2 (d) None of these

Ans. (b)

Q.21 The number of the terms in the A.P.: 7, 13, 19, ..., 205 are

(a) 34 (b) 39 (c) 35 (d) 33

Ans. (a)

Q.22 The value of x such that 8x + 4, 6x – 2, 2x + 7 will form an A.P. is

(a) 15 (b) 2 (c) $\frac{15}{2}$ (d) None of these

Ans. (c)

Q.23 The value of K for which the terms 7K + 3, 4K – 5, 2K + 10 are in A.P. is

(a) -13 (b) -23 (c) 13 (d) 23

Ans. (b)

Q.24 The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term a and common difference .

(a) a = 3, d = 2 (b) a = 4, d = 3 (c) a = 5, d = 4 (d) a = 6, d = 5

Ans. (a)

SELECTION OF TERMS OF AN ARITHMETIC PROGRESSION

When sum of terms of an A.P. is given

Number of Terms	Terms	Common Difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d
7	a– 3d, a – 2d, a – d, a, a + d, a + 2d, a + 3d	d

Q.25 Divide 24 into three parts which are in arithmetic progression (A.P.) and such that the product of the first two parts is 48. Find the three parts.

(a) 4, 8, 12 (b) 6, 12, 18 (c) 3, 6, 9 (d) 6, 8, 10

Ans. (d)

Trick : Go by Choice

Q.26 Divide 36 into three parts which are in A.P. and are such that the product of the first two parts is 108

(a) 9,12,15 (b) 1,2,3 (c) 11,12,15 (d) None of these

Ans. (a)

Tricks: Go by choice

Q.27 Four numbers form an arithmetic progression. The sum of the first and last terms is 8, and the product of the two middle terms is 15. Find the smallest number in the series.

(a) 4 (b) 3 (c) 1 (d) None of these

Ans. (c)

Q.28 Divide 12.50 into five parts in A.P. such that the first part and the last part are in the ratio of 2 : 3.

(a) 2, 2.25, 2.5, 2.75, 3 (b) –2, –2.25, –2.5, 2.75, –3

(c) 4, 4.5, 5, 5.5, 6 (d) –4, –4.5, –5, –5.5, –6

Ans. (a)

Q.29 Divide 69 into 3 parts which are in A.P and are such that the product of first two parts is 460

(a) 20,23,26 (b) 21,23,25 (c) 19,23,27 (d) 22,23,24

Ans. (a)

Q.30 Divide 144 into three parts which are in A.P. and such that the largest is twice the smallest, the smallest of three numbers will be:

(a) 48 (b) 36 (c) 13 (d) 32

Ans. (2)

Q.31 Divide 20 into five parts in A.P. such that the first and last parts are in the ratio 2 : 3.

(a) $\frac{24}{5}, \frac{29}{5}, 6, \frac{33}{5}, \frac{36}{5}$ (b) $6, \frac{36}{5}, \frac{33}{5}, \frac{24}{5}, \frac{27}{5}$

(c) $\frac{27}{5}, \frac{24}{5}, \frac{36}{5}, \frac{33}{5}$ (d) None of these

Ans. (d)

SUM OF THE FIRST N TERMS

Let S be the sum, a be the 1st term and l be the last term of an A.P.

If the number of terms are n, then l = an and d be the common difference of the A.P.

Thus, formula for Arithmetic Progressions are:

Sum of n term of an A.P., $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[a + a + (n-1)d]$

$= \frac{n}{2}[a + a_n]$

Or we can say

Sum of n terms of an A.P., $S_n = \frac{n(a+l)}{2}$, where $l = a_n$

Q.32 The sum of the series 1, 3, 5, 7, 9, ... to 100 terms is

(a) 1000 (b) 10,000 (c) 20,000 (d) None of these

Ans. (b)

Q.33 The number of terms of the series: 5 + 7 + 9... must be taken so that the sum is 480.

(a) 20 (b) 10 (c) 15 (d) 25

Ans. (a)

Q.34 The first and the last term of an AP are –4 and 146 respectively. The sum of the terms is 7171. The number of terms is

(a) 101 (b) 100 (c) 99 (d) None of these

Ans. (a)

Q.35 If the sum of 'n' terms of an AP is $2n^2$, the fifth term is

(a) 20 (b) 50 (c) 18 (d) 25

Ans. (c)

Q.36 Find the sum of the following arithmetic progression: to $3, \frac{9}{2}, 6, \frac{15}{2}, \ldots to$ 25 terms

(a) 450 (b) 500 (c) 525 (d) 800

Ans. (c)

Q.37 The sum of five terms of AP is 75, find the 3rd term.

(a) 20 (b) 30 (c) 15 (d) None of these

Ans. (c)

Q.38 The sum of a certain number of terms of an A.P. series –8, –6, –4 ... is 52, then the number of terms are

(a) 20 (b) –4 (c) 13 (d) None of these

Ans. (c)

Q.39 The sum of all natural numbers between 200 and 400 which are divisible by 7 is

(a) 7,730 (b) 8,729 (c) 7,729 (d) 8,730

Ans. (b)

Q.40 The first and the last term of an AP are –10 and 70 where the common difference is 10, then the numbers of terms in the AP are

(a) 8 (b) 9 (c) 11 (d) None of these

Ans. (b)

Q.41 A man saved ₹16500 in ten years. In each year after the first, he saved ₹100 more than he did in the preceding year. How much did he save in the first year?

(a) ₹1000 (b) ₹1100 (c) ₹1200 (d) ₹1500

Ans. (c)

Q.42 The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3rd term of the AP is

(a) $6\frac{4}{11}$ (b) 6 (c) $\frac{4}{11}$ (d) None of these

Ans. (a)

ARITHMETIC MEAN

If a, b, c are in A.P., then b is called an arithmetic mean between a and c, such that $b = \frac{a+c}{2}$

E.g.: If the numbers 2, 6 and 10 are in A.P., then 6 is the arithmetic mean between 2 and 10 since $\frac{2+10}{2} = 10$

Now, if 'k' arithmetic mean need to be inserted then

$A.M_K = a + kd$

i.e., $A.M_1 = a + d$

$A.M_2 = a + 2d$

$A.M_3 = a + 3d$ ...

$A.M_m = a + md$

Q.43 The arithmetic mean between 33 and 77 is

(a) 50 (b) 45 (c) 55 (d) None of these

Ans. (c)

Q.44 Find the 3 arithmetic means between 15 and 35.

(a) 16, 18, 20 (b) 10, 15, 20 (c) 20, 25, 30 (d) None of these

Ans. (c)

Q.45 The 4 arithmetic means between –2 and 23 are

(a) 3, 13, 8, 18 (b) 18, 3, 8, 13

(c) 3, 8, 13, 18 (d) None of these

Ans. (c)

- Sum of 1st n natural or counting numbers: $1, 2, 3, 4, 5, 6, 7, \ldots, n$

  $S = (\frac{n(n+1)}{2})$

- Sum of the Squares of the first, n natural numbers:

  $S = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

- Sum of Cubes of the first, n natural numbersL

  $S = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \{\frac{n(n+1)}{2}\}^2$

Q.46 Find out the sum of 1st n natural odd numbers.

(a) n^{2n} (b) $2n$ (c) n^2 (d) None of these

Ans. (c)

Q.47 The number of numbers between 74 and 25,556 divisible by 5 is

(a) 5,090 (b) 5,097 (c) 5,095 (d) None of these

Ans. (b)

Q.48 The sum of n terms of an AP is $3n^2 + 5n$. The series is

(a) 8, 14, 20, 26 (b) 8, 22, 42, 68

(c) 22, 68, 114,... (d) None of these

Ans. (a)

GEOMETRIC PROGRESSION (G.P.)

- If a sequence is such that the one term is the multiple of the previous term. It means that there is a common multiplier called a common ratio which is the ratio of a term to its previous term.

E.g.:

(1) $1, 3, 9, 27, 81 \ldots$ (2) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ (d) $1, -1, 16, -64, 256 \ldots$

In (1), the common ratio = $\frac{3}{1} = 3$

In (2), the common ratio = $\frac{1}{\frac{2}{1}} = \frac{1}{2}$

In (3), the common ratio = $-\frac{4}{1} = -4$

Now, if we need to generalize the Geometric progression, where first term = a, common ratio = r and number of terms = n, then

n^{th} term of G.P $t_n = t_n = ar^{n-1}$

common ratio: $r = \frac{Anyterm}{Precedingterm} = \frac{t_n}{t_{n-1}} = r$

Geometric Progression series = $a + ar + ar^2 + ar^3 + ar^4 + ar^5 + \cdots$

Q.49 If 5, 25, 125 ... is a G.P, then what is the common ratio and its 15^{th} term?

(a) $10, 5^{15}$ (b) $5, 5^{10}$ (c) $5, 5^{15}$ (d) None of these

Ans. (c)

Q.50 t_{12} of the series -128, 64, -32 is

(a) $\frac{-1}{6}$ (b) 16 (c) $\frac{1}{16}$ (d) None of these

Ans. (c)

Q.51 In the series 25, 5, 1, ..., $\frac{1}{3125}$, then which term is $\frac{1}{3125}$?

(a) 8^{th} term (b) 9^{th} term (c) 15^{th} term (d) None of these

Ans. (a)

Q.52 Which term of series 3, 9, 27,... is 243?

(a) 5 (b) 6 (c) 7 (d) None of these

Ans. (a)

Q.53 In a GP. If the fourth term is 3, then the product of first seven terms is

(a) 3^5 (b) 3^7 (c) 3^6 (d) 3^8

Ans. (b)

Q.54 Which term of the G.P. $\sqrt{3}, 3, 3\sqrt{3}$, ...is 729?

(a) 9th (b) 10th (c) 12th (d) None of these

Ans. (c)

Q.55 The last term of the series $x^2, x, 1, \ldots$ to 31 term is

(a) x^{28} (b) $\frac{1}{x}$ (c) $\frac{1}{x^{28}}$ (d) None of these

Ans. (c)

Q.56 Find the G.P where 3rd term is 9 and 7th term is $\frac{729}{256}$?

(a) $16, 12, 9, \frac{27}{4} \ldots$ (b) $-16, -12, -9, \frac{27}{4}$

(c) $1, 2, 9, 11 \ldots$ (d) None of these

Ans. (a)

Q.57 The 6th term of the series: 0.04, 0.2, 1, ... is

(a) 5 (b) 25 (c) 125 (d) 625

Ans. (c)

Q.58 Find the 6th term and the common ratio of the geometric progression (G.P.): 3, 9, 27, 81, ...

(a) 255, 9 (b) 343, 3 (c) 729, 3 (d) 2187, 9

Ans. (c)

Q.59 Which term of the series: 3, 9, 27, ... is 243?

(a) 3 (b) 4 (c) 5 (d) 6

Q.60 The last term of the series 1, 2, 4, ... to 10 terms is

(a) 512 (b) 256 (c) 1024 (d) None of these

Ans. (a)

Q.61 The 5th term of a G.P. is 32 and the 8th term is 256, then the common ratio of the G.P. is

(a) 2 (b) 4 (c) 6 (d) 16

Ans. (a)

Q.62 If the fifth and third terms of a G.P. are 256 and 16 respectively, the its 8th term is

(a) 16384 (b) 15560 (c) 14000 (d) None of these

Ans. (a)

Q.63 Which term of the geometric progression (G.P.) 2, 6, 18, ... is 1458?

(a) 6th (b) 7th (c) 8th (d) 9th

Ans. (b)

Q.64 In a geometric progression, the 3rd and 6th terms are respectively 1 and $-\frac{1}{8}$ The first term (a) and common ratio are respectively

(a) $4 \; and \; \frac{1}{2}$ (b) $4 \; and -\frac{1}{4}$ (c) $4 \; and -\frac{1}{2}$ (d) $4 \; and \; \frac{1}{4}$

Ans. (c)

SUM OF GEOMETRIC PROGRESSION

Let's take the sum of first terms of G.P. i.e. $S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$ Now, the formula of sum of G.P. has three cases:

1. If common ratio, $r > 1$, then $S_n = \frac{a(r^n-1)}{r-1}$
2. If common ratio, $r < 1$, then $S_n = \frac{a(1-r^n)}{1-r}$
3. If common ratio, $r = 1$, then each term of the G.P. is equal to 'a'

   Therefore, the sum of the G.P simplifies to:

$S_n = a + a + a + a + \cdots . + a$ (n terms)

$S_n = na$

4. Sum of infinite geometric series:

$S = \frac{a}{1-r}, r < 1$

Q.65 Find the sum of series 1 + 4 + 16+ ..... Up to 7 terms.

(a) 5121 (b) 5461 (c) 5752 (d) None of these

Ans. (b)

Q.66 The sum of terms of the G.P. 3, 6, 12, ... is 381. Find the value of n.

(a) 3 (b) 12 (c) 5 (d) 7

Ans. (d)

Q.67 How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}, \ldots$ be taken together to make $\frac{3069}{512}$?

(a) 7 (b) 8 (c) 10 (d) Cannot be determined

Q.68 The sum of the infinite G.P. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$ is

(a) 2 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) None of these

Ans. (b)

Q.69 If $y = 1 + x + x^2 + \ldots + \infty$, then $x =$

(a) $\frac{y-1}{y}$ (b) $\frac{y+1}{y}$ (c) $\frac{y}{y+1}$ (d) $\frac{y}{y-1}$

Ans. (a)

Q.70 The common ratio of a G.P. is 3, and the last term is 486. If the sum of these terms is 728, find the first term

(a) 2 (b) 5 (c) 6 (d) 3

Ans. (a)

Q.71 The series 3, 6, 9, 12, 15 are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Ans. (a)

Q.72 If the first term of a G.P. exceeds the second term by 2 and the sum to infinity is 50, the series is _____.

(a) $10, 8, \frac{308}{5}, \ldots$ (b) $10, 8, \frac{5}{2}, \ldots$ (c) $10, \frac{10}{3}, \frac{10}{9}, \ldots$ (d) None of these

Ans. (a)

Q.73 The sum of the series $\frac{1}{\sqrt{3}} + 1 + \frac{2}{\sqrt{3}} + \cdots$ to 18 terms is

(a) $\frac{9841(\sqrt{3}+1)}{\sqrt{3}}$ (b) 9841 (c) $\frac{9841}{\sqrt{3}}$ (d) None of these

Ans. (a)

Q.74 The sum of first 6 terms of the G.P.: 4, 12, 36, ... is

(a) 1450 (b) 1456 (c) 728 (d) 364

Ans. (a)

Q.75 The sum of the series 243, 81, 27, ... to 8 terms is

(a) 36 (b) $36\frac{13}{30}$ (c) $36\frac{1}{9}$ (d) None of these

Ans. (d)

Q.76 Find the sum of the infinite geometric progression 3, 1.5, 0.75, ...

(a) 4 (b) 6 (c) 8 (d) Cannot be determined

Ans. (b)

Q.77 If the sum of an infinite geometric progression is 10 and the common ratio is $\frac{1}{2}$, the first term is

(a) 5 (b) 7.5 (c) 2.5 (d) 10

Ans. (a)

GEOMETRIC MEAN

If we take terms a, b, c which are in G.P. then, $\frac{b}{a} = \frac{c}{b}$ i.e., $b^2 = ac$ thus b is called the geometric mean of a and c.

Q.78 Insert 3 geometric means between 1 and $\frac{1}{16}$

(a) $1, \frac{1}{2}, \frac{1}{4}$ (b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ (c) $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ (d) None of these

Ans. (b)

Q.79 The A.M. of two positive numbers is 40 and their G. M. is 24. The numbers are

(a) (72, 8) (b) (70, 10) (c) (60, 20) (d) None of these

Ans. (a)

Trick: Go by choices

SELECTION OF TERMS OF A GEOMETRIC PROGRESSION

When the product of terms of G.P. is given

Number of Terms	Terms	Common Ratio
3	$\frac{a}{r}, a, ar$	r

4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	r^2
5	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	r
6	$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$	r^2
7	$\frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar^2, ar^3$	r

Q.80Q.80 The product of the first three terms of a G.P. is -1. Find the middle term.

(a) -1 (b) 0 (c) 1 (d) 3

Ans. (a)

Q.81 The sum of 3 numbers of a G.P. is 39 and their product is 729. The numbers are

(a) 2, 4, 8 (b) 3, 9, 27 (c) 3, 9, 12 (d) None of these

Ans. (b)

Q.82 Three numbers are in A.P. and their sum is 21. If 1, 5, 15 are added to them respectively, they form a G.P. The numbers are

(a) 5, 7, 9 (b) 9, 5, 7 (c) 7, 5, 9 (d) None of these

Ans. (a)

Q.83 The sum of the infinite series $1 + \frac{2}{3} + \frac{4}{9} + \cdots$ is

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{2}{3}$ (d) None of these

Ans. (b)

Q.84 Sum of n terms of the series 2 + 22 + 222 + ... Is

(a) $\frac{2}{9}(10^{n+1} - 10 - 9n)$ (b) $\frac{2}{81}(10^{n+1} - 10 - 9n)$

(c) $2(10^{n+1} - 10 - 9n)$ (d) None of these

Ans. (b)

Q.85 Sum of n terms of the series 0.1 + 0.11 + 0.111 + ...

(a) $2\,(9n - 1 + (\frac{1}{10})^n)$ (b) $\frac{2}{81}(9n - 1 + (\frac{1}{10})^n)$

(c) $\frac{1}{81}(9n - 1 + (\frac{1}{10})^n)$ (d) None of these

Ans. (c)

Q.86 The sum of $1.03 + (1.03)^2 + (1.03)^3 + \ldots$ to n terms is

(a) $103\,\{(1.03)^n - 1\}$ (b) $\frac{103}{3}\{(1.03)^n - 1\}$

(c) $(1.03)^n - 1$ (d) None of these

Ans. (b)

Q.87 Given x, y, z are in G.P and $x^p = z^m$, then $\frac{1}{p}, \frac{1}{q}, \frac{1}{m}$ are in

(a) A.P (b) G.P. (c) Both A.P. and G.P. (d) None of these

Ans. (a)

Q.88 If unity is added to the sum of any number of terms of the A.P. 3, 5, 7, 9, ... the resulting sum is

(a) 'a' perfect cube (b) 'a' perfect square

(c) 'a' number (d) None of these

Ans. (b)

Q.89 Find the sum of the following series: 0.5 + 0.55 + 0.555 ... to n terms

(a) $\frac{5}{81}[n - \frac{(1-0.1^n)}{9}]$ (b) $\frac{5}{9}[n - \frac{(1-0.1^n)}{9}]$

(c) $\frac{1}{9}[n - \frac{(1-0.1^n)}{9}]$ (d) None of these

Ans. (b)

Q.90 The sum of square of first n natural number is

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)(2n+1)}{6}$

(c) $\frac{n(n-1)(n-1)}{6}$ (d) $\frac{n(n+1)(n+2)}{6}$

Ans. (b)

Q.91 Find the sum of n terms of the series 7 + 77 + 777 + ... to n terms.

(a) $\frac{7}{9}(10^{n+1} - 10) - \frac{7n}{9}$ (b) $\frac{7}{9}(10^{n+1} - 10) + \frac{7n}{9}$

(c) $\frac{7}{9}(\frac{10(10^n-1)}{9} - n)$ (d) $\frac{7}{81}(10^{n+1} - 10) + \frac{7n}{9}$

Ans. (c)

Q.92 If Arithmetic Mean and Geometric Mean between two number are 5 and 4 respectively, then these numbers are

(a) 2 and 3 (b) 2 and 8 (c) 4 and 6 (d) 1 and 16

Ans. (b)

Q.93 A person pays ₹975 by monthly installment each less than the former by ₹5. The first installment is ₹100. The time by which the entire amount will be paid is

(a) 10 months (b) 15 months (c) 14 months (d) None of these

Ans. (b)

Q.94 The population of a country was 55 crores in 2005 and is growing at 2% p.a. C.I. then the population in the year 2015 is estimated as

(a) 57 crores (b) 62 crores (c) 81 crores (d) None of these

Ans. (d)

Q.95 The m^{th} term of an A.P. is n and n^{th} term of an A.P. is m. the r^{th} term of an A.P. is

(a) $m + n + r$ (b) $n + m - 2r$ (c) $m + n + \frac{r}{2}$ (d) $m + n - r$

Ans. (d)

PREVIOUS YEAR QUESTIONS

SEQUENCE & SERIES

Q.1 Find the 17th term of an AP series if 15th and 21st terms are 30.5 and 39.5 respectively.
[Dec. 2023]
(a) 33.5
(b) 35.5
(c) 36.0
(d) 38.0
Ans. (a) 33.5

Q.2 If nth term of an AP series is 7n – 2, then sum of 'n' terms is:
[Dec. 2023]
(a) $0.5(7n^2 + 2n)$
(b) $0.5(7n^2 - 3n)$
(c) $0.5(7n^2 + 3n)$
(d) $0.5(7n^2 - 2n)$
Ans. (c) $0.5(7n^2 + 3n)$

Q.3 Given an infinite geometric series with first term 'a' and common ratio 'r'. If its sum is 4 and the second term is 3, then one of correct option is
[Dec. 2023]
(a) a = 1 and r = 1/4
(b) a = 3 and r = 3/4
(c) a = 3 and r = 1/4
(d) a = 1 and r = 1/2
Ans. (c) a = 3 and r = 1/4

Q.4 Find the value of 'x' for the following data.
1+7 + 13 + 19..+ x = 225
[Dec. 2023]
(a) 56
(b) 63
(c) 49
(d) 42
Ans. (c) 49

Q.5 If 4th , 7th and 10th terms of a Geometric Progressions are p, q and r, respectively, then
[June 2023]
(a) $p^2 = q^2 + r^2$
(b) $p^2 = qr$
(c) $q^2 = pr$

(d) $pqr + pq + 1 = 0$
Ans. (c) $q^2 = pr$

Q.6 How many numbers between 74 and 25,556 are divisible by 5?
[June 2023]
(a) 5090
(b) 5097
(c) 5095
(d) 5075
Ans. (b) 5097

Q.7 If 9^{th} and 19^{th} term of an Arithmetic Progression is 35 and 75 respectively then its 20^{th} term is
[June 2023]
(a) 78
(b) 79
(c) 80
(d) 81
Ans. (b) 79

Q.8 In a G.P. = 5^{th} term is 27 and 8^{th} term is 729. Find its 11^{th} term.
[Dec 2022]
(a) 729
(b) 6061
(c) 2187
(d) 19683
Ans. (d) 19683

Q.9 If p^{th} term of an AP is q and its q^{th} term is p, when what will be the value of $(p + q)^{th}$ term?
[Dec 2022]
(a) 0
(b) 1
(c) p + q - 1
(d) 2 (p + q – 1)
Ans. (a) 0

Q.10 The sum of first eight terms of geometric progression is five times the sum of the first four terms. The common ratio is
[Dec 2022]
(a) $\sqrt{2}$
(b) $\sqrt{3}$
(c) 4
(d) 2

Ans. (a) $\sqrt{2}$

Q.11 The first and last terms of an arithmetic progression are 5 and 905. Sum of the terms is 45, 955. The number of terms is
[June 2022]
(a) 99
(b) 100
(c) 101
(d) 102
Ans. (c) 101

Q.12 In a geometric progression, the second term is 12 and the sixth term is 192. Find the 11^{th} term.
[June 2022]
(a) 3072
(b) 1536
(c) 12288
(d) 6144
Ans. (d) 6144

Q.13 If the nth term of the arithmetic progression 9, 7, 5…… is same as the nth term of the arithmetic progression 15, 12, 9……., then n will be
[June 2022]
(a) 7
(b) 9
(c) 15
(d) 11
Ans. (a) 7

Q.14 The sum of square of any real positive quantity and its reciprocal is never less that
[July 2021]
(a) 1
(b) 2
(c) 3
(d) 4
Ans. (d) 4

Q.15 If the sum of 'n' terms of an AP (Arithmetic Progression) is $2n^2$, the fifth term is_____
[July 2021]
(a) 20
(b) 50
(c) 18
(d) 25

Ans. (c) 18

Q.16 The number of terms of the series: 5 + 7 + 9 +.... must be taken so that the sum may be 480
[July 2021]
(a) 20
(b) 10
(c) 15
(d) 25
Ans. (a) 20

Q.17 If the sun and products of three number in G.P. are 7 and 8 respectively,

(a) 6
(b) 4
(c) 8
(d) 16
Ans. (c) 8

Q.18 The sum of series 7+14+21+..... To 17^{th} term is:

(a) 1071
(b) 971
(c) 1171
(d) 1271
Ans. (a) 1071

Q.19 The number of terms of series: 5 + 7 + 9 + ... Must be taken so that the sum may be 480
[July. 21]
(a) 20
(b) 10
(c) 15
(d) 25
Ans. (a) 20

Q.20 If the sum of three numbers in a geometric progression is 28. When 7,2 and 1 are subtracted from the first, second and the third numbers respectively, then the resulting numbers are in arithmetic progression. What is the sum of squares of the original three numbers?
[July. 21]
(a) 510
(b) 456
(c) 400
(d) 336
Ans. (d) 336

Q.21 If the sum of 'n' terms of an AP (Arithmetic Progression) is 2n2, the fifth term is _____
[July 21]
(a) 20
(b) 50

(c) 18
(d) 25
Ans. (c) 18

Q.22 The number of integers from 1 to 100 which are neither divisible by 3, nor by 5 nor by 7, is
[Jan. 21]
(a) 67
(b) 55
(c) 45
(d) 33
Ans. (c) 45

Q.23 In a geometric progression, the 3rd and 6th terms are, respectively, 1 and –1/8. The first term (a) and common ratio are respectively.
[Jan. 2021]
(a) 4 and ½
(b) 4 and –1/4
(c) 4 and –½
(d) 4 and ¼
Ans. (c) 4 and –½

Q.24 The 20th terms of arithmetic progression whose 6th term is 38 and 10th term is 66 is _____
[Dec. 2020]
(a) 136
(b) 118
(c)178
(d) 210
Ans. (a) 136

Q.25 Divide 69 into 3 parts which are in A. P and are such that the product of first two parts is 460
[Dec. 2020]
(a) 20, 23, 26
(b) 21, 23, 25
(c) 19, 23, 27
(d) 22, 23, 24
Ans. (a) 20, 23, 26

Q.26 Three numbers is G.P. with their sum is 130 and their product is 27,000 are ____
[Dec. 2020]
(a) 90, 30, 10
(b) 10, 30, 90
(c) (a) & (b) Both
(d) 10, 20 ,30

Ans. (c) (a) & (b) Both

SUMMARY

- An ordered collection of numbers $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ is a sequence if there is some pattern and then an is called the term of the sequence, corresponding to any value of the natural number n.
- An expression of the form $a_1 + a_2 + a_3 + \ldots + a_n + \ldots$ which is the sum of the elements of the sequence is called a series.
- If the series contains a finite number of elements, it is called a finite series, otherwise called an infinite series.

ARITHMETIC PROGRESSION

- A sequence $a_1, a_2, a_3, \ldots$, is called an Arithmetic Progression (A.P.)

  when $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$
- This constant 'd' is called the common difference of the A.P.

  (a) If 3 numbers a, b, c are in A.P., then $b - a = c - b$ *or* $a + c = 2b$; b is called the arithmetic mean between a and c

  'or' A.M of a and b $= \frac{a+b}{2}$

  (b) nth term of A.P. $a_n = a + (n-1)d$ where a = first term, d= common difference
- Sum of n terms of AP: $S_n = \frac{n}{2}[2a + (n-1)d]$ *or* $S_n = \frac{n}{2}[a + l]$

  Where, l is the last term of A.P.
- Sum of 1st n natural or counting numbers $S = \frac{n(n+1)}{2}$
- Sum of 1st n odd numbers : $S = n^2$
- Sum of 1st n even numbers : $S = n(n+1)$
- Sum of the squares of the first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
- Sum of the cubes of the first n natural numbers is $\{\frac{n(n+1)}{2}\}^2$

GEOMETRIC PROGRESSION

- If in a sequence of terms, each term is a constant multiple of the preceding term, then the sequence is called a Geometric Progression (G.P.).
- nth term of a GP $= ar^{n-1}$
- The constant multiplier is called the common ration $= \frac{anyterm}{preceding\ term} = \frac{a_n}{a_{n-1}} = r$
- Sum of first n terms of a G.P:

  $S_n = a\frac{(1-r^n)}{1-r}$ where $r < 1$

  $S_n = a\frac{(r^n-1)}{r-1}$ where $r > 1$
- Sum of infinite geometric series $S_\infty = \frac{a}{1-r}, r < 1$
- If a, b, c are in G.P. we get $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$, b is called the geometric mean between a and c

CHAPTER-7

SETS, RELATIONS AND FUNCTIONS, BASIC OF LIMITS AND CONTINUITY FUNCTIONA

Unit -1: Sets, Relations and Functions

SETS

DEFINITION

A set is a collection of well-defined objects or elements. It is a fundamental concept in mathematics that allows us to organize and categorize objects based on common characteristics or properties.

- Objects of sets can be called as elements or members of a set.
- Sets are generally denoted by Capital letters like – A, B, C, D, etc.
- The elements of sets are generally denoted by small letters like a, b, c, d, p, q, r, s, etc.

  E.g.: A = {a, e, i, o, u}

  B = {2, 3, 5, 7, 11, 13, 17, 19}

  where, A is the set of vowels in the alphabet and a,e,i,o,u are its elements. Similarly, B is the set of prime numbers less than 20 where 2, 3, 5, 7, 11, 13, 17, 19 are its elements.

REPRESENTATION OF SETS

Roster Method: (Tabular form) or Bracket form: This method involves listing all the elements of a set, separating them by commas, and enclosing them in curly braces {}. The order of elements in a set does not matter. Each element is typically distinct and appears only once in the set.

E.g.: Set A = {1, 2, 3, 4, 5} represents a set with elements 1, 2, 3, 4 and 5.

Set B = {apple, banana, orange} represents a set with elements apple, banana, and orange.

Set Builder: or Algebraic Form or Rule Method: This method involves defining a common property or rule that all elements in the set possess, and no element outside the set possesses. The set is then represented by expressing the property or rule in a concise form.

E.g.: Set C = {x: x is an even number} represents a set of all even numbers. Here, the property or rule is that the elements must be even numbers.

Set D = {y: y > 0} represents a set of all positive numbers. Here, the property or rule is that the elements must be greater than zero.

FEW IMPORTANT AND WIDELY USED SETS

- N: The set of all natural numbers: 1, 2, 3, 4, 5, 6,.....
- Z: The set of all integers: .... −3, −2, −1, 0, 1, 2, 3,.....
- Q: The set of all rational numbers : $\frac{2}{5}, \frac{-3}{8}, \frac{-19}{5}, \ldots$
- R: The set of real numbers: $0.2, 1, 2, -3, \frac{2}{5}, \frac{-3}{8}, \ldots.$
- Z^+: The sets of positive integers: 1, 2, 3, 4, ...
- Q^+: The set of positive rational numbers: $\frac{2}{5}, \frac{3}{8}, \frac{19}{5}, \ldots \ldots \ldots$
- R^+: The set of positive real numbers: $0.2, 1, 2, \frac{2}{5}, \ldots \ldots \ldots$

Q.1 Write the following sets in the set-builder form:

A = {2, 4, 6, ……, 100} B = {P, R, I, N, C, A, L}

C = {a, e, i, o, u} D = {–10, –8, –6, –4, –2}

Q.2 Write the following set in Roster form.

A = {X: X is an integer and $\{-4 \leq x < 2\}$

(a) {–3, –2, –1, 0, 1} (b) {–4, –3, –2, –1, 0, 1}

(c) {0, 1} (d) None of these

Ans. (b)

Q.3 Write the following set in set-builder form.

{1, 4, 9, ..., 121}

(a) {x : x = 2n, n ∈ N and 1 ≤ x ≤ 11} (b) {x : x = n2, n ∈ N and 1 < x < 11}

(c) {x : x = n2, n ∈ N and 1 ≤ x ≤ 11} (d) None of these

Ans. (c)

Q.4 Write the set {x : x ∈ Z, –5 ≤ x ≤ 1} in roster form.

(a) {–5, –4, –3, –2, –1, 0, 1} (b) {–4, –3, –2, –1, 0}

(c) {–5, –4, –3, –2, –1, 0} (d) {–5, –4, –3, –2, –1}

Ans. (a)

SOME SPECIAL SETS

- **Finite sets:** A finite set is a set that contains a specific number of elements, which can be counted.

  E.g.: The set {1, 2, 3, 4} is a finite set with four elements.

- **Infinite sets:** An infinite set is a set that has an unlimited number of elements and cannot be counted.

  E.g.: The set of all natural numbers {1, 2, 3, 4, ...} is an infinite set.

- **Null (Empty) set:** The null set, also known as the empty set, is a set that does not contain any elements. It is denoted by the symbol φ or {}.

  E.g.: The set of all prime numbers less than 1 is an empty set.

- **Singleton set:** A singleton set is a set that contains only one element. It is a finite set with a cardinality of one.

  E.g.: {5} is a singleton set that contains the element 5.

- **Equal Set and Equivalent set:** Two sets are said to be equal if they have exactly the same elements.

  E.g.: If A = {1, 2, 3} and B = {3, 2, 1}, then A = B since they contain the same elements, even though the order may differ.

- On the other hand, two sets are said to be equivalent if they have the same cardinality, i.e., they contain the same number of elements. The elements in the equivalent sets may or may not be the same.

  E.g.: {1, 2, 3} and {4, 5, 6} are equivalent sets since both sets have three elements.

Note: All equal sets are equivalent sets but equivalent sets may not be equal sets.
E.g.: {1, 2, 3, 4} and {a, b, c, d} are equivalent sets but not equal.

- **Disjoint Sets:** Disjoint sets are sets that have no common elements. In other words, there is no element that belongs to both sets.
  E.g.: {1, 2, 3} and {4, 5, 6} are disjoint sets since they do not share any common elements.

Q.5 Which of the following sets is an infinite set?

(a) {1, 2, 3, 4, ... 50}
(b) {-5, -4, -3, -2, -1, 0, 1, 2}
(c) Set of months of a year
(d) Set of positive integers greater than 1000

Ans. (d)

Q.6 Which of the following is an example of a null set?

(a) Set of even prime numbers.
(b) Set of odd natural numbers divisible by 2
(c) Set of natural numbers greater than 5 but less than 7
(d) All of them

Ans. (b)

Q.7 Which of the following pairs of sets is disjoint?

(a) {1, 2, 3, 4} and {x : x is a natural numbers such that 4 x 6} ≤ ≤
(b) {a, e, i, o, u} and {c, d, e, f}
(c) {x : x is an even integer} and {x : x is an odd integer}
(d) None of these

Ans. (a)

Q.8 Which of the following pair of sets is equal?

(a) A = {1, 2, 3, 4} and B = {d, c, a, b}
(b) A = {1, 4, 9, 16} and B = {16, 1, 9, 4}
(c) A = {x : x is multiple of 10} and B = {10, 20, 25, 30, 40,...}
(d) None of these

Ans. (b)

Q.9 Check if the below given sets are equal sets or equivalent sets:

(I) {3, 5, 7} and {1, 3, 7}
(II) {8, 6, 10, 12} and {3, 2, 4, 6}
(III) {1, 4, 9, 16, 25} and {12, 22, 32, 42, 52}

OTHER FREQUENTLY USED SYMBOLS ARE

- ∈ **(belongs to):** The symbol "∈" is used to indicate that an element belongs to a set. For example, if x is an element of set A, we can write it as x ∈ A.

- ∉ **(Does not belong to):** The symbol "∉" is used to indicate that an element does not belong to a set. For example, if y does not belong to set B, we can write it as y ∉ B.

Q.10 If set, A = {2, 4, 7}, B = {1, 3, 5, 8, 9} and C = {11, 13, 15, 19, 22}. Fill in the blanks.

11 ∈ ___

8 ∈ ___

SUBSETS

A set A is said to be a subset of a set B if every element of A is also an element of B. In other words, if all the elements of set A are contained within set B, we represent it as A ⊆ B.

E.g.: A = {1, 2, 3} and B = {1, 2, 3, 4, 5}

Here, A is a subset of B, because every element of A (1, 2 and 3) is also present in B.

PROPERTIES

- If A ⊆ B and B ⊆ A, then it implies that A = B. This means that A and B are equal sets.
- Every set is a subset of itself, denoted as A ⊆ A.
- The empty set (φ) is a subset of every set. In other words, the empty set has no elements, and thus, it is contained within every set.

PROPER SUBSET AND SUPER SET

Proper set: If A is a subset of B, but A is not equal to B, then A is called a proper subset of B, denoted as A ⊂ B. It means that A is a subset of B, because B contains element/elements that are not present in A.

E.g.: A = {1, 2} and B = {1, 2, 3}

Here, A is a proper subset of B because A is a subset of B, but B contains an additional element (3) that is not in A.

Super set: If A is a subset of B, then B is a superset of A.

POWER SET

The power set of a set is the set of all possible subsets of the given set.

For example, if we have a set A = {1, 2}, then the power set of A, denoted as P(A), will contain all the subsets of A, including the empty set and the set itself.

E.g.: A = {1, 2}

P(A) = {φ, {1}, {2}, {1, 2}}

Here, P(A) contains all the possible subsets of A, which are the empty set (∅), the individual elements {1} and {2}, and the set itself {1, 2}.

Thus, the number of power sets in a set of cardinality 'n' = 2^n

Also, if n is the number of elements in a set,

- Total number of subsets = 2^n
- Total number of proper subsets = $2^n - 1$

SUBSETS OF SET OF REAL NUMBERS

We know that,

- N: Set of all natural numbers $= \{1, 2, 3, 4, 5, \ldots\ldots\}$
- Z: Set of all integers = {….., -4, -3, 0, 1, 2, 3, …….}
- Q: Set of all rational numbers $= \{\frac{p}{q}, where\ p\ and\ q\ are\ integers\ and\ q \neq 0\}$
- R: Set of real numbers = Sets of all numbers on a number line

  Then, $N \sqsubset Z \sqsubset Q \sqsubset R$

Q.11 The number of subsets of the set {2, 3, 5} is

(a) 3 (b) 8 (c) 6 (d) None of these

Ans. (b)

Q.12 If A = {x : x is a positive integer and x < 6} then the number of elements in the power set of A is

(a) 4 (b) 16 (c) 32 (d) 64

Ans. (c)

Q.13 Which of the following is a subset of the set A = {2, 4, 6, 8}?

(a) {1, 2, 3, 4} (b) {4, 8, 12} (c) {2, 4, 6} (d) {2, 4, 6, 8, 10}

Ans. (c)

Q.14 Which of the following is a universal set for the set A = {x : x is a prime number less than 10}?

(a) {1, 3, 4, 5, 6, 7, 8, 9}

(b) {2, 3, 5, 7, 11}

(c) {x: x is a positive integer less than 10}

(d) {x : x is a multiple of 2}

Ans. (b)

Q.15 The set of squares of positive integers is

(a) A finite set (b) An infinite set

(c) A null set (d) None of these

Ans. (b)

Q.16 The set of cubes of natural numbers is

(a) Null set (b) Finite set

(c) Infinite set (d) A finite set of three numbers

Ans. (c)

Q.17 Which of the following is an example of a singleton set?

(a) {a, b, c} (b) {x : x is a negative even number}

(c) {x : x is a vowel} (d) {π}

Ans. (d)

Q.18 Which of the following sets is finite?

(a) The set of months of a year.

(b) {1, 2, 3, ...}

(c) The set of positive integers greater than 100.

(d) None of these

Ans. (a)

Q.19 The null set is represented by

(a) (φ) (b) (O) (c) φ (d) None of these

Ans. (c)

Q.20 Write down all the subsets of the following set: {1, 2, 3}

(a) {1}, {2}, {3}

(b) {1}, {2}, {3}, {1, 2}, {2, 3}, {2, 3}, {1, 3}

(c) φ, {1}, {2}, {3}, {1, 2}, {2, 3}, {2, 3}, {1, 3}

(d) φ, {1}, {2}, {3}, {1, 2}, {2, 3}, {2, 3}, {1, 3}, {1, 2, 3}

Ans. (d)

Q.21 The number of the subsets of the set {0, 1, 2, 3} is

(a) 2 (b) 4 (c) 8 (d) 16

Ans. (d)

Q.22 If A = {0, 1} state which of the following statements are true:

(I) {1} ⊂ A (II) {1} ∈ A (III) f∈ A (IV) 0 ∈ A (V) 1 ⊂ A (VI) {0} ∈ A (VII) f⊂ A

(a) (I), (IV) and (VII) only are true

(b) (I), (IV) and (VI) only are true

(c) (II), (III) and (VI) only are true

(d) None of these

Ans. (a)

Q.23 The number of proper subsets of A ∩ B, if A = {1, 2, 3, 4, 5, 7, 8, 9, 10} and B = {2, 4, 6, 7, 9} is

(a) 8 (b) 15 (c) 16 (d) 64

Ans. (b)

Universal set: Universal set is a set which consists of all sets in a given situation. Generally denoted by the letter 'U'.

E.g.: Suppose we are working with the following sets of numbers:

- N: The set of natural numbers {1, 2, 3, 4, ...}
- Z: The set of integers {..., -3, -2, -1, 0, 1, 2, 3, ...}
- Q: The set of rational numbers { 135, , 0, ,..... 2 4 2 − }
- R: The set of real numbers (includes all rational and irrational numbers)

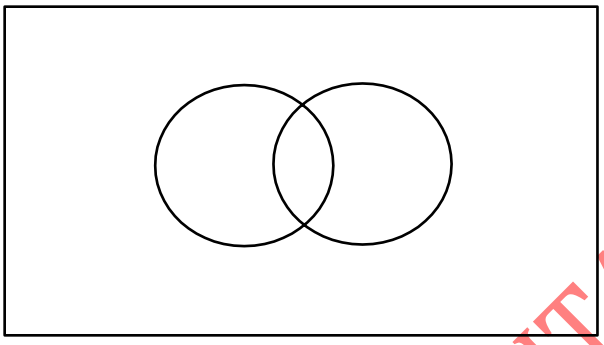
In this case, the universal set U would be the set of all real numbers, encompassing all the other sets:

In the above case, U = R

Thus, the universal set U in this example consists of all possible real numbers, including both rational and irrational numbers.

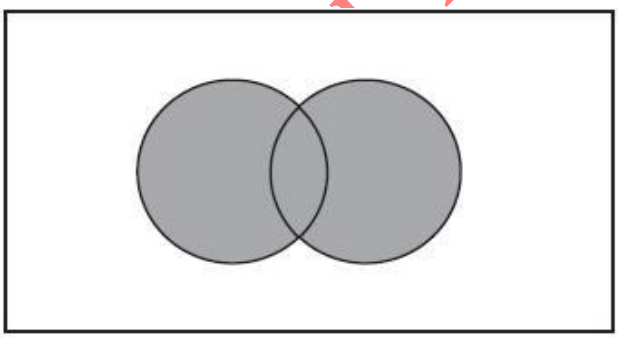
VENN DIAGRAM

Venn diagrams are named after the English logician, John Venn (1834 – 1883). These diagrams consist of rectangles and circles inside it. Universal set is a rectangle and other sets (which will eventually be subsets of the universal set.) are circles inside rectangles.



OPERATION ON SETS

Union of sets: Let's take sets A and B. Union of sets A and B is the set of all the elements either in A or B or in both, the common elements being taken only once. To denote union, we use the symbols '∪'. thus A union B will be written as A ∪ B.



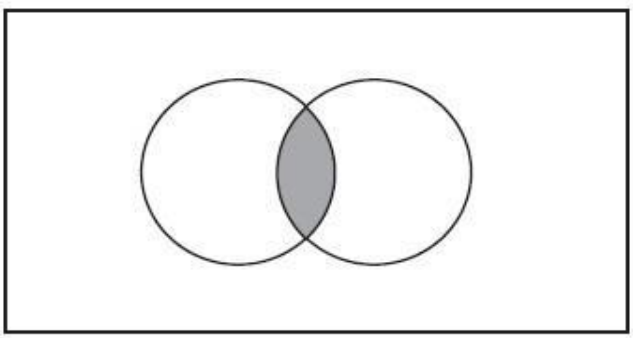
In set builder form, definition of A ∪ B = {x : x ∈ A or x ∈ B}.

E.g.: If A = {2, 4, 6, 8} and B = {5, 6, 8, 10}, then

A ∪ B = { 2, 4, 5, 6, 8, 10}

INTERSECTION OF SETS

Intersection of sets A and B is the set of all elements which are in A and in B. The symbol '∩' is used to denote the intersection. Thus, the intersection of A and B will be elements which are in both sets.



In set builder form definition of $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

If $A \cap B = \varphi$, then set is called Disjoint sets.

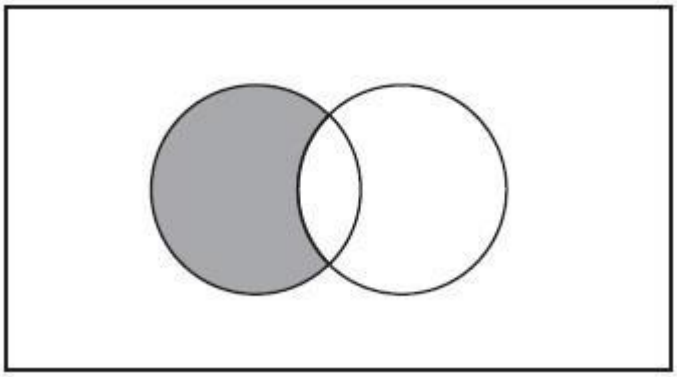
E.g.: (1) If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{3, 5, 7\}$, then

$A \cap B = \{3, 5, 7\}$

(2) If $C = \{2, 4, 6, 8, 10\}$ and $D = \{1, 3, 5, 7, 9\}$, then

$C \cap D = \varphi$ i.e., C and D are disjoint sets

DIFFERENCE OF TWO SETS



The difference of two sets A and set B, if taken in order A difference B, will consist of all elements in A leaving elements which are in intersection with B. It is denoted as A - B.

In set builder from, definition of $A - B = \{x : x \in A \text{ and } x \notin B\}$.

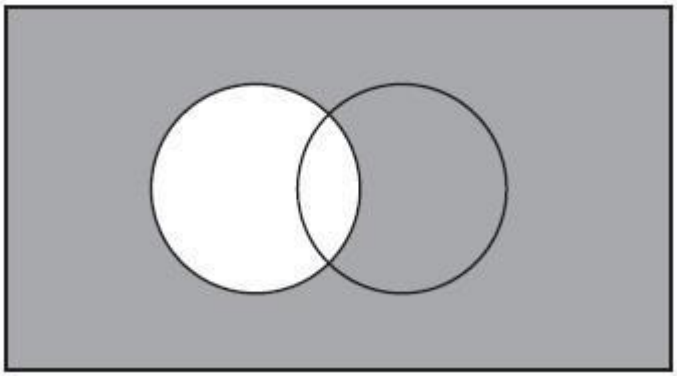
Note: $A - B \neq B - A$

E.g.: If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$

then $A - B = \{1, 3, 5\}$ and $B - A = \{8, 10\}$

Clearly, $A - B \neq B - A$

COMPLEMENT OF A SET



Complement of a set is denoted by A^c or A' and it includes all elements in the universal set except for elements in A i.e, $A^c = U - A$

In set builder form, definition of $A' = \{x : x \in U \text{ and } x \notin A\}$.

E.g.: If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 3, 5, 7, 9\}$ then

$A' = \{2, 4, 6, 8\}$

Properties: Complement laws

1. $A \cup A' = U$

2. $A \cap A' = \emptyset$

E.g.: If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 3, 5, 7, 9\}$ then $A' = \{2, 4, 6, 8\}$

Thus, $A \cup A' = \{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} = U$

Also, $A \cap A' = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8\} = \emptyset$

DE MORGAN'S LAW

1. $(A \cup B)' = A' \cap B'$

2. $(A \cap B)' = A' \cup B'$

Law of double complementation: $(A')' = A$

Laws of empty set and universal set: $\emptyset' = U$ and $U' = \emptyset$

Q.24 Find the union and intersection of below given pairs of sets.

(I) $A = \{7, 9, 12, 21\}$ and $B = \{5, 9, 12, 18\}$
(II) $C = \{a, b, c\}$ and $D = \{d, e, f\}$
(III) $E = \{x : n \in Z, x = 3n, -2 < n < 2\}$ and $F = \{x : x \in W, x < 5\}$

Q.25 If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $C = \{2, 4, 6\}$ then find:

(I) $A \cup B$ (II) $C \cup B$ (III) $A \cup B \cup C$ (IV) $A \cap C$
(V) $A - B$ (VI) $A \cap B \cap C$ (VII) $C - B$ (VIII) $B - C$

Q.26 Let $U = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{a, c, e, g, i\}$ and $B = \{b, c, d, e, f, g\}$

(I) A' (II) B' (III) $A' - B'$ (IV) $A' \cup B'$ (V) $A' \cap B'$

Q.27 The union of two sets $A = \{1, 2, 7, 9\}$ and $B = \{x : x2 - 10x + 16 = 0\}$ is
(a) $\{2\}$ (b) $\{1, 2, 7, 9\}$ (c) $\{1, 2, 7, 8, 9\}$ (d) $\{1, 2, 7, 8, 9, 10\}$
Ans. (c)

Q.28 If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. What is $(A - B) \cup (B - A)$?

(a) $\{1, 2, 3, 4, 5, 6\}$ (b) $\{3, 4, 5, 6\}$
(c) $\{1, 2, 3, 4\}$ (d) $\{1, 2, 5, 6\}$
Ans. (d)

Q.29 If $A \Delta B = (A - B) \cup (B - A)$ and $A = \{1, 2, 3, 4\}$, $B = \{3, 5, 7\}$ than $A \Delta B$ is

(a) $\{1, 2, 4, 5, 7\}$ (b) $\{3\}$
(c) $\{1, 2, 3, 4, 5, 7\}$ (d) None of these
Ans. (a)

Q.30 If $A = \{x : x$ is a prime number less than $10\}$, $B = \{x : x$ is an odd natural number less than $10\}$ then $(A \cap B)$ is
(a) $\{1, 3, 5, 7, 9\}$ (b) $\{2, 3, 5, 7\}$ (c) $\{3, 5, 7\}$ (d) None of these
Ans. (c)

Q.31 If A = {1, 2, 3, 4, 5, 6, 7, 8, 9}; B {1, 3, 4, 5, 7, 8}; C = {2, 6, 8}, then find (A – B) ∪ C

(a) {2, 6} (b) {2, 6, 8} (c) {2, 6, 8, 9} (d) None of these

Ans. (c)

Q.32 If U = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 2, 3, 8}, B = {3, 4, 5, 7} and C = {2, 3, 7, 8}. Find (A – B) ∪ C'.

(a) {1, 2, 3,5} (b) {1, 2, 4, 6, 8}

(c) {3} (d) {1, 2, 3, 4, 5, 6, 7, 8}

Ans. (b)

CARDINAL NUMBER

Let A and B be finite sets and if A ∩ B = ∅, then n(A ∪ B) = n(A) + n(B)

Let A and B be finite sets and if A ∩ B ≠ f, then

- n(A ∪ B) = n(A) + n(B) – n(A ∩ B)
- n(A ∪ B ∪ C) = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)
- n(B – A) = n(B) – n(A ∩ B)

Directions (Q. 32-34): By using the data

If P = {1, 2, 3, 5, 7}, Q = {1, 3, 6, 10, 15},

Universal Set S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

Q.33 The cardinal number of P ∩ Q is

(a) 3 (b) 2 (c) 0 (d) None of these

Ans. (b)

Q.34 The cardinal number of P ∪ Q is

(a) 10 (b) 9 (c) 8 (d) None of these

Ans. (c)

Q.35 n(P') is

(a) 10 (b) 5 (c) 6 (d) None of these

Ans. (a)

Q.36 If A = {1, 2, 3, 5, 7} and B = {x^2: x ∈A}

(a) n(B) = n(A) (b) n(B) > n(A) (c) n(A) < n(B) (d) None of these

Ans. (a)

Q.37 A ∩ E is equal to (E is a superset of A)

(a) A (b) E (c) φ (d) None of these

Ans. (a)

Q.38 If E = {1, 2, 3, 4, 5, 6, 7, 8, 9}, the subset of E satisfying 5 + x > 10 is **(ICAI)**

(a) {5, 6, 7, 8, 9} (b) {6, 7, 8, 9}

(c) {7, 8, 9} (d) None of these

Ans. (b)

Q.39 If A and B are two sets such that A has 50 elements. A ∪ B has 80 elements and A ∩ B has 15 elements, how many elements does B have?

(a) 25 (b) 50 (c) 75 (d) 45

Ans. (d)

Q.40 Out of 20 members in a family, 11 like to take tea and 14 like coffee. Assume that each one likes at least one of the two drinks. Find how many like both tea and coffee.

(a) 2 (b) 3 (c) 4 (d) 5

Ans. (d)

Q. 41 In a survey of 500 people, it was found that 350 people enjoy reading books and 250 people enjoy watching movies. How many people enjoy watching movies only?

(a) 150 (b) 120 (c) 100 (d) None of these

Ans. (a)

Q.42 In a group of 100 tourists, 55 can speak English and 85 can speak German.

How many people can speak both English and German?

(a) 40 (b) 50 (c) 60 (d) 70

Ans. (a)

Q.43 In a conference with 400 attendees, 300 attendees are from the finance industry and 200 attendees are from the marketing industries. How many are from the finance industries only?

(a) 70 (b) 150 (c) 200 (d) None of these

Ans. (c)

Q.44 The number of items in the set A is 40 in the Set B is 32; in the Set C is 50; in both A and B is 4; in both A and C is 5; in both B and C is 7; in all the set is 2. How many are in only one sets?

(a) 65 (b) 110 (c) 96 (d) 84

Ans. (c)

Q.45 In a group of 100 students, 60 students play football, 50 students play basketball, and 40 students play cricket. Out of these, 15 students play both football and basketball, 10 students play both basketball and cricket, 8 students play both football and cricket and 5 students play all three sports. How many students play cricket only?

(a) 14 (b) 29 (c) 34 (d) None of these

Ans. (b)

Q.46 X and Y are two sets such that n(X) = 17, n(Y) = 23 and n(X ∪ Y) = 38, find n(X ∩ Y).

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

Q.47 In a class of 40 students, 30 like cricket, 10 like both cricket and football. How many like football only and not cricket? How many like football?

(a) 20, 2 (b) 5, 10 (c) 20, 7 (d) 20, 10

Ans. (d)

Q.48 If P = {1, 2, 3, 5, 7}, Q = {1, 3, 6, 10, 15}, Universal set S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} then n(Q') is

(a) 4 (b) 10 (c) 4 (d) None of these

Ans. (b)

Q.49 If A = {1, 2, 3, 4, 5, 6, 7} and B = {2, 4, 6, 8}. Cardinal number of A - B is

(a) 4 (b) 3 (c) 9 (d) 7

Ans. (a)

Q.50 In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket and how much like tennis?

(a) 35, 25 (b) 25, 35 (c) 15, 25 (d) None of these

Ans. (b)

Q.51 In a survey of 100 students, 70 enjoy playing football, 60 enjoy playing basketball, and at least 50 students enjoy both sports. How many students enjoy both football and basketball?

(a) 70 (b) 30 (c) 50 (d) 80

Ans. (d)

ORDERED PAIR

An ordered pair is a simple way of keeping track of two elements by writing them in a specific order. It is denoted by (a, b).

CARTESIAN PRODUCT OF SETS

A set of all ordered pairs constructed by different sets. Let A and P be two sets with elements {a, b} and {p, q} respectively. The Cartesian product of A and P is denoted by A × P and is the set of all ordered pairs.

If we have two sets, A and P, with elements A = {a, b} and P = {p, q} respectively, the Cartesian product of A and P, denoted by A × P, is the set of all possible ordered pairs that can be formed by taking one element from A and one element from P.

So, the Cartesian product A × P would be:

A × P = {(a, p); (a, q); (b, p); (b, q)}

The Cartesian product P × A, on the other hand, would be:

P × A = {(p, a); (p, b); (q, a); (q, b)}

PROPERTIES

- $A \times P \neq P \times A$
- $A \times f = f$
- $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (Distributive law)
- $(A \cup B) \times C = (A \times C) \cup (B \times C)$ (Associative law)

Q.52 If A = {3, 5} and B = {4, 6} then A × B?

(a) {3, 5} (b) {(3, 4), (3, 6), (5, 4), (5, 6)}

(c) {4, 6} (d) None of these

Ans. (b)

Q.53 If A = {1, 3, 5} and B = {2, 3}, then find A × B.

(a) {(1, 2), (1, 3), (3, 2)}

(b) {(1, 2), (1, 3), (3, 3), (5, 5)}

(c) {(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)}

(d) None of these

Ans. (c)

Q.54 If P = {1, 5, 9} and Q = {2, 4) then P × Q?

(a) {(1, 2), (1, 4), (5, 2), (5, 4), (9, 2), (9, 4)}

(b) {(2, 1), (4, 1), (2, 5), (4, 5), (2, 9), (4, 9)}

(c) {(1, 2), (5, 2), (9, 2), (9, 4)}

(d) {(2, 1), (1, 4), (5, 2), (5, 4), (9, 2), (9, 4)}

Ans. (a)

Q.55 If A = {2, 3}, B = {4, 5} and C = {5, 6}, then find A × (B ∩ C).

(a) {(5, 2), (5, 3)} (b) {(2, 5), (3, 5)}

(c) {(2, 4), (5, 3)} (d) {(3, 5), (2, 6)}

Ans. (b)

- We can represent the product set of ordered pairs by points in the XY plane.
- If X = Y = the set of all natural numbers, the product set XY represent an infinite lattice of points in the first quadrant of the XY plane.

RELATION

A relation is any subset of the product of sets A and B. It is denoted by aRb, where 'a' belongs to set A and 'b' belongs to set B.

Suppose we define a relation R where set B (women) represents the mothers of set A (men):

Geeta has two sons, Ramesh and Brijesh; Babita has one son, Kamlesh; and Sunita has two sons, Suresh and Rajesh.

We can represent this relation as follows:

R = {(Ramesh, Geeta), (Brijesh, Geeta), (Kamlesh, Babita), (Suresh, Sunita), (Rajesh, Sunita)}

Now, let's find A × B, which represents the Cartesian product of sets A and B:

A × B = {(Ramesh, Geeta), (Ramesh, Babita), (Ramesh, Sunita), (Suresh, Geeta), (Suresh, Babita), (Suresh, Sunita), (Brijesh, Geeta), (Brijesh, Babita), (Brijesh, Sunita), (Rajesh, Geeta), (Rajesh, Babita), (Rajesh, Sunita), (Kamlesh, Geeta), (Kamlesh, Babita), (Kamlesh, Sunita)}

By comparing the relation R and the set A × B, we can see that they contain the same set of ordered pairs. The only difference is the representation and the context in which they are used.

Q.56 If A = {1, 2, 3, ..., 13, 14}. Find relation R in the set A defined by R = {(x, y) : 3x – y = 0}

KEY CONCEPTS

- If there are no elements that satisfy a particular relation, the relation is considered a void relation (R = φ).
- If R is a relation defined from A to A, it is called a relation defined on A.
- If R is a relation defined from a set A to a set B, and A × B contains all possible ordered pairs between elements of A and B, then R is called a universal relation. In other words, R includes all possible combinations of elements from A and B.
- If R is a relation defined from a set A to itself (A × A), it is called a relation defined on A. In this case, the elements of the relation are ordered pairs of elements from A.

  E.g.: Let A = {1, 2, 3} and B = {2, 5, 7}, and the relation is "the difference is odd":

  R = A × B = {(1, 2), (2, 5), (2, 7), (3, 2)}

  In this case, R is not a void relation because there are elements in the relation.

  Let A = {2, –2} and B = {4}, and the relation is "is square root of":

  R = A × B = {(2, 4), (–2, 4)}

  In this case, R is a universal relation because it includes all possible combinations of elements from A and B.

  If R is a relation defined from a set A to itself, such as A = {2, -2}, then the relation is defined on A.

DOMAIN AND RANGE OF A RELATION

For a relation from set A to set B i.e., aRb, all the elements of set A are called the domain of the relation R and the set of all second elements in a relation R from a set A to a set B is called the range of the relation R.

So, Domain (R) = {a : (a, b) ∈ R} and Range (R) = {b : (a, b) ∈ R}

E.g.: Consider the relation R defined on the set A = {1, 2, 3} as R = {(1, 2), (2, 3)}, then

Domain (R) = {1, 2}

Range (R) = {2, 3}

Q.57 Write the domain and range of the following relations.

R1 = {(1, 5), (2, 4), (2, 5), (1, 4), (1, 6), (1, 7)}

Q.58 Find the domain of the relation R defined by {(x, y): x, y ∈ N, xy = 8}

(a) {1, 2} (b) {1, 2, 4, 8} (c) {1, 2, 4, 8, 16} (d) None of these

Ans. (b)

Q.59 The range of the relation {(1, 0), (2, 0), (3, 0), (4, 0), (0, 0)} is

(a) {1, 2, 3, 4, 0} (b) {0} (c) {1, 2, 3, 4} (d) None of these

Ans. (b)

Q.60 The domain and range of $\{x, y : y = x^2\}$ where $x, y \in R$ is

(a) (reals, natural numbers) (b) (reals, non-negative reals)

(c) (reals, reals) (d) None of these

Ans. (b)

IDENTITY RELATION

The identity relation $I = \{(a, a) : a \in A\}$ is called the identity relation on A. In the identity relation, every element is related to itself.

For example, let A = {1, 2, 3} and the relation "is equal to":

I = {(1, 1), (2, 2), (3, 3)}

INVERSE RELATION

If R is a relation on A, then the relation R–1 on A, defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$, is called the inverse relation on A. In the inverse relation, the ordered pairs are reversed

For example, let A = {1, 2, 3} and R = {(1, 2), (2, 2), (3, 1), (3, 2)}:

Now, let's determine the values of the following:

1. Domain (R): The domain of relation R is the set of all first elements in the ordered pairs of R.

   Dom (R) = {1, 2, 3}

2. Range (R): The range of relation R is the set of all second elements in the ordered pairs of R.

   Range (R) = {1, 2}

3. R^{-1}: The inverse relation of R is obtained by reversing the ordered pairs of R.

   R^{-1} = {(2, 1), (2, 2), (1, 3), (2, 3)}

4. Dom (R^{-1}): The domain of the inverse relation R^{-1} is the set of all first elements in the ordered pairs of R^{-1} Dom (R^{-1}) = {1, 2}

5. Range (R^{-1}): The range of the inverse relation R–1 is the set of all second elements in the ordered pairs of R^{-1}

   Range (R^{-1}) = {1, 2, 3}

SOME IMPORTANT RELATIONS WHEN A RELATION IS DEFINED ON ITSELF

Let S = {a, b, c, …} be any set, and the relation R is a subset of the product set S × S.

1. If R contains all ordered pairs of the form (a, a) in S × S, then R is called Reflexive. In a reflexive relation, every element is related to itself.

   For example, the relation "is equal to" is reflexive because every element is equal to itself.

2. If (a, b) ∈ R implies (b, a) ∈ R for every a, b ∈ S, then R is called Symmetric. In a symmetric relation, if element a is related to element b, then element b is also related to element a.

   For example, the relation "is equal to" is symmetric because if a = b, then b = a.

3. If (a, b) ∈ R and (b, c) ∈ R implies (a, c) ∈ R for every a, b, c ∈ S, then R is called Transitive. In a transitive relation, if element a is related to element b and element b is related to element c, then element a is also related to element c.

   For example, the relation "is equal to" is transitive because if a = b and b = c, then a = c.

A relation that is reflexive, symmetric, and transitive is called an **Equivalence relation**. The relation "is equal to" is an equivalence relation.

Q.61 If a relation R is represented by {(8, 2), (6, 4), (1, 9), (3, 7)}, then the domain of R^{-1} is

(a) (1, 3, 6, 8) (b) (1, 2, 3, 4, 6, 7, 8, 9)

(c) (2, 4, 7, 9) (d) None of these

Ans. (c)

Q.62 A relation R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)} is

(a) Symmetric, Transitive but not Reflexive

(b) Reflexive, Symmetric but not Transitive

(c) Reflexive, Transitive but not Symmetric

(d) An Equivalence relation

Ans. (b)

Q. 63 If R be a relation on set A = {1, 2, 3, 4} such that R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} then R is

(a) Not Symmetric (b) Transitive

(c) Reflexive (d) Equivalence

Ans. (a)

Q.64 Which of the following is an example of an equivalence relation?

(a) {(x, y) : x < y} (b) {(x, y) : x > y}

(c) {(x, y) : x - y is an integer} (d) None of these

Ans. (c)

Q.65 "is perpendicular to" over the set of straight lines in a given plane is

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Ans. (b)

Q.66 Is equal to" over the set of all rational numbers is

(a) Reflexive (b) Symmetric (c) Transitive (d) Equivalence

Ans. (d)

Q.67 If R be the relation in the set {1, 2, 3} given by R = {(1, 3), (1, 2), (3, 2)}, then the relation is

(a) Reflexive (b) Reflexive and Symmetric

(c) Transitive (d) Symmetric and Transitive

Ans. (c)

Q.68 "Is the square of" over n set of real numbers is

(a) Reflexive (b) Symmetric (c) Transitive (d) None of these

Ans. (d)

FUNCTION

If we have two sets A and B, the relation 'f ' from set A to B is a function if and only if every element of A has a unique image in B. In other words, in a function, each element in the domain maps to a unique element in the codomain.

Q.69 Identify the function from the following:
(a) {(1, 1); (1, 2); (1, 3)}
(b) {(1, 1); (2, 1); (2, 3)}
(c) {(1, 2); (2, 2); (3, 2); (4, 2)}
(d) None of these

Ans. (c)

Q.70 If A = {2, 4, 6, 8} and B = {4, 64, 36, 16}, and the relation f is defined as "is square of" between elements of set A and set B. Which of the following represent the function?

(a) f = {(2, 4), (4, 16), (6, 36), (8, 64)}

(b) f = {(4, 2), (16, 4), (36, 6), (64, 8)}

(c) f = {(2, 4), (4, 8), (6, 12), (8, 16)}

(d) None of these

Ans. (a)

It can be framed also in a way, if we take a function f from set A to set B such that (x, y) ∈ f.

- Then f(x) = y is true only when for every element 'x' there is only one element 'y' or void.
- The function f from A to B is denoted by f : A → B and this is called mapping.

DOMAIN, CO-DOMAIN AND RANGE OF FUNCTION

For a function f from set A to set B (f : A → B), where f(a) = b, where a ∈ A and b ∈ B
The set A is called the domain of the function f.
The set B is called the codomain of the function f.
The set of images or the set f(A) = {f(x): x ∈ A} is called the range of the function f.
The function f is denoted as f: A → B, and this is called a mapping.

- For a function from set A to set B (f : A → B) i.e. f(a) = b where a ∈ A and b ∈ B
- All the elements of set A are called as the domain of the function f.
- While all the elements of set B are called as the co-domain of the function f and the set of image or set f(A) = {f(x): x ∈ A} is called the range of f.

Note: Range is always a subset of Co-domain, i.e. Range ⊆ Co-domain.

Q.71 If A = {2, 4, 6, 8}, B = {4, 64, 36, 16}

And let's say relation f "is square of" between elements of set A and set B, then domain, co-domain and range will be

Q.72 If A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 8, 10, 12} and a function is defined from set A to B where f(x) – 2x, then range of f is:

(a) {1, 2, 3, 4, 5} (b) {2, 4, 6, 8, 10}

(c) {1, 4, 9, 16, 25} (d) {2, 4, 6, 8, 10, 12}

Ans. (b)

Q.73 The range of the function f defined by $f(x) = \sqrt{16 - x^2}$ is

(a) [-4, 0] (b) [-4, 4] (c) [0, 4] (d) (-4, 4)

Ans. (c)

VARIOUS TYPES OF FUNCTIONS

One to One Function

A function is said to be one-to-one if each element in set A is uniquely mapped to a different element in set B. In other words, there are no repetitions in the mapping.

E.g.: Let A = {1, 2, 3} and B = {4, 5, 6}. The function f: A → B defined as f(x) = x + 3 is a one-to-one function because each element in set A is mapped to a unique element in set B without any repetitions.

Many to One Function

A function is called many-to-one if multiple elements in set A are mapped to the same element in set B. In this case, the mapping is referred to as a many-to-one mapping.

E.g.: Let A = {1, 2, 3, 4} and B = {1, 2}. The function f: A → B defined as f(x) = 1 for all x ∈ A is a many-to-one function because all elements in set A are mapped to the same element 1 in set B.

Note:

- If there is a greater-than sign (>) from set A to set B, it indicates a many-to-one function.
- If there is a less-than sign (<) from set A to set B, it implies that the relation is not a function.
- If the mapping is a straight line, it represents a one-to-one function.

Q.74 Let N be the set of natural numbers and the relation R be defined on N such that R = {(x, y): y = 2x, x, y ∈ N}. What is the domain, co domain and range of R? Is this relation a function?

Q.75 Let A = {–3, –1, 1, 3, 5, 7} and B = {–1, 1, 3, 5, 9, 25}

Consider the rule

(a) f(x) = x + 2, where x ∈ A (b) f(x) = x^2

Tell whether it is one to one or many to one function. Also, find the domain and range of the function.

Ans. (a)

ONTO OR SUBJECTIVE FUNCTIONS

Let f: A → B. If every element in B has at least one pre-image in A, then f is said to be an onto function. In other words, for every y ∈ B, there exists an x ∈ A such that y = f(x). A function f is onto if and only if the range of f is equal to the set B.

Q.76 Let N be the set of natural numbers and the relation R be defined on N such that R = {(x, y) : y = x, x, y ∈ N}. Tell whether it is onto function or not?

Sol. Given, the set of natural numbers N and the relation R defined on N such that

$R = \{(x, y) : y = x, x, y \in N\}$

In this case, every natural number y has at least one pre-image x such that y = x.

Therefore, the function defined by R is onto.

BIJECTIVE FUNCTION

A function that is both one-to-one (injective) and onto (subjective) is called a bijective function. A bijective function establishes a one-to-one correspondence between the elements of the domain and the elements of the co-domain.

E.g.: Let's consider the function $f : A \to B$ defined as $f(x) = 2x$, where A and B are sets of real numbers. This function is both one-to-one and onto. Each element in the domain A has a unique image in the co-domain B, and every element in B has a pre-image in A. Therefore, f is a bijective function.

IDENTITY FUNCTION

The identity function on a non-empty set A, denoted by $I : A \to A$, is defined as $I(x) = x$ for all $x \in A$. The identity function simply maps each element to it-self.

E.g.: Consider the set A = {1, 2, 3}. The identity function on A is defined as I(x) = x. So, I(1) = 1, I(2) = 2, and I(3) = 3. The identity function preserves the elements of the set.

Let A be a non-empty set. Then, the function I defined by $I : A \to A : I(x) = x$ for all $x \in A$ is called an identity function on A.

INTO FUNCTIONS

If there exists at least one element in the co-domain B that has no pre-image in the domain A, then the function $f: A \to B$ is said to be an into function.

E.g.: Let's consider the function $f : A \to B$ defined as f(x) = x2, where A and B are sets of real numbers. In this case, the function is not into because there are negative numbers in the co-domain B (e.g., -1) that do not have a pre-image in the domain A.

CONSTANT FUNCTION

A constant function is a function where all the elements in the domain have the same image in the co-domain.

E.g.: Let $f: A \to B$ be a constant function, where A = {1, 2, 3} and B = {5}. In this case, f(x) = 5 for all $x \in A$. Regardless of the input, the function always produces the same output, which is 5.

EQUAL FUNCTION

Two functions f and g are considered equal, denoted as f = g, if they have the same domain and satisfy the condition f(x) = g(x) for all x in their common domain.

E.g.: Let's consider two functions $f: A \to B$ and $g: A \to B$, where A = {1, 2} and B = {3, 6}. If f(x) = x + 2 and g(x) =3x, then f = g because both functions produce the same output for each input in their common domain.

INVERSE FUNCTION

If f is a one-to-one and onto function from set A to set B, then the inverse function f–1 is defined as f–1: B → A, where f–1 (y) = x if and only if f(x) = y. The inverse function undoes the action of the original function.

A FUNCTION IS INVERTIBLE IF AND ONLY IF F IS ONE-ONE AND ONTO

Q.77 Let A = {1, 2, 3} and B = {4, 5, 6, 7}. Consider the relation R from set A to set B, defined as R = {(1, 4), (2, 5), (3, 6)}. Then the relation is

(a) One-one function (b) Many-one function

(c) One-one and onto function (d) Many-one and onto function

Ans. (a)

Q.78 If A = {1, 2, 3, 4}, B = {2, 4, 6, 8}, f(1) = 2, f(2) = 4, f(3) = 6 and f(4) = 8 and f : A → B then f–1 is

(a) {(2, 1), (4, 2), (6, 3), (8, 4)} (b) {(1, 2), (2, 4), (3, 6), (4, 8)}

(c) {(1, 4), (2, 2), (3, 6), (4, 8)} (d) None of these

Ans. (a)

Q.79 The inverse function f^{-1} of $f(x) = 2x$ is

(a) $\frac{1}{2x}$ (b) $\frac{1}{x}$ (c) $\frac{1}{x}$ (d) None of these

Ans. (b)

Q.80 if f is an invertible function defined by $f(x) = \frac{3x-4}{6}$, then $f^{-1}(x) = ?$

(a) $2x + 3$ (b) $2x$ (c) $2x + \frac{4}{3}$ (d) None of these

Ans. (c)

COMPOSITE FUNCTION

A composite function is created when one function is substituted into another function. The composite function is formed by applying one function to the output of another function.

E.g.: Let's consider two functions f : A → B and g : B → C. The composite function (gof): A → C is defined as (gof) (x) = g(f(x)). The output of f(x) becomes the input for g(x)

Q.81 Given $f(x) = x^2 + 6$ *and* $g(x) = 2x - 1$ *find* $(fog)(x)$.

(a) $x^2 + x - 1$ (b) $4x^2 - 4x + 5$

(c) $x + 1$ (d) None of these

Ans. (b)

Q.82 If $f(x) = x^2$ and $g(x) = \sqrt{x}$, then

(a) gof(3) = 3 (b) gof(–3) = 9 (c) gof(9) = 3 (d) gof(–9) = 3

Ans. (a)

Q.83 If $f(x) = \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$. Find $fog(x)$.

(a) x (b) $\frac{1}{x}$ (c) $\frac{x}{\sqrt{1-x^2}}$ (d) $x\sqrt{1-x^2}$

Ans. (a)

Q.84 If $f(x) = x^2 - 1$ and $g(x) = |2x + 3|$, then $fog(3) - gof(-3) =$

(a) 71 (b) 61 (c) 41 (d) 51

Ans. (b)

Q.85 If A = {x, y, z}, B = {p, q, r, s}, which of the relations on A to B is a function?

(a) {(n, p), (x, q), (y, r), (z, s)} (b) {(s, x), (y, s), (z, s)}

(c) {(y, p), (y, q), (y, r), (z, s)} (d) {(x, p), (y, r), (z, s)}

Ans. (d)

Q.86 If $f(x) = \frac{3x+1}{2x+5}$ $for\ x \neq -\frac{5}{2}, then\ f^{-1}(2) =$

(a) 7 (b) -2 (c) 10 (d) -9

Ans. (d)

Q.87 $\{(x, y)|x - y = 3\}$, where $x, y \in R$ is a

(a) Not a function (b) a function

(c) One-one mapping (d) None of these

Ans. (c)

Q.88 $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$, then find $gof(x)$

(a) $8x$ (b) $2x$ (c) $2x^3$ (d) None of these

Ans. (b)

Q.89 Let the domain of x be the set {1}. Which of the following functions are equal to 1?

(a) $f(x) = x^2, g(x) = x$ (b) $f(x) = x, g(x) = 1 - x$

(c) $f(x) = x + x + 2, g(x) = (x + 1)^2$ (d) None of these

Ans. (a)

Q.90 If $f(x) = \frac{1}{1-x}$ $and\ g(x) = \frac{x-1}{x}$, then $gof(x)\ is$

(a) $x - 1$ (b) x (c) $\frac{1}{x}$ (d) None of these

Ans. (b)

Q.91 The range of the function f(x) $= log_{10}(1 + x)$ for the domain of real values of x when $0 \leq x \leq 9$ is

(a) [0, 1] (b) [0, 1, 2] (c) [0, 2] (d) None of these

Ans. (a)

Q.92 Let $A = R - \{3\}$ and B=R-{1}. Let f: A→B defined by $f(x) = \frac{x-2}{x-3}$. What is the value of $f^{-1}\left(\frac{1}{2}\right)$?

(a) − (b) $\frac{3}{4}$ (c) 1 (d) -1

Ans. (c)

Q.93 The $h^{-1}\ when\ h(x) = log_{10}x$ is

(a) $log_{10}x$ (b) 10^x (c) $log_{10}\left(\frac{1}{x}\right)$ (d) None of these

Ans. (b)

Q.94 "Is smaller than" over the set of eggs in a box is

(a) Transitive (T) (b) Symmetric (S)

(c) Reflexive (R) (d) Equivalence (E)

Ans. (a)

PREVIOUS YEAR QUESTIONS

SETS, RELATION & FUNCTIONS

Q.1 If B = {1,2,3,4,5}, then the number of proper subsets of B is [Dec. 2023]

(a)120 (b) 30 (c) 31 (d) 32

Ans. (c) 31

Q.2 If A = {1,2}, B = {3,4}, C = {5,6} then the value of A × $(B \cup C)$ [Dec. 2023]

(a) {(1,2),(3,4),(5,6)}

(b) {(1,3),(2,3),(1,4),(2,4),(2,5),(1,5),(1,6),(2,6)}

(c) {(1,3),(2,3),(1,4),(2,4),(2,5),(1,5)}

(d) {(3,1),(2,3),(4,1),(2,4),(2,5),(1,5),(1,6),(2,6)

Ans. (b) {(1,3),(2,3),(1,4),(2,4),(2,5),(1,5),(1,6),(2,6)}

Q.3 In a survey of 100 boys it was found that 50 used white shirts, 40 red shirts and 30 blue shirts. 20 were habituated in using both white and red shirts. 15 were using both red and blue shirts and 10 were using blue and white shirts. Find the number of boys who are using all colours.

[Dec. 2023]

(a) 20 (b) 25 (c) 30 (d) 35

Ans. (b) 25

Q.4If A = {2,4} and B = {1,2,3}, then (A ∪ B) × (A ∩ B) is equal to:

(a) {(1,2),(2,2),(3,2)}

(b) {(1,2),(2,2),(2,3),(2,4)}

(c) {(2,1),(2,2),(2,4)}

(d) {(1,2),(2,2),(3,2),(4,2)}

Ans. (d) {(1,2),(2,2),(3,2),(4,2)}

Q.5 If $f(x): N \to R$ is a function defined as $f(x) = 4x + 3, \forall x \in N$, then $f^{-1}(x)$ is :

[June 2023]

(a) $4 + \frac{x+3}{4}$ (b) $\frac{x+3}{4}$

(c) $\frac{x-3}{4}$ (d) $\frac{3x+4}{4}$

Ans. (c) $\frac{x-3}{4}$

Q.6 If $A = \{a, b, c\}, B = \{b, c, d\}$ and $C = \{a, d, c\}$, then $(A - B) \times (B \cap C)$ is equal to

[June 2023]

(a) {(a,d),(c,d)} (b) {(a,c),(a,d)}

(c) {(c,a),(d,a)} (d) {(a,c),(a,d),(b,d)}

Ans. (b) {(a,c),(a,d)}

Q.7 If R be a relation defined on the set of Natural numbers as " $xRy \Leftrightarrow (x - y)$ is divisible by 5" $\forall x, y \in N$, then the relation is

[June 2023]

(a) Equivalence

(b) Anti - Symmetric

(c) Symmetric But not transitive

(d) Symmetric but not reflexive

Ans. (a) Equivalence

Q.8 Given the relation $R = \{(1,2), (2,3)\}$ on the set $A = \{1,2,3\}$, the minimum number of ordered pairs which when added to R to make it an equivalence relation is
[June 2023]

(a) 5 (b) 6
(c) 7 (d) 8

Ans. (c) 7

Q.9 A survey shows that 74% of the Indians like grapes, whereas 68% like bananas. What percentage of the Indians like both grapes and bananas if everybody likes either fruit?
[June 2023]

(a) 42% (b) 26%
(c) 58% (d) 48%

Ans. (a) 42%

Q.10 The number of a subsets of the set {0, 1, 2, 3} is [Dec 2022]

(a) 2 (b) 4
(c) 8 (d) 16

Ans. (d) 16

Q.11 Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2,3), (1, 3)}. Then R is :

(a) symmetric and transitive (b) reflexive and transitive
(c) reflexive but not symmetric (d) neither symmetric nor transitive

Ans. (c) reflexive but not symmetric

Q.12 If A = {1, 2, 3, 4, 5, 7, 8,9} and B = {2, 4, 6, 7, 9} then how many proper subset of $A \cap B$ can be created.
[Dec 2022]

(a) 16 (b) 15
(c) 32 (d) 31

Ans. (b) 15

Q.13 Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. Then relation is
[Dec 2022]

(a) An Equivalence Relation (b) Reflexive and transitive only
(c) Reflexive only (d) Reflexive and Symmetric only

Ans. (b) Reflexive and transitive only

Q.14 If $f(p) = \frac{1}{1-p}$, then f^{-1} is [Dec 2022]

(a) 1 - p(b) (p – 1)/p (b) (p – 1)/p
(c) p/(p – 1) (d) 1/p

Ans. (b) (p – 1)/p

Q.15 Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are
[July 2022]

(a) 6,3 (b) 7,6
(c) 5,1 (d) 8,7

Ans. (a) 6,3

Q.16 Out of a group of 20 teacher in school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry, 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics, how may teach only Physics?

