

CORRELATION

→ Correlation: Statistical techniques used to measure the degree & direction of the relation b/w two variables.

- positive correlation: $x \uparrow y \uparrow$
- negative correlation: $x \downarrow y \downarrow$ (inverse relation)

→ Linear correlation.

• measure of linear correlation

1. graphical method (scatter plot diagram method)
2. non-graphical
 - a. Karl Pearson's co-efficient correlation
 - b. Spearman's Rank correlation
 - c. concurrent deviations methods.

1. graphical method.

- If the points plotted are concentrated from lower left corner to upper right corner, it is positive correlation.
 - $0 < r < 1$ → positive correlation
 - $r = 1$ → perfect positive correlation
- If the points plotted are concentrated from upper left to lower right corner, it is negative correlation.
 - $-1 < r < 0$ → negative correlation
 - $r = -1$ → perfect negative correlation
- If the points are scattered without any pattern the variables are un-correlated.
 - $r = 0$ → (unrelated, no correlation)

2. non-graphical

a. Karl Pearson's method.

(co-variance = $\text{cov}(x, y)$)

$$\text{cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{N} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

> $\text{cov}(x, y)$ can be any real number.

> change in origin - NO

> change in scale - YES

$$\sigma_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} \quad \sigma_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{N}}$$

$$r = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{N \sigma_x \sigma_y}$$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

$$r = \frac{\sum xy - \frac{\sum x \sum y}{N}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{N}}} \times \frac{1}{\sqrt{\sum y^2 - \frac{(\sum y)^2}{N}}}$$

$$\text{If } U = x_i - A \quad \& \quad V = y_i - A$$

$$r = \frac{\sum UV}{\sum U^2 - \frac{(\sum U)^2}{N}} \times \frac{1}{\sqrt{\sum V^2 - \frac{(\sum V)^2}{N}}}$$

$$r = \frac{\sum UV}{\sqrt{\sum U^2 - \frac{(\sum U)^2}{N}} \sqrt{\sum V^2 - \frac{(\sum V)^2}{N}}}$$

• Karl Pearson co-efficient of correlation.

• change in origin → no change

• change of scale → no change

→ [Spearman's Rank correlation / Spearman method]
used for relation b/w qualitative character
→ level of agreement b/w two judges.

$$r = 1 - \frac{6SD^2}{N^3 - N}$$

→ If some elements repeat.

$$r = 1 - \frac{6 \left[\sum D^2 + \frac{1}{12} (m_1^3 - m_1) + \frac{1}{12} (m_2^3 - m_2) + \frac{1}{12} (m_3^3 - m_3) \right]}{N^3 - N}$$

M_1, M_2, M_3 } = frequency of repeating numbers.

→ [concurrent deviation method]

$x \uparrow y \uparrow$ → concurrent
 $x \downarrow y \downarrow$ → concurrent
 $x \uparrow y \downarrow$ → non concurrent
 $x \downarrow y \uparrow$ → non concurrent

$$r = \pm \sqrt{\frac{\sum c - n}{n}}$$

c = Total concurrent deviation

n = Total no. of pairs

→ co-efficient of determination

• The coefficient of determination is used to explain the relationship between an independent and dependent variable.

• measures the amount of change in dependent variable due to change in independent variable.

formula

$$\text{COD} = r^2 = \frac{\text{Explained variance}}{\text{Total variance}}$$

→ co-efficient of non determination

$$= 1 - r^2$$

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$> 0 < r < 1$ → positive correlation
 $> r = 1$ → perfect positive correlation

- If the points plotted are concentrated from upper left to lower right corner, it is negative correlation

$> -1 < r < 0$ → negative correlation

> -1 → perfect negative correlation

- If the points are scattered without any pattern the variables are un-correlated

$> r = 0$ → (unrelated, no correlation)

2. non-graphical

a. Karl Pearson's method.

co-variance = $\text{cov}(x, y)$

$\text{cov}(x, y)$

Karl Pearson co-efficient of correlation.

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$\left. \begin{matrix} m_1 \\ m_2 \\ m_3 \end{matrix} \right\}$ = frequency of repeating numbers.

→ [Concurrent deviation method]

$x \uparrow y \uparrow$ → concurrent
 $x \uparrow y \downarrow$ → non concurrent
 $x \downarrow y \downarrow$ → concurrent

$$r = \pm \sqrt{\pm \left(\frac{\sum C - n}{n} \right)}$$

C = Total concurrent deviation

n = Total no of pairs

→ Co-efficient of determination

• The coefficient of determination is used to explain the relationship between an independent and dependent variable.

• measures the amount of change in dependent variable due to change in independent

$\gamma = 0 \rightarrow$ (unrelated, no correlation)

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a. Karl Pearson's method.

(co-variance = $\text{cov}(x, y)$)

$$\text{cov}(x, y) = \frac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{N} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

$\text{cov}(x, y)$ can be any real number.

change in origin - No

change in scale - Yes

$$\sigma_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} \quad \sigma_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{N}}$$

$$\gamma = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} = \frac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{N \sigma_x \sigma_y}$$

$$= \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \cdot \sqrt{\sum (y_i - \bar{y})^2}}$$

$$\gamma = \frac{\sum xy - \frac{\sum x \cdot \sum y}{N}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{N}}} \times \sqrt{\sum y^2 - \frac{(\sum y)^2}{N}}$$

$$U = x_i - A \quad \& \quad V = y_i - A$$

$$= \frac{\sum UV}{N} = \frac{\sum U \times \sum V}{N}$$

$$\sqrt{\sum U^2 - \frac{(\sum U)^2}{N}} \sqrt{\sum V^2 - \frac{(\sum V)^2}{N}}$$

- The coefficient of determination is used to explain the relationship between an independent and dependent variable.
- measures the amount of change in dependent variable due to change in independent variable.

formula

$$\text{COD} = \gamma^2 = \frac{\text{Explained variance}}{\text{Total variance}}$$

$$\rightarrow \text{co-efficient of non determination} = 1 - \gamma^2$$