Chapter 3 – Linear Inequalities

Introduction

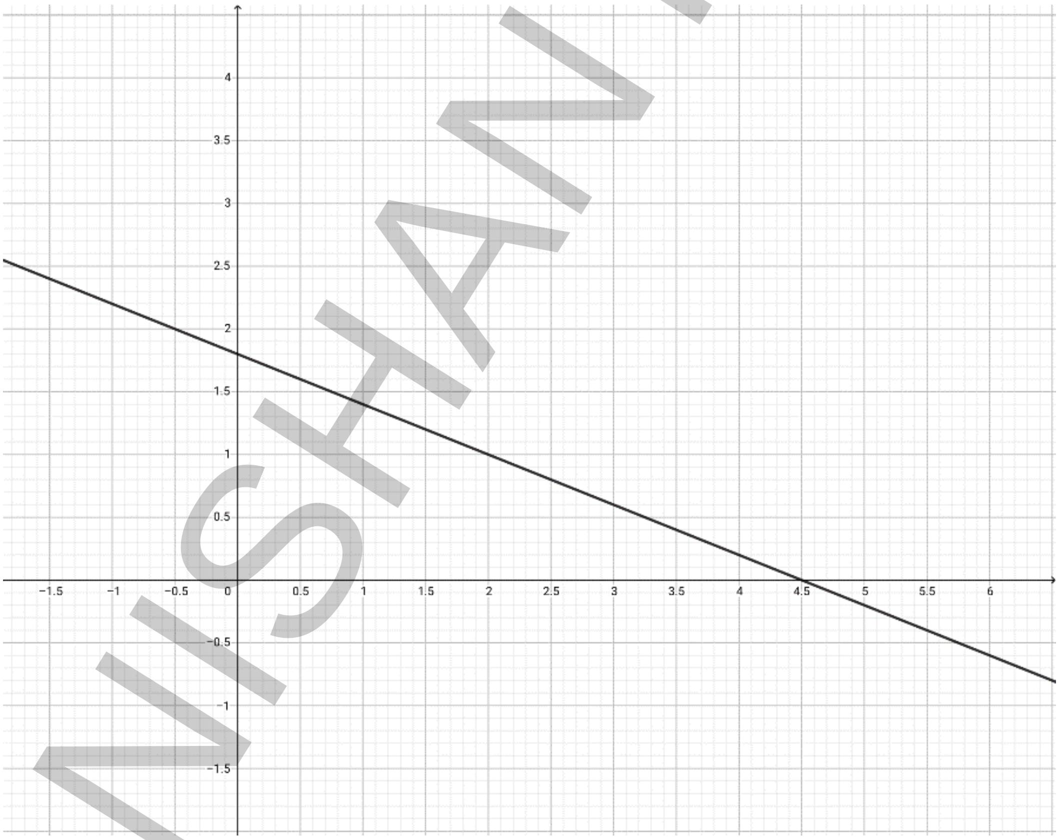
We have already studied Equations in the previous chapter. An example of a linear equation in one variable is $5x = 10$. Notice the "=" sign between the terms "$5x$" and "10". This is the equality sign which signifies that the term "$5x$" is **equal** to "10". This would give the value of x to be 2. This implies that the equation $5x = 10$ holds true only for $x = 2$. For all the other values of x, this equation won't hold true. Therefore, there's only 1 solution to the equation.

An **Inequality** on the other hand is of the type $5x < 10$. Notice the "<" sign between the terms "$5x$" and "10". This is the inequality sign which signifies that the term "$5x$" is always **less** than "10". On solving this, we'll get $x < 2$. This means that the inequality $5x < 10$ holds true for all the values of x which are less than 2. Thus, it is clear that while an equation has only 1 solution, an inequality has infinite solutions. These infinite solutions that an inequality has is called **Solution Space**. Again, since the highest power of the variables is 1, it is said to be a **Linear Inequality**.

Solving Linear Equations in Two Variables Graphically

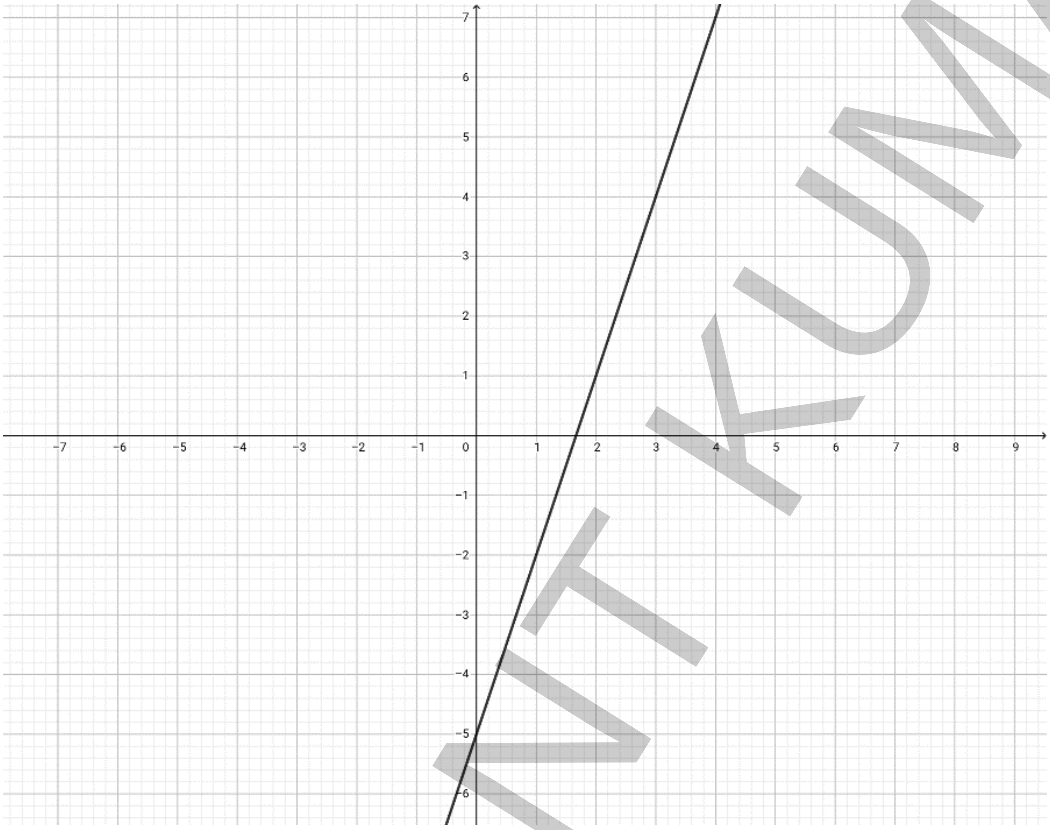
Any linear equation in two variables can be plotted on a graph paper. The graph of a linear equation in two variables is always a straight line.