[Dec 20

(a) 2,3 (b) 3,2
(c) 4,6 (d) 6,4
Ans. (a) 2,3

Q.16 If a is related to b if and only if the difference in a and b is an even integer. This relation is [Dec 2021]
(a) symmetric, reflexive but not transitive (b) symmetric, transitive but not reflexive
(c) transitive, reflexive but not symmetric (d) equivalence relation
Ans. (d) Equivalence relation

Q.19 If u(x)=$\frac{1}{1-x}$, then u^{-1} (x) is: [Dec 2022]
(a) $\frac{1}{x-1}$ (b) 1 – x
(c) $1-\frac{1}{x}$ (d) $\frac{1}{x}-1$

Ans **Q.17** $-\frac{1}{x}$

Q.19 If f(x) = x^2 –1 and g(x) = |2x + 3|, then fog(3) – gof (–3) [July 2021]
(a) 71 (b) 61
(c) 41 (d) 51
Ans. (b) 61

Q.20 Let A = R – {3} and B = R – {1}. Let f : A → B defined by f(x) = $\frac{x-2}{x-3}$ what is the value of $f^{-1}\left(\frac{1}{2}\right)$?
[July 2021]
(a) 2/3 (b) 3/4
(c) 1 (d) -1
Ans. (c) 1

Q.21 The range of the function f defined by $f(x) = \sqrt{16 - x^2}$ is [July 2021]
(a) [-4,0] (b) [-4,4]
(c) [0,4] (d) (-4,4)
Ans. (c) [0,4]

Q.22 Let U be the universal set, A and B are the subsets of U. If n(U) = 650, n(A) = 310, n(A ∩ B). = 95 and n(B) = 190, then $n(\bar{A} \cap \bar{B})$ is equal to ($\bar{A}$ and $\bar{B}$ are the complement of A and B, respectively)
[July 2021]
(a) 400 (b) 200
(c) 300 (d) 245
Ans. (d) 245

Q.23 Let f. R ⇒ R be defined by

$$f(x)\begin{cases} 2x \text{ for } & x > 3 \\ x^2 \text{ for } & 1 < x \leq 3 \\ 3x \text{ for } & x \leq 1 \end{cases}$$

The value of f(−1)+ f(2) + f(4) is [Jan. 2021]

(a) 9 (b) 14

(c) 5 (d) 6

Ans. (a) 9

Q.24 The set of cubes of natural numbers is [Jan. 2021]

(a) Null set (b) A finite set

(c) An infinite set (d) Singleton set

Ans. (c) An infinite set

Q.25 The inverse function f −1 of f(y) = 3y is _____ [Dec. 2020]

(a) 1/3y (b) y/3

(c) −3y (d) 1/y

Ans. (b) y/3

Q.26 The set of cubes of natural numbers is [Dec. 2020]

(a) Null set (b) Finite set

(c) Infinite set (d) A finite set of three numbers

Ans. (c) Infinite set

Q.27 Two finite sets respectively have x and y number of elements. The total number of sub sets of the first is 56 more than the total no. of sub sets of the second. The value of x, y are respectively _____ [Dec. 2020]

(a) 4 and 2 (b) 6 and 3

(c) 2 and 4 (d) 3 and 6

Ans. (b) 6 and 3

SUMMARY

- A set is defined to be a collection of well-defined distinct objects. This collection may be listed or described.
- Each object is called an element of the set. We usually denote sets by capital letters and their elements by small letters.
- Representation of Sets: Roster form & Set Builder Form
- Subset - Subset of a set is a collection of a few elements of a set.
- No. of sets for n number of elements = 2n subsets.
- Power Set - The collection of all possible subsets of a given set A is called the power set of A, to be denoted by P(A).
- Proper Subset and Super Set - A set containing n elements has 2n – 1 proper subsets.
- Equivalent Set - Two finite sets A & B are said to be equivalent if n(A) = n(B).
- Null Set - A set containing zero element is called null set.
- Singleton Set - A set containing one element is called a Singleton set.
- Equal Set - Two sets A & B are said to be equal, written as A = B if every element of A is in B and every element of B is in A.
- Universal Set - The set which contains all the elements under consideration in a particular problem is called the universal set denoted by S. Suppose that P is a subset of S.
- Venn Diagram - Venn diagrams are the diagrams that are used to represent the sets, relation between the sets and operation performed on them, in a pictorial way.
- Union of Sets - The union of two sets X and Y is equal to the set of elements that are present in set X, in set Y, or in both the sets X and Y. This operation can be represented as $X \cup Y = \{a : a \in X \text{ or } a \in Y\}$.
- Intersection of sets - Intersection of two sets A and B is the set containing elements that are common to both A and B. This can be expressed as: $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

- Complement of a set - If U is a universal set and A be any subset of U then the complement of A is the set of all members of the universal set U which are not the elements of A.
- The difference of two sets - A and B is defined as the lists of all the elements that are in set A but that are not present in set B. The set notation used to represent the difference between the two sets A and B is A – B or A, B
- A – B = {x | x ∈ A and x ∉ B}
- If A and B are two sets then, n(A ∪ B) = n(A) + n(B) – n(A ∩ B)
- If A and B are disjoint sets, then n(A ∪ B) = n(A) + n(B) as n(A ∩ B) = 0
- For three sets P, Q and R
- n(P ∪ Q ∪ R) = n(P) + n(Q) + n(R) – n(P ∩ Q) – R(Q ∩ R) – n(P ∩ R) + n(P ∩ Q ∩ R)
- When P, Q and R are disjoint sets n(P ∪ Q ∪ R) = n(P) + n(Q) + n(R)

RELATIONS

- Ordered Pair: Two elements a and b, listed in a specific order, form an ordered pair, denoted by (a, b).
- Cartesian Product of sets: If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B, is called the Cartesian product of A and B, to be denoted by A × B.
- If A = ***Φ***or B = ***Φ***, we define A × B = ***Φ***
- **Relation:** For x and y are two sets of ordered pairs. And set x has relation with set y, then the values of set x are called domain whereas the values of set y are called range.
- **Domain & Range of a relation:** If R is a relation from A to B, then the set of all first coordinates of elements of R is called the domain of R, while the set of all second coordinates of elements of R is called the range of R.
- So, Dom(R) = {a : (a, b) ∈ R} & Range (R) = {b : (a, b) ∈ R}

FUNCTION

- Any relation from X to Y in which no two different ordered pairs have the same first element is called a function.
- Let A and B be two non-empty sets. Then, a rule or a correspondence f which associates to each element x of A, a unique element, denoted by f(x) of B, is called a function or mapping from A to B and we write f : A → B
- The element f(x) of B is called the image of x, while x is called the pre-image of f(x).
- Let f : A → B, then A is called the domain of f, while B is called the co-domain of f.
- The set f(A) = {f(x): x ∈ A} is called the range of f.

RELATION AND FUNCTION

- **One-one Function:** Let f : A → B, If different elements in A have different images in B, then f is said to be a one-one or an injective function or mapping.
- **Onto or Surjective Functions:** Let f : A → B, If every element in B has at least one preimage in A, then f is said to be an onto function. If f is onto if and only if Range of f = B.
- **Bijection Function:** A one-one and onto function is said to be bijective. A bijective function is also known as a one-to-one correspondence.
- **Inverse Function** A function is invertible if and only if f is one-one onto.
- Let f be a one-one onto function from A to B. Then there exists an element x in A such that f(x) = y then f–1(y) = x.
- **Identity function:** For x ∈ A if f(x) = x, then f is identity function.
- **Constant function:** For x ∈ A if f(x) = c, then f is constant function.
- **Equal Functions:** Two functions f and g are said to be equal, written as f = g if they have the same domain and they satisfy the condition - f(x) = g(x), for all x.
- Let S = {a, b, c, ...} be any set then the relation R is a subset of the product set S × S

(i) If R contains all ordered pairs of the form (a, a) in S × S, then R is called reflexive. In a reflexive

relation 'a' is related to itself.

(ii) If $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in S$, then R is called symmetric.

(iii) If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \Rightarrow R$ for every $a, b, c, \in S$, then R is called transitive.

A relation which is reflexive, symmetric and transitive is called an equivalence relation or simply an equivalence.

LIMITS

In calculus, the concept of a limit is crucial for understanding how a function behaves as the input variable (commonly denoted as 'x') approaches a specific value ('a').

Let's explore the idea of limits with a new example.

Consider the function: g(x) = 2x + 3 and we want to determine its limit as x approaches 1.

The notation for the limit of a function is represented as:

$\lim_{x \to a} g(x) = L$

In the context of the function g(x) = 2x + 3, and as x approaches 1, we are interested in:

$\lim_{x \to 1}(2x + 3)$

This signifies our interest in understanding how the function 2x + 3 behaves as x gets closer and closer to 1 while going from 0 to 1 and while going from 2 to 1.

In this limit, when we approach g(x) from left hand side of 1:

When $x = 0.97$	$\lim_{x \to 0.97}(2x + 3) = 2(0.97) + 3 = 1.94 + 3 = 4.94$
When $x = 0.98$	$\lim_{x \to 0.98}(2x + 3) = 2(0.98) + 3 = 1.96 + 3 = 4.96$
When $x = 0.99$	$\lim_{x \to 0.99}(2x + 3) = 2(0.99) + 3 = 1.98 + 3 = 4.98$
When $x = 0.9999$	$\lim_{x \to 0.9999}(2x + 3) = 2(0.9999) + 3 = 1.9999 + 3 = 4.9998$
When $x = 1$	$\lim_{x \to 1}(2x + 3) = 2(1.0) + 3 = 2 + 3 = 5$

Now, when we approach g(x) from right hand side of 1:

When $x = 1.2$	$\lim_{x \to 0.97}(2x + 3) = 2(0.97) + 3 = 1.94 + 3 = 4.94$
When $x = 1.1$	$\lim_{x \to 1.1}(2x + 3) = 2(1.1) + 3 = 2.2 + 3 = 5.2$
When $x = 1.01$	$\lim_{x \to 1.01}(2x + 3) = 2(1.01) + 3 = 2.02 + 3 = 5.02$
When $x = 1.001$	$\lim_{x \to 1.001}(2x + 3) = 2(1.001) + 3 = 2.002 + 3 = 5.002$
When $x = 1.0$	$\lim_{x \to 1.0}(2x + 3) = 2(1.0) + 3 = 2 + 3 = 5$

From the above table, we can conclude that: When x gets closer and closer to 1, regardless of whether x approaches from left (x < 1) or x approaches from right (x > 1), the corresponding value of the function is approaching 5.

Thus, we conclude that the limit of the function g(x) = 2x + 3 as x approaches 1 is equal to 5. Therefore, we can write as:

$\lim_{x\to 1^-}(2x+3) = \lim_{x\to 1^+}(2x+3) = 5$

$\Rightarrow \lim_{x\to 1^-}(2x+3) = \lim_{x\to 1^+}(2x+3) = \lim_{x\to 1}(2x+3) = 5$

We can conclude that $\lim_{x\to a} f(x)$ is said to exist when;

- Both left-hand limit and right-hand limit exist and are equal.
- $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a} f(x)$

Let's consider the function, f(x) = {2x+1, if x ≠ 2}

= {4, if x = 2

Here, when x approaches from left – hand limit of 2 and right – hand limit of 2 then

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 2(2)+1 = 5$

But $\lim_{x\to 2} f(x) = 2$

In this case, both left-hand and right-hand limit exist and are equal but

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

Thus, we can conclude $\lim_{x\to 2^-} f(x)$ does not exist.

PROPERTIES OF LIMITS

Let $\lim_{x\to a} f(x) = l$ *and* $\lim_{x\to a} g(x) = m$ where l and m are finite quantities.

1. $\lim_{x\to a} f\{f(x)+g(x)\} = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = l+m$ that is limit of the sum of the difference of two functions is equal to the sum of their limits.
2. $\lim_{x\to a}\{f(x)-f(x)\} = \lim_{x\to a} f(x) - \lim_{x\to a} f(x) = l-m$ that is limit of the difference of two functions is equal to difference of their limits.
3. $\lim_{x\to a}\{f(x)g(x) = \lim_{x\to a} f(x).\lim_{x\to a} g(x)$ =lm that is limit of the product of two function is equal to the product of their limits.
4. $\lim_{x\to a}\{\frac{f(x)}{g(x)}\} = \{\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}\}$ = l/m if m≠ 0 that is limit of the quotient of two functions is equal to the quotient of their limits.
5. $\lim_{x-a} c = c$ where is constant that is limit of a constant is the constant
6. $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$
7. $\lim_{x\to a} F\{f(x)\} = F\{\lim_{x\to a} F(x)\} = F(a)$
8. $\lim_{x\to 0^+}\lim_{h\to 0}\frac{1}{h} \to +\infty(h>0)$

$\lim_{x\to 0^-}\frac{1}{-h} \to -\infty(h>0)$

∞ is a very – very large number called infinity

Thus $\lim_{x\to 0^+}\frac{1}{x}$ does not exist.

Q.1 Evaluating $\lim_{x\to 2} 9$

(a) 2 (b)9 (c)18 (d)Cannot be determined.

Ans. (b)

Q.2 Evaluate $\lim_{x\to -2}(3x+10)$

(a)1 (b)-2 (c)4 (d)10

Ans. (c)

Q.3 Evaluate $\lim_{x\to 1}(3x^2-2x)$.

(a)1 (b)2 (c)3 (d)4

Ans. (a)

Q.4 Evaluate $\lim_{x\to 2} 5x^2$

(a)2 (b)5 (c)10 (d)20

Ans. (d)

Q.5 Evaluate $\lim_{x\to 3} \sqrt[4]{x^2+7}$

(a) 2 (b)3 (c)4 (d)None of these

Ans. (a)

Q.6 Evaluate $\lim_{x\to 1} \frac{x^2-1}{x-1}$

(a) 3 (b)2 (c)4 (d)None of these

Ans. (b)

Q.7 Evaluate $\lim_{x\to 5} \frac{x^2-3x-10}{x^2-6x+5}$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{7}{4}$ (d) Cannot be determined

Ans. (c)

Q.8 Evaluate $\lim_{x\to 1}\{(x^2+4)(3x+2)\}$.

(a) 5 (b)25 (c)10 (d)Does not exist

Ans. (b)

Q.9 Evaluate $\lim_{x\to 4} \frac{1}{x-4}$

(a)4 (b)2 (c)0 (d)Does not exist

Ans. (d)

Q.10 Evaluate $\lim_{x\to 2} \sqrt{\frac{x^2+7x-6}{x+1}}$

(a)2 (b)4 (c)12 (d)Does not exist

SOME IMPORTANT LIMITS

1. $\lim_{x\to 0} \frac{(e^x-1)}{x} = 1$
2. $\lim_{x\to 0} \frac{a^x-1}{x} = log_e a \ (a > 0)$
3. $\lim_{x\to 0} \frac{log(1+x)}{x} = 1$
4. $\lim_{X\to x} (1+\frac{1}{x})^x = e$ or $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}}}{x} = e$
5. $\lim_{x\to 0} \frac{x^n-a^n}{x} = a^{n-1}$
6. $\lim_{x\to 0} \frac{(1+x)^n-1}{x} = n$

LIMITS AT INFINITY

Consider the function, f(x) = $\frac{1}{x}$

Now,

At $x = 1, f(x) = \frac{1}{1} = 1$

At $x = 10, f(x) = \frac{1}{10} = 0.1$

At $x = 100, f(x) = \frac{1}{100} = 0.01$

At $x = 1000, f(x) = \frac{1}{1000} = 0.001$

Thus, when x gets larger and larger, the values of $f(x)$ approaches to 0.

It can be represented as $\lim_{x\to\infty}\frac{1}{x}=0$

Similarly, as x gets smaller and smaller; the values of $f(x)$ approaches to 0.

It can be represented as $\lim_{x\to-\infty}\frac{1}{x}=0$

In general,

$\lim_{x\to\infty}\frac{1}{x^p}=0, where\ p>0$

Q.11Evaluate $\lim_{x\to2}\frac{\sqrt{x+2}-2}{x-2}=0$

(a) $\frac{1}{5}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) Does not exist

Ans. (c)

Q.12 Evaluate $\lim_{x\to0}\frac{e^{3x}-1}{x}$

(a)1 (b)3 (c)0 (d)Does not exist

Ans. (b)

Q.13Find $\lim_{x\to0}\frac{3x+|x|}{7x-5|x|}$

(a)2 (b)$\frac{1}{6}$ (c)2 or $\frac{1}{6}$ (d) Does not exist.

Ans. (d)

Q.14Find $\lim_{x\to0}\frac{e^{x}-e^{-x}}{x}$

(a) 0 (b)1 (c)2 (d)Does not exist

Ans. (c)

Q.15Find $\lim_{x\to\infty}\frac{2x^2+5x+1}{3x^3-2x+4}$

(a)2 (b)1 (c)0 (d)Does not exist

Ans. (c)

Q.16 Find $\lim_{x\to\infty}\frac{2x^2+5x^2+x+1}{3x^3-4x+5}$

(a)∞ (b)1 (c)0 (d)Does not exist

Q.17`Evaluate $\lim_{x\to4}\frac{x^2+16}{x-4}$

(a)8 (b)0 (c)4 (d)Does not exist

Ans. (b)

Q.18Evaluate $\lim_{x\to2}\frac{x^2+2x+1}{\sqrt{x^2+2}}$

(a) $\frac{7}{6}$ (b) $\frac{7}{\sqrt{6}}$ (c) 1 (d)Does not exist

Ans. (b)

Q.19 Evaluate $\lim_{x\to3}\sqrt[5]{x^2+7x+2}$

(a)5 (b)8 (c)10 (d)2

Ans. (d)

Q.20 Find $\lim_{x\to2}\frac{x^2-4}{4x}$

(a) 0 (b)1 (c)2 (d)Does not exist

Ans. (a)

Q.21Evaluate $\lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$ if $h>0$

(a) $\frac{1}{\sqrt{x}}$ (b) $\frac{1}{2\sqrt{x}}$ (c) $\frac{1}{x}$ (d) Does not exist

Ans. (b)

Q.22Evaluate: $\lim_{x\to 0}\frac{4x+2x^2}{2x-3x^2}$

(a)1 (b)2 (c)3 (d)None of these

Ans. (b)

Q.23If the function $y = f(x) = \frac{x^2-x-12}{x-4}$, then evaluate the limit of y as x→ 4

(a)∞ (b)3 (c)7 (d)Cannot be determined

Ans. (c)

CONTINUITY

In mathematics, the concept of continuity is a fundamental idea, particularly in calculus and real analysis. When we say a function is continuous, we mean that it can be smoothly drawn without any abrupt breaks or interruptions, much like the notion of sketching a curve without having to lift your pencil.

DEFINITION

A function $y = f(x)$ is continuous at $x = a$ if and only if

1. $f(a)$ exist i.e., f is defined at $x = a$
2. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$
3. $\lim_{x\to a} f(x) = f(x)$

Thus, a function is continuous over an interval if a function is continuous at each point of the interval. A function is not continuous at a point 'a' is said to be discontinuous at point 'a'.

PROPERTIES

1. A polynomial function is always continuous.
2. The sum, difference and product of two continuous functions is a continuous function.
3. The quotient of two continuous functions is a continuous function, provided that the denominator is not zero.

Q.24 Check if the function $f(x) = \begin{cases} 2x+3, x \le 0 \\ 3(x+1), x > 0 \end{cases}$ is continuous at $x = 0$

Ans. Given function: $f(x) = \begin{cases} 2x+3, x \le 0 \\ 3(x+1), x > 0 \end{cases}$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2x+3 = 2(0)+3 = 3$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 3(x+1) = 3(0+1) = 3(1) = 3$

$F(0) = 2(0)+3 = 3$

Therefore, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

Hence, the given function is continuous at $x = 0$

Q.25 Is the function $f(x) = \frac{x^2-9}{x-3}$ is continuous at $x = 3$?

Q.26 Find the points of discontinuity, if, any, of the function $f(x) = \frac{x^2-7}{x^3-x^2+x-1}$

Q.27 A function f is defined as $f(x) = \{x+1, if\ -1 \le x \le 0$

$\{x, if\ 0 < x \le 1$

$\{2-x, if\ 1 < x \le 2$

(a)$x = 1$ (b)$x = 0$ (3)Both (a) and (b) (d)None of these

Ans .(a)

Q.28The value of k such that the function is continuous at $x = 2$ is

f(x) = $\begin{cases} Kx^2, if\ x \le 2 \\ 3, if\ x > 2 \end{cases}$

(a)1 (b)3 (c)$\frac{3}{4}$ (d) None of these

Ans. (c)

Q.29 A function f is defined as

$f(x) = \{x+1, if\ -1 \le x \le 0$

$\{x, if\ 0 < x \le 0$ then, the function is discontinuous at

$\{2-x, if\ 1 < x \le 2$

(a)$x = 2$ (b)$x = 1$ (c)$x = 0$ (d) Everywhere

Ans. (c)

Q.29 The value of k such that the function is continuous at $x = 5$ is

$$f(x) = \begin{cases} kx+1, if\ x \le 5 \\ 3x-5, if\ x > 5 \end{cases}$$

(a)5 (b)$\frac{1}{5}$ (c)$\frac{9}{5}$ (d) None of these

Ans. (c)

Q.30 Find the points of discontinuity, if, any, of the function $f(x) = \frac{x^2+2x+5}{x^2-3x+2}$

(a)$x = 1$ (b)$x = 2$ (c)$x = 1\ or\ x = 2$ (d)None of these

Ans. (c)

Q.31The function $f(x)$ is defined by

$f(x) = \{5x-4, if\ 0\ N < x \le 1$

$= 4x^3 - 3x, if\ 1 < x < 2$ then the function is continuous at

(a)x=0 (b)x=1 (c)x=2 (d)None of these

Ans. (b)

Q.32The function $f(x) = \frac{x^2-9}{x-3}$ is to be continuous at $x = 3$. What value must be assigned to $f(3)$, if $f(x)$ is to be continuous at $x = 3$

(a)1 (b)3 (c)6 (d)Cannot be determined

Ans. (c)

Q.33Find $\lim_{x\to 1} f(x)$ *where* $f(x) = \begin{cases} x^2 - 1, x \le 1 \\ -x - 1, x > 1 \end{cases}$

(a)1 (b)0 (c)-2 (d)Does not exist

Ans. (c)

Q.34A function f is defined as

$f(x) = \{-x^2, if\ x \le 0$

$= \{5x-4, if\ 0 < x \le 1$

$= \{4x^2 - 3x, if\ 1 < x \le 2$ Then, the function is discontinuous at

$= \{3x+4, if\ x \ge 2$

(a)x=2 (b)x=1 (c)x=0 (d)None of these

Ans. (c)

Q.35Find the value of a so that the given function is continuous at $x = 2$

$f(x) = \{ax + 5, \text{if } x \le 2$

$\{x - 1, \text{if } x > 2$

(a) 2 (b)-2 (c)1 (d)None of these

Ans. (b)

SUMMARY

LIMIT

The limit of a function represents the value that the function approaches as the input (or variable) gets arbitrarily close to a certain point. This point may be a specific number or infinity. Formally, we say that the limit of a function $f(x)$ as x approaches a certain value c is L, denoted by:

$$\lim_{x \to c} f(x) = L$$

This notation indicates that as x gets arbitrarily close to c, the values of $f(x)$ get arbitrarily close to L. If $f(x)$ approaches a specific number as x approaches c, then that number is the limit. If $f(x)$ becomes arbitrarily large or small without bound, we say the limit is infinity or negative infinity, respectively.

PROPERTIES

1. $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a}(x) \pm \lim_{x \to a} g(x)$
2. $\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x).\ \lim_{x \to a} g(x)$
3. $\lim_{x \to a} f(x)\ \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$
4. $\lim_{x \to a}[cf(x)] = c \lim_{x \to a} f(x)$
5. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$
6. $\lim_{x \to 0^+} \frac{1}{x} = \lim_{h \to 0} \frac{1}{h} \to +\infty (h > 0)$
   $\lim_{x \to 0^+} \frac{1}{x} = \lim_{h \to 0} \frac{1}{h} \to -\infty (h > 0)$
   ∞ is a very-very large number called infinity
   Thus $\lim_{x \to 0^+} \frac{1}{x}$ does not exist.

CONTINUITY

Continuity is a fundamental concept that describes the smooth and unbroken nature of a mathematical object, particularly a function. A function is considered continuous at a point if, intuitively, the values of the function don't exhibit abrupt jumps or breaks as the input approaches that point.

Formally, a function f(x) is continuous at a point c in its domain if three conditions are met:

1. $f(c)$ is defined: The function must be defined at the point c
2. The limit of $f(x)$ as x approaches c exist: The $\lim_{x \to c} f(x)$ must exist.
3. The limit and the function value are equal: The value of the function at c i.e. $f(c)$ must be equal to the limit of the function as x approaches c i.e. $\lim_{x \to c} f(x)$

In mathematical notation, this can be expressed as:

$\lim_{x \to c} f(x) = f(c)$

If these three conditions are satisfied, then the function is said to be continuous at the point c. If a function is continuous at every point in its domain, it is simply referred to as a continuous function.

PROPERTIES

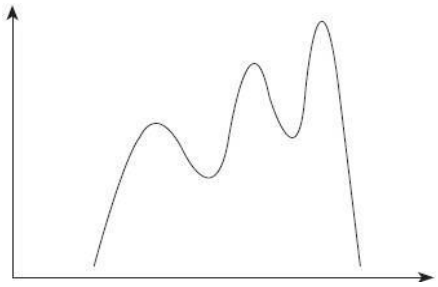
1. A polynomial function is always continuous.
2. The sum, difference and product of two continuous functions is a continuous function.
3. The quotient of two continuous functions is a continuous function, provided that the denominator is not zero.

CHAPTER-8
BASIC APPLICATION OF DIFFERENTIAL AND INTEGRAL CALCULUS IN BUSINESS AND ECONONICS

Unit – 1 Differential Calculus

INTRODUCTION

Differential calculus is a branch of calculus focused on studying the rates of change and derivatives of functions. It involves finding a function's derivative, representing its instantaneous rate of change at any point. The derivative measures how a function changes as the input variable changes, providing information about slope, direction, and concavity. In differentiation, "increment" signifies a small change in the independent variable, while "change" is the corresponding function change. As this change approaches zero, the derivative gives the exact rate of change. It's widely used in math, physics, engineering, economics, and more for function analysis, optimization, differential equations, rate understanding, and real-world modeling. Differential calculus is fundamental for understanding dynamic systems and solving various scientific problems.



To express the rate of change in any function
We introduce concept of derivative

DERIVATIVE OR DIFFERENTIAL COEFFICIENT

The derivative or differential coefficient of a function $f(x)$ is defined as the limit of the ratio of the change (increment) in the function corresponding to a small change (increment) in the independent variable x, as that change approaches zero. It represents the rate at which the function value changes with respect to the independent variable.

Let $y = f(x)$ be a function. If h $(or\ \Delta x)$ be the small increment in x and the corresponding increment in y or $f(x)$ be $\Delta y = f(x + h) - f(x)$, then the derivative of $f(x)$ is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This is denoted as $f'(x), \frac{dy}{dx}\ or\ \frac{df(x)}{dx}$

The derivative of $f(x)$ is also known as the differential coefficient of $f(x)$ with respect to x. This process of differentiation is called the first principle (or abinitio) (Ab-initio).

A function $f(x)$ is said to be differentiable at a specific point x = c if the derivative $f'(c)$ exists. This means that the function has a well-defined rate of change at that point.

i.e., $\lim_{h \to c} \frac{f(x)-f(c)}{x-c}$ exist which is denoted by $f(c)$ or $\left[\frac{dy}{dx}\right]_{x=c}$

FEW IMPORTANT FUNCTIONS AND THEIR DERIVATIVES

Function	Derivative of the Function
$f(x)$	$f'(x)$
x^n	nx^{n-1}
e^x	e^x
$log_e x$	$\frac{1}{x}$
a^x	$a^x.(\log a)$
$c\ (a\ constant)$	0
$cf(x)$	$cf'(x)$

BASIC LAWS FOR DIFFERENTION

- $h(x) = cf(x)$ where c is any real constant (Scalar multiple of a function)

Derivative of the function: $h'(x) = cf'(x)$

E.g.: $if\ h(x) = 3x^2$

then $h'(x) = \frac{d}{dx}(3x^2) = 3\frac{d}{dx}(x^2) = 3(2x^{2-1}) = 6x$

- $h(x) = f(x) \pm g(x)$ (Sum/Difference of function)

Derivative of the function: $h'(x) = f'(x) \pm g'(x)$

E.g.: $h(x = 2x + 5x^3)$

then $h'(x) = 2 + 5(5x^{3-1}) = 2 + 15x^2$

- Derivative of Product of functions: (Product Rule): $h(x) = f(x)g(x)$

If two function f and g are differentiable then

$$\frac{d}{dx}(f(x).g(x)) = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

$$\Rightarrow h'(x) = f(x)g'(x) + f'(x)g(x)$$

That is, the derivative of the product of two functions is the sum of the first function multiplied by the derivative of the second and the second function multiplied by the derivative of the first function.

E.g.: $h(x = e^x x^2)$

Then $h'(x) = e^x\frac{d(x^2)}{dx} + x^2\frac{d\ (e^x)}{dx}$

$h'(x) = e^x(2x) + e^x x^2$

$h'(x) = 2xe^x + e^x x^2$

$h'(x) = xe^x(2 + x)$

- Quotient Rule: $h(x) = \frac{f(x)}{g(x)}$

If $f(x)$ and $g(x)$ are two differentiable functions then

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$\Rightarrow h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

E.g.: $h(x) = \frac{x^2}{e^x}$

Then $h'(x) = \frac{e^x(2x) - x^2e^x}{(e^x)^2} = \frac{xe^x(2-x)}{(e^2)^2} = \frac{x(2-x)}{e^x}$

Q.1 Differentiate $x^{\frac{3}{2}}$ with respect to x

(a)$x^{\frac{3}{2}}$ (b)$\frac{3}{2}\sqrt{x}$ (c)$\frac{3}{2}x$ (d)None of these

Ans. (b)

Q.2 If $y = 4x^3 - 7x^4$, then $\frac{dy}{dx}$ is

(a)$2x(-14x^2 + 6x)$ (b)$2x(14x^2 - 6x)$

(c)$2x(14x^2 + 6x)$ (d)None of these

Ans. (a)

Q.3Differentiate each of the following function with respect to x and choose the correct option:

I.$5x^2 + 2x$

(a)$5x + 2$ (b)$10x + 2$ (c)$10x$ (d)$5x$

Ans. (b)

II.$xa^x + x^a + a^a$

(a)$a^x \log a + ax^{a-1}$ (b)$a^x + ax^{a-1}$

(c)$a^x \log a + x^{a-1}$ (d)$a^x + x^{a-1}$

Ans.(a)

III.$\frac{2}{3}x^3 - 7x^2 + 3x - 5logx + 3$

(a)$2x^2 - 14x + 3 - \frac{1}{x}$ (b)$6x^2 - 7x + 3 - \frac{5}{x}$

(c)$2x^2 - 14x + 3 - \frac{5}{x}$ (d)$6x^2 - 14x + 3 - \frac{5}{x}$

Ans. (c)

IV. $e^x \log x$

(a)$e^x(1 + x \log x)$ (b)$\frac{e^x}{x}(1 + \log x)$

(c)$e^x(1 + \log x)$ (d)$\frac{e^x}{x}(1 + x \log x)$

Ans. (d)Product rule of derivative is:

$\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v\frac{du}{dx}$

Using this, we get:

$\frac{d}{dx}(e^x \log x) = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^x)$

$\Rightarrow \frac{e^x}{x} + e^x \log x$

$\Rightarrow \frac{e^x}{x}(1 + xlog\, x)$

Hence, option (d) is correct.

V.$y \frac{e^x}{e^x - 1}$

(a)$\frac{-2e^x}{(e^x-1)^2}$ (b)$\frac{2e^x}{(e^x-1)^2}$

(c)$\frac{-2}{(e^x-1)^2}$ (d)None of these

Ans. (a)

Q.4If $f(x) = \frac{x^2}{e^x}$, then $f'(1)$ is equal to

(a)$-1/e$ (b)$\frac{1}{e}$ (c) e (d)None of these

Ans. (b)

Q.5Differentiate each of the following function with respect to x:

(i)$3^x x^4$ Ans. $(x^3 3^x\ (4 + x log 3)$

(ii)$\frac{x}{\log x}$ Ans. $\frac{2x}{\log x} - \frac{x}{(log x)^2}$

(iii)$\frac{e^x}{x}$ Ans. $\frac{e^x(x-2)}{x^2}$

(iv)$\frac{4x}{3x^2+5}$ Ans. $\frac{20-12x^2}{(3x^2+5)^2}$

(v) $\frac{x^3+1}{x^3}$ Ans. $2x - \frac{1}{x^2}$

(vi)$(x^2 - 3x + 7)(2x^2 + 5x + 9)$ Ans. $8x^3 - 3x^2 + 16x + 8$

DERIVATIVE OF A FUNCTION OF FUNCTION

If $y = f[h(x)]$, where $u = h(x)$, then $\frac{dy}{dx}$=?

If we have a function $y = f[h(x)]$, where $u = h(x)$, then the derivative $\frac{dy}{dx}$ can be calculated using the chain rule of differentiation.

The chain rule states that if $y = f(u)$, where u=h(x), then the derivative $\frac{dy}{dx}$ is given by: $\frac{dy}{dx} = \frac{dx}{du} \cdot \frac{du}{dx}$

In this case, $u = h(x)$, so we can rewrite the equation as:

$y = f(u) = f[h(x)]$

Now, let's differentiate both sides of the equation with respect to x:

$\frac{dy}{dx} = \frac{d}{dx}[f(h(x)]$

Using the chain rule, we have:

$\frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

Since $u = h(x)$, the derivative $\frac{dy}{dx}$ is simply the derivative of $h(x)$ with respect to x.

Now, the derivative $\frac{df}{dx}$ represents the derivative of the function f with respect to its argument u.

Thus, $\frac{dy}{dx} = f'(u) \times h'(x)$

Q.6Differentiate $log\ (2x + 3x^2)$ w.r.t.x.

(a) $\frac{1}{2x+3x^2}$ (b)$\frac{2+6x}{2x+3x^2}$ (c)$\frac{6x}{2x+3x^2}$ (d)$\frac{2+3x}{2x+3x^2}$

Ans. (b)

Q.7Differentiate e^{3x+8} $w.r.t.x$

(a)$3e^{3x+8}$ (b)e^{3x+8} (c)$(3x + 8)\ e^{3x+8}$ (d)$8\ e^{3x+8}$

Ans. (a)

Q.8If $y = e^{\sqrt{2x}}$, then $\frac{dy}{dx}$ is equal to _____

(a)$\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$ (b)$e^{\sqrt{2x}}$ (c)$e^{\frac{\sqrt{2x}}{2\sqrt{x}}}$ (d)None of these

Ans. (a)

Q.9If $y = \sqrt{x^2 + m^2}$, then $y \times y_1$ $(where\ y_1 = \frac{dy}{dx})$ is equal to

(a)$-x$ (b)x (c)$\frac{1}{x}$ (d)None of these

Ans. (b)

Q.10Differentiate $(5x + 4)^5$ with respet to x

(a)$25\ (5x + 4)^5$ (b)$25\ (5x + 4)^4$ (c)$5\ (5x + 4)^4$ (d)None of these

Ans. (b)

Q.11 If $y = log(5x^2 + 7)$, find $\frac{dy}{dx}$.

(a) $\frac{1}{5x^2+7}$ (b) $\frac{5}{5x^2+7}$ (c) $\frac{10x+7}{5x^2+7}$ (d) $\frac{10x}{5x^2+7}$

Ans. (d)

Q.12 Find the derivative of $(\log x)^3$

(a) $3 \log x^2$ (b) $\frac{3(logx)^2}{x}$ (c) $\frac{1}{x^3}$ (d) None of these

Ans. (b)

IMPLICIT FUNCTION

An implicit function is a function that is defined implicitly by an equation in the form $f(x, y) = 0$, where y cannot be directly expressed as a function of x.

For example: $x^2y^2 + 7xy + x + y$ where y cannot be directly defined as a function of x is called an implicit function of x.

Q.13 Find $\frac{dy}{dx}$ when, $x^3 + y^3 = xy$.

(a) $\frac{xy-3x^2}{3y^2-x}$ (b) $\frac{y-3x^2}{3y^2-x}$ (c) $\frac{y-3x^2}{3y^2-x^2}$ (d) None of these

Ans. (b)

Q. 14 If $x^5 + y^5 - 5xy = 0$, then $\frac{dy}{dx}$ is

(a) $\frac{y+x^4}{y+x}$ (b) $\frac{y-x^4}{y^4-x}$ (c) $\frac{x-y^4}{x-y}$ (d) $\frac{x+y^4}{x^4+y}$

Ans. (b)

Q.15 If $xy = 1$ then $y^2 + \frac{dy}{dx}$ is equal to

(a) 1 (b) 0 (c) -1 (d) None of these

Ans. (b)

Q.16 Find the derivative of the function: $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

(a) $\frac{y^{\frac{2}{3}}}{y^{\frac{2}{3}}}$ (b) $\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$ (c) $-\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}$ (d) None of these

Ans. (c)

Q.17 Let $xy = 1$, then $y^2 + \frac{dy}{dx}$=?

(a) 1 (b) 0 (c) 2 (d) None of these

Ans. (b)

Q.18 If $x^3 - 2x^2y^2 + 5x + y = 5$, then $\frac{dy}{dx}$ at $x = 1$ and $y = 1$ is

(a) $\frac{4}{3}$ (b) $-\frac{5}{4}$ (c) $\frac{4}{5}$ (d) $-\frac{4}{3}$

Ans. (a)

PARAMETRIC EQUATION

Parametric equations are a way of representing a curve or a set of points in terms of a parameter, usually denoted as "t" Instead of expressing the variables x and y directly in terms of each other, they are expressed individually as functions of the parameter.

For parametric equations $x = f(t)$ and $y = h(t)$, where $f(t)$and $h(t)$ are functions of the parameter t, the derivative $\frac{dy}{dx}$ can be obtained using the chain rule of differentiation.

Applying the chain rule to the parametric equation $x = f(t)$ and $y = h(t)$, we can find the derivative $\frac{dy}{dx}$ as follows:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt}.\frac{dt}{dx}$$

Q.19 If $x = ct$ and $y = \frac{c}{t}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{t}$ (b) $t.e^t$ (c) $-\frac{1}{t^2}$ (d) None of these

Ans. (c)

Q.20 Given $x = 2t + 4, y = t^2 - 2$, $\frac{dy}{dx}$ is calculated as

(a) t (b) $\frac{-1}{t}$ (c) $\frac{1}{t}$ (d) None of these

Ans. (a)

Q.21 If $x = at^2, y = 2$ at then $\left[\frac{dy}{dx}\right]_{t=2}$ is equal to

(a) $\frac{1}{2}$ (b) -2 (c) $-\frac{1}{2}$ (d) None of these

Ans. (a)

Q.22 Given $x = t + t^{-1}$ then the value of $\frac{dy}{dx}$ at $t = 2$ is

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{5}{3}$ (d) None of these

Ans. (c)

Q.23 Let $x = at^3, y = \frac{a}{t^2}$, then $\frac{dy}{dx}$ is

(a) $\frac{-3a}{t^6}$ (b) $= \frac{1}{t^6}$ (c) $\frac{1}{3at^2}$ (d) None of these

Ans. (a)

Q.24 If $x = t^2$ and $y = t^3$, then $\frac{dy}{dx} = ?$

(a) $\frac{3t}{2}$ (b) $\frac{2}{3t}$ (c) t (d) None of these

Ans. (a)

Q.25 If $x = 2t + 5$ and $y = t^2 - 5$, then $\frac{dy}{dx} = ?$

(a) t (b) $-\frac{1}{t}$ (c) $\frac{1}{t}$ (d) 0

Ans. (a)

LOGARITHMIC DIFFERENTIATION

The process of finding a derivative by taking logarithm in the first instance is called logarithmic differentiation. The procedure is convenient to adopt when the function to be differentiated involves a function in its power or when the function is the product of the number of functions.

Q.26 Differentiate x^x $w.r.t.x$

(a) $x^x(1 + \log x)$ (b) $x^x \log x$

(c) $x^x(1 - \log x)$ (d) None of these

Ans. (a) Let $y = x^x$

Taking logarithm on both sides, we get $log\ y = x\ log\ x$

Differentiating with respect to x

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x.\frac{1}{x} + log.(1)$

$\Rightarrow \frac{dy}{dx} = y(1 + \log x)$

$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$

Hence, the correct option is (a)

Q.27If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

(a) $\frac{\log x}{(1-\log x)^2}$ (b) $\frac{\log x}{(1+\log x)^2}$ (c) $\frac{\log x}{(1-\log x)}$ (d) $\frac{\log x}{(1+\log x)}$

Ans. (b)

Q.28 If $x^y . y^x = M$, where M is constant then $\frac{dy}{dx}$ is equal to

(a) $\frac{-y}{x}$ (b) $\frac{-y(y+x\log y)}{x(y\log x+x)}$ (c) $\frac{y+x\log y}{y\log x+x}$ (d)None of these

Ans. (b)

Q.29 Differentiate the following $w.r.t\ x$:

If $\log(\frac{x}{y}) = x + y$, $\frac{dy}{dx}$ is

(a) $\frac{y(1-x)}{x(1+y)}$ (b) $\frac{y}{x}$ (c) $\frac{1-x}{1+y}$ (d) None of these

Ans. (a)

Q.30 If $x^m . y^n = (x+y)^{m+n}$, then the value of $\frac{dy}{dx}$ is

(a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $\frac{1}{xy}$ (d) None of these

Ans .(b)

Q.31 If $y = x^x$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(a)0 (b)1 (c)-1 (d)2

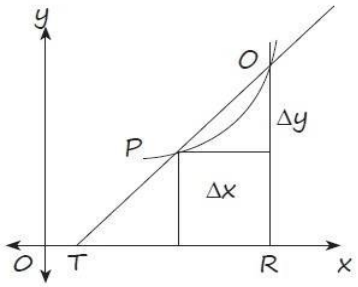
Ans. (b)

Q.32 If $x^y = y^x$ then $\frac{dy}{dx}$ will be equal to

(a) $\frac{x(x-y\log x)}{y(y-x\log x)}$ (b) $\frac{y(y-x\log y)}{x(x-y\log x)}$ (c) $\frac{y-x\log y}{x-y\log x}$ (d) None of these

Ans. (b)

GEOMETRICAL INTERPRETATION OF THE DERIVATIVE



Geometrically, the derivative of a function can be interpreted as the slope of the tangent line to the graph of the function at a given point. The tangent line touches the graph at that point and represents the best linear approximation to the function's behavior near that point.

Consider a function $f(x)$ and two points on its graph, $P\ (x, f(x))$ and $Q\ (x + \Delta x, f(x + \Delta x))$. The secant line passing through these two points represents the average rate of change of the function over the interval Δx.

The slope of the secant line is given by the formula:

$$m = \frac{\Delta y}{\Delta x} = \frac{(f(x + \Delta x) - f(x))}{\Delta x}$$

As Δx approaches zero, the secant line becomes closer and closer to the tangent line to the graph at the point $(x, f(x))$. The derivative of the function at that point is defined as the limit of the secant line slope as Δx approaches zero.

Mathematically, the derivative is given by:

$$f'(x) = \lim_{\Delta x \to 0} \frac{(f(x + \Delta x) - f(x))}{\Delta x}$$

The derivative represents the instantaneous rate of change of the function at the point (x, f(x)). It describes the slope of the tangent line at that point, which provides information about the function's behavior and how it is changing locally.

Q.33 The gradient of the curve $y = 2x^3 - 5x^2 - 3x \ at \ x = 0$ is

(a)3 (b)-3 (c)$\frac{1}{3}$ (d) None of these

Ans. (b)

Q.34 The gradient of the curve $y - xy + 2px + 3qy = 0$at the point (3, 2) is $\frac{-2}{3}$.The value of p and q are

(a) $(\frac{1}{-2}, \frac{1}{-2})$ (b)(2, 2) (c)$(\frac{-1}{2}, \frac{-1}{2})$ (d)$(\frac{1}{2}, \frac{1}{6})$

Ans. (d)

Q.35 The gradient of the curve $y = 2x^3 - 3x^2 - 12x + 8 \ at \ x = 0$ is

(a)-12 (b)12 (c)0 (d)None of these

Ans. (a)

Q.36 The slope of the tangent to the curve $y = \sqrt{4 - x^2}$ at the point, where the ordinate and the obscissa are equal is

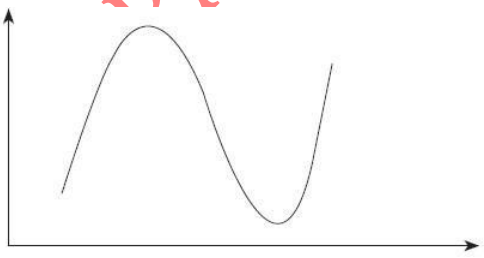
(a)-1 (b)1 (c)0 (d)None of these

Ans. (a)

CONCEPT OF MAXIMUM AND MINIMUM IN DIFFERENTIATION

How to Find Maximum and Minimum Points Using Differentiation?

The value of the function at a maximum point is called the maximum value of the function and the value of the function at a minimum point is called the minimum value of the function



How to find the maximum or minimum of a curve?

Steps for finding maximum or minimum:

1. Differentiate the given function $f(x)$ with respect to x to find its derivative $f'(x)$.
2. Set the derivative $f'(x)$ equal to zero and solve for x to find the critical points. These are the values of x where the slope of the curve is zero or undefined.
3. Find the second derivative $f''(x)$ by differentiating the derivative $f'(x)$ obtained in step 1.
4. Evaluate the second derivative $f''(x)$ at each critical point found in step 2.

- If $f''(x) < 0$ at a critical point, then the function has a local maximum at that point.
- If $f''(x) > 0$ at a critical point, then the function has a local minimum at that point.

- If $f''(x) = 0$ at a critical point, further investigation is needed to determine the nature of the point (it could be a point of inflection or a saddle point).

5. To find the maximum and minimum values of the function, substitute the x-values of the critical points (including any endpoints of the domain) into the original function $f(x)$ and compare the corresponding function values.

- The largest function value obtained is the maximum value of the function.
- The smallest function value obtained is the minimum value of the function.

Q.37 The maxima and minima of the function $y = 2x^3 - 15x^2 + 36x + 10$ occurs respectively at

(a)$x = 2$ and $x = 3$ (b)$x = 1$ and $x = 3$

(c)$x = 3$ and $x = 2$ (d)$x = 3$ and $x = 1$

Ans. (a)

Q.38 The minima of the function $y = x^5 - 5x^4 + 5x^3 - 1$ occur at

(a)$x = 1$ (b)$x = 3$ (c)$x = 5$ (d)None of these

Ans. (b)

Q.39 The maxima of the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 1$ is at

(a)$x = 1$ (b)$x = 3$ (c)$x = 5$ (d)None of these

Ans. (c)

Q.40 The maxima and minima of the function $f(x) = 2x^3 - 15x^2 - 36x + 18$ occur respectively at

(a)$x = 2$ and $x = 3$ (b)$x = 2$ and $x = -3$

(c)$x = -2$ and $x = -3$ (d) None of these

Ans. (a)

APPLICATION OF DIFFERENTIAL CALCULUS

Differentiation is a powerful mathematical tool that is applied in various scenarios to analyze and calculate important quantities related to rates of change and cost functions.

For example:

- Rate of change in price
- Change in demand with respect to change in output
- Change in revenue obtained with respect to change in price
- Change in demand with respect to change in income

Few things to be taken care of:

- Rate of Change in Quantities: Let there two quantities x and y such that rate of change in y is dependent of x then $\frac{dy}{dx}$ represents the rate of change of y with respect to x.
- Always remember that, "rate of Change of a function" means the derivative of the function.
- We can also write $f(x)$ in place of y and similarly $f'(x)$ in place of $\frac{dy}{dx}$

Cost Function: Total cost consists of two parts (i) Variable Cost (ii) Fixed Cost. If $C(X)$ denotes the cost producing x units of a product then $C(x) = V(x) + F(x)$, where $V(x)$ denotes the variable cost and $F(x)$ is the fixed cost. Variable cost depends upon the number of units produced (i.e value of x) whereas fixed cost is independent of the level of output

The formulas for average cost (AC or C), average variable cost (AVC), and average fixed cost (AFC) are as follows:

- Average cost (AC or C) $= \frac{C(X)}{X}$

- Average variable cost (AVC) $= \frac{V(X)}{X}$
- Average Fixed Cost (AFC) $= \frac{F(X)}{X}$

Marginal Cost: If C(x) the total cost producing x units then the increase in cost in producing one more unit is called marginal cost at an output level of x units and is given as Marginal Cost (MC).

- Marginal Cost (MC) = Rate of change in cost C per unit in output at an output level of x units = $\frac{dC}{dx}$
- To increase profits, a company may decide to increase its production. The question that concerns the management is how will the cost be affected by an increase in production. Economists use the marginal cost to answer the question.

REVENUE FUNCTION

The revenue function, $R(x)$ represents the total amount of money obtained (total turnover) by selling x units of a product. If x units are sold at a price of 'P' per unit, then the revenue function is given by: $R(x) = P.x$

MARGINAL REVENUE

The marginal revenue (MR) is the rate of change of revenue with respect to a unit change in output. If R is the revenue and x is the output, then the marginal revenue is given by:

$MR = \frac{dR}{dx}$

PROFIT FUNCTION

The profit function, $P(x)$, represents the difference between the total revenue, $R(x)$, and the total cost, $C(x)$. It is expressed as:

$P(x) = R(x) - C(x)$

MARGINAL PROFIT

The marginal profit is the rate of change of profit with respect to a unit change in output. It is given by:

Marginal Profit $= \frac{dP}{dx}$

Q.41 Find the marginal cost function if the total cost function to produce x units is defined by:

$C(x) = \frac{3x+2}{x+4}$

(a) $\frac{2x}{(x+4)^2}$ (b) $\frac{10}{(x+4)^2}$ (c) $\frac{10}{(x+4)}$ (d) None of these

Ans. (b)

Q.42 The cost function for the production of x units of a commodity is given by $C(x) = x^2 - 24x + 600$. The cost will be minimum when x will be

(a)12 (b)14 (c)15 (d)20

Ans. (a)

Q.43 If the cost function is given by $C(x) = 15x + 0.055x^2$ and the demand function is given by $p(x) = 100 - 0.055x$, where x is the number of units produced, then the marginal profit function when $x = 50$ is

(a)85 (b)11 (c)74 (d)25

Ans. (c)

Q.44 The cost function of a company $C(x) = 100x - 2x^2 + 120, 0 \leq x \leq 100$ and the revenue function $R(x) = 500 - x$, then the profit function is

(a) $2x^2 - 101x + 380$ (b) $2x^2 - 101x + 620$

(c) $x^2 - 101x + 380$ (d)None of these

Ans. (a)

Q.45 A computer software company wishes to start the production of floppy disks. It was observed that the company had to spend ₹2 lakhs for the technical informations. The cost of setting up the machine is ₹88,000 and the cost of producing each unit is ₹30, while each floppy could be sold at ₹45. Find:

(i)the total cost function for producing x floppies; and

(ii) the break-even point.

(a)$30x$ *and* 19,200 (b)$30x + 88{,}000$ *and* 19,200

(c)$30x + 2{,}88{,}000$ *and* 19,200 (d)None of these

Ans. (c)

Q.46 If the demand function of a product is given by P=200-5x, where p is the price per unit and x is the quantity demanded, then the marginal revenue function is given by

(a)$200x - 5x^2$ (b)$200 - 10x$ (c)$-10x$ (d)None of these

Ans. (b)

Q.47 If the cost function is given by $C(x) = (x^2 + 2)^3$, where x denotes the number of items produced, then the marginal average function at $x = 10$ is

(a)51626 (b)52726 (c)62726 (d)52420

Ans. (b)

Q.48 The cost function for the production of x units of a commodity is given by $C(x) = 2x^3 - 15x^2 + 36 + 15$. The cost will be minimum when 'x' equal to

(a)3 (b)2 (c)1 (d)4

Ans. (a)

Q.49 The cost function of production is given by $C(x) = 100x - 8x^2 + \frac{x^3}{3}$ where x denotes the number of items produced. The level of output for which marginal cost is minimum and level of output for which the average cost is minimum are given by, respectively.

(a)12 *and* 14 (b)8 *and* 12 (c)10 *and* 15 (d)None of these

Ans. (b)

Q.50 If the cost of producing x units is given by the function $C(x) = x^3 + 10x^2 - 30x + 50$, then the average cost function is

(a)$x^3 + 10x^2 - 30x + 50$ (b)$x^3 + 10x^2 - 30x$

(c)$3x^2 + 10x^2 - 30$ (d)$x^2 + 10x - 30x + \frac{50}{x}$

Ans. (d)

Q.51 The cost function of a company $C(x) = 100x - 2x^2 + 120.0 \le x \le 100$ and the revenue function $R(x) = 500 - x$, then the profit function is

(a)$2x^2 - 101x + 380$ (b)$2x^2 - 100x + 620$

(c)$x^2 + 101x + 380$ (d)None of these

Ans. (a)

Q.52 If the cost function of a firm is given by $C(x) = 3x^2 + 5x + 2$, where is the output. Find the average cost and the marginal cost.

(a)$3x + \frac{2}{x}$ *and* $6x + 5$ (b)$3x + 5 + \frac{2}{x}$ *and* $6x + 5$

(c)$3x^3 + 5x^2 + 2x$ *and* $6x^2 + 5$ (d)None of these

Ans. (b)

Q.53 If the demand function is $p(x) = 50 - 2x$, then the marginal revenue function is given by

(a)$50 - 4x$ (b)$50 - 2x$

(c) $50x - 4x^2$ (d)None of these

Ans. (a)

Q.54 The total cost function of a firm is: $C(x) = 0.0005x^3 - 0.03x^2 + 6x + 2000$, where x is the output. The marginal average cost will be

(a)$0.001x - 0.03 - \frac{2000}{x^2}$ (c)$0.0015x^2 - 0.03x$

(c)$0.001x - 0.03 + 6 - \frac{2000}{x^2}$ (d) None of these

Ans. (a)

Q.55 The profit gained by selling x units of an item is given by $P(x) = \frac{x^2}{x-1}$. Find the marginal profit if 10 units are sold.

(a)0.987 (b)20.87 (c)0.5 (d)10.5

Ans. (a)

Q.56 The cost function is given by $C(x) = 0.00002x^3 + 300x + 78000$ and the demand function is $p(x) = 500 - 0.05x$. Then the marginal profit at $x = 2500$ is

(a)-275 (b)-375 (c)575 (d)None of these

Ans. (a)

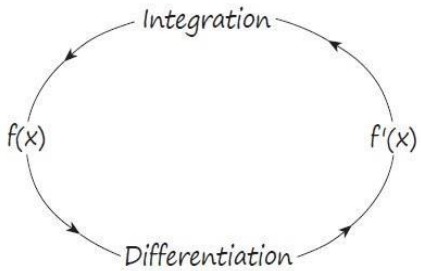
Q.57 The cost of production is given by $C(x) = \frac{x^3}{3} - 15x^2 + 36x$ where x denotes the number of items produced. The level of output for which marginal cost is minimum and level of output for which the average cost is minimum are given by, respectively.

(a)10 *and* 15 (d)10 *and* 12 (c)12 *and* 15 (d)15 *and* 10

Ans. (a)

What is Integration and why do we need to study?

The below figure represents the relation between Integratio and Differentiation.



We can say that integration is the inverse process of differentiation which is denoted with the symbol $\int$.

So, if we have to find integration of x^n

Then, integral of x^n with respect to x is equal to $\frac{x^{n+1}}{n+1}$

Thus, if we differentiate $\frac{x^{n+1}}{n+1}$ we will get back x^n

Again if we differentiate $\frac{x^{n+1}}{n+1} + C$, where C being a constant then we get back the same x^n

i.e., $\frac{d}{dx}\left[\frac{x^{n+1}}{n+1} + C\right] = x^n$

Hence, $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, where C is constant of integration.

Remember; always add constant c in an integration result.

Integral calculus was primarily invented to determine the area bounded by the curves dividing the entire area into an infinite number of infinitesimal small areas and taking the sum of all these small areas.

BASIC FORMULAE FOR INTEGRATION

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; n \neq 1$
2. $\int dx = x, since \int 1dx = \int x^0 dx = \frac{x^1}{1} = 15. \int \frac{dx}{x} = log_e x + c$
3. $\int e^x dx = e^x + c$
4. $\int e^{ax} dx = \frac{e^{ax}}{a} + c$
6. $\int a^x dx = \frac{a^x}{log_e a} + c$

ELEMENTARY RULE

$\int cf(x)dx = c\int f(x)dx$, where c is constrant

$\int\{f(x)dx \pm g(x\}dx = \int f(x)dx \pm \int g(x)dx$

Q.1 Evaluate $\int 5x^2 dx$

(a) $\frac{5}{3x^2} + k$ (b) $\frac{5x^3}{3} + k$ (c) $5x^3$ (d) None of these

Ans. (b) To evaluate the integral $\int 5x^2 dx$

We know that, $\int x^n dx = \frac{x^{n+1}}{n+1} + k$, where k is the constant of integration.

Thus, $\int 5x^2 dx = \frac{5x^3}{3} + k$

Hence, option (b) is correct.

Q.2 Evaluate $\int (x^2 - 1)dx$

(a)$\frac{x^5}{5}-\frac{2}{3}x^3+x+k$ (b)$\frac{x^3}{3}-x+k$

(c)$2x$ (d)None of these

Ans. (b) Given, $\int(x^2-1)dx$

As we know, if $f(x)=x^n$

$\Rightarrow \int f(x)dx=\frac{x^{n+1}}{n+1}+C$ [C→Arbitrary Constant]

$\Rightarrow \int(x^2-1)dx=\int x^2dx-1\int dx$

$=\frac{x^{n+1}}{2+1}-x+k$

$=\frac{x^3}{3}-x+k$ [K→Arbitrary Constant]

Hence, the correct option is (b) i.e., $\frac{x^3}{3}-x+k$.

Q.3Find the following integrals

1.$\int\sqrt{x}\,dx$

(a)$\frac{2}{3}x^{3/2}+k$ (b)$\frac{3}{2}x^{3/2}+k$

(c)$x^{3./2}+k$ (d)None of these

Ans. (a)

ii.$\int\frac{x\sqrt{x}}{2}$

(a)$\frac{2}{5}x^{\frac{5}{2}}+5$ (b)$\sqrt{2x}+k$

(c)$\frac{1}{3\sqrt{x}}+k$ (d)$\frac{1}{5}x^{\frac{5}{2}}+k$

Ans. (d)

iii.$\int[\sqrt{x}-\frac{1}{\sqrt{x}}]dx$

(a)$\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+k$ (b)$\frac{2}{3}\sqrt{x}-2\sqrt{x}+k$

(c)$\frac{1}{2\sqrt{x}}+\frac{1}{2x\sqrt{x}}+1$ (d)None of these

Ans. (a)

Q.4 $\int(1-3x)(1+x)dx$ is equal to

(a)$x-x^2-x^3$ (b)x^3-x^2-x

(c)$x-x^2-x^3+k$ (d)None of these.

Ans. (c)

Q.5Evaluate the following integral:

I.$\int(x+\frac{1}{x})^2dx$ Ans. $\frac{x^3}{3}+2x-\frac{1}{x}+c$

II.$\int\frac{1}{\sqrt{x}}dx$ Ans. $2\sqrt{x}+c$

III.$\int e^{3x}+e^{-3x}dx$ Ans. $\frac{e_{3x}}{3}-\frac{e_{-3x}}{3}+c$

IV. $\int 3^x dx$ Ans. $\frac{3^x}{\log_e 3}+c$

METHOD OF SUBSTITUTION (CHANGE OF VARIABLE)

It is sometimes possible by a change of independent variables to transform a function into another which can be reality integrated.

Q.6Find the value of $\int(4x\quad 5)^6dx$ is equal to

(a)$\frac{1}{7}(4x+5)^7+c$ (b)$\frac{1}{28}(4x+5)^7+c$

(c)$\frac{1}{4}(4x+5)^7+c$ (d)None of these

Ans. (b)

Q.7 Integrate w.r.t x, $(x^3+2)^2\ 3x^2$

(a)$\frac{1}{3}(x^3+2)^3+k$ (b)$3(x^3+2)^3+k$

(c)$3x^2(x^3+2)^3+k$ (d)$9x^2(x^3+2)^3+k$

Ans. (a)

Q.8 Evaluate $\int \frac{(logx)^2}{3}dx$

(a)$\frac{(logx)^3}{x^2}+c$ (b)$\frac{(logx)^3}{3}+c$

(c)$\frac{\log x}{x}+c$ (d)None of these

Ans. (b)

Q.9Find the following integral: $\int \frac{8x^2}{(x^3+2)^3}dx$

(a)$\frac{-4}{3}(x^3+2)^2+k$ (b)$\frac{-4}{3(x^3+2)^2}$

(c)$\frac{4}{3(x^3+2)^2}+k$ (c)None of these

Ans. (b)

INTEGRATION BY PART

Integral of product of two function:

= First function × Integral of second function

$-\int$(differentiation of first fuction × Integral of second function)dx

$$\int f(x).g(x)dx = f(x)g(x) - \int [\frac{df(x)}{dx} \times \int g(x)dx]dx$$

LAE method:

'L' stands for Logarithm function ($log\ x, \log x^3\ etc.$)

'A' stands for Algebraic function ($x^3, \sqrt{x}, x^5\ etc.$)

'E' stands for Exponential funtion ($e^x, e^{\sqrt{x}}, x^5\ etc.$)

Q.10 I.$\int xe^x dx$

(a)e^x+1+k (b)xe^x+e^x+k

(c)xe^x-e^x+k (d)None of these

Ans. (c)

II.$\int xlog\ xdx$

(a)$\frac{x^2}{2}+\log x+k$ (b)$\frac{x^2}{2}\log x-\frac{x^2}{4}+k$

(c)$\frac{x^2}{2}logx-\frac{x}{4}+k$ (c) None of these

III.$\int x^2e^{ax}$

(a)$\frac{xe^{ax}}{a}-\frac{2xe^{ax}}{a}+k$ (b)$\frac{xe^{ax}}{a}-\frac{2xe^{ax}}{a}+k$

(c)$x^2e^{ax}-2xe^{ax}+2e^{ax}+k$ (d)$\frac{x^2e^{ax}}{a}-\frac{2xe^{ax}}{a^2}+\frac{2}{a^3}e^{ak}+k$

Ans.(d)

IV.$\int(\log x)^2dx$

(a)$\int(\log x)^2-2x\log x+2x+k$ (b)$x\ (\log x)^2-2x+k$

(c)$2x\log x-2x+k$ (d)None of these

Ans. (a)

Q.11Evaluate $\int x^2 e^x dx$

(a)$[\frac{x^3}{3}]\, e^x + c$ (b)$(x^2+2)e^x + c$

(c)$(x^2 - 2x + 2)e^x + c$ (d)None of these

Ans.(c)

Q.12 Evaluate $\int x^3\,(logx)dx$

(a)$x^4\,(\frac{1}{4}\log x - \frac{1}{16}) + c$ (b)$x^4\,(\frac{1}{4}\log x + \frac{1}{16}) + c$

(c)$x^2\,(\frac{1}{4}logx) + c$ (d)None of these

Ans.(a)

Q.13 Evaluate $\int(1+5x)e^{2x}\,dx$

(a)$\frac{1}{2}e^{2x} + \frac{1}{4}e^{2x} + c$ (b)$\frac{5}{2}x^2 e^{2x} + c$

(c)$\frac{5}{2}xe^{2x} - \frac{3}{4}e^{2x} + c$ (d)None of these

Ans.(c)

SOME MORE IMPORTANT FORMULAE

1. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$
2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$
3. $\int \frac{dx}{\sqrt{x^2+a^2}} = log|x + \sqrt{x^2+a^2}| + c$
4. $\int \frac{dx}{\sqrt{x^2-a^2}} = log|x + \sqrt{x^2-a^2}| + c$
5. $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$
6. $\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x + \sqrt{x^2+a^2}) + c$
7. $\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x + \sqrt{x^2-a^2}) + c$
8. $\int \frac{f'(x)}{f(x)}dx = \log[f(x)] + c$

Q.14 Evaluate $\int \frac{1}{x^2-9}dx$

(a)$\frac{1}{9}log|\frac{x-3}{x+3}| + c$ (b)$\frac{1}{3}log|\frac{x+3}{x-3}| + c$

(c)$\frac{1}{6}log|\frac{x-3}{x+3}| + c$ (d)None of these

Ans. (c)

Q.15Evaluate $\int \frac{1}{\sqrt{x^2-25}}dx$

(a)$log|x + \sqrt{x^2-25}| + c$ (b)$\frac{1}{5}log|x + \sqrt{x^2-25}| + c$

(c)$\frac{1}{25}log|x + \sqrt{x^2-25}| + c$ (d)None of these

Ans.(a)

Q.16 Evaluate $\int \frac{e^x}{e^{2x}-4}dx$

(a)$\frac{1}{2}log|\frac{e^x+2}{e^x-2}| + c$ (a)$\frac{1}{2}log|\frac{e^x+4}{e^x-4}| + c$

(c)$\frac{1}{4}log|\frac{e^x+4}{e^x-4}| + c$ (d)$\frac{1}{4}log|\frac{e^x-2}{e^x+2}| + c$

Ans.(d)

Q.17 Evaluate $\int \frac{(2-x)e^x}{(1-x)^2}dx$ and the value is

(a)$\frac{e^x}{1-x} + k$ (b)$e^x + k$

(c)$\frac{1}{1-x} + k$ (d)None of these

Ans.(a)

Q.18 Evaluate $\int e^x(x^3 + 3x^2)dx$

(a)$xe^x + c$ (b)$x^2e^x + c$

(c)$x^3e^x + c$ (d)None of these

Ans. (c)

Q.19 Evaluate $\int \frac{1}{x^2-16}dx$

(a)$\frac{1}{8}log|\frac{x-4}{x+4}| + c$ (b)$\frac{1}{4}log|\frac{x+4}{x-4}| + c$

(c)$\frac{1}{8}log|x^2 - 16| + c$ (d)None of these

Ans.(a)

Q.20 Evaluate $\frac{1}{\sqrt{x^2-1}}dx$

(a)$log|x + \sqrt{x^2 - 1}|$ (d)$log|x + \sqrt{x^2 + 1}| + c$

(c)$log|x - \sqrt{x^2 + 1}| + c$ (d)None of these

Ans.(a)

Q.21 Evaluate $\int \frac{1}{\sqrt{9+x^2}}dx$

(a)$log|\sqrt{x^2 + 9}| + c$ (b)$log|x + \sqrt{x^2 + 9}| + c$

(c)$log|x - \sqrt{x^2 + 9}| + c$ (d)None of these

Ans.(b)

Q.22 $\int \frac{(x-1)e^x}{x^2}dx$ is equal to

(a)$\frac{e^x}{x} + k$ (b)$\frac{e^{-x}}{x} + k$

(c)$\frac{-e^x}{x} + k$ (d)None of these

Ans. (a)

METHOD OF PARTIAL FRACTION

Form of function	Form of partial fraction
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
$\frac{px+q}{(x-a)^2(x-b)}, a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{(x-a)} + \frac{BX+C}{x^2+bx+c}$

Q.23 Evaluate $\int \frac{1}{(x+1)(x+2)}dx$

(a)$log\frac{1}{(x+1)(x+2)} + c$ (b)$\log[(x + 1)(x + 2)] + c$

(c)$log|\frac{x+1}{x+2}| + c$ (d)None of these

Ans. (c)

Q.24 Integrate the following and choose the correct option: $\int \frac{(x+5)}{(x+1)(x+2)^2}dx$

(a)$4\log(x + 1) - 4\log(x + 2) + \frac{3}{x} + 2 + k$

(b)$4\log(x + 2) - \frac{3}{x} + 2 + k$

(c)$4\log(x + 1) - 4\log(x + 2)$

(d)None of these

Ans.(d)

Q.25 Evaluate the integral: $\int \frac{3x^2-2x+5}{(x-1)(x^2+5)}\,dx$

(a)$log|(x-1)(x^2-5)|+c$ (b)$log|(x^3-5)|+c$

(c)$log\left|\frac{x-1}{x^2+5}\right|+c$ (d)None of these

Ans. (a)

Q.26 Using method of partial fraction the integration of $f(x)$ when $f(x)=\frac{1}{x^2-a^2}$ and the answer is

(a) $\log x-\frac{a}{x-a}+k$ (b)$\log(x-a)-\log(x+a)+k$

(c)$\frac{1}{2a}log\left(\frac{x-a}{x+a}\right)+k$ (d)None of these

Ans.(c)

DEFINITE INTEGRATION

Let's say $f(x)$ is a function with s that varies from a to b

Then the function limit will be between $f(a)$ to $f(b)$

Thus, if we say $\int F(x)dx=f(x)+c$

By indefinite integral

$$\int_a^b F(x)dx=f(b)-f(a)$$

Where 'b' is the upper limit of integration and 'a' is the lower limit of integration. To find the difinite integral, we have to find the indefinite integral and then put values of limits.

In the process of evaluating definite integrals, the constant of integration (C) does not affect the final result because it cancels out when subtract the values at the limits. Therefore, theere is no need to consider the constant when finding definite integrals.

Q.27 $\int_0^2 3x^2\,dx$ is

(a)7 (b)-8 (c)8 (d)None of these

Ans.(c)

Q.28 $\int_0^1(2x^2-x^3)dx$

(a)$\frac{4}{3}+k$ (b)$\frac{5}{12}$ (c) $-\frac{4}{3}$ (d) None of these

Q.29 $\int_0^1(x^2-x)dx$

(a)$\frac{4}{3}$ (b) $-\frac{1}{6}$ (c) $-\frac{4}{3}$ (d) None of these

Ans. (b)

Q.30 $\int_1^2 x\log x\,dx$

(a) 2 log 2 (b) $\int_4^3$ (c) $2\log 2-\frac{3}{4}$ (d) None of these

Ans . (c)

Q.31 Evaluate $\int_1^4(2x+5)dx$ and the value is

(a)3 (b)10 (c)30 (d)None of these

Ans.(c) To evaluate : $\int_1^4(2x+5)dx$

$\therefore \int_1^4(2x+5)dx-\int_1^4(2x)dx+\int_1^4 5\,dx=2\int_1^4 x\,dx+5\int_1^4 dx$

We know that,

$\int x^n\,dx=\frac{x^{n+1}}{n+1},\quad \int dx=x+k$

Thus, $\int_1^4 (2x+5)dx = 2\left[\frac{x^2}{2}\right]_1^4 + 5[x]_1^4$

$= (4^2 - 1^2) + 5(4-1) = 15 + 15 = 30$

Hence, the correct option is (c)i.e., 30.

Q.32 $\int_1^2 \frac{2x}{1+x^2}dx$ is equal to

(a)$log_e\left(\frac{5}{2}\right)$ (b)$log_e 5 - log_e 2 + k$

(c)$log_e\left(\frac{2}{5}\right)$ (d)None of these

Ans.(a)

Q.33 $\int_0^4 \sqrt{(3x-4)}dx$ is equal to

(a)$\frac{9}{112}$ (b)$\frac{112}{9}$ (c)$\frac{11}{9}$ (d) None of these

Ans.(b)

Q.34 Evaluate $\int_0^2 \frac{x+2}{x+1}dx$

(a)$2 + log_e 2$ (b)$2 + log_e 3$ (c)$log_e 3$ (d)None of these

Ans. (b)

Q.35 Evaluate $\int_1^{e^2} \frac{1}{x(1+logx)^2}dx$ and the value is

(a)$\frac{3}{2}$ (b)$\frac{1}{3}$ (c)$\frac{26}{3}$ (d)$\frac{2}{3}$

Ans.(d)

Q.36 Evaluate the following integrals:

(i)$\int_2^3 x^2\,dx$ Ans. $\frac{19}{3}$

(ii)$\int_0^2 (2x^2+3x+1)dx$ Ans. $\frac{40}{3}$

(iii)$\int_2^3 \frac{1}{x}dx$ Ans. $log\frac{3}{2}$

(iv)$\int_0^1 \frac{2x}{1+x^2}dx$ Ans. $\log 2$

(v)$\int_0^1 xe^x dx$ Ans. 1

(vi)$\int_1^2 x log\, x\, dx$ Ans. $2\log 2 - \frac{3}{4}$

(vii)$\int_0^1 (e^x + e^{-x})dx$ Ans. $e - e^{-1}$

PROPERTIES OF DEFINITE INTEGRATION

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$
2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, a < c < b$
4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
5. $\int_0^a f(x)dx = \int_0^a f(a+0-x)dx = \int_0^a f(a-x)dx$
6. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ if $f(x)$ is an even function i.e., $f(-x) = f(x)$

   $= 0$ if $f(x)$ is an odd function i.e., $f(-x) = -f(x)$

Q.37 Evaluate $\int_{-1}^1 \log\left(\frac{2+x}{2-x}\right)dx$

(a) 0 (b)1 (c)$\log\frac{3}{2}$ (d)None of these

Ans. (a)

Q.38 Evaluate $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$

(a)1 (b)$\frac{1}{2}$ (c)2 (d)$\frac{3}{2}$

Ans.(a)

Q.39 $\int_{-1}^{1}(e^x - e^{-x})dx =$

(a) 0 (b)1 (c)12 (d)None of these

Ans.(b)

Q.40 Evaluate: $\int_0^2 x\sqrt{2-x}dx$

(a)$\frac{8\sqrt{2}}{15}$ (b)$\frac{16\sqrt{2}}{3} - \frac{8}{5}$ (c)$\frac{16\sqrt{2}}{15}$ (d) None of these

Ans.(c)

Q.41 $\int_{-1}^{1}\frac{|x|}{x}dx =$

(a)-1 (b)0 (c)1 (d)2

Ans. (b)

Q.42 The value of $\int_{-2}^{2} f(x)dx$, where $f(x) = 1 + x, x \leq 0$ and $f(x) = 1 - 2x$ when $x \geq 0$ is

(a)20 (b)-2 (c)-4 (d)0

Ans.(b)

Q.43 The equation of the curve which passes through the point (1,3) and has the slope $4x - 3$ at any point (1,3) is

(a)$y = 2x^3 - 3x + 4$ (b)$y = 2x^2 - 3x + 4$

(c)$y = 2x^2 - 3x + 4$ (d)None of these

Ans.(b)

Q.44 the value of $\int_2^3 f(5-x)dx - \int_2^3 f(x)dx$ is

(a) 1 (b)0 (c)-1 (d)None of these

Ans.(b)

Q.45 The gradient of the curve $y = 2x^3 - 5x^2 - 3x$ at $x = 0$ is

(a)3 (b)-3 (c)$\frac{1}{3}$ (d)None of these

Ans. (b)

Q.46 If $f(x) = e^{ax^2+bx+c}$ then $f'(x)$ is

(a)e^{ax^2+bx+c} (b)$e^{ax^2+bx+c}(2ax + b)$

(c)$2ax + b$ (d)None of these

Ans.(b)

Q.47 The derivative of the function $\sqrt{x^{2+\sqrt{x}}}$ is

(a)$\frac{2x+\frac{1}{\sqrt{x}}}{2\sqrt{x^2+\sqrt{x}}}$ (b)$1 + \frac{1}{2\sqrt{x}}$

(c)$\frac{1}{2\sqrt{x+\sqrt{x}}}(1 + \frac{1}{2\sqrt{x}})$ (d)None of these

Ans.(a)

Q.48 The cost function $C(x) = 125 + 500x - x^2 + \frac{x^3}{3}, 0 \leq x \leq 100$ and the demand function for the items is given by, $p(x) = 1500 - x$, then the marginal profit when 18 items are sold is

(a)751 (b)571 (c)676 (d)875

Ans. (c)

Q.49 If $f(x) = a(x^2 + x + 1)^2$ and $f'(-1) = 6$, then the value of a is

(a)1 (b)2 (c)3 (d)4

Ans.(c)

Q.50 If $y = x(x-1)(x-2)$, then $\frac{dy}{dx}$ is

(a) $-6x$ (b) $3x^2 - 6x + 2$ (c) $6x + 4$ (d) $3x^2 - 6$

Ans. (b)

Q.51 If $2x^2 - 2^y = 2^{x-y}$, then $\frac{dy}{dx}$ at $x = y = 2$

(a) 1 (b) 2 (3) 4 (d) 5

Ans. (a)

Q.52 $\int \log(a^x)dx$

(a) $\log a.\frac{x^2}{2} + c$ (b) $\log a.(\frac{x}{2}) + c$

(c) $x log\ a^x - x + c$ (d) $x \log a^x + c$

Ans. (a)

Q.53 $\int e^x(x^2 + 2x)dx =$

(a) $x^2.e^2 + c$ (b) $e^x.e^2 + c$

(c) $-e^x x^2 + c$ (d) $-e^x.\ x + c$

Ans. (a)

Q.54 $\int_1^2 e^x\ (\frac{1}{x} - \frac{1}{x^2})\ dx =$

(a) $e\ (\frac{e}{2} - 1)$ (b) $e(e-1)$

(c) a (d) $e^2\ (e-1)$

Ans. (a)

Q.55 $\int_{-1}^{1}(e^x - e^{-x})dx =$

(a) 0 (b) 1 (c) 12 (d) None of these

Ans. (a)

Q.56 $\int_1^2 \frac{1-x}{1+x}dx =$

(a) $2\log\frac{3}{2} - 1$ (c) $2\log 3 + 1$

(c) $\frac{1}{2}\log\frac{2}{2} - 1$ (d) $2\log 2 - 1 + k$

Ans. (a)

Q.57 Find the value of $\int(4x+5)^6\ dx$ is equal to

(a) $\frac{1}{7}(4x+5)^7 + c$ (b) $\frac{1}{28}(4x+5)^7 + c$

(c) $\frac{1}{4}(4x+5)^7 + c$ (d) None of these

Ans. (b)

Q.58 The value of $\int_1^2 \frac{1-x}{1+x}dx$ is equal to:

(a) $\log\frac{3}{2} - 1$ (b) $2\log\frac{3}{2} - 1$

(c) $\frac{1}{2}\log\frac{3}{2} - x$ (d) $\frac{1}{2}\log\frac{3}{2} - 1$

Ans. (b)

Q.59 $\int_0^{\sqrt{x}} \frac{2^{3\sqrt{x}}}{\sqrt{x}}dx$ is equal to

(a) $\frac{2\sqrt{2}}{log_e 3}$ (b) 0 (c) $\frac{2(3\sqrt{2}-1)}{log_e 3}$ (d) $\frac{3\sqrt{2}}{\sqrt{2}}$

Ans. (c)

Q.60 The value of $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}dx$ is

(a) 0 (b) 3 (c) 2 (d) 1

Ans. (d)

Q.61 Find the value of $\int xe^x dx$.

(a)$e^x(x-1)$ (c) $e^x(2x-1)+c$

(c)$e^x(x-1)$ (d) None of these.

Ans.(a)

Q.62 $\int 2e^{x^2} dx$ is equal to

(a)$2e^{x^2}+c$ (b)$\frac{1}{3}e^{x^2}+c$

(c)$\frac{1}{2}e^{x^2}+c$ (d)None of these

Ans.(c)

Q.63 $\int \log(a^x)dx =$

(a)$loga\ (\frac{x^2}{2})+c$ (b)$loga\ (\frac{x}{2})+c$

(c)$x\log(a^x)-x+c$ (d)$x\ \log(a^x)\ +c$

Ans.(a)

Q.64 $\int_0^1 \log(\frac{1}{x}-1)\,dx$ is equal to

(a)1 (b)0 (c)2 (d)-1

Ans. (b)

Q.65 The values of $\int_1^2 \frac{x}{x^2+1} dx$ is equal to

(a)$log_e(\frac{5}{2})$ (b)$\frac{1}{2}log_e(\frac{5}{2})$

(c)$log_e(5)-log_e 2+c$ (d)None of these

Ans. (b)

Q.66 $\int_{-1}^{1}(2x^2-x^3)dx =$

(a)14 (b)104 (c)$\frac{2x^3}{3}-\frac{x^4}{4}$ (d) __

Ans.(d)

Q.67 $\int e^x(x^2+2x)dx =$

(a)$x^2.e^x+c$ (b)$e^x.x+c$ (c)$-e^x.x^2+c$ (d)$-e^x.\ x\ +c$

Ans.(c)

Q.68 $\int_0^{2} \frac{3\sqrt{x}}{\sqrt{x}} dx$ is equal to

(a)$\frac{2\sqrt{2}}{log_e x}$ (b)0 (c)$\frac{2(3\sqrt{2}-1)}{log_e x}$ (d)$\frac{3\sqrt{2}}{\sqrt{2}}$

Ans.(c)

Q.69 $\int_2^4 \frac{xdx}{x^2-1}$ is

(a)$\frac{1}{2}\log(\frac{17}{5})$ (b)$2\log(\frac{17}{5})$

(c)$\frac{1}{2}\log(\frac{5}{17})$ (d)$2\log(\frac{5}{17})$

Ans.(a)

Q.70 Find the area under curve $f(x)=x^2+5x+2$ with the limits o to 1

(a)30833 (b)4.388 (c)4.833 (d)3.338

Ans.(c)

SUMMARY

DIFFERENTIATION RULE

$\frac{d}{dx}(f(x))=f'(x)$

- Product rule: $\{f(x)g(x)\}=f(x)g'(x)+f'(x)g(x)$

- Quotient rule: $\left\{\frac{f(x)}{g(x)}\right\}' = \frac{g(x)f^F(x)-g'(x)f(x)}{(g(x))^2}$
- Chain rule: $f(g(x))' = f'(g(x)) \times g'(x)$

DERIVATIVE OF VARIOUS FUNCTION

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(constant) = 0$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \log(a)$$

$$\frac{d}{dx}(log_e\, x) = \frac{1}{x}$$

Parametric equations: if $x = f(t) and\ y = h(t), then\ \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt}.\frac{dt}{dx}$

APPLICATIONS OF DIFFERENTIAL CALCULUS

- $C(x) = V(x) + F(x)$, where $V(x)$ denotes the variable cost and $F(x)$ is the fixed cost
- Average cost $(AC\ or\ C) = \frac{C(x)}{x}$
- Average variable cost $(AVC) = \frac{V(x)}{x}$
- Average Fixed Cost $(AFC) = \frac{F(x)}{x}$
- Marginal Cost $(MC) = \frac{dC}{dx}$
- Revenue function, $R(x) = P.x$
- Marginal revenue, $(MR) = \frac{dR}{dx}$
- Profit function, $P(x) = R(x) - C(x)$
- Marginal Profit $= \frac{dP}{dx}$

INTEGRATION RULES

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$
- $\int x^{-1} dx = \int \frac{1}{x} dx = ln|x| + c$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\log a} + c$

INTEGRATION BY PARTS: USING 'LAE' RULE

$$\int f(x).g(x)dx = f(x)\int g(x) - \int \left[\frac{df(x)}{dx} \times \int g(x)dx\right] dx$$

SOME MORE IMPORTANT FORMULAE

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$
2. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}log\frac{a+x}{a-x} + c$

3. $\int \frac{dx}{\sqrt{x^2+a^2}} = log|x + \sqrt{x^2 + a^2}| + c$
4. $\int \frac{dx}{\sqrt{x^2-a^2}} = log|x + \sqrt{x^2 - a^2}| + c$
5. $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$
6. $\int \sqrt{x^2 + a^2} = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log(x + \sqrt{x^2 + a^2}) + c$
7. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log(x - \sqrt{x^2 - a^2}) + c$
8. $\int \frac{f'(x)}{f(x)}dx = \log[f(x)] + c$

METHOD OF PARTIAL FRACTION

Form of function	Form of partial fraction
$\frac{px + q}{(x - a)(x - b)}, a \neq b$	$\frac{A}{(x - a)} + \frac{B}{(x - b)}$
$\frac{px + q}{(x - a)^2(x - b)}, a \neq b$	$\frac{A}{(x - a)} + \frac{B}{(x - a)^2} + \frac{C}{(x - b)}$
$\frac{px^2 + qx + r}{(x - a)(x^2 + bx + c)}$	$\frac{A}{(x - a)} + \frac{BX + C}{x^2 + bx + c}$

DEFINITE INTEGRAL

$$\int_a^b f(x)dx = \int_a^b f(t)dt$$

PROPERTIES

- $\int_a^b f(x)dx = \int_a^b f(t)dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$
- $\int_0^a f(x)dx = \int_0^a f(a + 0 - x)dx = \int_0^a f(a - x)dx$
- $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ when $f(-x) = f(x)$
  $= 0$ when $f(-x) = -f(x)$

CHAPTER-9

NUMBER SERIES, CODING-DECODING AND ODD MAN OUT

THIS CHAPTER CONSISTS OF 3 PARTS

- SERIES
- CODING DECODING
- ODD MAN OUT

NUMBER SERIES

Number series refers to a sequence of numbers that follows a specific pattern. In this type of series, one term is usually missing, and the task is to identify the missing term based on the given pattern. These questions test your ability to recognize numerical patterns and apply them to solve problems.

Here are a few examples of number series:

E.g.: 1, 2, 3, 4,__, 6, 7, 8

In this series, the numbers increase by 1 in each step, except for the missing term. The missing term is 5, as it follows the pattern of incrementing by 1.

E.g.:__, 9, 16, 25, 36, 49

E.g.,__, $3^2, 4^2, 5^2, 6^2, 7^2$

In this series, the numbers are squares of consecutive numbers.

The missing term is 4, as it is the square of 2 i.e., $2^2 = 4$

Q.1 What comes next in the sequence:7, 10, 14, 19, 25, ?

(a) 30 (b) 32 (c) 36 (d) 42

Ans. (b)

Q.2 What number should come next: 58, 52, 46, 40, 34, ...?

(a) 40 (b) 28 (c) 30 (d) 26

Ans. (b)

Q.3 What number should come next in the following sequence: 8, 16, 32, 64,... ?

(a) 80 (b) 96 (c) 128 (d) 256

Ans. (c)

Q.4 What comes next in the sequence: 16, 8, 4, 2, 1,.....?

