Chapter 2 – Equations

Introduction

- An equation is a mathematical statement of equality.
- The value of the variable which satisfies an equation is called the "solution" of the equation, or the "root" of the equation.
- An equation is said to be a Linear Equation, a Quadratic Equation, or a Cubic Equation depending on the highest power of the variable in it.
    - If the highest power of the variables in an equation is 1, it is said to be a Linear Equation.
    - If the highest power of the variables in an equation is 2, it is said to be a Quadratic Equation.
    - If the highest power of the variables in an equation is 3, it is said to be a Cubic Equation.

Simple Equation

- A simple equation is an equation with only one unknown in the form of $ax+b=0$.
- Here, a and b are constants and x is the variable which we need to find out. A simple equation has only one root.

Page 2.2 – Example

Find the value of x from $\frac{4x}{3}-1=\frac{14}{15}x+\frac{19}{5}$.

(a) 24 (b) 12 (c) 13 (d) None

Solution (b)

Exercise (A) – Question 1

The equation $-7x+1=5-3x$ will be satisfied for x equal to:

(a) 2 (b) –1 (c) 1 (d) None

Solution (b)

Exercise (A) – Question 2

The root of the equation $\frac{x+4}{4}+\frac{x-5}{3}=11$ is

(a) 20 (b) 10 (c) 2 (d) None

Solution (a)

Exercise (A) – Question 3

Pick up the correct value of x for $\frac{x}{30}=\frac{2}{45}$

(a) 5 (b) 7 (c) $1\frac{1}{3}$ (d) None

Solution (c)

Exercise (A) – Question 4

The solution of the equation $\frac{x+24}{5}=4+\frac{x}{4}$:

(a) 6 (b) 10 (c) 16 (d) None

Solution (c)

Exercise (A) – Question 5

8 is the solution of the equation:

(a) $\frac{x+4}{4}+\frac{x-5}{3}=11$ (b) $\frac{x+4}{2}+\frac{x+10}{9}=8$

(c) $\frac{x+24}{5}=4+\frac{x}{4}$ (d) $\frac{x-15}{10}+\frac{x+5}{5}=4$

Solution (b)

Exercise (A) – Question 6

The value of y that satisfies the equation $\frac{y+11}{6}-\frac{y+1}{9}=\frac{y+7}{4}$ is:

(a) –1 (b) 7 (c) 1 (d) $-\frac{1}{7}$

Solution (d)

Exercise (A) – Question 7

The solution of the equation $(p+2)(p-3)+(p+3)(p-4)=p(2p-5)$ is:

(a) 6 (b) 7 (c) 5 (d) None

Solution (a)

Exercise (A) – Question 8

The equation $\frac{12x+1}{4}=\frac{15x-1}{5}+\frac{2x-5}{3x-1}$ is true for:

(a) $x=1$ (b) $x=2$ (c) $x=5$ (d) $x=7$

Solution (d)

Exercise (A) – Question 9

Pick up the correct value of x for which $\frac{x}{0.5}-\frac{1}{0.05}+\frac{x}{0.005}-\frac{1}{0.0005}=0$.

(a) $x=0$ (b) $x=1$ (c) $x=10$ (d) None

Solution (c)

Page 2.3 – Illustration 1

The denominator of a fraction exceeds the numerator by 5 and if 3 be added to both the fraction becomes $\frac{3}{4}$. Find the fraction.

(a) $\frac{12}{17}$ (b) $\frac{13}{17}$ (c) $\frac{14}{18}$ (d) $\frac{15}{19}$

Solution (a)

Page 2.4 – Illustration 2

If thrice of A's age 6 years ago, be subtracted from twice his present age, the result would be equal to his present age. Find A's present age.