Consider the equation $2x + 5y = 9$. Following is its graph:



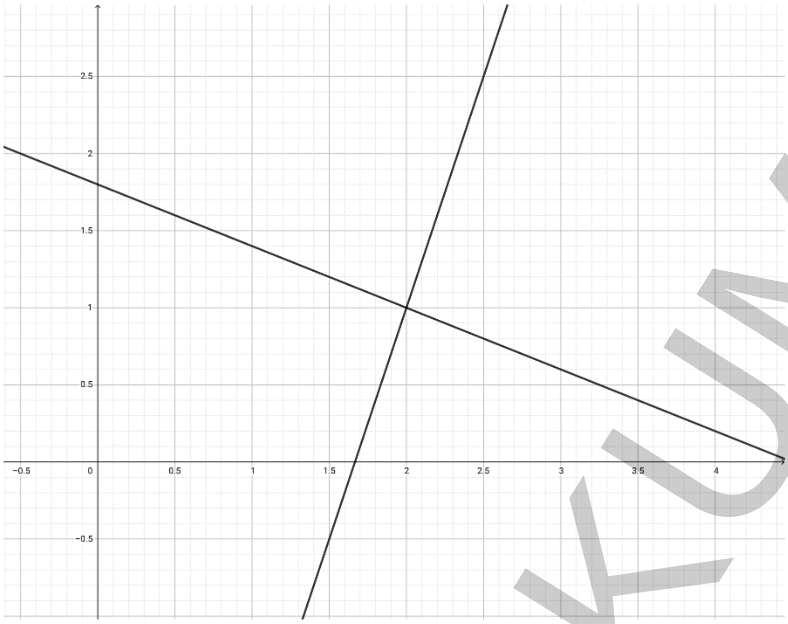
The above is the graph of the equation $2x + 5y = 9$. The straight line that you see is the line of the various solutions of this equation. This means that all the points falling on this straight line will solve the equation $2x + 5y = 9$. Take, for example, the point (4.5, 0). Putting 4.5 for x and 0 for y, we get the LHS $\rightarrow 2 \times 4.5 + 5 \times 0 = 9 + 0 = 9 =$ RHS. Therefore, we can see that the point (4.5, 0) is the solution of the equation $2x + 5y = 9$. Similarly, any such point falling on this line is the solution of this equation.

Now consider another equation: $3x - y = 5$. Following is its graph:



The above is the graph of the equation $3x - y = 5$. The straight line that you see is the line of the various solutions of this equation. This means that all the points falling on this straight line will solve the equation $3x - y = 5$. Take, for example, the point (0, –5). Putting 0 for x and –5 for y, we get the LHS $\rightarrow 3 \times 0 - (-5) = 0 + 5 = 5 =$ RHS. Therefore, we can see that the point (0, –5) is the solution of the equation $3x - y = 5$. Similarly, any such point falling on this line is the solution of this equation.

If we superimpose these two graphs, the lines will intersect at a point. This intersection point will give us the solution of these two equations when solved simultaneously.



As can be seen from the above graph, the lines intersect at the point (2, 1). Hence, this is the solution of the set of the equations: $2x + 5y = 9$; $3x - y = 5$.

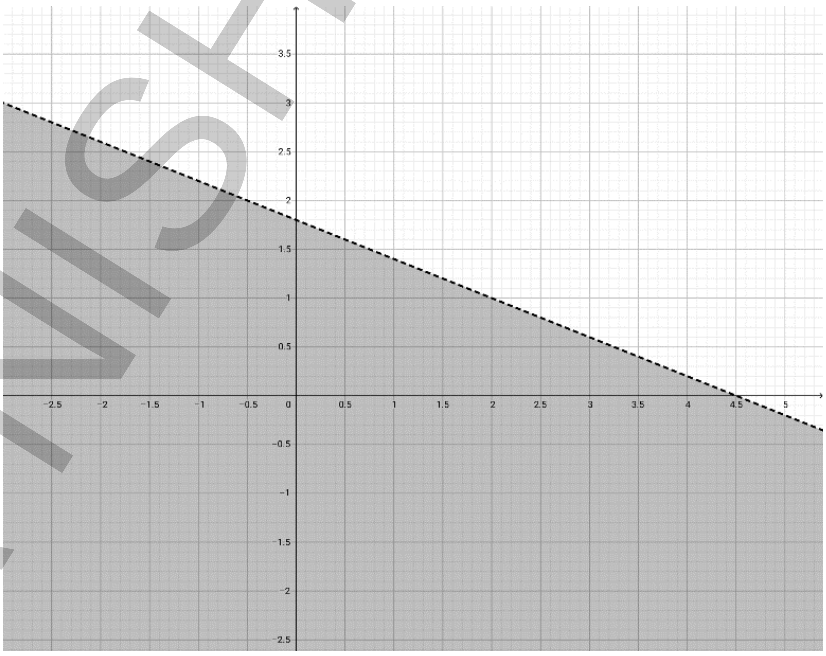
Question 1

Draw the graphs of the following equations:

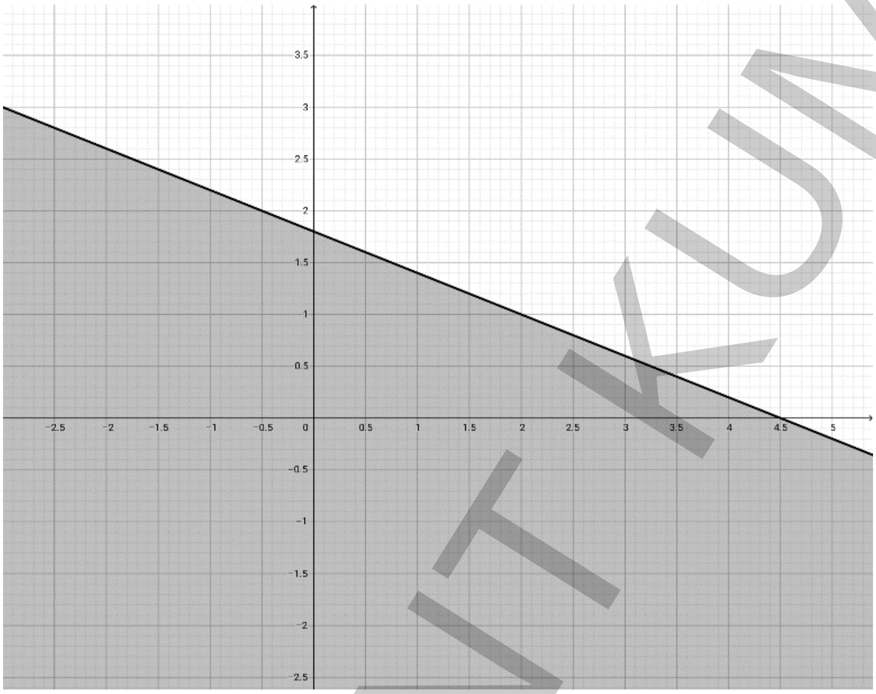
1. $x + 2y = 8$
2. $2x + 4y = 8$
3. $6x + 10y = 60$

Graphing Linear Inequalities in Two Variables

Consider the inequality $2x + 5y < 9$. Following is its graph:

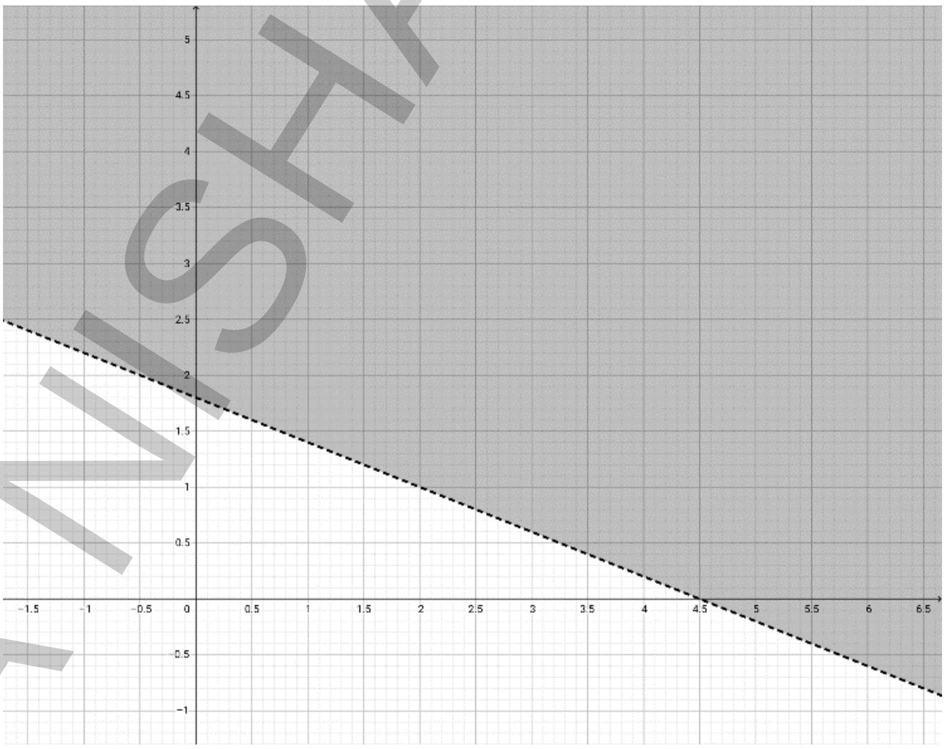


You can see that the line of the inequality is a dotted line and the area below it is shaded. The shaded area is the solution space of the inequality $2x + 5y < 9$. Any point lying in the shaded area will satisfy this inequality $2x + 5y < 9$. The dotted line indicates that any point lying on this line will not satisfy the inequality. However, if the inequality was $2x + 5y \leq 9$, then the line would not have been dotted:

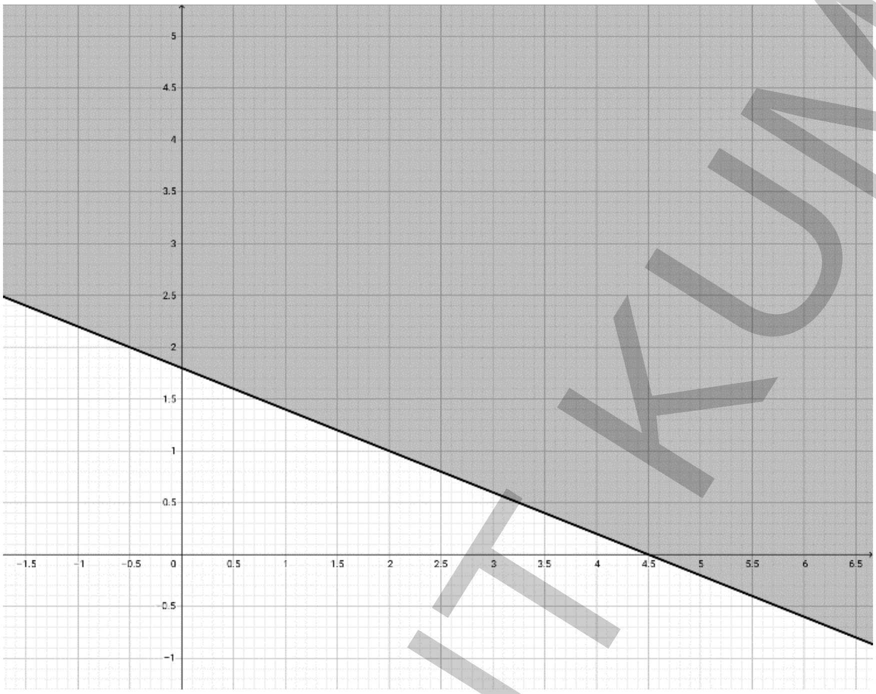


This means that any point lying on the fixed line, as well as in the shaded area will satisfy the inequality $2x + 5y \leq 9$.

Consider the inequality $2x + 5y > 9$. Its graph is:



Any point in the shaded area will satisfy the inequality $2x + 5y > 9$. The dotted line indicates that any point lying on this line will not satisfy the inequality $2x + 5y > 9$. However, this line would not have been dotted if the inequality was $2x + 5y \geq 9$. The graph in such a case would have been:



This means that all the points lying on the line as well as in the shaded area will satisfy the inequality $2x + 5y \geq 9$.