(a) 0 (b) –2 (c) 0.2 (d) 0.5

Ans. (d)

Q.5 What comes next in the sequence: 7, 10, 8, 11, 9, 12, ... ?

(a) 10 (b) 15 (c) 9 (d) None of these

Ans. (a)

Q.6 In the sequence, 8, 6, 9, 23, 87, ... What number should come next?

(a) 174 (b) 226 (c) 324 (d) 429

Ans. (d)

Q.7 The next two terms of the series: 9, 12, 11, 14, 13, 16, 15,.....?

(a) 14, 13 (b) 18, 21 (c) 14, 17 (d) 18, 17

Ans. (d)

Q.8 What number should come next: 3, 8, 27, 112, ... ?

(a) 256 (b) 408 (c) 565 (d) None of the above

Ans. (c)

Q.9 5, 2, 7, 9, 16, 25, 41,.... ?

(a) 65 (b) 66 (c) 67 (d) 68

Ans. (b)

Q.10 The number that comes next in the sequence 5.2, 4.8, 4.4, 4, ... is

(a) 3.3 (b) 3.6 (c) 3.8 (d) 4.2

Ans. (b)

Q.11 Find the missing number in the sequence, 165, 195, 255, 285, ?, 375

(a) 345 (b) 390 (c) 335 (d) 395

Ans. (a)

Q.12 120, 99, ?, 63, 48, 35

(a) 80 (b) 36 (c) 45 (d) 40

Ans. (a)

ALPHABET SERIES

A	B	C	D	E	I	F	G	H	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

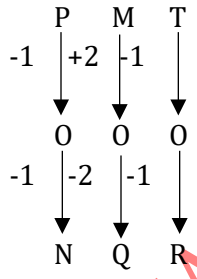
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

E.g.: B,D,F,H,___

To find the next term in this series, we need to consider the pattern. Here, each letter is two positions ahead of the previous one in the English alphabet. Applying this pattern, the next term is H + 2 = J.

Q.13 Complete this series PMT, OOS, NQR, ?

Ans. We have,



By examining the series, we can observe the following pattern:

The first letter in each term is moving in reverse alphabetical order.

The second letter in each term is increasing by 2 positions in the English alphabet.

The third letter in each term is also moving in reverse alphabetical order.

Applying this pattern, the next term in the series would be MSQ.

Therefore, the complete series would be: PMT, OOS, NQR, MSQ.

LETTER SERIES

Letter series involves a sequence of lowercase letters following a consistent pattern. The task is to identify the pattern and determine the missing term.

E.g.:____a b a____b a __a b

Here, the given series is following the pattern:

ab / ab / ab / ab / ab / ab

i.e., the pattern 'ab' is repeated.

Therefore, the pattern will be a b a b a b a b a b a b

Thus, the missing letters are: abbab

CODING AND DECODING

When we talk about Coding and Decoding, there are 2 types:

- Letter coding
- Number coding

LETTER CODING

In letter coding, the real alphabets in a word are replaced by certain other alphabets based on a specific rule. The task for the candidate is to identify the common rule and apply it to answer the given questions. This type of coding requires a keen understanding of the patterns and rules governing the replacement of alphabets.

E.g.: If GOLD is written as IQNF, WIND can be written in the same code as _____

Here, the pattern followed is:

Adding 2 to the place values of the letters.

G + 2 = I, O + 2 = Q, L + 2 = N, D + 2 = F

Therefore, WIND can be written as:

W + 2 = Y, I + 2 = K, N + 2 = P, D + 2 = F

Thus, WIND can be coded as YKPF.

Q.14 If TAP is coded as SZO, then how is FRIEND coded?

(a) EQJDNC (b) EQHDMC (c) GSIEND (d) None of above

Ans. (b)

Q.15 In certain code 'BILLION' is written as 'I B L L O I N'. How is 'HILTON' written in that code?

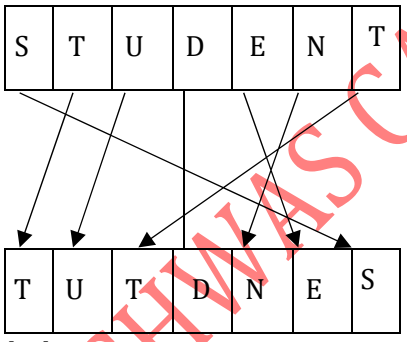
(a) I H T L O N (b) I H T L N O (c) O H T L I N (d) H I T L N O

Ans. (b)

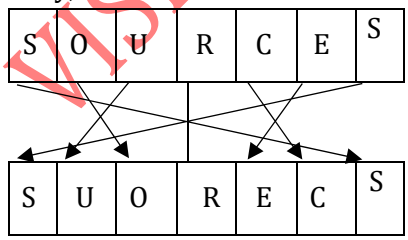
Q.16 In a certain code STUDENT is written as TUTDNES. How will SOURCES be written in that code?

(a) SOURECS (b) SOCRSEU (c) SUORECS (d) SORCESU

Ans.(c) STUDENT is written as TUTDNES,



Similarly,



Therefore, SOURCES can be written in that code as SUORECS.

Hence, the correct option is (c).

Q.17 In a certain code language, 'PICTURE' is written as 'QHDSVQF'. How would 'BROWSER' be written in that same code language?

(a) CQVVTDS (b) CQPVTDS (c) CQPUTDS (d) CQVPPDS

Ans. (b)

NUMBER CODING:

Number coding involves replacing alphabets with corresponding numerical values based on a specific rule. Each alphabet is assigned a unique numerical value, and the task is to decipher the code by understanding the assigned values and applying the given rule. There are two types in number coding:

- **When letter is given in a particular number code:**

Q.18 If 'GLOSSORY' is coded as '97533562' and 'GEOGRAPHY' is coded as '915968402', then 'GEOLOGY' can be coded as

(a) 915692 (b) 9157592 (c) 9057592 (d) 9157591

Ans.(b)

Q.19 In a certain code ATE is written as 145 and CHAIR is written as 09173, then how TEACHER can be written in the code?

(a) 4501953 (b) 4510953 (c) 4510934 (d) 4530943

Ans. (b)

Q.20 In a certain code, a number 18462 is written as BETKO and 7935 is written as RAHU. How is 43857 written in that code?

(a) THOEB (b) THROB (c) THKOB (d) THEUR

Ans. (d)

Q.21 If 'RAJU' is coded as 11-12-13-14 and 'JUNK' is coded as 13-14-10-9, then how will you code 'RANK'?

(a) 9-10- 11- 12

(b) 10- 11-12-9

(c) 11-12-10-9

(d) 12-11-10-9

Ans. (c)

Q.22 In a certain code, "TIGER" is written as "74159" and "LION" is written as "6247". How is "GOAT" written in that code?

(a) 1673 (b) 1467 (c) 4178 (d) 1437

Ans. (d)

Q.23 **LETTER:** L E A D I N G

CODE DIGIT: 3 6 1 4 2 5 7

Find out the correctly coded alternative for NGADLIA from amongst the given four alternatives.

(a) 5114312 (b) 4716321 (c) 5714321 (d) None of the above

Ans.(c)

Q.24 If CLOCK is coded 34235 and TIME is 8679. What will be the code of MOTEL?

(a) 72894 (b) 77684 (c) 72964 (d) 27894

Ans. (a)

ODD MAN OUT

Odd Man Out is a problem-solving concept where a series of elements is presented, and the task is to identify the element that does not fit the given pattern or follow the same rule as the other elements. To identify the Odd Man Out, you need to carefully analyze the given elements and look for any distinguishing features, characteristics, or patterns that set the odd element apart from the others. By identifying the unique element, you can determine the odd one out. **E.g.:** 2, 4, 6, 8, 10, 12, 15 In this series of numbers, all the elements are

even except for "15." The odd number "15" stands out and does not follow the same pattern as the other even numbers. Therefore, "15" is the Odd Man Out in this series.

Q.25 Find the odd man out of the following given numbers.

(a) 12 (b) 13 (c) 19 (d) 17

Ans. (a)

Q.26 Find odd man out of the following: 6, 9, 15, 21, 24, 28, 30

(a) 28 (b) 21 (c) 24 (d) 30

Ans. (a)

Q.27 Select the odd man out:

(a) Cricket (b) Volleyball (c) Fencing (d) Baseball

Ans. (c)

Q.28 Find odd one out of the following:

41, 43, 47, 53, 61, 71, 75, 83

(a) 75 (b) 73 (c) 71 (d) 53

Ans. (a)

Q.29 There are four groups of letters in each. Three of these groups are alike in some way while one is different. Find the one which is different. TRP, YWU, SQO, TVX

(a) TRP (b) YWU (c) SQO (d) TVX

Ans. (d)

Q.30 Find odd man out of the following: 2, 5, 10, 17, 26, 37, 50, 64

(a) 50 (b) 26 (c) 37 (d) 64

Ans. (d)

Q.31 Find the odd one out of the following: 1, 5, 7, 11, 14, 17, 21

(a) 5 (b) 11 (c) 14 (d) 21

Ans. (c)

Q.32 Find the odd one out of the following: 9, 27, 64, 81, 125, 216, 343

(a) 27 (b) 81 (c) 216 (d) 343

Ans. (b)

Q.33 Find odd man out of the following: January, May, July, November

(a) January (b) May (c) July (d) November

Ans. (d)

Q.34 Find odd man out of the following: 6, 9, 15, 21, 24, 28, 30

(a) 28 (b) 21 (c) 24 (d) 30

Ans. (a)

Q.35 Find the odd man out of the following: 13, 14, 18, 27, 32, 43, 68

(a) 27 (b) 43 (c) 32 (d) 68

Ans. (c)

Q.36 In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?

(a) 318826 (b) 318286 (c) 618826 (d) 338816

Q.37 If LOSE is coded as 1357 and GIAN is coded as 2468, what does figure 82146 stands for?

(a) NGLAI (b) NGLIA (c) GNLIA (d) GNIA

Ans. (a)

Q.38 If PLAY is coded as 8123 and RHYME is coded as 49367. What will be the code of MALE?

(a) 6217 (b) 6198 (c) 6395 (d) 6285

Ans.(a)

Q.39 Find out the next number in the following series 7, 11, 13, 17, 19, 23, 25, 29 ?

(a) 30 (b) 31 (c) 32 (d) 33

Ans. (b)

Q.40 If HONEY is coded as JQPGA. What is the code for VCTIGVU?

(a) XEVKIXW (b) TRAPETS (c) TARGETS (d) UMBRELU

Ans. (a)

Q.41 Find odd out of the following series: 15, 21, 63 ,81, 69.

(a) 15 (b) 21 (c) 63 (d) 81

Ans. (d)

Q.42 Find the odd man out of the following series 7, 9, 13, 17, 19.

(a) 7 (b) 9 (c) 19 (d) 13

Ans. (b)

Q.43 Find the next number of the series: 7, 23, 47, 119, 167,...

(a) 211 (b) 223 (c) 287 (d) 319

Ans. (b)

Q.44 Find the odd man out of the following: 13, 14, 18, 27, 32, 43, 68.

(a) 27 (b) 43 (c) 32 (d) 68

Ans. (c)

Q.45 If in a certain language, MADRAS is coded as NBESBT, How is BOMBAY coded in that language?

(a) CPNCBX (b) CPNCBZ (c) CPOCBZ (d) CQOCBZ

Ans.(b)

Q.46 Which of the following is an odd one out?

(a) CEHL (b) KMPT (c) OQTX (d) NPSV

Ans.(d)

Q.47 Look at this series:2, 1, $\frac{1}{2}$, $\frac{1}{4}$, ... What number should come next?

(a) $\frac{1}{3}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{16}$

Ans. (b)

Q.48 Look at the series: 7, 10, 8, 11, 9, 12, ... What number should come next?

(a) 7 (b) 10 (c) 12 (d) 13

Ans. (b)

Q.49 Look at the series: 36, 34, 30, 28, 24, ... What number should come next?

(a) 20 (b) 22 (c) 23 (d) 26

Ans. (b)

Q.50 Look at the series: 22, 21, 23, 22, 24, 23, ... What number should come next?

(a) 22 (b) 24 (c) 25 (d) 26

Ans. (c)

Q.51 Look at the series: 53, 53, 40, 40, 27, 27, ... What number should come next?

(a) 12 (b) 14 (c) 27 (d) 53

Ans. (b)

Q.52 Look at the series: 21, 9, 21, 11, 21, 13, 21, ... What number should come next?

(a) 14 (b) 15 (c) 21 (d) 23

Ans. (b)

Q.53 Look at the series: 58, 52, 46, 40, 34, ... What number should come next?

(a) 26 (b) 28 (c) 30 (d) 32

Ans. (b)

Q.54 Look at the series: 3, 4, 7, 8, 11, 12, ... What number should come next?
(a) 7 (b) 10 (c) 14 (d) 15
Ans. (d)
Q.55 Look at the series: 8, 22, 8, 28, 8, ... What number should come next?
(a) 9 (b) 29 (c) 32 (d) 34
Ans. (d)
Q.56 Look at the series: 31, 29, 24, 22, 17, ... What number should come next?
(a) 15 (b) 14 (c) 13 (d) 12
Ans. (a)
Q.57 Look at the series: 2, 4, 6, 8, 10, ... What number should come next?
(a) 11 (b) 12 (c) 13 (d) 14
Ans. (b)
Q.58 Look at the series: 201, 202, 204, 207, ... What number should come next?
(a) 205 (b) 208 (c) 210 (d) 211
Ans. (d)
Q.59 Look at the series: 544, 509, 474, 439, ... What number should come next?
(a) 404 (b) 414 (c) 420 (d) 445
Ans. (a)
Q.60 Look at the series: 2, 6, 18, 54, ... What number should come next?
(a) 108 (b) 148 (c) 162 (d) 216
Ans. (c)
Q.61 Find the missing number: 7, 26, 63, 124, 215, ?, 511
(a) 342 (b) 343 (c) 442 (d) 421
Ans. (a)
Q.62 Look at the series: 5.2, 4.8, 4.4, 4, ... What number should come next?
(a) 3 (b) 3.3 (c) 3.5 (d) 3.6
Ans. (d)
Q.63 Look at the series: 8, 6, 9, 23, 87, ... What number should come next?
(a) 128 (b) 226 (c) 324 (d) 429
Ans. (d)
Q.64 The next two terms of the series: 28, 25, 5, 21, 18, 5, 14, ...
(a) 11, 5 (b) 10, 7 (c) 11, 8 (d) 5, 10
Ans. (a)
Q.65 The next two terms of the series: 8, 11, 21, 15, 18, 21, 22,...
(a) 25, 18 (b) 25, 21 (c) 25, 29 (d) 24, 21
Ans. (b)
Q.66 The next two terms of the series: 9, 16, 23, 30, 37, 44, 51,.....
(a) 59, 66 (b) 56, 62 (c) 58, 66 (d) 58, 65
Ans. (d)
Q.67 The next two terms of the series: 2, 8, 14, 20, 26, 32, 38,...
(a) 2, 46 (b) 44, 50 (c) 42, 48 (d) 40, 42
Ans. (b)
Q.68 The next two terms of the series: 9, 11, 33, 13, 15, 33, 17, ...
(a) 19, 33 (b) 33, 35 (c) 33, 19 (d) 15, 33
Ans. (a)
Q.69 The next two terms of the series: 2, 3, 4, 5, 6, 4, 8,...

(a) 9, 10 (b) 4, 8 (c) 10, 4 (d) 9, 4

Ans. (d)

Q.70 The next two terms of the series: 17, 17, 34, 20, 20, 31, 23, ...

(a) 26, 23 (b) 34, 20 (c) 23, 33 (d) 23, 28

Ans. (d)

Q.71 The next two terms of the series: 6, 20, 8, 14, 10, 8, 12, ...

(a) 14, 10 (b) 2, 18 (c) 4, 12 (d) 2, 14

Ans. (d)

Q.71 The next two terms of the series: 21, 25, 18, 29, 33, 18, ...

(a) 43, 18 (b) 41, 44 (c) 37, 18 (d) 37, 41

Ans. (d)

Q.72 The next two terms of the series: 75, 65, 85, 55, 45, 85, 35, ...

(a) 25, 15 (b) 25, 85 (c) 35, 25 (d) 85, 35

Ans. (b)

Q.73 The next two terms of the series 11, 16, 21, 26, 31, 36, 41, ... are

(a) 47, 52 (b) 46, 52 (c) 45, 49 (d) 46, 51

Ans. (d)

Q.74 The next two terms of the series 3, 8, 13, 18, 23, 28, 33, ... are

(a) 39, 44 (b) 28, 44 (c) 38, 43 (d) 37, 42

Ans. (c)

Q.75 The next two terms of the series: 84, 78, 72, 66, 60, 54, 48, ...

(a) 44, 34 (b) 42, 36 (c) 42, 32 (d) 40, 34

Ans. (b)

Q.76 The next two terms of the series: 20, 20, 17, 17, 14, 14, 11, ...

(a) 11, 8 (b) 11, 11 (c) 11, 14 (d) 8, 9

Ans. (a)

Q.77 The next two terms of the series: 61, 57, 50, 61, 43, 36, 61,...

(a) 29, 61 (b) 29, 22 (c) 31, 61 (d) 22, 15

Ans. (b)

Q.78 The next two terms of the series: 4, 8, 22, 12, 16, 22, 20, 24, ...

(a) 28, 32 (b) 28, 22 (c) 22, 28 (d) 32, 36

Ans. (c)

Q.79 The next two terms of the series: 40, 40, 31, 31, 22, 22, 13, ...

(a) 13, 4 (b) 13, 5 (c) 4, 13 (d) 9, 4

Ans. (a)

Q.80 The next two terms of the series: 1, 10, 7, 20, 13, 30, 19, ...

(a) 26, 40 (b) 29, 36 (c) 40, 25 (d) 25, 31

Ans. (c)

Q. 1 The next two terms of the series: 10, 20, 25, 35, 40, 50, 55, ...

(a) 70, 65 (b) 60, 70 (c) 60, 75 (d) 65, 70

Ans. (d)

PREVIOUS YEAR QUESTIONS

NUMBER SERIES, CODING-DECODING ODD MAN OUT SERIES

Q.1 In a certain language SIKKIM is written as THLJJL, then how is TRAINING written in the code? [Dec. 2023]

(a) SQBHOHOH (b) UQBHOHO
(c) UQBJOHOH (d) UQBJOHOM

Ans. (b) UQBHOHO

Q.2 In certain language, PEAR is coded as 7519, and TOIL is coded as 2693, then how DOCTOR be written in that code? [Dec. 2023]

(a) 463293 (b) 463239
(c) 463269 (d) 463296

Ans. (c) 463269

Q.3 Find odd one out of the series: 16, 25, 36, 72, 144, 196 and 225. [Dec. 2023]

(a) 36 (b) 72
(c) 196 (d) 225

Ans. (b) 72

Q.4 Find the missing number 2,3,8, ?, 3968 [Dec. 2023]

(a) 65 (b) 63
(c) 70 (d) 80

Ans. (a) 65

Q.5 In a certain system of coding the word "STATEMENT" is written as "TNEMETATS". In the same system of coding the word "POLITICAL" written as: [Dec. 2023]

(a) LACITILOP (b) LACTILIOP
(c) OPILITACL (d) LACATILOP

Ans. (b) LACTILIOP

Q.6 Find the odd man out of the following data?

190,145,136,352,460,324,631,244 [June 2023]

(a) 460 (b) 244
(c) 136 (d)324

Ans. (d)324

Q.7 In a certain code 'MENTION' is written as LNEITNO, how is PRESENT written in that code? [June 2023]

(a) QFSTUM (b) ONESEPP
(c) QRESTNO (d)OERESTN

Ans. (d)OERESTN

Q.8 Out of the following 41,43,47,53,61,71,83,95 the odd man out shall be [June 2023]

(a) 95 (b) 83
(c) 71 (d)53

Ans. (a) 95

Q.9 Find the next number in the series: Q1F, S2E, U6D, W21C, ? [June 2023]

(a) Y66B (b) Y44B

(c) Y88B (d) Z66B

Ans. (c) Y88B

Q.10 The number in place of question mark in 7, 26, 63,124,215, ? ,511 is [June 2023]

(a) 342 (b) 432

(c) 441 (d)421

Ans. (a) 342

Q.11 If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, what will be the code for SEARCH? [Dec 2022]

(a) 246173 (b) 214673

(c) 216473 (d) 214763

Ans. (b) 214673

Q.12 Find the next number in the given sequence?

11, 17, 39, 85, ?, 281, 447 [Dec 2022]

(a) 133 (b) 143

(c) 153 (d)163

Ans. (d)163

Q.13 In certain code language, if TOUR, is written as 1234, CLEAR is written 56784 and SPARE is written as 90847, Find the code for CARE ? [Dec 2022]

(a) 1247 (b) 4847

(c) 5247 (d) 5847

Ans. (d)5847

Q.14 Find the missing number in the following series ?

3, 5, 5, 19, 7, 41, 9,?, 11, 109 [Dec 2022]

(a) 71 (b) 61

(c) 69 (d) 70

Ans. (a) 71

Q.15 Find the odd man out:

34, 105, 424, 2123, 12756. [Dec 2022]

(a) 12756 (b) 2123

(c) 424 (d) 34

Ans. (b) 2123

Q.16 If 'FROZEN' is decoded as 'OFAPSG'. Tick the right option that depicts 'MOLTEN' written in this way ? [Dec 2022]

(a) OFPOMN (b) OFSMPN

(c) OFUMPN (d) OFUNPN

Ans. (c) OFUMPN

Q.17 In a coded language, if 'EARTH' is written as 34215 and 'VENUS' is written as 73089. What is the code for "SATURN"? [June 2022]

(a) 941012 (b) 941820
(c) 914281 (d) 912418

Ans. (b) 941820

Q.18In a certain code 'TELEPHONE' is written as ENOHPELET. Then ALIGATOR is written as [June 2022]
(a) ROTAGILA (b) ROTAGAIL
(c) ROTAGILE (d) TOTAGILA

Ans. (a) ROTAGILA

Q.19 7, 26, 63, 124, 215, 342 ?- Find the missing number. [June 2022]
(a) 391 (b) 421
(c) 481 (d) 511

Ans. (d) 511

Q.20What is the missing number in the sequence given below?
12, 9,13.50, 30.375, ?, 341.71875 [June 2022]
(a) 91.125 (b) 89.145
(c) 90.475 (d) 92.485

Ans. (a) 91.125

Q.219,27,31,155,161,1127, ?- Find missing number? [June 2022]
(a) 1316 (b) 1135
(c) 1288 (d) 2254

Ans. (b) 1135

Q.22If in a certain code "THANKS" is written as "SKNTHA", then how is "STUPID" written? [DEC 2021]
(a) DIPUTS (b) DISPUT
(c) DIPUST (d) DIPSTU

Ans. (d) DIPSTU

Q.23Find the odd one from the following : [DEC 2021]
(a) Zebra (b) Giraffe
(c) Horse (d) Tiger

Ans. (d) Tiger

Q.24The missing term of the series 4,13, ..., 49, 76 is: [DEC 2021]
(a) 26 (b) 28
(c) 30 (d) 75

Ans. (b) 28

Q.25What comes at the last place in R, U, X, A, D, ... ? [DEC 2021]
(a) E (b) F
(c) G (d) H

Ans. (c) G

Q.26If MOUSE is coded as 34651 and KEY is coded as 217, then how will YES [DEC 2021]
(a) 715 (b) 517
(c) 175 (d) 571

Ans. (a) 715

Q.27If CLOCK is coded as 34235 and TIME as 8679 , then MOTEL is coded as [July 2021]

(a) 27894 (b) 72964
(c) 72894 (d) 77684

Ans. (c) 72894

Q.28 If FRAME is coded as 0618011305 then ARISE is coded as [July 2021]

(a) 0118091905 (b) 0119091805
(c) 0118190905 (d) 0118091805

Ans. (a) 0118091905

Q.29If DELHI is coded as EFMIJ then JAIPUR is coded as [July 2021]

(a) JQVSBK (b) QVSKBJ
(c) BJQVSK (d) KBJQVS

Ans. (d)KBJQVS

Q.30The wrong term in the series ……., 225,196,169,144,121,100,77,64 is [July 2021]

(a) 121 (b) 77
(c) 100 (d) 69

Ans. (b) 77

Q.31Choose the missing term in the series 1,1,8,4,27 , ……, 64,16 [July 2021]

(a) 27 (b) 11
(c) 9 (d) 125

Ans. (c) 9

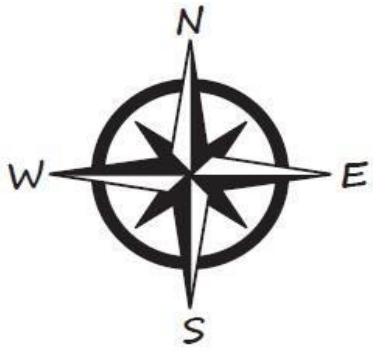
SUMMARY

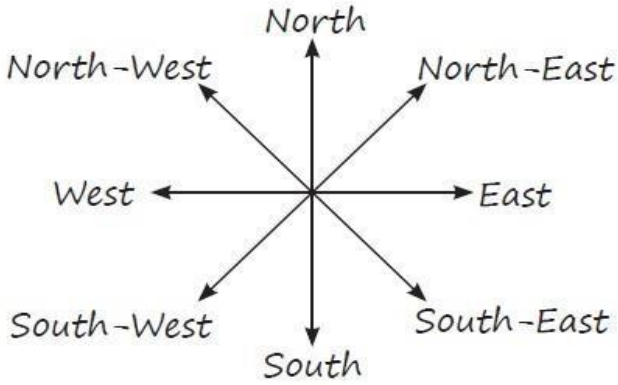
- **Number series:** Number series is a series of numbers following a particular pattern, and we need to find missing terms while few terms are given.
- **Coding and decoding:** There are two kinds of coding Letter coding and number coding.
- **Letter coding:** In this type the real alphabets in a word are replaced by certain other alphabets according to a specific rule to form its code.
- **Number coding:** Here either letters are given a particular number code, or number is given a particular letter code.
- **Odd-man out:** When there are few words, numbers or letters are given to you, such that all have a relation except one and you need to find that particular word.

CHAPTER-10

DIRECTION TESTS

- Direction sense is a concept that we have been familiar with since childhood. It involves understanding and navigating the different directions in our surroundings. The four major directions that we commonly know are **North**, **South**, **East**, and **West**.
- In addition to these cardinal directions, we have also established four more directions in between them, known as the intermediate directions. These intermediate directions are **North-East**, **North-West**, **South-East**, and **South-West**.





- While questions related to direction sense are generally easy, they can sometimes be confusing if not approached systematically. One effective strategy is to visualize yourself moving in different directions and mentally picture the diagram of the directions mentioned above. By having a clear mental image of the diagram, you can easily determine the relationships and positions of different directions.

Q.1 Laxman went 15 km north then he turned west and covered 10 km. Then he turned south and covered 5 km, finally turning east he covered 10 km. In which direction he is moving now?

(a) East (b) West (c) North (d) South

Ans. (a)

Q.2 You go north, turn right then go to the left. In which direction are you now?

(a) South (b) West (c) East (d) North

Ans. (d)

Q.3 Rohit drove towards the North for 20 km. Then he turned left and drove another 30 km. After a while, he again turned left and drove 20 km and took some rest. Once more he turned left and drove 30 km to reach his town. In which direction is he driving now?

(a) West (b) East (c) North (d) South

Ans. (b)

Q.4 A man started to walk east, and after moving a certain distance, he turned to his right. After moving some distance, he turned to his right again. After moving a little, he turned in the end to his left. In which direction was he going now?

(a) East (b) West (c) North (d) South

Ans. (d)

Q.5 Rohan walked 8 km towards the East. Then he turned back and walked 13 km. Again, he turned to his left and walked 4 km. Now, he walked 5 km after turning left. At last, he turned to his left and walked 3 km. How far and in which direction is he from the starting point ?

(a) 3 km West (b) 1 km North (c) 1 km South (d) 4 km South

Ans. (c)

Q.6 Five friends A, B, C, D and E are staying in the same locality. B's house is to the east of A's house and to the north of C's house C's house is to the west of D's house. D's house is in which direction with respect to A's house?

(a) North-East (b) South-East (c) North-West (d) South-West

Ans. (b)

Q.7 Kailash walks 3 km to the East and turns South and walks 4 km. After a while, he turns West and walks 6 km. How far is he from the starting point?

(a) 7 km (b) 5 km (c) 6 km (d) 3 km

Ans. (b)

Q.8 A and B start moving towards each other from two places apart. After walking 60m, B turns left and goes then 20 m then he turns right and goes 40 m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now?

(a) 80 m (b) 70 m (c) 40 m (d) 60 m

Ans. (c)

Q.9 Anand walked 20 m towards the north direction. After a while, he turned right and walked 30 m. Again, he turns to his right and walks 35 m. Then he turns left and walks 15 m. At last he turns to his left and walks 15 m. In which direction is Anand from the starting position?

(a) East (b) West (c) North (d) South

Ans. (a)

Q.10 One morning, after sunrise, Vikram and Shailesh were standing in a lawn with their backs towards each other. Vikram's shadow fell exactly towards left hand side. Which direction was Shailesh facing?

(a) South-West (b) West (c) South (d) South-East

Ans. (c)

Q.11 From her home Prerna wishes to go to school. From home she goes towards North and then turns left and then turns right, and finally she turns left and reaches school. In which direction her school is situated with respect to her home?

(a) North-East (b) North-West (c) South-East (d) South-West

Ans. (b)

Q.12 Anita travels 15 km to the North. After taking rest for some time, she turned to West and covered 10 km. Then she turned to the South and covered 5 km. At the end, she turned to East and covered 10 km. In which direction is she traveling now?

(a) East (b) West (c) North (d) South

Ans. (a)

Q.13 Prerna walked 20 km from West towards East. After a while, she once again turned to her right and walked 10 km and again turned right and walked 9 km. Then, she turned to left and walked 5 km and again turned to her left and covered a distance of 12 km. Finally she turned to her left and walked 6 km. In what direction is she moving?

(a) East (b) North (c) West (d) South

Ans. (b)

Q.14 Karuna walked 10 feet from A to B in the East direction. Then she turned to her right and walked 3 feet. Now again she turned to her right and walked 14 feet. In which direction is she from her starting point A?

(a) North-East (b) South-West (c) South (d) None of these

Ans. (b)

Q.15 Anand ride his bicycle and travels meters towards the East direction. Then he turns to his left and travel meters and again turns to his right and travels some distance. Towards which direction is he now moving?

(a) North (b) East (c) West (d) South

Ans. (b)

Q.16 There are four towns P, Q, R and T. Q is to the south-west of P, R is to the east of Q and south east of P, and T is to the north of R in line with QP. In which direction of P is T located?

(a) North (b) North-East (c) East (d) South-East

Ans. (b)

LET US UNDERSTAND THE CONCEPT OF ROTATION AND DEGREE

When we talk about rotation, it refers to the act of turning or revolving around a central point. In mathematics and geometry, rotations are usually measured in degrees or radians. A complete rotation or a full circle is equal to 360 degrees.

In the context of directions, we often encounter clockwise and anticlockwise rotations. Clockwise rotation refers to the movement or turn in the direction that follows the hands of a clock. It is a rightward or clockwise movement. On the other hand, anticlockwise rotation refers to the movement or turn in the opposite direction, counter clockwise or leftward.

Clockwise
Mean turn right

Anti clockwise
Mean turn left

Q.17 Man is facing west. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anti-clockwise direction. Which direction is he facing now?

(a) South (b) North–west (c) West (d) South-west

Ans. (d)

Q.18 Ana is facing the north direction. She turns 90° in the clockwise direction, Again after a while she turns 180° in the anticlockwise direction and then another 90° in the same direction. Which direction is she facing now?

(a) East (b) West (c) North (d) South

Ans. (d)

Q.19 Maya starts at point T, and walks straight to point U which is 4 feet away. She turns left at 90° and walks to W which is 4 feet away, turn 90° right and goes 3 feet to P, turns 90° right and walks 1 feet to Q, turns left at 90° and goes to V, which is 1 feet away and once again turn 90° right and goes to R, 3 feet away. What is the distance between T and R?

(a) 4 feet (b) 5 feet (c) 7 feet (d) 8 feet

Ans. (d)

Q.20 Sahil started from his school towards the north. After walking a distance of 18 m, he turned to the left and walked 14 m. He then turned to the left again and walked 30 m. After this, he is to turn left at 180 degrees and to cover 12 m. In which direction is he now as per his initial position?

(a) 14 m East (b) 14 m West (c) 14 m South (d) 14 m North

Ans. (b)

Q.21 Madhuri moved a distance of 75 m towards the north. She then turned to the left and walked for about 25 m, turned left again, and walked 80 m. Finally, she turned to the right at an angle of 45^0. In which direction was she moving finally?

(a) South –East (b) South –West (c) North-West (d) North –East

Ans. (b)

Q.22 A man is facing west. He turns 45 degree in the clockwise direction and then another 180 degrees in the same direction and then 270 degrees in the anticlockwise direction. Find which direction he is facing now?

(a) South-East (b) West (c) South (d) South-West

Ans. (d)

MORE TYPES OF QUESTION

Q.23 Ravi wants to go to the university. He starts from his home which is in the east and comes to a crossing. The road to the left ends in a theatre straight ahead is the hospital. In which direction is the university?

(a) North (b) South (c) East (d) West

Ans. (a)

Q.24 A man is performing yoga with his head down and legs up. His face is towards the west. In which direction will his left hand be?

(a) North (b) South (c) East (d) West

Ans. (a)

Q.25 Raman starts walking in the morning facing the sun. After some time, he turned to the left, and later again he turned to his left. In which direction is Raman moving now?

(a) East (b) West (c) South (d) North

Ans. (b)

Q.26 Sunita rode her Scotty northwards, then turned left, and then again rode to her left 4 km. She found herself exactly 2 km west of her starting point. How far did she ride northwards initially?

(a) 2 km (b) 4 km (c) 6 km (d) 5 km

Ans. (b)

Q.27 One evening A & B were standing under the sun talking face to face. The shadow of A fell exactly towards its left hand side. What direction does B face?

(a) North (b) South (c) East (d) West

Ans. (a)

Q.28 A is located to the West of B. C is located to the North in between A and B. D is exactly to the south of B and also in line with B. In which direction of C is D located?

(a) South (b) South-East (c) West (d) South-West

Ans. (b)

Q.29 It is 3 o'clock on a watch. If the minute hand points towards the North-East, then the hour hand will point towards the

(a) South (b) South-West (c) North-West (d) South-East

Ans. (d)

Q.30 Rohan walks a distance of 3 km towards the north then turns to his left and walks for 2 km. He again turns left and walks 3 km. At this point, he turns to his left and walks for 3 km. How many kilometers is he from the starting point?

(a) 1 km (b) 2 km (c) 3 km (d) 5 km

Ans. (a)

Q.31 Namita walks 14 km towards the west then turns to her right and walks 14 km and then turns to her left and walks 10 km. Again turning to her left she walks 14 km. What is the shortest distance (in km) between her starting point and the present position?

(a) 10km (b) 24 km (c) 28 km (d) 38 km

Ans. (b)

Q.32 Amit walked 30 meters towards the east, took a right turn and walked 40 meters, then he took a left turn and walked 30 meters. In which direction is he now from the starting point?

(a) North – East (b) East (c) South – East (d) South

Ans. (c)

Q.33 A walks 10 meters in front and 10 meters to the right. Then every time turning to his left he walks 5, 15 and 15 meters respectively. How far is he now from his starting point?

(a) 5 meters (b) 10 meters (c) 15 meters (d) 20 meters

Ans. (a)

Q.34 Of the five villages P,Q,R,S and T are situated close to each other, P is to the west of Q, R is to the south of P, T is to the north of Q and S is to the east of T. Then, R is in which direction with respect to S?

(a) North –west (b) South –east (c) South –west (d) Data inadequate

Ans. (c)

Q.35 Mohan starts from point A and walks 1 km towards south, turns left and walks 1 km. Then he turns left again and walks 1 km. Now he is facing_____direction.

(a) East (b) West (c) North (d) South-West

Ans. (c)

Q.36 A man starts from a point, walks 4 miles towards north and turns left and walks 6 miles, turns right and walks for 3 miles and again turns right and walks 4 miles and takes rest for 30 minutes. He gets up and walks straight 2 miles in the same direction and turns right and walks one mile. What is the direction he is facing?

(a) North (b) South (c) South-East (d) West

Ans. (b)

Q.37 A boy starts from home in the early morning and walks straight for 8km facing the sun. Then, he takes a right turn and walks straight for 3km. Then, he turns right again and walks for 2km, and then turns left and walks for 1km. Then, he turns right, travels 1 km, and then turns right and travels for 4km straight. How far is he from the starting point?

(a) 5 km (b) 6 km (c) 2 km (d) 4 km

Ans. (a)

Q.38 Raju drives 25 km north and turns left and travels 5 km and reaches point 'O'. He then turns right and covers another 5km. Afterward, he turns to the east and drives 5 km. How much distance does he has to travel to go back to the starting point?

(a) 30 km (b) 20 km (c) 25 km (d) 35 km

Ans. (a)

Q.39 Pinky walks a distance of 600 m towards the east, turns left and moves 500m, then turns left and walks 600m, and then turns left again and moves 500m and halts. At what distance (in m) is she from the starting point?

(a) 0 (b) 600 (c) 500 (d) 2200

Ans. (a)

Q.40 Pratap starts from school and walks 7km towards the east. He takes a left and walks 4km, then takes a right and walks 2km, and again takes a right and walks 3km. Which direction is he facing now?

(a) South (b) North (c) East (d) West

Ans. (a)

Q.41 Laxman went 15 km to the north, then he turned west and covered 10 km. Then he turned south and covered 5 km, finally turning to east he covered 10 km. In which direction he is moving now?

(a) East (b) West (c) North (d) South

Ans. (a)

Q.42 A man is facing east, then he turns left and goes 10 meters then turns right and goes 5 meters then goes 5 meters to the south and from there 5 meters to west. In which direction is he from his original place?

(a) East (b) West (c) North (d) South

Ans. (c)

Q.43 X walks southwards and then turns right then left and then right. In which direction is he moving now?

(a) South (b) North (c) West (d) South West

Ans. (c)

Q.44 Mr. Raman starts walking in the morning facing the sun. After some time, he turned to the left, later again he turned to his left. In which direction is Raman moving now?

(a) East (b) West (c) South (d) North

Ans. (b)

Q.45 I stand with my right hand extended side-ways towards the south. Towards which direction will my back be?

(a) North (b) West (c) East (d) South

Ans. (b)

Q.46 You go north, turn then right then go to the left. In which direction are you now?

(a) South (b) West (c) East (d) North

Ans. (d)

Q.47 K is a place which is located 2 km away in the north-west direction from the capital P. R is another place that is located 2 km away in the south-west direction from K. M is another place and that is located 2 km away in the north-west direction from R. T is yet another place that is located 2 km away in the south-west direction from M. In which direction is T located in relation to P?

(a) South-West (b) North-West (c) West (d) North

Ans. (c)

Q.48 Anoop starts walking towards south after walking 15 meters he turns towards north. After walking 20 meters he turns towards east and walks 10 meters. He then turns towards south and walks 5 meters. In which direction is he from the original position?

(a) North (b) South (c) East (d) West

Ans. (c)

Q.49 Rahim started from point X and walked straight 5 km. West, then turned left and walked straight 2 km. and again turned left and walked straight 7 km. In which direction is he from point X?

(a) North – East (b) South- West (c) South-East (d) North-West

Ans. (c)

Q.50 A man started to walk east, after moving a certain distance, he turned to his right. After moving some distance, he turned to his right again. After moving a little he turns now to his left, currently he is going in _____ direction.

(a) East (b) West (c) North (d) South

Ans. (d)

Q.51 Manu wants to go to the market. He starts from his house towards the north and reaches a crossing after 30 m. He turns towards east, goes 10 m till the second crossing and turns again, moves towards south straight for 30 m where the marketing complex exits. In which direction is the market from his house?

(a) North (b) South (c) East (d) West

Ans. (c)

Q.52 Sangeeta leaves from her home, she first walks 30 meters in North-West direction and then 30 meters in south west direction, next she walks 30 meters in south east direction. Finally, she turns towards her house. In which direction is she moving?

(a) North –west (b) North –east (c) South –east (d) South west

Ans. (b)

Q.53 When a person faces north and walks 25m right and she turns left and walks 20m and she again turns right and walks 25m, turns right 25m and turns right and walks 40m , in which direction is he now from his starting point?

(a) North –west (b) North-east (c) South –east (d) South –west

Ans. (c)

Q.54 Laxman went 15 km to the North, then he turned West and covered 10 km. Then he turned south and covered 5 km, finally turning to East he covered 10 Km. In which direction in which he is now moving?

(a) East (b) West (c) North (d) South

Ans. (a)

Q.55 Sandesh starts driving from point A and drives 12 km toward North. He takes a right turn and drives 20 km. Now drive 8 km after taking a left turn. Finally, he takes a left turn and drives 20 km and stops at point B. How far is point A with respect to point B?

(a) 20 km (b) 10 km (c) 5 km (d) 9 km

Ans. (a)

Q.56 Sohan drove 15 km to the west from his house, then turned left and drove 20km. Again, turned east and drove 25 km and finally turning left covered 20 km. How far is he from his house?

(a) 40 km (b) 80 km (c) 5 km (d) 10 km

Q.57 Swati starts from a point, walks 2 km towards North, turns toward her right and walks 2 km, turns right again and walks. In which direction is she going now?

(a) North (b) S-E (c) South (d) West

Ans. (c)

Q.58 Rashmi leaves from her home. She first walks 30 m in N-W direction and then 30 m in S-W direction. Next she walks 30 m in S-E direction. Finally, she turns towards her house. In which direction is she moving?

(a) N-E (b) N-W (c) S-E (d) S-W

Ans. (a)

Q.59 A walks Southwards, then turns right, then left and then right. In which direction is he from the straight point?

(a) South (b) East (c) South-West (d) North

Ans. (c)

Q.60 I started walking down a road in the morning facing the Sun. After walking for some time I turned to my left. Then I turned to my right. In which direction was I going then?

(a) East (b) West (c) North (d) south

Ans. (a)

Q.61 Mr. A walks Southwards and then turn right, then left and then right. In which direction is he from his starting point ?

(a) South-West (b) North (c) South (d) South-East

Ans. (a)

Q.62 Gopal started walking 2 km straight from his school. Then he turned right and walked 1 km. Again, he turned right and walked 1 km to reach his house. If his house is southeast from his school, then in which direction did Gopal start walking from the school?

(a) East (b) West (c) South (d) North

Ans. (a)

Q.63 A taxi driver commenced his journey from a point and drove 10 km toward north and turned to his left and drove another 5 km. After waiting to meet a friend here, he turned to his right and continued to drive another 10 km. He has covered a distance of 25 km so far, but in which direction would he be now?

(a) South (b) North (c) East (d) South-east

Ans. (b)

Q.64 A car traveling from the south covers a distance of 8 km, then turns right and runs another 9 km and again turns to the right and stops. Which direction does it face now?

(a) South (b) North (c) West (d) East

Ans. (a)

Q.65 Five boys A, B, C, D, and E are sitting in a park in a circle. A is facing South-West, D is facing South-East, B, and E are right opposite A and D respectively and C is equidistant between D and B. Which direction is C facing?

(a) West (b) South (c) North (d) East

Ans. (d)

Q.66 Sikha starts walking north and after a while, she turns to her right. After walking some distance, she turns to his left and walks a distance of 1 km. She then turns to her left again. In which direction is she moving now?

(a) North (b) West (c) East (d) South

Ans. (b)

Q.67 Kunal starts walking North, then turns left and covers some distance, then he turns right and walks. After some time, he turns to his right and then turns left. In which direction he is walking now?

(a) East (b) South (c) North (d) South-East

Ans. (c)

Q.68 X walks southwards and then turns right, then left, and then right. In which direction is he moving now?

(a) South (b) North (c) West (d) South-West

Ans. (c)

PREVIOUS YEAR QUESTIONS

DIRECTION TEST

Q.1 Kamal walks 10 Km north from there he walks 6 Km towards south. Then he walks 3 Km towards east. How far and in which direction is he with reference to starting point? [Dec. 2023]

(a) 5 Km south - east (b) 7 Km north – east

(c) 5 Km north -east (d) 7 Km south – west

Ans. (c) 5 Km north -east

Q.2 Sunita walked 30 meters towards the East, took a right turn and walked 40 meters. Then she took a left turn and walked 30 meters. In which direction is she now from the starting point? [Dec. 2023]

(a) North-East (b) East

(c) South -East (d) South

Ans. (c) South -East

Q.3 Mr. X walks 14 kilometers towards north. From there, he walks 8 kilometers towards South. Then he walks 8 kilometers towards the West. How far and in which direction is he with reference to his starting point? [Dec. 2023]

(a) 10 Kilometers North –West (b) 10 Kilometers West

(c) 7 Kilometers East (d) 7 Kilometers West

Ans. (a) 10 Kilometers North –West

Q.4 Mr. X walks northwards. After a while, he turns to his rights and a little further to his left. Finally, after walking 500 Meter, he turns to his left again. In which direction is he moving now. [Dec. 2023]

(a) North (b) South
(c) East (d) West

Ans. (d) West

Q.5 Deepika starts walking straight towards east. After walking 65 m, she turns to the left and walks 25 m straight. Again, she turns to the left and walks a distance of 40 m. At what distance and in which direction currently she is from the initial position? [June 2023]

(a) 35.35m in North-East (b) 35.35m in South-West
(c) 25m in North (d) 25m in West

Ans. (a) 35.35m in North-East

Q.6 Sunita walks a distance of 2 km towards East, turns left and moves 1 km, then turn left and moves 2 km and then turns left again and moves 1 km, then halts. At what distance Sunita is now from the beginning point? [June 2023]

(a) 0km (b) 1km
(c) 2km (d) 6km

Ans. (a) 0km

Q.7 Mr. Kartik puts his time piece on the table in such a way that at 6:00 PM hour hand points to north. In which direction the minute hand will point at 9:15 PM? [June 2023]

(a) South-East (b) East
(c) West (d) South-West

Ans. (c) West

Q.8 Shrikant is facing East and turns 120° in the clockwise direction and then turns 180° in the anticlockwise direction. Which direction is Shrikant facing now? [June 2023]

(a) East (b) North-East
(c) North (d) South-west

Ans. (b) North-East

Q.9 Five boys Ajay, Brijmohan, Chandru, Dheeraj and Ehsan are sitting in park in a circle facing the centre. Ajay is facing south west; Dheeraj is facing south east. Brijmohan and Ehsaan are right opposite Ajay and Dheeraj respectively and Chandru is equidistant from Dheeraj and Brijmohan. Which direction is Chandru facing? [June 2023]

(a) West (b) South
(c) North (d) East

Ans. (d) East

Q.10 Radha moves towards South-East a distance of 7 km, then she moves towards West and travels a distance of 14 km. from here she moves towards North –West a distance of 7 km and finally she moves a distance of 4 km towards east. How far is she now from the starting point? [Dec 2022]

(a) 3km (b) 4km
(c) 10km (d) 11km

Ans. (c) 10km

Q.11P, Q, R and S are playing a game of carom P, R and S, Q are partners, 'S' is to the right of 'R'. If 'R' is facing West, then 'Q' is facing which direction? [Dec 2022]

(a) South (b) North
(c) East (d) West

Ans. (b) North

Q.12One morning a boy starts walking in a particular direction for 6 Km and then takes a left turn and walks another 5 Km. thereafter he again takes left turn and walks another 5 Km and at last he takes right turn and walks 5 Km. Now he sees his shadow in front of him. What direction he did start initially? [Dec 2022]

(a) South (b) North
(c) West (d) East

Ans. (b) North

Q.13 It is 3'o clock in a watch. If the minute hand points towards the North-East then the hour hand will point towards the [Dec 2022]

(a) South (b) South-west
(c) North-west (d) South-east

Ans. (d) South-east

Q.14 A man is facing west. He turns 45 degrees in the clockwise direction and then another 180 degrees in the same direction and then 270 degrees in the anticlockwise direction. Find which direction he is facing now? [Dec 2022]

(a) South-east (b) West
(c) South (d) South-west

Ans. (d) South-west

Q.15 If P × Q means P is to the south of Q : P + Q means P is to the north of Q : P % Q means P is to the east of Q : P – Q means P is to the west of Q : then in case of A % B + C – D, D is in which direction with respect to B? [June 2022]

(a) North-West (b) South-East
(c) North-East (d) South-East

Ans. (d) South-East

Q.16 I am facing west, turning to the left I go 20 m, then turning to the left I go 20m and turning to the right I go 20m, then again turning to the right I go 40 m and then again, I go 40 m to the right. In which direction am I from my original position? [June 2022]

(a) North (b) West
(c) South (d) East

Ans.

Q.17 A, B, C, D, E, F, G, H and I are nine poles. C is 2 km east of B. A is 1 km north of B and H is 2 km south of A.G is 1 km west of H while D is 3 km East of G and F is 2 km north of G. I is situated right in the middle of B and C while E is just in the middle of H and I. The Distance between B and I is? [June 2022]

(a) 1km
(b) 1.41km
(c) 2km
(d) 3km

Ans. (a)1km

Q.18 Starting from a point, Rani walked 12m South, she turned left and walked 10m, she again turned left and walked 12m, then she turned right and walked 5m. how far is she now and in which direction from the starting point? [June 2022]

(a) 27m towards East
(b) 5m towards East
(c) 10m towards West
(d) 15m towards East

Ans. (d) 15m towards East

Q.19 Puru was driving his car and at a circle there was direction pole, which was showing all the four correct directions. But due to the wind, it turns in such a manner that now North pointer is showing West. Puru went in the wrong direction thinking that he was travelling East. In what direction he was actually travelling? [June 2022]

(a) West
(b) East
(c) North
(d) South

Ans. (c) North

Q.20 A person walks 1 km (kilometer) towards West and then he turns to South and walks 5 km. Again, he turns to West and walks 2 km. After this he turns to North and walks 9 km. How far is he from his starting point? [Dec 2021]

(a) 3km
(b) 4km
(c) 5km
(d) 7km

Ans. (c) 5km

Q.21 Daily in the morning the shadow of a Clock Tower installed on Railway Station falls on high rise Mall and in the evening the shadow of the same mall falls on the Clock Tower installed on Railway Station exactly. So, in which direction is Clock Tower to Mall? [Dec 2021]

(a) Eastern side
(b) Western side
(c) Northern side
(d) Southern side

Ans. (a) Eastern side

Q.22 R's office is 4 km in East direction from his home and club is 4 km. in North direction from his home. On midway from office to club, R starts moving towards his home. In which direction is he facing his back? [Dec 2021]

(a) South-East
(b) North-East
(c) North-West
(d) South-West

Ans. (b) North-East

Q.23 A man starts from a point walks 4 miles towards North and turn left and walks 6 miles, turns right and walks for 3 miles and again turns right and walks 4 miles and takes rest for 30 minutes. He gets up and walks

straight 2 miles in the same direction and turns right and walks one mile. What is the direction he is facing? [Dec 2021]

(a) North (b) South
(c) South-East (d) West

Ans. (b) South

Q.24 The hour hand of a clock is in west direction when time is 3′o clock. What is the direction of minutes hand when time is 6: 45 ? [Dec 2021]

(a) East (b) West
(c) North (d) South

Ans. (a) East

Q.25 A and B start moving towards each other from two places 200 m apart. After walking 60 m, B turns left and goes 20 m, then he turns right and goes 40 m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now? [July 2021]

(a) 80m (b) 70m
(c) 40m (d) 60m

Ans. (c) 40m

Q.26 There are four towns P, Q, R and T, Q is to the south-west of P, R is to the east of Q and south east of P, and T is to the north of R in line with QP. In which direction of P is T located? [July 2021]

(a) North (b) North-East
(c) East (d) South-East

Ans. (b) North-East

Q.27 Five friends A, B, C, D and E are staying in the same locality, B's house is to the east of A's house and to the north of C's house C's house is to the west of D‘s house. D's house is in which direction with respect to A 's house? [July 2021]

(a) North-East (b) South-East
(c) North-West (d) South-West

Ans. (b) South-East

Q.28 One morning, after sunrise, Vikram and Shailesh standing in a lawn with their backs towards each other. Vikram's shadow fell exactly towards left hand side. Which direction was Shailesh facing? [July 2021]

(a) South-West (b) West
(c) South (d) East-South

Ans. (c) South

SUMMARY

- Picturising the directions through a diagram is the best way to solve the directions.
- Pythagoras theorem : In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides i.e. h2 = p2 + b2
- When we are in the North, turning left is West and turning right is East.
- When we are in the West, turning left will be South, and turning right will be North.
- When we are in the East, turning to left will be North, and turning to right will be South.
- When we are in the South, turning to the left is East and turning to the right is West
- If we move to the right, the entire direction will be clockwise. So, every right movement will lead to the East, South, and West directions, which are clockwise when starting from the right point, at North.
- Similarly, if the movement begins from the left, the entire directions that follow would be anti-clockwise. So, if you turn to the left from North, the directions would be West, South, and East, which are in a reverse pattern.

INTRODUCTION

In these questions, you have to arrange a group of persons fulfilling certain conditions. Here we can classify these problems into 4 types:

1. Linear Arrangement
2. Two row sequence Arrangement
3. Circular Arrangement
4. Polygon Arrangement

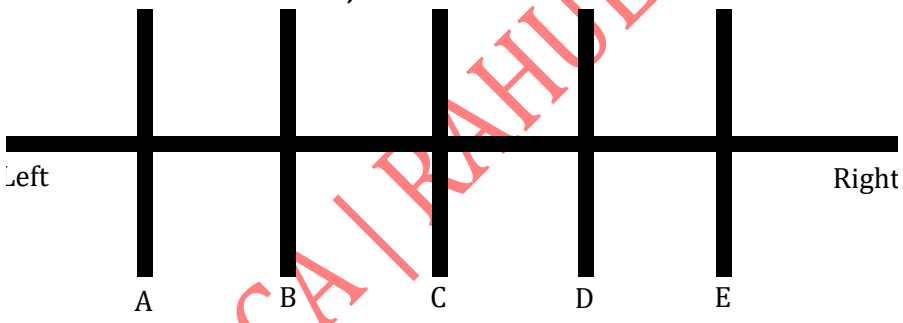
LINEAR ARRANGEMENT

Here the arrangement of the persons is linear i.e., you have to arrange them in a line. Here generally a single row of arrangement is formed.

There will be two types of questions for linear arrangement.

1. When you don't know the direction of facing
2. When you know the direction of facing, (for example: Upward, Downward, East, North, South, West etc…)

(a) When direction of face is not clear, then we take our self as below:



Now, there are a few conclusions which we have to make from above. But first let us know some definition:

1. **Immediate left**: It refers to the position or object located just to the left side of the reference point, without any other object in between.

   E.g.: A is immediate left to B.

2. **Immediate right**: It refers to the position or object located just to the right side of the reference point, without any other object in between.

   E.g.: D is immediate right to C.

3. **To the left**: It means moving towards the left side of the reference point, which can include objects located at different distances or positions.

   E.g.: A, B, C are to the left of D and E.

4. **To the right:** It means moving towards the right side of the reference point, which can include objects located at different distances or positions.

   E.g.: B, C, D and E are to the right of A.

5. **In between:** It refers to objects or positions that are located between two specified points or objects.

   E.g.: B is between A and C.

   Now, the conclusion from the above diagram are:

   - B, C, D, E are right of A but only B is to the immediate right of A.
   - D, C, B, A are right of E but only D is to the immediate left of E.
   - C, D, E are right of B but Only C is immediate right of B
   - C is in the middle of line.
   - A and E are sitting at the extreme end of line.

Q.1 In a park, there are five trees labeled A, B, C, D and E. Tree A is to the right of Tree B. Tree C is to the left of Tree D. Also, Tree D is to the right of Tree E and Tree B is to the left of Tree C. Which tree is in the middle?
(a) Tree B (b) Tree A (c) Tree D (d) Tree C
Ans. (d)

Q.2 There are five houses P, Q, R, S, T, P is the immediate right of Q and T is the immediate left of R and immediate right of P. Q is on the right of S. Which house is located at the extreme left end?
(a) P (b) R (c) S (d) T
Ans. (c)

Q.3 Five students P, Q, R, S and T are positioned in a row facing North. Among them, S is seated between T and Q, and Q is immediately to the left of R. P is immediately to the left of T. Who is positioned at the second right?
(a) S (b) T (d) R (d) Q
Ans. (d)

Q.4 Five students A, B, C, D and E are standing in a row. D is on the right of E, B is on the left of E but on the right of A. D is next to C on his left. The student in the middle is
(a) B (b) E (c) C (d) A
Ans. (b)

Q.5 In a conference, seven participants labeled A, B, C, D, E, F, and G are seated in a row. The following statements are given:
(I) C is seated to the left of B, but on the right of D.
(II) A is seated to the right of B.
(III) F is seated to the right of E, but to the left of D.
(IV) H is seated to the left of E.
Which person is seated at the extreme right end?
(a) H (b) D (c) A (d) None of these
Ans. (c)

Q.6 P, Q, R, S and T are sitting in a line facing West. P and Q are sitting together. R is sitting at the south end and S is sitting at the North end. T is neighbor of Q and R. Who is sitting in the middle?
(a) P (b) Q (c) R (d) S
Ans. (b)

Q.7 Five friends A, B, C, D, and E are sitting on a bench.
I. A is sitting next to B.

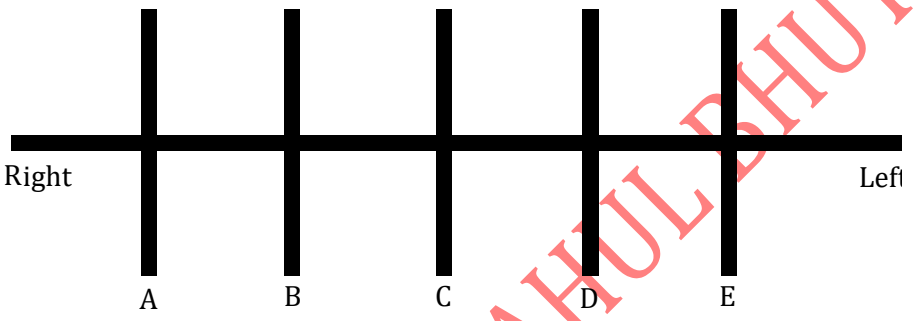
II. C is sitting next to D.
III. D is not sitting with E.
IV. E is the last end of the bench.
V. C is on the 2nd position from the right.
VI. A is on the right of B and E.
VII. A and C are sitting together.
In which position A is sitting?
(a) Between B and D (b) Between B and C
(c) Between E and D (d) Between C and E

Ans. (b)

Q.8 In a college party, 5 girls are sitting in a row. F is to the left of M and to the right of O. R is sitting to the right of N but to the left of O. Who is sitting in the middle?
(a) O (b) R (c) P (d) M

Ans. (a)

When you know the direction of facing. (For example – upward, Downward, East, north, south, west etc……) If it faced South, then



Q.9 P, Q, R, S, T, U, V and W are sitting in a row facing south.
a. P is fourth to the right of T
b. W is fourth to the left of S
c. R and U, which are not at the ends, are neighbours of Q and T respectively.
d. W is next to the left of P and P is the neighbor of Q.
Who are sitting at the extreme ends?
(a) T and S (b) T and P (c) P and S (d) Q and T

Ans. (a)

Q.10 Five boys are standing in a row facing East. Pavan is left of Tavan. Vipin and Chavan to the left of Nakul. Chavan is between Tavan and Vipin. Vipin is fourth from the left, then how far is Tavan to the right?
(a) First (b) Second (c) Third (d) Fourth

Ans. (d)

Q.11 Five boys A_1, A_2, A_3, A_4, A_5 are sitting in a stair in the following way:
(i) A_5 is above A_1
(ii) A_4 is above A_2
(iii) A_2 is above A_1
(iv) A_4 is between and A_2 and A_3
Who is in the lowest position of the stairs?
(a) A_1 (b) A_3 (c) A_5 (d) A_2

Ans. (b)

Q.12 In a residential complex, five families named A, B, C, D, and E are living in a multi-storeyed building. The following statements are given:
Family A lives in a flat above Family B.
Family C lives in a flat below Family D.
Family B lives in a flat above Family D.
Family E lives in a flat below Family C. Which family lives in the middle?
(a) Family A (b) Family B (c) Family C (d) Family D

Ans. (d)

Double Row Arrangement: In these questions, there will be two groups of persons. You have to arrange one group in one row and the other group in the other row. The persons in these rows normally face each other.

Right A B C Left

Left P Q R Right

Q.13 Six persons A, B, C, D, E and F are sitting in two rows. Three in each row.
1. E is not at the end of any row
2. D is second to the left of F
3. C, the neighbour of E, is sitting diagonally opposite to D
4. B is the neighbour of F.

Which of the following are in one of the two rows?
(a) D, B, and F (b) C, E and B (c) A, E and F (d) F, B and C

Ans. (a)

Q.14 Eight persons P to W are sitting in front of one another in two rows. Each row has four persons. P is between U and V and facing North Q, who is to the immediate left of M is facing W. R is between T and M and W is to the immediate right of V. Who is to the immediate right of R?
(a) M (b) U (c) M or T (d) None of these

Ans. (d)

Q.15 Six individuals named A, B, C, D, E, and F are seated in front of one another facing each other in two rows. Each row has three people. The following statements provide their seating arrangement:
(I) A is seated between D and F and is facing South.
(II) B, who is to the immediate left of E, is facing F.
(III) C is seated to the immediate right of E. Who is sitting in front of C?
(a) E (b) B (c) D (d) C

Ans. (c)

Q.16 Six persons A, B, C, D, E and F are sitting in two rows with three persons in each row. Both rows are in front of each other. E is not at the end of the any row and D is second left to the F, C is neighbor of E and diagonally opposite to D if B is neighbour F who is in front of C then who is sitting diagonally to F?
(a) C (b) E (c) A (d) D

Ans. (c)

- **Circular Arrangement:**
  In the circular arrangement, we need to arrange around a circular table and check if the arrangement is to the left or right or facing outwards or inwards.

Q.17 Four girls A, B, C, D are sitting around a circle facing the centre. B and C infront of each other, which of the following is definitely true?
(a) A and D infront of each other (b) A is not between B and C

(c) D is left of C (d) A is left of C

Ans. (a)

Q.18 Parikh is sitting between Narendra and Babita, Charu is to the left of Babita, Pankaj is sitting between Charu and Ashma, they all sitting around a circle facing the center then who is sitting to the right of Babita?

(a) Parikh (b) Ashma (c) Charu (d) Narendra

Ans. (a)

Q.19 Bunty, Dev, Manav, Kavya, Payal, Qasturba, Wasir and Himmat are sitting around a circle facing at the center. Manav is to the immediate right of Bunty who is 4th to the right of kavya. Payal is 2nd to the left of Bunty and is 4th to the right of Wasir. Qasturba is 2nd to the right of dev who is 2nd to the right of Himmat. Who is 3rd to the right of Bunty?

(a) Wasir (b) Manav (c) Himmat (d) None of these

Ans. (c)

Q.20 Eight executives H, I, J, K, L, M, N and P are sitting around a circular table for a meeting. M is third to the right of J and second to the left of K. H is to the immediate left of I. P is to the immediate right of K, L is third to the left of N. Find who will be a neighbour to I?

(a) H and M (b) L and K (c) H and J (d) M and J

Ans. (a)

Q.21 Six friends are seated in a circular arrangement, all facing the center. Deepa is positioned between Prakash and Pankaj. Priti is located between Mukesh and Lalit and Mukesh is sitting right to Priti. Prakash and Mukesh are seated opposite each other. Who is sitting immediate left to Prakash ?

(a) Mukesh (b) Deepa (c) Pankaj (d) Lalit

Ans. (b)

Q.22 Six girls, named P, Q, R, S, T, and V, are seated in a circle, all facing the center. The following statements are given:

A. T is not seated between Q and S but instead, between two other individuals.

B. P is positioned immediately to the left of V.

C. R is located four seats to the right of P.

Which of the following statements is not true?

(a) V is seated just to the right of P.

(b) T is seated just to the right of V.

(c) R is positioned second to the left of T.

(d) P is seated second to the right of R.

Ans. (c)

Q.23 Eight girls are seating around a square table, they are facing centre,

- A sits 6th to the left of B, who sits third to the right of F.
- E sits 2nd to the left of F.
- G sits in between A and B.
- C sits immediate left of F, who is immediate left of D.

(I) Who sits second to the left of D?

(a) E (b) A (c) C (d) F

Ans. (c)

(II) What is the position of E with the Respect to H?

(a) 2nd to the left (b) 2nd to the right

(c) Fourth to the right (d) Third to the left

Ans. (c)

DIRECTION (Q24 – Q27):

I. A, B, C, D, E, F, G, and H are sitting in a row facing North.
II. A is fourth to the right of E.
III. H is fourth to the left of D.
IV. C and F, who are not at the ends are neighbours of B and E, respectively
V. H is next to the left of A and A is the neighbours of B.

Q.24 Who are sitting at the ends?
(a) E and C (b) E and D (c) G and D (d) None of these
Ans. (b)

Q.25 Which of the following statements is not true?
(a) H is second to the right of F.
(b) E is fourth to the left of A.
(c) D is fourth to the right of H.
(d) None of these
Ans. (d)

Q.26 Who is/are the neighbor(s) of D?
(a) F alone (b) C alone (c) B and C (d) Cannot be determined
Ans. (b)

Q.27 Which of the following statements is not true?
(a) G is the neighbours of H and F (b) B is the next to the right of A
(c) E is at left end (d) D is next to the right of B
Ans. (d)

Q.28 What is the position of F?
(a) Next to the right of E. (b) Next to the right of G.
(c) Sixth to the right of D (d) Between G and H
Ans. (a)

DIRECTION (Q29 – Q32): Seven friends P, Q, R, S, T, U and V are standing in a straight line facing north but not necessarily in the same order.

- U stands second from the left end of the line.
- Only two people stand between U and T.
- Only one person stands between P and R.
- V stands third to the left of R.
- Neither R nor U is an immediate neighbors of S.

Q.29 Which of the following pairs stand at the extreme ends of the line?
(a) Q, S (b) Q, P (c) Q, T (d) V, S
Ans. (d)

Q.33 What is the position of V with respect to U?
(a) Immediate left (b) Second to the right
(c) Third to the right (d) Immediate right
Ans. (a)

Q.31 Four of the following five form a group as per the given arrangement. Which of the following does not belong to the group?
(a) R Q (b) T P (c) V T (d) U V
Ans. (c)

Q.32 Which of the following is TRUE regarding Q?
(a) P stands second to the left of Q.

(b) S is one of the immediate neighbor of Q.
(c) Only one person stands between Q and T.
(d) Q stands at one of the extreme ends of the line.

Ans. (c)

Q.33 In a gathering seven members are sitting in a row. 'C' is sitting left to 'B' but on the right to 'D'. 'A' is sitting right to 'B', 'F' is sitting right to 'E' but left to 'D'. 'H' is sitting left to 'E'. Find the person sitting in the middle.

(a) C (b) D (c) E (d) F

Ans. (b)

Q.34 Six children A, B, C, D, E and F are standing in a row. B is between F and D. E is between A and C. A does not stand next to either F or D. C does not stand next to D. F is between which of the following pairs of children?

(a) B and E (b) B and C (c) B and D (d) B and A

Ans. (b)

Q.35 Five senior citizens are living in a multi-storeyed building. Mr. Manu lives in a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat above Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives the topmost flat?

(a) Mr. Lokesh (b) Mr. Gaurav (c) Mr. Manu (d) Mr. Rakesh

Ans. (c)

Q.36 There are eight books kept one over the other. Two books are on Organisation Behaviour, two books on TQM, three books on Industrial Relations and one book is on Economics. Counting from the top, the second, fifth and sixth books are on Industrial Relations. Two books on Industrial Relations are between two books on TQM. One book of Industrial Relations is between two books on Organizational Behaviour while the book above the book of Economics is a book of TQM. Which book is the last book from the top?

(a) Economics
(b) TQM
(c) Industrial Behaviour
(d) Organizational Behaviour

Ans. (a)

Directions (Q37 – Q40): A, B, C, D, E, F and G are sitting in a row facing North:

I. F is to the immediate right of E
II. E is 4th to the right of G
III. C is the neighbour of B and D
IV. Person who is third to the left of D is at one of ends

Q.37 Who is at immediate right of D?

(a) E and F only (b) G, B, and C
(c) E, F and A (d) G and B only

Ans. (c)

Q.38 Who are neighbours of F?

(a) A and E (b) E and A (c) D and E (d) B and D

Ans. (b)

Q.39 The third person from the right extreme end is

(a) E (b) D (c) C (d) F

Ans. (a)

Q.40 Person immediately to the left of B is

(a) C (b) G (c) F (d) D

Ans. (b)

Q.41 Four children are sitting in a row. A is occupying a seat next to B but not next to C. If C is not sitting next to D? Who is occupying the seat adjacent to D?

(a) B (b) B and A

(c) Not enough information (d) A

Ans. (d)

Q.42 Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. Who is sitting immediate right to Reeta?

(a) Seema (b) Rani (c) Bindu (d) Mary

Ans. (d)

Q.43 5 children are sitting in a row. S is sitting next to P but not T. K is sitting next to R, who is sitting on the extreme left. T is not sitting next to K. Who are sitting adjacent to S?

(a) K & P (b) R & P (c) only P (d) P & T

Ans. (a)

Directions (Q44 – Q47): Four Indian, A, B, C and D and four Chinese E, F, G and H are sitting in a circle around a table facing each other in a conference. No two Indians or Chinese are sitting side by side, C who is sitting between G and E is facing D, F is between D and A and facing G, H is to the right of B.

Q.44 Who is sitting left of A?

(a) E (b) F (c) G (d) H

Ans. (a)

Q.45 Who is sitting third right of D?

(a) A (b) C (c) G (d) E

Ans. (d)

Q.46 Person sitting between B & C is

(a) G (b) H (c) A (d) E

Ans. (a)

Q.47 Looking clockwise Person sitting second next to H is

(a) A (b) F (c) E (d) B

Ans. (b)

Q.48 Five boys A, B, C, D and E are sitting in a row. A is to the right of B and E is to the left of B but to the right of C. A is to the left of D. Who is second from the left end?

(a) D (b) A (c) E (d) B

Ans. (c)

Q.49 Five senior citizens are living in a Multi-Storeyed building. Mr. Muan lives in a flat above Mr. Ashokan, Mr. Lokesh in a flat below Mr. Gaurav, Mr. Ashokan lives in a flat below Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who lives in the top most flat?

(a) Mr. Lokesh (b) Mr. Gaurav

(c) Mr. Muan (d) Mr. Rakesh

Ans. (c)

Directions (Q50 – Q53): Statement: In a straight line there are six person sitting in a row. B is between F and D. E is between A and C. A does not stand next to F or D, C does not stand next to D.

Q.50 F is between which of the following persons?

(a) B and E (b) B and C (c) B and D (d) B and A

Ans. (b)

Q.51 Who are sitting at extreme ends?

(a) B and E (b) D and E (c) D and A (d) B and E

Ans. (c)

Q.52 Who is fourth from the left extreme end?

(a B (b) C (c) B (d) F

Ans. (d)

Q.53 Which group is followed from given information?

(a) F C E (b) F E A (c) D F E (d) F E A

Ans. (a)

Q.54 5 persons are standing in a line. One of the 2 persons at the extreme ends is a professor and the other a businessman. An advocate is standing to the right of student. An author is to the left of the businessman. The student is standing between the professor and advocate. Counting from the left, the author is at which place?

(a) 2nd (b) 3rd (c) 4th (d) None of these

Ans. (c)

Q.55 Four Indian A, B, C and D and four Chinese E, F, G and H are sitting in a circle around a table facing each other in the conference. No two Indians or Chinese are sitting side by side, C who is sitting between G and E is facing D, F is between D and A and facing G, H is to the left of B.Who is sitting left to A?

(a) E (b) F (c) G (d) H

Ans. (a)

Q.56 A, B, C and D are playing cards, A and B are partners. D faces towards the North. If A faces west, then who faces south?

(a) C (b) B (c) D (d) Data is inadequate

Ans. (a)

Q.57 Eight persons A, B, C, D, E, F, G and H are sitting in a line. E sits second right to D. H sits fourth left to D. C and F are immediate neighbors, but C is not immediate neighbor of A. G is not neighbor of E. Only two persons sit between A and E. The persons on left end and right end respectively are

(a) G and E (b) B and E (c) H and E (d) G and B

Ans. (a)

PREVIOUS YEAR QUESTIONS

SEATING ARRANGEMENT

Q.1 Seven friends O, P, Q, R, S, T and U are watching movie sitting in a row, S is sitting at one extreme end. Q is sitting second to the right of S. P is sitting between O and Q. Both U and O are not sitting at extreme end. R is sitting immediate left of T. Who is sitting in the middle? [Dec. 2023]

(a) O (b) P

(c) Q (d) U

Ans. (b) P

Q.2 A, B, C, D, E, F, G, H are sitting in a circle facing the center. D sits 3rd to left of A. E sits to the immediate right of A.B is third to left of D.G is second to right of B. C is neighbourer of B. C is 3rd to left of H. Who is sitting exactly in between F and E [Dec.2023]

(a) C (b) E

(c) H (d) A

Ans. (d) A

Q.3 Five persons are living in a five-story building. Mr. Mahesh lives in flat above Mr. Ashok. Mr. Lokesh lives in a flat below Mr. Gaurav and Mr. Rakesh lives in a flat below Mr. Lokesh. Who possibly lives in the ground floor? [Dec. 2023]

(a) Mr. Rakesh
(b) Mr. Lokesh
(c) Mr. Mahesh
(d) Mr. Gaurav

Ans. (a) Mr. Rakesh

Nine friends, J, K, L, M, N, O, P, Q and R, are sitting in a row. L is to the right of M and is at the third place to the right of N. K is at one end of the row. Q is seated adjacent to both O and P. O is at the third place to the left of K. J is right next to L and the left of O.

Answer the following questions: (Q. No. 62 and Q. No. 63)

Q.4 Who is immediate left to M? [Dec. 2023]

(a) N
(b) R
(c) L
(d) J

Ans. (b) R

Q.5 Who is sitting at the center of the row?" [Dec. 2023]

(a) I
(b) J
(c) O
(d) Q

Ans. (b) J

Q.6 Seven friends A, B, C, D, E, F and G are sitting in a row. E is sitting at one extreme end. C is sitting second to E. B is sitting between A and C. G is not sitting at any extreme end. A is not at any extreme end. D is sitting immediate to F. Who is sitting in the middle? [June 2023]

(a) C
(b) D
(c) G
(d) B

Ans. (d) B

Q.7 Pran, Komal, Ravi, Shalu, Trilok, Urvi, Vasu and Walter are sitting in a row facing north.

(i) Pran is fourth to the right of Trilok
(ii) Walter is fourth to the left of Shalu
(iii) Ravi and Urvi, which are not at the ends, are the neighbors of Komal and Trilok respectively
(iv) Walter is immediate left of Pran and Pran is the neighbor of Komal Identify who is sitting at the extreme ends? [June 2023]

(a) Pran and Walter
(b) Trilok and Urvi
(c) Trilok and Shalu
(d) Shalu and Pran

Ans. (c) Trilok and Shalu

Q.8 Six friends A, B, C, D, E and F are sitting around a circular table facing towards the centre of the circle. E is not sitting between B and D. A sit to the left of F and C is fourth right of A. D is immediate right of E. Who sits second to the right of F ? [June 2023]

(a) C
(b) A
(c) D
(d) B

Ans. (c) D

Q.9 P, Q, R, S and T are sitting in a line facing West. P and Q are sitting together. R is sitting at south end and S is sitting at North end. T is neighbor of Q and R. Who is sitting the middle? [Dec 2022]

(a) P
(b) Q
(c) R
(d) S

Ans. (b) Q

Q.10 Six persons A, B, C, D, E and F are sitting in two rows with three persons in each row. Both rows are in front of each other. E is not at the end of the any row and D is second left to the F, C is neighbor of E and diagonally opposite to D. If B is neighbor F who is in front of C then who is sitting diagonally to F? [Dec 2022]

(a) C
(b) E
(c) A
(d) D

Ans. (c) A

Q.11 Six friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the center of the hexagonal. P is second to the left of U. Q is neighbor of R and S. T is second to the left of S. Which one is sitting opposite to P? [June 2022]

(a) R
(b) Q
(c) T
(d) S

Ans. (d) S

Q.12 Five persons are sitting on a bench to be photographed, S is to the left of N and to the right of B. M is to the right of N. R is between N and M. Who is sitting immediate right to R? [June 2022]

(a) B
(b) N
(c) M
(d) S

Ans. (c) M

Q.13 Eight persons E, F, G, H, I, J, K and L are seated around a square table, facing table - two on each side. J is between L and F; G is between I and F; H a lady member is second to the left of J; F a male member is seated opposite to E, a lady member. There is a lady member between F and I. who among the following is to the immediate left of F? [June 2022]

(a) G
(b) I
(c) J
(d) H

Ans. (c) J

Q.14Six friends Surya. Bhanu. Dinkar, Ravi. Suraj and Dinesh Are sitting in a circle and are facing the centre of the circle, Dinesh is between Dinkar and Suraj. Bhanu is between Ravi and Surya. Dinkar and Ravi are opposite to each other. Who are the immediate neighbors of Ravi? [June 2022]

(a) Suraj and Dinesh
(b) Dinkar and Bhanu
(c) Surya and Dinesh
(d) Bhanu and Suraj

Ans. (d) Bhanu and Suraj

Q.15In a line, P is sitting 13th from left. Q is sitting 24th from the right and 3rd left from P. How many people are sitting in the line? [Dec 2021]

(a) 34
(b) 31
(c) 32
(d) 33

Ans. (d) 33

Q.16Persons M, N, O, P, Q, R, S and T are sitting on a compound wall facing North, O sits fourth left of S; P sits second to the right of S; only two people sit between P and M; N and R are immediate neighbours of each other. N is not an immediate neighbour of M; T is not a neighbour of P. How many persons are seated between M and Q ? [Dec 2021]

(a) 1
(b) 2
(c) 3
(d) 4

Ans. (a) 1

Q.17Four Ladies A, B, C and D and four Gentlemen E, F, G and H are sitting in a circle around a table facing each other.

i) No two Ladies or gentlemen are sitting side by side.
ii) C, who is sitting between G and E, facing D.
iii) F is between D and A and facing G.
iv) H is to the right of B.

Who is immediate neighbor of B ? [Dec 2021]

(a) G and H
(b) E and F
(c) E and G
(d) A and B

Ans. (a) G and H

Q.18Five persons A, B, C, D, E are sitting in a row. A sit S left to C and C sit S left to B. E sits rights to B, D sits in between E and B. Who is sitting in the middle? [Dec 2021]

(a) B
(b) C
(c) E
(d) D

Ans. (a) B

Q.19Six children, named as P, Q, R, S, T and U, are sitting in a row, Q is between U and S. T is between P and R; P does not sit next to either U or S; B does not sit next to S. So, U is sitting between the pairs…… of children.

[Dec 2021]

(a) Q and T
(b) Q and R
(c) Q and S
(d) Q and P

Ans. (b) Q and R

Q.20Six friends P, Q, R, S, T and U are sitting around the hexagonal table each at one corner and are facing the centre of the hexagonal. P is second to the left of U. Q is neighbor of R and S. T is second to the left of S. Which one is sitting opposite to S ? [July 2021]

(a) R
(b) P
(c) Q
(d) T

Ans. (b) P

Q.21Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary. Who is sitting immediate right to Reeta ?

[July 2021]

(a) Seema
(b) Rani
(c) Bindu
(d) Mary

Ans. (d) Mary

Q.22A, B, C, D, E, F and G are sitting in a row facing North :

i) F is to the immediate right of E.
ii) E is 4th to the right of G.
iii) C is the neighbor of B and D.
iv) Person who is third to the left of D is at one of ends.

Who are to the right of D? [July 2021]

(a) E and F only
(b) G, B and C
(c) E, F and A
(d) G and G only

Ans. (c) E, F and A

SUMMARY

- We have to arrange a group of person according to given conditions, and we classify the arrangement into 4 types:
  - Linear arrangement
  - Two row arrangement

- Circular arrangement
- Polygon arrangement

- **Linear arrangement:** Here we have to arrange given people in a line.
  - Here, either we know the direction of facing; in this case we consider all possibilities.
  - Or we know the direction of facing such as (facing upward, downward, east, west, …)
- **Two row arrangement:** There will be two groups of persons. Here, we arrange one group in one row and another group in another row. and persons in these rows face each other.
- **Circular arrangement:** In the circular arrangement, we need to arrange around a circular table and check if the arrangement is to the left or right or facing outwards or inwards.

- Blood relation refers to the connection between people based on their shared family ancestry, not through marriage or other relationships.
- In blood relation questions, certain information about the members of a family is given, and based on that information, you need to determine the relationship between specific family members.

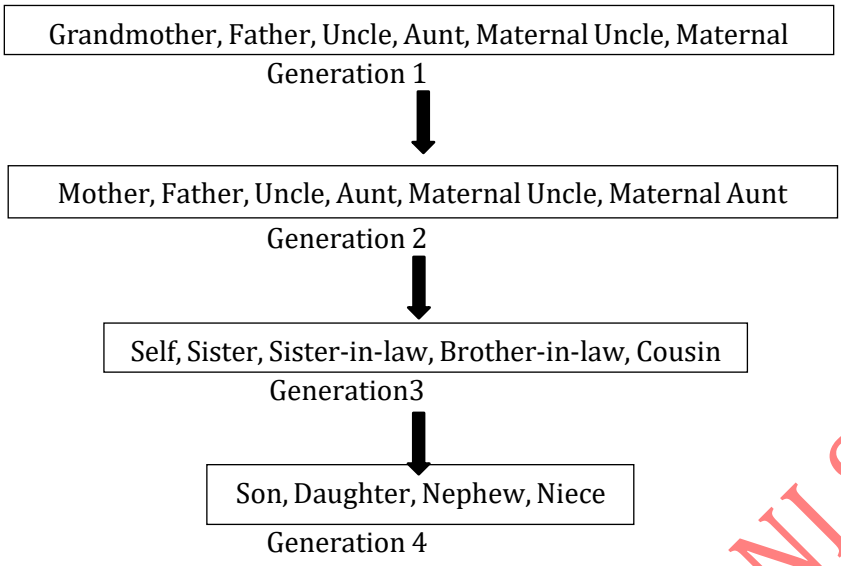
S.No	Let's see some Relations:
1.	Mother's or Father's Father: Grandfather (Maternal Grandfather/Paternal Grandfather)
2.	Mother's or Father's Mother: Grandmother (Maternal Grandmother/Paternal Grandmother)
3.	Mother's or Father's Brother: Uncle
4.	Mother's or Father's Sister: Aunt
5.	Wife's Father or Husband's Father: Father-in-law
6.	Wife's Mother or Husband's Mother: Mother-in-law
7.	Sons of Wife: Daughter-in-law
8.	Daughter's Husband: Son-in-law
9.	Husband's or Wife's Brother: Brother-in-law
10.	Husband's or Wife's Sister: Sister-in-law
11.	Brother's or Sister's Daughter: Niece
12.	Brother's or Sister's Son: Nephew
13.	Uncle's or Aunt's Son or Daughter: Cousin

Note: Husband and Wife can be referred to as "spouse" only.