(a) 8 (b) 9 (c) 10 (d) 11

Solution (b)

Page 2.4 – Illustration 3

A number consists of two digits. The digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number, the digits are reversed. Find the number.
(a) 63 (b) 84 (c) 42 (d) 21

Solution (c)

Page 2.4 – Illustration 4

For a certain commodity, the demand equation giving demand *'d'* in kg, for a price *'p'* in rupees per kg. is $d = 100(10 - p)$. The supply equation giving the supply *s* in kg. for a price *p* in rupees per kg. is $s = 75(p - 3)$. The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.
(a) 10, 400, 400 (b) 9, 500, 500
(c) 8, 340, 440 (d) 7, 300, 300

Solution (d)

Exercise (B) – Question 1

The sum of two numbers is 52 and their difference is 2. The numbers are:
(a) 17 and 15 (b) 12 and 10 (c) 27 and 25 (d) None

Solution (c)

Exercise (B) – Question 2

The diagonal of a rectangle is 5 cm and one of at sides is 4 cm. Its area is:
(a) 20 sq. cm. (b) 12 sq. cm. (c) 10 sq. cm. (d) None

Solution (b)

Exercise (B) – Question 3

Divide 56 into two parts such that three times the first part exceeds one third of the second by 48. The parts are:

(a) (20, 36) (b) (25, 31) (c) (24, 32) (d) None

Solution (a)

Exercise (B) – Question 4

The sum of the digits of a two-digit number is 10. If 18 be subtracted from it, the digits in the resulting number will be equal. The number is:

(a) 37 (b) 73 (c) 75 (d) None

Solution (b)

Exercise (B) – Question 5

The fourth part of a number exceeds the sixth part by 4. The number is:

(a) 84 (b) 44 (c) 48 (d) None

Solution (c)

Exercise (B) – Question 6

Ten years ago, the age of a father was four times of his son. Ten years hence, the age of the father will be twice that of his son. The present ages of the father and the son are:

(a) (50, 20) (b) (60, 20) (c) (55, 25) (d) None

Solution (a)

Exercise (B) – Question 7

The product of two numbers is 3200 and the quotient when the larger number is divided by the smaller is 2. The numbers are:

(a) (16, 200) (b) (160, 20) (c) (60, 30) (d) (80, 40)

Solution (d)

Exercise (B) – Question 8

The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator, the fraction increases by unity. The fraction is:

(a) $\frac{5}{7}$ (b) $\frac{1}{3}$ (c) $\frac{7}{9}$ (d) $\frac{3}{5}$

Solution (d)

Exercise (B) – Question 9

Three persons Mr. Roy, Mr. Paul and Mr. Singh together have ₹51. Mr. Paul has ₹4 less than Mr. Roy and Mr. Singh has got ₹5 less than Mr. Roy. They have the money as:

(a) (₹20, ₹16, ₹15) (b) (₹15, ₹20, ₹16)
(c) (₹25, ₹11, ₹15) (d) None

Solution (a)

Exercise (B) – Question 10

A number consists of two digits. The digits in the ten's place is 3 times the digit in the unit's place. If 54 is subtracted from the number, the digits are reversed. The number is:

(a) 39 (b) 92 (c) 93 (d) 94

Solution (c)

Exercise (B) – Question 11

One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so, the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number was:

(a) 320 (b) 400 (c) 480 (d) None

Solution (c)

Exercise (B) – Question 12

If a number of which the half is greater than $1/5^{th}$ of the number by 15, then the number is:

(a) 50 (b) 40 (c) 80 (d) None

Solution (a)

Simultaneous Linear Equations in Two Variables

Page 2.6 – Example 1

Solve $2x+5y=9$ and $3x-y=5$.

(a) (2, 1) (b) (3, 2) (c) (2, 2) (d) None

Solution (a)

Page 2.7 – Example 2

Solve $3x+2y+17=0$ and $5x-6y-9=0$.

(a) (–3, –4) (b) (3, 2) (c) (–3, 2) (d) None

Solution (a)

Exercise C – Question 1

The solution of the set of equations $3x + 4y = 7$, $4x – y = 3$ is

(a) (1, –1) (b) (1, 1) (c) (2, 1) (d) (1, –2)

Solution (b)

Exercise C – Question 2

The values of x and y satisfying the equations $\frac{x}{2}+\frac{y}{3}=2$, $x+2y=8$ are given by pair:

(a) (3, 2) (b) (–2, –3) (c) (2, 3) (d) None

Solution (c)

Exercise C – Question 3

$\frac{x}{p}+\frac{y}{q}=2$, $x+y=p+q$ are satisfied by the values given by the pair:

(a) $(x = p, y = q)$ (b) $(x = q, y = p)$
(c) $(x = 1, y = 1)$ (d) None

Solution (a)