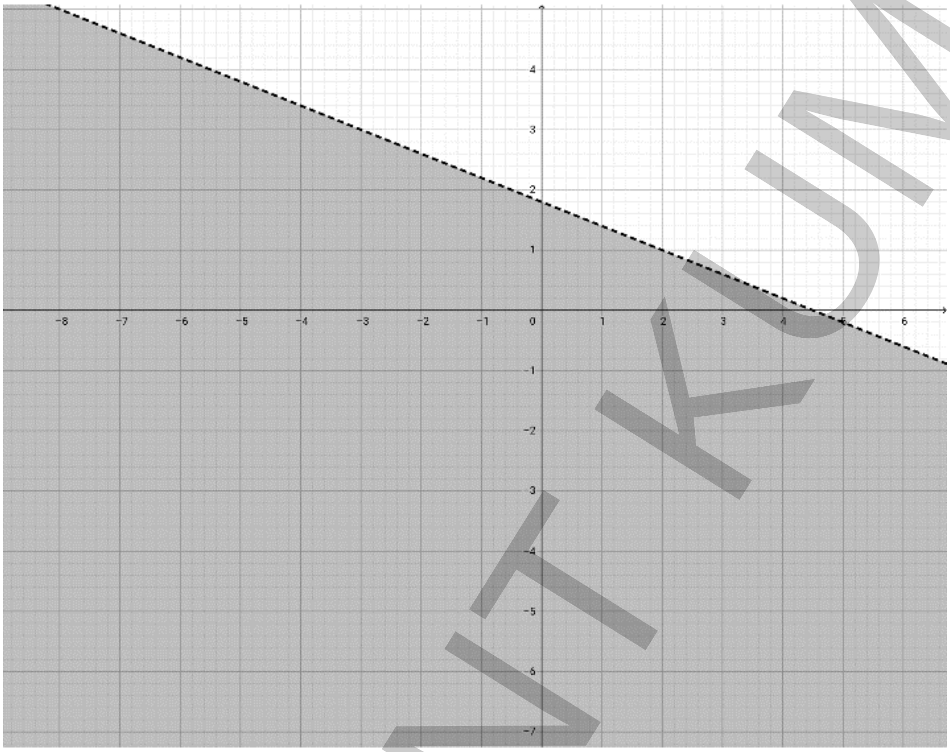
Points to be Noted

1. If the inequality sign is $>$ or $<$, a dotted line is drawn.
2. If the inequality sign is $\geq$ or $\leq$, a fixed line is drawn.
3. If the sign is $<$ or $\leq$, the area towards 0 is shaded.
4. If the sign is $>$ or $\geq$, the area away from 0 is shaded.

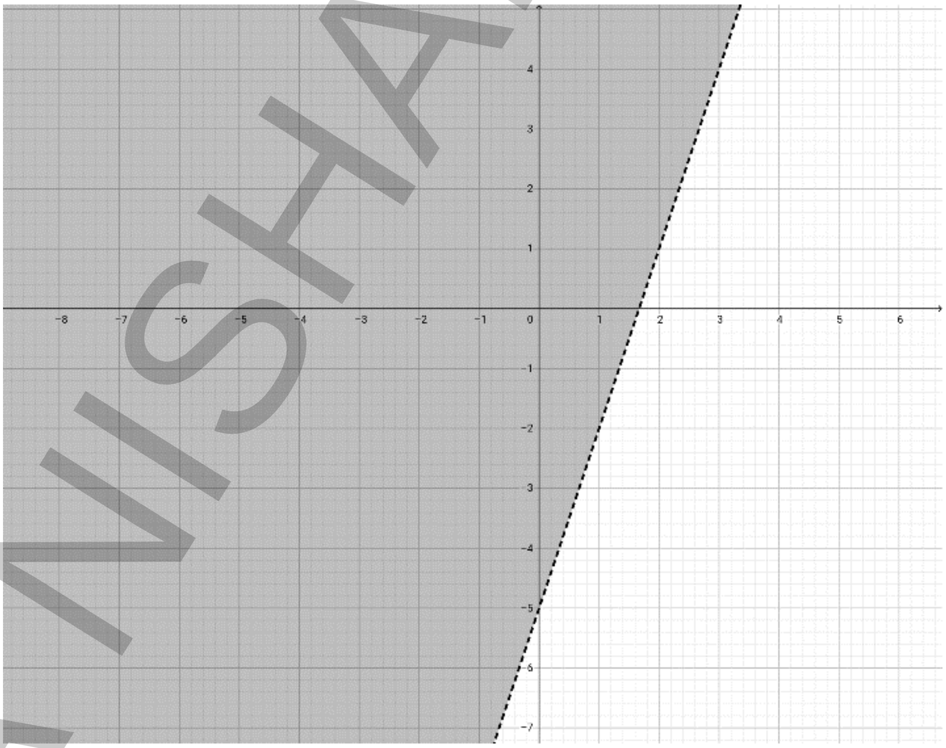
Solving a System of Linear Inequalities in Two Variables Graphically

Consider the following system of Linear Inequalities in Two Variables: $2x + 5y < 9$; $3x - y < 5$.