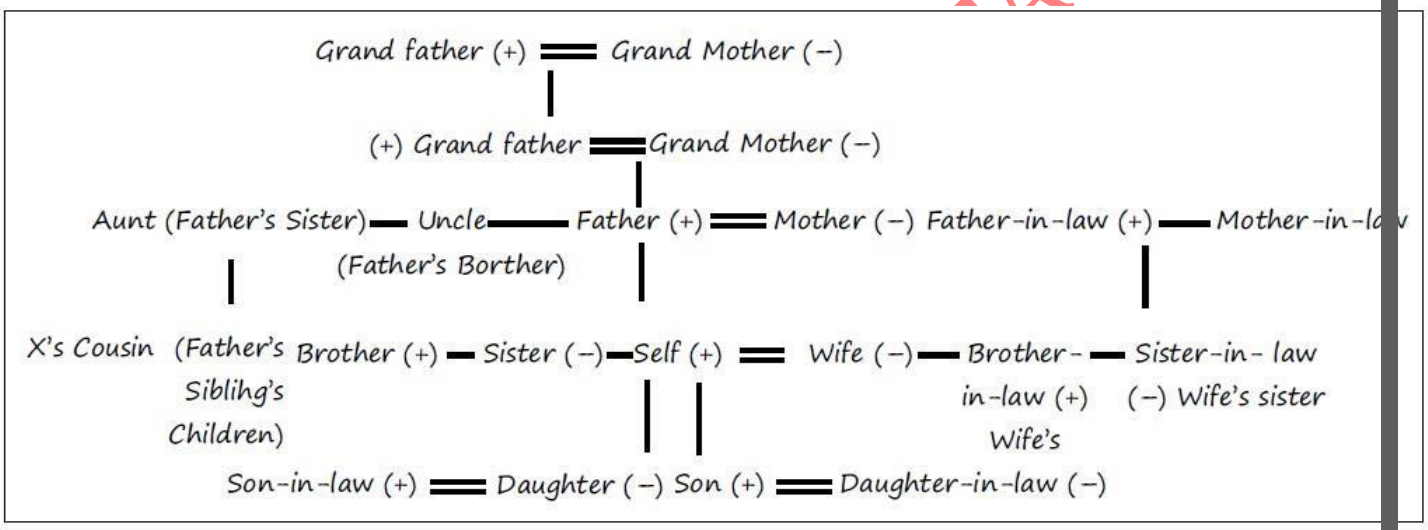
Now, if we directly attempt to solve the questions, it can become complicated and confusing. Therefore, it is essential to follow a systematic approach.

Symbols	Representation
Plus +	Male
Minus –	Female
Horizontal Line ——	Same generation
Verticle Line |	One generation gap
═	Husband/Wife

Also keep in mind about different generation.



Now if all above is clear we can draw the main family diagram called as Family Tree.



—	Sibling's Relation
=	Husband – Wife Relation
|	Next Generation
(+) Male	(-) Female

By using all of the above, we can solve questions of blood relatives.

SOME TIPS BEFORE WE START

- **Break the question into small parts:** Long statements can be confusing. To avoid confusion, break down each statement into smaller sub-statements and analyses them individually before proceeding to the next part of the question.
- **Avoid assuming gender based on names:** It's important not to make assumptions about a person's gender solely based on their name. Read the question carefully and gather information about relationships before drawing any conclusions about gender.

- **Draw clear diagrams:** Visual representation can greatly aid in solving blood relation questions. Draw a clear and organized family tree diagram as you solve the question. Place individuals of the same generation at the same level, creating a hierarchical structure. This will help you visualize and understand the relationships more easily.

BASED ON PUZZLES

Q.1 If A is the husband of B, B is the sister of C, and C is the daughter of D, then what is the relationship of A to Do?

(a) Father-in-law (b) Father (c) Uncle (d) Brother

Ans. (a)

Q.2 A, B and C are sisters. D is the brother of E and E is the daughter of B. How is A related to D?

(a) Sister (b) Cousin (c) Niece (d) Aunt

Ans. (d)

Q.3 Given that:

I. A is the mother of B.
II. C is the son of A.
III. D is the brother of E.
IV. E is the daughter of B.

Who is grandmother of D?

(a) A (b) B (c) C (d) D

Ans. (a)

Q.4 If P is the husband of Q and R is the mother of S and Q. What is R to P?

(a) Mother (b) Sister (c) Aunt (d) Mother-in-law

Ans. (d)

Q.5 A and B are married couples. X and Y are brothers. X is the brother of A. How is Y related to B?

(a) Brother-in-law (b) Brother
(c) Cousin (d) None of these

Ans. (a)

Q.6 I. F is the brother of A.
II. C is the daughter of A.
III. K is the sister of F.
IV. G is the brother of C.

Who is the uncle of G?

(a) A (b) C (c) F (d) K

Ans. (c)

Q.7 E is the son of A, D is the son of B, E is married to C. C is B's daughter. How is D related to E?

(a) Brother (b) Uncle (c) Father-in-law (d) Brother-in-law

Ans. (d)

Q.8 P is the father of T. T is the daughter of M. M is the daughter of K. What is P to K ?

(a) Father (b) Father-in-law (c) Brother (d) Son-in-law

Ans. (d)

Q.9 Ram and Mohan are brothers; Shankar is Mohan's father. Chhaya is Shankar's sister. Priya is Shankar's niece. Shubhra is Chhaya's granddaughter. Then Ram is Shubhra's

(a) Brother (b) Uncle (c) Cousin (d) Nephew

Ans. (b)

Q.10 Aryan is the brother of Bhavana. Deepak is the brother of Chetna. Bhavana is the daughter of Deepak. How is Aryan related to Chetna?

(a) Nephew (b) Father (c) Son (d) Brother

Ans. (a)

Q.11 C is wife of B. E is the son of C, A is the brother of B and father of D. What is the relationship of E to D?

(a) Mother (b) Sister (c) Brother (d) Cousin

Ans. (d)

Q.12 If X is the wife of Y and Z is the father of W and Y. What is Z to X?

(a) Father (b) Brother (c) Uncle (d) Father-in-law

Ans. (d)

Q.13 A boy goes to see a movie and sees a man sitting to his left and finds out that the man is his relative. The man is the husband of the sister of his mother. How is the man related to the boy?

(a) Nephew (b) Brother (c) Uncle (d) Father

Ans. (c)

BASED ON DIALOGUE OR CONVERSATION

Similarly, few dialogues and in parallel relations to it:

- Only son of my Grandfather: My Father
- Only son of my Grandmother: My Father
- Only daughter of my Grandmother: My Mother
- Only daughter of my Grandfather: My Mother
- Sister of my Mother: My Aunt
- Son /daughter of my Husband: My Son /Daughter
- Son /Daughter of my Wife: My Son /Daughter
- Only Daughter of my Grandfather's only Son: My Sister/Self
- Grandmother of my Father's only Son: My Grandmother
- Father of my Daughter's Father: My Father/Father-in-law
- Father of my Son's Father: My Father/Father-in-law
- My Son's Sister: My Daughter
- Daughter-in-law of Grandmother of my Father's only Son: My Mother
- A is the Father of B but B is not the Son of A: B is Daughter of A

Q.14 B is the husband of P. Q is the only grandson of E, who is the wife of D and mother–in-law of P. How is B related to D?

(a) Nephew (b) Cousin (c) Son –in-law (d) Son

Ans. (d)

Q.15 Maya introduces Rajesh as the son of the only daughter of her mother's brother. How is Maya related to Rajesh?

(a) Cousin (b) Sister (c) Aunt (d) Niece

Ans. (c)

Q.16 Pointing to a photograph, a man said "His mother's husband's sister is my aunt," Then what is the relation between a man and him?

(a) Son (b) Uncle (c) Nephew (d) Brother

Ans. (d)

Q.17 Pointing to the old man Kailash said "his son is my son's uncle". How is Kailash related to an old man?

(a) Brother (b) Either son or son in law

(c) Father (d) Grand father

Ans. (b)

Q.18 Sanjay's mother said to him, "My brother has a son whose daughter is Tina." How is Tina related to Sanjay?
(a) Cousin (b) Sister (c) Aunt (d) Niece
Ans. (d)
Q.19 Pointing to a photograph Vikas said "She is the daughter of my grandfather's only son". How is the lady in the photograph related to Vikas in the photograph?
(a) Father (b) Brother (c) Sister (d) Mother
Ans. (c)
Q.20 Rohan's mother said to Rohan "My mother has a son whose son is Vansh". Rohan is related to Vansh as
(a) Uncle (b) Cousin (c) Nephew (d) Grandfather
Ans. (b)
Q.21 Kalyani is the mother-in-law of Veena, who is the sister-in-law of Ashok. Dheeraj is the father of Sandeep, the only brother of Ashok. How is Kalyani related to Ashok?
(a) Mother (b) Wife (c) Mother-in-law (d) Aunt
Ans. (a)
Q.22 Vinod introduces Vishal as the son of the only brother of his father's wife. How is Vinod related to Vishal?
(a) Cousin (b) Brother (c) Son (d) Uncle
Ans. (a)
Q.23 A and B are sisters. A is the mother of D. B has a daughter C who is married to F. G is the husband of A. How is C related to D?
(a) Cousin (b) Niece (c) Aunt (d) Sister-in-law
Ans. (a)
Q.24 Rahul and Robin are brothers. Promod is Robin's father. Sheela is Pramod's sister. Prema is Promod's niece. Shubha is Sheela's grand-daughter. How is Rahul related to Shubha?
(a) Brother (b) Cousin (c) Uncle (d) Nephew
Ans. (c)
Q.25 Suresh introduces a man as "He is the son of the woman who is the mother of the husband of my mother". How is Suresh related to the man?
(a) Uncle (b) Son (c) Cousin (d) Grandson
Ans. (b)
Q.26 Pointing to a picture, Abhishek said, she is the mother of my son's wife's daughter. How is the lady related to Abhishek?
(a) Uncle (b) Cousin (c) Daughter (d) None
Ans. (d)
Q.27 Ramu's mother said to Ramu,"My mother has a son whose son is Achyut". How is Achyut relation to Ramu?
(a) Uncle (b) Cousin (c) Brother (d) Nephew
Ans. (b)
Q.28 A woman said, "The man in the picture is my nephew's maternal grandfather." How is the man in the picture related to the woman?
(a) Brother (b) Son (c) Grandfather (d) Father
Ans. (d)
Q.29 Lalita said to Tina, "You are the daughter-in-law of the grandmother of my father's only son". How is Lalita related to Tina?
(a) Aunt (b) Sister (c) Mother (d) Indeterminable
Ans. (d)

Q.30 Pointing to a photograph, Sonia said, "His mother's only daughter is my mother". How does Sonia relate to that man?

(a) Nephew (b) Sister (c) Wife (d) Niece

Ans. (d)

Q.31 Pointing to a man in a photograph, a woman said, "the father of his brother is the only son of my grandfather", how is the woman related to the man in the photograph?

(a) Mother (b) Aunty (c) Daughter (d) Sister

Ans. (d)

SYMBOLICALLY CODED RELATION

- In symbolically coded relations questions, the relationships between certain members of the family are indicated by symbols like @, £, $, %, &, +, –, etc. The candidate is required to understand the codes/symbols assigned to the relationships and decipher the other relationship expressed to such codes or express another given relationship in the symbolic code form.

Q.32 Read the following information carefully and answer the questions given below:

1. A + B means A is the son of B;
2. A – B means A is the wife of B;
3. A × B means A is the brother of B;

What does P + R - Q mean?

(a) Q is the uncle of P (b) Q is the son of P

(c) Q is the father of P (d) Q is the brother of P

Ans. (c)

Q.33 Read the following carefully and answer the questions given below:

1. A $ B means A is the father of B;
2. A # B means A is the sister of B;
3. A * B means A is the daughter of B and
4. A @ B means A is the brother of B.

Which of the following indicates that M is the wife of Q ?

(a) Q $ T # T @ M (b) Q $ R @ T # M

(c) Q $ R * T # M (d) Q $ R @ T * M

Ans. (d)

Q.34 Read the following information and answer the following question:

1. A $ B means A is mother of B
2. A # B means A is father of B
3. A @ B means A is husband of B
4. A % B means A is daughter of

Which of the following expressions indicate 'R is the sister of H'?

(a) H $ D @ F # R (b) H % D @ F $ R

(c) R $ D @ F # H (d) R % D @ F $ H

Ans. (d)

Answer the below question based on the following information:

1. A + B means A is the mother of B.
2. A - B means A is the sister of B.
3. A * B means A is the father of B.
4. A ? B means A is the brother of B.

Q.35 Which of the following means Q is the grandfather of P?

(a) P + N * M * Q (b) Q * N * M + P

(c) Q ? M ? N * P (d) None of these

Ans. (a)

Q.36 If P + Q means P is the mother of Q. P ÷ Q means P is the father of Q. P – Q means P is the sister of Q. Then which of the following relationship shows that M is the daughter of R?

(a) R ÷ M + N (b) R + N + M

(c) R + M ÷ N (d) None of these

Ans. (a)

Q.37 Which of the following means that N is the maternal uncle of M?

(a) N ? P – L + E – M (b) N – Y + A ? M

(c) M – Y * P – N (d) N ? C + F * M

Ans. (a)

Q.38 A reads a book and finds the name of the author familiar. The author 'B' is the paternal uncle of C.C is the daughter of A. How is B related to A?

(a) Brother (b) Sister (c) Father (d) Uncle

Ans. (a)

Q.39 P and Q are brother's, R and S are sisters. P's son is R's brother. How is Q related to R?

(a) Uncle (b) Brother (c) Father (d) Grandfather

Ans. (a)

Q.40 Six persons are seen together in a group. They are A, B, C, D, E and F, B is the brother of D, but D is not brother of B. F is brother of B, C and A are married together, F is son of C, but C is not mother of F, E is brother of A, then the number of female members in the group is

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

Q.41 Pointing to a girl, Prasan said, she is the only granddaughter of my wife's grandfather's only child. How is the girl related to Prasan?

(a) Sister (b) Niece (c) Daughter (d) Cannot be determined

Ans. (c)

Q.42 G is the father of K, who is the brother of B. K is married to U. B is the daughter of C. D is the father of U and M is the only son of D. How is U related to C?

(a) Daughter (b) Daughter-in-law (c) Grand Daughter (d) Son–in– law

Ans. (d)

Directions (Q43 – Q44): These are seven members A, C, D, E, F, G and H in a family. There are two fathers, one mother, two sisters and four brothers. E is the sister–in–law of D. G is a daughter of C. F is the brother of E. A is a grandfather of G. E is a mother of H?

Q.43 How many are related to A?

(a) Grandson (b) Granddaughter

(c) Son (d) Cannot be determined

Ans. (a)

Q.44 How many male members in the family?

(a) 4 (b) 5 (c) 3 (d) Data inadequate

Ans. (b)

Directions (Q45 – Q46): There are six children taking part in an essay competition, namely A, B, C, D, E, and F. A and E are brothers. F and D are the sisters of E. C is the only son of A's uncle. B and D are the brothers of C's father.

Q.45 How is D related to A?

(a) Uncle (b) Sister

(c) Niece (d) Cousin

Ans. (b)

Q.46 How many male competitors are there?

(a) 6 (b) 5 (c) 4 (d) 3

Ans. (d)

Directions (Q47 – Q49):

1. P × Q means P is the brother of Q.
2. P ÷ Q means Q is the mother of P.
3. P – Q means P is father of Q.
4. P + Q means Q is the sister of P.

Q.47 Which of the following means M is the daughter of T?

(a) M + N ÷ J – T (b) T – J × R + M

(c) M – J × T ÷ K (d) M + W × R ÷ T

Ans. (b)

Q.48 How is K related to R in the expression R ÷ T + K?

(a) Daughter (b) Sister (c) Niece (d) None of these

Ans. (d)

Q.49 Which of the following means D is grandfather of W?

(a) D – K × T – W (b) D ÷ K × T ÷ W

(c) D – K × T – W (d) D – K × T ÷ W

Ans. (a)

Q.50 "Rahul said, 'She is the daughter-in-law of my father's only child.' How is Rahul related to the girl?"

(a) Uncle (b) Husband (c) Brother (d) None of these

Ans. (b)

Q.51 A woman said, "The man in the picture is my nephew's maternal grandfather." How is the man in the picture related to the woman?

(a) Brother (b) Son (c) Grandfather (d) Father

Ans. (d)

Q.52 In a joint family, there are father, mother, 3 married sons and one unmarried daughter. Out of the sons, two have 2 daughters each and one has a son only. How many female members are there in the family?

(a) 3 (b) 6 (c) 9 (d) 8

Ans. (c)

Q.53 There are six children playing football namely A,B,C,D,E and F, A & E are brothers, F is sister of E, C is the only son of As uncle, B & D are daughter of the brother of C's father. How D is related to A?

(a) Uncle (b) Cousin (c) Nice (d) Sister

Ans. (d)

Q.54 When Rani saw Vinit, she recollected that "He is the brother of my grandfather's son". How is Rani related to Vinit?

(a) Aunt (b) Daughter (c) Sister (d) Niece

Ans. (d)

Q.55 Suman is daughter-in-law of Rakesh and sister-in-law of Rajesh, Ramesh is the son of Rakesh and only brother of Rajesh. Find the relation of Suman with Ramesh.

(a) Sister-in-law (b) Cousin (c) Aunt (d) Wife

Ans. (d)

Q.56 Pointing to a man in the photograph, Khushi says, "This man's son's sister is my mother –in-law," How is Khushi's husband related to the man in the photograph?

(a) Grandson (b) Son (c) Son in law (d) Cousin

Ans. (a)

PREVIOUS YEAR QUESTIONS

BLOOD RELATION

Q.1 There are six persons A, B, C, D, E and F in family. B is the son of C but C is not the mother of B. A and C are a married couple. E is the brother of C. D is the daughter of A. F is the brother B. Who is the mother of B? [Dec. 2023]

(a) A (b) E
(c) D (d) F

Ans. (a) A

Q.2 When Mr. P saw Mr. Q. he recalled, "He is the son of the father of my daughter's mother." Who is Mr. Q to Mr. P? [Dec. 2023]

(a) Brother (b) Cousin
(c) Nephew (d) Brother-in-law

Ans. (d) Brother-in-law

Q.3 Introducing a boy, Akshat said, "His mother is the only daughter of my mother-in-law". How is Akshat related to the boy? [Dec. 2023]

(a) Uncle (b) Father
(c) Brother (d) Husband

Ans. (b) Father

Q.4 Showing the man playing the cricket, Ms. P said, "He is the brother of my uncle's daughter". Who is the man to Ms. P? [Dec. 2023]

(a) Son (b) Cousin
(c) Uncle (d) Brother –in-law

Ans. (b) Cousin

Q.5 In a family, there are six members A, B, C, D, E and F. A & B are married couple. A being the male member. D is only son of C and is brother of A. E is sister of D.B. is daughter –in- law F, whose husband is died. How C is related to B? [Dec. 2023]

(a) Brother (b) Nephew
(c) Brother-in-law (d) Sister –in-law

Ans. Not given in options (Father-in-law)

Q.6 Pointing to a lady, Suresh said " She is the mother of my son's wife's daughter". How is Suresh related to lady? [Dec. 2023]

(a) Uncle (b) Cousin
(c) Daughter –in-law (d) Father-in-law

Ans. (d) Father-in-law

Q.7 Pointing to lady, a man said "The son of her only brother is brother of my wife". How is the lady related to man [Dec. 2023]

(a) Mother –in-law (b) Sister of father –in-law
(c) Mother of Father –in-law (d) Cousin

Ans. (b) Sister of father –in-law

Q.8 If A is the brother of B, B is the daughter of C and D is the father of A, the how is C related to D ? [June 2023]

(a) Husband (b) Wife
(c) Grand daughter (d) Grand father

Ans. (b) Wife

Q.9 Pointing to a photograph, a woman says "This man's son's sister is my mother-in-law." How is the woman's husband related to the man in the photograph? [June 2023]

(a) Son (b) Son-in-law
(c) Grandson (d)Nephew

Ans. (c) Grandson

Q.10 There are six persons A, B, C, D, E and F in a family. A and B are married couple and A is the male member. D is the only son of C who is the brother of A. E is the sister of D. B is the daughter in law of F whose husband has died. Who is the mother of C ? [June 2023]

(a) A (b) E
(c) D (d)F

Ans. (d)F

Q.11 Neelam, who is Deepak's daughter, says to Deepika that, your Mother in law Rekha is younger daughter of Ramlal, who is my Grandfather. How Neelam is related to Deepika? [June 2023]

(a) Cousin (b) Niece
(c) Sister-in-law (d)Aunt

Ans. (c) Sister-in-law

Q.12 Based on the statements given below, find out who is the uncle of P ?

(i) K and J are brothers
(ii) K' s sister is M
(iii) P and N are siblings
(iv) N is the daughter of J [June 2023]

(a) K (b) J
(c) N (d)M

Ans. (a) K

Q.13 X is the husband of Y. W is the daughter of X. Z is husband of W. N is daughter of Z. What is the relationship of Y to N ? [June 2023]
(a) Cousin (b) Mother
(c) Daughter (d) Grandmother

Ans. (d) Grandmother

Q.14 P, Q, R, S, T, U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor who is mother of R and U. Q the lawyer is married to P. P has one son and one grandson. Of the two married ladies one is a housewife. There is also one student and one male engineer in the family. Which of the following is true about the grandson of the family? [June 2023]
(a) He is a lawyer (b) He is an engineer
(c) He is a student (d) He is a doctor

Ans. (b) He is an engineer

Q.15 Pointing to a man in the photograph, Khushi says, "This man's son's sister is my mother –in-law," How is the Khushi's husband related to the man in the photograph? [Dec 2022]
(a) Grandson (b) Son
(c) Son-in-law (d) Cousin

Ans. (a) Grandson

Q.16 Suman is daughter-in-law of Rakesh and sister-in-law of Rajesh. Ramesh is the son of Rakesh and only brother of Rajesh. Find the relation of Suman with Ramesh. [Dec 2022]
(a) Sister-in-law (b) Cousin
(c) Aunt (d) Wife

Ans. (d) Wife

Q.17 Annanya is mother of Satya and Shyam is the son of Bhima. Shiva is brother of Annanya. If Satya is sister of Shyam, how Bhima is related to Shiva? [Dec 2022]
(a) Son (b) Cousin
(c) Brother-in-law (d)Son-in-law

Ans. (c) Brother-in-law

Q.18 When Rani saw Vinit, she recollected that "He is the brother of my grandfather's son". How is Rani related to Vinit? [Dec 2022]
(a) Aunt (b) Daughter
(c) Sister (d) Niece

Ans. (d) Niece

Q.19 In a joint family, there are father, mother, 3 married sons and one unmarried daughter. Out of the sons, two have 2 daughters each one has a son only. How many female members are there in the family ?[Dec 2022]

(a) 3 (b) 6
(c) 9 (d) 5

Ans. (c) 9

Q.20 There are six children playing football namely A, B, C, D, E and F. A & E are brothers. F is sister of E. C is the only son of A's uncle. B & D are daughters of the brother of C's father. How D is related to A ? [Dec 2022]

(a) Uncle (b) Cousin
(c) Niece (d) Sister

Ans. (b) Cousin

Q.21 Suresh's sister is the wife of Ram. Ram is Rani's brother. Ram's father is Madhur. Sheetal is Ram's grandmother. Rama is Sheetal's daughter-in-law. Rohit is Rani's brother's son. Who is Rohit to Suresh ? [Dec 2022]

(a) Brother-in-law (b) Son
(c) Brother (d) Nephew

Ans. (d) Nephew

Q.22 Rani told Jaya, "The girl I met yesterday at the beach was the youngest daughter of the brother-in-law of my friend's mother." How is the girl related to Rani's friend? [June 2022]

(a) Cousin (b) Daughter
(c) Mother (d) Aunt

Ans. (a) Cousin

Q.23 Ravi is a son of Aman's father's sister. Sahil is the son of Divya who is the mother of Gaurav and grand other of Aman. Ashok is the father of Tanya and grand father of Ravi Divya is the wife of Ashok. How is Ravi related to Divya? [June 2022]

(a) Nephew (b) Son
(c) Grandson (d) Father

Ans. (c) Grandson

Q.24 B and C are siblings. M has two children and he is son of E, who is father-in-law of H. H has only one son. C is not grand daughter of E. How's B related to E? [June 2022]

(a) Daughter (b) Son
(c) Grandson (d)Grand daughter

Ans. (d) Grand daughter

Q.25 If X + Y means X is the mother of Y; X – y means X is the brother y: X % Y means X is the father of Y and X × Y means X is the sister of y, Which of the following shows that A is the maternal uncle of B? [June 2022]

(a) B + D × C – A (b) B – D % A
(c) A – C + D × B (d) A + C × D - B

Ans. (c) A – C + D × B

Q.26 A woman going with a boy is asked by another woman about the relationship between them. The woman replied " My maternal uncle and the uncle of his maternal uncle is the same". How is the lady related with that boy? [June 2022]

(a) Grandmother and grandson (b) Mother and son
(c) Brother and sister (d) Aunt and nephew

Ans. (b) Mother and son

Q.27 P, Q, R, S, T, U are 6 members of a family in which there are two married couples. T, a teacher is married to a doctor who is mother of R and U, Q the lawyer is married to P. P has one son and one grandson. Of the two married ladies one is a housewife. There is also one student and one male engineer in the family. Which of the following is true about the granddaughter of the family? [Dec 2021]

(a) She is a lawyer (b) She is an engineer
(c) She is a student (d) She is a doctor

Ans. (c) She is a student

Q.28 R told to M as, "the girl, I met at the beach, was the youngest daughter of the brother-in-law of my friend's mother". How is the girl related to R's friend? [Dec 2021]

(a) Cousin (b) Daughter
(c) Niece (d)Aunt

Ans. (a) Cousin

Q.29 It is given that "A is the mother of B; B is the sister of C; C is the father of D ". How is A related to D? [Dec 2021]

(a) Mother (b) Grandmother
(c) Aunt (d)Sister

Ans. (b) Grandmother

Q.30 Introducing a boy a girl said, "He is the son of the daughter of the father of my uncle". Who is the boy to the girl? [Dec 2021]

(a) Brother (b) Nephew
(c) Uncle (d)Son-in-law

Ans. (a) Brother

Q.31 D is daughter of E. A is son of D. C is a brother of A and B is sister of A. F is brother of D. How F is related to B ? [Dec 2021]

(a) Father-in-law (b)uncle
(c) Brother (d)Mother-in-law

Ans. (b) uncle

Q.32 A is the son of C; C and Q are sisters; Z is the mother of Q and P is the son of Z Which of the following statements is true? [July 2021]

(a) A and P are cousins (b) C and P are sisters
(c) P is the maternal uncle of A (d) A is the maternal uncle of P

Ans. (c) P is the maternal uncle of A

Q.33 Amit said "This girl is the wife of the grandson of my mother". How Amit related to the girl? [July 2021]
(a) Father-in-law (b) Grandson
(c) Father (d) Son

Ans. (a) Father-in-law

Q.34 Shyam's mother said to Shyam "my mother has a son whose son is Ram". Shyam is related to Ram as [July 2021]
(a) Uncle (b) Cousin
(c) Nephew (d) Grandfather

Ans. (b) Cousin

Q.35 Pointing towards "A", "B", said : "Y our mother is the younger sister of my mother", "A" is related to "B" as [July 2021]
(a) Uncle (b) Cousin
(c) Nephew (d) Father

Ans. (b) Cousin

SUMMARY

- Any relation in the world which either by birth or by marriage is called a Blood Relation.
- Different ways in which the blood relation is described :
  - Dialogue/ Conversation Based - One person describes his/her relation with another person (this may or may not be related to the person with whom the conversation is being made). •
  - Based on Puzzles - To make the questions complex, blood relation questions are also being asked in the form of a puzzle. A piece of brief information about multiple people being interrelated is given and sub-questions based on the same may be asked. •
  - Symbolically coded relations: the relationships between certain members of the family are indicated by symbols like £, $. %, &. +. - etc.
- Family tree is the best way to represent blood relations and related answers to them. The following points help in drawing a family tree.
- All the family members of the upper generation are represented above in the family tree. **E.g.:** father, mother, uncle, aunt etc. The logic can be extended by representing the grandparents above the parents in the family tree. **E.g.:** grandfather and grandmother.
- Similarly the relation between two members of the family is shown by connecting a double-headed arrow.
- All the family members of the same generation are represented in the middle of the family tree. **E.g.:** brothers, Sisters, cousins, wife, husband, etc.
- All the family members of the next generation are represented below in the family tree. **E.g.:** Daughter, son, niece, nephew.

PART- C

STATISTICS

CHAPTER-13

STATISTICAL DESCRIPTION OF DATA AND SAMPLING

Unit – 1: Statistical Description of Data

TOPICS TO COVER

- Introduction of Statistics
- Collection of Data
  - Primary Data
  - Secondary Data
- Presentation of Data
  - Frequency Distribution
  - Cumulative Frequency
- Presentation of Data Graphically
  - Line Chart or Histogram
  - Frequency Polygon
  - Pie Chart

WHAT ARE STATISTICS AND WHY WE NEED TO STUDY IT?

Statistics refers to the collection, analysis, interpretation, presentation, and organization of data. It involves using mathematical techniques to draw meaningful conclusions and make informed decisions based on data. The study of statistics is important because it provides us with the tools and methods to analyze and understand data, which is crucial in various fields. Where have we heard this word Statistics?

We often encounter the word "statistics" in different contexts. **E.g.:** we may hear it when referring to the weather report, which is based on statistical analysis of historical weather data to predict future conditions. Similarly, when someone mentions that the Indian population will be the highest by 2025, this statement is likely based on statistical projections and demographic data analysis. Furthermore, when discussing business goals, such as the next year's target for a company, statistics can be used to analyze past performance and set realistic objectives.

The term "statistics" has different origins, including the Latin word "status," the Italian word "Statista," the German word "statistik," and the French word "statistique." However, its exact origin remains uncertain.

MEANING OF STATISTICS

When defined in Plural sense: Statistics refer to the collection of qualitative and quantitative data in a way that allows us to draw meaningful conclusions from it. When defined in Singular sense: Statistics represent a scientific method used to collect, organize, and analyze data, ultimately leading to drawing statistical inferences about important characteristics. It can be considered the "science of counting" or the "science of averages."

APPLICATION OF STATISTICS

Statistics is applicable in every field where understanding and analysis of data are crucial. In the context of commerce, economics, business management, and industry, statistics provides the means to analyze economic data, make informed decisions, develop theories, conduct surveys, project future outcomes, and maximize profit. It offers various tools and techniques for data analysis, allowing businesses to gain a competitive advantage and make effective strategies based on statistical insights.

ECONOMICS

1. **Many branches of economics,** such as time series analysis, index number analysis, and demand analysis, are essentially branches of statistics. These branches use statistical methods to analyze economic data and draw meaningful conclusions.
2. **Econometrics** is another important application of statistics in economics. It involves the quantitative application of statistical and mathematical models using data to develop theories or test existing hypotheses in economics.
3. **Socio–economic surveys** play a crucial role in understanding various socio–economic factors. Statistical methods are used to conduct these surveys and analyze the derived data.
4. **Regression analysis** is used in economics to project future outcomes, such as sales, production, and prices, based on statistical analysis of past data.

BUSINESS MANAGEMENT

1. **How do managers work today?** In modern business management, decisions are no longer based solely on instincts. Managers now rely on quantitative data analysis, combined with statistical methods and operational research techniques, to make calculative and informed decisions.
2. **Sampling helps us to create criteria for making strategy**. In complex business environments like e–commerce, statistics plays a vital role in formulating strategies at every point by analyzing relevant data.
3. **Statistical decision theory** is employed to analyze complex strategies and assist businesses in making fruitful decisions by considering the merits and demerits of different options.

STATISTICS IN COMMERCE AND INDUSTRY:

1. **Getting an Edge in a competitive business environment.** In the competitive business environment, statistics can provide a significant edge. Business collects, compare, and analyze data on sales, production, profit, and other relevant factors to gain insight and formulate effective strategies.

2. **To Maximize Profit:** Maximizing profit is a common goal in commerce and industry. Statistical analysis is used to examine data on previous sales, wages, raw materials, and competitor products. By comparing and analyzing this data, business can identify opportunity to maximize profit.

3. **Various tools used in Commerce and Industry of Statistics,** including measures of central tendency (such as mean, median, and mode), measures of dispersion (such as range and standard deviation), sampling techniques, correlation and regression analysis, and index number and time series analysis. These tools help in analyzing data, identifying patterns, and making informed decisions.

LIMITATIONS OF STATISTICS

1. **Study of Quantitative data only:** Statistics studies only such facts as can be expressed in numerical terms. It does not study qualitative phenomena like honesty, friendship, wisdom, health, patriotism, justice, etc.
2. **Study of Aggregates only:** Statistics studies only the aggregates of quantitative facts. It does not study statistical facts relating to any particular. **E.g.:** It may be a statistical fact that your class teacher earns 50,000 per month. But if you want to find statistics of average salary of different schools, you will study aggregates of salary of each school
3. **Homogeneity of Data, an essential Requirement:** To compare data, it is essential that statistics are uniform in quality. Data of diverse qualities and kinds cannot be compared. For **E.g.:** we cannot compare the statistics of banana and air pressure in tire

4. **Results are True only on an Average:** Most statistical findings are true only as averages. They are not always valid under all conditions. For instance, if it is said that per capita income in India is 50,000 per annum, it does not mean that the income of each and every Indian is 50,000 per annum. Some may have more and some may have less.
5. **Results may Prove to be Wrong:** In order to understand the conclusions precisely, it is necessary that the circumstances and conditions under which these conclusions have been drawn are also studied. Otherwise, they may prove to be wrong.
6. **Can be used only by the Experts:** Statistics can be used only by those persons who have special knowledge of statistical methods. Those who are ignorant about these methods cannot make sensible use of statistics. In the words of Yule and Kendall, "Statistical methods are the most dangerous tools in the hands of an inexpert.

COLLECTION OF DATA

On the nature of data, we can define data as follows:

- **Quantitative Data:** This refers to data that represents numeric values of quantitative data include measurements, counts, and numerical ratings.
- **Qualitative Data:** This refers to data that represents characteristics or qualities of qualitative data include descriptive information, opinions, and categorical variables. If we want to analyze qualitative information using statistics, it needs to be converted into quantitative information by assigning a numeric description to the given characteristic. Quantitative data is collected in the form of variables.

WHAT IS VARIABLE?

A variable is a characteristic or attribute that can take on different values. It is the unit of measurement for collecting data.

TYPES OF VARIABLES

1. Discrete Variable
2. Continuous Variable
   - **Discrete Variable:** A discrete variable is one that can only take on specific, countable values. **E.g.:** the number of children in a family, the number of students in a class, or the number of cars in a parking lot.
   - **Continuous Variable:** A continuous variable is one that can take on any value within a given interval or range. It can have decimal values and is not limited to specific points.
     E.g.: height, weight, temperature, or time.

On the basis of source of data, we can define data as follows:

- **Primary Data:** Primary data refers to data collected by the investigator for their own purpose, from the beginning to the end of the research.
  - These are collected from the source of origin.
  - Primary data is considered original and specific to the research being conducted of primary data collection method include surveys, interviews, experiments, and observations.
- **Secondary Data:** Secondary data refers to data that already exists and has been collected for some other purpose.
  - These data are considered second-hand as they have been collected by someone else.
  - Secondary data can be obtained from published or unpublished reports, articles, books, databases, or other sources of secondary data include government reports, academic studies, industry statistics, and historical records.

COLLECTION OF PRIMARY DATA CAN BE DONE BY 4 WAYS

- Interview method

- Mailed questionnaire method
- Observation method
- Questionnaires filled and sent by enumerators.

INTREVIEW METHOD

- Direct Interview Method or Personal Interview Method
- Indirect Interview Method
- Telephonic Interview Method

This method involves direct interaction between the investigator and the respondent. There are different types of interview methods, such as direct interview, indirect interview, and telephonic interview. Direct interview provides more accurate data, while telephonic interview allows for faster data collection over a wider area. However, the telephonic interview method may have a higher number of non–responses.

E.g.: Let's say a school wants to gather feedback from its students regarding their satisfaction with the school facilities and extracurricular activities. They decide to use the interview method to collect data directly from the students.

Note:

- In the first two methods data collected will be more accurate but if you have to collect data faster and at a wider area then you need to collect data by Telephonic Interview method.
- The number of non-responses is maximum for this third method of data collection

MAILED QUESTIONNAIRE METHOD

In this method, questionnaires are mailed to the informants. The questionnaire is accompanied by a letter explaining the purpose of the enquiry and ensuring the confidentiality of the information. The informants fill out the questionnaires and return them to the investigator. This method allows for covering a wide area, but it may also result in a higher number of non–responses.

E.g.: Suppose a local government wants to gather feedback from residents about their satisfaction with public transportation services in the city. They decide to use the mailed questionnaire method to reach a wide range of residents.

OBSERVATION METHOD

This method involves direct observation by the investigator to collect information. It is considered one of the best methods for data collection, but it can be time–consuming and covers only a small area.

E.g.: A researcher is interested in studying the playground behavior of children in a local park. They choose to use the observation method to gather data directly by observing the children during their playtime.

QUESTIONAIRES FILLED AND SENT BY ENUMERATORS

Under this method, enumerators are appointed to approach the informants and fill out the questionnaires. This method is suitable for covering a wider range of respondents but can be costly.

E.g.: A market research company wants to collect data on consumer preferences for a new product. They decide to use the questionnaires filled and sent by enumerators method to gather information from a diverse group of respondents.

SOURCES OF SECONDARY DATA

Secondary data refers to data that has been previously collected by someone else for a different purpose but can be utilized for the current research or analysis. Here are some important sources of secondary data.

There are many sources of getting secondary data. Some important sources are listed below:

- **International sources:** These include organizations like the World Health Organization (WHO), International Labor Organization (ILO), International Monetary Fund (IMF), World Bank, and others. These global institutions collected and provide data on various topics such as health, labor, economy, and development.
  - **Government sources:** Government often collect extensive data on various aspect of society and the economy. Statistical agencies, such as the Central Statistical Office (CSO) in different countries, publish reports and statistical abstracts that provide valuable data. Ministries and departments related to agriculture, education, labor, and more also release official statistics.
  - **Private and quasi-government sources:** Private organizations and quasi-governmental institutions also generate and maintain datasets relevant to specific sectors. These sources can include research institutes, industry associations, academic institutions, and specialized agencies.
  - **Unpublished sources:** Researchers and experts often conduct studies or collect data that may not be widely available. These unpublished sources can include research papers, reports, surveys, and studies conducted by various institutions, researchers, or organizations.

SCRUTINY OF DATA

Scrutiny of data refers to the process of carefully examining the collected data to ensure its accuracy, reliability, and consistency. It involves reviewing the data for any errors, inconsistencies, or outliers that may affect the validity of the analysis. Researchers scrutinize the data by performing data cleaning and validation procedures. This includes checking for missing values, outliers, data entry errors, and logical inconsistencies. By carefully reviewing the data, researchers can ensure the quality and integrity of the dataset before proceeding with analysis.

PRESENTATION OF DATA

- After collecting and verifying the quality of data, it should be presented in a clear and concise manner.
- Effective data presentation is important for effectively communicating finding and facilitating understanding.
- Common methods of data presentation include tables, charts, graphs, and visualizations.
- Researchers select the appropriate presentation format based on the nature of the data and the research objectives.
- The presentation should emphasize the essential features of the data to make it easier for the audience to interpret and derive meaningful insights.

CLASSIFICATION OF DATA OR ORGANIZING OF DATA

Classification of data involves organizing raw data into meaningful groups or classes based on their characteristics. This process enables researchers to draw conclusions and identify patterns or relationships within the data. Data can be classified based on various attributes such as age groups, income brackets, geographical regions, or product categories. By categorizing data into relevant classes, researchers can analyze and interpret data more effectively.

E.g.: A market researcher conducting a survey on customer preferences for a product might classify the responses into different age groups (e.g., 18–25, 26–35, 36–45) to understand how preferences vary across different demographic segments.

Classification of data helps in summarizing and simplifying complex data sets, making it easier to identify trends, make comparisons, and draw meaningful conclusions.

OBJECTIVES OF CLASSIFICATION

1. **Simplification and Briefness:** Classification aims to simplify complex data by grouping them into categories or classes. It provides a condensed and organized representation of data, making it easier to analyze and interpret.
2. **Comparability:** Classification enables the comparison of data within and across different categories or classes. It allows for identifying patterns, trends, and relationships between different groups of data.
3. **Statistical Analysis:** Classification facilitates statistical analysis by organizing data into meaningful groups. It enables researchers to apply various statistical techniques and methods to explore the characteristics and relationships within each category or class.
4. **Makes data more understandable:** Classification makes data more understandable by presenting them in a structured manner. It provides a clear framework for data interpretation and helps in deriving insights and making informed decisions.

DATA MAY BE CLASSIFIED AS

1. **Chronological Data or Temporal or Time Series:** This classification arranges data based on their time intervals or chronological order. It helps in studying patterns and trends over time.

   E.g.: Classifying monthly sales data of a product over the past year into different time intervals such as quarters or seasons.

2. **Geographical or Spatial Series Data:** This classification groups data based on their geographical or locational differences. It helps in analyzing variations across different regions or areas.

   E.g.: Classifying population data of different cities or states into regional groups such as North, South, East, and West.

3. **Qualitative or Ordinal Data:** This classification categorizes data based on their qualities or attributes. It involves assigning data to specific categories or classes based on subjective characteristics.

   E.g.: Classifying survey responses into categories such as "Satisfied," "Neutral," and "Dissatisfied" based on customer satisfaction levels.

4. **Quantitative or Cardinal Data:** This classification organizes data into classes or groups based on their numerical values. It involves creating intervals or ranges to represent different levels or quantities.

   E.g.: Classifying test scores of students into different grade ranges such as A, B, C, etc., based on their numerical value ranges.

FREQUENCY DATA

Frequency of a particular data value is the number of times the data value occurs and data in which we can count frequency is called as frequency data

E.g.: Recording the number of times a specific word appears in a document or the number of customers who visited a store on different days of the week.

NON–FREQUENCY DATA

Non–frequency data refers to data where the individual values and their specific identities are important and need to be preserved.

E.g.: Recording the names and addresses of customers in a database for a marketing campaign, where each customer's information is unique and distinct.

THERE ARE GENERALLY THREE FORMS OF PRESENTATION OF DATA

Textual or Descriptive Presentation

In the textual presentation, data is described using paragraphs of text. This method is commonly used in official reports, where the activities, plans, or programs of a project are explained in detail, with relevant facts and figures inserted within the text. This form of presentation is suitable when the amount of data is relatively small.

However, statisticians generally prefer other methods as it can be dull, monotonous, and difficult to compare different fields of data.

E.g.: A project report describing the progress, achievements, and future plans of a company, including relevant statistical data within the narrative.

Tabular Presentation: Data is organized in rows and columns in a tabular presentation. A statistical table provides a systematic and structured representation of data in a concise format.

Considerations when creating a table:

- Designing of Table: Ensuring a clear and organized layout with appropriate column headings and row labels.
- Comparable Data: Presenting data in a format that allows for easy comparison and analysis.
- Beautiful Presentation: Enhancing the visual appeal of the table through proper formatting, font usage, and gridlines.
- Descriptive Notes: Including explanatory notes or footnotes to provide additional context or clarify any assumptions made.

Here's an example of a tabular presentation depicting the sales performance of a company across different product categories:

Product Category	Q1 Sales (in INR)	Q2 Sales (in INR)	Q3 Sales (in INR)
Electronics	5,00,000	4,50,000	5,50,000
Apparel	3,50,000	4,00,000	3,80,000
Home Goods	2,80,000	3,20,000	3,00,000
Beauty	2,00,000	1,80,000	2,20,000

DESCRIPTION OF EACH CONSIDERATION

1. **Designing of Table:** The table is designed with clear column headings ("Product Category," "Q1 Sales," "Q2 Sales," "Q3 Sales") and appropriate row labels for each product category.
2. **Comparable Data:** The data is presented in a format that allows easy comparison of sales across different quarters for each product category. This enables analysis and identification of trends or variations.
3. **Beautiful Presentation:** The table is properly formatted, with consistent alignment, appropriate font usage, and visible gridlines, making it visually appealing and easy to read.

4. **Descriptive Notes:** No specific descriptive notes are provided in this example, but they could be included below the table to explain any abbreviations, define terms, or provide additional context if needed.

FORMAT OF TABLE

Table Number: _____

Title: _____

Stub (Row Heading)	**Caption (Column Heading)**				**Total (Rows)**
	Sub-head		**Sub-head**		
	Column - head	**Column - head**	**Column - head**	**Column - head**	
Stub Entries (Row Entries)					
• • •					
• • •		BODY			
• • •					
• • •					
Total (Column)					

Source Note:

Footnote:

Main Parts of Table:

- **Table number:** It's like an ID for the table, so you can refer to it easily.
- **Title:** Think of it as a short summary that tells you what the table is about.
- **Headnote:** Gives you some extra info or context right at the beginning.
- **Captions/Column Heading:** Labels for columns, helping you understand what each column represents.
- **Stubs/Row Heading:** These are like labels for rows, making it clear what each row is talking about.
- **Body of table:** This is where the main information is – the numbers or data you're interested in.
- **Source note:** Tells you where the information in the table comes from.
- **Footnote:** Extra details or explanations found at the bottom of the table.

The tabulation method is generally preferred over textual presentation because

1. It allows for easy comparison of data across different categories or variables.
2. It provides a structured and organized format that facilitates data analysis.
3. It presents data in a concise and condensed manner, making it easier to grasp key information.
4. It allows for the inclusion of mathematical calculations and statistical measures within the table.

DIAGRAMMATIC REPRESENTATION OF DATA

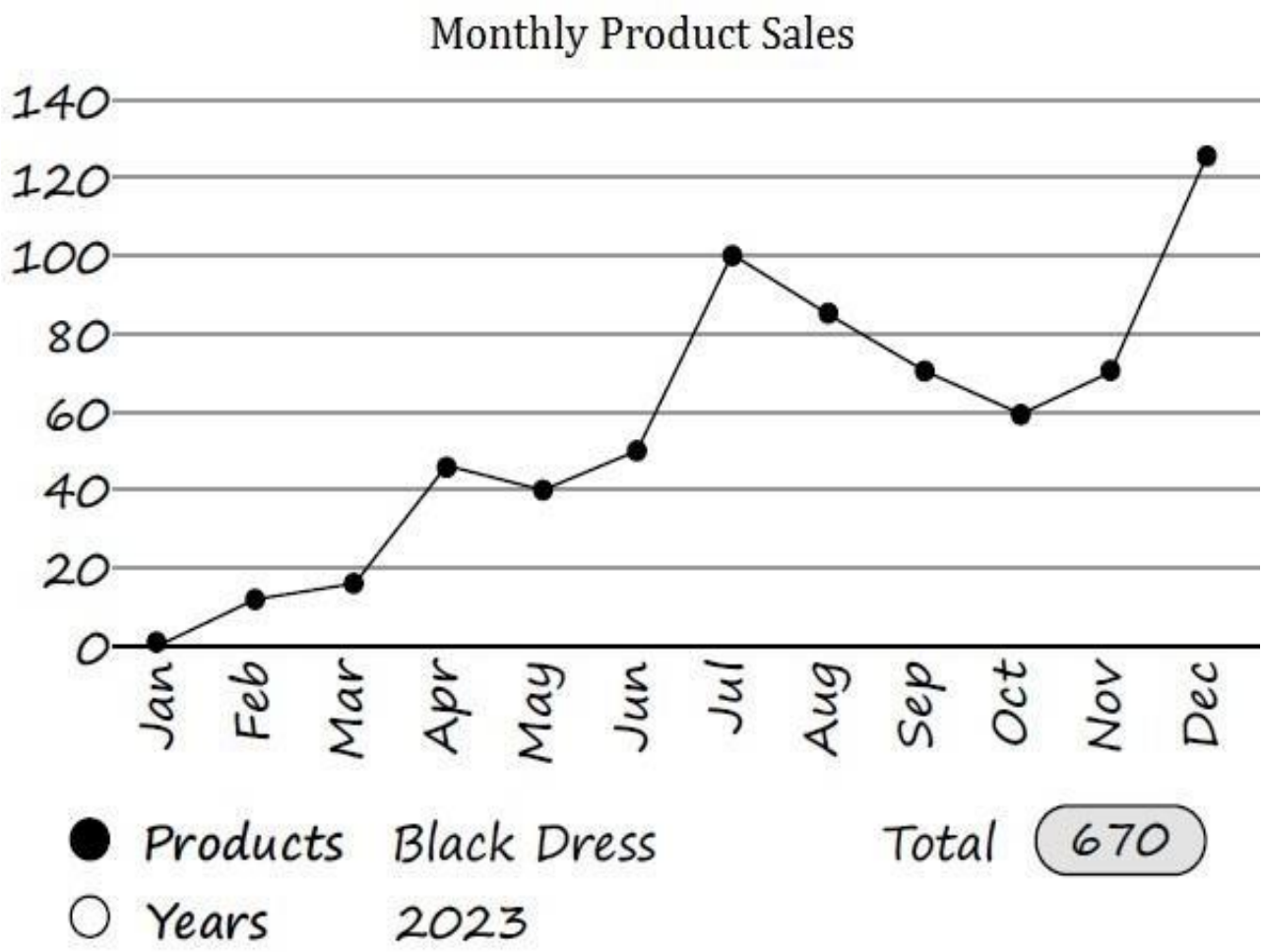
Diagrammatic presentation of data is a highly effective method of visually representing information in a concise manner. It involves the use of various types of diagrams to convey data effectively. In this context, we will focus on three commonly used diagrams.

LINE DIAGRAM OR HISTORIAGRAM

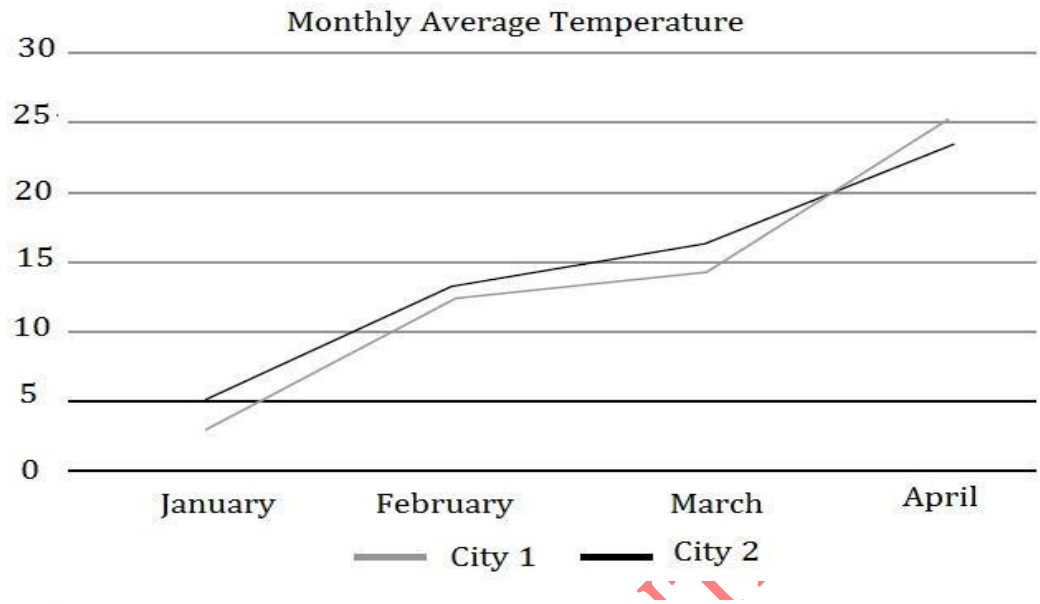
A line diagram, also known as a historiagram, is used to represent data that changes over time. It involves plotting pairs of values on a graph. By connecting these points with lines, we can visualize the trend or pattern of the data over time.

- If we have to represent multiple data (similar data in same unit) varying with time, we can use Multiple line chart.
- If we have to represent multiple data (with different unit) varying with time, we can use Multiple axis chart.

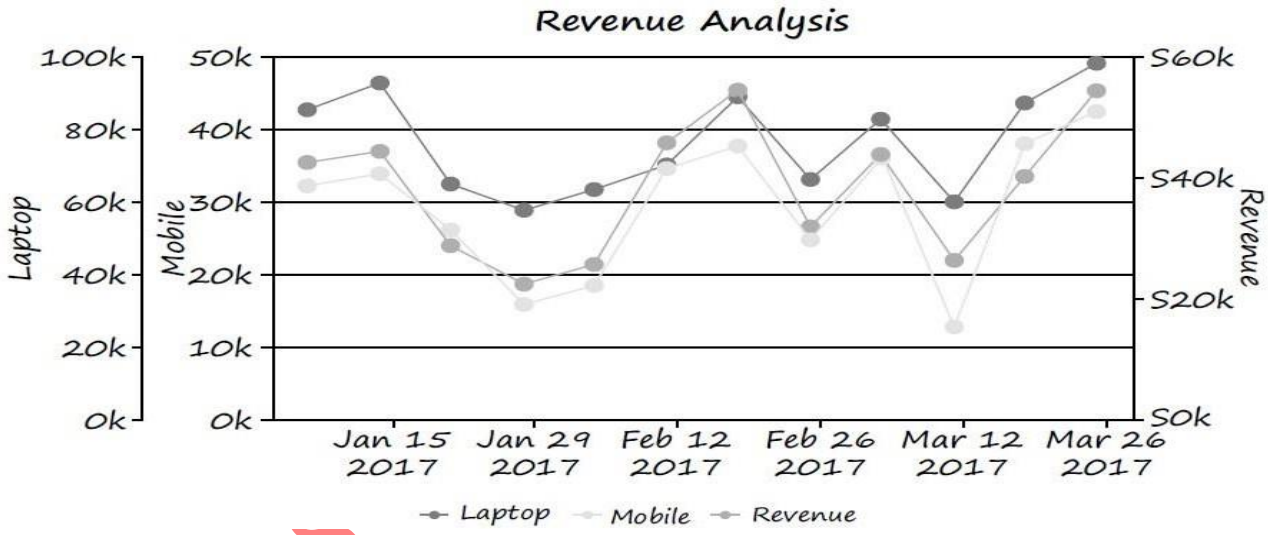
Line Chart: Suppose we have data on the monthly product (Black dress) sale in the year 2022. We can use a line diagram to plot the sale (y-axis) against the corresponding months (x-axis), allowing us to observe the sale trend over time.



Multiple Line Chart



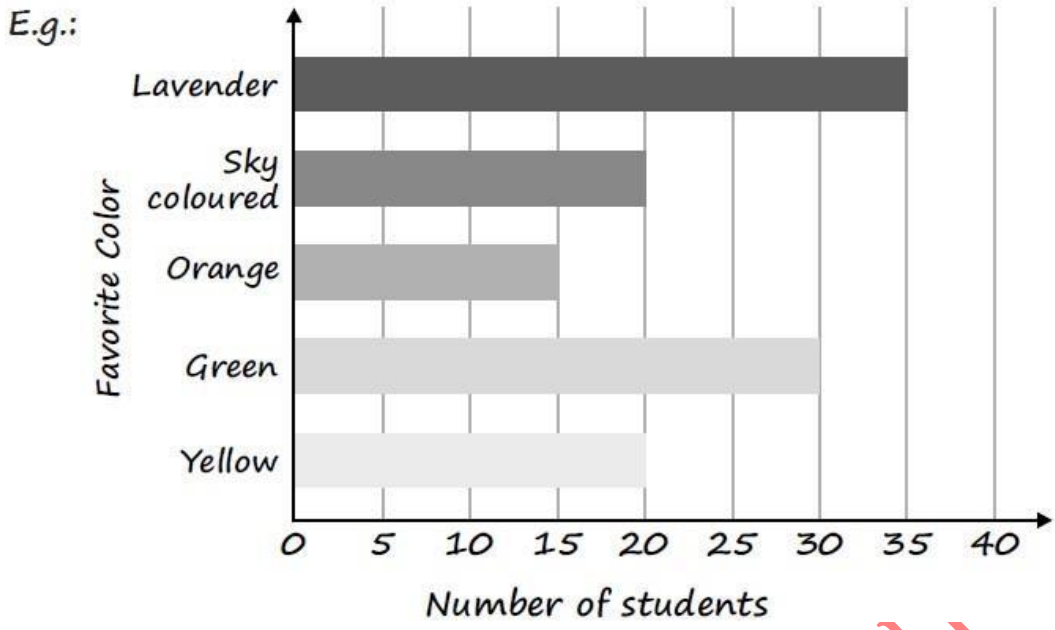
Multiple Axis Chart:



BAR DIAGRAM

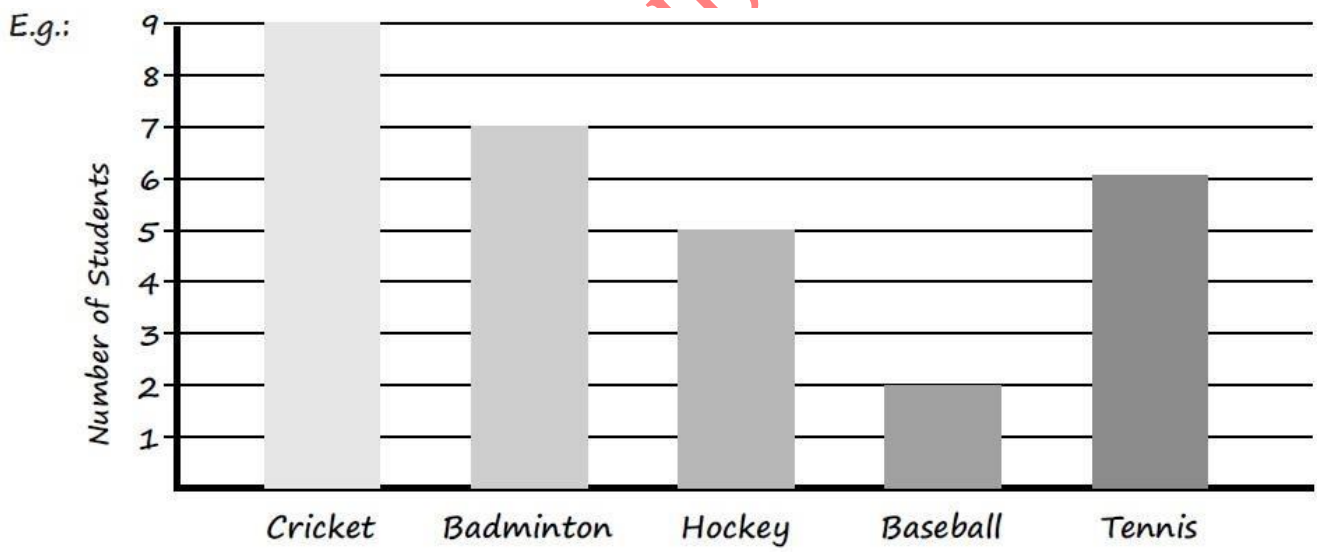
A bar diagram is a popular choice for comparing different categories or variables. It involves the use of bars to represent the magnitude of each category or variable. There are different types of bar diagrams depending on their orientation and purpose.

- **Horizontal Bar Diagram:** In a horizontal bar diagram, the categories or variables are represented on the y–axis, while the corresponding values or frequencies are shown on the x–axis. The lengths of the horizontal bars represent the magnitude of the variables.



Note: It is used for Qualitative data or data varying over space.

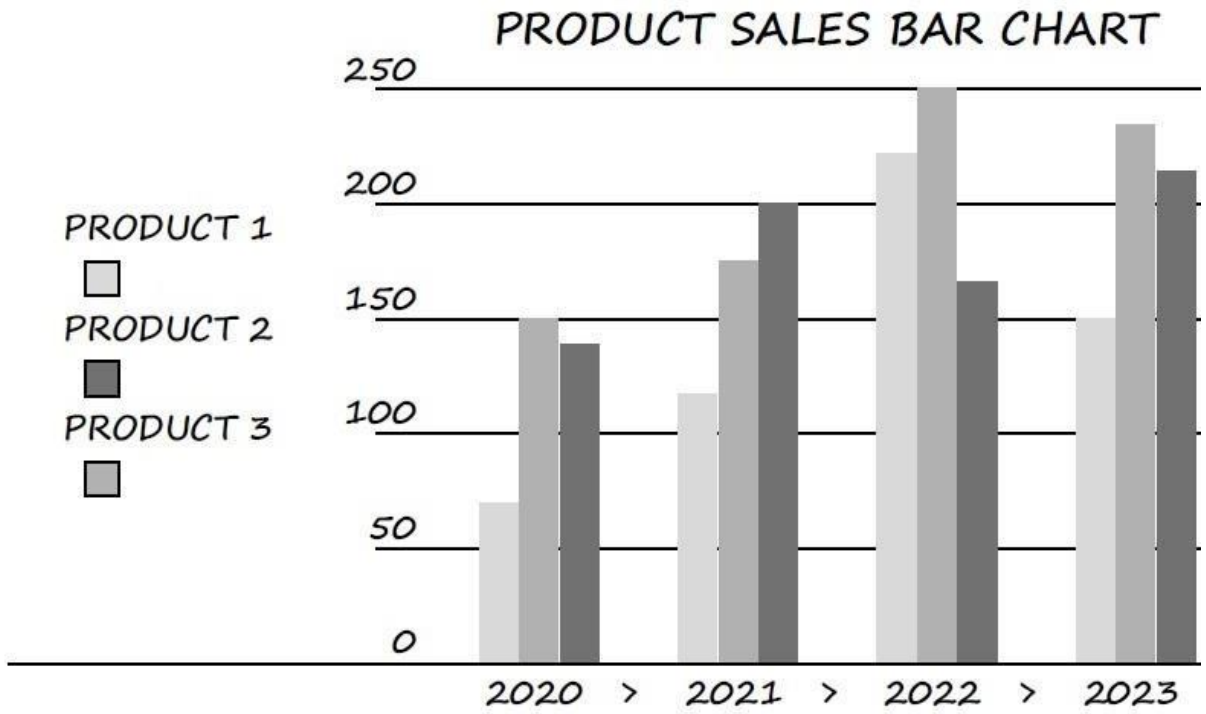
- **Vertical Bar Diagram:** In a vertical bar diagram, the categories or variables are represented on the x–axis, while the corresponding values or frequencies are shown on the y–axis. The lengths of the vertical bars represent the magnitude of the variables.



Note: It is used for Quantitative data or time series data.

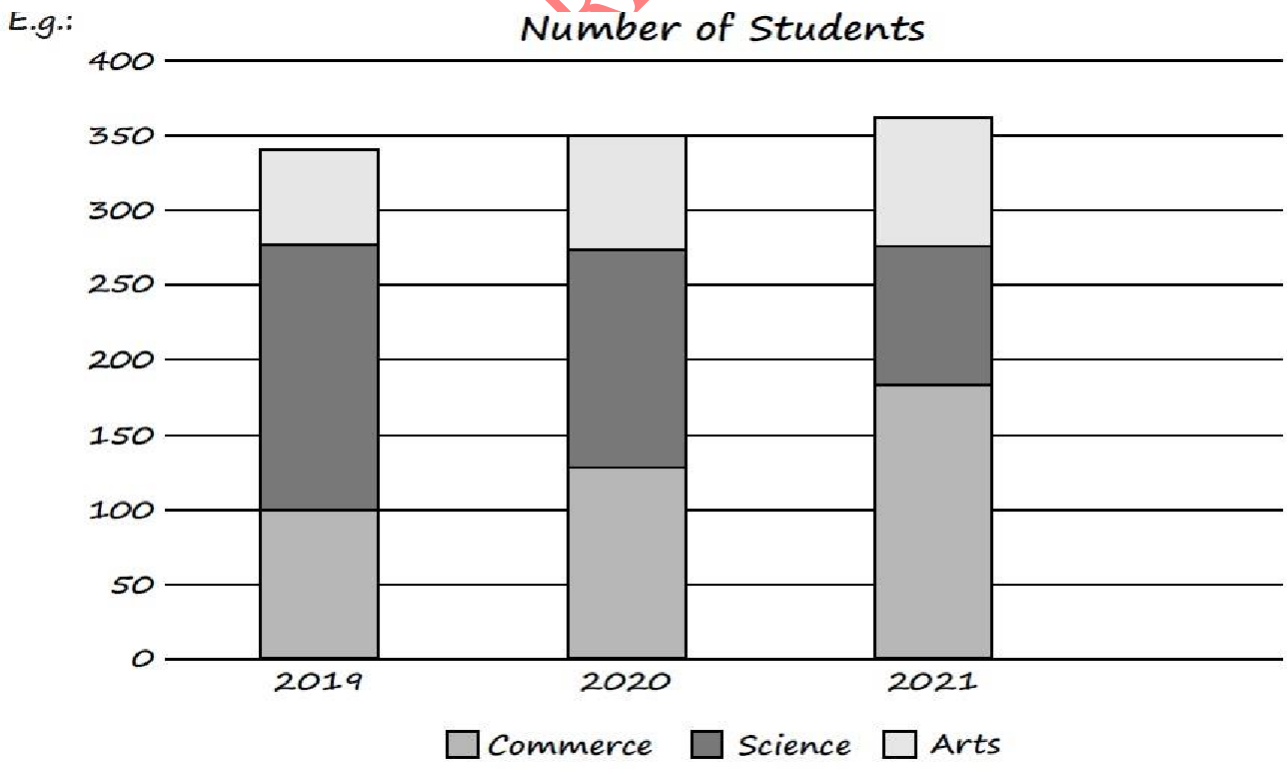
- **Multiple or Grouped Bar Diagrams:** Multiple or grouped bar diagrams are used when we need to represent multiple datasets that have the same unit and vary with time. It allows for easy comparison between the different datasets.

E.g.:

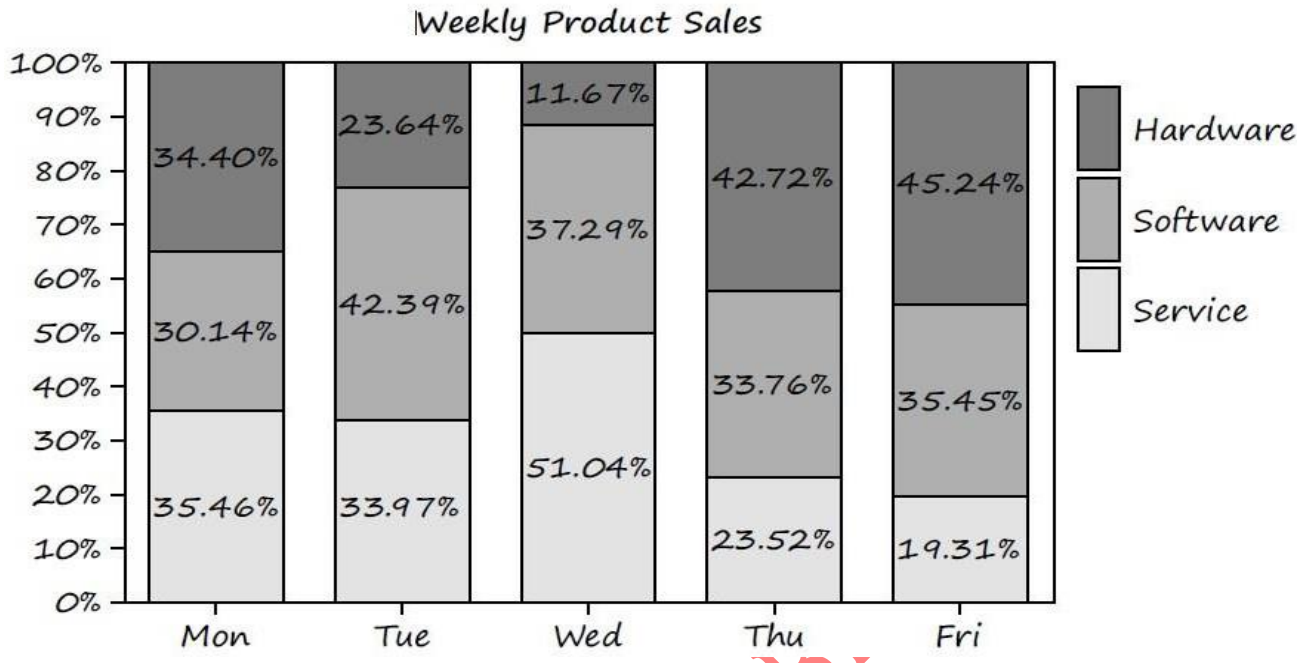


- **Component or Sub-divided Bar Diagrams:** Component or sub-divided bar diagrams are used when we need to represent multiple datasets that have different units but still vary with time. This type of diagram uses multiple axes to accommodate the different units.

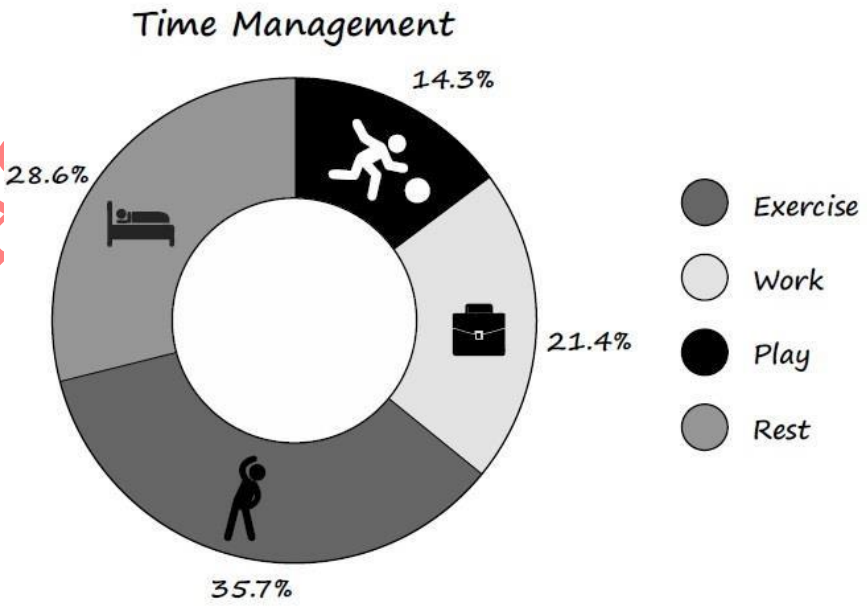
E.g.:



- **Divided Bar charts or Percentage Bar diagrams:** Comparing different components of a variable and also the relating of the components to the whole.



- **Pie Chart:** A Pie chart is used to represent data as a circular chart divided into sectors. Each sector represents a specific category or variable, and its size or angle represents the proportion or percentage of that category. Pie charts are especially useful for illustrating proportions or composition.
  E.g.: The below Pie chart represents the time management. The size of each sector in the pie chart corresponds to the proportion of time dedicated to that specific category, providing a clear visual overview of how time is distributed across these four essential aspects of life.

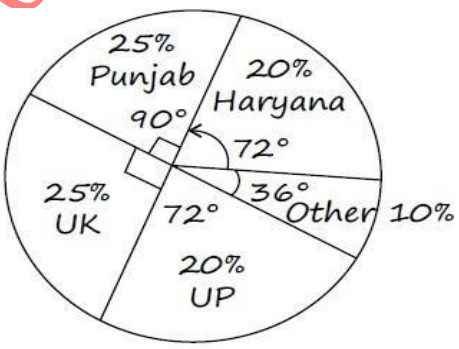


E.g.: The production of Wheat by different States of India are as shown below:

State	Production
Haryana	20%
Punjab	25%
Uttarakhand	25%
U.P	20%
Others	10%
Total	**100%**

Draw the suitable diagram to represent the information.
Here, according to the given data, we have
To find the corresponding percentage, we have $\frac{Row\ Value}{Total} \times 360^\circ$

State	Production	
Haryana	20%	$\frac{20}{100} \times 360^\circ = 72^\circ$
Punjab	25%	$\frac{25}{100} \times 360^\circ = 90^\circ$
Uttarakhand	25%	$\frac{25}{100} \times 360^\circ = 90^\circ$
U.P	20%	$\frac{20}{100} \times 360^\circ = 72^\circ$
Others	10%	$\frac{10}{100} \times 360^\circ = 36^\circ$
Total	**90**	**360°**



In summary, diagrammatic representation of data provides an effective way to present information visually. Line diagrams show trends over time, bar diagrams compare variables, and pie charts depict proportions or compositions.

Q.1 Cost of sugar in a month under the heads raw materials, labour, direct production and others were 12, 20, 35 and 23 units respectively, What is the difference between the central angles for the largest and smallest components of the cost of sugar?

(a) 72° (b) 48° (c) 56° (d) 92°

Ans. (d)

Q.2 A pie diagram is used to represent the following data.

Source	Customer	Excise	Income Tax	Wealth Tax
Revenue in Million	120	180	240	180

The central angles corresponding to Excise is

(a) 120° (b) 80° (c) 90° (d) 60°

Ans. (c)

Q.3 Mode of a distribution can be obtained from

(a) Histogram (b) Less than type ogives

(c) More than type ogives (d) Frequency polygon

Ans. (a)

Q.4 The most appropriate diagram to represent the data relating to the monthly expenditure on different items by a family is

(a) Histogram (b) Pie-diagram (c) Frequency polygon (d) Line graph

Ans. (b)

Q.5 The column headings of a table are known as:

(a) Body (b) Stub (c) Box-head (d) Caption

Ans. (d)

FREQUENCY DISTRIBUTION

- Frequency distribution is a method of organizing data to provide insights into how often certain values occur.
- It can be classified in two ways: as frequency data classified by categories or as frequency data classified by intervals.

Frequency represents the number of times an observation occurs in the data.

E.g.: In the list of numbers 1, 2, 3, 4, 6, 9, 9, 8, 5, 1, 1, 9, 9, 0, 6, 9, the frequency of the number 9 is 5.

Frequency distribution involves tabulating data in a table, where the total frequency is distributed among different classes or intervals. The classes or intervals should be mutually exclusive and cover the entire range of data.

UNGROUPED FREQUENCY DISTRIBUTION

When the data consists of discrete variables, we can create an ungrouped frequency distribution table. This table presents the frequency counts for each unique value in the dataset.

E.g.: Suppose we have a dataset representing the number of pets owned by individuals: 2, 4, 1, 3, 2, 4, 2, 1, 3, 4. The ungrouped frequency distribution table would show the frequency count for each unique value:

Number of Pets	Frequency
1	2
2	3
3	2
4	3

GROUPED FREQUENCY DISTRIBUTION

When the data consists of continuous variables, we can create a grouped frequency distribution table. This table categorizes the data into intervals or classes and displays the frequency count for each interval.

E.g.: Suppose we have a dataset representing the heights (in centimeters) of a group of individuals: 156, 168, 174, 160, 162, 170, 168, 172, 158, 164. To create a grouped
frequency distribution, we can group the data into intervals (e.g., 150–160, 161–170, 171–180) and count the frequency in each interval:

Height (in cm)	Frequency
150-160	2
160-170	5
170-180	3

HOW TO MAKE A FREQUENCY DISTRIBUTION TABLE FOR DISCRETE VARIABLE

I. Find the largest and smallest observations and obtain the difference between them.
II. Form a number of classes depending on the number of isolated values assumed by a discrete variable.
III. Present the class in a table known as frequency distribution table
IV. Apply 'tally mark' i.e., a stroke against the occurrence of a particulars value in a class.
V. Count the tally marks and present these numbers in the next column, known as frequency column, and finally check whether the total of all these class frequencies tally with the total number of observations.
E.g.: Let's say you survey a number of households and find out how many pets they own. The results are 3, 0, 1, 4, 4, 1, 2, 0, 2, 2, 0, 2, 0, 1, 3, 1, 2, 1, 1, 3. Let's distribute data in frequency table.

Number of Pets	Tally Marks	Frequency
0	||||	4
1	𝍸	6
2	𝍸||	5
3	|||	3
4	||	2

How to make a frequency distribution table for continuous variable: Creating a Frequency Distribution Table for Discrete Variables To create a frequency distribution table for a discrete variable, follow these steps:

1. Identify the largest and smallest observations in the dataset and calculate the range by finding the difference between them.
2. Determine the number of classes or categories to be used in the frequency distribution table. This depends on the number of distinct values assumed by the discrete variable.
3. Present the classes in a table, known as the frequency distribution table. The classes should cover the entire range of values and be mutually exclusive.

4. Use tally marks (vertical strokes) to represent the occurrence of each value within a class. Each value gets a tally mark against the corresponding class.
5. Count the tally marks and record the frequencies in the next column of the frequency distribution table. This column represents the frequency of each class.
6. Finally, verify whether the total of all class frequencies matches the total number of observations in the dataset.

Q.6 Following are the heights (in cm) of B. Com students of St. Xavier's College.
161, 150, 154, 165, 168, 161, 154, 162, 150, 151, 162, 164, 171, 165, 158, 154, 156, 172, 160, 170, 153, 159, 161, 170, 162, 165, 166, 168, 165, 164, 154, 152, 153, 156, 158, 162, 160, 161, 173, 166, 161, 159, 162, 167, 168, 159, 158, 153, 154, 159
Construct a frequency distribution of heights, taking class length as 5.
Sol. Here,
Smallest weight 150 cm
Largest weight 173 cm
Range: 173-150=23
Number of classes $= \frac{Range}{Class\ length} = \frac{23}{5} = 4.6$ (round up to 5 classes)
Therefore, the frequency distribution table is as follows.

Heights (in cm)	Frequency
150-154	12
155-159	9
160-164	14
165-169	10
170-174	5
	50

SOME IMPORTANT TERMINOLOGIES

- **Class Limit:** The class limit refers to the minimum and maximum values that define a class interval. The lower–class limit (LCL) is the minimum value, and the upper–class limit (UCL) is the maximum value within a class interval.

  E.g.: in the class interval 10–20, 10 is the lower–class limit, and 20 is the upper–class limit.

- **Class boundary:** Class boundaries are used to establish a common boundary between adjacent class intervals when there is a difference between the upper class limit of one interval and the lower class limit of the next interval. By using class boundaries, we ensure a smooth transition and maintain consistency in data representation.

- Please note that for overlapping class interval – Class boundaries and Class limits are same.

- Lower class boundary (LCB) $= LCL - \frac{D}{2}$

- Upper class boundary (UCB) $= UCL - \frac{D}{2}$ where, D is difference between the LCL of the next class interval and UCL of the given class interval.

  Eg.: Let's consider the following class intervals for a dataset 10-15, 15-20, 20-25, 25-30.

  Here, the class intervals are overlapping thus Class boundaries and Class limits are same.

  E.g.: Let's consider the following class intervals for a dataset: 10-14, 15-19, 20-24, 25-29.

  Here, the class intervals are non-overlapping.

  Thus, first class limits:10-15

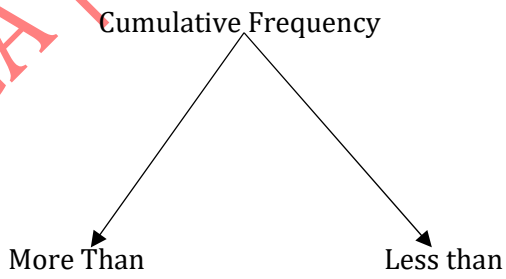
  And class boundaries of first class:

  LCB $= 10 - \frac{1}{2} = 9.5, HCB = 14 + \frac{1}{2} = 14.5$

  Thus, the first class boundaries is 9.5-14.5.

- **Class Midpoint:** The class midpoint (or class mark) is a specific point in the center of the class interval in a frequency distribution table.
  Mid-Point = (LCL+UCL)/2 = (LCB + UCB) / 2
  E.g.: For class interval 10-14
  Mid-point $= \frac{10+14}{2} = \frac{24}{2} = 12$
- **Width or Size of Class interval:** The class width is the difference between the upper – class boundary to the lower – class boundary of consecutive classes
  E.g.: Width of the interval 10-14 is 14-10 =4

Cumulative Frequency: Cumulative frequency is defined as a running total of requencies.



LESS THAN CUMULATIVE FREQUENCY

Height (in cm)	Frequency	Less than Cumulative frequency
15-154	12	12
155-159	9	12+9=21
160-164	14	21+14=35
165-169	10	35+10=45

170-174	5	45+5=50
	50	

MORE THAN CUMULATIVE FREQUENCY

Height (in cm)	Frequency	More than Cumulative frequency
150-154	12	50
155-159	9	50-12=38
160-164	14	38-9=29
165-169	10	29-14=15
170-174	5	15-10=5
	50	

- **Frequency Density of Class Interval:** Frequency density is a measure that helps to standardize the frequencies of different class intervals. It is calculated by dividing the frequency of a class interval by its width or size. The frequency density gives us an idea of the concentration of data within each class interval, taking into account the varying widths of the intervals.

  Frequency Density = Frequency / Width

- **Relative Frequency or Percentage Frequency of Class Interval:** Relative frequency, also known as percentage frequency, is the proportion of the total frequencies that each class interval represents. It is calculated by dividing the frequency of a class interval by the total number of observations and multiplying by 100 to express it as a percentage.

  Formula: Relative Frequency = (Frequency / Total number of observations) × 100

GRAPHICAL REPRESENTATION OF FREQUENCY DISTRIBUTION

HISTOGRAM

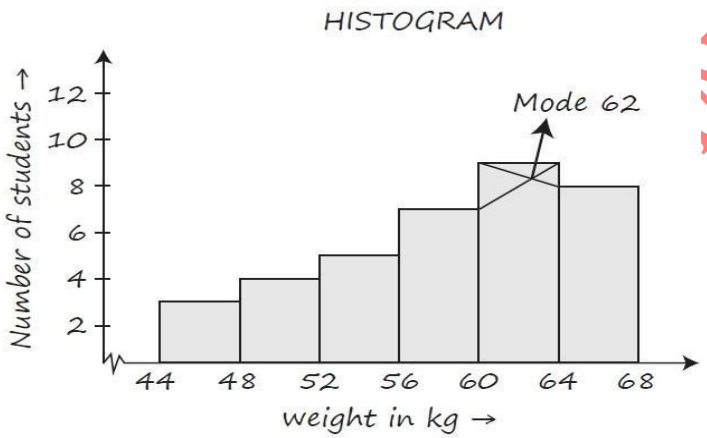
- This is very convenient way to represent a frequency distribution. Comparison among class interval is possible in this mode of diagrammatic representation
- A two–dimensional graphical representation of a continuous frequency distribution is called a histogram.
- In histogram, the bars are placed continuously side by side with no gap between adjacent bars.
- That is, in histogram rectangles are erected on the class intervals of the distribution. The areas of rectangle are proportional to the frequencies.
- We can also find the mode from Histogram

Let's draw histogram for the following data:

FREQUENCY DISTRIBUTION OF WEIGHT OF 36 BBA STUDENT

Weight in kg (class interval)	Tally marks	No. of students (frequency)
44-48	|||	**3**

49-52	IIII	**4**
52-56	IIII	**5**
56-60	IIIII	**7**
60-64	IIIIIII	**9**
64-68	IIIIII	**8**
Total		**36**



FREQUENCY POLYGON

As name says, we have to plot frequencies on the graph. For single frequency distribution we can do it easily. By plotting points and joining the points with the line.

- But for grouped frequency distribution, frequency polygon can be drawn only if the class interval is uniform. Then we take the value of xi as the midpoint of the class intervals. And fi being the frequencies of the classes respectively we can draw frequency polygon by plotting the (xi, fi) and then joining it with the line.

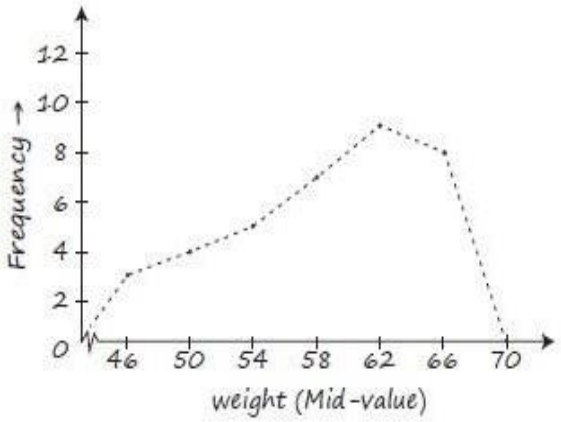
  E.g.: Let's draw for the below table

FREQUENCY DISTRIBUTION OF WEIGHTS OF 36 BBA STUDENTS

Weight in kg (class interval)	Tally marks	No. of students (frequency)
44-48	III	**3**
48-52	IIII	**4**
52-56	IIII	**5**

56-60	𝍸 ||	**7**
60-64	𝍸 ||||	**9**
64-68	𝍸 |||	**8**
Total	-	**36**

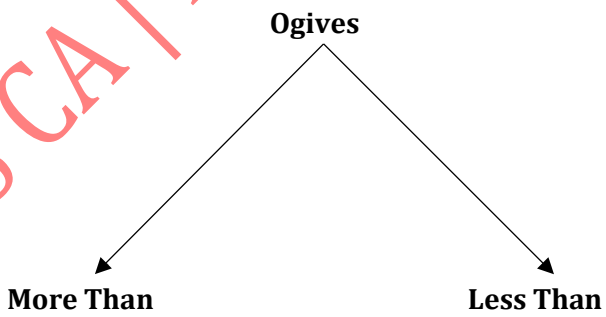
Mid – point	Frequency	Frequency Polygon
46	**3**	
50	**4**	
54	**5**	
58	**7**	
62	**9**	
66	**8**	



OGIVES OR CUMULATIVE FREQUENCY GRAPH

By plotting cumulative frequency against the respective class boundary, we get ogives.