Exercise C – Question 4

The solution for the pair of equations $\frac{1}{16x}+\frac{1}{15y}=\frac{9}{20}$, $\frac{1}{20x}-\frac{1}{27y}=\frac{4}{45}$ is given by:

(a) $\left(\frac{1}{4},\frac{1}{3}\right)$ (b) $\left(\frac{1}{3},\frac{1}{4}\right)$ (c) (3, 4) (d) (4, 3)

Solution (a)

Exercise C – Question 5

Solve for x and y: $\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10}$ and $3xy=10(y-x)$

(a) (5, 2) (b) (–2, –5) (c) (2, –5) (d) (2, 5)

Solution (d)

Exercise C – Question 6

The pair satisfying the equations $x+5y=36$, $\frac{x+y}{x-y}=\frac{5}{3}$ is given by:

(a) (16, 4) (b) (4, 16) (c) (4, 8) (d) None

Solution (a)

Exercise C – Question 7

Solve for x and y: $x - 3y = 0$, $x + 2y = 20$

(a) $x = 4$, $y = 12$ (b) $x = 12$, $y = 4$ (c) $x = 5$, $y = 4$ (d) None

Solution (b)

Exercise C – Question 8

The simultaneous equations $7x - 3y = 31$, $9x - 5y = 41$ have solutions given by:

(a) (–4, –1) (b) (–1, 4) (c) (4, –1) (d) (3, 7)

Solution (c)

Exercise C – Question 9

$1.5x + 2.4y = 1.8$, $2.5(x + 1) = 7y$ have solutions as:

(a) (0.5, 0.4) (b) (0.4, 0.5) (c) $\left(\frac{1}{2},\frac{2}{5}\right)$ (d) (2, 5)

Solution (b)

Exercise C – Question 10

The values of x and y satisfying the equations $\frac{3}{x+y}+\frac{2}{x-y}=3,\ \frac{2}{x+y}+\frac{3}{x-y}=3\frac{2}{3}$ are given by:

(a) (1, 2) (b) (–1, –2) (c) $\left(1, \frac{1}{2}\right)$ (d) (2, 1)

Solution (d)

Exercise D – Question 1

$1.5x + 3.6y = 2.1,\ 2.5(x + 1) = 6y$

(a) (0.2, 0.5) (b) (0.5, 0.2) (c) (2, 5) (d) (–2, –5)

Solution (a)

Exercise D – Question 2

$\frac{x}{5}+\frac{y}{6}+1=\frac{x}{6}+\frac{y}{5}=28$

(a) (6, 9) (b) (9, 6) (c) (60, 90) (d) (90, 60)

Solution (c)

Exercise D – Question 7

$\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10},\ 3xy=10(y-x)$

(a) (2, 5) (b) (5, 2) (c) (2, 7) (d) (3, 4)

Solution (a)

Exercise D – Question 8

$\frac{x}{0.01}+\frac{y+0.03}{0.05}=\frac{y}{0.02}+\frac{x+0.03}{0.04}=2$

(a) (1, 2) (b) (0.1, 0.2) (c) (0.01, 0.02) (d) (0.02, 0.01)

Solution (c)

Word Problems

Page 2.12 – Illustration 1

If the numerator of a fraction is increased by 2 and the denominator by 1, it becomes 1. Again, if the numerator is decreased by 4 and the denominator by 2, it becomes 1/2. Find the fraction.

(a) 2/3 (b) 4/5 (c) 7/8 (d) None

Solution (c)

Page 2.12 – Illustration 2

The age of a man is three times the sum of the ages of his two sons and 5 years hence his age will be double the sum of their ages. Find the present age of the man?

(a) 23 (b) 45 (c) 78 (d) None

Solution (b)

Page 2.13 – Illustration 3

A number consist of three digits of which the middle one is zero and the sum of the other digits is 9. The number formed by interchanging the first and third digits is more than the original number by 297. Find the number.

(a) 306 (b) 207 (c) 702 (d) None

Solution (a)

Exercise E – Question 1

Monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenses are in the ratio 7 : 9. If each saves ₹50 per month find their monthly incomes.

(a) (500, 400) (b) (400, 500) (c) (300, 600) (d) (350, 550)

Solution (b)

Exercise E – Question 2

Find the fraction which is equal to 1/2 when both its numerator and denominator are increased by 2. It is equal to 3/4 when both are increased by 12.