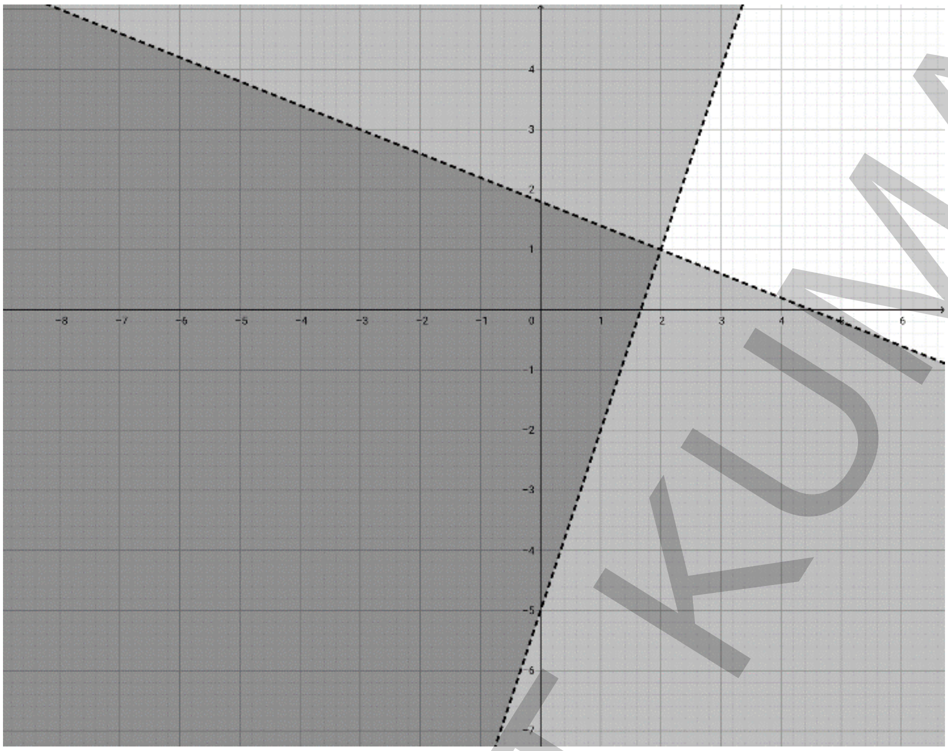
The graph of $2x + 5y < 9$ is:



The graph of $3x - y < 5$ is:



On superimposing these two graphs, we get:



The dark shaded portion in the above graph is the solution space of both the inequalities simultaneously, i.e. any point lying in the dark shaded portion will satisfy both the inequalities $2x + 3y < 9$, as well as $3x - y < 5$.

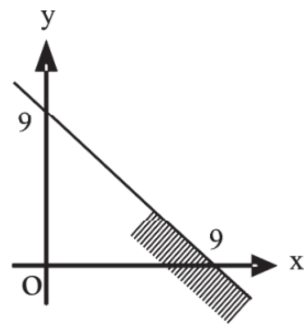
Points to Remember

1. $x > 0$ means shading towards the right of the y axis.
2. $y > 0$ means shading above the x axis.
3. x and y are always greater than or equal to 0.

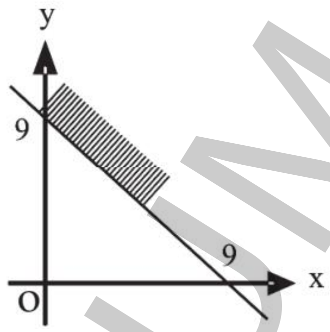
Exercise 3A – Question 1 (v)

The graph to express the inequality $x + y \leq 9$ is

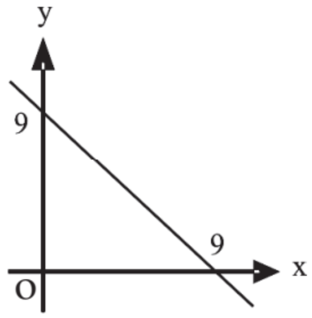
(a)



(b)



(c)



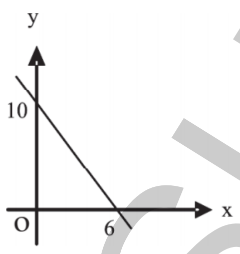
(d) None

Solution (a)

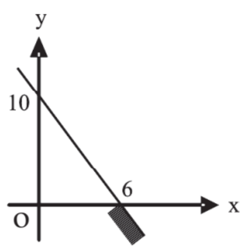
Exercise 3A – Question 1 (vi)

The graph to express the inequality $5x + 3y \geq 30$ is:

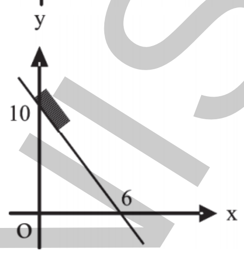
(a)



(b)



(c)



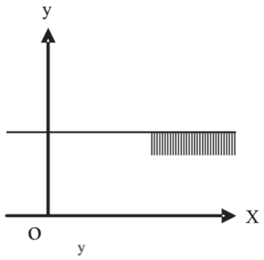
(d) None

Solution (c)

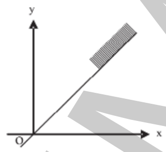
Exercise 3A – Question 1 (vii)

The graph to express the inequality $y \leq \left(\frac{1}{2}\right)x$ is indicated by:

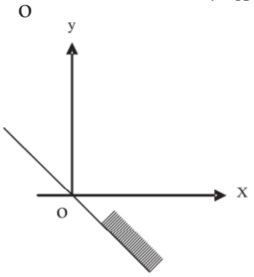
(a)



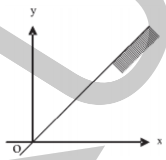
(b)



(c)

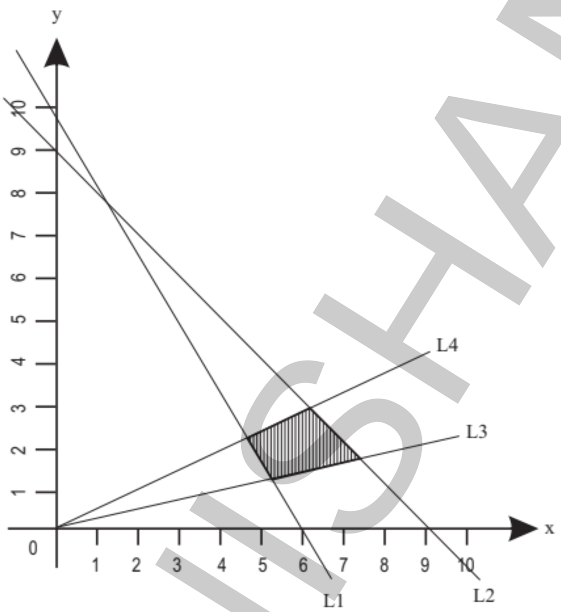


(d)



Solution (d)

Exercise 3A – Question 1 (viii)