As Cumulative Frequency Graph can be drawn as less than type and more than type. Therefore



E.g.: Let's draw ogive for the below table

Frequency distribution of weights of 36 BBA Students

Weight in kg (class interval)	Tally marks	No. of students (frequency)
44-48	|||	**3**
49-53	||||	**4**
54-58	𝍸	**5**

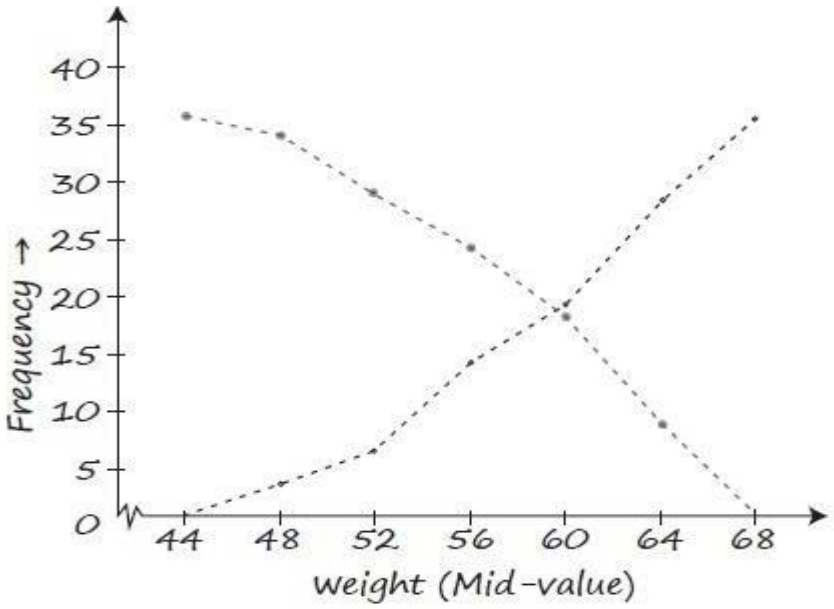
59-63	𝍸 II	**7**
64-68	𝍸 IIII	**9**
64-68	𝍸 III	8
64-68	𝍸 III	**8**
Total	**-**	**36**

Less than Ogive

Weight	No. of students	Commutative frequency
Less than 48	3	3
Less than 52	4	7
Less than 56	5	12
Less than 60	7	19
Less than 64	9	28
Less than 68	8	36
Total	36	

More than Ogive

Weight	No. of students	Commutative frequency
More than 44	3	36
More than 48	4	36-3=33
More than 52	5	36-3-4=29
More than 56	7	36-3-4-5=24
More than 60	9	36-3-4-5-7=17
More than 64	8	36-3-4-5-7-9=8
Total	36	



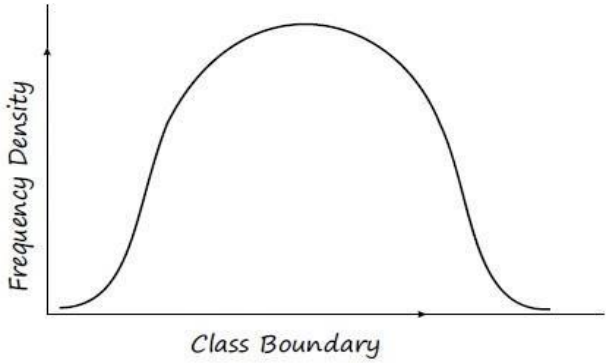
e: Ogives help us to find out Median. If a perpendicular is drawn from the point of intersection of the two ogives on the horizontal axis, then the x–value of this point gives us the value of median

FREQUENCY CURVE

A frequency curve is a graphical representation of the frequency distribution of a dataset. It is a smooth curve obtained by joining the midpoints of the upper side of each vertical bar in a histogram. The frequency curve depicts the relationship between the frequency density on the vertical axis and the class boundary on the horizontal axis. The total area under the frequency curve is taken to be unity, meaning the graph represents the relative frequencies of the data.

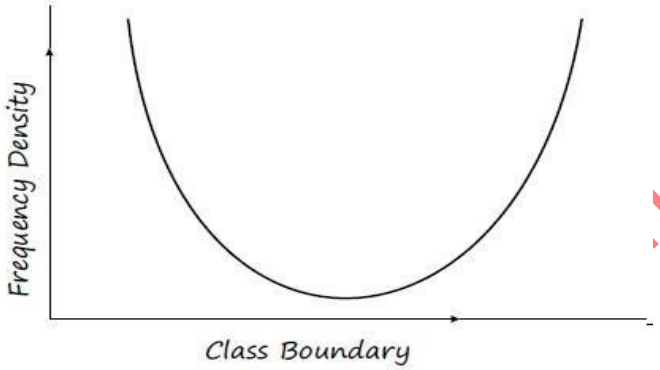
There are four main types of frequency curves:

1. **BELL SHAPED CURVE:** The bell–shaped curve, also known as the normal distribution, exhibits a symmetrical shape. It starts with a low frequency value, increases to a maximum at the center, and then decreases to a low frequency value at the other extreme. The bell–shaped curve is commonly observed in natural phenomena and human attributes, such as the distribution of heights, weights, exam scores, or profits.



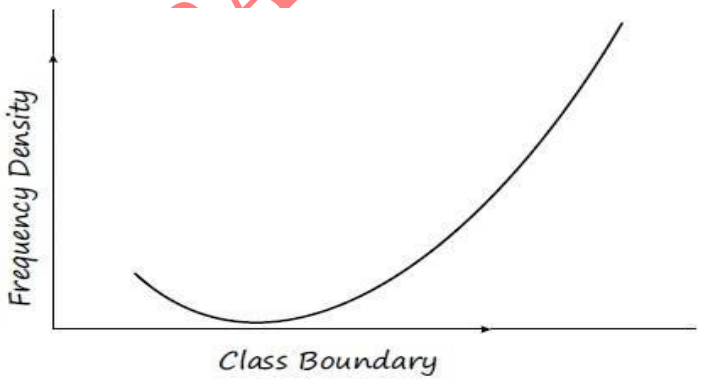
E.g.: The distribution of human heights in a population often follows a bell–shaped curve. In this distribution, the majority of individuals tend to cluster around the average height, with fewer individuals at the extreme ends (very short or very tall).

2. **U SHAPED CURVE:** The U–shaped curve represents a situation where the frequency is minimum near the central part and gradually increases to the maximum at the two extremities. This type of curve indicates a pattern where there is a higher concentration of observations at the extremes and fewer observations in the middle.



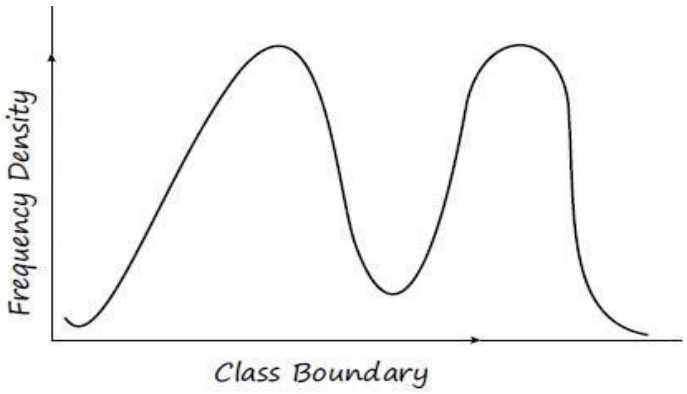
E.g.: The traffic flow in a metropolitan city during office timings often exhibits a U–shaped curve. The traffic volume is minimal during non–peak hours, increases significantly during rush hours, and then decreases again during the late evening.

3. **J–SHAPED CURVE:** The J–shaped curve starts with a minimum frequency and gradually increases to reach its maximum frequency at the other extremity. This type of curve suggests a pattern where there are initially fewer observations, but as we move towards the extreme, the frequency increases rapidly.



E.g.: The traffic on roads between 5 am to 10 am in a busy city like Delhi often follows a J–shaped curve. Initially, there is minimal traffic in the early morning, but as people start commuting to work, the traffic volume gradually increases, peaking during the morning rush hour.

4. **MIXED CURVE:** In a mixed curve, different portions of the distribution may resemble a bell–shaped curve, a U–shaped curve, a J–shaped curve, or other shapes. This indicates that the dataset is a combination of different subgroups or populations, each with its own distribution pattern.



E.g.: Let's consider a dataset representing the heights of individuals from two different populations, such as male and female. If we combine the heights of both populations in a single frequency distribution, the resulting frequency curve may exhibit characteristics of both a bell–shaped curve and a U–shaped curve. The female heights may follow a bell– shaped curve, while the male heights may show a U–shaped curve. The overall distribution would then be a mixed curve, reflecting the characteristics of both populations.

Q.7 Which of the following statements is false?

(a) Statistics is derived from the Latin word 'Status'

(b) Statistics is derived from the Italian word 'Statista'

(c) Statistics is derived from the French word 'Statistik'

(d) None of these

Ans. (b)

Q.8 Statistics is defined in terms of numerical data in the

(a) Singular sense (b) Plural sense

(c) Either (a) or (b) (d) Both (a) and (b)

Ans. (c)

Q.9 Statistics is concerned with

(a) Qualitative information (b) Quantitative information

(c) (a) or (b) (d) Both (a) and (b)

Ans. (d)

Q.10 Annual income of a person is

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) (b) or (c)

Ans. (b)

Q.11 Marks of a student is an example of

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) None of these

Ans. (b)

Q.12 Drinking habit of a person is

(a) An attribute (b) A variable

(c) A discrete variable (d) A continuous variable

Ans. (a)

Q.13 Which of the following is an example of qualitative data?

(a) Temperature in degree Celsius.
(b) Number of cars in a parking lot.
(c) Rating of customer satisfaction (Excellent, Good, Average, Poor).
(d) Height in centimeters.

Ans. (c)

Q.14 Data collected on religion from the census report are

(a) Primary data (b) Secondary data
(c) Sample data (d) (a) or (b)

Ans. (b)

Q.15 The weights of a group of individuals were recorded using a weighing scale. Based on this information, identify the nature of the data collected

(a) Primary data (b) Secondary data
(c) Discrete data (d) Continuous data

Ans. (a)

Q.16 Which of the following methods is considered the most efficient for collecting primary data quickly? **(ICAI)**

(a) Personal interview (b) Indirect interview
(c) Telephone interview (d) Direct observation

Ans. (c)

Q.17 The most effective approach for data collection during a natural disaster is

(a) Personal interview (b) Indirect interview
(c) Questionnaire method (d) Direct observation method.

Ans. (a)

Q.18 When studying historical events and analyzing past records, which method of data collection is commonly used to extract information?

(a) Personal interview (b) Secondary data analysis
(c) Questionnaire method (d) None of these

Ans. (b)

Q.19 Which data collection method involves gathering information from a large sample of individuals by mailing them questionnaires to be completed and returned?

(a) Telephone interview method (b) Mailed questionnaire method
(c) Direct interview method (d) All of these

Ans. (b)

Q.20 Which method of data collection involves recording information from documents, reports, or other existing sources?

(a) Personal interview (b) Secondary data analysis
(c) Questionnaire method (d) Direct observation method

Ans. (b)

Q.21 Some important sources of secondary data are

(a) International and Government sources
(b) International and primary sources
(c) Private and primary sources
(d) Government sources

Ans. (a)

Q.22 Sweetness of sweet dish is

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) A variable

Ans. (a)

Q.23 The accuracy and consistency of data can be verified by

(a) Internal checking (b) External checking

(c) Scrutiny (d) Both (a) and (b)

Ans. (c)

Q.24 There were 200 employee is an office in which 150 were married. Total male employees were 160 out of which 120 were married. What was the number of female unmarried employees?

(a) 30 (b) 40 (c) 50 (d) 10

Ans. (d)

Q.25 The distribution of scores in a test is an example of the frequency distribution of

(a) A discrete variable (b) A continuous variable

(c) An attribute (d) (a) or (c)

Ans. (a)

Q.26 Which of the following is suitable for the graphical representation of a Cumulative frequency distribution?

(a) Frequency polygon (b) Histogram

(c) Ogive (d) Pie chart

Ans. (c)

Q.27 Which method is commonly used to present detailed information?

(a) Charts (b) Graphs

(c) Tables (d) Visualizations

Ans. (c)

Q.28 The column headings of a table are known as:

(a) Body (b) Stub

(c) Box-head (d) Caption

Ans. (d)

Q.29 The number of "Frequency distribution" is

(a) Two (b) One (c) Five (d) Four

Ans. (b)

Q.30 (class frequency)/ (width of the class) is defined as

(a) Frequency density (b) Frequency Distribution

(b) Both (d) None of these

Ans. (a)

Q.31 Tally Marks determines

(a) class width (b) class boundary

(c) class limits (d) class frequency

Ans. (d)

Q.32 Most of the Commonly used distributions provide

(a) Bell – Shaped (b) U Shaped

(c) J- Shaped Curve (d) Mixed Curve

Ans. (a)

Q.33 Unequal widths of classes in the frequency distribution can create difficulties in the construction of:

(a) Ogive (b) Frequency polygone

(c) Histogram (d) None of these

Ans. (c)

Q.34 The number of types of cumulative Frequency is

(a) One (b) Two (c) Three (d) Four

Ans. (b)

Q.35 Classes with zero Frequency are called

(a) nil class (b) Empty class (c) class (d) None of these

Ans. (b)

Q.36 For calculating class frequencies, it is important that these classes are

(a) Mutually exclusive (b) Non–overlapping

(c) Independent (d) None of these

Ans. (b)

Q.37 The value exactly at the middle would ever be included in a class interval are called

(a) class mark (b) mid value

(c) both (d) None of these

Ans. (c)

Q.38 For the non–overlapping classes 1–20,21–40,41–60,61–80,81–100, the class mark of the class 61–80 is

(a) 10.5 (b) 70.5 (c) 90.5 (d) None

Ans. (b)

Q.39

Class	0-10	10-20	20-30	30-40	40-50
Frequency	10	12	8	15	6

For the class 20-30, cumulative frequency is

(a) 22 (b) 8 (c) 51 (d) 30

Ans. (d)

Q.40 The curve obtained by joining the points, whose x – coordinates are the upper limits of the class – intervals and y coordinates are corresponding cumulative frequencies is called

(a) Ogive (b) Histogram (c) Frequency Polygon (d) Frequency curve

Ans. (a)

Q.41 In Histogram, the classes are taken

(a) Overlapping (b) Non–overlapping (c) Both (d) None of these

Ans. (a)

Q.42

Class	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	20	8	3

For the class 20-30, cumulative frequency is

(a) 10 (b) 26 (c) 30 (d) 41

Ans. (c)

Q.43 Find the number of observations between 250-300 from the following data:

Value more than:	200	250	300	500
No. of observation:	56	38	15	0

(a) 38 (b) 23 (c) 15 (d) None of the above

Ans. (b)

Q.44 Data Collection on religion from census reports are:

(a) Primary data (b) Secondary data

(c) Sample data (d) (a) or (b)

Ans. (b)

Q.45 What is a exclusive series ?

(a) In which both upper and lower limit are not included in class frequency.

(b) In which lower limit is not included in class frequency.

(c) In which upper limit is not included in class frequency

(d) None of the above

Ans. (c)

Q.46 If the class intervals are 10 – 14, 15 – 19, 20 – 24, ..., then the first class boundary is

(a) 9.5 – 14.5 (b) 10 – 15 (c) 9 – 15 (d) 10.5 – 15.5

Ans. (a)

Q.47 The following data related to the marks of group of students

Marks No of Students	
More than 70%	7
More than 60%	18
More than 50%	40
More than 40%	60
More than 30%	75
More than 20%	100

How many students have got marks less than 50%?

(a) 60 (b) 82 (c) 40 (d) 53

Ans. (a)

Q.48 Out of 1000 persons, 25 percent were industrial workers and the rest were agricultural workers. 300 persons enjoyed world cup matches on T.V. 30 percent of the people who had not watched world cup matches were industrial workers. What is the number of agricultural workers who had enjoyed world cup matches on TV?

(a) 230 (b) 250 (c) 240 (d) 260

Ans. (d)

Q.49 A pie diagram is used to represent the following data.

Source:	Customers	Excise	Income Tax	Wealth Tax
Revenue in Millions:	120	180	240	180

The central angles corresponding to Income Tax and Wealth Tax are

(a) 90°, 120° (b) 120°, 90° (c) 60°, 120° (d) 90°, 60°

Ans. (b)

Q.50 A sample study of the people of an area revealed that total number of women were 40% and the percentage of coffee drinkers were 45 as a whole and the percentage of male coffee drinkers was 20. What was the percentage of female non-coffee drinkers?

(a) 10% (b) 15% (c) 18% (d) 20%

Ans. (b)

Q. 1 In 2000, out of total of 1,750 workers of a factory 1,200 were members of a trade union. The number of women employed was 200 of which 175 did not belong to a trade union. In 2004, there were 1,800 employees who belong to a trade union 50 who did not belong to trade union. Of all the employees in 2004, 300 were women of whom only 8 were non-trade members. On the basis of this information, the ratio of female member of the trade union in 2000 and 2004 is

(a) 292 : 25 (b) 8 : 175 (c) 175 : 8 (d) 25 : 292

Ans. (d)

PREVIOUS YEAR QUESTIONS
STATISTICAL DESCRIPTION OF DATA

Q.1 By plotting cumulative frequency against the respective class boundary, we get [Dec. 2023]

(a) Frequency curve (b) Ogives

(c) Frequency polygon (d) Histogram

Ans. (b) Ogives

Q.2 The frequency of visitor in an office is given below:

Time	9 AM – 11 AM	11 AM – 1 PM	1 PM – 3PM	3 PM – 5 PM
Frequency	5	18	7	12

Find the cumulative frequency of visitors for the time 11 AM – 1 PM? [Dec. 2023]

(a) 5 (b) 23

(c) 18 (d) 30

Ans. (b) 23

Q.3 Consider the following data where class length is given as 5. Calculate the number of class intervals.
59, 68, 78, 57, 44, 73, 40, 60, 70, 47 [Dec. 2023]

(a) 5 (b) 6

(c) 7 (d) 8

Ans. (d) 8

Q.4 In a cumulative frequency curve, what is represented on the Y-axis? [Dec. 2023]

(a) Class interval (b) Cumulative frequency

(c) Frequency density (d) Relative frequency

Ans. (b) Cumulative frequency

Q.5 In a frequency distribution, the relative frequency of the class is: [Dec. 2023]

(a) The ratio of the class frequency to the total number of classes

(b) The ratio of the class frequency to the total frequency

(c) The ratio of the class frequency to the total number of data points.
(d) The ratio of the class mid point to the class frequency

Ans. (b) The ratio of the class frequency to the total frequency

Q.6 Frequency density corresponding to a class interval is ratio of : [Dec. 2023]
(a) Class frequency to class length
(b) Class frequency to total frequency
(c) Class frequency to Cumulative frequency
(d) Class length to class frequency

Ans. (a) Class frequency to class length

Q.7 The share holding pattern of *ABC* Itd. is as follows :-

Share holders	Promoters	FII	MF	Others	Public
No. of shares in Millions	120	25	20	20	15

What is the difference between central angles of Promoters and Public in pie chart? [June 2023]
(a) 216 (b) 189
(c) 180 (d) 99

Ans. (b) 189

Q.8 What does an Ogive curve represent? [June 2023]
(a) The cumulative frequency and class boundary
(b) The cumulative frequency and frequency
(c) The frequency and cumulative frequency
(d) the frequency and class interval

Ans. (a) The cumulative frequency and class boundary

Q.9 For tabulation caption is [June 2023]
(a) The upper part of the table
(b) the lower of the table
(c) the main part of the table
(d) The upper part of the table that describes the rows and sub rows

Ans. (a) The upper part of the table

Q.10 The mode of presentation of data are [June 2023]
(a) Textual, Diagrammatic and Internal presentation
(b) Tabular, Textual and Internal Presentation
(c) Tabular, Textual and Diagrammatic presentation
(d) Tabular, Diagrammatic and Internal presentation

Ans. (c) Tabular, Textual and Diagrammatic presentation

Q.11 The following is the data of the daily income of 86 persons

Income in `	500 – 999	1000 – 1499	1500 – 1999	2000 – 2499
No. of persons	15	28	36	7

What is the percentage of persons earning at least ₹1500 per day? [June 2023]

(a) 50% (b) 45%

(c) 40% (d) 60%

Ans. (a) 50%

Q.12Ogive for more than type and less than type distributions intersect at [Dec 2022]

(a) Mean (b) Median

(c) Mode (d) Origin

Ans. (b) Median

Q.13 The suitable formula for computing the number of class intervals is : [Dec 2022]

(a) 3.322 log N (b) 0.322 log N

(c) 1+3.322 log N (d) 1-3.322 log N

Ans. (c) 1+3.322 log N

Q.14Which of the left part of the table providing the description of the rows ? [Dec 2022]

(a) Caption (b) Box head

(c) Stub (d) Body

Ans. (c) Stub

Q.15Which one of the following is a source of primary data ? [Dec 2022]

(a) Government records

(b) Research Articles

(c) Journals

(d) Questionnaire filled by enumerators

Ans. (d) Questionnaire filled by enumerators

Q.16 Which one of the following is a continuous variable? [Dec 2022]

(a) The quantum of days to get a cure from illness

(b) The quantum of oxygen cylinders used to treat a patient

(c) The quantum of drug injected in to a patient

(d) The quantum of tablets prescribe to a patient

Ans. (b) The quantum of oxygen cylinders used to treat a patient

Q.17 Which one of the following is not a mode of presentation of data?

(a) Textual Presentation (b) Tabular presentation

(c) External presentation (d) Diagrammatic representation

Ans. (c) External presentation

Q.18 Histograms are drawn only when

(a) Frequencies in various class interval are equal

(b) Class intervals are equal

(c) Class interval are inequal

(d) For less than type cumulative frequencies

Ans. (b) Class intervals are equal

Q.19 The collected information on which of the following characteristic do not form data? [June 2022]

(a) The number of files audited are 'less than 6'. 'between 5 and 10' and 'more than 9'

(b) The number of files audited are 'very less'. 'between 5 and 10' and 'more than 9'

(c) The number of files audits in a file

(d) The number of auditors who audited a file

Ans. (b) The number of files audited are 'very less'. 'between 5 and 10' and 'more than 9'

Q.20 Types of research data are [June 2022]

(a) The number of auditors who audited a file

(b) Qualitative data and quantitative data

(c) Processed data and unprocessed data

(d) Discrete data and continuous data

Ans. (b) Qualitative data and quantitative data

Q.21 We get____by plotting cumulative frequency against the respective class boundary. [June 2022]

(a) Histogram (b) Polygon

(c) Pie chart (d) Ogives

Ans. (d) Ogives

Q.22 Median of a distribution can be obtained from [June 2022]

(a) Frequency polygon (b) Histogram

(c) Pie chart (d) Less than type ogives

Ans. (d) Less than type ogives

Q.23 Sweetness of a sweet dish is [June 2022]

(a) An attribute (b) A discrete variable

(c) A continuous variable (d) A variable

Ans. (a) An attribute

Q.24 The following data realte to the marks of a group of students: (SOD)

Marks:	Below 10	Below 20	Below 30	Below 40	Below 50
No. of Student:	15	38	65	84	100

How many students got marks more than 30?

(a) 65 (b) 50

(c) 35 (d) 43

Ans. (c) 35

Q.25 The following data relate to the marks of 48 students in Statistics: (SOD)

56	10	54	38	21	43	12	22
48	51	39	26	12	17	36	19
48	36	15	33	30	61	57	17
5	17	45	46	43	55	57	38
43	28	32	35	54	27	17	16
11	43	45	2	16	46	28	45

What are the frequency densities for the class intervals 30-39, 40-49, 50-59?

(a) 0.20, 0.50, 0.90 (b) 0.70, 0.90, 1.10

(c) 0.1875, 0.1667, 0.2083 (d) 0.90, 1.00, 0.80

Ans. (d) 0.90, 1.00, 0.80

Q.26 A student marks in five subjects S1, S2, S3, S4 and S5 are 86, 79, 90, 88 and 89. If we need to draw a Pie chart to represent these marks, then what will be Central angle for S3? (S.O.D)

(a) 130.2^0 (b) 75^0

(c) 105.6^0 (d) 94.8^0

Ans. (b) 75^0

Q.27 Multiple axis line chart is considered when (SOD)

(a) There is more than one time series

(b) The units of the variables are different.

(c) In any case

(d) If there are more than one time series and unit of variables are different.

Ans. (b) The units of the variables are different

Q.28 A National Institute arrange its student's data in accordance with difference States. The arrangement of data is known as

(a) Temporal Data (b) Geographical Data

(c) Ordinal Data (d) Cardinal Data

Ans. (b) Geographical Data

Q.29 In a study about the male and female students of Commerce and Science departments of a college in 5 years, the following data were obtained:

1995	2000
70% female students	75% female students
65% read Commerce	40% read Science
20% of male students read Science	50% of female students read Commerce Students
3000 Total No. of students	3600 Total No. of student

After combining 1995 and 2000 if x denotes the ratio of female commerce student to female Science student and y denotes the ratio of male commerce student to male Science student, then

(a) $x = y$ (b) $x > y$

(c) $x < y$ (d) $x \geq y$

Ans. (c) $x < y$

Q.30 A graph that uses vertical bars to represent data is called a [July 2021]

(a) Line graph (b) Scatter plot

(c) Vertical graphs (d) Bar graph

Ans. (d) Bar graph

Q.31 Frequency density of a class interval is the ratio of_____. [July 2021]

(a) Class frequency to the total frequency

(b) Class length to class frequency

(c) Class frequency to the cumulative frequency

(d) Frequency of that class interval to the corresponding class length

Ans. (d) Frequency of that class interval to the corresponding class length

Q.32 ____means separating items according to similar characteristics grouping them into various classes. [July 2021]

(a) Classification (b) Editing

(c) Separation (d) Tabulation

Ans. (a) Classification

Q.33Which of the following diagram is the most appropriate to represents various heads in total cost? [July 2021]

(a) Pie chart (b) Bar graph

(c) Multiple line chart (d) Scatter plot

Ans. (a) Pie chart

Q.34 There were 200 employees in an office in which 150 were married. Total male employees were 160 out of which 120 were married. What was the number of female unmarried employees? [July 2021]

(a) 30 (b) 40

(c) 50 (d) 10

Ans. (d) 10

Q.35 A bar chart is drawn for [Jan. 2021]

(a) Continuous date (b) Nominal data

(c) Time series data (d) Comparing different components

Ans. (d) Comparing different components

Q.36 A tabular presentation can be used for [Jan. 2021]

(a) Continuous series data (b) Nominal data

(c) Time series data for longer period (d) Primary data

Ans. (d) Primary data

Q.37 A variable qualitative characteristic is known as [Jan. 2021]

(a) Quality variable (b) An attribute

(c) A discrete variable (d) A continuous variable

Ans. (b) An attribute

Q.38 The accuracy and consistency of data can be verified by [Jan. 2021]

(a) Scrutiny (b) Internal Checking

(c) External Checking (d) Double Checking

Ans. (a) Scrutiny

Q.39 From a histogram one cannot compute the approximate value of [Jan. 2021]

(a) Mode (b) Standard deviation

(c) Median (d) Mean

Ans. (b) Standard deviation

Q.40 The left part of a table providing the description of rows is called [Jan. 2021]

(a) Caption (b) Box – head

(c) Stub (d) Body

Ans. (c) Stub

Q.41 Mode can be obtained from_____. [Jan. 2021]

(a) Frequency polygon (b) Histogram

(c) Ogive (d) All of the above

Ans. (b) Histogram

Q.42 Most of the commonly used distributions provide a [Jan. 2021]

(a) Bell-shaped (b) U-shaped

(c) J – shaped curve (d) Mixed Curve

Ans. (a) Bell-shaped

Q.43 Which of the following is suitable for the graphical representation of a cumulative frequency distribution? [Jan. 2021]

(a) Frequency polygon (b) Histogram

(c) Ogive (d) Pic Chart

Ans. (c) Ogive

Q.44 Sweetness of sweet dish is [Jan. 2021]

(a) An Attribute (b) A discrete Variable
(c) A Continuous Variable (d) A Variable

Ans. (a) An Attribute

Q.45 The average of salaries in a factory is Rs. 47,000. The statement that the average salary Rs. 47,000 is _____ [Dec. 2020]

(a) Descriptive static (b) Inferential
(c) Detailed (d) Undetailed

Ans. (a) Descriptive statics

Q.46 Statistics cannot deal with_____data. [Dec. 2020]

(a) Quantitative (b) Qualitative
(c) Textual (d) Attribute

Ans. (b) Qualitative

Q.47 Sweetness of a sweet dish is _____ [Dec. 2020]

(a) Attribute (b) Discrete Variable
(c) Continuous Variable (d) Variable

Ans. (a) Attribute

Q.48 Census reports are used as a source of_____data [Dec. 2020]

(a) Secondary (b) Primary
(c) Organize (d) Confidential

Ans. (a) Secondary

Q.49 Types of cumulative frequencies are _____ [Dec. 2020]

(a) 1 (b) 2
(c) 3 (d) 4

Ans. (b) 2

Q.50 You are an auditor of a firm and the firm earns a profit Rs. 67,000/– you stated to them that the annual profit is Rs. 67,000. This is_____type of statics. [Dec. 2020]

(a) Descriptive (b) Detailed
(c) Non detailed (d) Inferential

Ans. (a) Descriptive

Q.51The_____are used usually when we want to examine the relationship between two variables. [Dec. 2020]

(a) Bar Graph (b) Pie Chart
(c) Line Chart (d) Scatter Plot

Ans. (d) Scatter Plot

SUMMARY

- Statistics deals with the aggregates. An individual, to a statistician has no significance except the fact that it is a part of the aggregate.
- Statistics is concerned with quantitative data. However, qualitative data also can be converted to quantitative data by providing a numerical description to the corresponding qualitative data.
- We can broadly classify data as
  - Primary
  - Secondary
- Scrutiny of Data : Data scrutiny focuses on inference, the process of deriving a conclusion based solely on what is already known by the researcher. Data needs to be curated, cleansed, enriched and translated into actionable insights before it can be put to work doing something meaningful.
- Mode of Presentation of Data
  (a) Textual presentation
  (b) Tabular presentation or Tabulation;
  (c) Diagrammatic representation.
- The type of diagrams:
  (a) Line diagram or Histogram
  (b) Bar diagram
  (c) Pie chart
- Frequency Distribution of a Variable
  (a) Find the largest and smallest observation and obtain the difference between them, known as Range,
  (b) In the case of discrete variable, Form a number of classes depending on the number of isolated value assumed by a discrete variable.
  (c) In case of a continuous variable, find the number of class intervals using the relation,
  No. of class interval × class length = Range.
  (d) Cumulative frequency – less than and more than types
- Graphical Representation of Data
  (i) Histogram or Area diagram;
  (ii) Frequency Polygon;
  (iii) Ogives or cumulative Frequency graphs.
- Frequency Curve
  (a) Bell-shaped curve; (b) U-shaped curve;
  (c) J-shaped curve; (d) Mixed curve.
- Lower Class Boundary (LCB) = Lower Class Length $-\frac{D}{2}$
  Upper Class Boundary = Upper Class Length $+\frac{D}{2}$
- Mid - point= $\frac{LCB}{2}+\frac{LCL+UCL}{2}$
- Frequency density of a class interval = frequency of that class interval/corresponding class length
- Relative frequency and percentage frequency of a class interval = Class frequency/total frequency

INTRODUCTION TO SAMPLING

Sampling is a fundamental technique in statistics that enables researchers to draw meaningful conclusions about a large population by studying a smaller, manageable subset. This approach acknowledges that it is often impractical, if not impossible, to collect data from an entire population. Instead, it provides a systematic way to select a representative group, known as the sample, which is carefully chosen to reflect the characteristics of the larger population. Sampling is essential for various fields, including market research, scientific experiments, public opinion polling, and quality control, as it offers an efficient means of making inferences about a broad range of phenomena.

E.g.: When a person goes to market to buy rice, firstly, he takes some rice in his hand from a bag containing rice and then examining it, he takes the decision whether to buy it or not.

PRINCIPLES OF SAMPLE SURVEY

A sample survey is a research method where data is collected from a subset (sample) of individuals or items from a larger group (population). It's used to make inferences about the entire population by studying the characteristics and behaviors of the selected sample.

Basic principles of Sample survey comparison the components such as:

- **Law of Statistical Regularity:** This principle is based on the notion that in large and random samples, patterns and regularities emerge. It means that in sufficiently large samples, the characteristics of the sample start to mirror the characteristics of the population it's drawn from.
- **Principle of Inertia:** This principle suggests that, unless there's a good reason to the contrary, researchers often prefer not to change their methods or sample size. It helps in maintaining consistency and comparability in survey results over time.
- **Principle of Optimization:** This principle focuses on maximizing the information gained from a sample while minimizing the cost and effort of collecting data. It's about finding the right balance between sample size and data quality.
- **Principle of Validity:** Validity is all about measuring what you intend to measure. In sampling surveys, this principle emphasizes the importance of using methods and questions that accurately capture the information you're interested in. It's essential to ensure that your survey results are meaningful and truly represent the characteristics of the population.

TYPES OF ERRORS IN SAMPLE SURVEY

There are two types of errors in sample survey:

(a) Sampling errors (b) Non-sampling errors

- Sampling errors are discrepancies between the characteristics of a sample and the larger population, usually due to random chance.
  - **(a) Errors arising out of defective sampling** occur when the sampling method used is flawed, leading to an unrepresentative sample.
  - **(b) Errors arising out due to substitution** involve issues when intended survey participants are replaced with others, impacting data accuracy.
  - **(c) Errors owing to faulty demarcation of units** result from misidentifying or misclassifying survey units, affecting the overall sample.

(d) **Errors owing to the wrong choice of statistic** occur when an inappropriate statistical method is used, leading to misleading or inaccurate results.

(e) **Variability in the population** is an error related to the natural differences that exist within a population. It arises when the sampled units exhibit greater diversity than expected, leading to less precise estimates.

Non-sampling errors are inaccuracies in survey data that aren't a result of the sampling process. They encompass a wide range of errors, including coverage errors, nonresponse errors, measurement errors, and processing errors. These errors can be caused by factors such as undercoverage, nonresponse from participants, errors in data collection, and mistakes during data processing. Non-sampling errors can significantly affect the quality and reliability of survey results.

POPULATION AND SAMPLE

1. **Population:** The population is the entire group or collection of individuals, objects, or data that the researcher wants to draw conclusions about. It represents the larger entity under study and can be finite or infinite, depending on the research context.

2. **Sample:** A sample is a subset of the population selected for a research study. It is chosen to represent the population because studying the entire population may be impractical or too costly. The characteristics and behaviors of the sample are analyzed to make inferences about the entire population. The process of selecting a sample should be done systematically to ensure it is a representative and unbiased reflection of the population.

S_No	Population	Sample
1.	Student of the school	Commerce student of grade 12
2.	All books in a library	Book

3. **Parameter:** A parameter is a characteristic of a population, determined by considering all units within that population. Statistical inferences about population parameters are made by analyzing sample observations drawn from the same population.

Here,

- When we are interested in 'Population Mean' then it is represented as:
  $\mu = \frac{\sum_{a=1}^{n} x_a}{N}$
  Where n denotes the population size and x_a represent ath member of the population.
- When we are interested in 'Population Proportion' then it is represented as:
  $p = \frac{X}{N}$
  Where, X represent the count possessing the required attribute.
- Another important parameter namely the population variance, to be denoted by σ^2 is given by:
  $\sigma = \frac{\sum(X-\mu)}{N}$
  Also, $SD = \sigma = \sqrt{\frac{\sum(X_a-\mu)^2}{N}}$

STATISTICS

It is defined as a statistical measure of sample observation.

It is a function of sample observations. If the sample observations are denoted by $x_1, x_2, x_3, \ldots x_n$ then a statistics T may be expressed as $T = f(x_1, x_2, x_3, \ldots x_n)$.

A statistic is used to estimate a particular population parameter. The estimates of population mean, variance and population proportion are given by

$\bar{x} = \hat{\mu} = \frac{\sum x_i}{n}$

$S_2 = \hat{\sigma}^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

and $p = \hat{p} = \frac{x}{n}$

Here x (in the last formula) denotes the number of units in the sample in possession of the attribute under discussion.

UNDERSTANDING SAMPLING DISTRIBUTION AND STANDARD ERROR

Imagine starting with a group of N items. By taking several samples of a fixed size (let's call it n), we can explore the variability in our results. If we draw samples with replacement, the possibilities are endless, but without replacement, the total number of potential samples is ${}^{N}C_{n}$.

When we calculate a statistic, like the mean, it's natural for the values to differ across samples. This variation is what we call "Sampling Fluctuations."

If we manage to gather all possible values of a statistic (let's call it T) from samples of a fixed size, along with their probabilities, we can create a probability distribution. This distribution, treating the statistic as a random variable, is the sampling distribution of the statistic. Similar to theoretical probability distributions, this sampling distribution has key characteristics. The mean of the statistic in its sampling distribution is known as the "Expectation," and the standard deviation of the statistic (T) is called the "Standard Error (SE)" of T. The SE acts as a measure of precision achieved through sampling and is inversely proportional to the square root of the sample size. This implies that as the sample size increases, the SE decreases, indicating increased precision.

SE is inversely proportional to the square root of sample size.

$SE() = \frac{\sigma}{\sqrt{n}}$ for simple random sampling with replacement.

$= \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$ for simple random sampling without replacement.

STANDARD ERROR FOR PROPORTION

$SE(p) = \sqrt{\frac{pq}{n}}$ for simple random sampling with replacement $= \sqrt{\frac{pq}{n}} \sqrt{\frac{N-n}{N-1}}$ for simple random sampling without replacement.

The factor $\sqrt{\frac{N-n}{N-1}}$ is known as finite population correction (fpc) or finite population multiplier and may be ignored as it tends to 1 if the sample size (n) is very large or the population under consideration is infinite when the parameters are unknown, they may be replaced by the corresponding statistics.

SAMPLING

The process of selecting a sample from the given population is called Sampling.

Broadly it is of three types:

(a) Probability sampling

(b) Non-probability sampling

(c) Mixed sampling

SIMPLE RANDOM SAMPLING (SRS)

- Probability-based; each member has a fixed chance to be part of the sample.
- Units selected independently; can be with or without replacement.
- Effective for small, homogenous populations.

STRATIFIED SAMPLING

- Suitable for large, heterogeneous populations.
- Divides population into strata; minimal variation within strata.
- Enhances precision, provides representation for all sub-populations.

MULTI-STAGE SAMPLING

- Complex design with multiple stages of sampling units.
- **E.g.:** State, district, police station, household.
- Cost-effective, flexible, and covers a large population, but may be less accurate.

SYSTEMATIC SAMPLING

- Units selected at regular intervals after a random start.
- Convenient and less time-consuming; suitable with an updated sampling frame.
- Prone to bias if there is an undetected periodicity in the sampling frame.

PURPOSIVE OR JUDGMENT SAMPLING

- Non-probabilistic; dependent on the discretion of the sampler.
- Subjective, varies among individuals; unsuitable for statistical hypothesis testing.

NOTEWORTHY POINTS

MIXED SAMPLING

1. Combines probabilistic and pre-decided rules.
2. Systematic sampling is an example.

SIMPLE RANDOM SAMPLING (SRS) CONSIDERATIONS

1. Effective if the population is not very large, sample size is reasonable, and population is homogenous.
2. Free from sampler's biases; foundational for tests of significance.

STRATIFIED SAMPLING CONSIDERATIONS

1. Advisable for large, heterogeneous populations with prior information available.
2. Involves proportional allocation or Bowley's allocation for sample sizes.

MULTI-STAGE SAMPLING COVERAGE

1. Extensive coverage, computational labor savings, and cost-effectiveness.
2. Adds flexibility but may be less accurate compared to stratified sampling.

SYSTEMATIC SAMPLING DRAWBACK

1. Prone to bias if periodicity in the sampling frame exists.
2. No statistical inference about population parameters due to non-probabilistic nature.

Q.1 Sampling can be described as a statistical procedure
(a) To infer about the unknown universe from a knowledge of any sample
(b) To infer about the known universe from a knowledge of a sample drawn from it
(c) To infer about the unknown universe from a knowledge of a random sample drawn from it
(d) Both (a) and (b)

Ans. (c)

Q.2 The basis for making a statistical decision about an unknown universe is:
(a) Sample observations (b) A sampling frame
(c) Sample survey (d) Complete enumeration

Ans. (a)

Q.3 Which sampling method provides flexibility in the sampling process?
(a) Simple random sampling (b) Multistage sampling
(c) Stratified sampling (d) Systematic sampling

Ans. (c)

Q.4 Which sampling method is most impacted if the sampling frame reveals an undetected periodicity?
(a) Simple random sampling (b) Stratified sampling
(c) Multistage sampling (d) Systematic sampling

Ans. (d)

Q.5 If a random sample of size 2 with replacement is taken from the population containing the units 3, 6 and 1, then the samples would be
(a) (3, 6), (3, 1), (6, 1)
(b) (3, 3), (6, 6), (1, 1)
(c) (3, 3), (3, 6), (3, 1), (6, 6), (6, 3), (6, 1), (1, 1), (1, 3), (1, 6)
(d) (1, 1), (1, 3), (1, 6), (6, 1), (6, 2), (6, 3), (6, 6), (1, 6), (1, 1)

Ans. (c)

Q.6 If from a population with 25 members, a random sample without replacement of 2 members is taken, the number of all such samples is
(a) 300 (b) 625 (c) 50 (d) 600

Ans. (a)

Q.7 A population comprises 5 members. The number of all possible samples of size 2 that can be drawn from it with replacement is
(a) 100 (b) 15 (c) 125 (d) 25

Ans. (d)

Q.8 The population of roses in Salt Lake City is an example of
(a) A finite population (b) An infinite population

(c) A hypothetical population (d) An imaginary population

Ans. (b)

Q.9 As the sample size increases, standard error

(a) Increases (b) Decreases

(c) Remains constant (d) Decreases proportionately

Ans. (b)

Q.10 A parameter is a characteristic of

(a) Population (b) Sample

(c) Both (a) and (b) (d) (a) or (b)

Ans. (a)

Q.11 A statistic is

(a) A function of sample observations

(b) A function of population units

(c) A characteristic of a population

(d) A part of a population

Ans. (a)

Q.12 In a survey of 50 households, a sample of 10 households is randomly chosen without replacement. How many different samples are possible?

(a) 1,102,100 (b) 102,722,500 (c) 102,722 (d) 10,000

Ans. (b)

Q.13 If a random sample of size two is drawn without replacement from a population containing the units a, b, c, and d, what are the possible samples?

(a) (a, b), (a, c), (a, d)

(b) (a, b), (b, c), (c, d)

(c) (a, b), (b, a), (a, c), (c, a), (a, d), (d, a)

(d) (a, b), (a, c), (a, d), (b, c), (b, d), (c, d)

Ans. (d)

SUMMARY

- Sampling is a vital statistical method enabling meaningful conclusions about a large population by studying a smaller, representative subset. It's crucial in various fields like market research and public opinion polling.
- **Principles of sample surveys:** Statistical regularity, Inertia, Optimization, and Validity. It aims for accurate representation while minimizing costs.
- **Errors in Surveys:**
  - **Sampling:** Defective sampling, Substitution, Faulty demarcation of units, Wrong choice of statistic, Population variability
  - **Non-Sampling:** Coverage, Nonresponse, Measurement, and processing Errors.
- **Population** is the entire group under study, while a sample is a subset representing it.
- **Parameters** are population characteristics, analyzed through sample observations for statistical inferences.

- **Types of Sampling:**
  - Probability sampling
  - Non-probability sampling
  - Mixed sampling
- **Sampling Distribution and Standard Error:**
  - Sampling distribution of a statistic (T) is a probability distribution of all possible values.
  - Standard Error (SE) measures precision, inversely proportional to the square root of sample size.
- **Sampling Methods:**
  1. Simple Random Sampling (SRS)
  2. Multi-Stage Sampling
  3. Systematic Sampling
  4. Stratified Sampling

Unit- 1: Measure of Central Tendency

WHAT IS CENTRAL TENDENCY?

Let's take an example, if you talk about the weight or height of students in your class, what would you observe?

As we can see the majority of data is tending to cluster towards the center which we call the central tendency and the value which represents the center is called as measure of central tendency.

In the simplest cases, the measure of central tendency is an average of a set of measurements.

That's why, when our marks in exams come – we see average marks of students or when we go to work – we see the average salary of employees.

Central Tendency refers to the tendency of data to cluster around a central or typical value. It represents the value that best represents the center of a distribution. In other words, it gives us an idea of where most of the data points are concentrated.

LET'S SEE SOME DIFFERENT MEASURES OF CENTRAL TENDENCY

- **Arithmetic Mean (AM):** It is also known as the average, which is the sum of all the values divided by the total number of values.
- **Median (Me):** It is the middle value in a sorted dataset. It divides the dataset into two equal halves, with half of the values below and half above the median.
- **Mode (Mo):** It is the value that occurs most frequently in a dataset. It represents the value that appears with the highest frequency.
- **Geometric Mean (GM):** It is the nth root of the product of n values. It is often used for calculating growth rates or when dealing with exponential data.
- **Harmonic Mean (HM):** It is the reciprocal of the arithmetic mean of the reciprocals of a set of values. It is useful when dealing with rates, ratios, or averages of rates.

Now, let's understand all the above measures of central tendency in detail.

ARITHMETIC MEAN

Arithmetic Mean (Mean) $= \frac{Sum\ of\ observations}{Number\ of\ observation}$

Let's variable x has n values: $x_1, x_2, x_3, x_4, x_5, \ldots \ldots x_n$, then Arithmetic Mean is denoted by $\bar{x}$, which is equal to $\frac{x_1+x_2+\cdots+x_n}{n}$

Q.1 The following figures give the marks of 10 students in a class test:

Marks obtained: 12, 8, 17, 13, 15, 9, 18, 11, 6, 1. Find the arithmetic mean.

(a) 8 (b) 9 (c) 10 (d) 11

Ans. (d)

Q.2 Following are the Monthly salary in multiple of 1000 INR of a sample of 10 employees: 41, 32, 45, 65, 67, 78, 90, 75, 33, 44. Compute the mean wage (in 1000 INR).

(a) 40 (b) 45 (c) 57 (d) 59

Ans. (c)

ANOTHER METHODS TO FIND MEAN

1. Direct method: If $x_1, x_2, x_3, \ldots x_n$ be n observation with respective frequencies $f_1, f_2, f_3 \ldots f_n$, then $Mean = \frac{f_1x_1+f_2x_2+\cdots+f_nx_n}{f_1+f_2+\cdots+f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
2. Step-deviation method: When the data values are quite, the step-deviation method is used to find the mean. $a + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$

   Where, $a = assumed\ mean$

   $x_i = class - marks = \frac{upper\ limit + Lower\ limit\ class}{2}$

   $d_i = x_i - a =$ deviation from ith class

   $u_i = \frac{x_i - a}{h}\ and\ h = length\ of\ class$

Q.3 In a test of 10 marks, 100 students get marks as below:

Marks obtained	Number of Students
1	2
2	3
3	5
4	10
5	30
6	10
7	18
8	12
9	6
10	4

What is the average marks obtained?

(a) 4.89 (b) 5.89 (c) 5.79 (d) 6.99

Ans. (b)

Q.4 The following data give the daily earning (in `) of 20 workers in a factory:

Daily earnings (in `)	100	140	170	200	250
No. of workers:	5	2	6	4	3

Calculate the average daily earnings using:

(i) Direct Method

(ii) Step – deviation Method.

(a) 167.5 (b) 165.85 (c) 510.70 (d) 166.5

Ans. (a)

Q.5 Computer the mean weight of your friend's group:

Weight in kg	41-45	46-50	51-55	56-60	61-65	66-70
No. of friends:	4	3	9	8	7	2

(a) 55 (b) 54.2727 (c) 56.7272 (d) 59.2882

Ans. (b)

Q.6 Given below is the distribution of marks obtained by 140 students in an examination:

Marks:	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of students:	7	15	18	25	30	20	16	7	2

(a) 67.5 (b) 56.85 (c) 51.070 (d) 50.714

Ans. (d)

Q.7 Calculate mean from the following data:

Marks	No. of students
0-10	4
10-20	16
20-30	40
30-40	76
40-50	96
50-60	112
60-70	120
70-80	125

(a) 50.20 (b) 53.60 (c) 51.70 (d) 66.5

Ans. (b)

Q.8 The mean of the following frequency distribution is 50 but the frequencies f_1 and f_2 in class 20-40 and 60-80 are missing. Find the missing frequencies.

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency:	17	f_1	32	f_2	19	120

(a) 28, 24 (b) 26, 26 (c) 22, 30 (d) 32, 20

Ans. (a)

PROPERTIES OF ARITHMETIC MEAN

1. If all the observation of the dataset are constants, let 'a', then the arithmetic mean will also be equal to 'a'.

   E.g.: If the weight of each student of group of 6 students of grade 10 is 60 kg, then the mean weight will also be 60 kg.

2. The algebraic sum of deviations of the given set of observations from their arithmetic mean is zero.

   That is, $(x - \bar{x}) = 0$ or for a frequency distribution $\sum f\,(x - \bar{x}) = 0$

E.g.: For the given set of observations 2, 3, 4, 5, 6, then $\bar{x} = \frac{2+3+4+5+6}{5} = 4$

Thus, $\sum(x - \bar{x}) = (2-4) + (3-4) + (4-4) + (5-4) + (6-4)$

$= -2 - 1 + 0 + 1 + 2 = 0$

i.e., $\sum(x - \bar{x}) = 0$

3. If each observation of a dataset is increased or decreased by a constant 'k', then the arithmetic mean of new data will also gets increased or decreased by the same constant 'k'.

   E.g.: The arithmetic mean of 5 observations: 5, 10, 15, 20 and 25 is 15. If each observation is increased by 3, then the new mean will also increase by 3 i.e., new arithmetic mean = 15 + 3 = 18

4. AM is affected with the change of origin and scale i.e., if the variable 'x' is changed to a new variable 'y' by a change of origin 'a' and scale 'b'

   $\Rightarrow y = a + bx$, then

   $AM_{y=}a + b \times AM_x \; or \bar{y} = a + b\bar{x}$

   E.g.: If x and y are related by the equation $3x + y = 6 \; and \; \bar{x} = 1$ then

   $\Rightarrow \bar{y} = 6 - 3(1) \Rightarrow \bar{y} = 3$

5. **Combined mean:** If the arithmetic means of two or more sets of data are known, then the arithmetic mean of the combined data can be obtained.

   If n_1 and n_2 are the number of observation and $\bar{x}_1$ and $\bar{x}_2$ are their respective means of two sets of data then the combined mean with observation is given by $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$

Q.9 The sum of deviations of a certain number of observations measured from 4 is 72 and the sum of observations of the same value from 7 is -3. Find the number of observations and their mean.

(a) 24, 7.5 (b) 24, 6.77 (c) 25, 6.88 (d) 25, 6.5

Ans. (c)

Q.10 The arithmetic mean of a set of 5 observation 5, 10, 15, 20 and 25 is 15 However, if each item is increased by 3, then the arithmetic mean will be

(a) 18 (b) 18.5 (c) 19 (d) 19.5

Ans. (a)

Short-cut method: Since, the arithmetic mean of the given set of observation is 15. Now, each observation is increased by 3, thus they will also increase by 3.

Therefore, new mean = 15 + 3 = 18

Hence, the correct option is (a).

Q.11 The arithmetic mean of a set of 5 observations: 2, 4, 6, 8, 10 is 6. If each item is multiplied by 2, then the new arithmetic mean will be?

(a) 18 (b) 12 (c) 15 (d) 20

Ans. (b)

Short-cut method: Since, the arithmetic mean of the given set of observations is 6.

Now, each observation gets multiplied by 2, thus the mean will also be multiplied by 2.

Therefore, new mean = 6 × 2 = 12

Hence, the correct option is (b).

Q.12 If it is known that 2 variables x and y are related by equation 2x + 3y = 5 and x = 1, then y is

(a) 1 (b) 0 (c) 5 (d) 2

Ans. (a)

Q.13The mean salary for a group of 40 female workers is ` 52000 per month and that for a group of 50 male workers is ` 68000 per month. What is the combined mean salary?

(a) ` 65000 (b) ` 60888.88 (c) ` 51070 (d) ` 16650

Ans. (b)

Q.14A distribution consists of three components with total frequencies of 200, 250 and 300 having means 25, 10 and 15 respectively. Find the mean of the combined distribution.

(a) 4 (b) 8 (c) 12 (d) 16

Ans. (d)

Q.15The mean weight of 150 students (boys and girls) in a class is 60 kg. The mean weight of boy students is 70 kg and that of girl students is 55 kg. The number of boys and girls respectively in the class are

(a) 50, 100 (b) 100, 50 (c) 120, 30 (d) 30, 120

Ans. (a)

Q.16Fifty students took up a test. The result of those who passed the test is given below:

Marks	4	5	6	7	8	9
Number of Students:	8	10	9	6	4	3

If the average for all 50 students was 5.16 marks, find the average of those who failed.

(a) 2.1 (3.9) (1.1) (3.3)

CORRECTING INCORRECT MEAN

Q.17The mean marks of 100 students were found to be 40. Later on, it was discovered that a score of 53 was misread as 83. Find the correct mean corresponding to the correct score.

(a) 67.5 (b) 39.7 (c) 51.70 (d) 66.5

Ans. (b)

Note: We can use the direct formula for the above:

i.e., $\bar{x}_c = \bar{x}_i + \frac{x_c - x_{in}}{n}$

where, $\bar{x}_c$ represents the correct mean, in $\bar{x}_i$ is the incorrect mean, x_c is the correct observation and x_{in} is the incorrect observation.

Q.18The mean salary paid to 1000 employees of an establishment was found to be `180.40. Later on, after disbursement of salary, it was discovered that the salary of the two employees was wrongly entered as `297 and `165. Their correct salaries were `197 and `185. Find the correct mean salary.

(a) 168.5 (b) 179.85 (c) 510.70 (d) 180.32

Ans. (d)

WEIGHTED ARITHMETIC MEAN

Let $w_1, w_2, \ldots \ldots w_n$ be the weight attached to n observations $x_1, x_2 \ldots x_n$ respectively then the weighted arithmetic mean, denoted by $\bar{x}w$ is given by

Weighted mean $= \frac{\sum x_i w_i}{\sum w_i}$

Q.19 A candidate obtained the following percentage of marks in different subjects in the Half-yearly examination: English 46%, Statistics 67%, Cost Accountancy 72%, Economics 58%, Income Tax 53%. It is agreed to give double weights to marks in English and Statistics as compared to other subjects. What is the simple and weighted arithmetic mean?

(a) 56.4, 59.42 (b) 58.42, 59.2 (c) 59.2, 58.42 (d) None of these

Ans. (a)

Q.20 Measures of central tendency for a given set of observations measures

(a) The scatterness of the observations (b) The central location of the observations

(c) Both (a) and (b) (d) One of these

Ans. (b)

Q.21 While computing the AM from a grouped frequency distribution, we assume that

(a) The classes are of equal length
(b) The classes have equal frequency
(c) All the values of a class are equal to the mid-value of that class
(d) None of these.

Ans. (c)

Q.22 Which of the following statements is wrong?

(a) Mean is rigidly defined

(b) Mean is not affected due a sampling fluctuation

(c) Mean has some mathematical properties

(d) All of these

Ans. (b)

Q.23 If there are 3 observations 15, 20, 15 then the sum of deviation of the observations from their AM is

(a) 0 (b) 5 (c) -5 (d) None of these

Ans. (a)

Q.24 If there are two groups containing, 30 and 20 observations and having 50 and 60 as arithmetic means, then the combined arithmetic mean is

(a) 55 (b) 56 (c) 54 (d) 52

Ans. (c)

Q.25 The average salary of a group of unskilled workers is `10,000 and that of a group of skilled workers is `15,000. If the combined salary is `12,000, then what is the percentage of skilled workers?

(a) 40% (b) 50% (c) 60% (d) None of these

Ans. (a)

Q.26If the relationship between two variables u and v are given by 2u + v + 7 = 0 and if the AM of u is 10, then the AM of v is

(a) 17 (b) –17 (c) –27 (d) 27

Ans. (c)

Q.27The weights of a group of individuals are recorded in kilograms: 65, 70, 75, 80, 85, 90, 95. Compute the mean weight.

(a) 75 kg (b) 80 kg (c) 85 kg (d) 90 kg

Ans. (a)

Q.28 Find the mean salary of 60 workers in a factory from the following table.

Salary in `	Number of Workers
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1

(a) `5065.50 (b) `6000 (c) `5083.33 (d) None of the above

Ans. (c)

Q.29The mean of a set of numbers is 8. If each number is multiplied by 3, then the new mean is_____.

(a) 24 (b) 8 (c) 3 (d) 12

Ans. (a)

Q.30What is the value of mean for the following data:

Marks	5-14	15-24	25-34	35-44	45-54	55-64
No. of Students	10	18	32	26	14	10

(a) 30 (b) 29 (c) 33.68 (d) 34.21

Ans. (c)

Q.31Find the combined mean from the following data:

	Series X	Series Y
Arithmetic mean	12	20
Number of items	80	60

(a) 10.54 (b) 15.43 (c) 3.50 (d) None of these

Ans. (b)

Q.32The mean of 20 observations is 85, but it was later found that two of the observations were wrongly read as 75 and 70 instead of 57 and 60. Find the correct mean.

(a) 80.7 (b) 75.5 (c) 63.5 (d) 83.6

Ans. (d)

Q.33Calculate weighted mean from the following data:

Value	10	12	15	18	20
Weight	2	5	12	4	7

(a) 18.74 (b) 17.55 (c) 1350 (d) 15.73

Ans. (d)

CRITERIA FOR AN IDEAL MEASURE OF CENTRAL TENDENCY

- It should be properly and unambiguously defined
- It should be easy to comprehend.
- It should be simple to compute.
- It should be based on all the observations
- It should have certain desirable mathematical properties.
- It should be least affected by the presence of extreme observations

MERITS AND DEMERITS OF ARITHMETIC MEAN

Merits

Arithmetic mean possesses the following merits

1. It is rigidly defined.
2. It is easy to calculate and simple to understand.
3. It is based on all the observations.
4. It is suitable for further mathematical treatment.
5. Of all the average, arithmetic mean is affected least by fluctuation of sampling.

Demerits

Arithmetic mean has the following drawbacks:

1. It is very much affected by extreme values.
2. In a distribution with open-end classes the value of mean cannot be computed without making assumptions regarding the size of the class' interval of the open-end classes.
3. It can neither be determined by inspection nor can it be located graphically.
4. It cannot be computed for qualitative data such as honesty, beauty, intelligence etc.
5. It may lead to wrong conclusions if the details of the data from which it is obtained are not available.

MEDIAN

- Median, for a given set of observations, when arranged in an ascending order or a descending order of magnitude. It may be defined as the middle-most value.
- As distinct from the arithmetic mean, which is based on all the items of the distribution, the median is what is called a positional average.
- The position of the median in a distribution is such that the number of observations above it is equal to the number of observations below it.

CALCULATION OF MEDIAN: INDIVIDUAL OBSERVATIONS

For ungrouped data consisting of n observations, the calculation of median involves the following steps:

Step1. Arrange the given set of observation in an ascending or descending order of magnitude.

Step2. The median is given by

(a) When n is odd, the value is given by: $(\frac{n+1}{2})\,th$ observation $= \frac{\frac{n}{2}th\ observation + (\frac{n}{2}+1)th\ observation}{2}$

Q.34What is the median for the following observations?

10, 2, 7, 9, 13

(a) 5 (b) 9 (c) 6 (d) 11

Ans. (b)

Q.35The following number of goals were scored by a team in a series of 10 matches:

2, 3, 4, 5, 0, 1, 3, 3, 4, 3. Find the median of these scores.

(a) 5 (b) 6 (c) 3 (d) None of these

Ans. (c)

Q.36The median of the observations 42, 72, 35, 92, 67, 85, 72, 81, 51, 56 is

(a) 69.5 (b) 72 (c) 64 (d) 61.5

Ans. (a)

Q.37The median of $x, \frac{x}{2}, \frac{x}{3}, \frac{x}{5}$ is 10. Find x, where $x > 0$

(a) 24 (b) 32 (c) 8 (d) 16

Ans. (a)

CALCULATION OF MEDIAN: DISCRETE SERIES

In the case of discrete series, where the variable takes the values $X_1, X_2, \ldots \ldots X_n$ with respect frequencies $f_1, f_2, \ldots f_n$ with $\sum f = N$, median is the size of $(\frac{N+2}{2})\,th$ observation.

In this case, the calculation of median involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency $(c.f)$ distribution.

Step 2: Find $(\frac{N+1}{2})$

Step 3: See the $c.f$. just greater than or equal to $\frac{N+1}{2}$

Step 4: The value of the variable corresponding to the $c.f$ obtained in Step 3 gives the required median.

Q.38 For the distribution

X	1	2	3	4	5	6
F	6	9	10	14	12	8

The value of median is

(a) 3.5 (b) 3 (c) 4 (d) 5

Ans. (c)

Q.39 Calculate median from the following data

X	10	20	30	40	50	60	70
F	1	5	12	20	19	9	4

(a) 50 (b) 70 (40) (d) Cannot be determined

Ans. (c)

CALCULATION OF MEDIAN – CONTINUOUS SERIES

In the case of continuous series, median is the size of $\frac{N}{2}$th observation, where $N = \sum f$ is the total frequency.

The calculation of median in this case involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency $(c.f)$ distribution.

Step 2: Find $\frac{N}{2}$

Step 3: See the c.f. just greater than or equal to $\frac{N}{2}$

Step 4: Find the class corresponding to the c.f. obtained in Step 3. This is called the median class.

Step 5: Applying the following interpolation formula for calculating the median.

$$Median = l + \frac{\frac{N}{2} - c.f}{f} \times h$$

Where, l =lower limit of the median class, f =frequency of the median class, $c.f.$= cumulative frequency of the class preceding the median class, and h =size of width of the median class.

Q.40 The marks obtained by 100 students in a certain examination are given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students:	10	9	25	30	16	10

Calculate the median marks.

(a) 32 (b) 50 (c) 30 (d) None

Ans. (a)

PROPERTIES OF MEDIAN

1. If x and y are two variables, to be related by $y = a + bx$ for any two constants a and b, then the median of y is given by $y_{me} = a + bx_{me}$
2. For a set of observations, the sum of absolute deviations is minimum when the deviations are taken from the median. This property states that $\sum |x - A|$ is minimum if we choose A as the median.

Q.41 Consider two variables, x and y, related by the equation y = 3x + 5. If the median of x is 10, what is the median of y?

(a) 20 (b) 40 (c) 37 (d) 35

Ans. (d)

Q.42 Two variables x and y are given by $y = 2x + 3$. If the median of x is 20, what is the median of y?

(a) 20 (4) 40 (c) 37 (d) 35

Ans. (c)

Q.43 In case of an even number of observations, which of the following is median?

(a) Any of the two middle – most value

(b) The simple average of these two middle values

(c) The weighted average of these two middle values

(d) Any of these

Ans. (b)

Q.44 What is the median for the following observation?

5, 8, 6, 9, 11, 4

(a) 6 (b) 7 (c) 8 (d) None of these

Ans. (b)

Q.45 Find the median for the following data: 58 49 64 70 91 34

(a) 58 (b) 70 (c) 61 (d) 19

Ans. (c)

Q.46 What is the value of median for the following data?

Marks	5-14	15-24	25-34	35-44	45-54	55-64
No. of Students:	10	18	32	26	14	10

(a) 28 (b) 30 (c) 32.94 (d) 33.18

Ans. (c)

Q.47 Two variables x and y are given by $y - 5x - 5 = 0$. If the median of x is 20, What is the median of y?

(a) 100 (b) 105 (c) 1105 (d) None of these

Ans. (b)

Q.48 Find the missing frequency from the following data, given that the median mark is 23.

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students:	5	8	?	6	3

(a) 10 (b) 16 (c) 5 (d) 18

Ans. (a)

Q.49 The median for the following data

Profit in '000`:	Below 5	Below 10	Below 15	Below 20	Below 25	Below 30
No. of Firms:	10	25	45	55	62	65

(a) 11.60 (b) 11556 (c) 11875 (d) 11.50

Ans. (c)

Q.50 Given below is the distribution of marks obtained by 140 students in an examination:

Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of Students:	7	15	18	25	30	20	16	7	2

Find the median of the distribution.

(a) 55.60 (b) 65.75 (c) 51.17 (d) 61.50

Ans. (c)

Merits of Median

Median possesses the following merits

1. It is rigidly defined
2. It is easy to calculate and simple to understand.
3. It can be computed while dealing with a distribution with open end classes
4. Being a- positional average, it is not much affected by extreme observations.
5. it is the most appropriate average to be used while dealing with qualitative data.
6. It can sometimes be located by inspection and can also be determined graphically.

Demerits

Median has the following limitations:

1. Median, being a positional average, is not based on each and every item of the distribution.
2. It is not suitable for further mathematical treatment. For example, it is not possible to find the combined median of two or more groups.
3. It cannot be determined exactly for an ungrouped data consisting of an even number of observations. It is determined approximately as the mid-point of two middle 1 observations.
4. In comparison to arithmetic mean, it is much affected by sampling fluctuations.
5. For calculating the median, it is necessary to arrange the data in order of magnitude.

PARTITION VALUE OR QUARTILES OR DECILES OR PERCENTILES

Partition values, such as quartiles, deciles, and percentiles, are statistical measures that divide a given set of observations into equal parts.

- **Quartiles**: Quartiles divide the data into four equal parts, each containing 25% of the observations. The three quartiles are denoted as Q_1, Q_2 (which is the median), and Q_3. Q_1 represents the value below which 25% of the data falls, Q_2 is the median (50th percentile), and Q_3 represents the value below which 75% of the data falls.
- **Deciles:** Deciles divide the data into ten equal parts, each containing 10% of the observations. Deciles are often used to analyze income distributions or rank data. The first decile D_1 represents the value below which 10% of the data falls, the second decile D_2 represents the value below which 20% of the data falls, and so on until the ninth decile (D_9).
- **Percentiles:** The value which divides the data into hundred equal parts, each containing 1% of the observations. There are percentiles which are Percentiles are used to understand the relative position of a particular value in a dataset.

  For example, the 75th percentile (P_{75}) represents the value below which 75% of the data falls, and the 90th percentile (P_{9C}) represents the value below which 90% of the data falls. Thus, is the median.

CALCULATION OF QUARTILES – INDIVIDUAL OBSERVATIONS

For ungrouped data consisting of n observations (not necessarily all distinct), the calculation of kth quartile Q_k $(k = 1, 2, 3)$ involves the following stepsL

Step 1: Arrange the given data in an ascending order of magnitude.

Step 2: The value of kth quartile Q_k is given by $\frac{k(n+1)}{4} th$ observation.

Thus, $Q_1 = \frac{(n+1)}{4} th$ observation

$Q_2 = \frac{2(n+1)}{4} th$ observation

$Q_3 = \frac{3(n+1)}{4} th$ observation

Q.51 Consider the following set of observations: 19, 12, 27, 14, 21, 18, 9, 15. What is the value of the first quartile?

(a) 11.5 (b) 12 (c) 12.5 (d) 13

Ans. (c)

CALCULATION OF DECILES = INDIVIDUAL OBSERVATIONS

For ungrouped data consisting on n observation (not necessarily all distinct), the calculation of kth decile D_k (k=1, 2, … ,9) involves the following steps:

Step 1: Arrange the given data in an ascending order of magnitude.

Step 2: The value of kth decile D_k is given by $\frac{k(n+1)}{10}th$ observation.

Thus, $D_1 = \frac{(n+1)}{10}th$ observation

$D_2 = \frac{2(n+1)}{10}th$ observation

$D_3 = \frac{3(n+1)}{10}th$ observation and so on

CALCULATION OF PERCENTILES – INDIVIDUAL OBSERVATIONS

For ungrouped data consisting of n observations (not necessarily all distinct), the calculation of kth percentile P_k(k=1, 2, 3, …, 99) involves the following steps:

Step 1: Arrange the given data in an ascending order of magnitude.

Step 2: The value of kth percentile P_k is given by $\frac{k(n+1)}{100}th\ observation$.

Thus, $P_1 = \frac{(n+1)}{100}th$ observation

$P_2 = \frac{2(n+1)}{100}th$ observation

$P_3 = \frac{3(n+1)}{100}th$ observation and so on

Q.52 Consider the set of numbers: 14, 8, 19, 22, 17, 9, 13, 16. What is the value of the third decile?

(a) 11.8 (b) 13.4 (c) 13.5 (d) 14

Ans. (a)

Q.53 Consider the following set of observations: 19, 12, 27, 14, 21, 18, 9, 15. What is the value of the first quartile?

(a) 11.5 (b) 12 (c) 12.5 (d) 13

Ans. (c)

Q.54 The marks obtained by 9 students in a test are 25, 20, 15, 45, 18, 7, 10, 38 and 12. Find the value of P_{70}.

(a) 25 (b) 7 (c) 10 (d) None of these

Ans. (a)

CALCULATION OF QUARTILES – DISCRETE SERIES

In case of discrete frequency distribution where the variable X takes the values $X_1, X_2, \ldots X_N$ with respective frequencies $f_1, f_2, \ldots f_n$ with $\sum f = N$, the calculation of each quartile $Qk(k = 1, 2, 3)$ involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{k(N+1)}{4}$

Step 3: See the c.f. just greater than or equal to $\frac{k(N+1)}{4}$

Step 4: The value of X corresponding to the c.f. obtained in Step 3 gives the required value of Q_k

CALCULATION OF DECILES – DISRETE SERIES

In case of discrete frequency distribution where the variable X takes the value $X_1, X_2, \ldots X_n$ with respective frequencies $f_1, f_2, \ldots f_n$ with $\sum f = N$, the calculation of each decile D_k($k = 1, 2, 3, \ldots 9$) involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{k(N+1)}{10}$

Step 3: See the c.f. just greater than or equal to $\frac{k(N+1)}{10}$

Step 4: The value of X corresponding to the c.f. obtained in Step 3 gives the required value of D_k

CALCULATION OF PERCENTILES – DISCRETE SERIES

In case of discrete frequency distribution where the variable X takes the values $X_1, X_2, \ldots X_n$ with respective frequencies $f_1, f_2, \ldots f_n$ with $\sum f = N$, the calculation of each percentile P_k $(k = 1, 2, 3, \ldots 99)$ involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{k(N+1)}{100}$

Step3 : See the c.f. just greater than or equal to $\frac{k(N+1)}{100}$

Step 4: The value of X corresponding to the $c.f.$ obtained in Step 3 gives the required value of P_k.

Q.55 Calculate the value of Q_1 and P_{65} from the following data:

X	10	5	7	11	8
f	15	20	15	18	12

(a) 7, 10 (b) 35, 52.65 (c) 10, 7 (d) None of these

Ans. (a)

CALCULATION OF QUARTILES – CONTINUOUS SERIES

In case of continuous frequency distribution, the calculation of Q_K $(k = 1, 2, 3)$ involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{KN}{4}$, where $N = \sum f$ is the total frequency.

Step 3: See the c.f. just greater than or equal to $\frac{KN}{4}$

Step 4: Find the class, the class corresponding to c.f. obtained in Step 3.

Step 5: The value of Q_k is then obtained by using the following interpolation formula:

$$Q_k = l + \frac{\frac{kN}{4} - C}{f} \times h$$

Where l = lower limit of Q_k class

$C = c.f.$ of the class preceding the Q_k class

F = frequency of the Q_k class, and h= size or width of Q_k class.

CALCULATION OF DECILES – CONTINUOUS SERIES

In case of continuous frequency distribution, the calculation of $D_k(k = 1, 2, \ldots, 9)$ involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{kN}{4}$, *where* $N = \sum f$ is the total frequency.

Step 3: See the c.f. just greater than or equal to $\frac{kN}{10}$

Step 4: Find the D_k class, the class corresponding to c.f. obtained in Step 3.

Step 5: The value of Q_k is then obtained by using the following interpolation formula.

$$Q_k = l + \frac{\frac{kN}{10} - c}{f} \times h$$

Where $l = lower\ limit\ of\ D_k$ class

$C =$ $c.f.$ of the class preceding the D_k class.

$f =$ frequency of the D_k class, and h=size or width of D_k class.

CALCULATION OF PERCENTILES – CONTINUOUS SERIES

In case of continuous frequency distribution, the calculation of P_K $(k = 1, 2, \dots, 99)$ involves the following steps:

Step 1: Prepare the 'less than' cumulative frequency distribution.

Step 2: Find $\frac{kN}{100}$, where $N = \sum f$ is the total frequency.

Step 3: See the $c.f.$ just greater than or equal to $\frac{kN}{100}$

Step 4: Find the P_K class, the class corresponding to $c.f.$ obtained in Step 3.

Ste[5: The value of is then obtained by using the following interpolation formula:

$$P_k = l + \frac{\frac{kN}{100} - c}{f} \times h$$

Where $l = lower\ limit\ of\ P_k$ class

$C =$ $c.f.$ of the class preceding the P_k class.

$f =$ frequency of the P_k class, and h=size or width of P_k class.

Q.56The third quartile and 65th percentile for the following data are:

Profit in '000`	Less than 10	10-19	20-29	30-39	40-49	50-59
No. of firms	5	18	38	20	9	2

(a) `33,500 and `29,184 (b) `33,000 and `28,680

(c) `33,600 and `29,000 (d) `33,250 and `29,250

Ans. (a)

Q.57What is the value of the first quartile for observation 15, 18, 10, 20, 23, 28, 12, 16?

(a) 17 (b) 16 (c) 12.75 (d) 12

Ans. (c)

Q.58The third decile for the numbers 15, 10; 20, 25, 19, 11, 9, 12 is

(a) 13 (b) 10.70 (c) 11 (d) 11.50

Ans. (b)

Q.59Following are the wages of the laborer's:

`82, `56, `90, `50, `120, `75, `75, `80, `130, `65

Find D_6 and P_{82}.

(a) 16.2 and 100.2 (b) 81.2 and 120.2

(c) 46.2 and 100.2 (d) None of these

Ans. (a)

Q.60Quartiles are the values dividing a given set of observations into

(a) Two equal parts (b) Four equal parts

(c) Five equal parts (d) None of these

Ans. (b)

Q.61 From the following data, calculate the median and the first and third quartile wages.

Daily wages(`)	No. of workers	Daily wages(`)	No. of workers
30-32	2	40-42	62
32-34	9	42-44	39
34-36	25	44-46	20
36-38	30	46-48	11
38-40	49	48-50	3

(a) 37.7667 and 42.5385
(b) 30.5660 and 24.5385
(b) 37.7667 and 32.7380
(d) None of these.

Ans. (a)

Q.62Find the 45th and 57th percentiles for the following data on marks obtained by 100 students:

Marks	20-25	25-30	30-35	35-40	40-45	45-50
No. of Students	10	20	20	15	15	20

(a) 30 and 37.34
(b) 33.35 and 37.34
(c) 40.5 and 44.6
(d) None of these

Ans. (b)

MODE

Mode is a measure of central tendency that represents the value or values in a dataset that occur most frequently. It is the data point with the highest frequency or the data points with equal highest frequencies.

E.g.: In dataset 1, 2, 3, 4, 4, 5, 2, 3, 4, the mode is 4 because it appears three times, which is more than any other value in the dataset.

Bimodal Distribution: If there are two modes in a distribution it is called Bimodal Distribution.

E.g.: 1, 2, 2, 2, 4, 5, 8, 6, 6, 6 then the modes are 2 and 6, as both values appear equally most frequently.

Multi-Modal Distribution

When we have multiple modes in a distribution it is called Multi- Modal Distribution.

E.g.: Dataset 1, 2, 2, 4, 4, 6, 8, 6 has three modes: 2, 4, and 6.

No Mode: When all observations have the same frequencies then the distribution has no mode.

E.g.: 1, 2, 3, 4, 5 has no mode as each value appears only once and has the same frequency.

CALCULATION OF MODE: DISCRETE SERIES

In discrete frequency distribution, mode can be determined just by inspection. It is the value of the variable corresponding to the maximum frequency

E.g.: Consider the following distribution:

X	1	2	3	4	5	6	7

F	2	3	12	5	2	6	3

Here, the highest frequency is 12 which corresponds to X = 3.

Therefore, Mode for the given distribution is 3.

Note: While determining mode by inspection in the case of discrete frequency distribution, an error of judgement is possible when the difference between the greatest frequency and the frequency preceding it or succeeding it is very small and the values are heavily concentrated on either side.

Q.63 What is the modal value for the number 5, 8, 6, 4, 10, 15, 18, 10?

(a) 18 (b) 10 (c) 14 (d) None of these

Ans. (b)

Q.64 The heights (in centimeters) of a group of students in a class were recorded. The data set is as follows: 141, 150, 154, 152, 162, 162, 142, 155, 160, 167, 147, 152, 165. What is the mode(s) of the heights in the data set?

(a) 142 (b) 165

(c) 152 and 160 (d) No mode

Ans. (c)

Q.65 What is the mode for the following set of numbers? 7, 9, 12, 15, 18, 21, 24, 27

(a) 7 (b) 27 (c) 15, 18 (d) No mode

Ans. (d)

Q.66 Modal group is:

Height in cms:	1	2	3	4	5
No. of students:	2	3	12	5	2

(a) 65-67 (b) 69-71 (c) 63-25 (d) None

Ans. (a)

CALCULATION OF MODE - CONTINUOUS FREQUENCY DISTRIBUTION

The first step is to find the modal class, i.e., the class corresponding to the maximum frequency. The value of mode is then obtained by applying the following interpolation formula:

$$Mode = l + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times c$$

Where, $l = LCB$ of the modal class,

f_0 =frequency of modal class,

f_{-1} =frequency of pre-modal class,

f_1 =frequency of post-modal class,

$c =$ class-length of the modal class

Q.67 The mode for the following frequency distribution.

Class interval	350-369	370-389	390-409	410-429	430-449	450-469

Frequency	15	27	31	19	13	6

(a) 390 (b) 390.50 (c) 394.50 (d) 394

Ans. (c)

Q.68 Following is an incomplete distribution having modal mark as 44.

Marks:	0-20	20-40	40-60	60-80	80-100
No. of Students:	8	18	?	12	5

What would be the mean marks?

(a) 45 (b) 46 (c) 47 (d) 48

Ans. (c)

It may be remarked that the above formula for computing mode is based on the following assumptions:

1. The frequency distribution must be continuous with exclusive type classes without any gaps. If the data are not given in the form of continuous classes, it must first be converted into continuous classes before applying the above formula.

2. The class intervals must be uniform throughout, i.e., the size of all the class intervals must be the same. If they are unequal, they should first be made equal on the assumption that frequencies are uniformly distributed over all the classes.

PROPERTIES OF MODE

- If $y = a + bx$ then $y_{mode} = a + bx_{mode}$

EMPIRICAL RELATIONSHIP OF MEAN, MEDIAN AND MODE

For a grouped frequency distribution, we may consider the following empirical relationship between mean, median and mode:

Mean – Mode = 3(Mean – Median)

⇒ Mean – Mode = 3 Mean – 3 Median

⇒ 3 Median = 3 Mean – Mean + Mode

⇒ 3 Median = 2 Mean + Mode

Q.69 If mean $(\bar{X})$ is 10 and mode (Z) is 7, then find out the value of median (M).

(a) 9 (b) 17 (c) 3 (d) 4.33

Ans. (a)

Q.70 For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean mark and median mark were found to be 55.60 and 52.40. What is the modal mark?

(a) 46 (b) 16 (c) 51 (d) 66

Ans. (a)

Q.71 If $y = 2 + 1.50x$ and mode of x is 15, what is the mode of y?

(a) 7.5 (b) 24.5 (c) 10.7 (d) 17.5

Ans. (b)

MERIT AND DEMERITS OF MODE

Merits

Mode possesses the following merits:

1. It is simple to understand and easy to calculate.
2. In some cases it can be located merely by inspection.
3. It can be determined graphically from a histogram.
4. It is not at all affected by extreme observations and can be calculated even if extreme values are not known.
5. It can be conveniently determined for distribution with open end classes.

Demerits

Mode has the following drawbacks:

1. It is not rigidly defined
2. It is not based on all the observations.
3. It is not suitable for further mathematical treatment.
4. As compared to mean, mode is affected to a greater extent by the fluctuations of sampling.
5. The value of mode cannot always be determined. In some cases, we may have a bi-modal distribution.

Q.72 What is the modal value for the numbers 5, 8, 6, 4, 10, 15, I8, 10?

(a) 18 (b) 10 (c) 14 (d) None the these

Ans. (b) Given data: 5, 8, 6, 4, 10, 15, 18, 10

We know that,
For a given set of observations, mode may be defined as the value that occurs the maximum number of times.
Here, 10 occurs maximum number of times (2 times).
Therefore, the modal value is 10.
Hence, the correct option is (b)

Q.73 If x and y are related by $x - y - 10 = 0$ and mode of x is known to be 23, then the mode of y is

(a) 20 (b) 13 (c) 3 (d) 23

Ans. (b)

Q.74 Which of the following measure(s) satisfies (satisfy) a linear relationship between two variables?

(a) Mean (b) Median (c) Mode (d) None of these

Ans. (d)

Q.75 Which of the following measures of central tendency is based on only fifty percent of the central values?

(a) Mean (b) Median (c) Mode (d) Both (a) and (b)

Ans. (b)

Q.76 The mode of the numbers 7, 7, 7, 9, 10, 11, 11, 11, 12 is

(a) 11 (b) 12 (c) 7 (d) 7 & 11

Ans. (d)

Q.77 Calculate mode from the following data:

Height in inches:	56	58	59	60	61	62	63	64	66	68
No. of persons:	3	7	6	9	20	22	24	5	3	1

(a) 68 (b) 20 (c) 63 (d) 56

Ans. (c)

Q.78Given below is the distribution of weights of a group of 60 students in a class:

Weight (in kg):	30-34	35-39	40-44	45-49	50-54	55-59	60-64
No. of Students:	3	5	12	18	14	6	2

Find the mode of the distribution.

(a) 47.5 (b) 65.85 (c) 10.70 (d) 46.59

Ans. (a)

GEOMETRIC MEAN

The geometric mean, usually abbreviated as G.M., of a set of n observations $x_1, x_2, \ldots\ldots, x_n$ is the nth root of their product. That is,

$GM = (x_1 x_2 x_3 \ldots\ldots\ldots\ldots x_n)^{\frac{1}{n}}$

Note: It may be noted that if there are two observations, G.M. can be computed by taking the square root; if there are three observations, G.M. can be computed by taking the cube root of their product and so on. **E.g.:**,

$G.M.$ of two number 4 and 9 is $\sqrt{4 \times 9} = 2 \times 3 = 6$

$G.M.$ of three numbers 1, 4 and 128 is $\sqrt[3]{1 \times 4 \times 128} = 2^3 = 8$

Using calculator tricks we can find Log and antilog for any number:

To find Logarithm	To find Anti-logarithm
1. Type number and press root for 15 times	1. Multiply number by 0.000070274
2. Subtract 1 after Step 1	2. Add 1 after step 1
3. Divide number by 0.000070274	3. Press multiply & =”button for 15 times

Q.79What is the geometric mean (GM) for the numbers 2, 8 and 32?