(a) 3/8 (b) 5/8 (c) 2/8 (d) 2/3

Solution (a)

Exercise E – Question 3

The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.

(a) 60 years (b) 52 years (c) 51 years (d) 50 years

Solution (d)

Exercise E – Question 4

A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed find the number.

(a) 54 (b) 53 (c) 45 (d) 55

Solution (c)

Exercise E – Question 5

The wages of 8 men and 6 boys amount to ₹33. If 4 men earn ₹4.50 more than 5 boys determine the wages of each man and boy.

(a) (₹1.50, ₹3) (b) (₹3, ₹1.50)
(c) (₹2.50, ₹2) (d) (₹2, ₹2.50)

Solution (b)

Exercise E – Question 6

A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reversed. The number is:

(a) 63 (b) 35 (c) 36 (d) 60

Solution (c)

Exercise E – Question 7

Of two numbers, 1/5th of the greater is equal to 1/3rd of the smaller and their sum is 16. The numbers are:

(a) (6, 10) (b) (9, 7) (c) (12, 4) (d) (11, 5)

Solution (a)

Exercise E – Question 8

y is older than x by 7 years. 15 years back, x's age was $3/4^{th}$ of y's age. Their present ages are:

(a) (x = 36, y = 43) (b) (x = 50, y = 43) (c) (x = 43, y = 50) (d) (x = 40, y = 47)

Solution (a)

Exercise E – Question 9

The sum of the digits in a three digit number is 12. If the digits are reversed, the number is increased by 495 but reversing only of the tens and units digits increases the number by 36. The number is:

(a) 327 (b) 372 (c) 237 (d) 273

Solution (c)

Exercise E – Question 10

Two numbers are such that twice the greater number exceeds twice the smaller one by 18 and $1/3^{rd}$ of the smaller and $1/5^{th}$ of the greater number are together 21. The numbers are:

(a) (36, 45) (b) (45, 36) (c) (50, 41) (d) (55, 46)

Solution (b)

Exercise E – Question 11

The demand and supply equations for a certain commodity are $4q + 7p = 17$ and $p = \frac{q}{3} + \frac{7}{4}$ respectively where p is the market price and q is the quantity. The equilibrium price and quantity are:

(a) $2, \frac{3}{4}$ (b) $3, \frac{1}{2}$ (c) $5, \frac{3}{5}$ (d) None

Solution (a)

Simultaneous Linear Equations in Three Variables

Page 2.7 – Example 1

Solve for x, y and z: $2x - y + z = 3$, $x + 3y - 2z = 11$, $3x - 2y + 4z = 1$

(a) (3, 2, –1) (b) (3, 2, 1) (c) (3, –2, 1) (d) None

Solution (a)

Page 2.8 – Example 2

Solve for x, y and z: $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=5$, $\frac{2}{x}-\frac{3}{y}-\frac{4}{z}=-11$, $\frac{3}{x}+\frac{2}{y}-\frac{1}{z}=-6$

(a) $\left(\frac{1}{2}, -\frac{1}{3}, \frac{1}{6}\right)$ (b) $\left(\frac{1}{2}, -\frac{1}{3}, -\frac{1}{6}\right)$

(c) $\left(-\frac{1}{2}, -\frac{1}{3}, \frac{1}{6}\right)$ (d) None

Solution (a)

Page 2.9 – Example 3

Solve for x, y and z: $\frac{xy}{x+y}=70$, $\frac{xz}{x+z}=84$, $\frac{yz}{y+z}=140$

(a) (105, 210, 420) (b) (105, 420, 210)
(c) (420, 105, 210) (d) None

Solution (a)

Exercise D – Question 3

$\frac{x}{4}=\frac{y}{3}=\frac{z}{2}$; $7x + 8y + 5z = 62$

(a) (4, 3, 2) (b) (2, 3, 4) (c) (3, 4, 2) (d) (4, 2, 3)

Solution (a)

Exercise D – Question 4

$$\frac{xy}{x+y}=20;\ \frac{yz}{y+z}=40;\ \frac{zx}{z+x}=24$$

(a) (120, 60, 30) (b) (60, 30, 120)
(c) (30, 120, 60) (d) (30, 60, 120)