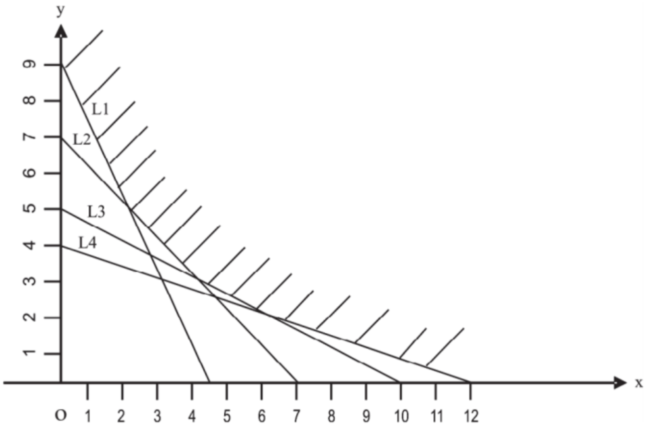


$L1: 5x+3y=30;\ L2: x+y=9;\ L3: y=x/3;\ L4: y=x/2$

(a)	$5x+3y \leq 30$	(b)	$5x+3y \geq 30$	(c)	$5x+3y \geq 30$	(d)	$5x+3y > 30$
	$x+y \leq 9$		$x+y \leq 9$		$x+y \geq 9$		$x+y < 9$
	$y \leq 1/5x$		$y \geq x/3$		$y \leq x/3$		$y \geq 9$
	$y \leq x/2$		$y \leq x/2$		$y \geq x/2$		$y \leq x/2$
			$x \geq 0,\ y \geq 0$		$x \geq 0,\ y \geq 0$		$x \geq 0,\ y \geq 0$

Solution (b)

Exercise 3A – Question 3



$L1: 2x + y = 9$; $L2: x + y = 7$; $L3: x + 2y = 10$; $L4: x + 3y = 12$

(a) $2x + y \le 9$, $x + y \ge 7$, $x + 2y \ge 10$, $x + 3y \ge 12$

(b) $2x + y \ge 9$, $x + y \le 7$, $x + 2y \ge 10$, $x + 3y \ge 12$

(c) $2x + y \ge 9$, $x + y \ge 7$, $x + 2y \ge 10$, $x + 3y \ge 12$, $x \ge 0$, $y \ge 0$

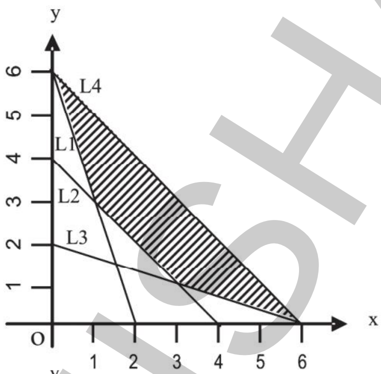
(d) None

Solution (c)

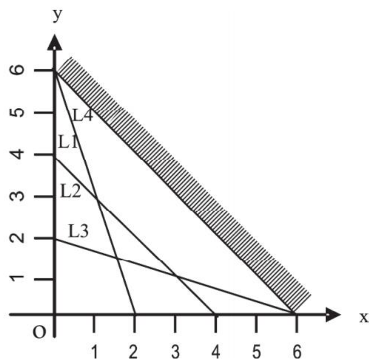
Exercise 3A – Question 4

The common region satisfied by the inequalities $L1: 3x + y \ge 6$, $L2: x + y \ge 4$, $L3: x + 3y \ge 6$, $L4: x + y \le 6$

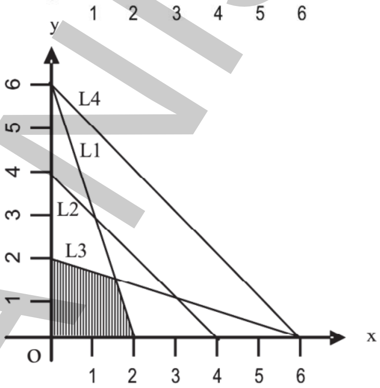
(a)



(b)



(c)

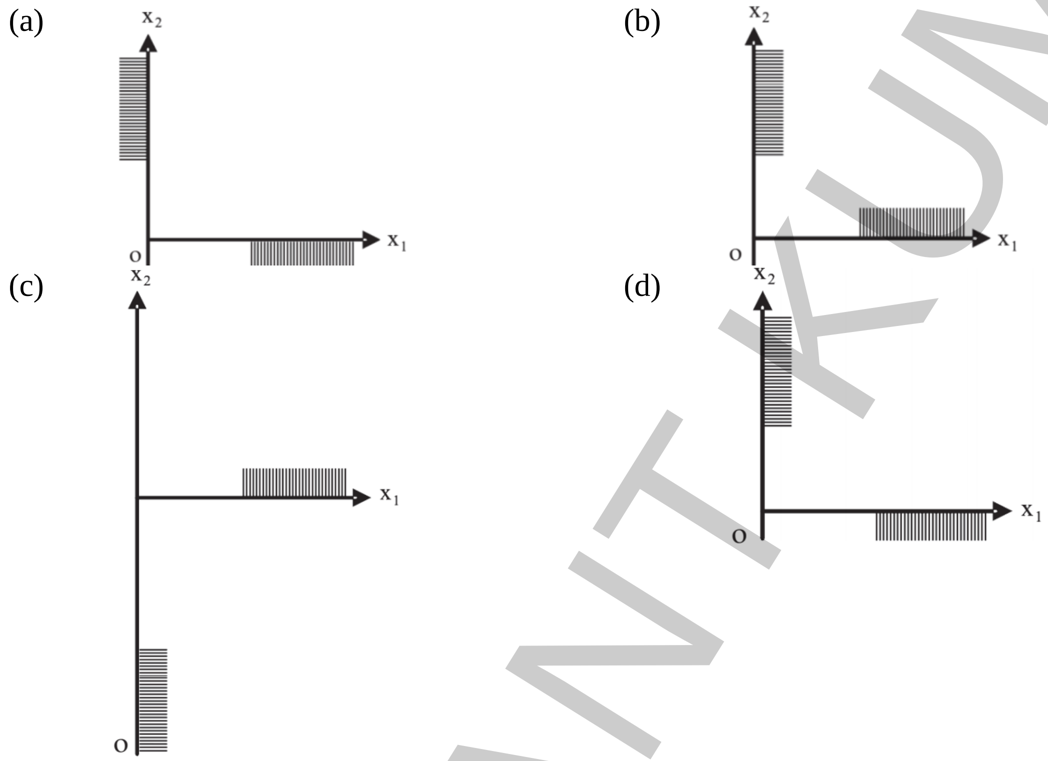


(d) None

Solution (a)

Exercise 3A – Question 6(i)