(a) 4 (b) 6 (c) 8 (d) 12

Ans. (c)

Q.80What is the GM for the number 8, 24 and 40?

(a) 24 (b) 12 (c) $8\sqrt[3]{15}$ (d) 10

Ans. (c)

Q.81The geometric mean of three numbers is 12 and two of the numbers are 4 and 16. What is the third numbers?

(a) 12 (b) 32 (c) 27 (d) 48

Ans. (c)

CALCULATION OF GEOMETRIC MEAN – DISCRETE SERIES

The geometric mean of a set of n observations $X_1, X_2, \ldots X_n$ with their respective frequencies $f_1, f_2, \ldots\ldots\ldots f_n$ is given by:

$G.M = (X_1^{f1} \times X_2^{f2} \times \ldots X_n^{fn})^{\frac{1}{N}}$

Where $N = \sum f$ is the total frequency.

Taking logarithm of both sides, we obtain

$$Log\ (G.M.) = \frac{1}{N}\log(X_1^{f_1} \times X_2^{f_2} \times \dots X_n^{f_n})^{1/N} = \frac{1}{N}[log(X_1^{f_1} \times X_2^{f_2} \times \dots X_n^{f_n})]$$

$$= \frac{1}{N}[f_1 logX_1 + f_2 logX_2 + \cdots \dots . . + f_n logX_n] = \frac{1}{N}\sum flogX$$

$$G.M. = AL\ [\frac{1}{N}\sum flogX]$$

CALCULATION OF GEOMETRIC MEAN – CONTINUOUS SERIES

In the case of grouped or continuous frequency distribution, $G.M.$ is given by $G.M. = AL\ [\frac{1}{N}\sum flogX]$

Where $X_1, X_2, \dots \dots, X_n$ are the class marks (or mid - values) of a set of grouped data with corresponding class frequencies $f_1, f_2, \dots \dots \dots, f_n$

Q.82Find the geometric mean from the following data:

Diameter (mm):	130	135	140	145	143	148	149	150
No. of Screws:	3	4	6	6	3	5	2	2

(a) 142 mm (b) 165 mm (c) 110 mm (d) 149 mm

Ans. (a)

PROPERTIES OF GEOMETRIC MEAN

1. Logarithm of G for a set of observation is the AM of the logarithm of the observations i.e., $\log G = \frac{1}{r}\sum \log X$
2. If all the observations assumed by a variable are constants, (k), then the GM of the observations is also k.
3. GM of the product of two variables is the product of their GM's i.e., if $z = xy\ then$ GM of $z = GM\ of\ x \times GM\ of\ y$
4. GM of the ratio of two variables is the ratio of the GM's of two variable i.e., if $z = \frac{x}{y}$ then GM of z= $\frac{GM\ of\ x}{GM\ of\ y}$

USES OF GEOMETRIC MEAN

1. Geometric mean is used primarily to average data for which the ratio of consecutive terms remains approximately constant. This occurs, for example, with such data as rates of change, ratios, percent increase in sales, population sizes over consecutive time periods and the like.
2. It is the most appropriate average to be used in the construction of index numbers.
3. It is the most suitable average to be used when it is desired to give more weightage to smaller items and vice-versa.

MERITS AND DEMERITS OF GEOMETRIC MEAN

Merits

1. It is rigidly defined.
2. It is based on all the observations.

3. It is suitable for further mathematical treatment. For example, if the geometric means of two or more sets of data are known, then the geometric mean of the combined data can also be obtained. If G_1 and G_2 are geometric means of two sets of data with number of observations n_1 and n_2 respectively, then geometric mean G of the combined data with $n_1 + n_2$ observation is given by:
$logG = \frac{n_1 logG_1 + n_2 logG_2}{n_1 + n_2}$
4. It gives less weight to large items and more to small ones than does the arithmetic mean.
5. Unlike AM, geometric mean is affected to a lesser extent by extreme observations.
6. It is not affected much by fluctuations of sampling.

Demerits

Geometric mean has the following drawbacks.

1. It is difficult to understand.
2. It is not easy to calculate for a non-mathematical person.
3. If any of the observations is zero, the geometric mean becomes zero and if any one of the observations is negative, the value of GM cannot be calculated.

Q.83Which of the following measures of the central tendency is difficult to computer?

(a) Mean (b) Median (c) Mode (d) GM

Ans. (d)

Q.84If GM of x is 10 and GM of y is 15, then the GM of xy is

(a) 150 (b) log 10 × log 15 (c) log 150 (d) None of these

Ans. (a)

Q.85Find the geometric mean of 2, 4, 8, 12, 16 and 24.

(a) 29.49 (b) 5.86 (c) 8.16 (d) 6.56

Ans. (c)

Q.86Find the GM for the following distribution:

x	2	4	8	16
F	2	3	3	2

(a) 2 (b) $4\sqrt{2}$ (c) 5 (d) $6\sqrt{3}$

Ans. (b)

Q.87The rates of returns from three different shares are 100%, 200%, 200% and 400% respectively, the average rate of return will be_____.

(a) 350% (b) 233.33% (c) 200% (d) 300%

Ans. (c)

Q.88Find the geometric mean for the following distribution:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	3	4	6	6	3	5

(a) 21.57 (b) 25.5 (c) 26.73 (d) None of these

Ans. (c)

HARMONIC MEAN

The harmonic mean, usually abbreviated as H.M., is defined as the reciprocal of the arithmetic mean of the reciprocals of the given set of observations.

Symbolically,

$$HM = \frac{n}{\sum \frac{1}{X}}$$

CALCULATION OF HARMONIC MEAN: INDIVIDUAL OBSERVATIONS

The harmonic mean of a set of n observations $X_1, X_2, \ldots\ldots\ldots, X_n$ (not necessarily all distinct) is given by

$$HM = \frac{n}{\frac{1}{X_1} + \frac{1}{X_2} + \cdots\ldots\ldots + \frac{1}{X_n}} = \frac{n}{\sum \frac{1}{X}}$$

Q.89Find the harmonic mean of 5 numbers 4, 5, 6, 10 and 12.

(a) 2.5 (b) 6.25 (c) 1.50 (d) None of these.

Ans. (b)

Q.90The harmonic mean for the numbers 2, 3, 5 is

(a) 2.00 (b) 3.33 (c) 2.90 (d) $-\sqrt[3]{30}$

Ans. (c)

Q.91What is the HM of $1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots \frac{1}{n}$?

(a) n (b) $2n$ (c) $\frac{2}{(n+1)}$ (d) $\frac{n(n+1)}{2}$

Ans. (c)

CALCULATION OF HARMONIC MEAN: DISCRETE SERIES

The harmonic mean of a set of n observations $X_1, X_2, \ldots\ldots\ldots, X_n$ with their respective frequencies $f_1, f_2, \ldots\ldots\ldots, f_n$ is given by

$$HM = \frac{n}{\sum [\frac{f}{X}]}$$

CALCULATION OF HARMONIC MEAN: CONTINUOUS SERIES

In the case of continuous series, we take $X_1, X_2, \ldots\ldots\ldots, X_n$ are the class marks (or mid-values) of a set of grouped data with corresponding class frequencies $f_1, f_2, \ldots\ldots, f_n$ then harmonic mean is given by:

$$HM = \frac{n}{\sum [\frac{f}{X}]}$$

Q.92Find the harmonic mean for the following distribution.

X	10	15	20	25	30
f	2	5	4	9	5

(a) 10.15 (b) 15.41 (c) 19.73 (d) None of these

Ans. (c)

USES OF HARMONIC MEAN

The harmonic mean is used

1. In averaging speeds when equal distances are covered with varying speed
2. In finding the average cost of some commodity when several different purchases are made by investing the same amount of money each time.

Q.93An aeroplane flies from A to B at the rate of 500 km/hour and comes back from B to A at the rate of 700 km/hour. The average speed of the aeroplane is

(a) 100 km/hour (b) 583.33 km/hour

(c) 100 $\sqrt{35}$ $km/hour$ (d) 620 km/hour

Ans. (b)

Q.94A fire engine rushes to a place of fire accident with a speed of 110 kmph and after the completion of operation returned to the base at a speed of 35 kmph. The average speed per hour in per direction is obtained as_____of those speeds.

(a) Speed average of (b) HM of

(c) GM of (d) Half of HM of

Ans. (b)

WEIGHTED HARMONIC MEAN

Let $w_1, w_1, \ldots, w_n$ be the weights attached to n observations $X_1, X_2, \ldots, X_n$ respectively. Then the weighted Harmonic Mean, denoted by $H.M_{.w}$, is defined as

$$H.M_{.w} = \frac{\sum w_i}{\sum (\frac{w_i}{x_i})}$$

USE OF WEIGHTED HARMONIC MEAN

- In actual practice, the weighted harmonic mean is most frequently used in averaging speeds when different distances are covered with varying speeds.
- In finding the average cost of some commodity when several different purchases are made by putting in a different amount of money each time.

PROPERTIES

If all the observations taken by a variable are constants, say k, then the HM of the observations is also k.

If there are two groups with n_1 and n_2 observations and H_1 and H_2 as respective HM's then

Combined HM is given by: $\frac{n1+n2}{\frac{n1}{H_1}+\frac{n2}{H_2}}$

Q.95The harmonic mean for 40 observations of group 1 data is 520 and for 50 observations of group 2 data is 680. What is the combined harmonic mean?

(a) 450.60 (b) 598.20 (c) 800.00 (d) None of these

Ans. (b)

Q.96If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations, then the combined HM is given by

(a) 65 (b) 70.36 (c) 70 (d) 71

Ans. (c)

MERITS AND DEMERITS OF HARMONIC MEAN

Merits

Harmonic mean possess the following merits.

1. It is rigidly defined.
2. It is based on all the observations.
3. Since the reciprocals of the value of the observations are involved, it is not as much affected by one or two big observations.
4. It is not affected very much by fluctuations of sampling.

Demerits

Harmonic mean has the following drawbacks.

1. It is not easy to understand.
2. It is difficult to calculate.
3. Its value cannot be computed if any one of the observations is zero.
4. It gives the largest weighted to the smallest item. This is generally not a desirable feature and as such this average is not very useful for the analysis of economic data.

Q.97 Which of the following measure(s) possesses (possesses) mathematical properties?

(a) AM (b) GM (c) HM (d) All of these.

Ans. (d)

RELATIONSHIP BETWEEN AM, GM, AND HM

1. For any set of data, the value of A.M, G.M., and H.M are connected by the following relation: $AM \geq GM \geq HM$
   - If all the values in the dataset are equal then $AM = GM = HM$
   - If all the values in the dataset are distinct then $AM > GM > HM$
2. For two numbers, we have $(G.M.)^2 = A.M \times H.M.$

Q.98 The A.M. and G.M. of two numbers are 15 and 12 respectively. Find the H.M. of the numbers.

(a) 2 (2) 15 (c) 20 (d) 9.6

Ans. (d)

Q.99 If the AM and GM for 10 observations are both 15, then the value of HM is

(a) Less than 15 (b) More than 15

(c) 15 (d) Cannot be determined.

Ans. (c)

Q.100 If the AM and GM for two numbers are 6.50 and 6 respectively, then the two numbers are

(a) 6 and 7 (b) 9 and 4 (c) 10 and 3 (d) 8 and 5

Ans. (b)

Q.101 If the AM and HM for two numbers are 5 and 3.2 respectively, then the GM will be

(a) 16.00 (b) 4.10 (c) 4.05 (d) 4.00

Ans. (d)

Q.102 Which of the following results hold for a set of distinct positive observations?

(a) $AM \geq GM \geq HM$ (b) $HM \geq GM \geq AM$

(c) $AM > GM > HM$ (d) $GM > AM > HM$

Ans. (c)

Q.103 For a moderately skewed distribution, which of the following relationships holds?

(a) Mean - Mode = 3 (Mean - Median) (b) Median - Mode 3 (Mean - Median)

(c) Mean Median = 3 (Mean Mode) (d) Mean - Median = 3 (Median - Mode)

Ans. (a)

Q.104 Find the harmonic mean of the following numbers: 2, 3, 6, 8, 10

(a) 4.08 (b) 5.18 (c) 9.04 (d) None of these

Ans. (a)

Q.105 Two values yielded an arithmetic mean of 24 and a harmonic mean of 6. The geometric mean of these values is_____.

(a) 8 (b) 12 (c) 14 (d) 16

Ans. (b)

Q.106 In a moderately skewed distribution, the mode and median are 20 and 24 respectively. The value of mean will be_____.

(a) 21 (b) 26 (c) 30 (d) None

Ans. (b)

Q.107 The mean of 'n' observation is 'X'. If 'K' is added to each observation, then the new mean is _____.

(a) X (b) XK (c) X - K (d) X + K

Ans. (d)

Q.108 If the AM and HM for two numbers are 5 and 3.2 respectively then the GM will be

(a) 16.00 (b) 4.10 (c) 4.05 (d) 4.00

Ans. (d)

Q.109 For a distribution Mean, Median and Mode are 23, 24 and 25. 5 respectively, then it is most likely _____ skewed distribution.

(a) Positively (b) Symmetrical (c) Asymptotically (d) Negatively

Ans. (d)

Q.110 There are n numbers. When 50 is subtracted from each of these number the sum of the numbers so obtained is -10. When46 subtracted from each of the original n numbers, then the sum of numbers so obtained is 70. What is the mean of the original n numbers?

(a) 56.8 (b) 25.7 (c) 49.5 (d) 53.8

Ans. (c)

Q.111 Calculate the mode for the following data.

Monthly wages(`)	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of workers	4	6	20	12	33	17	8	2

(a) 259.50 (b) 350.78 (c) 400.42 (d) 428.38

Ans. (d)

Q.112 An incomplete distribution is given below:

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Total Frequency	10	____	30	____	25	20	125

(a) 10 and 15 (b) 15 and 20 (c) 15 and 25 (d) 20 and 25

Ans. (c)

Q.113 A survey was conducted by a group of students as a part of their environmental awareness program, in which they collected the following data regarding the number of plants in 200 houses in a locality. Find the mean number of plants per house.

Class Interval	0-2	2-4	4-6	6-8	6-8	8-10	12-14
Total Frequency	1	2	1	5	6	2	3

(a) 162 (b) 20 (c) 15.5 (d) 8.1

Ans. (d)

CHAPTER-14 Unit - 2

MEASURE OF DISPERSION

WHAT IS DISPERSION?

The central tendency measures we've explored so far provide valuable insights into data, but they alone cannot offer a comprehensive description. Regardless of which measure we use, it doesn't reveal the distribution pattern of the data. We may encounter situations where different sets of observations share the same mean but exhibit substantial variations in their measurements around this average..

E.g.: consider the following three sets of observations, each containing 9 items:

Mean										Total	*Mean*
Set-A	20	20	20	20	20	20	20	20	20	180	20
Set-B	16	17	18	19	20	21	22	23	24	180	20
Set-C	12	14	16	18	20	22	24	26	28	180	20

All the three sets have the same mean i.e. 20 yet they are quite different.

Therefore, we can say that we need some more measures in addition to the central tendency to describe the data completely.

Definition of Dispersion: Dispersion in statistics is a way of describing how spread out a set of data is. It may be defined as the amount of deviation of the observations.

MEASURES OF DISPERSION

The degree to which the numerical data tends to deviate from the average value is called the variation or dispersion of the data.

The measure of dispersion can be classified as:

1. Absolute measures of dispersion
2. Relative measures of dispersion

ABSOLUTE MEASURE OF DISPERSION.	**RELATIVE MEASURE OF DISPERSION**
▪ Absolute measures are dependent on the unit of the variable under consideration. ▪ Easy to comprehend and compute. **Different Measure ways:** ○ Range ○ Mean Deviation ○ Standard Deviation ○ Quartile Deviation	▪ Relative measures of dispersion are unit free ▪ For comparing two or more distributions, relative measures of dispersion are considered. **Different Measure ways:** ○ Coefficient of Range ○ Coefficient of Mean Deviation ○ Coefficient of Variation ○ Coefficient of Quartile Deviation.

Q.1 Dispersion measures
(a) The scatterness of a set of observations.
(b) The concentration of a set of observations
(c) Both (a) and (b)
(d) Neither (a) nor (b)

Ans.(a)

Q.2 When it comes to comparing two or more distributions, we consider
(a) Absolute measure of dispersion (b) Relative measures of dispersion
(c) Both (a) and (b) (b) Either (a) or (b)

Ans. (c)

RANGE

DEFINITION

The range of a set of data is defined as the difference between the largest and the smallest value in the set.

Range = Largest value - Smallest value

For a grouped frequency distribution, it is the difference between upper limit of the highest class and lower limit of the smaller class.

Range = Upper class boundary (U.C.B) – Lower class boundary (L.C.B)

Q.3 Following are the wages of 10 workers expressed in INR. 45, 72, 78, 90, 65, 20, 90, 65, 50, 70. Find the range

(a) 60 (b) 59 (c) 63.63 (d) None of the above

Ans. (d)

Q.4 The following data represents the heights (in centimeters) of a group of students in a class:

Height (cm)	Frequency
100-120	5
120-140	8
140-160	12
160-180	10
180-200	6

What is the range of heights for the given grouped frequency data?

(a) 100 cm (b) 120 cm (c) 160 cm (d) 200 cm

Ans. (a)

COEFFICIENT OF RANGE

The range is an absolute measure of dispersion and is expressed in the unit of measurement of values of a distribution. Hence, it cannot be used to compare two distributions expressed in different units.

To overcome this difficulty, we need a relative measure which is independent of the units of measurement. This relative measure, called the coefficient of range, is defined as follows:

$$\text{Coefficient of Range} = \frac{\text{Range}}{\text{Sum of the largest and the smallest values}}$$

i.e., $\text{Coefficient of range} = \frac{L-S}{L+S} \times 100$

where, L is the largest value and S is the smallest value

Q.5 Following are the wages of 10 workers expressed in INR. 45, 72, 78, 90, 65, 20, 90, 65, 50, 70. Find the coefficient of range.

(a) 60 (b) 59 (c) 63.63 (d) None of these

Ans. (c)

Q.6 The following data represents the weights the weights (in kilograms) of a group of individuals in a gym:

Weight (Kg)	Frequency
41-50	6
51-60	12
61-70	15
71-80	8
81-90	5

Find the coefficient of range for the given grouped frequency data

(a) 1.13% (b) 3.82% (c) 4.29% (d) None of these

Ans. (b)

Q.7 What is the coefficient of range for the following distribution?

Class Interval	10-19	20-29	30-39	40-49	50-59
Frequency	11	25	16	7	3

(a) 22 (b) 50 (c) 72.46 (d) 75.82

Ans. (c)

PROPERTIES OF RANGE

Range remains unaffected due to a change of origin but affected in the same ratio due to a change in scale i.e., for any two constants a and b, two variables x and y are related by $y = a + bx$

Then, the range of y is given by $R_y = |b| \times R_x$

Q.8 If the relationship between x and y is given by $2x + 5y = 10$ and the range of x is 5, what would be the range of y?

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b)

Q.9 If R_x and R_y denote the range of x and y respectively where x and y are related by $3x + 2y + 10 = 0$, What would be the relation between x and y?

(a) $R_x = R_y$ (b) $2R_x = 3R_y$ (c) $3R_x = 2R_y$ (d) $R_x = 2R_y$

Ans. (c)

Q.10 What is the coefficient of range for the wages of 8 workers?

`80, `65, `90, `75, `70, `72, `80

(a) `30 (b) `20 (c) `30 (d) `20

Ans. (d)

Q.11 Find the range of the daily wages of 10 persons given below:

`240, `180, `250, `160, `200, `280, `220, `170, `170, `210 and `270

(a) `130 (a) `140 (c) `120 (d) `150

Ans. (c)

Q.12 If the range of x is 2, what would be the range of $-3x + 50$?

(a) 2 (b) 6 (c) -6 (d) 44

Ans. (b)

Q.13 The following are the prices of shares of a company from Monday to Saturday?

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Price (in `)	55	54	52	53	56	58

Calculate the range and the coefficient of range.

(a) 5 and 0.064 (b) 6 and 0.054 (c) 8 and 0.034 (d) None of these

Ans. (b)

Q.14 Find the coefficient of range for the following frequency distribution:

x	3-5	6-8	9-11	12-14
f	3	2	2	3

(a) 60.5% (b) 65.80% (c) 70.59% (d) None of these

Ans. (c)

MERITS

The range possesses the following merits

1. It is simple to understand and easy to calculate.
2. It requires minimum time to calculate the value of range.
3. It is useful in studying fluctuations in the share prices.

DEMERITS

The range has the following drawbacks:

1. It is not based on all the observations.
2. Range is a poor measure of variation. It considers only the extreme values and tells us nothing about the distribution of numbers in between.
3. It is very much affected by fluctuations of sampling. Its value varies widely from sample to sample
4. It cannot be calculated for grouped frequency distribution with open-end classes.
5. It is not suitable for further mathematical treatment.

CHARACTERISTICS FOR AN IDEAL MEASURE OF DISPEERSION

An ideal measure of dispersion should be

- Properly defined

- Easy to comprehend
- Based on all the observations
- Unaffected by sampling fluctuations and emenable to some desirable mathematical treatment.

MEAN DEVIATION

As we saw, range is not based on all the observations. Moreover, it does not show any scatterness around an average. If we wish to measure variation in the sense of showing the scatter around an average, we must include the deviations of each and every item from an average.

- Mean deviation or the Average deviation helps us in achieving this goal. As the name suggests, this measure of dispersion is obtained by taking the average (arithmetic mean) of the deviations of the given values from a measure of central tendency.
- Mean Deviation (about an average A) $= \frac{1}{n}\sum|x_i - A|\ where, |x_i - A|$ is the modulus value or the absolute value of the deviation from A, ignoring $\pm$ signs.

Usually, we obtain mean deviation about any one of the three averages Mean (M), Median (M_d) or Mode (M_0). As we know mode is generally ill-defined, in practice, mean deviation is computed about mean or median and if we calculate mean deviation about median, it will be much beneficial because the sum of the deviations of items from median is least when signs are ignored. But mostly, the mean is more frequently used in computing the average deviation and this is the reason why it is more commonly referred to as mean deviation.

PROCEDURE FOR COMPUTING THE MEAN DEVIATION

We now outline the procedure for computing the mean deviation:

Step1. Calculate the average A about which mean deviation is to be computed, by the methods discussed earlier.

Step2. Find the deviation of each observation X from A and denote it by D. That is, find $D = X - A$.

Step3. Find the absolute value of the deviation of each observation from A ignoring the signs and denote it by $|D|$

Step4. Find the sum of all absolute deviations obtained in Step 3 to get $\sum|D|$

Step5. Divide the sum obtained in Step 4 by the number of observations to get the required mean deviation about the average A.

Q.15What is the mean deviation for the following numbers?

15, 18, 20, 9, 12, 16

(a) 2 (b) 3 (c) 7 (d) 8

Ans. (b)

Q.16What is the value of mean deviation about mean for the following numbers?

5, 8, 6, 3, 4

(a) 5.20 (b) 7.20 (c) 1.44 (d) 2.23

Ans. (c)

Q.17Given the observations: 4, 9, 11, 14m 37. The mean deviation about the median is

(a) 11 (b) 8.5 (c) 7.6 (d) 7.45

Ans. (c)

Q.18What is the value of mean deviation about mean for the following observation?

50, 60, 50, 50, 60, 60, 60, 50, 50, 50, 60, 60, 50

(a) 5 (b) 7 (c) 35 (d) 10

Ans. (a)

Q.19The mean deviation about mode for the number $\frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \frac{9}{11}, \frac{12}{11}, \frac{8}{11}$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{11}$ (c) $\frac{6}{11}$ (d) $\frac{5}{11}$

Ans. (a)

COMPUTATION OF MEAN DEVIATION –DISCRETE SERIES

- In case of discrete series where the variable X takes the Values $X_1, X_2, \ldots \ldots, X_n$ with respective frequencies $f_1, f_2, \ldots \ldots, f_n$, the mean deviation about an average A is given by
- Mean Deviation (about an average A) $= \frac{\sum f_i|X_i - A|}{N}$

PROCEDURE FOR COMPUTING THE MEAN DEVIATION

Step1. Calculate the average A about which mean deviation is to be computed.

Step2. Take the deviation of each observation from A and denote it by D. That is, find $D = X - A$.

Step3. Find the absolute value of the deviation of each observation from A ignoring $\pm$ signs and denote it by $|D|$.

Step4. Multiply each absolute deviation $|D|$ by the corresponding frequency f_i to get $\sum |D|$.

Step5. Add all the products obtained in Step 4 to get $\sum f_i |D|$.

Step6. Divide the sum obtained in Step 5 by N, the total frequency, to get the required mean deviation.

Q.20 Calculate the mean deviation about the mean for the following data:

X	10	11	12	13	14	Total
f	3	12	18	12	3	48

(a) 12 (b) 0.75 (c) 15.5 (d) None of these

Ans. (b)

COMPUTATON OF MEAN DEVIATION: CONTINUOUS SERIES

- The computation of the mean deviation in the case of continuous series is exactly the same as discussed above for discrete series.

- The only difference is that here we have to obtain the class marks (or mid=values) of the various class and take absolute deviations of these values from the average A.
- Thus, if $X_1, X_1, \ldots \ldots, X_n$, then the mean deviation about an average A is given by:

  Mean Deviation (about an average A)$= \frac{\sum f_i|X_i - A|}{N}$

Q.21 Calculate mean deviation from the median for the following data:

Marks	10-20	20-30	40-50	50-60	60-70	80-90
No. of students	2	6	18	25	20	7

(a) 15.57 (n) 12.95 (c) 16.25 (d) None of these

Ans. (b)

Q.22What is the coefficient of mean deviation for the following distribution of heights? Take deviation from AM

Height in inches	60-62	63-65	66-68	69-71	72-74
No. of students	5	22	28	17	3

(a) 2.30 inches (b) 3.45 inches (c) 3.82 inches (d) 2.48 inches

Ans. (b)

COEFFICIENT OF MEAN DEVIATION

The relative measure corresponding to the mean deviation, called the coefficient of mean deviation, is given by

- Coefficient of M.D $= \frac{1}{n}\sum|x_i - A|$
- Coefficient of M.D. about mean $= \frac{1}{n}\sum|x_i - \bar{x}|$ where, $\bar{x}$ is mean
- Coefficient of M.D. about median $= \frac{1}{n}\sum|x_i - \text{Median}|$

Coefficient of mean deviation is a pure number independent of the units of measurement and can be used to compare two distributions expressed in different units.

PROPERTIES

- Mean deviation takes its minimum value when the deviations are taken from the median.
- Also mean deviation remains unchanged due to a change of origin but changes in the same ratio due to a change in scale i.e., if y = a + bx, a and b being constants, then MD of $y = |b| \times MD\ of\ x$

Q.23If the relation between x and y in $5y - 3x = 0$ and the mean deviation about mean for x is 12, then the mean deviation is y about mean is

(a) 7.20 (b) 6.80 (c) 20 (d) 18.80

Ans. (a)

MERITS AND DEMERITS OF MEAN DEVIATION

Merits

1. It is easy to understand and simple to calculate.

2. It is based on each and every item of the data.
3. It is rigidly defined.
4. As compared to standard deviation, it is less affected by extreme observations.
5. Since deviations are taken from a central value, comparison about formation of different distributions can easily be made.

Demerits

1. The major drawback of mean deviation is that algebraic signs are ignored while taking the deviations of the items.
2. It is not suitable for further mathematical treatment.
3. It cannot be computed for distribution with open-end classes.
4. It is rarely used in sociological studies.

Q.24 If two variables x and y are related by the equation 2x - 3y + 4 = 0 and the mean and mean deviation about mean of x are 4 and 0.6 respectively, then the coefficient of mean deviation of y about its mean is

(a) 5 (b) 8 (c) 10 (d) None of these

Ans. (c)

Q.25 The coefficient of mean deviation about mean for the first 9 natural number is

(a) $\frac{200}{9}$ (b) 80 (c) $\frac{400}{9}$ (d) 50

Ans. (c)

Q.26 If two variable x and y are related by $2x + 3y - 7 = 0$ and the mean and mean deviation about mean of ▯ are 1 and 0.3 respectively, then the coefficient of mean deviation of y about its mean is

(a) -5 (b) 12 (c) 50 (d) 4

Ans. (b)

Q.27 If x and y are related as $3x + 4y + 7 = 0$ and mean deviation of x is 6.40, what is the mean deviation of y?

(a) 2.52 (b) 4.18 (c) 6.40 (d) None of these

Ans. (b)

Q.28 Find the mean deviation about the median for the following data:

82, 56, 75, 70, 52, 80, 68

(a) 21.5 (b) 12.45 (c) 17.68 (d) None of these

Ans. (b)

Q.29 The mean deviation of weights about median for the following data:

Weight (lb)	131-140	141-150	151-160	161-170	171-180	181-190
No. of persons	3	8	13	15	6	5

(a) 10.97 (b) 8.23 (c) 9.63 (d) 11.45

Ans. (a)

Q.30 What is the mean deviation about median for the following data?

X	3	5	7	9	11	13	15

F	2	8	9	16	14	7	4

(a) 2.50 (b) 2.46 (b) 2.43 (d) 2.37

Ans. (d)

Q.31 Calculate coefficient of mean deviation from the median for the following data:

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of Students	2	6	12	18	25	20	10	7

(a) 0.24 (b) 0.95 (c) 1.50 (d) None of these

Ans. (a)

QUARTILE DEVIATION OR SEMI INTER-QUARTILE RANGE

Inter-quartile Range is an absolute measure of dispersion defined by the formula:

Inter-quartile range = $Q_3 - Q_1$ where Q_1 and Q_3 are the first (lower) and the third (or upper) quartiles respectively.

Quartile deviation, also called semi inter-quartile range, is an absolute measure of dispersion defined by the formula:

Quartile Deviation (Q.D.)$=\frac{Q3-Q1}{2}$

- In a symmetrical distribution, the two quartiles Q_1 and Q_3 are equidistant from the median, meaning their difference is equal.
- The quartile deviation provides a measure of the spread or dispersion of the data around the median.
- The quartile deviation can be used to estimate the spread of data and compare the variability between different datasets.
- The range of values from (Median - $Q.D.$) to (Median + $Q.D.$) covers exactly 50% of the data, making it useful for analyzing the central half of the distribution.

COEFFICIENT OF QUARTILE DEVIATION

The coefficient of quartile deviation is a relative measure of dispersion that provides a standardized measure of the spread of data. It is defined by the formula:

Coefficient of Quartile Deviation $= \frac{\text{Quartile Deviation}}{\text{Median}} \times 100 = \frac{Q_3-Q_1}{Q_3+Q_1} \times 100$

The coefficient of quartile deviation is a pure number that is independent of the units of measurement used for the data. This makes it useful for comparing the variability of different distributions, even if they are expressed in different units.

Therefore, Quartile deviation can be computed from the distribution having open-end classes. It is affected considerably by the sampling fluctuations. Quartile deviation remains affected due to change of origin but is affected in the same ratio due to change in scale.

Q.32 When 1st quartile =20, 3rd quartile =30, the value of quartile deviation is

(a) 7 (b) 4 (c) -5 (d) 5

Ans. (d)

Q.33 The first quartile of a dataset is 25 and the third quartile is 75. What is the interquartile range of the dataset?

(a) 25 (b) 50 (c) 75 (d) 100

Ans. (b)

Q.34 Quartile Deviation for the data 1, 3, 4, 5, 6, 6, 10 is

(a) 3 (b) 1 (c) 6 (d) 1.5

Ans. (d)

Q.35If median =5, Quartile deviation – 1.5, then the coefficient of quartile deviation is

(a) 33 (b) 35 (c) 30 (d) 20

Ans. (c)

Q.36The quartile deviation for the data is:

x	2	3	4	5	6
y	3	4	8	4	1

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

Ans. (c)

Q37 Following are the marks of the 10 students: 56, 48, 65, 35, 42, 75, 82, 60, 55, 50. Find the coefficient of quartile deviation.

(a) 16.50 (b) 18 (c) 18.42 (d) None of these

Ans. (c)

Q.38 If x and y are related as $2x + 5y = 30$ and the quartile deviation of x is 10, then the quartile deviation of y is:

(a) 2 (b) 4 (c) 5 (d) 6

Ans. (b)

Q.39Following are the marks of the 10 students: 34, 28, 45, 26, 24, 47, 38, 36, 49, 50. Find the quartile deviation.

(a) 20 (b) 30 (c) 10 (d) None of these

Ans.(a)

Q.40The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is

(a) 10 (b) 20 (c) 25 (d) 8.30

Ans. (a)

Q.41If the first quartile is 56 and the third quartile is 77, then the coefficient of quartile deviation is

(a) 18.09 (b) 15.79 (c) 63.80 (d) 56.71

Ans. (b)

Q.42If the first quartile is 48 and quartile deviation is 6, find the median. (Assuming the distribution to be symmetrical)

(a) 54 (b) 48 (c) 42 (d) 6

Ans. (a)

Q.43Find the interquartile range and the coefficient of quartile deviation from the following data:

Marks leass than	10	20	30	40	50	60	70	80
No. of Students	4	16	40	76	96	112	120	125

(a) 50 and 25 (b) 20 and 25 (c) 50 and 30 (d) None

Ans. (b)

Q.44 If the quartile deviation of x is 5 and $2x + 5y = 10$, what is the quartile deviation of y?

(a) 2 (b) 4 (c) 1 (d) Cannot be determined

Ans. (a)

Q.45 Calculate quartile deviation from the following distribution:

X	5-7	8-10	11-13	14-16	17-19
Frequency	14	24	38	38	20

(a) 1.55 (b) 2.27 (c) 3.05 (d) None of these

Ans. (b)

Q.46 Find the value of the third quartile if the values of first quartile and quartile deviation are 104 and 18 respectively.

(a) 100 (b) 110 (c) 120 (d) 140

Ans. (d)

MERITS

- Quartile deviation provides the best measure of dispersion for open-end classification. In fact, it is the only measure of dispersion which can be obtained while dealing with a distribution having open-end classes.
- It is also less affected due to sampling fluctuations. It is not affected at all by extreme observations as it ignores 25% of the data from the beginning of the distribution and another 25% of the data from the top end.
- Quartile deviation is useful especially when it is desired to study variability in the central half part of the data.
- Like other measures of dispersion, quartile deviation remains unaffected due to a change of origin but is affected in the same ratio due to change in scale.

DEMERITS

1. Quartile deviation is not based on all the observations. In fact, it ignores 25% of the data at the lower end and 25% of the data at the upper end. Hence it cannot be considered as a good measure of dispersion.
2. Quartile deviation is not suitable for further mathematical treatment.
3. It is affected considerably by sampling fluctuations.

STANDARD DEVIATION

The standard deviation, abbreviated as S.D. of a given set of observations is defined as the positive square root of the arithmetic mean of the squares of deviations of the observations from their arithmetic mean. It is denoted by the Greek letter (read as sigma).

Thus, standard deviation of a set on n observations $X_1, X_2, \dots, X_n$ is given by

$$\sigma = \sqrt{\frac{\sum(X-\bar{X})^2}{N}} - where\ \bar{X} = \frac{\sum X}{n}$$

If $X_1, X_2, \dots \dots X_n$ are the class marks of a set of grouped data with class frequencies $f_1, f_2, \dots f_n$, then the standard deviation is given by

$$\sigma = \sqrt{\frac{\sum f_i(x_i-\bar{x})^2}{n}}$$

Q.47 The heights (in centimeters) of a group of students are as follows: 150, 160, 165, 155, 170, 155, 160, 155. What is the standard deviation of the heights?

(a) 4.16 (b) 5.20 (c) 6.62 (d) 7.07

Ans. (d)

Q.48 What is the standard deviation of 5, 5, 9, 9, 9, 10, 5, 10, 10?

(a) $\sqrt{14}$ (b) $\frac{\sqrt{42}}{3}$ (c) 4.50 (d) 8

Ans. (b)

Q.49 Calculate the standard deviation for the following data:

X	20	30	40	50	60	70
Frequency	8	12	20	10	6	4

(a) 13.75 (b) 14.50 (c) 10.15 (d) None of these

Ans. (a)

Q.50 Calculate the standard deviation from the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of Students	10	15	25	25	10	10	5

(a) 11.65 (b) 13.39 (c) 14.40 (d) 15.94

Ans. (d)

DIFFERENT METHODS OF CALCULATING STANDARD DEVIATION: UNGROUPED DATA

The standard deviation of an ungrouped data consisting on N observations $X_1, X_2, \ldots \ldots, X_n$, is given by

$$\sigma = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

The computation of standard deviation is very effective if X is an integer. However, if X comes out to be in fraction, its computation becomes very cumbersome and time-consuming. In that case we apply the following short-cut method which is very effective and reduces the numerical calculations to a great extent.

DIFFERENT METHODS OF CALCULATING STANDARD DEVIATION - GROUPED DATA

All the methods discussed earlier for calculating standard deviation in the case of ungrouped data can also be used in the case of grouped data. However, in practice it is the step deviation method that is mostly used.

Standard deviation is given by: $\sigma = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$

Q.51 From the following information, find standard deviation of X and Y variables:

$\sum X = 235, \sum X = 250, \sum X^2 = 6750, \sum Y^2 = 6840$ N = 10

(a) 11.08 and 7.68 (b) 12.55 and 8.06

(c) 29.50 and 16.76 (d) None of the these

Ans. (a)

Q.52From the following data, calculate the standard deviation:

X	10	11	12	13	14	15	16	17	18
Frequency	2	7	10	12	15	11	10	6	3

(a) 1.50 (b) 1.986 (c) 2.576 (d) None of these

VARIANCE

The variance of a given set of observations is defined as the square of its standard deviation and is denoted by σ^2

Thus,

For individual observations, $\sigma = \frac{\Sigma(x-\bar{x})}{n}$ or $\sigma^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

For Ungrouped / Grouped Observations, $\sigma^2 = \frac{\Sigma f_i(x_i-\bar{x})^2}{n}$

Q.53$\sum x^2 = 3390$, n=30, $\sigma = 7$ then $\bar{x} =$_____

(a) 113 (b) 210 (c) 8 (d) None of these

Ans. (c)

Q.54Find variance if $\sum D^2 = 150$ and N=6 Deviations are taken from actual mean

(a) 5 (b) 25 (c) 36 (d) None of these

Ans. (b)

Q.55If the standard deviation for the marks obtained by a student in monthly test is 36, then the variance is

(a) 36 (b) 6 (c) 1296 (d) None of these

Ans. (c)

CORRECTING INCORRECT VALUES OF MEAN AND STANDARD DEVIATION

Q.56The mean and variance of 100 items were worked out as 40 and 25 respectively by a student. By mistake an item 50 was wrongly taken as 5 in calculating the above. You are required to find the correct mean and correct standard deviation.

(a) 2.54 (b) 3.05 (c) 3.68 (d) None of these

Ans. (c)

COMBINED STANDARD DEVIATION

If two sets of data contain n_1 and n_2 observations having means $\bar{x}_1$ and $\bar{x}_2$ and standard deviations σ_1 and σ_2 respectively, then the standard deviation, σ, of the combined data with $n_1 + n_2$ observations is given by:

- $\sigma = \frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}$

- $\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2}$

The result can be generalized to more than two sets of data.

E.g.: If $\bar{X}_1, \bar{X}_2 \ldots X_k$ be the means, $\sigma_1, \sigma_2, \ldots \ldots \ldots \ldots \sigma_k$ be the standard deviations and $n_1, n_2, \ldots\ldots\ldots\ldots\ldots, n_k$ be the number of observation in each set, then the standard deviation of the combined data with $n_1 + n_2 + \cdots \ldots\ldots\ldots + n_k$ observations is given by

- $\sigma = \frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)+\cdots\ldots+n_k(\sigma_k^2+d_k^2)}{n_1+n_2+\cdots+n_k}$

  Where $d_1 = \bar{X}_1 - \bar{X}$, $d_2 = \bar{X}_2 - \bar{X} \ldots, d_k = \bar{X}_k - \bar{X}$

- $\bar{x} = \frac{n_1\bar{x}_1+n_2\bar{x}_2+\cdots+n_k\bar{x}_k}{n_1+n_2+\cdots+n_k}$ is the combined mean.

Q.57 If two samples of sizes 30 and 20 have means as 55 and 60, and variances as 16 and 25 respectively, then what would be the SD of the combined sample of size 50?

(a) 5.00 (b) 5.06 (c) 5.23 (d) 5.35

Ans. (b)

COEFFICIENT OF VARIATION

- The standard deviation is an absolute measure of dispersion, depending upon the units of measurement. It does not tell us much about the variability of a single set of data. The coefficient of standard deviation, based on standard deviation, is a relative measure of dispersion.
- This is a pure number independent of the units of measurement and hence can be used to compare the variability of two distributions expressed in different units.
- Perhaps a more appropriate measure is the coefficient of variation (C.V.), defined by

  Coefficient of Variation $= \frac{S.D}{\bar{x}} \times 100$

  The above expression expresses the standard deviation as a percentage of the mean.
- Since C.V. is a measure of relative variation expressed as a percent, the coefficient of variation can be used to compare the variability of two or more sets of data even when observations are expressed in different units of measurement.
- A distribution for which the coefficient of variation is smaller is said to be less variable or more consistent, more uniform, more stable or more homogeneous.
- On the other hand, the distribution for which the coefficient of variation is greater is said to be more variable or less consistent, less uniform, less stable or less homogeneous.

Q.58 If Mean = 5, Standard deviation = 2.6, then the coefficient of variation is

(a) 49 (b) 51 (c) 50 (d) 52

Ans. (d)

Q.59 If the coefficient of variation and standard deviation are 30 and 12 respectively, then the arithmetic mean of the distribution is

(a) 40 (b) 36 (c) 25 (d) 19

Ans. (a)

Q.60 What is the coefficient of variation of the following numbers?

(a) 8.09 (b) 18.08 (c) 20.23 (d) 24.45

Ans. (a)

Q.61 The sum of squares of deviation from mean of 10 observations is 250. Mean of the data is 10. Find the co-efficient of variation.

(a) 10% (b) 25% (c) 50% (d) 0%

Ans. (c)

Q.62 Mean of a series is equal to 100, co-efficient of variation is 45% then the S.D. is

(a) 45 (b) 0.45 (c) 4.5 (d) 40.5

Ans. (a)

PROPERTIES FOR STANDARD DEVIATION

Standard deviation is independent of change of origin but not of scale, i.e., if there are two variables x and y related as $y = a + bx$ for any two constants a and b, the SD of y is given by: $S_y = |b| \times S_x$

Q.63 If x and y are related by $2x + 3y + 4 = 0$ and SD of x is 6, then SD of y is

(a) 22 (b) 4 (c) 5 (d) 9

Ans. (b)

Q.64 If AM and C.V of a random variable x are 10 & 40 respectively, the variance of $(-15 + \frac{3x}{2})$

(a) 64 (b) 81 (c) 49 (d) 36

Ans. (d)

Note:

- Standard deviation is suitable for further mathematical treatment. For instance, if we know the sizes, means and standard deviations of two or more sets of data, then we can obtain the standard deviation of the combined data.
- The standard deviation of first natural numbers is $\sqrt{\frac{n^2-1}{12}}$
- If all the observations assumed by a variable are constant i.e., equal, then the SD is zero. This means that if all the values taken by a variable x is k, say, then $s = 0$. This result applies to range as well as mean deviation.

Q.65 If the $S.D.$ of the 1st n natural number is $\sqrt{30}$, then the value of n is

(a) 19 (b) 20 (c) 21 (d) None of these

Ans. (a)

Q.66 The mean and SD for a, b and 2 are 3 and $\frac{2}{\sqrt{3}}$ respectively, The value of ab would be

(a) 5 (b) 6 (c) 11 (d) 3

Ans. (c)

Q.66 Which of the following companies A or B is more consistent so far the payment of dividend is concerned?

Dividend paid by A	5	9	6	12	15	10	8	10
Dividend paid by B	4	8	7	15	18	9	6	6

(a) A (b) B (c) Both A & B (d) Neither A nor B

Ans. (a)

Q.67 For any two numbers, SD is always

(a) Twice the range (b) Half of the range
(c) Square of the range (c) None of these

Ans. (b)

Q.68 If all the observations are increased by 10, then

(a) SD would be increased by 10
(b) Mean deviation would be increased by 10
(c) Quartile deviation would increased by 10
(d) All these three remain unchanged.

Ans. (d)

Q.69 If all the observations are multiplied by 2, then

(a) New SD would be also multiplied by 2
(b) New SD would be half of the previous SD
(c) New SD would be increased by 2
d) New SD would be increased by 2

Ans. (a)

Q.70 What is the standard deviation from the following data relating to the age distribution of 200 person?

Age (year)	20	30	40	50	60	70	80
No. of people	13	28	31	46	39	23	23

(a) 15.29 (b) 16.87 (c) 18.00 (d) 17.52

Ans. (b)

Q.70 What is the coefficient of variation for the following distribution of wages?

(a) `14.73 (b) 14.73 (c) 26.53 (d) 20.82

Ans. (c)

Q.71 The mean and SD of a sample of 100 observation were calculated as 40 and 5.1 respectively by a CA student who took one of the observation as 50 instead of 40 by mistake. The current value of SD would be

(a) 4.90 (b) 5.00 (c) 4.88 (d) 4.85

Ans. (b)

Q.71 The sum of mean and SD of series is $a + b$, if we add 2 to each observation of the series then the sum of mean and SD is

(a) $a + b + 2$ (b) $6 - a + b$ (c) $4 + a - b$ (d) $a + b + 4$

Ans. (a)

Q.72 If the coefficient of quartile deviation is $\frac{1}{4}$, then $\frac{Q3}{Q_1}$ is

(a) 5/3 (b) 4/3 (c) ¾ (d) 3/5

Ans. (a)

Q.73 For a symmetric distribution

(a) Mean = Median = Mode (b) Mode = 3 median – 2 mean
(c) Mode $\frac{1}{3}$ median = $\frac{1}{2}$ Mean (d) None of these

Ans. (a)

Q.74 If the profits of a company remain some for the last ten months, then the S.D of profit of the company would be:

(a) Positive (b) Negative (c) Zero (d) (a) or (b)

Ans. (c)

Q.75S.D of first five consecutive natural numbers is

(a) $\sqrt{10}$ (b) $\sqrt{8}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

Ans. (d)

Q.76Standard deviation is_____times of $\sqrt{MD \times QD}$

(a) $2\sqrt{3}$ (b) 4/5 (c) $\sqrt{\frac{15}{8}}$ (d) $\sqrt{\frac{8}{15}}$

Ans. (c)

Q.77In a moderately skewed distribution the values of mean & median are 12 & 8 respectively. The value of mode is

(a) 0 (b) 12 (c) 15 (d) 30

Ans. (a)

Q.78The AM of 15 observations is 9 and the AM of first 9 observations is 11 and then AM or remaining observation is

(a) 11 (b) 6 (c) 5 (d) 9

Ans. (b)

Q.79If $\sigma^2 = 100$ and coefficient of variation = 20% then $\bar{X}$=

(a) 60 (b) 70 (c) 80 (d) 50

Ans. (d)

Q.80Find the interquartile range for the following dataset representing the scores of 10 students in a mathematics test:

35, 42, 48, 55, 60, 63, 68, 70, 72, 78

(a) 12 (b) 24 (c) 46.5 (d) None of these

Ans. (b)

Q.81If in a moderately skewed distribution, the values of mode and mean are 32.1 and 35.4 respectively, then value of the median is

(a) 34.3 (b) 33.3 (c) 34 (d) 33

Ans. (a)

Q.82Standard deviation for the marks obtained by a student in monthly test in mathematics (out of 50) as 30, 35, 25, 20 and 15 is

(a) 25 (b) $\sqrt{50}$ (c) $\sqrt{30}$ (d) 50

Ans. (b)

Q.83If the range of a set of values is 65 and maximum values in the set is 83, then the minimum values in the set is

(a) 74 (b) 9 (c) 18 (d) None of these

Ans. (c)

Q.84The Geometric mean of 3, 6, 24, and 48 is

(a) 8 (b) 12 (c) 24 (d) 20

Ans. (b)

Q.85The mean of 20 items of a data is 5 and If each item is multiplied by 3, then the new mean will be

(a) 5 (b) 10 (c) 15 (d) 20

Ans. (c)

Q.86The median of the data 5, 6, 7, 7, 8, 9, 10, 11, 11, 12, 15, 18, 18, and 19 is

(a) 10.5 (b) 10 (c) 11 (d) 11.5

Ans. (a)

Q.87The average of a series of overlapping averages, each of which is based on a certain number of items within a series is known as

(a) Moving average (b) Weighted average
(c) Simple average (d) None of these

Ans. (a)

Q.88 If each item is reduced by 15, then A.M is

(a) Reduced by 15 (b) Increased by 15
(c) Reduced by 10 (d) None

Ans. (a)

Q.89 $\frac{(Q3-Q1)}{(Q_3+Q_1)}$ is known as

(a) Coefficient of Range. (b) Coefficient of Q.D.
(c) Coefficient of S.D (d) Coefficient of M.D.

Ans. (b)

Q.90 Mean deviation is the least when deviation are taken from

(a) Mean (b) Median (c) Mode (d) Harmonic mean

Ans. (b)

PREVIOUS YEAR QUESTIONS

SETS, RELATION & FUNCTIONS

Q.1 The AM and HM of two numbers are 5 and 3.2 respectively, then GM will be: [Dec. 2023]

(a) 4.4 (b) 4.2
(c) 4 (d) 3.8

Ans. (c) 4

Q.2 If mode of grouped data is 10 and median is 6, then what is the value of mean? [Dec. 2023]

(a) 2 (b) 4
(c) 6 (d) 8

Ans. (b) 4

Q.3 If mean of 5 observations x + 1, x + 3, x + 5, x + 7 and x + 9 is given 15, then the value of x will be: [Dec. 2023]

(a) 10 (b) 12
(c) 8 (d) 11

Ans. (a) 10

Q.4 The mean of the first three terms is 17 and mean of next four terms is 21. Calculate the mean of seven terms. [Dec. 2023]

(a) 18.28 (b) 19.78
(c) 19.58 (d) 19.28

Ans. (d) 19.28

Q.5 The mean of a set of 20 observations in 18.3. The mean is reduced by 0.6 when a new observation is adde d to the set. The new observation is: [Dec. 2023]

(a) 17.6 (b) 18.9
(c) 5.7 (d) 24.6

Ans. (c) 5.7

Q.6 If A.M. and G.M. of two positive numbers a and b are 12 and 12, respectively, find the numbers. [Dec. 2023]

(a) 18 and 6 (b) 15 and 9
(c) 16 and 8 (d) 12 and 12

Ans. (d) 12 and 12

Q.7 A perpendicular drawn from the point of intersection of two Ogive on the horizontal axis gives the value of [Dec. 2023]

(a) 2nd Quartile (b) 3rd Quartile
(c) Mode (d) 1st Quartile

Ans. (a) 2nd Quartile

Q.8 For a given set of normally distributed data, the following statistical parameters are known: Mean = 6; Standard deviation = 2.6; Median = 5 and Quartile deviation = 1.5, then the coefficient of quartile deviation equals to [June 2023]

(a) 30 (b) 32
(c) 25 (d) 39

Ans. (a) 30

Q.9 The mean and variance of a group of 100 observations are 8 and 9 respectively. Out of the 100 observations, the mean and standard deviation of 60 observations is 10 and 2 respectively. Find the standard deviation of remaining 40 observations? [June 2023]

(a) 4.5 (b) 3.5
(c) 2.5 (d) 1.5

Ans. (d) 1.5

Q.10 If the standard deviation of data 2,4,5,6,8,17 is 4.47 , then standard deviation of the data 4,8,10,12,16,34 is [June 2023]

(a) 4.47 (b) 8.94
(c) 13.41 (d) 2.24

Ans. (b) 8.94

Q.11 If the first quartile is 42.75 and the third quartile is 74.25 , then the coefficient of quartile deviation is: [June 2023]

(a) 29.62 (b) 15.75
(c) 17.57 (d) 26.92

Ans. (d) 26.92

Q.12 What is the mean deviation about mean for the data. 12,16,24,30,35,39,40 [June 2023]

(a) 9.14 (b) 9.41
(c) 8.91 (d) 9.81

Ans. (a) 9.14

Q.13 If x and y are related as $4x + 3y + 11 = 0$ and mean deviation of y is 7.2 then the mean deviation of x is [June 2023]

(a) 2.70 (b) 7.20
(c) 4.50 (d) 5.40

Ans. (d) 5.40

Q.14 If the mean of two numbers is 30 and geometric mean is 24 , then what will be the Harmonic mean of two numbers? [June 2023]

(a) 19.2 (b) 21.8
(c) 22.3 (d) 18.4

Ans. (a) 19.2

Q.15 For the given data set: 5,10,3,6,4,8,9,3,15,2,9,4,19,11,4 find the median [June 2023]

(a) 8 (b) 6
(c) 4 (d) 9

Ans. (b) 6

Q.16 Find the mode of the following data

X	$25-30$	$30-35$	$35-40$	$40-45$	$45-50$	$50-55$
$f(x)$	20	53	42	42	41	43

[June 2023]

(a) 31.75 (b) 30.75
(c) 33.75 (d) 35.75

Ans. (c) 33.75

Q.17 The median of the following set of observations: 24,18,36,42,30,28,21,29,25,33 is [June 2023]

(a) 26.5 (b) 27.5
(c) 28.5 (d) 29.5

Ans. (c) 28.5

Q.18 The Geometric Mean of 3,7,11,15,24,28,30,0 is [June 2023]

(a) 6 (b) 0
(c) 9 (d) 12

Ans. (b) 0

Q.19 For a moderately skewed distribution of marks in Statistics for a group of 200 students, the mean marks and median marks were found to be 55.60 and 52.40 respectively. What are the modal marks? [June 2023]

(a) 54.43 (b) 48
(c) 53.56 (d) 46

Ans. (d) 46

Q.20 Find the mean of the following data

Class Interval	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$	$60-70$	$70-80$
Frequency	9	13	6	4	6	2	3

[June 2023]

(a) 23.7 (b) 35.7
(c) 39.7 (d) 43.7

Ans. (b) 35.7

Q.21 A professor has given an assignment to students in a Statistics class. A student computes the arithmetic mean and standard deviation for a set of 100 observations as 50 and 5 respectively. Later on, she points out that she has made a mistake in taking one observation as 100 instead of 50. What would be the correct mean if the wrong observation is corrected? [June 2023]

(a) 50.5 (b) 49.9
(c) 49.5 (d) 50.1

Ans. (c) 49.5

Q.22 The average age of 15 students in a class is 9 years. Out of them, the average age of 5 students is 13 years and that 8 students is 5 years. What is the average of remaining 2 students? [Dec. 2022]

(a) 5 years (b) 9 years
(c) 10 years (d) 15 years

Ans. (d) 15 years

Q.23 If Arithmetic mean between two numbers is 5 and Geometric mean is 4 then what is the value of Harmonic mean? [Dec. 2022]

(a) 3.2 (b) 3.4
(c) 3.5 (d)3.6
Ans.

Q.24 If Arithmetic Mean and Geometric Mean between two number are 5 and 4 respectively, then these numbers are [Dec. 2022]

(a) 2 & 3 (b) 2 & 8
(c) 4 & 6 (d) 1 & 16

Ans. (b) 2 & 8

Q.25 The mean of 50 observations is 36. If two observations 30 and 42 are to be excluded, then the mean of the remaining observations will be: [Dec. 2022]

(a) 36 (b) 38
(c) 48 (d)50

Ans. (a) 36

Q.26 If the sum of square of the value equals to 3390, Number of observations are 30 and Standard deviation is 7, what is the mean value of the above observation? [Dec. 2022]

(a) 14 (b) 11
(c) 8 (d) 5
Ans. (c) 8

Q.27 The median of the observations 42, 72, 35, 92, 67, 85, 72, 81, 51, 56 is [Dec. 2022]

(a) 69.5 (b) 72
(c) 64 (d) 61.5

Ans. (a) 69.5

Q.28 If the first quartile in 56.50 and the third quartile is 77.50 then the co-efficient of quartile deviation is [June 2022]

(a) 618.09 (b) 15.67
(c) 63.80 (d) 156.71

Ans. (b) 15.67

Q.29 Mean deviation is minimum when deviations are taken from: [June 2022]

(a) Mean (b) Median
(c) Mode (d) Range

Ans. (b) Median

Q.30 Which measure of dispersion is based on the absolute deviation only? [June 2022]

(a) Range (b) Standard Deviation
(c) Mean Deviation (d) Quartile Deviation

Ans. (c) Mean Deviation

Q.31 The arithmetic mean and coefficient of variation of data set X are respectively, 10 and 30. The variance of 30 – 2X is [June 2022]

(a) 28 (b) 32
(c) 34 (d) 36

Ans. (d) 36

Q.32 Find the standard deviation and coefficient of variation of 1, 6, 5, 9, 8. [June 2022]

(a) 2.78 and 40.83 respectively (b) 2.45 and 47.93 respectively
(c) 2.78 and 47.93 respectively (d) 2.87 and 49.37 respectively

Ans. (c) 2.78 and 47.93 respectively

Q.33 Following are the ages of 8 employees of a small old age home expressed in 96, 50, 67, 75, 71, 69, 64, 66. Find the range and its coefficient. [June 2022]

(a) 46, 31.51 respectively (b) 51, 37.67 respectively
(c) 43, 29.49 respectively (d) 49, 36.42 respectively

Ans. (a) 46, 31.51 respectively

Q.34 What is mean deviation about mean of the following numbers? 11, 8, 10, 10, 12, 9 [June 2022]

(a) 2 (b) 1
(c) 1.5 (d) 1.8

Ans. (b) 1

Q.35 Calculate the value of 3rd quartile from the following data 40, 35, 51, 30, 21, 25, 16, 29, 27, 32. [June 2022]

(a) 36.25 (b) 30.25
(c) 25 (d)35

Ans. (a) 36.25

Q.36 The Mean of 100 students was 45. Later on, it was discovered that the marks of two students were misread as 85 and 54 instead of 58 and 45. Find out the correct mean. [June 2022]

(a) 68 (b) 36
(c) 44.64 (d) 52

Ans. (c) 44.64

Q.37 A seller of pearls kept the pearls in seven boxes labelled from one to seven. At the end of a day, he found that J labelled box contained J pearls, the average number of pearls per box is [June 2022]

(a) 4 (b) 6.5
(c) 7.5 (d)8

Ans. (a) 4

Q.38 Which one of the following is not a measure of central tendency? [June 2022]

(a) Median (b) Range
(c) Arithmetic Mean (d) Harmonic Mean

Ans. (b) Range

Q.39 The coefficient of deviation based on 25th and 75th percentiles of 6, 9, 3, 8, 4, 5, 8 and 4 is [June 2022]

(a) 50 (b) 100/3
(c) 30 (d) 25

Ans. (b) 100/3

Q.40 _____ Mean is calculated, when the values in series do not have equal importance. [June 2022]

(a) Arithmetic (b) Harmonic
(c) Geometric (d) Weighted

Ans. (d) Weighted

Q.41 There are n numbers. When 50 is subtracted from each of these number the sum of the numbers so obtained is –10. When 46 is subtracted from each of the original numbers, then the sum of numbers, so obtained is 70. What is the mean of the original n numbers? [June 2021]

(a) 56.8 (b) 25.7
(c) 49.5 (d) 53.6

Ans. (c) 49.5

Q.42 The mean of 'n' observation is 'X'. If K is added to each observation, then the new mean is _____ [June 2021]

(a) X (b) XK
(c) X –K (d) X + K

Ans. (d) X + K

Q.43 If y = 3 + 1.9x, and mode of x is 15, then the mode of y is: [June 2021]

(a) 15.9 (b) 27.8
(c) 35.7 (d) 31.5

Ans. (d) 31.5

Q.44 From the records on sizes of shoes sold in a shop, one can compute the following to determine the most preferred shoe size. [Jan. 2021]

(a) Mean (b) Median
(c) Mode (d) Range

Ans. (c) Mode

Q.45 Which of the following measure does not possess mathematical properties? [Jan. 2021]

(a) Arithmetic mean (b) Geometric mean

(c) Harmonic mean (d) Median

Ans. (d) Median

Q.46 If y = 3 + (4.5)x and the mode for x-value is 20, then the mode for y–is [Jan. 2021]

(a) 3.225 (b) 12

(c) 24.5 (d) 93

Ans. (d) 93

SUMMARY

MEASURES OF CENTRAL TENDENCY

- **Arithmetic mean:** Sum of the observation divided by the number of observations

  Arithmetic Mean (Mean) $= \frac{Sum\ of\ observations}{Number\ of\ observations}$

  Methods to find mean:

  (i) **Direct method:** Mean $= \frac{f1x1+f2x2+\cdots+fnxn}{f_1+f_2+\cdots+f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

  (ii) **Step-deviation method:** $Mean = a + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h$

- **Geometric mean:**

  (i) For individual series: GM = $(x_1 x_2 x_3 \ldots x_n)^{1/n}$

  (ii) For discrete series: G.M.= $AL\ [\frac{1}{N}\sum f \log X]$

  (iii) For continuous series G.M = $AL\ [\frac{1}{N}\sum f \log X]$

- **Harmonic mean:**

  (i) For individual series: H.M. $= \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\cdots+\frac{1}{x_n}}$

  (ii) For discrete series: $HM = \frac{n}{\sum \frac{f}{X}}$

  (iii) For continuous series: $HM = \frac{n}{\sum \frac{f}{X}}$

- Mode: The value that occurs the maximum number of times.

  For continuous frequency distribution: $Mode = l + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times c$

- Median:

  (i) **For individual series**: Arrange the dataset in ascending order or descending order, then

  (a) If n is odd, the value is given by: $(\frac{n+1}{2})\ th$ observation

  (b) If n is even, the value is given by: $\frac{\frac{n}{2}th\ observation + (\frac{n}{2}+1)th\ observation}{2}$

  (ii) For discrete series:

  Median is the size of $(\frac{N+1}{2})$th observation

  (iii) For continuous series:

  $Median = l + \frac{\frac{N}{2} - c.f}{f} \times h$

- **Quartiles** divide the entire dataset into four equal parts. So, there are three quartiles; first second and third represented by Q_1, Q_2 and Q_3 respectively. Q_1 is the lower quartile and median of the lower half of the data set. Q_2 is the median.
- **Deciles** divide the entire dataset into ten equal parts. There are 9 deciles
- **Percentiles** divide the entire dataset into hundred equal parts. There are 99 percentiles.

TABLE FOR PARTITION VALUE

Calculation for	Quartiles	Deciles	Percentiles
Individual Observation	$Q_k = size\ of\ \frac{k(n+1)}{4}\ th$ observation	$D_k = \frac{k(n+1)}{10}\ th$ observation	$P_k = \frac{k(n+1)}{100}\ th$ Observation
Discrete Series	The value of X corresponding to the c.f. Just greater than or equal To $\frac{k(n+1)}{4}$	The value of X corresponding to the c.f. just greater than or equal to $\frac{k(n+1)}{10}$	The value of X corresponding to the c.f. just greater than or equal to $\frac{k(n+1)}{100}$
Continuous Series	$Q_k = l + \frac{\frac{KN}{4} - C}{f} \times h$ $l = lower\ limit\ of\ Q_k\ class$ $C = c.f.of\ the\ class\ Preceding\ the\ Q_k\ class,\quad f = frequency\ of\ the\ Q_k\ class,$ $h = size\ or\ width\ of\ Q_k\ class.$	$D_k = l + \frac{\frac{KN}{10} - C}{f} \times h$	$P_k = l + \frac{\frac{KN}{100} - C}{f} \times h$

- Algebraic sum of deviation of a set of observation from their AM is zero i.e. $\sum x_i - \bar{x} = 0$
- AM is affected due to a change of origin and/or scale which implies that if the original variable is changed to another variable by effecting a change of origin,
- For given two positive numbers, $A.M \times H.M = (G.M)^2$
- $AM \geq GM \geq HM$ The equality sign occurs, as we have already seen, when all the observations are equal.
- $Mode = 3\ Median - 2\ Mean$

MEASURES OF DISPERSION

Range, interquartile range, and standard deviation are the three commonly used measures of dispersion.

- **Range** is the difference between the largest and the smallest observation in the data.
  - (i) For individual series: Range = Largest value - Smallest value
  - (ii) For grouped frequency distribution: Range = Upper class boundary (U.C.B) - Lower class boundary (L.C.B)
- **Coefficient of Range**:

$$Coefficient\ of\ Range\ = \frac{Range}{Sum\ of\ the\ largest\ and\ the\ lowest\ values}$$

$$i.e., Coefficient\ of\ range\ = \frac{L-S}{L+S} \times 100$$

- **Quartile Deviation or Semi Inter-Quartile Range:**

(i) **Interquartile range** is defined as the difference between the 25th and 75th percentile (also called the first and third quartile). Hence the interquartile range describes the middle 50% of observations. If the interquartile range is large it means that the middle 50% of observations are spaced wide apart.

$$Inter-quartile\ range = Q_3 - Q_1$$

(ii) **Quartile deviation,** also called semi inter-quartile range, is an absolute measure of dispersion defined by the formula

$$Quartile\ Deviation\ (Q.D.) = \frac{Q_3-Q_1}{2}$$

(iii) Coefficient of Quartile Deviation

$$Coefficient\ of\ Quartile\ Deviation = \frac{Quartile\ Deviation}{Median} \times 100 = \frac{Q_3-Q_1}{Q_3+Q_1} \times 100$$

- **Mean Deviation**

(i) Discrete Series: Mean Deviation (about an average A) $= \frac{\sum f_i|x_i-A|}{N}$

(ii) Continuous series: Mean Deviation (about an average A) $\frac{\sum fi|xi-A|}{N}$ where $x_1, x_2, \ldots, x_n$ are the class marks (or mid-values) of a set of grouped data.

Coefficient of Mean Deviation

The relative measure corresponding to the mean deviation, called the coefficient of mean deviation, is given by

1. $Coefficient\ of\ M.D. = \frac{1}{n}\sum|x_i - A|$
2. $Coefficient\ of\ M.D.\ about\ mean = \frac{1}{n}\sum|x_i - \bar{x}|, where, \bar{x}\ is\ mean$
3. $Coefficient\ of\ M.D.\ about\ mean = \frac{1}{n}\sum|x_i - Median|$

- **Standard Deviation** (SD) is the most commonly used measure of dispersion. It is a measure of spread of data about the mean. SD is the square root of the sum of squared deviation from the mean divided by the number of observations.

(i) For individual series $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$ where $\bar{x} = \frac{\sum X}{n}$

(ii) For grouped data: $\sigma = \sqrt{\frac{\sum f_i(x_i-\bar{x})^2}{n}}$

(iii) For ungrouped data: $\sigma = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$

Standard deviation of first natural numbers $= \sqrt{\frac{n^2-1}{12}}$

- **Variance:** It is the square of the standard deviation.

  (i) For individual observations, $\sigma^2 = \frac{\Sigma(x_i-\bar{x})^2}{n}$ or $\sigma^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

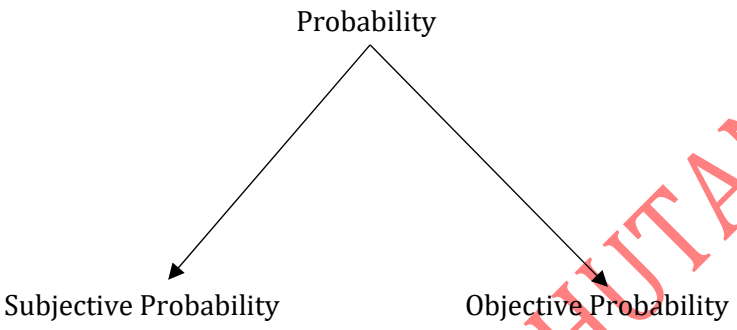
  (ii) For Ungrouped / Grouped observations, $\sigma^2 = \frac{\Sigma f_i(x_i-\bar{x})^2}{n}$

- Coefficient of Variation $= \frac{S.D}{X} \times 100$

If we were to consider an India vs. Pakistan cricket match today, predicting the outcome involves probability.

PROBABILITY

- The terms 'Probably' 'in all likelihood', 'chance', 'odds in favor', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics, known as Probability.
- Thus, it is a branch of mathematics that deals with quantifying uncertainty and analyzing the likelihood of events occurring.



SUBJECTIVE PROBABILITY

- Subjective probability is based on an individual's personal beliefs, judgments, and opinions about the likelihood of an event.
- It is often used when there is limited or no historical data available to estimate probabilities objectively.
- Subjective probabilities are influenced by personal experiences, biases, and perceptions.

OBJECTIVE PROBABILITY

- Objective probability, also known as frequentist probability, is based on observed or historical data and the relative frequency of an event occurring in a large number of trials or observations.
- It is considered a more objective and data-driven approach to probability, as it relies on empirical evidence.
- Objective probability can be expressed as a ratio of the number of favorable outcomes to the total number of possible outcomes.

In order to develop a sound knowledge about probability, it is necessary to get ourselves familiar with a few terms.

- **EXPERIMENT:** An experiment refers to the performance of certain tasks to produce certain results.
- **RANDOM EXPERIMENT**: A random experiment is one in which the results depend solely on chance and cannot be predicted with certainty.
- **EVENTS:** Events are the results or outcomes of a random experiment. Sometimes, it gives a combination of outcomes.

For example, {HH} represents the event of getting 2 heads.

- **MUTUALLY EXCLUSIVE EVENTS OR INCOMPATIBLE EVENTS** These are events that cannot occur simultaneously.

E.g.: Consider rolling a six-sided die. The events "rolling a 1" and "rolling a 2" are mutually exclusive because it is impossible for both events to occur simultaneously.

- Two events A and B are mutually exclusive if $P(A \cap B) = 0$, meaning there are no common outcomes between A and B.
- Similarly, three events A, B, and C are mutually exclusive if $P(A \cap B \cap C) = 0$, meaning there are no common outcomes among all three events.

EXHAUSTIVE EVENTS

Exhaustive events are events that together cover all possible outcomes of an experiment.

E.g.:

1. In a coin toss, there are two exhaustive events: "Getting heads" or "Getting tails".

2. In throwing a die, there are 6 exhaustive events: $\{1, 2, 3, 4, 5, 6\}$.

(a) Two events A and B are exhaustive if $P(A \cup B) = 1$, meaning all possible outcomes are covered by A and B.

(b) Similarly, three events A, B, and C are exhaustive if $P(A \cup B \cup C) = 1$, meaning all possible outcomes are covered by A, B, and C.

EQUALLY LIKELY EVENTS or Mutually Symmetric Events or Equi-Probable Events: Equally likely events are events that have the same probability of occurring.

E.g.: In a fair coin toss, getting heads or tails is an example of equally likely events.

- Three events A, B, and C are equally likely if $P(A) = P(B) = P(C)$, meaning the probabilities of each event are the same.

CLASSICAL DEFINITION OF PROBABILITY OR A PRIOR DEFINITION

FOR FINITE ELEMENTARY EVENTS

Let's consider a random experiment that results in n finite elementary events, which are assumed to be equally likely. If $n_A (\leq n)$ events are favorable to an event A, then the probability of occurrence of event A is defined as the ratio of the number of events favorable to A to the total number of events. This can be expressed as:

$$P(A) = \frac{n_A}{n} = \frac{Number\ of\ events\ favorable\ to\ A}{Total\ number\ of\ events}$$

FOR COMPOSITE EVENTS

In the case of composite events that are mutually exclusive, exhaustive, and equally likely, we can consider $m\ (\leq n)$ such events. If $m_A\ (\leq nA)$ represents the number of mutually exclusive, exhaustive, and equally likely events favourable to A, then the probability of event A is given by:

$$P(A) = \frac{m_A}{m} = \frac{Number\ of\ mutually\ exclusive, exhaustive, and\ equally\ likely\ events\ favorable\ to\ A}{Total\ number\ of\ mutually\ exclusive, exhaustive, and\ equally\ likely\ events}$$

E.g.: Consider the rolling of a dice once. The sample space S is given by $S = \{1, 2, 3, 4, 5, 6\}$.

We define events A, B and C as follows:

A: The event of getting an even number: $A = \{2, 4, 6\}$

B: The event of getting an odd number: $B = \{1, 3, 5\}$

C: The event of getting a multiple of 3: $C = \{3, 6\}$

Calculation of Probabilities

Now, let's apply the classical definition of probability to the given example:

- $P(A)$ is the probability of getting an even number. Since there are 3 even numbers in S

  And a total of 6 sample points, we have $P(A) = \frac{3}{6} = \frac{1}{2}$
- $P(B)$ is the probability of getting an odd number. There are 3 odd numbers in S, so $P(B) = \frac{3}{6} = \frac{1}{2}$
- P(C) is the probability of getting a multiple of 3. Among the 6 sample points, 2 of them are multiples of **3.** Therefore, $P(C) = \frac{2}{6} = \frac{1}{3}$
- Also, since $P(A \cap B) = 0$ since $A \cap B = \{2, 4, 6\} \cap \{1, 3, 5\} = \emptyset$
  Thus, A and B are mutually exclusive events.

Q.1 A die is thrown, then the probability of getting a prime number is

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) None of these

Ans. (b)

Q.2 The probability that exactly one head appears in a single throw of two fair coins is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) None of these

Ans. (b)

Q.3 A bag contains 15 one-rupee coins, 25 two-rupee coins and 10 five-rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one-rupee coin is

(a) 0.30 (b) 0.70 (c) 0.25 (d) 0.20

Ans. (b)

Q.4 Three coins are tossed together. The probability of getting three tails is

(a) 5/8 (b) 3/8 (c) 1/8 (d) None of these

Ans. (c)

Q.5 A coin is tossed three times. What is the probability of getting:

(I) 2 tails (II) at least 2 tails

(a) $\frac{3}{8}, \frac{5}{8}$ (b) $\frac{3}{8}, \frac{3}{8}$ (c) $\frac{3}{8}, \frac{1}{8}$ (d) None of these

Ans. (d)

Some key points related to the classical definition of probability:

1. The probability of an event lies between 0 and 1, inclusive. It cannot be negative or greater than 1.

2. The non-occurrence of event A is denoted by A' or A^c or $\bar{A}$ and is known as the complementary event of A. The event A and its complementary event A' from a set of mutually exclusive and exhaustive events.

3. The ratio of the number of favourable events to the number of unfavourable events is known as the odds in favour of event A. Its inverse ratio is known as the odds against event A. i.e. If 'p' be the number of favourable outcomes of an event and 'q' be the number of unfavourable outcomes of the event, then

$Probability = \frac{p}{p+q}$ *where* $\frac{p}{q}$ is the odds in favour of event and $\frac{q}{p}$ is the odds against the event.

Q.6 if $P(A) = \frac{5}{9}$, then the odds against the event A is

(a) 5:9 (b) 5:4 (c) 4:5 (d) 5:14

Ans. (c)

Q.7 Three unbiased coins are tossed simultaneously, then the probability of getting at least 2 heads is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

Ans. (b)

Q.8 A dice is rolled twice. What is the probability of getting a difference of 5 points?

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{9}$ (d) None of these

Ans. (a)

Q.9 Find the probability that a four-digit number comprising the digits 2, 5, 6 and 7 would be divisible by 4

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{24}$ (d) None of these

Ans. (b)

Q.10 A committee of 6 members is to be formed from a group comprising 6 gentlemen and 4 ladies. What is the probability that the committee would comprise:

(I) 3 ladies (II) at least 3 ladies

(a) $\frac{8}{21}, \frac{19}{42}$ (b) $\frac{24}{110}, \frac{19}{42}$ (c) $\frac{1}{21}, \frac{5}{21}$ (d) None of these

Ans. (a)

Q.11 A coin in tossed twice, what is the probability that at least one tail occurs?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) 1

Ans. (c)

Q.12 Two broad divisions of probability are

(a) Subjective probability and objective probability

(b) Deductive probability and non- deductive probability

(c) Statistical probability and Mathematical probability

(d) None of these

Ans. (a)

Q.13 A die is thrown, then the probability of getting a number greater than or equal to 3 is

(a) $\frac{4}{5}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

Ans. (d)

Q.14 A committee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise 2 ladies?

(a) $\frac{37}{210}$ (b) $\frac{24}{110}$ (c) $\frac{140}{429}$ (d) None of these

Ans. (c)

Q.15 Three events, A, B and C are mutually exclusive, exhaustive and equally likely. What is the probability of the complementary event of $(A' - B)$?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) None of these

Ans. (b)

Q.16 Two dice are thrown simultaneously. What is the probability that the sum of the numbers rolled in a prime number?

(a) $\frac{5}{12}$ (b) $\frac{1}{4}$ (c) $\frac{7}{12}$ (d) 1

Ans. (a)

THIS CLASSICAL DEFINITION OF PROBABILITY HAS THE FOLLOWING DEMERITS OR LIMITATIONS

1. It is applicable only when the total number of events is finite
2. It can be used only when the events are equally likely orequi-probable. This assumption is made well before the experiment is performed.
3. This definition has only a limited field of application like coin tossing, dice throwing, drawing cards etc. where the possible events are known well in advance. In the field of uncertainty or where no prior knowledge is provided, this definition is inapplicable.

RELATIVE FREQUENCY DEFINITION OF PROBABILITY

KEY POINTS TO STUDY

- Concept of Relative frequency was first developed by the British mathematicians in connection with the survival probability of a group of people.
- If a random experiment is performed by repeating "n times" (under an identical set of condition) then the probability of an event A can be defined as limiting value of the ratio of event occurrence (f_A) to number of times experiment is being repeated (n)

$$i.e.\ P(A) = \lim_{n\to\infty} \frac{f_A}{n}$$

This statistical definition is applicable if the above limit exists and tends to a finite value.

Q.17 The following data relate to the distribution of salary of a group of employee:

Salary (thousand `)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Workers	12	23	24	32	19	11	8

If an employee is selected at random from the entire group of employee, what is the probability that

(I) His Salary would be less than `30,000?

(a) 0 (b) 0.5 (c) 3 (d) 0.7

Ans. (a)

(II) His Salary would be less than `60,000?

(a) 0.3 (b) 0.457 (c) 0.8 (d) 0.44

Ans. (b)

(III) His Salary would be more than `100,000?

(a) 0 (b) 0.4 (c) 0.2 (d) 0.55

Ans. (a)

(IV) His salary would be between `40,000 and `80,000?

(a) 0.66 (b) 0.759 (c) 0.99 (d) 0.1

Ans. (b)

AXIOMATIC OR MODERN DEFINITION OF PROBABILITY

The axiomatic or modern definition of probability states that for a sample space S and an event A defined on S, the probability of A, denoted a $P(A)$, is determined by the following axioms:

1. The probability of any event A is always greater than or equal to zero, i.e., $P(A) \geq 0$ for every $A \sqsubseteq S$ (subset).
2. The probability of the entire sample space S is equal to 1, i.e., $P(S) = 1$
3. For any sequence of mutually exclusive events $A_1, A_2, A_3, \ldots..$, the probability of their union is equal to the sum of their individual probabilities, i.e.,

   $P(A_1 \cup A_2 \cup A_3 \ldots) = P(A_1) + P(A_2) + P(A_3) + \cdots$

ADDITION THEOREMS OR THEOREMS ON TOTAL PROBABILITY

- **THEOREM 1:** For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B. i.e. $P(A \cup B)$ or $P(A + B) = P(A) + P(B)$ ..... or $P(A \; or \; B)$ whenever A and B are mutually exclusive.

  E.g.: Let's consider the events A and B representing the outcomes of rolling a fair six-sided die. If A is the event of getting an even number $(2, 4 \; or \; 6)$, and B is the event of getting an odd number $(1, 3 \; or \; 5)$, then A and B are mutually exclusive. The probability of either getting an even number or an odd number is given by:

  $P(A \cup B) = P(A) + P(B) = \frac{3}{6} + \frac{3}{6} = 1$

This means that the probability of getting either an even number or an odd number when rolling the die is 1, which is the total probability of the entire sample space.

Q.18If A and B are two mutually exclusive events such that $P(A \cup B) = \frac{2}{3}, P(A) = \frac{2}{5}, then\ P(B) =$

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{15}$

Ans. (a)

Q.19A number is selected from the first 20 natural numbers. What is the probability that is would be divisible by 3 or 8?

(a) 0 (b) 0.4 (c) 0.33 (d) 0.75

Ans. (b)

- **THEOREM 2:** For any $k(\geq 2)$ mutually exclusive events $A_1, A_2, A_3 \ldots, A_k$ then probability that at least one of the them occurs is given by the sum of the individual probabilities of the K events.
  i.e., $P(A_1 \cup)A_2 \cup A_3 \cup \ldots \cup A_k) = P(A_1) + P(A_2) + \cdots + P(A_k)$
- **THEOREM 3:** Fir any two events A and B, the probability that either A or B occurs is given by the sum of individual probability of A and B less the probability of simultaneous occurs of the events A and B
  I.e., $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Q.20A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 3 or 7?

(a) 0 (b) 0.426 (c) 0.33 (d) 0.75

Ans. (b)

- **THEOREM 4.** For any three events A, B and C, the probability that at least one of the events occurs is given by:
  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Q.21There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(a) 1 (b) 0.9 (c) 0.5 (d) None of these

Ans. (b)

Q.22Which of the following pairs of events are mutually exclusive?

(a) A: The team wins the football match.
B: The team lost the football match.

(b) A: The card drawn is a heart.
B: The card drawn is a red card.

(c) A: Anita is 20 years old.
B: She is a great dancer.

(d) A: The dice shows an even number.
B: The dice shows a prime number.

Ans. (a)

Q.23If two events A and B, $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ *and* $P(A \cup B) = \frac{2}{3}$ *then find* $P(A \cap B)$

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Ans. (b)

Q.24If A and B are mutually exclusive events, then

(a) $P(A) = P(A - B)$ (b) $P(B) = P(A - B)$

(c) $P(A) = P(A \cap B)$ (d) $P(B) = P(A \cap B)$

Ans. (a)

Q.25A coin is tossed thrice. What is the probability of getting 2 or more tails?

(a) 0 (b) 0.4 (c) 0.5 (d) 0.75

Ans. (c)

Q.26A certain problem has odds of 5 to 2 against A solving it, and odds of 3 to 1 in favor of B solving it. What is the probability of the problem being solved if both A and B attempt it?

(a) $\frac{13}{14}$ (b) $\frac{15}{28}$ (c) $\frac{9}{14}$ (d) None of these

Ans. (a)

Q.27An event that can be split into further events is known as

(a) Complex event (b) Mixed event

(c) Simple event (d) Composite event

Ans. (d)

Q.28Which of the following pairs of events are mutually exclusive?

(a) A: The student reads in a school. B: He studies Philosophy.

(b) A: Raju was born in India. B: He is a fine Engineer.

(c) A: Ruma is 16 years old. B: She is a good singer.

(d) A: Peter is under 15 years of age. B: Peter is a voter of Kolkata.

Ans. (b)

Q.29If a number is selected at random from the first 50 natural numbers, what will be the probability that the selected number is a multiple of 3 and 4?

(a) 5/50 (b) 2/25 (c) 3/50 (d) 4/25

Ans. (b)

Q.30The probability that an applicant for an Accountant's job has a B.Com. degree is 0.75. The probability that they have knowledge of Tally is 0.40. The probability that they have a B.Com. degree or knowledge of Tally is 0.30. Out of 1000 applicants, how many would be B.Com. experts in Tally?

(a) 0 (b) 850 (c) 1150 (d) 750

Ans. (b)

Q.31A, B, C are three mutually independent with probabilities 0.3, 0.2 and 0.4 respectively.

What is $P(A \cap B \cap C)$?

(a) 0.400 (b) 0.240 (c) 0.024 (d) 0.500

Ans. (c)

Q.32There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years is 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

(a) 1.00 (b) 0.28 (c) 0.45 (d) 0.90

Ans. (d)

Q.33A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6?

(a) 0.30 (b) 0.25 (c) 0.20 (d) 13

Ans. (d)

CONDITIONAL PROBABILITY AND COMPOUND THEOREM OF PROBABILITY

COMPOUND PROBABILITY OR JOINT PROBABILITY

The probability of an event, discussed so far, is technically known as unconditional or marginal probability. But if there are two or more events occurring simultaneously, how to calculate the probability.

The probability of occurrence of two events A and B simultaneously is known as the Compound Probability or Joint Probability of the events A and B and is denoted by $P(A \cup B)$.

In a similar manner, the probability of simultaneous occurrence of k events $A_1, A_2, A_3 \ldots, A_K$, is denoted by $P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_K)$

CONDITIONAL PROBABILITY

Let A and B be two events and S be the sample space, then the probability of event B given that event A has already occurred is called the conditional probability of B given A.

It is denoted by $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

E.g.: Let two unbiased coins be tossed then Samples space, $S = \{HH, HT, TH, TT\}$

Now,

Let A be the event of getting at least one head = $HH, HT, TH\}$

B be the event of getting both head = $\{HH\}$

then, $P(A) = \frac{3}{4}$ *and* $P(B) = \frac{1}{4}$

If it is known that A has already happened, then it is sure that TT cannot occur.

Thus, $P\left(\frac{B}{A}\right) = \frac{1}{3}$

Also, $\frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

Similarly, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

Q.34If $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cap B) = \frac{1}{6}$, then $P\left(\frac{A}{B}\right)$ is

(a) $\frac{1}{6}$ (b) $\frac{2}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

Ans. (b)

Q.35If $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$, then $P\left(\frac{B}{A}\right)$ is

(a) $\frac{1}{6}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

Ans. (c)

Q.36If $P(\overline{A \cup B}) = \frac{5}{6}, P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$, what is $P(A \cup B)$?

(a) 1 (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

Ans. (a)

INDEPENDENT EVENTS

Independent events are events in which the occurrence or outcome of one event does not affect the probability of the other event. The probability of the second event happening is the same whether the first event has occurred or not.

E.g.: Coin Flips Suppose you flip a fair coin twice. The outcome of the first coin flip (e.g., getting heads) has no influence on the outcome of the second coin flip. The probability of getting heads on the second flip remains $\frac{1}{2}$, regardless of whether you got heads or tails on the first flip. These events are independent.

Thus, if A and B are two independent events, then

$P\left(\frac{A}{B}\right) = P(A) and\ P\left(\frac{B}{A}\right) = P(B)$

If A and B are independent events, then

$P \cap (A \cap B) = P(A) \times P(B)$

Similarly,

- $P(A \cap C) = P(A) \times P(C)$
- $P(B \cap C) = P(B) \times P(C)$
- $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

If events A and B are independent, the pairs A and B', A' and B, and A' and B' are also independent, where A' represents the complement of event A and B' represents the complement of event B.

Q.37For any two events A_1, A_2 : Let $P(A_1) = \frac{2}{3}, P(A_2) = \frac{3}{8}, P(A_1 \cap A_2) = \frac{1}{4}$, then A_1, A_2 are

(a) Mutually Exclusive but not independent events

(b) Mutually Exclusive and independent events

(c) Independent but not mutually exclusive

(d) None of these

Ans. (c)

Q.38If events A and B are given to be independent such that $P(A) = 0.2, P(A \cup B = 0.6, then\ P(B) is$

(a) 0.4 (b) 0.5 (c) 0.7 (d) None of these

Ans. (b)

Q.39A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem will be solved?

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{5}$ (d) None of these

Ans. (b)

Q.40Ronaldo is known as one of the football players to hit 4 goals out of 10 shots whereas Messi is known to hit 5 goals out of 11 shots. What is the probability that the target would be hit once they both have hit for the penalty shootout?

(a) 0 (b) 0.67 (c) 0.33 (d) 0.75

Ans. (b)

DEPENDENT EVENTS

Dependent events are events in which the outcome of one event does affect the probability of the other event. The probability of the second event happening is influenced by the outcome of the first event.

E.g.: Drawing Cards

Suppose you have a deck of 52 playing cards. If you draw a card and do not replace it before drawing the next card, these events are dependent. For example, if you draw an Ace of Spades as the first card, there are now only 51 cards left in the deck, and the probability of drawing another Ace of Spades as the second card is $\frac{1}{51}$, not $\frac{1}{52}$ as it was initially. The probabilities change because the events are dependent on each other.

THEOREMS OF COMPOUND PROBABILITY

- **Theorem 5:** For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred i.e.,

  $P(A \cap B) = P(A) \times P(\frac{B}{A}), provided\ P(A) > 0$

- **Theorem 6:** For any three events A, B and C, the probability that they occur jointly is given by

  $P(A \cap B \cap C) = P(A) \times P(\frac{B}{A}) \times P(\frac{C}{A \cap B}), provided\ P(A \cap B) > 0$

Q.41If for two events A and B, $P(A \cap B) \neq P(A) \times P(B)$, then the two events A and B are

(a) Independent (b) Dependent

(c) Not equally likely (d) Not exhaustive

Ans. (b)

Q.42In a group of 15 males and 10 females, 5 males and 7 females are service holders. What is the probability that a person selected at random from the group is not a service holder given that the selected person is a female?

(a) 0 (b) 0.5 (c) 0.3 (d) 0.75

Ans. (c)

Q.43A card is drawn from a pack of 52 cards, the card drawn is a red card. What is the probability of its being a card of diamond?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{5}$

Ans. (b)

Q.44A pair of dice is thrown together and the sum of points of the two dice is noted to be 9. What is the probability that one of the two dice has shown the point 4?

(a) 0 (b) 0.5 (c) 0.33 (d) 0.75

Ans. (b)

Q.45Events A and B are given to be independent such that $P(A) = 0.40$ *and* $P(A \cup B) = 0.70$ then P(B)=

(a) 0.2 (b) 0.4 (c) 0.5 (d) 1

Ans. (c)

Q.46In connection with a random experiment, it is found that

$P(A) = \frac{1}{3}, P(B) = \frac{2}{5}$ *and* $P(A \cup B) = \frac{3}{4}$

Evaluate the following probabilities:

(I) $P(A/B)$ Ans. 1318

(II) $P(B/A)$ Ans. 1320

(III) $P(A'/B)$ Ans. 518

(IV) $P(A/B')$ Ans. 712

(V) $P(A'/B')$ Ans. 512

Q.47The probability of the occurrence of a number greater than 2 in a throw of a die if it is known that only even numbers can occur is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) None of these

Ans. (c)

Q.48Given that for two events A and B, $P(A) = \frac{3}{5}, P(B) = \frac{2}{3}$ *and* $P(A \cup B) = \frac{3}{5}$, *what is* $P\left(\frac{A}{B}\right)$?

(a) 0.655 (b) $\frac{13}{60}$ (c) $\frac{60}{31}$ (d) 0.775

Q.49If $P(A \cap B) = 0$, then the two events A and B are

(a) Mutually exclusive (b) Exhaustive

(c) Equally likely (d) Independent

Ans. (a)

Q.50If A, B and C are mutually exclusive, independent and exhaustive events then what is the probability that they occur simultaneously?

(a) 1 (b) 0.50 (c) 0 (d) any value between 0 and 1

Ans. (c)

Q.51If for two independent events A and B, $P(A \cup B) = \frac{2}{3}$ *and* $P(A)$ $\frac{2}{5}$, *what is* $P(B)$?

(a) $\frac{4}{15}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{15}$

Ans. (a)

Q.52For three events A, B and C, the probability that only A occur is

(a) $P(A)$ (b) $P(A \cup B \cup C)$

(c) $P(A' \cap B \cap C)$ (d) $P(A \cap B' \cap C')$

Ans. (d)

Q.53The probability that is leap year has 53 Monday is:

(a) $\frac{1}{7}$ (b) $\frac{2}{3}$ (c) $\frac{2}{7}$ (d) $\frac{3}{5}$

Ans. (c)

Q.54There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of the same colour?

(a) $\frac{89}{729}$ (b) $\frac{97}{729}$ (c) $\frac{82}{729}$ (d) $\frac{23}{32}$

Ans. (a)

Q.55A problem in probability was given to three CA students A, B and C whose chance of solving it are $\frac{1}{3}, \frac{1}{5}$ *and* $\frac{1}{2}$ respectively. What is the probability that the problem would be solved?

(a) $\frac{4}{15}$ (b) $\frac{7}{8}$ (c) $\frac{8}{15}$ (d) $\frac{11}{15}$

Ans. (d)

Q.56For a group of subjects 30%, 40% and 50% failed in Physics, Chemistry and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Ans. (a)

Q.57An article consists of two parts A and B. The manufacturing process of each part is such that probability of defect in A is 0.08 and that B is 0.05. What is the probability that the assembled product will not have any defect?

(a) 0.934 (b) 0.864 (c) 0.85 (d) 0.874

Ans. (d)

Q.58If for two events A and B, $P(A \cap B) \neq P(A) \times P(B)$, then the two events A and B are

(a) Independent (b) Dependent

(c) Not equally likely (d) Not exhausted

Ans. (b)

Q.59If $P\left(\frac{A}{B}\right) = P(A)$, *then*

(a) A is independent of B (b) B is independent of A

(c) B is dependent of A (c) Both (a) and (b)

Ans. (a)

Q.60If $P(A - B) = P(B - A)$, then the two events A and B satisfy the condition

(a) $P(A) = P(B)$ (b) $P(A) + P(B) = 1$

(c) $P(A \cap B) = 0$ (d) $P(A \cup B) = 1$

Ans. (a)

Q.61If $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}$ *and* $P(A \cup B) = \frac{11}{12}$, *then* $P\left(\frac{B}{A}\right)$ is

(a) $\frac{1}{6}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

Ans. (c)

Q.62A machine is made of two parts A and B. The manufacturing process of each part is such that probability of defective in part A is 0.08 and that B is 0.05. What is the probability that the assembled part will not have any defect?

(a) 0.934 (b) 0.864 (c) 0.85 (d) 0.874

Ans. ()

Q.63IF $P\left(A \cup \bar{B}\right) = \frac{5}{6}, P\left(\bar{A}\right) = \frac{1}{2}$ *and* $P\left(\bar{B}\right) = \frac{2}{3}$, *what is* $P(A \cup B)$

(a) 1 (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

Ans. (a)

Q.64In a class 40% students read Mathematics, 25% Biology and 15% both Mathematics and Biology. One student is selected at random. The probability that he reads Mathematics if it is known that he reads Biology is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) None of these

Ans. (b)

RANDOM VARIABLE - PROBABILITY DISTRIBUTION

A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.

A random variable is denoted by a capital letter.

E.g.: Consider the experiment of tossing a coin three times. Let X represent the number of heads obtained. In this case, the sample space is given by:

$\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$.

The different values of X would be:

X = 0 (no heads)

X = 1 (one head)

X = 2 (two heads)

X = 3 (three heads)

TYPE OF RANDOM VARIABLE

Discrete Random Variable	Continuous Random Variable
A random variable from the discrete sample space is called a discrete random variable. For example, consider the experiment of tossing a coin three times. Let X represent the number of heads obtained. In this case, the sample space is given by: $\{HHH, HHT, HTH, THH, TTH, TTT\}$ The different values of X would be: X= 0 (No heads) X=1 (one head) X=2 (two heads) X=3 (three heads)	A random variable from the continuous sample space is called a discrete random variable. For example, consider the experiment of measuring the height of a randomly selected person. Let X represent the height in centimeters. The values of X would be any real number within a certain range, such as 150 cm to 200 cm. Since the height can take on any value within the interval, X is a continuous random variable.

PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE

It may be defined as a statement where we take different values for random variables with their corresponding probabilities. Writing it mathematically, if a random variable X assumes n finite values $X_1, X_2, X_3, \dots, X_n$ with corresponding probabilities $P_1, P_2, P_3, \dots, P_n$ such that

1. $P_i \geq 0$ (for every i)
2. $\sum P_i = 1$(over all i)

Then the probability distribution of the random variable X is given by

X:	X_1	X_2	X_3	...	X_n	Total
P:	P_1	P_2	P_3	...	P_n	1

For example, if an unbiased coin is tossed three times and if X denoted the number of heads then, as we have already discussed, X is a random variable and its probability distribution is given by:

Probabilities Distribution of X (Number of heads when a coin is tossed thrice)

X:	0	1	2	3	Total
P:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

PROBABILITY MASS FUNCTION (PMF) OF X

If a function $f(x)$ exists which defines the probability (P) as a function of X, where X is discrete random variable, where $f(X)$ satisfies the below given condition:

1. $f(x) \geq 0$ for every X
2. $\sum f(x) = 1$

Where, $f(X)$ is given by $f(X) = P(X = x)$

Q.65Which of the following set of function define a probability space on $S = \{a_1, a_2, a_3\}$?

(a) $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{2}, P(a_3) = \frac{1}{4}$

(b) $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{6}, P(a_3) = \frac{1}{6}$

(c) $P(a_1) = \frac{2}{3}, P(a_2) = \frac{2}{3}, P(a_3) = \frac{1}{4}$

(d) None of these

Ans. (b)

Q.66Let P be a probability function on $S = \{X_1, X_2, X_3\}$ if $P(X_1) = \frac{1}{4}, P(X_3) = \frac{1}{3}$ then $P(X_2)$ is

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{3}{4}$ (d) None of these

Ans. (a)

Q.67The probability distribution of a random variable is as follows:

X	1	2	3	4	5
P	3k	2k	3k	k	k

Find the value of k and $P(x \leq 3)$

(a) $\frac{1}{10}, 0.2$ (b) $\frac{1}{10}, 0.5$ (c) $\frac{1}{10}, 1.5$ (d) None of these

Ans. (b)

Q.68A random variable X taking values 0, 1, 2 has the following probability distribution for some number k.

$P(X) = k \; if \; X = 0$

$= 2k \; if \; X = 1$

$= 3k \; if \; X = 2$

Find the value of k.

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) None of these

Ans. (c)

Q.69A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	2k	3k	k	2k	k^2	$7k^2$	$2k^2 + k$

Find the value of k.

(a) 10 (b) -1 (c) $\frac{1}{10}$ (d) None of these

Ans. (c)

PROBABILITY DENSITY FUNCTION (PDF)

When x is a continuous random variable defined over an interval $[\alpha, \beta]$, where $\beta > \alpha$, then x can assume an infinite number of values from its interval. And in such cases we assign intervals of values to probability rather than assigning individual probability to every mass point x.

Then, if a function $f(x)$ exists such that it defines the probability. It will be called as probability density function if it satisfies the below given condition:

1. $f(x) \geq 0 for \; x \in [\alpha, \beta]$
2. and if the probability that x lies between two specified values a and b where $\alpha \leq a \leq b \leq \beta$ given by $\int_a^b f(x)dx$

Q.70If $f(x) = \{^{cx^2, 0 < x < 1}_{0, elsewhere}\}$, elsewhere is the probability density function of a continuous random variable X, find the value of c.

(a) 3 (b) 2 (c) 1 (d) None of these

Ans. (a)

EXPECTED VALUE OF A RANDOM VARIABLE

Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

Note: Expected value gives the mean of all values.

Hence, if a random variable x assumes n values $x_1, x_2, x_3, \ldots, x_n$ with corresponding probabilities

$p_1, p_2, p_3, \ldots, p_n$, where p_i satisfies

1. $p_i(x_i) \geq 0$ for every X
2. $\sum p_i x_i = 1$

Then the expected value of x is given$\mu = E(x) = \sum p_i x_i$

Expected value of x^2 is given by $E = (x^2) = \sum p_i x_i^2$

In particular expected value of a monotonic function $g(x)$ is given by $E[g(x)] = \sum p_i E(x_i)$

Variance of x, to be denoted by, σ^2 is given by

$V(x) = \sigma^2 = E(x-\mu)^2 = E(x^2) - \mu^2$

The positive square root of variance is known as standard deviation and is denoted by σ if $y = a + bx$, for two random variables x and y and for a pair of constants a and b, then the mean i.e., expected value of y is given by:

$\mu_y = a + b\mu_x$

And the standard deviation of y is given by:

$\sigma_y = |b| \times \sigma_x$

Now, when x is discrete random variable and $f(x)$ is the probability mass function, then the expected value (i.e. Mean) is given by

$\mu = E(x) = \sum xf(x)$

Its variance is given by $\sigma^2 = E(x^2) - \mu^2$, where, $E(x^2) = \sum x^2 f(x)$

If x is continuous random variable defined in $(-\infty, \infty)$, then the expected value is given by $E(x) = \int_{-\infty}^{\infty} xf(x)dx$ *and* $\sigma^2 = E(x^2) - \mu^2$

Where,$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

PROPERTIES OF EXPECTED VALUES

1. Expectation of a constant k is k i.e., if all values of x is equal to k, then the expected value will be equal to

   $E(k) = k$ for any constant k.
2. Expectation of sum of two random variables is the sum of their expectations.

   i.e., $E(x+y) = E(x) + E(y)$ for any two random variables x and y
3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.

   i.e., $E(kx) = k.E(x)$ for any constant k.
4. Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.

   i.e., $E(xy) = E(x) \times E(y)$ where x and y are independent.

Q.71If x and y are random variables having expected values as 4.5 and 2.5 respectively, then the expected value of $(x - y)$ is

(a) 2 (b) 7 (c) 6 (d) 0

Ans. (a)

Q.72The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively. If A, B and C prepare 60, 70 and 90 such statements, then the expected number of correct statement is

(a) 170 (b) 176 (c) 178 (d) 180

Ans. (c)

Q.73A wholesaler can make a profit of `12,000 or incur a loss of `8,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.8 and 0.2 respectively. What is his expected profit?

(a) `8,000 (b) `12,000 (c) `4,000 (d) None of these

Ans. (a)

Q.74A number is selected at random from a set containing the first 50 natural numbers and another number is selected at random from another set containing the first 100 natural numbers. What is the expected value of

(I) the sum (II) the product

(a) $76, \frac{5151}{2}$ (b) $66, \frac{8151}{2}$ (c) $50, \frac{9158}{2}$ (d) $45, \frac{5190}{2}$

Ans. (a)

Q.75A random variable has the following probability distribution:

X:	4	5	7	8	10
P:	0.15	0.20	0.40	0.15	0.10

Find $E[X - E(X)]^2$. Also obtain $v(3x - 4)$.

(a) 3.04, 17.36 (b) 3.04, 27.36

(c) 4.15, 37.65 (d) None of these

Ans. (b)

Q.76A box contains 10 tube lights of which 3 are defective. A man select four tube lights at random. What is the expected number of defective tube lights in his selection?

(a) 1.21 (b) 1.69 (c) 1.71 (d) 1.20

Ans. (d)

Q.77Raju draws 2 balls from a bag containing 3 white and 5 Red balls. He gets `500 if he draws a white ball and `200 if he draws a red ball. What is his expectation? But here are the twists, for each game he has to pay `400 for participating in the game, would he consider it a fair game and participate?

(a) Yes, the game is fair and should participate

(b) No, the game is not fair

(c) Cannot be determined

(d) None of these

Ans. (a)

Q.78A dice is thrown repeatedly till a '1' appears. Write down the sample space. Also find the expected number of throws.

(a) 0 (b) 1 (c) 6 (d) None of these

Ans. (c)

Q.79A random variable x has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	$2k$	$3k$	K	$2k$	k^2	$7k^2$	$2k^2 + k$

Find:

(I) $P(x \leq 2)$ (II) $P(x > 3)$ (III) $P(3 \leq x < 6)$

(a) 0.50, 0.40, 0.30 (b) 0.40, 0.40, 0.30

(c) 0.50, 0.40, 0.30 (d) 0.50, 0.40, 0.30

Ans. (d)

Q.80If all the values taken by a random variable are equal then

(a) its expected value is zero

(b) its standard deviation is zero

(c) its standard deviation is positive

(d) its standard deviation is a real number

Ans. (b)

Q.81If x and y are independent, then

(a) $E(xy) = E(x)\ x\ E(y)$

(b) $E(xy) = E(x) + E(y)$

(c) $E(x - y) = E(x) + E(y)$

(d) $E(x - y) = E(x) + x\ E(y)$

Ans. (a)

Q.82If two random variables x and y are related by y = 2 – 3x, then the SD of y is given by

(a) $-3 \times SD\ of\ x$ (b) $3 \times SD\ of\ x$

(c) $9 \times SD\ of\ x$ (d) $2 \times SD\ of\ x$

Ans. (b)

Q.83If an unbiased die is rolled once, the odds in favour of getting a point which is a multiple of 3 is

(a) 1:2 (b) 2:1 (c) 1:3 (d) 1

Ans. (a)

Q.84if a random variable x assumes the value 0, 1 and 2 with probabilities 0.30, 0.50 and 0.20, then its expected value is

(a) 1.50 (b) 3 (c) 0.90 (d) 1

Ans. (c)

Q.85When 2 – dice are thrown simultaneously then the probability of getting at least one 5 is

(a) $\frac{11}{36}$ (b) $\frac{5}{36}$ (c) $\frac{8}{15}$ (d) $\frac{1}{7}$

Ans. (a)

Q.86If a coin is tossed 5 times then the probability of getting tail and head occurs alternatively is

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{1}{64}$

Ans. (b)

Q.87Two event A and B are such that do not occurs simultaneously then they are called_____events

(a) Mutually exhausted (b) Mutually exclusive

(c) Mutually independent (d) Equally likely

Ans. (b)

Q.88Ram is known to hit a target in 2 out of 3 shots whereas Shyam is known to hit the same target in 5 out of 11 shots. What is the probability that the target would be hit if they both try?

(a) $\frac{9}{11}$ (b) $\frac{3}{11}$ (c) $\frac{10}{33}$ (d) $\frac{6}{11}$

Ans. (a)

Q.89If two random variable x and y are related by $y = 2 - 3x$, then the SD of y is given by

(a) $-3 \times SD\ of\ x$ (b) $3 \times SD\ of\ x$

(c) $9 \times SD\ of\ x$ (d) $2 \times SD\ of\ x$

Ans. (b)

Q.90Variance of random variable x is given by

(a) $E(X-\mu)^2$ (b) $E[(E-E(X)]^2$

(c) $E(X^2-\mu)$ (d) (a) or (b)

Ans. (d)

Q.91The theorem of compound probability states that for any two events A and B

(a) $P(A \cap B) = P(A) \times P(B/A)$ (b) $P(A \cup B) = P(A) \times P(B/A)$

(c) $P(A \cap B) = P(A) \times P(B)$ (d) $P(A \cup B) = P(A) \times P(B) - P(A \cap B)$

Ans. (a)

Q.92The term "chance" and probability are synonyms

(a) True (b) False (c) Both (d) None of these

Ans. (a)

Q.93Two broad divisions of probability are

(a) Subjective probability and objective probability

(b) Deductive probability and mathematical Probability

(c) Statistical probability and mathematical probability

(d) None of these

Ans. (a)

Q.94Given $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ *and* $P(A \cap B) = \frac{1}{4}$, the value of $P(A/B)$ is

(a) ½ (b) 1/6 (c) 2/3 (d) ¾

Ans. (d)

Q.95The Probability distribution of the demand for a commodity is given below:

Demand (x)	5	6	7	8	9	10
Probability [P(x)]	0.05	0.10	0.30	0.40	0.10	0.05

The expected value of demands will be

(a) 7.55 (b) 7.85 (c) 1.25 (d) 8.35

Ans. (a)

Q.96Let A and B are two events with $P(A) = \frac{2}{3}, P(B) = \frac{1}{4}$ *and* $P(A \cap B) = \frac{1}{12}$, *then* $P(B/A)$ will be

(a) 7/8 (b) 1/3 (c) 1/8 (d) 8/7

Ans. (c)

Q.97For two events, A, B let $P(A) = \frac{2}{3}, P(B) = \frac{3}{8}$ *and* $P(A \cap B) = \frac{1}{4}$ then A and B are

(a) Mutually exclusive but not independent

(b) Independence but not mutually exclusive

(c) Mutually exclusive and independent

(d) None of these

Ans. (b)

Q.98Three coins are tossed together, the probability of getting exactly two head is:

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{8}$ (d) None of these

Ans. (b)

Q.99If a random variable x assume the values x_1, x_2, x_3, x_4 with corresponding probabilities p_1, p_2, p_3, p_4; then the expected value of x is

(a) $p_1 + p_2 + p_3 + p_4$ (b) $x_1p_1 \times x_2p_3 \times x_3p_2 \times x_4p_4$

(c) $p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$ (d) None of these

Ans. (c)

Q.100If there are 20 cars, 14 of them are SUVs and 6 of them are sedans, then the probability of randomly selecting 4 cars that include 2 SUVs and 2 sedans is

(a) 0.04 (b) 0.17

(c) 0.28 (d) 0.23

Ans. (c)

Q. 1Let X be a random variable having following Probability distribution:

x	-3	6	9
P(x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$

(a) $\frac{11}{2}, \frac{93}{2}$ (b) $\frac{17}{2}, \frac{75}{2}$ (c) $\frac{11}{3}, \frac{97}{3}$ (d) None of these

Ans. (a)

PREVIOUS YEAR QUESTIONS

PROBABILITY

Q.1 In a survey of 100 boys it was found that 50 used white shirts, 40 red shirts and 30 blue shirts. 20 were habituated in using both white and red shirts. 15 were using both red and blue shirts and 10 were using blue and white shirts. Find the number of boys who are using all colours. [Dec. 2023]

(a) 20 (b) 25
(c) 30 (d) 35

Ans. (b) 25

Q.2 If P(A) = ½ and P(B) = 1/3 and P (A ∪ B) = 2 / 3 then find P (A ∩ B) [Dec. 2023]

(a) ¼ (b) 2/3
(c) 1/6 (d)1/2

Ans. (c) 1/6

Q.3 If six coins are tossed simultaneously. The probability of obtaining exactly two heads are. [Dec. 2023]

(a) 0.2343 (b) 0.9841
(c) 0.1268 (d) 0.0156

Ans. (a) 0.2343

Q.4 A box contain 20 electrical bulbs out of which 4 are defective. Two bulbs are chosen at random from this box. The probability that at least one of them is defective. [Dec. 2023]

(a) 7/19 (b) 4/19
(c) 12/19 (d) 15/19

Ans. (d) 15/19

Q.5 If a card is drawn at random from a pack of 52 cards, what is the chance of getting a Club or a King? [Dec. 2023]

(a)13/52 (b) 4/52
(c) 14/52 (d) 16/52

Ans. (b) 4/52

Q.6 A number is selected from the first 30 natural numbers. What is the probability that it would be divisible by 3 or 8? [Dec. 2023]

(a) 0.2 (b) 0.4
(c) 0.6 (d) 0.8

Ans. (b) 0.4

Q.7 A number is selected at random f rom the first 100 natural numbers. What is the probability that it would be a multiple of 3 or 7? [Dec. 2023]

(a) 33/100 (b) 4/100
(c) 21/100 (d) 43/100

Ans. (d) 43/100

Q.8 If the mean and median of a moderately asymmetrical series are 26.8 and 27.9 respectively, then the most probability mode is: [Dec. 2023]

(a) 35.4 (b) 30.1
(c) 34.3 (d) 70.8

Ans. (b) 30.1

Q.9 If mean and variance of a random variable which follows the Binomial Distribution are 7 and 6 respectively, then the probability of success is: [Dec. 2023]

(a) 6/7 (b) 36/49
(c) 1/7 (d) 1/49

Ans. (c) 1/7

Q.10 On a commodity exchange when booking trades with provision, the trader can make a profit of `50,000 or incur a loss of `20,000. The probability of making profit and loss, from the past experience, are known to be 0.75 and 0.25 respectively. The expected profit to be made by trader should be [June 2023]

(a) 32,500 (b) 35,000
(c) 30,000 (d) 37,500

Ans. (a) 32,500

Q.11 The probability that a four-digit number comprising the digits 2,5,6 and 7 without repetition of digits, would be divisible by 4 is [June 2023]

(a)1/2 (b) 3/4
(c) 1/4 (d) 1/3

Ans. (d) 1/3

Q.12 For any two events A and B it is know that $P(A) = 2/3, P(B) = 3/8$ and $P(A \cap B) = 1/4$, then the events A and B are [June 2023]

(a) Mutually Exclusive and Independent
(b) Mutually not Exclusive and Independent
(c) Mutually Exclusive But not Independent
(d) Neither Mutually Exclusive nor Independent

Ans. (b) Mutually not Exclusive and Independent

Q.13 The probability distribution of x is given below

X	1	0	Total
Probability	P	1 – P	1

[June 2023]

(a) P (b) 1-P
(c) 0 (d) 1

Ans. (a) P

Q.14Company A produces 10% defective products, company B produces 20% defective products and company C produces 5% defective products. If choosing a company is an equally likely event, what is probability that product chosen is free from defect? [June 2023]

(a) 0.88 (b) 0.80
(c) 0.79 (d) 0.78

Ans. (a) 0.88

Q.15Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The probability that exactly 2 of them are children is [June 2023]

(a) 10/21 (b) 1/12
(c) 1/5 (d) 1/9

Ans. (a) 10/21

Q.16 If a random variable X has the following probability distribution, then the expected value of X is: If a random variable X has the following probability distribution, then the expected value of X is:

X					
f(x)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{3}$

[June 2023]

(a) 3/2 (b) 1/2
(c) 1/6 (d) 1/5

Ans. (c) 1/6

Q.17 If $P(A) = 1/3, P(B) = 1/4$ and $P(A/B) = 1/6$, the probability P(B/A) is [June 2023]

(a) 1/8 (b) 1/4
(c) 3/8 (d) ½

Ans. (a) 1/8

Q.18 If three coins are tossed simultaneously, what is the probability of getting two heads together?

[Dec 2022]

(a) ¼ (b) 1/8
(c) 5/8 (d) 3/8

Ans. (d) 3/8

Q.19 If a number is selected at random from the first 50 natural numbers, what will be the probability that the selected number is a multiple of 3 and 4 ? [Dec 2022]

(a) 5/50 (b) 2/25
(c) 3/50 (d) 4/25

Ans. (b) 2/25

Q.20 The Theorem of Compound Probability states that for any two events A and B. [Dec 2022]

(a) $P(A \cap B) = P(A) \times P(\frac{B}{A})$ (b) $P(A \cup B) = P(A) \times P(\frac{B}{A})$

(c) $P(A \cap B) = P(A) \times P(B)$ (d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Ans. (a) $P(A \cap B) = P(A) \times P(\frac{B}{A})$

Q.21 Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let A' and B' be their complements. Which one of the following statements in FALSE? [Dec 2022]

(a) $P(A \cap B) = P(A)P(B)$ (b)) $P(\frac{A}{B}) = P(A)$

(c) $P(A \cup B) = P(A) + P(B)$ (d) $P(A' \cap B') = P(A')P(B')$

Ans. (c) $P(A \cup B) = P(A) + P(B)$

Q.22 The probability that a leap year has 53 Monday is : [Dec 2022]

(a) 1/7 (b) 2/3
(c) 2/7 (d) 3/5

Ans. (c) 2/7

Q.23 If $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$ *then* $P(\frac{B}{A})$ *is* : [Dec 2022]

(a) 1/6 (b) 4/9
(c) 1/½ (d) 1/8

Ans. (c) 1/2

Q.24 A machine is made of two parts A and B. The manufacturing process of each part is such that probability of defective in part A is 0.08 and that B is 0.05. What is the probability that the assembled part will not have any defect? [Dec 2022]

(a) 0.934 (b) 0.864
(c) 0.85 (d) 0.874

Ans. (d) 0.874

Q.25 The odds in favour of event A in a trial is 3:1. In a three independent trials, the probability of no occurrence of the event A is [June 2022]

(a) 1/64 (b) 1/32
(c) 1/27 (d) 1/8

Ans. (a) 1/64

Q.26 The odds in favour of an event A is 2 : 3 and odds against an event B is 6 : 4 the probability that only one of A and B occurs is y/25 where y is [June 2022]

(a) 12 (b) 15
(c) 18 (d) 9

Ans. (a) 12

Q.27 Thirty balls are serially numbered and placed in a bag. Find chance that the first ball drawn is a multiple of 3 or 5. [June 2022]

(a) 8/15 (b) 2/15
(c) ½ (d) 7/15

Ans. (d) 7/15

Q.28 Two balanced dice are rolled. The probability of getting 1 in at least one dice is x/36 where x is [June 2022]

(a) 12 (b) 1
(c) 11 (d) 2

Ans. (c) 11

Q.29 What is the chance that a leap year selected at random will contain 53 Fridays? [June 2022]

(a) 3/7 (b) 1/7
(c) 2/7 (d) 4/7

Ans. (c) 2/7

Q.30 If P (A) = 0.3, P (B) = 0.8 and P (B/A) = 0.5. Find P (A∪B). [June 2022]

(a) 0.7 (b) 0.95
(c) 0.60 (d) 0.59

Ans. (b) 0.95

Q.31 There are 3 boxes with the following composition :

Box I : 7 Red +5 White +4 Blue balls
Box II : 5 Red +6 White +3 Blue balls
Box III : 4 Red +3 White +2 Blue balls
One of the boxes is selected at random and a ball is drawn from it. What is the probability the drawn ball is red? [Dec 2021]

(a) 1249/3024 (b) 1247/3004
(c) 1147/3024 (d) ½

Ans. (a) 1249/3024

Q.32 In a group of 20 males and 15 females, 12 males and 8 females are service holders. What is the probability that a person selected at random from the group is a service holder given that the selected person is a male? [Dec 2021]

(a) 0.4 (b) 0.6
(c) 0.45 (d) 0.55

Ans. (b) 0.6

Q.33 Assume that the probability for rain on a day is 0.4 . An umbrella salesman can earn Rs. 400 per day in case of rain on that day and will lose Rs. 100 per day if there is no rain. The expected earnings (in Rs.) per day of the salesman is [Dec 2021]

(a) 400 (b) 200
(c) 100 (d) 0

Ans. (c) 100

Q.34 The probability that a football team loosing a match at Kolkata is 3/5 and winning a match at Bengaluru is 6/7, the probability of the team winning at least one match is [July 2021]

(a) 3/35 (b) 18/35
(c) 32/35 (d) 17/35

Ans. (c) 32/35

Q.35 A bag contains 7 Blue and 5 Green balls. One ball is drawn at random. The probability of getting a blue ball is [July 2021]

(a) 5/12 (b) 12/35
(c) 7/12 (d) 0

Ans. (c) 7/12

Q.36 If in a class, 60% of the student study Mathematics and Science and 90% of the student study Science, then the probability of a student studying Mathematics given that he/she is already studying Science is [July 2021]

(a) ¼ (b) 2/3
(c) 1 (d) ½

Ans. (b) 2/3

Q.37 If there are 48 marbles marked with numbers 1 to 48, then the probability of selecting a marble having the number divisible by 4 is [July 2021]

(a) ½ (b) 2/3

(c) 1/3 (d) ¼

Ans. (d) ¼

Q.38 If there are 16 phones, 10 of them are Android and 6 of them are of Apple, then the probability of 4 randomly selected phones to include 2 Android and 2 Apple phone is [July 2021]

(a) 0.47 (b) 0.51

(c) 0.37 (d) 0.27

Ans. (c) 0.37

Q.39 A biased coin is such that the probability of getting a head is thrice the probability of getting a tail. If the coin is tossed 4 times, what is the probability of getting a head all the times? [July 2021]

(a) 2/5 (b) 81/128

(c) 81/256 (d) 81/64

Ans. (c) 81/256

SUMMARY

- **Experiment:** An experiment may be described as a performance that produces certain results.
- **Random Experiment:** An experiment is defined to be random if the results of the experiment depend on chance only.
- **Events:** The results or outcomes of a random experiment are known as events. Sometimes events may be a combination of outcomes.
- **The events are of two types:**
  (i) Simple or Elementary,
  (ii) Composite or Compound.
- **Mutually Exclusive Events or Incompatible Events:** A set of events $A_1, A_2, A_3, \ldots\ldots$is known to be mutually exclusive if not more than one of them can occur simultaneously.
- **Exhaustive Events:** The events A_1, A_2, A_3, … are known to form an exhaustive set if one of these events must necessarily occur.
- **Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events:** The events of a random experiment are known to be equally likely when all necessary evidence is taken into account, no event is expected to occur more frequently as compared to the other events of the set of events.
- The probability of occurrence of the event is defined as the ratio of the number of events favorable to A to the total number of events. Denoting this by $P(A)$, we have
  $P(A) = \frac{No.of\ equally\ likely\ favorable\ events}{Total\ no.of\ equally\ likely\ events}$
- The probability of an event lies between 0 and 1, both inclusive i.e., $0 \leq P(A) \leq 1$.
- When $P(A) = 0$, A is known to be an impossible event and when $P(A) = 1$, A is known to be a sure event.

- Non – occurrence of event A is denoted By A' or A^c or $\bar{A}$ and it is known as complimentary event of A. The event A along with its complementary A' forms a set of mutually exclusive and exhaustive events.
- The ratio of no. of favorable events to the no. of unfavorable events is known as odds in favor of the event and its inverse ratio is known as odds against event A.
  i.e. odds in favor of $A = m_A : (m - m_A)$
  and odds in against $A = (m - m_A) : m_A$
- For any two mutually exclusive events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B.

  i.e. or $P(A + B) = P(A) + P(B)$
- For any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

  i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  Conditional Probability : $P(B/A) = \frac{P(A \cap B)}{P(A)}$
- For any two events A and B, the probability that A and B occur simultaneously is given by the product of the unconditional probability of A and the conditional probability of B given that A has already occurred i.e., Compound Probability or Joint Probability Provided.
- A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from R and assigning a real number to each and every sample point of the random experiment.
- Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

THEORETICAL DISTRIBUTION OF PROBABILITY

A probability distribution is a way of describing the probabilities of different outcomes in a random experiment or event. It is similar to a frequency distribution, where we distribute different frequencies over class intervals, but in a probability distribution, we distribute different probabilities.

1. In the case of a **Discrete random variable**, which takes on a finite or countable number of values, the probability distribution is typically represented by a probability mass function (PMF). The PMF gives the probability of each possible outcome.

   E. g.: Binomial Distribution and the Poisson Distribution

2. In the case of a **Continuous random variable**, which can take on any value within a certain range, the probability distribution is typically represented by a probability density function (PDF). The PDF gives the probability density at each point. The area under the PDF represents the probability of an event occurring within a certain range.

   E.g.: Normal Distribution
   As a probability distribution exists only in theory, it is called the theoretical distribution of Probability.

WHY DO WE NEED TO STUDY?

- We generally do sampling by frequency distribution, and based on the same samples if we need to find some results. We need to learn probability distribution.
- For example: Finding the age of different tube lights
- Theoretical probability distribution will help you create future projections.
- For Example: Finding the projection of profitability of tube lights in next 10 years
- Statistical analysis is possible only on the basis of theoretical probability distribution.
- A probability distribution also possesses all the characteristics of an observed distribution.

Like finding mean (μ), median (μ), mode(μ_0), standard deviation (σ) etc. exactly the same way we have done earlier.

THINGS TO STUDY

- Discrete Probability Distributions:
  - Binomial Distribution
  - Poisson Distribution
- Continuous Probability Distributions:
  - Normal Distribution

BINOMIAL DISTRIBUTION

- It is derived from a particular type of random experiment known as Bernoulli process named after the famous mathematician Bernoulli.
- When 'trial' is attempted to produce a particular outcome which is neither certain nor impossible.
- The characteristics of Bernoulli trials are stated below:

(a) Trial is associated with two mutually exclusive and exhaustive outcomes

(b) The occurrence of one of which is known as a 'success' and as such its non occurrence as a 'failure'.

As an example, flipping a coin multiple times and counting the number of heads.

- The trial is independent.
- The probability of a success, usually denoted by p, and hence that of a failure, usually denoted by $q = 1 - p$, remain unchanged throughout the process
- The number of trials is a finite positive integer.

Under such conditions, A discrete random variable x is defined to follow binomial distribution with parameters n and p, to be denoted by $x \sim B(n, p)$, if the probability mass function of x is given by

$f(x = P(X = x) = {}^{n}C_{x}(p)^{2}(q)^{n-x} or\ x = 0, 1, 2, \dots, n$

$= 0$ otherwise

Q.1 What is the probability of getting 3 heads if 6 unbiased coins are tossed simultaneously?

(a) 0.50 (b) 0.25 (c) 0.3125 (d) 0.6875

Ans. (c)

Q.2 What is the probability of making 3 correct guesses in 5 True – False answer type question?

(a) 0.3125 (b) 0.5676 (c) 0.6875 (d) 0.4325

Ans. (a)

IMPORTANT POINTS IN CONNECTION WITH BINOMIAL DISTRIBUTION

1. As $n > 0$, $p, q \geq 0$, it follows that $f(x) \geq 0$ for every x. Also $\sum f(x) = f(0) + f(a) + f(b) + \cdots + f(n) = 1$
2. Binomial distribution is known as bi-parametric distribution as it is characterized by two parameters n and p. This means that if the values of n and p are known, then the distribution is known completely.
3. The mean of the binomial distribution is given by $\mu = np$.
4. Depending on the values of the two parameters, binomial distribution may be unimodal or bi-modal μ_0, the mode of binomial distribution, is given by: μ_0 =the largest integer contained in:

   (a) $(n + 1)p\ if\ (n + 1)p$ is a non-integer.

   (b) $(n + 1)p\ and\ (n + 1)p - 1\ if\ (n + 1)p$ is an integer.
5. The variance of the binomial distribution is given by $\sigma^2 = npq$

   ⇒ Variance of a binomial variable is always less than its mean. Since p and q are numerically less than or equal to 1 and Variance is maximum at $p = q = 0.5$ and the maximum value is $\frac{n}{4}$.
6. Additive property of binomial distribution. If X and Y are two independent variable such that $X \sim B(n_1, P)$ and $Y \sim B(n_2, P) then\ (X + Y) \sim B(n_1 + n_2, P)$.

APPLICATION OF BINOMIAL DISTRIBUTION

- Binomial distribution is applicable when the trial are independent and each trial has just two outcomes: success and failure. It is applied in coin tossing experiments, sampling inspection plans, genetic experiment and on.

Q.3 If Binomial distribution, $np = 9$ and $npq = 2.25$ then q is equal to

(a) 0.25 (b) 0.75 (c) 1 (d) None

Ans. (a)

Q.4 X is a binomial variable with n=20. What is the mean of X if it is known that X is symmetric?

(a) 5 (b) 10 (c) 2 (d) 8

Ans. (b)

Q.5 What is the number of trials of a binomial distribution having mean and SD as 3 and 1.5 respectively?

(a) 2 (b) 4 (c) 8 (d) 12

Ans. (d)

Q.6 If X is a binomial variate with parameter 15 and $\frac{1}{3}$, what is the value of mode of the distribution?

(a) 5 and 6 (b) 5 (c) 5.50 (d) 6

Ans. (b)

Q.7 If it is known that the probability of a missile hitting a target is $\frac{1}{8}$, what is the probability that out of 10 missile fired, at least 2 will hit the target?

(a) 0.4258 (b) 0.3968 (c) 0.5238 (d) 0.3611

Ans. (d)

Q.8 Assuming that one-third of the population is tea drinkers and each of 1000 enumerators takes a sample of 8 individuals to find out whether they are tea drinkers or not, how many enumerators are expected to report that five or more people are tea drinkers?

(a) 100 (b) 95 (c) 88 (d) 90

Ans. (c)

Q.9 In 10 independent rollings of a biased die, the probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. What is the probability that an even number will appear twice when the die is rolled 8 times?

(a) 0.0304 (b) 0.1243 (c) 0.2315 (d) 0.1926

Ans. (a)

Q.10 What are the parameters of binomial distribution?

(a) n (b) p (c) Both n and p (d) None of these

Ans. (c)

Q.11 The Standard Deviation of Binomial distribution is

(a) npq (b) $\sqrt{npq}$ (c) np (d) $\sqrt{np}$

Ans. (b)

Q.12 If X is a binomial variate with $p = \frac{1}{3}$ for the experiment of 90 trials, then the standard deviation is equal to

(a) $-\sqrt{5}$ (b) $\sqrt{5}$ (c) $2\sqrt{5}$ (d) $\sqrt{15}$

Ans. (c)

Q.13 An unbiased die is tossed 500 times. The mean of the number of 'sixes' in these 500 tosses is

(a) $\frac{50}{6}$ (b) $\frac{500}{6}$ (c) $\frac{5}{6}$ (d) None of these

Ans. (b)

Q.14If in a Binomial distribution if $mean = 20, S.D = 4$, then n is equal to

(a) 80 (b) 100 (c) 90 (d) None of these

Ans. (b)

Q.15In Binomial distribution if $n = 4\ and\ p = \frac{1}{3}$ then the value of variance is

(a) $\frac{8}{3}$ (b) $\frac{8}{9}$ (c) $\frac{4}{3}$ (d) None of these

Ans. (b)

POISSON DISTRIBUTION

- Poisson distribution is a theoretical discrete probability distribution which can describe many processes. Simon Denis Poisson of France introduced this distribution way back in the year 1837.

  Let us think of a random experiment under the following conditions:
- The probability of finding success in a very small-time interval $(t, t + dt)$ is kt, where $k\ (> 0)$ is a constant.
- The probability of having more than one success in this time interval is very low.
- The probability of having success in this time interval is independent of t as well as earlier successes.
- The above model is known as the Poisson Model. The probability of getting x successes in a relatively long-time interval T containing m small time intervals t i.e. $T = kt$ is given by $e^{-kt}\frac{(kt)^x}{x!}\ for\ x = 0, 1, 2, \ldots$

  Taking $kt = m$, the above form is reduced to $\frac{e^{-m}m^x}{x!} for\ x = 0, 1, 2, \ldots$

DEFINITION OF POISSON DISTRIBUTION

- A random variable X is defined to follow Poisson distribution with parameter m, to be denoted by $X \sim P(m)$ if the probability mass function of x given by

  $f(x) = P(X = x) = \frac{e^{-m}m^x}{x!}\ for\ x = 0, 1, 2, \ldots.$

  Here, e is a transcendental quantity with an approximate value as 2.71828

Q.16If 1.5 percent of items produced by a manufacturing units are known to be defective, what is the probability that a sample of 200 items would contain no defective items?

(a) 0.05 (b) 0.15 (c) 0.20 (d) 0.22

Ans. (a)

Q.17X is a Poisson variate satisfying the following condition $9\ P(X = 4) + 90\ P(X = 6) = P(X = 2)$. What is the value of $P(X \leq 1)$?

(a) 0.5655 (b) 0.6559 (c) 0.7358 (d) 0.8201

Ans. (c)

Q.18If 1 percent of an airline's flights suffer a minor equipment failure in an aircraft, what is the probability that there will be exactly two such failure in the next 100 such flights?

(a) 0.50 (b) 0.184 (c) 0.265 (d) 0.256

Ans. (b)

It is wiser to remember the following important points in connection with Poisson distribution:

1. Since $e^{-m} = \frac{1}{e^m} > 0$, whatever may be the value of $m, m > 0$, it follows that $f(x) \geq 0$ for every x, Also it can be established that $f(x) = 1$

   i.e., $f(0) + f(a) + f(b) + \cdots = 1$

2. Poisson distribution is known as a uniparametric distribution as it is characterized by only one parameter m.

3. The mean of Poisson distribution is given by m i.e. $\mu = m$

4. The variance of Poisson distribution is given by $\sigma^2 = m$

5. Like binomial distribution, Poisson distribution could be also unimodal or bimodal depending upon the value of the value of the parameter m.

   We have $\mu_0 =$

   (a) The largest integer contained in m is a non-integer

   (b) m and $m - 1$ if m is an integer

6. Poisson approximation to Binomial distribution:

   The Poisson distribution can be used as an approximation to the binomial distribution under certain conditions. When the number of trials in a binomial distribution (n) is large and the probability of success (p) is small, the binomial distribution becomes computationally challenging. In such cases, the Poisson distribution provides a simpler and more convenient approximation.

   (a) The conditions for using the Poisson approximation to the binomial distribution are as follow:

   (b) The number of trials (n) is large, typically greater than 20.

   (c) The probability of success (p) is small, typically less than or equal to 0.05.

   (d) The events occur independently of each other.

   Then, $B(n, p)$P $P(m)$

7. Additive property of Poisson distribution:

   The Poisson distribution has an additive property, which means that if we have two or more independent Poisson-distributed random variables, the sum of these variables will also follow a Poisson distribution.

   Mathematically, if X and Y are two independent Poisson-distributed random variables with parameters m_1 and m_2 respectively, then the sum $Z = X + Y$ will be a Poisson distributed random variable with parameter $m = m_1 + m_2$.

   i.e., If $X \sim P(m_1)$ *and* $Y \sim P(m_2)$ and X and Y are independent then

   $Z = X + Y \sim P(m_1 + m_2)$

Application of Poisson distribution:

Poisson distribution is applied when the total number of events is pretty large but the probability of occurrence is very small. Thus, we can apply Poisson distribution, rather profitably, for the following cases:

1. The distribution of the number of printing mistakes per page of a large book.
2. The distribution of the number of road accidents on a busy road per minute.
3. The distribution of purchases of number of shoe laces per minute at shoe store.
4. The distribution of the number of demands per minute for health centers and so on.

FITTING A POISON DISTRIBUTION

As explained earlier, we can apply the method of moments to fit a Poisson distribution to an observed frequency distribution. Since Poisson distribution is uniparametric, we equate m, the parameter of Poisson distribution, to the arithmetic mean of the observed distribution and get the estimate of m.

i.e $\hat{m} = \bar{x}$

The fitted Poisson distribution is then given by $\hat{f}(x) = \frac{e^{-m}(\hat{m})^x}{x!} for\ x = 0, 1, 2, \ldots$

Q.19 If the standard deviation of a Poisson variate X is 2, what is $P(1.5 < X < 2.9)$?

(a) 0.231 (b) 0.158 (c) 0.15 (d) 0.146

Ans. (d)

Q.20 For a Poisson variate X, $P(X-1) = P(X=2)$. What is the mean of X?

(a) 1.00 (b) 1.50 (c) 2.00 (d) 2.50

Ans. (c)

Q.21 If X is a Poison variate such that, $P(X=1) = 0.7$, $P(X=2) = 0.3$, then $P(X=0)$

(a) $e^{\frac{6}{7}}$ (b) $e^{-\frac{6}{7}}$ (c) $e^{\frac{2}{3}}$ (d) $e^{-\frac{1}{3}}$

Ans. (b)

Q.22 If for a Poisson variable X, $f(2) = 3f(4)$, what is the variance of X?

(a) 2 (b) 2 (c) $\sqrt{2}$ (d) 3

Ans. (a)

Q.23 A Company has two cars which it hires out during the day. The number of Cars demand with mean 1.5. Then percentage of days on which only one car was in demand is

(a) 23.26 (b) 33.47 (c) 44.62 (d) 46.40

Ans. (b)

Q.24 If the parameter of Poisson distribution is m and $mean + S.D. = \frac{6}{25}$ then find.

(a) $\frac{3}{25}$ (b) $\frac{1}{25}$ (c) $\frac{4}{25}$ (d) $\frac{3}{5}$

Ans. (b)

Q.25 In a certain manufacturing process, 5% of the tools produced turn out to be defective. Find the probability that in a sample of tools, at most 2 will be defective. Given $e^{-2} = 0.135$

(a) 0.555 (b) 0.932 (c) 0.785 (d) 0.675

Ans. (d)

Q.26The number of accidents in a year attributed to taxi drivers in a locality follows Poisson distribution with an average 2. Out of 500 taxi drivers of that area, what is the number of drivers with at least 3 accident in a year?

(a) 162 (b) 180 (c) 201 (d) 190

Ans. (a)

Q.27A renowned hospital usually admits 200 patients every day. One percent of patients, on an average, require special room facilities. On one particular morning, it was found that only one special room was available. What is the probability that more than 3 patients would require special room facilities?

(a) 0.1428 (b) 0.1732 (c) 0.2235 (d) 0.3450

Ans. (a)

Q.28A random variable X follows Poisson distribution and its coefficient of variation is 50. What is the value of $P\left(\frac{x>1}{x>0}\right)$

(a) 0.1876 (b) 0.2341 (c) 0.9254 (d) 0.8756

Ans. (c)

Q.29A car hire firm has 2 cars which are hired out everyday. The number of demands per day for a car follows Poisson distribution with mean 1.20. What is the proportion of day on which some demand is refused? Given $e^{1.20} = 3.32$

(a) 0.25 (b) 0.3012 (c) 0.12 (d) 0.03

Ans. (d)

Q.30In Poisson distribution, which of the following is same?

(a) Mean and variance (b) Mean and SD

(c) Both (d) None of these

Ans. (a)

Q.31Let X be a Poisson random variable with parameter λ Then $p(X)$ is equal to

(a) $\frac{e^{\lambda}-e^{-\lambda}}{2}$ (b) $\frac{e^{\lambda}-e^{-\lambda}}{2}$ (c) $\frac{e^{2\lambda}-1}{2}$ (d) $\frac{\lambda^x.e^{-\lambda}}{x!}$

Ans. (d)

Q.32Which one of the following has Poisson distribution?

(a) The number of days to get a complete cure.

(b) The number of defects per meter on long roll of coated polythene sheet.

(c) The errors obtained in repeated measuring of the length of a rod.

(d) The number of claims rejected by an insurance agency.

Ans. (b)

Q.33Mean of Poisson distribution is 6 then variance is_____.

(a) 6 (b) $\sqrt{6}$ (c) 4 (d) 3

Ans. (a)

Q.34If the overall percentage of success is an exam is 60, what is the probability that out of a group of 4 students, at least one has passed?

(a) 0.6525 (b) 0.9744 (c) 0.8704 (d) 0.0256

Ans. (b)

Q.35The probability that a random variable x following Poisson distribution would assume a positive value is $(1 - e^{-2.7})$. What is the mode of the distribution?

(a) 1 (b) 1 (c) 2 (d) None of these

Ans. (c)

Q.36Find the mean and standard deviation of x, where x is a Poisson variate satisfying the condition

$P(x = 3) = P(x = 4)$

(a) 4, 2 (b) 4, 4 (c) 2, 2 (d) None of these

Ans. (a)

Q.37For a Poisson distributed variable, we have $P(X = 7) = 8P(X = 9)$ the mean of the distribution is

(a) 4 (b) 3 (c) 7 (d) 9

Ans. (b)

Q.38What is the standard deviation of the number of recoveries among 48 patients when the probability of recovering is 0.75?

(a) 36 (b) 81 (c) 9 (d) 3

Ans. (d)

NORMAL OR GAUSSIAN DISTRIBUTION

- The two distributions discussed so far, namely binomial and Poisson, are applicable when the random variable is discrete.
- In the case of a continuous random variable like height or weight, it is impossible to distribute the total probability among different mass points because between any two unequal values, there remains an infinite number of values.
- Thus, a continuous random variable is defined in term of its probability density function $f(x)$, provided, of course, such a function really exists, $f(x)$ satisfies the following condition:

  $f(x) \geq 0$ or for $x \in (-\infty, \infty)$ and $\int_{-\infty}^{\infty} f(x)dx = 1$
- It is also known as Gaussian Distribution as Karl Gauss was instrumental for deriving normal distribution and as such normal distribution but the contribution of De-Moivre, Laplace was significant,
- A continuous random variable x is defined to follow normal distribution with parameters μ and σ^2, to be denoted by $X \sim N(\mu, \sigma^2)$
- If the probability density function of the random variable x is given by

  $f(x = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ for $-\infty < x < \infty$ where μ and σ are constants and $\sigma > 0$.

Q.39The mean and the variance of a random variable X having the probability density function

$P(X = x) = \frac{1}{\sqrt{\pi}} e^{-(x-4)^2}, -\infty < x < \infty$ is

(a) $4, \frac{1}{2}$ (b) $4, \frac{1}{\sqrt{2}}$ (c) 2, 2 (d) $2, \frac{1}{2}$

Ans. (a)

Q.40What is the coefficient of variation of x, characterized by the following probability density function

$$f(x) = \frac{1}{4\sqrt{2\pi}} e^{\frac{-(X-10)^2}{32}} \; for -\infty < x < \infty$$

(a) 50 (b) 60 (c) 40 (d) 30

Ans. (c)

Q.41What is the first quartile of X having the following probability density function?

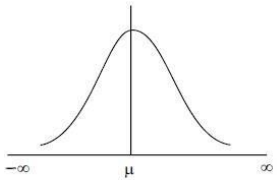
$$f(x) = \frac{1}{\sqrt{72\pi}} e^{\frac{-(x-10)^2}{72}} \; for -\infty < x < \infty$$

(a) 4 (b) 5 (c) 5.95 (d) 6.75

Ans. (c)

SOME IMPORTANT POINTS RELATING TO NORMAL DISTRIBUTION ARE LISTED BELOW

1. The name Normal Distribution has its origin some two hundred years back as the then mathematicians were in search for a normal model that can describe the probability distribution of most of the continuous random variables.
2. If we plot the probability function $y = f(x)$, then the curve, known as probability curve, takes the following shape:



- **Curve is symmetric:** If you look above curve is the symmetrical curve which is bell shaped and has one peak representing the mode of normal distribution.
- **The line of symmetry is $x = \mu$** which is similar to binomial distribution, which is symmetric about $p = 0.5$

Tails of the Curve: If we look at the two tails of the normal curve, it extends indefinitely on both sides of the curve and both the left and right tails never touch the horizontal axis. The total area of the normal curve or for that any probability curve is taken to be unity i.e. one. Since the vertical line drawn through $x =$ divides the curve into two equal halves, it automatically follows that The area between $-\infty \; to \; \mu =$ the area between $\mu \; to \; \infty = 0.5$

When the mean is zero, we have

Area between $-\infty \; to \; 0 =$ Area between $0 \; to \; \infty = 0.5$

$\Rightarrow$ If we take $\mu = 0 \; and \; \sigma = 1$, we have

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \text{ for } -\infty < z < \infty$$

The random variable z is known as Standard normal variate (or variable) or standard normal deviate.

The probability that a standard normal variate X would take a value less than or equal to a particular value say $X = x$ is given by

$\emptyset(x) = F(X \leq x)$

$\emptyset(x)$ is known as the cumulative distribution function.

We also have $\emptyset(0) = P(X \leq 0)$

$=$ Area of the standard normal curve between $-\infty$ and 0

$= 0.5$

The normal distribution is known as biparametric distribution as it is characterized by two parameters μ and σ^2. Once the two parameters are known, the normal distribution is completely specified.

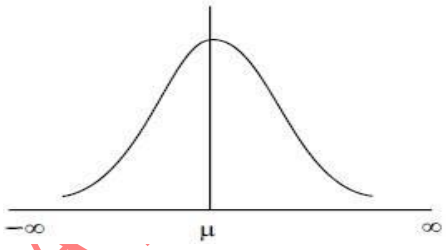
PROPERTIES OF NORMAL DISTRIBUTION

- Since $\pi = \frac{22}{7}, e^{-\theta} = \frac{1}{e^{\theta}} > 0$ whatever maybe, it follows that $f(x) = 1$
- The mean of the normal distribution is given by μ. Further, since the distribution is symmetrical about $x = \mu$, it follows that the mean, median and mode of a normal distribution coincide, all being to μ.
- The standard deviation of the normal distribution is given by σ.
- Mean deviation of normal distribution is $\sigma\sqrt{\frac{2}{\pi}} = 0.8\sigma$
- The first and third quartiles are given by

  $Q_1 = m - 0.675\,\sigma \; and \; Q_3 = m + 0.675\sigma$

So, Quartile Deviation $= 0.675\sigma$

- The normal distribution is symmetrical about $x = \mu$. As such, its skewness is zero i.e. the normal curve is neither inclined to move towards the right (negative skewed) nor towards the left (positively skewed).
- The normal curve $y = f(x)$ has two points of inflexion to be given by $x = \mu - \sigma$ and $x = \mu + \sigma$ and $x = \mu + \sigma$ i.e., at these two points, the normal curve changes its curvature from concave to convex and from convex to concave.

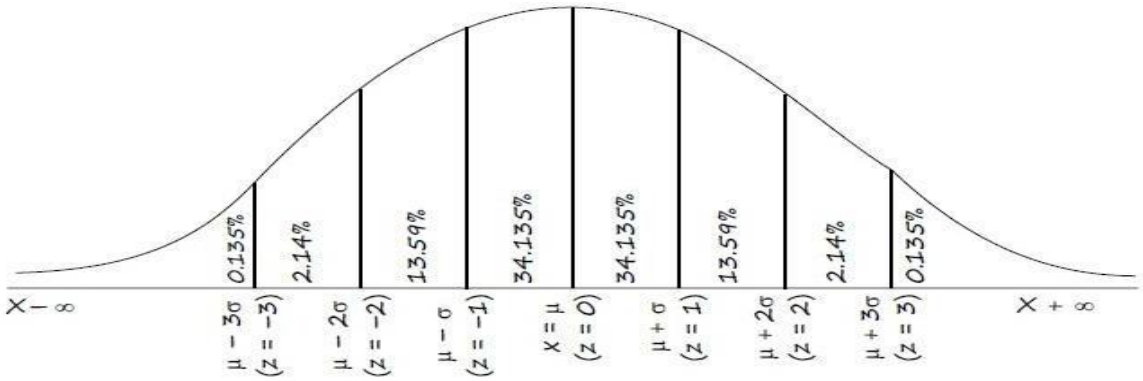


- If $x \sim N(\mu, \sigma^2)$ then $z = \frac{(X-\mu)}{\sigma} \sim N(0, 1), z \sim N(0, 1), z$ is known as standardized normal variate or normal deviate.
- We also have $P(z \leq k) = \emptyset(k)$ which is known as the cumulative distribution function.
- The values of $\emptyset(k)$ for different k are given in a table known as "Biometrika." Because of symmetry, we have $\emptyset(-k) = 1 - \emptyset(k)$

We can evaluate the different probabilities in the following manner:

- $P(x < a) = P\left[\frac{x-u}{\sigma} < \frac{a-\mu}{\sigma}\right] = P(z < k)$ where, $k = \frac{a-\mu}{\sigma} = \emptyset(k)$
- Also $P(x \leq a) = P(x < a) as\ x\ is\ continuous.$
- $= P(x > b) = 1 - P(x < b) = 1 - \emptyset\left(\frac{b-\mu}{\sigma}\right)$
- $P(a < x < b) = \emptyset\left(\frac{b-\mu}{\sigma}\right) - \emptyset\left(\frac{a-\mu}{\sigma}\right)$
- Also, $\emptyset(-k) = \emptyset(k)$

- The value of $\emptyset(k)$ for different k are also provided in the Biometrika Table.
- Area under the normal curve is shown in the following figure:



- If x and y are independent normal variable with means and standard deviations as μ_1 and μ_2 and σ_1 and σ_2 respectively, the $z = x + y$ also follows normal distribution with mean $\mu_1 + \mu_2$ and SD= $\sqrt{\sigma_1^2 + \sigma_2^2}$ respectively.

SOME IMPORTANT PROPERTIES OF STANDARD NORMAL VARIATE

If we take $\mu = 0$ and $\sigma = 1$, we have $f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^{2}/2} \quad -\infty < z < \infty$

1. z has mean, median and mode all equal to zero.
2. The standard deviation of z is 1. Also, the approximate values of mean deviation and quartile deviation are 0.8 and 0.675 respectively.
3. The standard normal distribution is symmetrical about $z = 0$
4. The two points of inflexion of the probability curve of the standard normal distribution are -1 and 1
5. The two tails of the standard normal curve never touch the horizontal axis.
6. The upper and lower p per cent points of the standard normal variable z is given by

   $P(Z > z_p) = P(Z < z_1 - p) = p \; ie., P(Z < z_p) = p$ respectively since for a standard normal distribution.

   $(z_1 - p = -z_p)$

   Selecting $P = 0.005, 0.025, 0.01$ and 0.05 respectively, We have

   $z_{0.005} = 2.58$

   $z_{0.025} = 1.96$

   $z_{0.01} = 2.33$

   $z_{0.05} = 1.645$

APPLICAATION OF NORMAL DISTRIBUTION

The applications of normal distribution are not restricted to statistics only. Many science subjects, social science subjects, management, commerce etc. find many applications of normal distributions. Most of the continuous variables like height, weight, wage, profit etc. follow normal distribution. If the variable under study does not follow normal distribution, a simple transformation of the variable, in many cases, would lead to the normal distribution of the changed variable.

Q.42If the two quartiles of $N\ (\mu, \sigma^2)$ are 14.6 and 25.4 respectively, what is the standard deviation of the distribution?

(a) 9 (b) 6 (c) 10 (d) 8

Ans. (d)

Q.43If the mean deviation of a normal variable is 16, what is its quartile deviation?

(a) 10.00 (b) 13.50 (c) 15.00 (d) 12.05

Ans. (b)

Q.44if the quartile deviation of a normal curve is 4.50, then its mean deviation is

(a) 5.26 (b) 6.24 (c) 4.24 (d) 4.80

Ans. (d)

Q.45If the first quartile and mean deviation about median of a normal distribution are 13.25 and 8 respectively, then the mode of the distribution is

(a) 20 (b) 10 (c) 15 (d) 12

Ans. (a)

Q.46If the inflexion points of a normal distribution are 6 and 14. Find its standard deviation?

(a) 4 (b) 6 (c) 10 (d) 12

Ans. (a)

Q.47If the area of standard normal curve between z=0 to z=1 is 0.3413, then the value of $\varphi(a)$ is

(a) 0.5000 (b) 0.8413 (c) -0.5000 (d) 1

Ans. (b)

Q.48Area of the normal curve

(a) between $-\infty\ to\ \mu\ is\ 0.50$ (b) between $\mu\ to\ -\infty\ is\ 0.50$

(c) between $-\infty\ to\ \infty\ is\ 0.50$ (d) both (a) and (b)

Ans. (d)

Q.49The cumulative distribution function of a random variable X is given by

(a) $F(x) = P(X \leq x)$ (b) $F(x) = P(X < x)$

(c) $f(x) = P(X \geq x)$ (d) $F(x) = P(X = x)$

Ans. (a)

Q.50if X follows normal distribution with $\mu = 50$ and $\sigma = 10$, what is the valueof P $(\frac{x \leq 60}{x > 50})$?

(a) 0.8413 (b) 0.6828 (c) 0.1587 (d) 0.7256

Ans. (b)

Q.51For a certain type of mobiles, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. A person owns one of those mobiles and wants to know the probability that length of time will be between 50 & 70 hours is (Given: ∅ (1.33) = 0.9082, ∅ (0) = 0.5)

(a) -0.4082 (b) 0.5 (c) 0.4082 (d) -0.5

Ans. (c)

Q.52X and Y are two independent Normal variables, then the distribution of $X + Y$ is _____

(a) Normal Distribution (b) t-distribution

(c) Chi-Square distribution (d) F-distribution

Ans. (a)

Q.53If the two quartile of $N\ (\mu, \sigma^2)$ are 47.30 and 52.70 respectively, what is the mean deviation about the median of the distribution?

(a) 9.0 (b) 6.5 (c) 3.20 (d) 8.45

Ans. (c)

Q.54In normal distribution 95% observation lies between_____&_____.

(a) $(\mu - 2\sigma, \mu + 2\sigma)$ (b) $(\mu - 3\sigma, \mu + 3\sigma)$

(c) $(\mu - 1.96\sigma, \mu + 1.96\sigma)$ (d) $(\mu - 2.58\sigma, \mu + 2.58\sigma)$

Ans. (c)

Q.55The mean of a binomial distribution with parameter n and p is

(a) $n(1-p)$ (b) $n(1-p)$ (c) np (d) $\sqrt{np(1-p)}$

Ans. (c)

Q.56The variance of a binomial distribution with parameter n and p is

(a) $np^2(1-p)$ (b) $\sqrt{np(1-p)}$ (c) $nq(1-d)$ (d) $n^2p^2(1-d)$

Ans. (c)

Q.57The maximum value of the variance of a binomial distribution with parameters n and p is

(a) $\frac{n}{2}$ (b) $\frac{n}{4}$ (c) $np(1-p)$ (d) $2n$

Ans. (b)

Q.58The method usually applied for fitting a binomial distribution is known as

(a) Method of least square

(b) Method of moments.

(c) Method of probability distribution

(d) Method of deviations

Ans. (b)

Q.59The normal curve is

(a) Positive skewed. (b) Negatively skewed

(c) Symmetrical (d) All of these

Ans. (c)

Q.60The mean and mode of a normal distribution

(a) May be equal (b) May be different

(c) Are always equal (d) (a) or (b)

Ans. (c)

Q.61The interval $(\mu - 3\sigma, \mu + 3\sigma)$ covers

(a) 95% are of a normal distribution

(b) 96% area of a normal distribution

(c) 99% area of a normal distribution

(d) all but 0.27% area of a normal distribution

Ans. (d)

Q.62The probability mass function of binomial distribution is given by

(a) $f(x) = P^x q^{n-x}$ (b) $f(x) = {}^nC_x p^x q^{n-x}$

(c) $f(x) = {}^nC_x q^x p^{n-x}$ (d) $f(x) = {}^nC_x p^{n-x} q^x$

Ans. (b)

Q.63If $X \sim P(m)$ and its coefficient of variation is 50, what is the probability that X would assume only non-zero values?

(a) 0.018 (b) 0.982 (c) 0.989 (d) 0.976

Ans. (b)

Q.64If the points of inflexion of a normal curve are 40 and 60 respectively, then its mean deviation is

(a) 8 (b) 5 (c) 10 (d) 6

Ans. (a)

Q.65The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is

(a) 54.24 (b) 23.20 (c) 0.275 (d) 2.70

Ans. (d)

Q.66The salary of workers of a factory is known to follow normal distribution with an average salary of `10,000 and standard deviation of salary as `2,000. If 50 workers receive salary more than `14,000, then the total number of workers in the factory is

(a) 2193 (b) 2000 (c) 2200 (d) 2500

Ans. (a)

Q.6750 percent of a certain product has weight 60 kg or more whereas 10 per cent have weight 55 kg or less. On the assumption of normality, what is the variance of weight? Given $\varphi(1.28) = 0.90$

(a) 15.21 (b) 9.00 (c) 16.00 (d) 22.68

Ans. (a)

Q.68If the weekly wages of 5000 workers in a factory follows normal distribution with mean and standard deviation as ` 700 and ` 50 respectively, what is the expected number of workers with wages between ` 660 and ` 720?

Given, $\emptyset(0.8) = 0.2881$, $\emptyset(0.4) = 0.1554$

(a) 2,050 (b) 2,200 (c) 2,218 (d) 2,300

Ans. (c)

Q.69In a sample of 800 students, the mean weight and standard deviation of weight are found to be 50 kg and 20 kg respectively. On the assumption of normality, what is the number of students weighing between 46 kg and 62 kg? Given area of the standard normal curve between $z = 0$ to $z = 0.20 = 0.0793$ and area between $z = 0\ to\ z = 0.60 = 0.2257$.

(a) 250 (b) 244 (c) 240 (d) 260

Ans. (b)

Q.70Standard deviation of binomial distribution is:

(a) $(npq)^2$ (b) $\sqrt{npq}$ (c) $(np)^2$ (d) $\sqrt{np}$

Ans. (c)

Q.71The wages of workers of factory follows:

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) Chi-square distribution

Ans. (c)

Q.72If for a Binomial distribution $B(n, p)$; $n = 4$ and also $P(x = 2) = P(x = 3)$ then the value of p is

(a) 9/11 (b) 1 (c) 1/3 (d) 1/9

Ans. (c)

Q.73An example of a bi-parametric discrete probability distribution is:

(a) Binomial distribution (b) Poisson distribution

(c) Normal distribution (d) Both (a) and (b)

Ans. (a)

Q.74If $X \sim N(50, 16)$, then which of the following is not possible?

(a) $P(X > 60) = 0.30$ (b) $P(X < 50) = 0.50$

(c) $P(X < 60) = 0.40$ (d) $P(X > 50) = 0.50$

Ans. (c)

Q.75In normal distribution, Mean, Median and Mode are

(a) Zero (b) Not Equal (c) Equal (d) Null

Ans. (c)

Q.76Probability distribution may be

(a) discrete (b) continuous (c) infinite (d) (a) or (b)

Ans. (d)

Q.77Which one of the following is an uniparametric distribution?

(a) Poisson (b) Normal (c) Binomial (d) Hyper geometric

Ans. (a)

Q.78For a Poisson variate X, $P(X = 2) = 3P(X = 4)$, then the standard deviation of X is

(a) $\sqrt{2}$ (b) 3 (c) 4 (d) 5

Ans. (a)

Q.79The mean of the Binomial distribution $B\left(4, \frac{1}{3}\right)$ is equal to

(a) 3/5 (b) 4/3 (c) 8/3 (d) 3/4

Ans. (b)

Q.80If for a normal distribution $Q_1 = 54.52$ and $Q_3 = 78.86$, then the median of the distribution is

(a) 12.17 (b) 39.43 (c) 66.69 (d) None of these

Ans. (c)

Q.81What is the mean of X having the following density function?

$$f(x)=\frac{1}{4\sqrt{2\pi}} e^{-\frac{(x-10)^2}{32}} \quad for -\infty < x < \infty$$

(a) 4 (b) 10 (c) 40 (d) None of the above

Ans. (a)

Q.82The probability that a student is not a swimmer is 1/5, then the Probability that out of five students four are swimmer is

(a) (4/5)(1/5) (b) ${}^5C_1\,(1/5)^4(4/5)$

(c) ${}^5C_4\left(\frac{4}{5}\right)^1(1/5)^4$ (d) None of these

Ans. (c)

Q.834 coins were tossed 1600 times. What is the probability that all 4 coins do not turn head upward at a time?

(a) $1600\,e^{-100}$ (b) $1000\,e^{-100}$ (c) $100\,e^{-1600}$ (d) e^{-100}

Ans. (d)

Q.84If mean and variance are 5 and 3 respectively then relation between p and q is

(a) $p > q$ (b) $p < q$ (c) $p = q$ (d) p is symmetric

Ans. (b)

Q.85In a Poisson distribution if P(x=4)=P(x=5) then the parameter of Poisson distribution is

(a) 4/5 (b) 5/4 (c) 4 (d) 5

Ans. (d)

Q.86Area between =1.96 to 1.96+1.96 in a normal distribution is:

(a) 95.45% (b) 95% (c) 96% (d) 99%

Ans. (b)

Q.87A coin with probability for head as is tossed 100 times. The standard deviation of the number of head 5 turned up is

(a) 3 (b) 2 (c) 4 (d) 6

Ans. (c)

Q.88If Poisson distribution is such that $P(X = 2) = P(X = 3)$ then the variance of the distribution is

(a) $\sqrt{3}$ (d) 3 (c) 6 (d) 9

Ans. (b)

SUMMARY

- **Binomial distribution:** Trials are independent and each trail has only two outcomes Success & failure. $P(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

  Denoted by $X \sim B\ (n, p)$

  Mean : $\mu = np$

  Variance : $\sigma^2 = npq$

  $p + q = 1$

- Poisson distribution: Trials are independent and probability of occurrence is very small is given time $P(X = x) = \frac{e^{-\lambda}\, \lambda^{x}}{x!}$

  Denoted by $X \sim P(m)$

  Mean: $\mu = m$

  Variance: $\sigma^2 = m$

- Normal distribution: When distribution is symmetric $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

  Mean = Median = Mode = μ

  Variance = σ^2

- We note that 99.73 percent of the values of a normal variable lies between $\mu - 3\sigma$ & $\mu + 3\sigma$.

- If x and y are independent normal variables with mean and standard deviation as μ_1 and μ_2 and σ_1, and σ_2 respectively, the $z = x + y$ also follows normal distribution with mean $\mu_1 + \mu_2$ and $SD = \sigma_1^2 + \sigma_2^2$ respectively.

- If a continuous random variable z follows standard normal distribution, to be denoted by $z \sim N(0, 1)$, then the probability density function of z is given by $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\pi^{2/2}} for -\infty < z < \infty$

- Some important properties of z are listed below:

  (i) z has mean, median and mode all equal to zero.

  (ii) The standard deviation of z is 1. Also the approximate value of mean deviation and quartile deviation are 0.8 and 0.675 respectively.

  (iii) The standard normal distribution is symmetrical about $z = 0$

  (iv) The two points of inflexion of the probability curve of the standard normal distribution are -1 and 1

CORRELATION

Correlation is a fundamental concept in statistics that allows us to measure the statistical relationship between two variables. It helps us understand the extent to which changes in one variable are associated with changes in another. Imagine it as a way to quantify how two things interact or influence each other.

BIVARIATE DATA

In real life, we generally deal with more than one variable.

E.g.:

	Math-Enthusiast	Math-Neutral	Math-Disliker	Marginal (English)
English-Lover	10	5	3	18
English-Neutral	3	8	2	13
English-Disliker	1	2	7	10
Marginal (Math)	14	15	12	41

- The above type of data is called a Bivariable data.
- When we involve more than two variables in our study, that type of data is called Multivariate data.

However, in real-life situations, we often deal with more than just two variables, leading to what we call multivariate data. Multivariate data involves the measurement or observation of multiple variables for each individual or object in a study.

Thus, in the given scenario, the data of the singing competition involves two variables which represent bivariate data. If we further consider additional variables such as the age, gender, or singing experience of the participants, it would be an example of multivariate data.

MARGINAL DISTRIBUTION

The marginal distribution of a variable in a bivariate frequency distribution is the distribution of that variable alone, without considering the other variable. It is obtained by summing (or adding) the frequencies along one of the margins (rows or columns) of the distribution table.

E.g.: Consider a bivariate frequency distribution table representing the preferences of students regarding two subjects, Math and English. The table might look like this:

	Math-Enthusiast	Math-Neutral	Math-Disliker	Marginal (English)
English-Lover	**10**	**5**	**3**	**18**
English-Neutral	**3**	**8**	**2**	**13**
English-Disliker	**1**	**2**	**7**	**10**
Marginal (Math)	**14**	**15**	**12**	**41**

1. The marginal distribution of the variable "English preference" is obtained by summing the frequencies along the "Marginal (English)" column.

2. The marginal distribution of the variable "Math preference" is obtained by summing the frequencies along the "Marginal (Math)" row.

CONDITIONAL DISTRIBUTION

The conditional distribution of a variable in a bivariate frequency distribution is the distribution of that variable under the condition that a specific value of the other variable is known. It is calculated by dividing the joint frequency by the corresponding marginal frequency.

To find the conditional distribution of "Math preference" given that a student is an "English Lover," divide each frequency in the "English Lover" row by the marginal frequency in the "Marginal (English)" column.

	Math-Enthusiast	Math-Neutral	Math-Disliker	Marginal (English)
English-Lover	$\frac{10}{18}$	$\frac{5}{18}$	$\frac{3}{18}$	18

CORRELATION

Correlation refers to the statistical relationship between two variables. It measures the extent to which the change in one variable is accompanied by a change in the other. Correlation helps us understand the association or connection between different variables and how they interact with each other.

E.g.: If we consider the height of students as variable X and their weight as variable Y, we would expect that large values of X (tall students) correspond to large values of Y (higher weight), while small values of X (short students) correspond to small values of Y (lower weight). This indicates a correlation between height and weight, as they tend to change together.

Let's see one more example to understand correlation. We might find a high degree of relationship between the price of a product and consumer demand for that product. As the price increases, the demand may decrease, and vice versa. This demonstrates the correlation between price and demand.

POSITIVE AND NEGATIVE CORRELATION

- **Positive Correlation:** It indicates that as one variable increases (or decreases), the other variable also increases (or decreases). For instance, in a business context, if the amount spent on digital advertising (X) by a firm increases, there is a corresponding increase in total annual sales (Y). Conversely, a reduction in advertising expenditure is associated with a decrease in total sales.

- **Negative Correlation:** It indicates that as one variable increases (or decreases), the other variable decreases (or increases). Consider the price of a product (X) and its demand (Y).

If there is a negative correlation, an increase in the product's price leads to a decrease in demand. Conversely, a decrease in price tends to result in an increase in demand for the product.

METHODS OF STUDYING CORRELATION

We shall discuss the following methods of measuring the linear relationship between two variables:

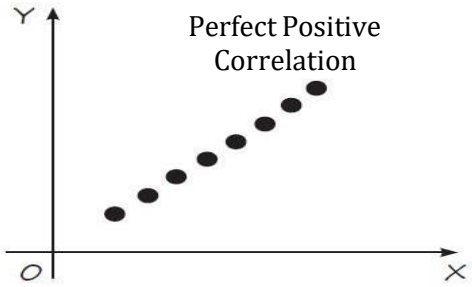
1. Scatter Diagram
2. Karl Pearson's Coefficient of Correlation
3. Spearman's Rank Correlation Coefficient
4. Coefficient of Concurrent deviations

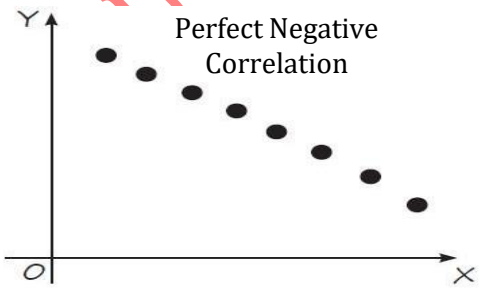
SCATTER DIAGRAM

A scatter diagram is a graphical presentation of bivariate data {(Xi, Yi): i = 1, 2, … n} on two quantitative variables X and Y that allows us to show two variables together, one on each axis, each pair being represented by a point on the graph as in coordinate geometry.

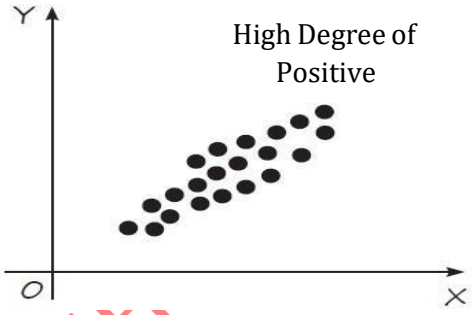
SCATTER DIAGRAM

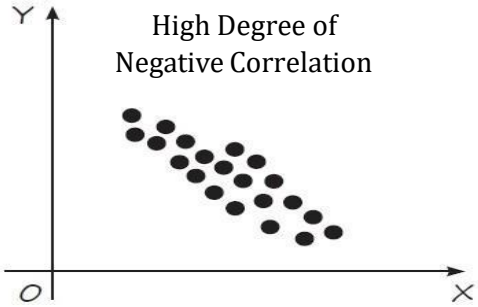
- **Perfect correlation**



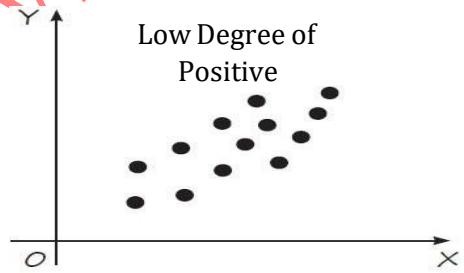


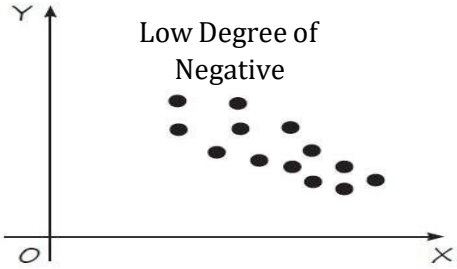
- **Very high degree of correlation**



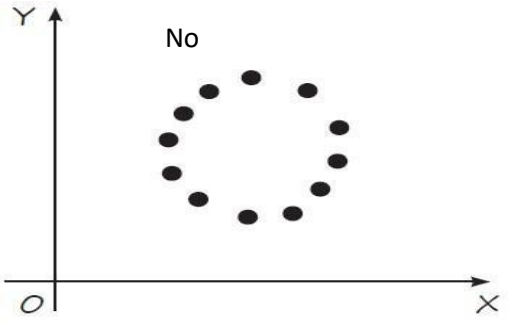


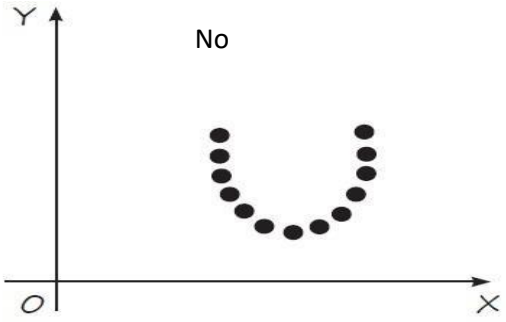
- **Low degree correlation**





- **No correlation**





1. If $r = 1$, then all the points of the scatter diagram lie on a straight line having positive slope and we say that a perfect positive linear relationship exists between the two variables. Similarly, if $r = -1$, then all the points of the scatter diagram lie on a straight line having a negative slope and we say that a perfect negative linear relationship exists between the two variables.
2. If r is close to $+1$, then all the points of the scatter diagram closely follow a straight line having positive slope and we say that a high positive correlation exists between the two variables. Similarly, if r is close to – 1, then all the points of the scatter diagram closely follow a straight line having a negative slope and we say that a high negative correlation exists between the two variables.
3. If r is close to 0, the linear relationship between the two variables is weak or perhaps non-existent.