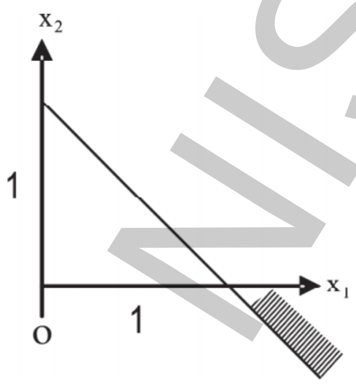
The inequalities $x_1 \geq 0$, $x_2 \geq 0$ are represented by one of the graphs shown below:



Solution (b)

Exercise 3A – Question 6(ii)

The following region is expressed as:



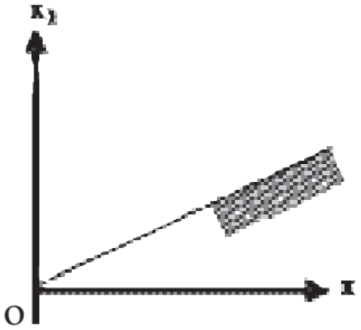
(a) $x_1 - x_2 \geq 1$ (b) $x_1 + x_2 \leq 1$ (c) $x_1 + x_2 \geq 1$ (d) None

Solution (c)

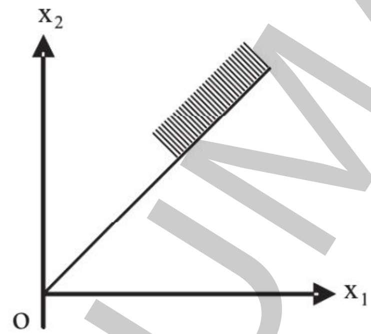
Exercise 3A – Question (iii)

The inequality $-x_1 + 2x_2 \le 0$ is indicated on the graph as:

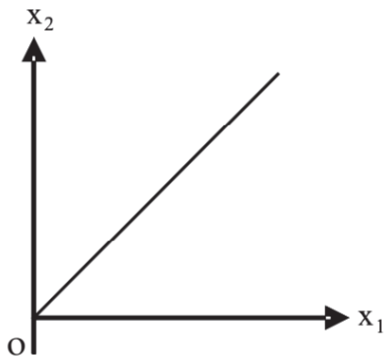
(a)



(b)



(c)

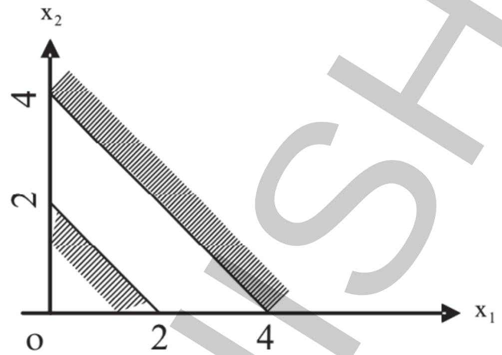


(d) None

Solution (a)

Exercise 3A – Question 5

The region indicated by the shading in the graph is expressed by inequalities



(a)
$x_1 + x_2 \le 2$
$2x_1 + 2x_2 \ge 8$
$x_1 \ge 0$, $x_2 \ge 0$

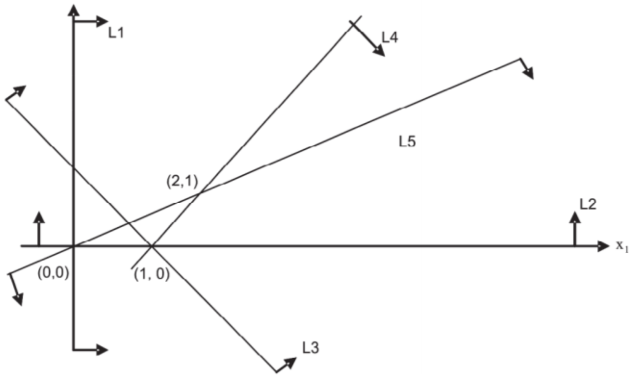
(b)
$x_1 + x_2 \le 2$
$x_2 x_1 + x_2 \le 4$

(c)
$x_1 + x_2 \ge 2$
$2x_1 + 2x_2 \ge 8$

(d)
$x_1 + x_2 \le 2$
$2x_1 + 2x_2 > 8$

Solution (a)

Exercise 3A – Question 7



The common region indicated on the graph is expressed by the set of five inequalities:

(a) $L1: x_1 \geq 0$
$L2: x_2 \geq 0$
$L3: x_1 + x_2 \leq 1$
$L4: x_1 - x_2 \geq 1$
$L5: -x_1 + 2x_2 \leq 0$

(b) $L1: x_1 \geq 0$
$L2: x_2 \geq 0$
$L3: x_1 + x_2 \geq 1$
$L4: x_1 - x_2 \geq 1$
$L5: -x_1 + 2x_2 \leq 0$

(c) $L1: x_1 \leq 0$
$L2: x_2 \leq 0$
$L3: x_1 + x_2 \geq 1$
$L4: x_1 - x_2 \geq 1$
$L5: -x_1 + 2x_2 \leq 0$

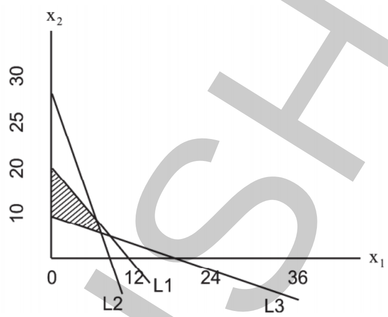
(d) None

Solution (b)

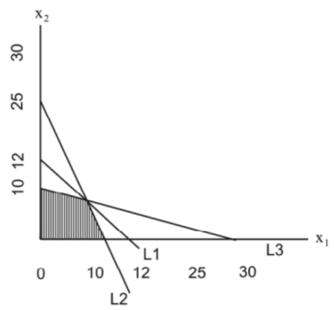
Exercise 3A – Question 9

The set of inequalities $L1: x_1 + x_2 \leq 12$, $L2: 5x_1 + 2x_2 \leq 50$, $L3: x_1 + 3x_2 \leq 30$, $x_1 \geq 0$, and $x_2 \geq 0$ is represented by:

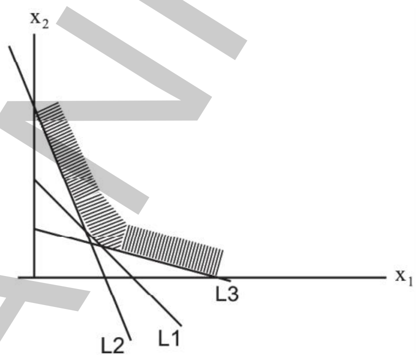
(a)



(b)



(c)



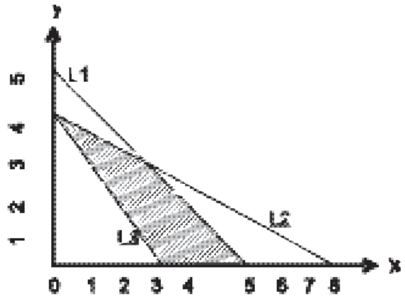
(d) None

Solution (b)

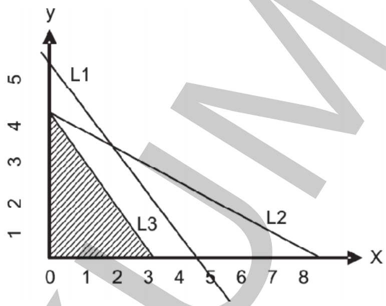
Exercise 3A – Question 10

The common region satisfying the set of inequalities $x_1 \geq 0$, $x_2 \geq 0$, $L1: x+y \leq 5$, $L2: x+2y \leq 8$, and $L3: 4x+3y > 12$ is indicated by:

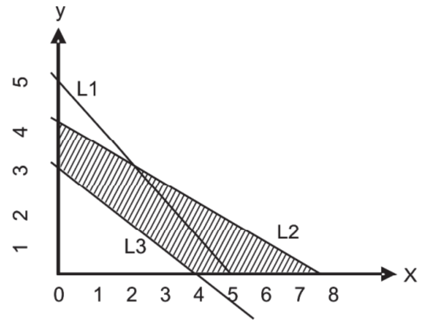
(a)



(b)



(c)



(d) None

Solution (a)

Additional Question Bank – Question 1

On solving the inequalities $2x+5y \leq 20$, $3x+2y \leq 12$, $x \geq 0$, $y \geq 0$, we get the following situation:

(a) $(0,\ 0)$, $(0,\ 4)$, $(4,\ 0)$, $\left({}^{20}\!/_{11},\ {}^{36}\!/_{11}\right)$

(b) $(0,\ 0)$, $(10,\ 0)$, $(0,\ 6)$, $\left({}^{20}\!/_{11},\ {}^{36}\!/_{11}\right)$

(c) $(0,\ 0)$, $(0,\ 4)$, $(4,\ 0)$, $(2,\ 3)$

(d) $(0,\ 0)$, $(10,\ 0)$, $(0,\ 6)$, $(2,\ 3)$

Solution (a)

Additional Question Bank – Question 2

On solving inequalities $6x+y \geq 18$, $x+4y \geq 12$, $2x+y \geq 10$, we get the following situation:

(a) $(0,\ 18)$, $(12,\ 0)$, $(4,\ 2)$, $(2,\ 6)$

(b) $(3,\ 0)$, $(0,\ 3)$, $(4,\ 2)$, $(7,\ 6)$

(c) $(5,\ 0)$, $(0,\ 10)$, $(4,\ 2)$, $(7,\ 6)$

(d) $(0, 18), (12, 0), (4, 2), (0, 0), (7, 6)$

Solution (a)

Exercise 3A – Question 1(i)

An employer recruits experienced (x) and fresh workmen (y) for his firm under the condition that he cannot employ more than 9 people. x and y can be related by the inequality:

(a) $x+y\neq 9$
(b) $x+y\leq 9$; $x\geq 0,\ y\geq 0$
(c) $x+y\geq 9,\ x\geq 0,\ y\geq 0$
(d) None

Solution (b)

Exercise 3A – Question 1(ii)

On the average experienced person does 5 units of work while a fresh one 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as:

(a) $5x+3y\leq 30$
(b) $5x+3y>30$
(c) $5x+3y\geq 30$; $x\geq 0,\ y\geq 0$
(d) None

Solution (c)

Exercise 3A – Question 1(iii)

The rules and regulations demand that the employer should employ not more than 5 experienced hands to 1 fresh one and this fact can be expressed as:

(a) $y\geq x/5$
(b) $5y\leq x$
(c) $5y\geq x$
(d) None

Solution (a) or (c)

Exercise 3A – Question 1(iv)

The union however forbids him to employ less than 2 experienced persons to each fresh person. This situation can be expressed as:

(a) $x\leq y/2$
(b) $y\leq x/2$
(c) $y\geq x/2$
(d) $x>2y$

Solution (b)

Exercise 3A – Question 2

A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per kg. of each food is shown below:

	A	B	C	D
Food I	2	1	1	2
Food II	1	1	2	3

Assuming *x* units of food I is to be mixed with *y* units of food II the situation can be expressed as:

(a) $2x + y \le 9$, $x + y \le 7$, $x + 2y \le 10$, $2x + 3y \le 12$, $x > 0,\ y > 0$

(b) $2x + 3y \ge 30$, $x + y \le 7$, $x + 2y \ge 10$, $x + 3y \ge 12$

(c) $2x + y \ge 9$, $x + y \ge 7$, $x + y \le 10$, $x + 3y \ge 12$

(d) $2x + y \ge 9$, $x + y \ge 7$, $x + 2y \ge 10$, $2x + 3y \ge 12$, $x \ge 0,\ y \ge 0$

Solution (d)

Exercise 3A – Question 8

A firm makes two types of products: Type A and Type B. The profit on product A is ₹20 each and that on product B is ₹30 each. Both types are processed on three machines M1, M2 and M3. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Product A	Product B	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

The constraints can be formulated taking x_1 = number of units A and x_2 = number of units of B as:

(a) $x_1 + x_2 \le 12$, $5x_1 + 2x_2 \le 50$, $2x_1 + 6x_2 \le 60$

(b) $3x_1 + 3x_2 \ge 36$, $5x_1 + 2x_2 \le 50$, $2x_1 + 6x_2 \ge 60$, $x_1 \ge 0,\ x_2 \ge 0$

(c) $3x_1 + 3x_2 \le 36$, $5x_1 + 2x_2 \le 50$, $2x_1 + 6x_2 \le 60$, $x_1 \ge 0,\ x_2 \ge 0$

(d) None

Solution (c)