THE CORRELATION COEFFICIENT

Correlation analysis attempts to measure the strength or closeness of linear relationships between two variables by means of a single number called a correlation coefficient.

Definition. The quantitative measure of strength in the linear relationship between two variables is called the correlation coefficient. It is denoted by r.

- Thus, the correlation coefficient r measures the extent to which the points cluster about a straight line.
- The correlation coefficient ranges from + 1 to – 1 i.e. $-1 \leq r \leq 1$.
- If two variables have no linear relationship, the correlation between them is zero.
- Consequently, the more correlation differs from zero, the stronger the linear relationship between the two variables.

The following table shows degrees of correlation according to various values of r.

Degree of Correlation	Positive	Negative
Perfect Correlation	+1	-1
Very high degree of correlation	+0.9 to +1	-0.9 to -1
Fairly high degree of correlation	+0.75 to +0.9	-0.75 to -0.9
Moderate degree of correlation	+0.50 to +0.75	-0.50 to -0.75
Low degree of correlation	+0.25 to +0.50	-0.25 to -0.50
Very low degree of correlation	0 to +0.25	-0.25 to 0
No correlation	0	0

Q.1 The covariance between two variables is

(a) Strictly positive (b) Strictly negative

(c) Always 0 (d) Either positive or negative or zero

Ans. (d)

COVERIANCE

- **Definition:** Consider a set of n pairs of observations $(x_1, y_1), (x_1, y_1), \ldots\ldots\ldots (x_n n, y_n)$ on two quantitative variables x and y, where $x_1, x_2, \ldots\ldots\ldots$ denote observed values of the variable x, and $y_1, y_2, \ldots\ldots\ldots$ those of y.

  The covariance between x and y, denoted by $Cov(x, y)$, is given by:

  $Cov(x, y) = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n} = \frac{\sum x_i y_i}{n} = \overline{xy}$

Q.2 Find $Cov\ (x, y)$ between x and y if

X	3	4	5	6	7
y	8	7	6	5	4

(a) 2 (b) 3 (c) -2 (d) -1

Ans. (c)

Q.3 Find the $Cov(x, y)$ between x and y if

X	3	4	5	6	7
y	8	7	6	5	4

(a) 2 (b) 3 (c) -2 (d) -1

Ans. (c)

Q.4 Calculate the covariance between X and Y for the following data:

X	1	2	3	4	5	6	7	8	9	10
y	6	9	6	7	8	5	12	3	17	1

(a) 2 (b) 0.2 (c) -0.4 (d) 0.4

Ans. (d)

Q.5 Find the covariance between X and Y, given that $\sum X = 60,\ \sum Y = 60,\ \sum XY = 574$ and $n = 10$

(a) 2 (b) 3.4 (c) 3.2 (d) 3.6

Ans. (b)

KARL PEARSON'S COEFFICIENT OF CORRELATION

The Karl Pearson's coefficient of correlation, also called the Pearson's product - moment correlation coefficient, is the most widely used method of measuring the linear correlation between two variables.

Definition: The Karl Pearson's coefficient of correlation between two variables x and y, denoted by r, is defined by

$$r = \frac{Cov(xy)}{\sigma_x * \sigma_y}$$

Where var. x and var. y are the variance of the values of x and y respectively, while σ_x and σ_y are their standard deviations.

Q.6 If for two variables x and y, the covariance, variance of x and variance of y are 40, 16 and 256 respectively, what is the value of the correlation coefficient?

(a) 0.01 (b) 0.625 (c) 0.4 (d) 0.5

Ans. (b)

Q.7 The covariance between the length and weight of five items is 6 and their standard deviations are 2.45 and 2.61 respectively. Find the coefficient of correlation between length and weight.

(a) 0.9383 (b) 1.9385 (c) 0.2583 (d) 3.6353

Ans. (a)

Q.8 The coefficient of correlation between x and y is 0.5, the covariance is 16. If the standard deviation is x is 4 the standard deviation of y is

(a) 4 (b) 8 (c) 16 (d) 64

Ans. (b)

Q.9 If covariance of 10 pairs of items is 7, variance of X is 36 and $\sum(Y - \bar{Y})^2 = 90$. Find the value of r.

(a) 1.389 (b) 0.389 (c) 0.258 (d) 1.635

Ans. (b)

Q.10 The coefficient of correlation between two variables x and y is 0.4 and their covariance is 10. If the variance of x series is 9, find the standard deviation of y series.

(a) 4.389 (b) 69.445 (c) 8.33 (d) 7.635

Ans. (c)

CHARACTERISTICS OF THE CORRELATION COEFFICIENT

- Correlation coefficient is independent of the units of measurement.
- The correlation coefficient r is independent of change of origin and scale under consideration depending on the sign of scale factors.

  That is, if the variables X and Y are replaced by the variables U and V such that

  $U = aX + b \; and \; V = cY + d$, where a and c are positive constants.

  However, if a and c are arbitrary constants, then.

  $r(aX + b, cY + d) = \frac{a \times c}{|a \times c|} r_{XY}$

  'or'

$r_{uv} = \frac{|a \times c|}{a \times c} r_{XY}$

Thus, $r_{XY} = r_{UV}$ when a and c have the same signs and $r_{XY} - r_{UV}$ when a and c have the opposite signs.

The correlation coefficient lies between -1 and 1, including both the limits.

That is, $-1 \leq r \leq 1$

Q.11What are the limits of the correlation coefficient?

(a) No limit (b) -1 and 1, excluding the limits

(c) 0 and 1, excluding the limits (d) -1 and 1, including the limits

Ans. (d)

Q.12Correlation coefficient is_____of the units of measurement.

(a) Dependent (b) Independent (c) Both (c) None of these

Ans. (b)

Q.13If the correlation between X and Y is r, $U = \frac{X-5}{10}$ and $V = \frac{Y-7}{2}$, then r_{uv} is

(a) r (b) -r (c) $\frac{r-5}{2}$ (d) $\frac{r-7}{10}$

Ans. (a)

Q.14If r=0.58, Correlation coefficient of $u = -5x + 3$ and $v = y + 2$ is

(a) 0.58 (b) -0.58 (c) 0.62 (d) None

Ans. (b)

Q.15The coefficient of correlation between X and Y is 0.6. U and V are two variables defined as U=$\frac{X-3}{2}$, $V = \frac{Y-2}{3}$, then the coefficient of correlation between U and V is

(a) 0.6 (b) 0.4 (c) 0.8 (d) -0.8

Ans. (a)

Q.16If x denotes height of a group of students expressed in cm and y denotes their weight expressed in kg, then the correlation coefficient between height and weight would be shown

(a) in kg (b) in cm (c) in kg and cm (d) free from any unit

Ans. (d)

Q.17The correlation coefficient between two variables X and Y is found to be 0.4. What is the correlation coefficient between 2X and (- Y)?

(a) 0.4 (b) –0.8 (c) 0.8 (d) –0.4

Ans. (d)

Q.18If for two variables A and B, the covariance, variance of A, and variance of B are 60, 25, and 400 respectively, what is the value of the correlation coefficient?

(a) 0.2 (b) 0.5 (c) 0.4 (d) 0.6

Ans. (d)

Q.19The covariance between two variable X and Y is 8.4 and their variance are 25 and 36 respectively. Calculate Karl Pearson's coefficient of correlation between them.

(a) 0.82 (b) 0.25 (c) 0.01 (d) 0.09

Ans. (b)

Q.20If correlation coefficient between x and y is 0.5, then the correlation coefficient between $2x - 3$ and $3 - 5y$ is

(a) 0.5 (b) -0.5 (c) 2.5 (d) -2.5

Ans. (b)

Q.21The coefficient of correlation between X and Y series is -0.38. The linear relation between U and V are $3X + 5U = 3$ and $-8Y - 7V = 44$, what is coefficient between U & V?

(a) 0.38 (b) -0.38 (c) 0.40 (d) None of these

Ans. (b)

COMPUTING THE CORRELATION COEFFICIENT

Various formulas for computing the correlation coefficient between the two variables X and Y:

- The correlation coefficient, r, between two variables X and Y is given by

$$r = \frac{cov(X,\ Y)}{\sqrt{Var\ X}.\sqrt{var\ Y}}$$

If the variable x takes on the values $x_1, x_2, \ldots, x_n$ and the variable y takes on the values $y_1, y_2, \ldots, y_n$ then we have

$$Cov(X,Y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} or\ \frac{\Sigma xy}{n} - \bar{x}\bar{y}$$

$$Var(X) = \frac{\Sigma(x-\bar{x})^2}{n} or\ \frac{\Sigma x^2}{n} - (\bar{x})$$

$$VarY = \frac{\Sigma(y-\bar{y})^2}{n} or\ \frac{\Sigma y^2}{n} - (\bar{y})^2$$

We obtain another formula for r:

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2.\Sigma(y-\bar{y})^2}}$$

Q.22If the sum of the product of the deviation of X and Y from their means is zero. Then the correlation coefficient between X and Y is

(a) Zero (b) Positive (c) Negative (d) 10

Ans. (a)

Q.23Calculate coefficient of correlation from the following results: $n = 100$

$$\sum X = 100, \sum Y = 150, \sum(X-10)^2 = 180\ \sum(Y-15)^2 = 215 \text{ and } \sum(X-10)(Y-15) = 60$$

(a) 0.463 (b) 2.15 (3) 0.305 (d) -0.7618

Ans. (c)

Q.24Find the covariance between X and Y for the following data:

X	66	67	68	69	70	71	72
y	68	67	70	70	69	70	69

(a) 0.67 (b) 2.4 (c) 0.25 (d) 1.6

Ans. (a)

Q.25The coefficient of correlation between X and Y series from the following data is

	X series	Y series
Number of pairs of observation	15	15
Arithmetic mean	25	18
Standard Deviation	3.01	3.03
Sum of squares of dev. From mean	136	138

Sum of the product of the deviations of X and Y series from their respective means = 122

(a) 0.89 (b) 0.99 (c) 0.69 (d) 0.91

Ans. (a)

Q.26Given the following information: $r = 0.8, \sum xy = 60, \sigma_y = 2.5$ and $\sum x^2 = 90$. Where x and y are the deviation from the respective means. Find the number of items (n)

(a) 8 (b) 11 (c) 14 (d) 10

Ans. (d)

ANOTHER FORMULA

Also,

$$r = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{\frac{\sum xy}{n} - \frac{\sum x \sum y}{n \; n}}{\sqrt{\frac{\sum x^2}{n} - (\frac{\sum x}{n})} \sqrt{\frac{\sum y^2}{n} - (\frac{\sum y}{n})^2}}$$

$$r = \frac{\frac{n\sum xy - \sum x \sum y}{n^2}}{\sqrt{\frac{\sum x^2}{n} - \frac{(\sum x)^2}{n^2}} \sqrt{\frac{\sum y^2}{n} - \frac{(\sum y)^2}{n^2}}} = \frac{\frac{n\sum xy - \sum x \sum y}{n^2}}{\frac{n\sum x^2 - (\sum x)^2}{n^2} \sqrt{\frac{n\sum y^2 - (\sum y)^2}{n^2}}}$$

$$r = \frac{\frac{n\sum xy - \sum x \sum y}{n^2}}{\frac{\sqrt{n\sum x^2 - \sum (x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}{n^2}}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}$$

FORMULAE OF CORRELATION COEFFICIENT

- $r = \frac{Cov(X,Y)}{\sqrt{Var\ X}.\sqrt{VarY}}$
- $r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$
- $r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2-(\sum x)^2}\sqrt{n\sum y^2-(\sum y)^2}}$

Q.27For the set of observation {(1, 2), (2, 5), (3, 7), (4, 8), (5, 10)}, the value of Karl-Pearsons' coefficient of correlation is approximately given by

(a) 0.755 (b) 0.655 (c) 0.525 (d) 0.985

Ans. (d)

Q.28The coefficient of correlation between x and y where

X	64	60	67	59	69
Y	57	60	73	62	68

Is

(a) 0.65 (b) 0.68 (c) 0.73 (d) 0.758

Ans.(a)

Q.29What would be the correlation between u and v?

X	10	15	25	20	35
y	-24	-36	-42	-48	-60

(a) -6.0 (b) -0.3224 (c) -0.93 (d) 0.93

Ans. (c)

Q.30The coefficient of correlation between X and Y if $cov(x, y)$=16.5, $Var(X) = 8.25$ and $Var(Y) = 33$

(a) 0 (b) 1 (c) 2.5 (d) 4.3

Ans. (b)

Q.31The coefficient of correlation between X and Y for the following data:

$n = 25, \sum X = 55, \sum Y = 40, \quad \sum X^2 = 385, \quad \sum Y^2 = 192, \quad \sum XY = 185$

(a) 0.068 (b) -0.068 (c) 0.186 (d) None of these

Ans. (b)

Q.32Find the covariance between X and Y, given that $\sum X = 60, \sum Y = 90, \sum XY = 574$ and $n = 10$

(a) 1.5 (b) 2.7 (c) 3.4 (d) None of these

Ans. (c)

Q.33What is the coefficient of correlation between the ages of husbands and wives from the following data?

Age of Husband (Year)	46	45	42	40	38	35	32	30	27	25
Age of wife (Year)	37	35	31	28	30	25	23	19	19	18

(a) 0.58 (b) 0.98 (c) 0.89 (d) 0.92

Ans. (b)

SPEARMAN'S COEFFICIENT OF RANK CORRELATION

- In situations where we have a series of items that cannot be measured numerically, such as qualities like beauty, intelligence, leadership ability, honesty, etc., we can still compare the rankings of individuals within a group.
- But the individuals in the group can be arranged in order thereby obtaining for each individual a number indicating its rank in the group.
- If we have a group of individuals ranked according to two different qualities, it is natural to ask the following question: "Is there an association between the rankings?"
- To answer this question, we need to use a formula known as Spearman's coefficient of rank correlation.
- The Spearman's correlation coefficient is nothing but Karl Pearson's correlation coefficient between the ranks and is interpreted in much the same way.

The Spearman's correlation coefficient, denoted by "p" will range from -1 *to* $+1$. A value of $+1$ indicates perfect association for identical rankings and a value of -1 indicates perfect association for reverse rankings.

Rank R_1	Rank R_2
1	7
2	8
3	6
4	1
6	2
8	4
7	3
5	5

In this example, we have two sets of ranking, R_1 and R_2. The values in the table represent the ranks assigned to the individuals for each quality. By applying Spearman's coefficient of rank correlation formulae, we can determine the extent of association between these rankings.

COMPUTING THE RANK CORRELATION COEFFICIENT

Case I: We shall actual ranks are given

In this case the following steps are involved:

1. Compute D, the difference between the two ranks given to each individual.
2. Compute D^2 and obtain the $\sum D^2$

Apply the formula: $\rho = 1 - \frac{6\sum D^2}{n(n^2-1)}$ where, n is the number of boservations.

Q.34Determine Spearman's rank correlation coefficient from the given data $\sum D^2 = 30$ $N = 10$.

(a) R = 0.82 (b) R=0.32 (c) R=0.40 (d) None of these

Ans. (a) Here, $\sum D^2 = 30,\ N = 10$

Spearman's rank correlation is;

$$\rho = 1 - \frac{6\sum D^2}{N(N^2-1)} = 1 - \frac{6\times 30}{10(10^2-1)} = 1 - \frac{180}{990} = 1 - \frac{2}{11} = \frac{9}{11} = 0.82$$

Therefore, R=0.82

Hence, the correct answer is option (a), i.e, R=0.82

Q.35If the sum of squares of the rank differences of 9 pairs of value is 80, find the correlation coefficient between them

(a) 0.33 (b) 0.39 (c) -0.33 (d) 0.039

Ans. (a)

Q.36Ten competitors in following in a beauty contest are ranked by two judges in the following order:

I Judge	1	6	5	10	3	2	4	9	7	8
II Judge	6	4	9	8	1	2	3	10	5	7

Calculate the Spearman's rank correlation coefficient.

(a) 0.6364 (b) 1.395 (c) -0.9894 (d) 0.7618

Ans. (a)

Q.37In a bivariate data of n pairs of observations, the sum of squares of differences between the ranks of observed values of two variables is 231 and the rank correlation coefficient is -0.4. Find the value of n.

(a) 10 (b) 13 (c) $-5 \pm \sqrt{74i}$ (d) 5

Ans. (a)

Q.38The coefficient of rank correlation of the marks obtained by 10 students in Statistics and Accountancy was found to be 0.2. It was later discovered that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 9 instead of 7. Find the correct value of coefficient of rank correlation.

(a) 0.3334 (b) 0.3939 (c) -0.3334 (d) -0.3939

Ans. (b)

Case II: When actual ranks are not given:

- Sometimes we are given the actual bivariate data on two variables and not the ranks. In such situations, it is necessary to assign the ranks.
- Ranks can be assigned by taking either the highest value as 1 or the lowest value as 1. The next highest or the next lowest value is given rank 2 and so on.
- But whether we start with the lowest value or the highest value, we must follow the same method in case of both the variables.

Q.39The marks obtained by 9 students in Mathematics and Accountancy are as follows:

Marks in Mathematics (X)	30	33	45	23	8	49	12	4	31
Marks in Accountancy (Y)	35	23	47	17	10	43	9	6	28

Calculate Spearman's rank correlation coefficient.

(a) 0.4 (b) 0.39 (c) -0.34 (d) 0.9

Ans. (d)

WHEN RANKS ARE EQUAL

- If there are two or more items with the same rank in either series, then it is customary to assign common rank to each repeated item.
- The common rank is the average of the ranks which these items would have got if they were different from each other and the next item will bet the rank next to the rank used in computing the common rank.
- For example, suppose there are two items at rank 4. In this case, the common rank assigned to each item would be 4.5.
- The next item in line would then be assigned rank 5.
- Similarly, if there are three items at rank 7, the common rank assigned to each item will be 7.
- The next rank to be assigned will be 10.
- When equal ranks are assigned to certain items, an adjustment is made in the formula for calculating the Spearman's rank correlation coefficient. This adjustment involves adding a correction factor is to be added for each repeated item in both the series. The formulae can thus be written as follows:

$$\rho = 1 - \frac{6\sum D^2}{n(n^2-1)}$$

Q.40Calculate Spearman's coefficient of rank correlation from the following data:

X	57	16	24	65	16	16	9	40	33	48
Y	19	6	9	20	4	15	6	24	13	13

(a) 0.7515 (b) 0.3009 (c) -0.3674 (d) 0.7899

Ans. (a)

MERITS AND DEMERITS OF SPEARMAN'S RANK CORRELATION METHOD

Merits

The rank correlation method has the following merits

1. It is easy to understand and simple to apply.
2. The Spearman's rank correlation method is the only method that can be used to find correlation coefficients if we are dealing with data of qualitative characteristics like beauty, intelligence, honesty, etc.
3. This is the only method that can be used where we are given the ranks and not the actual bivariate data on two variables.

DEMERITS

The rank correlation method has the following limitations:

1. This method cannot be used for finding correlation in the case of bivariate frequency distribution.

2. This method is very difficult to apply when the number of items is more than 30.

Q.41If the sum of square of differences of rank is 50 and the number of items is 8, then what is the value of the rank correlation coefficient?

(a) 0.59 (b) 0.40 (c) 0.36 (d) 0.63

Ans. (b)

Q.42If the rank correlation coefficient between marks in Management and Mathematics for a group of students is 0.6 and the sum of the squares of the difference in rank is 66 . Then what is the number of students in the group?

(a) 9 (b) 10 (c) 11 (d) 12

Ans. (b)

Q.43The ranks of five participants given by two judges are:

Participants

	A	B	C	D	E
Judge I	**1**	**2**	**3**	**4**	**5**
Judge II	**5**	**4**	**3**	**2**	**1**

(a) 1 (b) 0 (c) -1 (d) 12

Ans. (c)

Q.44The coefficient of rank correlation between debenture prices and share prices of a company is found to be 0.143. If the sum of the squares of the difference in ranks is 48, find the value of n

(a) 5 (b) 6 (7) 7 (d) None of these

Ans. (c)

COEFFICIENT OF CONCURRENT DEVIATION

The coefficient of concurrent deviation is a statistical measure used to assess the degree of simultaneous variation or deviation in two variables. It helps in understanding how two sets of data move together or diverge from each other. The formula for the coefficient of concurrent deviation is as follows

$$r_c = \pm\sqrt{\pm\frac{2c-m}{m}}$$

where, $'n'$ represents the number of concurrent deviation, $'m'$ represents the total number of deviations (which must be one less than the number of pairs of x and y values)

Also, if $(2c-m) > 0$, then we take the positive sign both inside and outside the radical sign and if $(2c-m) < 0$, we take the negative sign both inside and outside the radical sign.

Q.45What is the coefficient of concurrent deviation for the following data

Supply	68	43	38	78	66	83	38	23	83	63	53
Demand	65	60	55	61	35	75	45	40	85	80	85

(a) 0.82 (b) 0.85 (c) 0.89 (d) -0.81

Ans. (c)

Q.46For 10 pairs of observations, number of concurrent deviations was found to be 4. What is the value of the coefficient of concurrent deviation?

(a) $\sqrt{0.2}$ (b) $-\sqrt{0.2}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

Ans. (d)

Q.47The coefficient of concurrent deviation for P pairs of observations was found to be 13. If the number of concurrent deviations was found to be 6, then the value of P is

(a) 10 (b) 9 (c) 8 (d) None of these

Ans. (a)

REGRESSION

Regression, in a literal sense, refers to a process of moving backward or returning to an average value. In the context of statistics and mathematics, it involves creating a mathematical equation to predict the value of one variable based on known values of one or more other variables. It is a method for understanding relationships and making forecasts.

E.g.: Consider predicting a student's final exam score based on the number of hours spent studying. Regression analysis could help develop an equation that models this relationship, allowing educators to make predictions about a student's potential performance based on their study hours. The regression equation would provide a tool for forecasting final exam scores using the known variable of study hours.

- **Dependent Variable:** The variable whose value is to be predicted is called the dependent variables or explained variable.

- **Independent Variable:** The variables which are used to predict the values of a dependent variable are called independent variables or explanatory variables.
- **Simple Regression Analysis and Simple Linear Analysis:** The regression analysis confined to the study of only two variables, a dependent variable and an independent variable, is called simple regression analysis.

  When the relationship between the dependent variables and the independent variable is linear, the technique for prediction is called simple linear regression.

  If let say y depends on x, then equation will be:-

  $\Rightarrow y = a + bx$
- **METHOD OF LEAST SQUARE: REGRESSION**: Thus, if a line of best fit approximating the given data has the equation $Y = a + bX$ then the method of least squares requires that we must determine constants a and b the so as to minimize

  $e_i = \sum(y_i - \hat{y})^2 = \sum y_i - (a + bx_i)^2$ where $\hat{y} = a + bx_i$

  Where, e_i is the error of estimation.
- The equation known as the normal equation for estimating a and b, are given by

  $$\sum y = an + b\sum x$$

  $$\sum xy = a\sum x + b\sum x^2$$

  Multiply first eq. by $\sum x$ and second eq. by n, we get

  $\sum x \sum y = an\sum x + b(\sum x)^2$

  And $n\sum xy = an\sum x + bn\sum x^2$

  On subtracting the above equations, we get

  $\sum x \sum y = an\sum x + b(\sum x)^2$

  $\sum x \sum y = an\sum x + nb\sum x^2$

  $\sum x \sum y - n\sum x\, y = b(\sum x)^2 - nb\sum x^2$

  $(n\sum x\, y - \sum x \sum y) = b(n\sum x^2 - (\sum x)^2)$

  $\Rightarrow b = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$

  $\Rightarrow b = \frac{n\sum xy - \sum x\sum y/n}{n\sum x^2 - (\sum x)/n^2}$

  $\Rightarrow b = \frac{\frac{\sum xy}{n^x} - \frac{\sum x\sum y}{n^2}}{\frac{\sum x^2}{n^2} - \frac{(\sum x)^2}{x^2}}$

  $\Rightarrow b = \frac{\frac{\sum xy}{n} - \frac{\sum x\sum y}{n}\frac{n}{n}}{\frac{\sum x^2}{n} - (\frac{\sum x}{n})^2} = \frac{cov(x,y)}{\sqrt{x^2}}$

  Solving equation simultaneously for a and b, we obtain

  $b_{yx} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$

Hence, the line of best fit approximating n pair of observations $(x_1, y_1), (x_2, y_2) \ldots \ldots, (x_n y_n)$is y on x

$Y = a + bx$, where a, b are the constants.

The line of best fit given is called the least squares line of regression of y on x. The constant b is called the regression coefficient of y on x is denoted by b_{yx}

It measures the change in y corresponding to a unit change in x.

- Thus, b_{yx} represent the slope of the line of regression of Y on X and is given by

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

On the other hand, if we wish to estimate a value of X for a given value of Y, we have to obtain a regression line of X on Y:

$X = bY + a.$

The line of best fit given is called the least squares line of regression of X on Y. The constant b is called the regression coefficient of X on Y is denoted by b_{XY}.

It measures the change in X corresponding to a unit change in Y.

Thus, b_{XY}. Represent the slope of the line of regression of Y on X and is given by $b_{xy} = \frac{n\sum xy - \sum x \sum y}{N\sum y^2 - (\sum y)^2}$

The equation of the line of regression of Y on X can also be written as $(y - \bar{y} = b_{yx}(x - \bar{x})$

REGRESSION LINES

Since, $\sum y = an + b\sum x$

$(\frac{\sum y}{n}) = \frac{na + b\sum x}{n}$

$(\frac{\sum y}{n}) = \frac{na}{n} + \mathrm{b}\frac{\sum x}{n}$

$\Rightarrow \bar{y} = \mathrm{a} + \mathrm{b}\bar{x}$

$\Rightarrow a = \bar{y} - b\bar{x}$ ...(i)

$\Rightarrow b_{yx} = \mathrm{b} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$

(y on x)

$b_{yx} = \frac{cov(x,y)}{(\sigma_x)^2}$

$b \text{ or } b_{yx} = \mathrm{r}(\frac{\sigma_y}{\sigma_x})$ ...(ii)

Regression line of y on x is given by

$y = a + bx$

From (i)

$\Rightarrow y = (\bar{y} - b\bar{x}) + bx$

$\Rightarrow y - \bar{y} = b(x - \bar{x})$

From (ii)

$\Rightarrow y - y = r(\frac{\sigma_y}{\sigma_x})(x - \bar{x})$

(Regression line in correlation coefficient)

Q.48Calculate the regression coefficient from the following information:

$\sum X = 50, \sum Y = 30, \sum XY = 1000, \sum X^2 = 3000, \sum Y^2 = 1800$ and $n = 10$

(a) 0.497 and 0.309 (b) 0.307 and 0.3009

(c) 0.586 and -0.367 (d) None of these

Ans. (a)

Q.49In the regression equation X on Y, $X=\frac{35}{8} - \frac{2Y}{5}\, b_{xy}$ is equal to

(a) $-\frac{2}{5}$ (b) $\frac{35}{8}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$

Ans. (a)

Q.50The regression equation of y on x for the following data:

X	41	82	62	37	58	96	127	74	123	100
Y	28	56	35	17	42	85	105	61	98	73

(a) $y = 12x - 15$ (b) $y = 12x + 15$

(c) $y = 0.93x - 14.68$ (d) $y = 1.5x - 10.89$

Ans. (c)

Q.51Following table gives the age of cars of a certain make and annual maintenance costs. Obtain the regression equation for cost related to age:

Age of cars in year:	2	4	6	8
Maintenance cost (in hundred)	10	20	25	30

Also estimate the annual maintenance cost for the ten-year –old car.

(a) $3.792x - y - 1.28 = 0\ and\ 30$

(b) $0.72x - 1.30y - 1.238 = 0\ and\ 40$

(c) $3.25x - y + 5.0 = 0\ and\ 37.5$

(d) $0.792x - y - 1.28 = 0\ and\ 37.5$

Ans. (c)

Q.52If mean of X and Y variables is 20 and 40 respectively and the regression coefficient Y on X is 1.608, then the regression line of Y on X is:

(a) $Y = 1.608\,X + 7.84$ (b) $Y = 1.56\,X + 4.84$

(c) $Y = 1.608\,X + 4.84$ (d) $Y = 1.56\,X + 7.84$

Ans. (a)

Q.53If Y=18X+5 is the regression line of X on Y, the value of b_{xy} is

(a) $\frac{5}{18}$ (b) 18 (c) 5 (d) $\frac{1}{18}$

Ans. (d)

Q.54The regression equation of Y on X is, 2X+3Y+50=0. The value of b_{yx} is

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{-3}{2}$ (d) None of these

Ans. (b)

Q.55The two regression coefficients for the following data are:

X	38	23	43	33	28
Y	28	23	43	38	8

(a) 1.2 and 0.4 (b) 1.6 and 0.8 (c) 1.7 and 0.8 (d) 1.8 and 0.3

Ans. (a)

SOME MORE FORMULAS - REGRESSION

I. Formulas for Regression Coefficient in terms of Covariance and Variances:

By definition, the regression coefficient of Y and X is given by

$$b_{yx} = \frac{Cov\,(x,y)}{\sigma_x^2} \; or \; b_{yx} = r.\frac{\sigma_y}{\sigma_x}$$

Similarly, the regression coefficient of X and Y is given by

$$b_{yx} = \frac{Cov\,(x,y)}{\sigma_y^2} \; or \; b_{yx} = r.\frac{\sigma_x}{\sigma_y}$$

The reader may also recall that the covariance between X and Y & the variance of X and Y values are respectively given by

$$Cov(X,Y) = \frac{\sum(x-\bar{x})(y-\bar{y})}{n} \; or \; \frac{\sum xy}{n} - \bar{x}.\bar{y}$$

$$\sigma_x^2 = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \; or \; \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

$$\sigma_y^2 = \sqrt{\frac{\sum(y-\bar{y})^2}{n}} or \sqrt{\frac{\sum y^2}{n} - (\bar{y})^2}$$

Formulas for regression coefficient in terms of deviation of $X-$ and $Y-$ values from their respective means.

By definition, the covariance between x and y is given by

$$Cov(x,y) = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{N} = \sqrt{\frac{x_i y_i}{N}} = \bar{x}\,\bar{y}$$

Further, the variance of x and y values are respectively given by

$$\sigma_x^2 = \frac{Cov(X,Y)}{b_{yx}} \; and \; \sigma_y^2 = \frac{Cov(X,Y)}{b_{xy}}$$

Thus, using formulas, we obtain

$$\sigma_y^2 = \frac{Cov(X,Y)}{b_{xy}} \; and \; b_{xy} = \frac{Cov(X,Y)}{\sigma_y^2}$$

RELATION OF REGRESSION COEFFICIENTS AND CORRELATION COEFFICIENT

$$r = \pm\sqrt{\pm b_{XY} \times b_{YX}} \text{ and } r. = \frac{Cov(x.y)}{\sigma_x \sigma_y}$$

Q.56Given $\sigma_x = 20, \sigma_y = 20 \; and \; cov(X,Y) = -100$ find:

1. Correlation coefficient
2. Both the regression Coefficients.

(a) 0.25, 30, 78

(b) -0.25 -0.25, -0.25

(c) 25 0.55 0.10

(d) -0.45 0.25 0.72

Ans. (b)

Q.57Given $x = 50$ $y = 20, \sigma_x = 20$ and $\sigma_y = 20$, find both the regression Coefficient.

(a) $0.25x + y - 32.5 = 0, x + 0.25y - 55 = 0$

(b) $0.25x + y - 32.5 = 0, x + 0.25y - 55 = 0$

(c) $x + 0.25y - 55 = 0, 0.792x - 1.033y - 1.228 = 0$

(d) $-0.45x + 0.25y - 55 = 0, 0.792x - 1.0y - 1.28 = 0$

Ans. (b)

Q.58Find the regression coefficient b_{yx} and b_{xy} of Y on X and X on Y respectively, if standard deviations of X and Y are 4 and 3 respectively, and coefficient of correlation between X and Y is 0.8

(a) 2.4, 3.8 (b) 8.2, 4.8 (c) 1.2 2.2 (d) 3.2, 2.4

Ans. (d)

Q.59 The coefficient of correlation between the ages of husband and wives in a community was found to be +0.8. The average of the husband's age of 25 years and that of the wives age is 22 years. Their standard deviations were 4 and 5 respectively. Find with the help of a regression equation, the expected age of husband when wife's age is 18 years.

(a) 12 (b) 13 (c) 10 (d) 11

Ans. (d)

PROPERTIES OF REGRESSION COEFFICIENTS

SOME IMPORTANT PROPERTIES OF REGRESSION COEFFICIENTS

- **Property 1:** The coefficients of correlation and two regression coefficient have the same signs.
- **Property 2:** The coefficients of correlation are the geometric mean between the regression coefficients.
- **Property 3:** If one of the regression coefficients is greater than unity, the other must be less than the unity.
- **Property 4:** The two lines of regression intersect at the point $(\bar{x}, \bar{y})$ where x and y are the variables under consideration.
- **Property 5:** The regression coefficients are independent of change of origin but not for scale.

Determining the line of regression of y on x and that of x on y out of the given two regression lines:

Sometimes, it required to find the line of regression of y on x or x on y out of the given two regression lines in such a case we below the following steps:

Step 1: Choose any one of the two regression lines as the lines of regression of y on x and the other as the line of regression of x on y.

Step 2: Find two regression coefficients b_{xy} and b_{yx}

Step 3: Compute the product b_{xy}. b_{yx} . if b_{xy}. $b_{yx} \leq 1$, then the assumption made in step 1 is correct, otherwise the assumption is wrong.

Q.60 Regression coefficient are _____

(a) dependent of change of origin and of scale.

(b) independent of both change of origin and of scale.

(c) dependent of change of origin but not of scale.

(d) independent of change of origin but not of scale.

Ans. (d)

Q.61 If the regression line of y on x and of x on y are given by $2x + 3y = -1$ and $5x + 6y = -1$, then the arithemetic mean of x and y are given by

(a) $(1,-1)$ (b) $(-1,1)$ (c) $(-1,-1)$ (d) $(2,3)$

Ans. (a)

Q.62If u=2x+5 and v=-3y-6 and regression coefficient of y on x is 2.4, what is the regression coefficient of v on u? (a) 3.6 (b) -3.6 (c) 2.4 (d) -2.4

Ans. (b)

Q.63If $4y - 5x = 15$ is the regression line of y on x and the coefficient of correlation between x and y is 0.75, what is the value of the regression coefficient of x and y?

(a) 0.45 (b) 0.9375 (d) 0.6 (d) None of these

Ans. (a)

Q.64If the regression line of y on x and of x on y are given be $2x + 3y = -1$ and $5x + 6y = -1$, then the arithmetic means of x and y are given by

(a) (1, -1) (b) (-1, 1) (c) (-1, -1) (d) (2, 3)

Ans. (a)

Q.65The regression equation are $2x + 3y + 1 = 0$ and $5x + 6y + 1 = 0$, then Mean of x and y respectively are

(a) $-1,-1$ (b) $-1,1$ (c) $1,-1$ (d) $2,3$

Ans. (c)

Q.66The equations of the two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the correlation coefficient between x and y

Ans. (a)

COEFFICIENT OF DETERMINATION AND NON-DETERMINATION

As we know, Correlation coefficient measures a linear relationship between the two variables which tells the amount of variation in one variable with respect to another variable. Thus, a better measure of correlation is indicated by square of the correlation coefficient, which is known as 'Coefficient of Determination'' given by

Coefficient of determination = r^2

For coefficient of correlation, $r = 0.4$, then $r^2 = (0.4)^2 = 0.16$ indicated 16% of variation which is accounted by ther factor under consideration and remaining 84% is due to other factors which we call it as ''Coefficient of Non-determination'' given by $1 - r^2$.

i.e., Coefficient of Non-determination = $1 - r^2$

Q.67What is a spurious correlation?

(a) It is a bad relation between two variables.

(b) It has very low correlation between the two variables.

(c) It is the correlation between two variables having no causal relation.

(d) It is a negative correlation.

Ans. (c)

Q.68Scatter diagram helps us to

(a) Find the nature correlation between two variables

(b) Compute the extent of correlation between two variables

(c) Obtain the mathematical relationship between two variables

(d) Both (a) and (c)

Ans. (d)

Q.69When r=1, all the points in a scatter diagram would lie

(a) On a straight line directed from lower left to upper right

(b) On a straight line directed from upper left to lower right

(c) On a straight line

(d) Both (a) and (b)

Ans. (d)

Q.70What are the limits of the correlation coefficient?

(a) No limit (b) −1 and 1, excluding the limits

(c) 0 and 1, including the limits (d) −1 and 1, including the limits

Ans. (d)

Q.71If the coefficient of correlation between two variables is -0.9, then the coefficient of determination is

(a) 0.9 (b) 0.81 (d) 0.1 (d) 0.19

Ans. (b)

Q.72If the coefficient of correlation between two variables is 0.7, then the percentage of variation unaccounted for is

(a) 70% (b) 30% (c) 51% (d) 49%

Ans. (d)

Q.73What are the limits of the two regression coefficients?

(a) No limit

(b) Must be positive

(c) One positive and the other negative

(d) Product of the regression coefficient must be numerically less than unity.

Ans. (d)

Q.74If the regression line of Y on X is given by Y=X+2 and Karl Pearson's coefficient is 0.5 then $\frac{\sigma_y^2}{\sigma_x^2}$?

(a) 3 (b) 2 (c) 4 (d) None

Ans. (c)

Q.75Determine spearman's rank correlation coefficient from the given data $\sum d^2 = 30, n = 10$

(a) $r = 0.82$ (b) $r = 0.32$ (c) $r = 0.40$ (d) None of these.

Ans. (a)

Q.76Which of the following statements is not true about scatter diagram?

(a) It finds the type of correlation

(b) It helps to identify whether variables are correlated or not

(c) It determines the linear or not – linear correlation

(d) If finds the numerical value of correlation coefficient

Ans. (b)

Q.77A.M of regression coefficient is

(a) Equal to r (b) Greater than or equal to r

(c) Half of r (d) None

Ans. (b)

Q.78If the correlation coefficient between the variables X and Y is 0.5, then the correlation coefficient between the variables $2x - 4$ and $3 - 2y$ is

(a) 1 (b) 0.5 (c) -0.5 (d) 0

Ans. (c)

Q.79If r=0.6 then the coefficient of non – determination will be:

(a) 0.40 (b) -0.60 (c) 0.36 (d) 0.64

Ans. (d)

Q.80The correlation coefficient (r) is the____of the two regression coefficient (b_{yx} *and* b_{xy})

(a) AM (b) GM (c) HM (d) Median

Ans. (b)

Q.81The coefficient of determination is defined by the formula

(a) $r^2 = \frac{unexplained\ variance}{Total\ variance}$ (b) $r^2 = \frac{explained\ variance}{Total\ variance}$

(c) Both (a) and (b) (d) None of these

Ans. (c)

Q.82Correlation coefficient is_____of units of measurement.

(a) Dependent (b) Independent (c) Both (d) None of these

Ans. (b)

Q.83The two lines of regression intersect at the point

(a) Mean (b) Mode (c) Median (d) None of these

Ans. (a)

Q.84If the two lines of regression are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$, then the regression line of y on x is:

(a) $x + 2y - 5 = 0$ (b) $2x + 3y - 8 = 0$

(c) $x + 2y = 0$ (d) $2x + 3y = 0$

Ans. (a)

Q.85If the sum of square of the difference of ranks, given by two judges X and Y of 10 students is 28, what is the value of the rank correlation coefficient?

(a) 0.725 (b) 0.650 (c) 0.750 (d) 0.873

Ans. (d)

Q.86For 12 pairs of observations, the number of concurrent deviations was found to be 3. What is the value of the coefficient of concurrent deviation?

(a) $\sqrt{\frac{5}{11}}$ (c) $-\sqrt{\frac{5}{11}}$ (d) None of these

(b) $\frac{5}{11}$

Ans. (c)

Q.87If the two regression lines are 3X=Y and 8Y=6X, then the value of correlation coefficient is

(a) -0.5 (b) 0.5 (c) 0.75 (d) -0.80

Ans. (b)

SUMMARY

There are four ways to find Correlation:

- **Scatter Diagram:** A scatter diagram is a graphical presentation of bivariate data $\{(X_i, Y_i): i = 1, 2, \dots n\}$ on two quantitative variables X and Y that allows us to show two variables together.
- **Perfect Positive Correlation:** If the points of the scatter diagram fall on a straight line and have a positive(upward) slope, then the correlation is said to be perfectly positive;
- **Positive Correlation:** When the points of the scatter diagram cluster around a straight line (upward slope from left to right), then the correlation is said to be positive.
- **Perfect Negative Correlation:** If the points of the scatter diagram fall on a straight line and have a negative(downward) slope, then the correlation is said to be perfectly negative
- **Negative Correlation:** When the points of the scatter diagram cluster around a straight line (downward/negative slope), then the correlation is said to be negative.
- **No Correlation:** When the points of the scatter diagram are scattered in a haphazard manner, then there is zero or no correlation.
- **Correlation coefficient (r):**

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

Degree of Correlation	Positive	Negative
Perfect correlation	$+1$	-1
Very high degree of correlation	$+0.9\ to\ +1$	$-0.9\ to\ -1$
Fairly high degree of correlation	$+0.75\ to\ +0.9$	$-0.75\ to\ -0.9$
Moderate degree of correlation	$+0.50\ to\ +0.75$	-0.50 to -0.75
Low degree of correlation	$+0.25\ to\ +0.50$	$-0.25\ to\ -0.5$
Very low degree of correlation	$0\ to\ +0.25$	$-25\ to\ 0$
No correlation	0	0

REGRESSION

- Linear regression establishes the linear relationship between two variables based on a line of best fit. Linear regression is thus graphically depicted using a straight line with the slope defining how the change in one variable impacts a change in the other.

- The y −intercept of a linear regression relationship represents the value of one variable when the value of the other is zero
- Formulated by : $y = mx + b$

  where m is the slope and b is the intercept

  $r = \pm\sqrt{b_{yx} \times b_{xy}}$

  $b_{yx} = \frac{q}{p} \times b_{vu}$ where $u = \frac{x-a}{p}$ and $v = \frac{y-c}{q}$

  Coefficient of determination: r^2.

INDEX NUMBER

An Index Number is a statistical measure that is used to track and represent changes in a variable or a collection of related variables over time, space, or other factors. It provides a way to summarize and compare data to understand the overall trend or performance.

In the context of financial markets, Indices play a crucial role in measuring the performance of stock markets. For example, the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE) in India provide indices such as the Sensex and Nifty, respectively. These indices represent the collective performance of a specific set of stocks listed on the respective exchanges.

Market Summary > NIFTY 50

Market Summary > BSE SENSEX

Definition

An index number is a ratio of two or more time periods involved, one of which is the base time period. The value at the base time period serves as the standard point of comparison.

Types of Index Number

Types	Significance
Price Index Numbers	Shows Movements in the price levels between the base year and other periods. Increase in Price Level is called Inflation whereas decrease refers to Deflation.
Quantity Index Number	Shows the movement in value levels between two periods. Value = Price × Quantitly It is used for computing Growth rate of an Economy etc.
Value Index Numbers	Shows the movement in value levels between two period. Value = Price × Quantity It is used for computing Growth rate of an Economy etc.

Index Time Series: An index Time Series is a list of index numbers for two or more periods of time, where each index number employs the same base year.

- **Issues involved in Construction of Index Numbers:**

I. **Selection of Data:** Choosing the right data is crucial. For instance, if you're making an index to measure the cost of living, you should focus on prices that directly affect living expenses while excluding prices for things like machinery.

II. **Base Period:** The base period serves as your reference point. It should be a stable time, not influenced by unusual events like wars. A relatively recent base period is often better, and there are different methods for choosing it.

III. **Selection of Weights:** Every variable in your index needs a weight based on its importance for your specific purpose. For example, if you're calculating a cost-of-living index, essentials like cereals might weigh more than non-essentials like sugar.

IV. **Use of Averages:** Averages are vital. The type of average you pick, be it geometric or arithmetic, depends on the nature of your index.

V. **Choice of Variables:** Deciding what variables to use, like price or quantity, is essential. For a price index, you need to choose between wholesale or retail prices and decide on the relevant time frame for prices.

VI. **Selection of Formula:** The formula you use is critical. Different formulas applied to the same data can yield different results. So, it's important to choose the right one for your index.

- **CONSTRUCTION OF INDEX NUMBERS:**

Notation: If we have to take the prices of 3 different commodity for n^{th} period then it can be written as

$P_n(1), P_n(2), P_n(3)$

While if we take price corresponding to base period for 3 different commodities, it will be written as

$P_0(1), P_0(2), P_0(3)$

Now let's say we have commodity 'j' where j is variable and varying from 1 to k then:

Summation for all prices of nth period can be written as $\sum_{j=1}^{k} P_n(j) or \ \sum P_n(j)$.

RELATIVES

1. Price relative: As we are discussing the prices, let's talk about relative prices which is called as price relative. Price relative can be defined as the ratio of Price of a single commodity in one time period to the price of base period or reference period.

It is written as: Price relative $= \frac{P_n}{P_0}$

If we have to express it is form of percentage, it can be multiplied by 100:

Price relative $= \frac{P_n}{P_0} \times 100$

E.g.: Let's consider the price of a particular product, a smartphone, in two time periods: the base period (2010) and the current period (2021). In 2010. the smartphone was priced at `20,000. While in 2021, it was priced at `48,000. To calculate the price relative, we can use the formula:

$$\Rightarrow Price\ Relative = \frac{Price\ in\ 2021}{Price\ in\ 2010} \times 100 = \frac{48{,}000}{20{,}000} \times 100 = 2.4 \times 100 = 240\%$$

This indicates that the price of the smartphone has increased by 240% from the base period to the current period in terms of Indian Rupees.

As we discussed for Price relative, the same discussion can happen in terms of quantity, volume of consumption etc.

Then, in that case relatives will be:

2. **Quantity Relative:** The quantity relative compares the quantity of a particular commodity in one time period to the quantity in a base or reference period.

$$\text{Quantity Relative} = \frac{Q_n}{Q_0}$$

3. **Value relative:** The value relative considers the value of a commodity by multiplying the price and quantity relatives together.

$$\text{Value Relative} = \frac{V_n}{V_0} = \frac{P_n}{P_0} \times \frac{Q_n}{Q_0} = \frac{P_n Q_n}{P_0 Q_0}$$

Link Relative: When we take ratio of successive price or quantities, then it is called the link relatives i.e.,

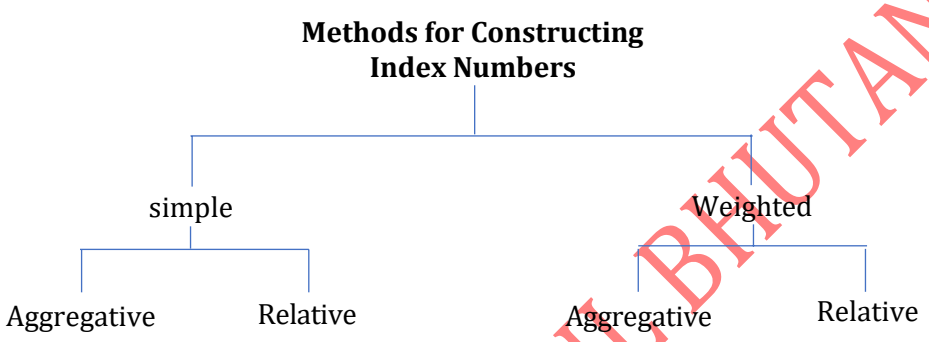
$$\frac{P_1}{P_0}, \frac{P_2}{P_1}, \frac{P_3}{P_2}, \frac{P_n}{P_{n-1}}$$

Commodity	Milk (per litre)	Link Relative
2005	50	$\frac{50}{50} \times 100 = 100$
2010	55	$\frac{55}{50} \times 100 = 110$
2015	60	$\frac{60}{50} \times 100 = 109.09$
2020	70	$\frac{70}{50} \times 100 = 116.67$

Chain Relative: When the ratio is taken in respect to base price then it is called the chain relatives i.e.,

$$\frac{P_1}{P_0}, \frac{P_2}{P_0}, \frac{P_3}{P_0}, \frac{P_n}{P_0},$$

Commodity	Milk (per litre)	Link Relative
2005	50	$\frac{50}{50} \times 100 = 100$
2010	55	$\frac{55}{50} \times 100 = 110$
2015	60	$\frac{60}{50} \times 100 = 120$
2020	70	$\frac{70}{50} \times 100 = 140$



Let's start our discussion by understanding methods:

SIMPLE AGGREGATIVE

In this method of computing a price index, we express the total of commodity prices in a given year as a percentage of total commodity price in the base year.

Simple Aggregative price index $= \frac{\sum P_n}{\sum P_0} \times 100$

Where, $\sum P_n$ is the sum of all commodity prices in the current year and $\sum P_0$ is the sum of all commodity prices in the base year.

Example of Simple Aggregate

Commodity	2010	2015	2020
Milk (per litre)	50	60	70
Atta (per kg)	10	12	15
Banana (dozen)	30	45	50
Aggregate	90	117	135
Index	100	130	150

Simple Aggregate Index for 2015 over 2010 $= \frac{117}{90} \times 100 = 130$ and for 2020 over 2010 $= \frac{135}{90} \times 100 = 150$

DEMERITS OF ABOVE METHOD

- It shows that the first commodity exerts greater influence than the other two because the price of the first commodity is higher than that of the other two.
- Further, if units are changed then the Index numbers will also change.

SIMPLE RELATIVE OR SIMPLE AVERAGE

If we change the actual price for each variable into percentage of the base period. These percentages are called relatives because they are relative to the value for the base period and the index number formed is simple relative.

E.g.:

Commodity	$2010(P_0)$	$2015(P_1)$	$2020(P_2)$
Milk (per litre)	50	60	70
Atta (per kg)	10	12	15
Potato (dozen)	20	30	30

Will become

Commodity	2010	2015	2020
Milk (per litre)	$\frac{50}{50} \times 100 = 100$	$\frac{60}{50} \times 100 = 120$	$\frac{70}{50} \times 100 = 140$
Atta (per kg)	$\frac{10}{10} \times 100 = 100$	$\frac{12}{10} \times 100 = 120$	150
Banana (dozen)	$\frac{20}{20} \times 100 = 100$	$\frac{30}{20} \times 100 = 120$	150
Aggregate	300	390	440
Index	$\frac{300}{3} = 100$	$\frac{390}{3} = 130$	$\frac{440}{3} = 146.66$

ADVANTAGE OF SIMPLE RELATIVE METHOD

- Index number computed from relatives will remain the same regardless of the units by which the prices are quoted.

DISADVANAGE OF SIMPLE RELATIVE METHOD

- This amount to giving undue weight to a commodity which is used in a small quantity because the relatives which have no regard to the absolute quantity will give weight more than what is due from the quantity used.

WEIGHTED AGGREGATIVE INDEX

While calculating the index number we will take care of the number of quantities or portions of the commodity.

E.g.:

Commodity	Quantity for2010	2010 (Price)	Quantity for 2020	2020 (Price)
Milk (per litre)	2 litres	50	2.5 litres	70
Atta (per kg)	1.5 kg	10	1 kg	15
Banana (dozen)	3 dozen	30	4 dozen	50

SOME OF THE IMPORTANT FORMULA TO CALCULATE IT ARE

- Laspeyre's Index number $= \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$
- Paasche's Index number $= \frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100$
- Marshall – Edgeworth Index number $= \frac{\sum P_n (Q_0 + Q_n)}{\sum P_0 (Q_0 + Q_n)} \times 100$
- Fisher's Index number $= \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{P_n Q_n}{\sum P_0 Q_n}} \times 100$
- Dorbish and Bowley's Price Index $\frac{\frac{\sum P_n Q_0}{\sum P0Q0} + \frac{\sum P_n Q_n}{\sum P0Q_n}}{2} \times 100$

WEIGHTED AVERAGE OF RELATIVE METHOD

To overcome the disadvantage of a simple average of relative method, we can use weighted average of relative method.

Let's understand with same example:

Commodity	Quantity for2010	2010 (Price)	Quantity for 2020	2020 (Price)
Milk (per litre)	2 litres	50	2.5 litres	70
Atta (per kg)	1.5 kg	10	1 kg	15
Banana (dozen)	3 dozen	30	4 dozen	50

We can solve it as:

Commodity	Quantity for2010 (Q_0)	2010 (Price) (P_0)	Quantity for 2020 (Q_n)	2020 (Price) (P_n)	$P_0 Q_0$	$P_0 Q_n$	$P_n Q_0$	$P_n Q_n$
Milk (per litre)	2 litres	50	2.5 litres	70	100	125	140	175
Atta (per kg)	1.5 kg	10	1 kg	15	15	10	22.5	15
Banana (dozen)	3 dozen	30	4 dozen	50	90	120	150	200

Then,

- Laspeyre's Index number $= \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100 = \frac{312.5}{205} \times 100 = 152.44$
- Paasche's Index number $= \frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100 = \frac{390}{255} \times 100 = 152.94$
- Marshall-Edgeworth Index number $= \frac{\sum P_n (Q_0 + Q_n)}{\sum P_0 (Q_0 + Q_n)} \times 100 = \frac{312.5+390}{205+255} \times 100 = 152.71$
- Fisher's Index number $r = \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{P_n Q_n}{\sum P_0 Q_n} \times 100} = \frac{390}{255} \times \frac{312.5}{205} + 100 = 152.68$

For Weighted Average of Relative Method:

Commodity	Quantity for2010 (Q_0)	2010 (Price) (P_0)	Quantity for 2020 (Q_n)	2020 (Price) (P_n)	$P_0 Q_0$	$\frac{P_n}{P_0}$	$\frac{P_n}{P_0} \times P_0 Q_0$
Milk (per litre)	2 litres	50	2.5 litres	70	100	$\frac{70}{50}$	$\frac{70}{50} \times 100 = 140$
Atta (per kg)	1.5 kg	10	1 kg	15	15	$\frac{15}{10}$	$\frac{15}{10} \times 100 = 22.5$
Banana (dozen)	3 dozen	30	4 dozen	50	90	$\frac{50}{30}$	$\frac{50}{30} \times 100 = 150$
						$\sum P_0 Q_0 = 205$	$\sum \frac{P_n}{P_0} \times P_0 Q_0 = 312.5$

Thus, Weighted Average of Relative $= \frac{\sum \frac{P_n}{P_0} \times P_0 Q_0}{\sum P_0 Q_0} \times 100$

$= \frac{312.5}{205} \times 100 = 152.44$

Relationship between Fisher's Index, Paasche's Index number and Laspeyre's Index:

Fisher's Ideal Index Number $= \sqrt{Paasche\ index\ number \times Laspeyre's\ index\ number}$

Relationship between Dorbish and Bowley's Price Index, Paasche's Index number and Laspeyre's Index:

Dorbish and Bowley's Price Index Number $\frac{Laspeyre^{'}s Price\ Index + Paasche^{'}s Price\ Index}{2}$

PRICE AND QUANTITY INDEX

	Method	Price Index	Quantity Index
1.	Simple Aggregate	$\frac{\sum P_n}{\sum P_0}$	$\frac{\sum Q_n}{\sum Q_0}$
2.	Simple Average of Relative	$\frac{\sum \frac{P_n}{P_0}}{n}$	$\frac{\sum \frac{Q_n}{Q_0}}{n}$
3.	Weighted Aggregate		
	(a) With base year weight (Laspeyre's index)	$\frac{\sum P_nQ_0}{\sum P_0Q_0}$	$\frac{\sum Q_nP_0}{\sum Q_0P_0}$
	(b) With current year weight (Paasche's index)	$\frac{\sum P_nQ_n}{\sum P_0Q_n}$	$\frac{\sum Q_nP_n}{\sum Q_0P_n}$
	(c) Fisher's Ideal [Geometric mean of Laspeyre's and Paasche's]	$\sqrt{\frac{\sum P_nQ_0}{\sum P_0Q_0}\times\frac{\sum P_nQ_n}{\sum P_0Q_n}}$	$\sqrt{\frac{\sum Q_nP_0}{\sum Q_0P_0}\times\frac{\sum Q_nP_n}{\sum Q_0P_n}}$
4.	Weighted average of Relative W= Weight = Base Year or Current Year Price Weight	$\frac{\sum \frac{P_n}{P_0}w}{\sum w}$	$\frac{\sum (\frac{Q_n}{Q_0})}{\sum w}$

CHAIN INDEX NUMBERS

- So far we concentrated on a fixed base but it does not suit when conditions change quite fast.
- Under this method the relatives of each year are first related to the preceding year called the link relatives and then they are chained together by successive multiplication to form a chain index.
- Chain Index $=\frac{\textit{Link Relative of current year} \times \textit{Chain index of the previous year}}{100}$

Let us understand from the below examples:

year	Price of commodity	Link relative	Chain index
2010	10	$\frac{10}{10}\times 100 = 100$	100
2012	12	$\frac{12}{10}\times 100 = 120$	$\frac{120\times 100}{100} = 120$
2014	14	$\frac{14}{12}\times 100 = 116.67$	$\frac{116.67\times 120}{100} = 140$
2016	15	$\frac{15}{14}\times 100 = 107.14$	$\frac{107.14\times 140}{100} = 150$
2018	17	$\frac{17}{15}\times 100 = 113.33$	$\frac{113.33\times 150}{100} = 170$
2020	20	$\frac{20}{17}\times 100 = 117.65$	$\frac{117.65\times 170}{100} = 200$
2022	22	$\frac{22}{20}\times 100 = 110$	$\frac{200\times 110}{100} = 220$

VALUE INDEX

We know, $Value = Price \times Quantity$ *i.e.*, $\sum \frac{V_n}{\sum V_0} = \frac{\sum P_n \times Q_n}{\sum P_0 \times Q_0} \times 100$

E.g.: The Value Index for the following data:

Commodity	Quantity (units)		Price in (`)	
	1995 Q_0	1999 Q_n	1995 P_0	1999 P_n
A	100	150	500	900
B	80	100	320	500
C	60	72	120	360
D	30	33	360	297

We have,

Commodity	Quantity (units)		Price in (`)			
	1995 Q_0	1999 Q_n	1995 P_0	1999 P_n	P_0Q_0	P_nQ_n
A	100	150	500	900	50000	135000
B	80	100	320	500	25600	50000
C	60	72	120	360	7200	25920
D	30	33	360	297	10800	9801
					93600	220721

We know, Value is Price multiplied by Quantity

Thus, $\frac{\sum Vn}{\sum V_0} = \frac{\sum P_n \times Q_n}{\sum P_0 \times Q_0} \times 100$

$= \frac{220721}{93600} \times 100 = 253.813$

Deflating time series using Index: Deflated Value $= \frac{\textbf{Current value}}{\textbf{Price index of current year}}$

E.g.: From the table, compute the real GNP.

Year	Wholesale Price index	GNP at current Price
1970	113.1	1499
1971	116.3	7935
1972	121.2	8657
1973	127.7	9323

Thus, we have

1970	113.1	7499	$\frac{7499}{113.1} \times 100 = 6630$
1972	121.2	8657	$\frac{8657}{121.5} \times 100 = 7143$

SHIFTING AND SPLICING OF INDEX NUMBERS

Shifting of index number means that base period of the index has to be shifted:

Shifted Price index $= \frac{Original\ Price\ index}{Price\ Index\ of\ the\ year\ on\ which\ it\ has\ to\ be\ shifted} \times 100$

From the following Index numbers with 1980 =100 Shift the Index number to 1990 as the base

	1980		1982		1984		1956		1988		1990	
	100		106		110		115		125		140	

Thus, on shifting the index number to 1990 as the base, we get

Year	1980	1981	1982	1983	1984	1985	1956	1987	1988	1989	1990	1991
Original Price index	100	104	106	108	110	112	115	117	125	131	140	147
Shifting Base index (1990)	$\frac{100}{140} \times 100 = 71.4$	$\frac{104}{140} \times 100 = 74.28$	75.7	76.4	78.57	80	82.1	83.6	89.28	93.5	100	105

SPLICING OF INDEX NUMBERS

The following represent two series of index numbers:

1. One series with 1990 as base which is discontinued with the year 1995.

2. The second series with 1995 as the base is started from the year 1995, the year in which the old index is discontinued.

Year	Old Price Index [1990 = 100]	Revised Price Index [1995 = 100]
1990	100.00	____
1991	102.30	____
1992	105.30	____
1993	107.60	____
1994	111.90	____
1995	114.20	100.00
1996	____	102.50
1997	____	106.40
1998	____	108.30
1999	____	111.70
2000	____	117.80

LIMITATION OF INDEX NUMBER

1. As the index are constructed mostly from deliberate samples, chances of errors creeping in cannot be always avoided.

2. Since index numbers are based on some selected items, they simply depict the broad trend and not the real picture.

3. Since many methods are employed for constructing index numbers, the result gives different values and this at time create confusion

USEFULNESS OF INDEX NUMBERS

1. Framing suitable policies in economics and business. They provide guidelines to make decisions in measuring intelligence quotients, research etc.

2. They reveal trends and tendencies in making important conclusions in cyclical forces, irregular forces etc.

3. They are important in forecasting future economic activity. They are used in times series analysis to long term trends, seasonal variations and cyclical developments.

4. Index numbers are very useful in deflating i.e., they are used to adjust the original data for price changes and thus transform nominal wages into real wages.

TEST OF ADEQUACY

1. **Unit Test:** This test requires that the formula should be independent of the unit in which or for which prices and quantities are quoted. Except for the simple (weighted) aggregative index all other formulae satisfy this test.

2. **Time Reversal Test:** It means if periods are reversed and indices are multiplied it should result in unity. $P_{01} \times P_{10} = 1$ where P_{01} is the index for time 1 on 0 and P_{10} is the index for time 0 on 1. Laspeyre's method and Paasche's method do not satisfy this test, but Fisher's Ideal formula does.

   Proof:

   I. Laspeyre's Index:

   We know that, $P_{01} = \frac{\sum P_1Q_0}{\sum P_0Q_0} P_{10} = \frac{\sum P_0Q_1}{\sum P_1Q_1}$

   Thus, $P_{01} \times P_{10} = \frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_0Q_1}{\sum P_1Q_1} \neq 1$

   II. Paasche's Index:

   We know that , $P_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_1}, P_{10} = \frac{\sum P_0Q_0}{\sum P_1Q_0}$

   $\therefore P_{01} \times P_{10} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0} \neq 1$

   III. Fisher's Index:

   We know, $P_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}, P_{10} = \sqrt{\frac{\sum P_0Q_1}{\sum P_1Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0}}$

   $\therefore P_{01} \times P_{10} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum P_0Q_1}{\sum P_1Q_1} \times \frac{\sum P_0Q_0}{\sum P_1Q_0}} = 1$

3. Factor Reversal Test: This holds when the product of the price Index and the quantity index should be equal to the corresponding value index, i.e. $P_{01} \times Q_{01} = V_{01}$

   Proof:

   (I) Paasche's Index:

   To check: $P_{01} \times Q_{01} = V_{01}$

   We know,

   $$P_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_1}$$

   $$Q_{01} = \frac{\sum Q_1P_1}{\sum Q_0P_1}$$

$$P_{01} \times Q_{01} = \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum Q_1P_1}{\sum Q_0P_1} \neq V_{01}$$

Similarly, we can prove that Laspeyre's Index number do not satisfy the Factor Reversal test.

(II) Fisher's Index:

We know, $P_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$

$$Q_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1} \times \frac{\sum P_1Q_0}{\sum P_0Q_0} \times \frac{\sum P_1Q_1}{\sum P_0Q_1}}$$

$$\Rightarrow \frac{\sum(P_1Q_1)^2}{\sum(P_0Q_0)^2} = \frac{\sum P_1Q_1}{\sum P_0Q_0}$$

Fisher's Index satisfies Factor Reversal test.

Since, Fisher's Index number satisfies both the test in (2) and (3), thus it is called an Ideal Index number.

Fisher's index is the Ideal Index Number.

4. **Circular Test:** It is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base. i.e., $P_{01} \times P_{12} \times P_{20} = 1$

It is satisfied by:

(I) Weighted aggregative with fixed weighted average.

(II) Simple geometric mean of price relatives.

Q.1 A series of numerical figures which show the relative position is called

(a) Index number (b) Relative number

(c) Absolute number (d) None of these

Ans. (a)

Q.2 Index number for the base period is always taken as

(a) 200 (a) 50 (c) 1 (d) 100

Ans. (d)

Q.3 Price relative is equal to

(a) $\frac{Price\ the\ given\ year \times 100}{Price\ in\ the\ base\ year}$ (b) $\frac{Price\ in\ the\ base\ year \times 100}{Price\ in\ the\ base\ year}$

(c) Price in the given year × 100 (d) Price in the base year × 100

Ans. (a)

Q.4 _____is an extension of the time reversal test.

(a) Factor Reversal test (b) Circular test

(c) Both (d) None of these

Ans. (b)

Q.5 Index number show_____changes rather than absolute amounts of change.

(a) Relative (b) Percentage (c) Both (d) None of these

Ans. (a)

Q.6 The_____makes index number time-reversible.

(a) A.M. (b) G.M. (c) H.M. (d) None of these

Ans. (b)

Q.7 _____play a very important part in the construction of index numbers.

(a) Weights (b) Classes (c) Estimations (d) None of these

Ans. (a)

Q.8 Index number is equal to

(a) Sum of Price Relatives (b) Average of the price relative

(c) Product of Price Relative (d) None of these

Ans. (b)

Q.9 The_____of group indices gives the General Index.

(a) H.M. (b) G.M. (c) A.M. (d) None of these

Ans. (c)

Q.10 Circular Test is one of the test of

(a) Index numbers (b) Hypothesis

(b) Both (d) None of these

Ans. (a)

Q.11The cost-of-living index is always

(a) Price index number (b) Quantity index number

(c) Weighted Index number (d) Value index number

Ans. (c)

Q.12_____is particularly suitable for the construction of index numbers.

(a) H.M. (b) A.M. (c) G.M. (d) None of these

Ans. (c)

Q.13Weighted G.M. of relative formula satisfy_____test.

(a) Time Reversal Test (b) Circular test

(c) Factor Reversal Test (d) None of these

Ans. (a)

Q.14Factor Reversal test is satisfied by

(a) Fisher's Ideal Index (b) Laspeyre's Index

(c) Paasche's Index (d) None of these

Ans. (a)

Q.15G.M of Laspeyre's and Paasche's Price Index number is_____price index number.

(a) Kelly's (b) Fisher's (c) Bowley's (d) None of these

Ans. (b)

Q.16Laspeyre's formula does not satisfy

(a) Factor Reversal Test (b) Time Reversal Test

(c) Circular Test (d) All of the above

Ans. (d)

Q.17A ratio or an average of ratio expressed as a percentage is called

(a) A relative number (b) An absolute number

(c) An index number (d) None of these

Ans. (c)

Q.18The consumer price index goes up from 120 to 180 when salary goes up from 240 to 540, what is the increase in real terms?

(a) 80 (b) 150 (c) 120 (d) 240

Ans. (c)

Q.19An index time series is a list of numbers for two or more periods of time.

(a) Index (b) Absolute (c) Relative (d) None of these

Ans. (a)

Q.20Index numbers are often constructed from the

(a) Frequency (b) Class (c) Sample (d) None of these

Ans. (c)

Q.21 Fisher's index number is called as ideal index number because it satisfies

(a) Factor reversal test

(b) Time reversal test

(c) Both factor and time reversal test

(d) Circular test

Ans. (c)

Q.22_____is a point of reference in comparing various data describing individual behavior.

(a) Sample (b) Base period (c) Estimation (d) None of these

Ans. (b)

Q.23 The ratio of price of single commodity in a given period to its price in the preceding year price is called the

(a) Base period (b) Price ratio (c) Relative price (d) None of these

Ans. (c)

Q.24 Chain index is equal to

(a) $\frac{Link\ relative\ of\ current\ year \times chain\ index\ of\ the\ current\ year}{100}$

(b) $\frac{Link\ relative\ of\ previous\ year \times chain\ index\ of\ the\ current\ year}{100}$

(c) $\frac{Link\ relative\ of\ Current\ year \times chain\ index\ of\ the\ Previous\ year}{100}$

(d) $\frac{Link\ relative\ of\ Previous\ year \times chain\ index\ of\ the\ Previous\ year}{100}$

Ans. (b)

Q.25 If Laspeyre's Index is 119 and Paasche's Index is 112. Then Fisher's index number will be

(a) 113.99 (b) 115.45 (c) 115.89 (d) 151.98

Ans. (b)

Q.26 P_{01} is the index for time

(a) 1 on 0 (b) 0 on 1 (c) 1 on 1 (d) 0 on 0

Ans. (a)

Q.27 P_{10} is the index for time

(a) 1 on 0 (b) 0 on 1 (c) 1 on 1 (d) 0 on 0

Ans. (b)

Q.28 If $\sum P_0Q_0 - 240,\ \sum P_1Q_1 = 480, \sum P_1Q_0 = 600, \sum P_0Q_1 = 192$ then the Laspeyre's index number is

(a) 250 (b) 300 (c) 350 (d) 200

Ans. (a)

Q.29 In the year 2010, the price index for a particular item is 150 with the base year 2005. What does this index value indicate?

(a) The prices of the item have decreased by 50% since 2005.

(b) The prices of the item have increased by 50% since 2005.

(c) The prices of the item have increased by 150% since 2005.

(d) The prices of the item have increased by 50 units since 2005.

Ans. (b)

Q.30 When the product of price index and the quantity index is equal to the corresponding value index then the test that holds is

(a) Unit Test (b) Time Reversal Test

(c) Factor Reversal Test (d) None holds

Ans. (c)

Q.31 The Index number in wholesale prices is 152 for August 1999 compared to August 1998. During the year there is net increase in prices of wholesale commodities to the extent of

(a) 45% (b) 35% (c) 52% (d) 48%

Ans. (c)

Q.32 Laspeyre's method and Paasche's method do not satisfy

(a) Unit Test (b) Time Reversal Test

(c) Circular test (d) both (b) and (c)

Ans. (d)

Q.33 An index number that can serve many purposes is known as a

(a) General purpose index (b) Special purpose index

(c) Both (a) and (b) are incorrect (d) Both (a) and (b) are correct

Ans. (a)

Q.34 Which of the following statements is true?

(a) Paasche's Index number is based on the base year quantity

(b) Fisher's Index number is the arithmetic mean of Laspeyre's Index number and Paasche's Index Numbers

(c) Arithmetic Mean is the most appropriate average for constructing the index number

(d) Fisher's Index number is an Ideal Index Number

Ans. (d)

Q.35 The index number is a special type of average.

(a) False (b) True (c) Both (d) None of these

Ans. (b)

Q.36 The cost of living index numbers in years 2015 and 2018 were 97.5 and 115 respectively. The salary of a worker in 2015 was `19,500. How much additional salary was required for him in 2018 to maintain the same standard of living as in 2015?

(a) 3000 (b) 4000 (c) 3500 (d) 4500

Ans. (c)

Q.37 Fisher's Ideal Formula does not satisfy _____

(a) Unit Test (b) Circular Test (c) Time Reversal Test (d) Factor Reversal Test

Ans. (b)

Q.38_____satisfies circular test.

(a) G.M. of price relatives or the weighted aggregate with fixed weight

(b) A.M of price relative or the weighted aggregate with fixed weight

(c) H.M of price relative or the weighted aggregate with fixed weight

(d) None of these

Ans. (a)

Q.39Laspeyre's and Paasche's method_____time reversal test.

(a) Satisfy (b) Do not satisfy (c) Are (d) Are not

Ans. (b)

Q.40The number of tests of adequacy is

(a) 2 (b) 5 (c) 3 (d) 4

Ans. (d)

Q.41Theoretically, G.M. is the best average in the construction of index numbers but in practice, mostly the A.M. is used.

(a) False (b) True (c) Both (d) None of these

Ans. (b)

Q.42Laspeyre's or Paasche's or the Fisher's ideal index do not satisfy

(a) Time Reversal Test (b) Unit Test

(c) Circular Test (d) None of these

Ans. (c)

Q.43_____is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base.

(a) Unit Test (b) Circular Test

(b) Time Reversal Test (d) None of these

Ans. (b)

Q.44The test of shifting the base is called

(a) Unit Test (b) Time Reversal Test

(c) Circular Test (d) None of these

Ans. (c)

Q.45 The formula for conversion to current value

(a) Deflated value = $\frac{\text{Price Index of the current year}}{\text{Previous value}}$

(b) Deflated value = $\frac{\text{Current value}}{\text{Price Index of the current year}}$

(c) Deflated value = $\frac{\text{Price Index of the Previous year}}{\text{Previous value}}$

(d) Deflated value = $\frac{\text{Price Index of the Previou year}}{\text{Previous value}}$

Ans. (b)

Q.46Shifted price Index = $\frac{\text{Original Price} \times 100}{\text{Price Index of the year on which it has to be shifted}}$

(a) True (b) False (c) Both (d) None of these

Ans. (a)

Q.47The weighted aggregative price index number for 2001 with 2000 as the base year using Paasche's index number is

Commodity	Price in `		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 112.32 (b) 112.38 (c) 112.26 (d) 112.20

Ans. (d)

Q.48The weighted aggregative price index number for 2001 with 2000 as the base year using Marshal – Edgeworth Index number is

Commodity	Price in `		Quantities	
	2000	2001	2000	2001
A	10	12	20	22
B	8	8	16	18
C	5	6	10	11
D	4	4	7	8

(a) 112.26 (b) 112.20 (c) 112.32 (d) 112.38

Ans. (a)

Q.49From the following data

	Commodity	A	B	C	D
1992 Base year	Price	3	5	4	1
	Quantity	18	6	20	14
1993 Current year	Price	4	5	6	3
	Quantity	15	9	26	15

The Paasche's price index number is

(a) 146.41 (b) 120.50 (c) 164.82 (d) None of these

Ans. (a)

Q.50The prices and quantities of 3 commodities in base and current year are as follows:

p_0	p_1	q_0	q_1
12	14	10	20
10	8	20	30
8	10	30	10

The Laspeyre's Index number is:

(a) 118.13 (b) 107.14 (c) 120.10 (d) None of these

Ans. (b)

Q.51With the base year 1960 the C.L.I in 1972 stood at 250. x was getting a monthly salary of ` 500 in 1960 and `750 in 1972. In 1972 to maintain his standard of living in 1960, x has to receive as extra allowances of

(a) `600 (b) `500 (c) ` 300 (d) None of these

Ans. (b)

Q.52If the ratio between Laspeyres index number and Paasche's index number is 28:27. Then the missing figure in the following table P is

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
X	L	10	2	5
Y	L	5	P	2

(a) 1 (b) 4 (c) 3 (d) 0

Ans. (b)

Q.53If Laspeyre's index number is 250 and Paasche's index number is 160, then fisher's index number is

(a) 40000 (b) $\frac{25}{16}$ (c) 200 (d) $\frac{16}{25}$

Ans. (c)

Q.54Which of the following statements is true?

(a) Paasche's index number is based on the base year quantity

(b) Fisher's index number is the Arithmetic mean of Laspeyre's index number and Paasche's index number

(c) Arithmetic mean is the most appropriate average for constructing the index number

(d) Fisher's index number is an ideal index number

Ans. (d)

Q.55The number of tests of Adequacy is

(a) 2 (b) 5 (c) 4 (d) 3

Ans. (c)

Q.56If the 1970 index with base with 1965 is 200 and 1965 index with base 1900. Is 150, the index 1970 on base 1960 will be

(a) 700 (b) 300 (c) 500 (d) 600

Ans. (b)

Q.57Circular test is satisfied by

(a) Laspeyres's index number

(b) Paasche's index number

(c) The simple geometric mean of price relatives and the weighted aggregative with fixed weights.

(d) None of these

Ans. (c)

Q.58The circular test is an extension of

(a) The time reversal test (b) The factor reversal test

(c) The unit test (d) None of these

Ans. (a)

Q.59If $\sum P_0Q_0$ =1360, $\sum P_nQ_0 = 1900$ $\sum P_0Q_n = 1344$, $\sum P_nQ_n = 1880$, then Laspeyre's index number is

(a) 0.71 (b) 1.39 (c) 1.75 (d) None of these

Ans. (b)

Q.60The cost-of-living index is always

(a) Price index number (b) Quantity index number

(c) Weighted Index number (d) Value index number

Ans. (c)

Q.61P_{01} is the index for time

(a) 1 on 0 (b) 0 on 1 (c) 1 on 1 (d) 0 on 0

Ans. (a)

SUMMARY

Index numbers are the most important need in the study of statistics. If you were to modify the variable in the estimation of any particular statistics, just imagine how it would be without these data! The technique will be wholly ineffectual in and of itself. The measurement of any change in a variable or variables over a predetermined period is therefore done via index numbers. These figures represent a broad relative shift rather than a specific quantifiable value. A percentage is used to express an index number.

Learn more about the Index numbers so that we may explore their significance, traits, categories, and restrictions. Continue reading the material to learn more about the supplementary part that we have included.

IMPORTANCE OF INDEX NUMBER

In studies of a region's economic situation, index numbers are most frequently utilized. The level of a variable in relation to its level across a specific time period is defined by the index number, as was previously indicated. These index values are used to analyse how the impact of all the variables that cannot be directly measured or approximated vary over time.

Due to their effectiveness in determining the magnitude of economic changes over a given time period, index numbers therefore have a significant position. The impact of such changes owing to variables that cannot be assessed directly are studied.

THE FOLLOWING ARE SOME OF THE KEY DISTINGUISHING CHARACTERISTICS OF INDEX NUMBERS

- When absolute measurement cannot be done, this specific category of average is used to assess relative changes.
- Only speculative changes in variables that may not be immediately measurable are shown by index numbers. It provides a broad overview of the relative changes.
- The index number measurement technique varies from one connected variable to another.
- It facilitates comparing the levels of a phenomena on a given day to those from earlier dates.

- It serves as an example of an uncommon average, particularly for a weighted average.
- Index numbers are useful everywhere. You may also utilize the index that is used to determine price fluctuations.

SIMPLE AGGREGATE METHOD

This approach is predicated on the idea that different things and their pricing are given in the same units. Each thing is given the same importance. The following is the formula for a straightforward aggregative pricing index $P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$

Where,

$\sum P_1$ is the total of current year's prices for the various items.

$\sum P_0$ is the total of base year's prices for the various items.

VARIOUS INDEX NUMBER TYPES

Different sorts of index numbers are used in certain ways. To understand the same, we shall study the various index numbers. The students will get an understanding of the significance of each form of Index number in relation to the assignment they are practicing for from this lesson on the many sorts of Index numbers.

- **Price index:** The ratio of the aggregate value for a given period to the aggregate value found in the base period yields a value index number. The value index is used, among other things, for inventory, sales, and international commerce.

  Price relative $= \frac{}{P} \times 100$

- **Quantity Index:** When measuring changes in the volume or number of items produced, consumed, and sold within a given time frame, a quantity index number is utilized. It displays the relative change over a time period for certain product quantities. An illustration of a quantity index is the Index of Industrial Production (IIP)

  Quantity Relative $= \frac{Q_0}{Q_0} \times 100$

- Value Relative: The value relative considers the value of a commodity by multiplying the price and quantity relatives together.
  Value Relative $= \frac{V_n}{V_0} = \frac{P_n}{P_0} \times \frac{Q_n}{Q_0} = \frac{P_n Q_n}{P_0 Q_0}$

USE OF INDEX NUMBER IN STATISTICS

In several straightforward to complex research, index numbers are helpful. Like it is used in the basic study of human population in a country and also it is used to determine the extinction rate of the rare animals in a particular region. There are many more applications for index numbers; here are some examples:

- It aids in gauging changes in both pricing levels and living standards.
- Regulation of wage rates is compatible with shifts in the level of prices. Wage rates may change when pricing levels are established.
- The index number of prices is used to frame government policy. Index numbers serve as the foundation for the built-in price stability of fiscal and economic policy.
- It provides a starting point for comparing many economic factors internationally, such as the living standards of two nations.

ADVANTAGES OF INDEX NUMBER

- Index numbers' benefits are closely related to how they are used. In conclusion, the benefits are as follows: It makes cost-effective adjustments to primary data, which is helpful for deflation. It makes the switch from nominal wage to real wage easier.
- In economics, index numbers are frequently used to aid in the formulation of effective policies. These results also aid in the development of research.
- It is useful for patterns like making conclusions for cyclical and irregular forces.
- In the event that economic activity develops in the future, index numbers can be useful. To identify patterns and cyclical processes, this time series analysis is used.
- The figure is helpful in gauging changes in living standards across various nations over a predetermined time frame.

ISSUES IN THE CONSTRUCTION OF INDEX NUMBERS

The following are some considerations for the development of an index number that should be made:

- Purpose of Index number
- Selection of Items
- Choice of Average
- Assignment of weights
- Choice of Base year

LASPEYRE'S METHOD: $(\frac{\sum P_1Q_1}{\sum P_0Q_1}) \times 100$

Here, $\sum P_1Q_1$ is the sum of current year prices multiplied by current year quantities take as weights, and $\sum P_0Q_1$ is the sum of base year prices multiplied by current year quantities taken as weights.

MARSHALL=EDGEWORTH METHOD $\frac{\sum Pn(Q0+Qn)}{\sum P_0(Q_0+Q_n} \times 100$

FISHER'S METHOD: $\sqrt{[(\frac{\sum P_1Q_0}{\sum P_0Q_0}) \times (\frac{\sum P_1Q_1}{\sum P_0Q_1})]} \times 100$

Fisher's ideal index number is an index number.

DORBISH AND BOWLEY'S PRICE INDEX

Dorbish and Bowley's Price Index = $\frac{\frac{\sum P_nQ_0}{\sum P0Q0} + \frac{\sum P_nQ_n}{\sum P0Qn}}{2} \times 100$

LIMITATIONS OF INDEX NUMBER

We are aware that everything in the world has both benefits and drawbacks. Although index numbers offer many benefits, this is also where some of its drawbacks appear. The following are some index numbers' restrictions:

- Given that index numbers are generated from samples, there is a risk for inaccuracy. These samples are assembled after careful consideration, which increases the possibility of mistakes. It can also be found in base periods, weights, etc.

- It is always determined using the things. Items that are so carefully chosen may not accurately reflect current trends, which leads to erroneous analysis.
- The index numbers provide a rough idea of the relative changes that take place.
- The choice of representative goods could be biased.

MAIN CHARACTERISTICS OF INDEX NUMBERS

Index numbers are a particular kind of average that

1. give a measurement of relative changes in the frequency of a certain phenomena;
2. are stated in terms of percentages to demonstrate the magnitude of relative change
3. quantify relative changes.
4. They are also capable of measuring changes that are not readily quantifiable.

Economic barometers are index numbers. They aid in the creation of economic policy, planning, and other things. They are employed in the analysis of trends and patterns. Indicators like index numbers can be used to predict future economic activity. They gauge the value of money in terms of purchases.

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