Solution (d)

Exercise D – Question 5

$2x + 3y + 4z = 0$, $x + 2y - 5z = 0$, $10x + 16y - 6z = 0$

(a) (0, 0, 0) (b) (1, –1, 1) (c) (3, 2, –1) (d) (1, 0, 2)

Solution (a)

Exercise D – Question 6

$$\frac{1}{3}(x+y)+2z=21;\ 3x-\frac{1}{2}(y+z)=65;\ x+\frac{1}{2}(x+y-z)=38$$

(a) (4, 9, 5) (b) (2, 9, 5) (c) (24, 9, 5) (d) (24, 9, 5)

Solution (c)

Exercise D – Question 9

$$\frac{xy}{y-x}=110,\ \frac{yz}{z-y}=132,\ \frac{zx}{z+x}=\frac{60}{11}$$

(a) (12, 11, 10) (b) (10, 11, 12)
(c) (11, 10, 12) (d) (12, 10, 11)

Solution (b)

Exercise D – Question 10

$3x - 4y + 70z = 0$, $2x + 3y - 10z = 0$, $x + 2y + 3z = 13$

(a) (1, 3, 7) (b) (1, 7, 3) (c) (2, 4, 3) (d) (–10, 10, 1)

Solution (d)

Quadratic Equations

- A quadratic equation is an equation in which the highest power of the variables is 2.
- A quadratic equation is of the form $ax^2+bx+c=0$.
- x is a variable while a, b and c are constants.
- A quadratic equation has two solutions/roots.

Methods of Solving Quadratic Equations

There are three methods of solving any quadratic equation:

1. Factorization Method

   Example: Solve the equation $x^2-5x+6=0$ using factorization method.

2. Quadratic Formula

   Quadratic Formula = $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

   If we call the roots α, and β, then,

   $$\alpha=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

   $$\beta=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

   Example: Solve the equation $x^2-5x+6=0$ using quadratic formula.

   Sum of Roots $(\alpha+\beta)=-\dfrac{b}{a}$

   Product of Roots $\alpha\beta=\dfrac{c}{a}$

3. Fastest Method

   Solve the equation $x^2-5x+6=0$ using fastest method.

   Here, a = 1; b = –5; c = 6

   Sum of Roots $=-\dfrac{b}{a}=-\dfrac{-5}{1}=5$

   Product of Roots $=\dfrac{c}{a}=\dfrac{6}{1}=6$

   Now, take the sum of the roots, divide it by half, and add x to it. You'll get $\left(\dfrac{5}{2}+x\right)$.

   Similarly, take the sum of the roots, divide it by half, and subtract x from it. You'll get $\left(\dfrac{5}{2}-x\right)$. Multiply these two and equate with the product, i.e. 6.

   $$\left(\frac{5}{2}+x\right)\left(\frac{5}{2}-x\right)=6$$

   $$\Rightarrow\left(\frac{5}{2}\right)^2-x^2=6$$

   $$\Rightarrow\frac{25}{4}-x^2=6$$

$$\Rightarrow x^2 = \frac{25}{4} - 6$$

$$\Rightarrow x^2 = 6.25 - 6$$

$$\Rightarrow x^2 = 0.25$$

$$\Rightarrow x = \sqrt{0.25}$$

$$\Rightarrow x = 0.5$$

Now, put the value of x = 0.5 in the factors $\left(\frac{5}{2} + x\right)$, and $\left(\frac{5}{2} - x\right)$. You'll get the roots.

Therefore, $\alpha = \frac{5}{2} + 0.5 = 3$; $\beta = \frac{5}{2} - 0.5 = 2$.

This method applies to complicated roots as well.

Page 2.18 – Illustration 3

Solve for x: $4^x - 3.2^{x+2} + 2^5 = 0$

(a) 2, 3 (b) 4, 5 (c) 1, –1 (d) None

Solution (a)

Page 2.18 – Illustration 4

Solve $\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = 7\frac{1}{4}$.

(a) 2, ½ (b) 2, 1 (c) –2, –1 (d) None

Solution (a)

Page 2.19 – Illustration 5

Solve for x: $2^{x-2} + 2^{3-x} = 3$

(a) 2, 3 (b) 4, 5 (c) 1, –1 (d) None

Solution (a)

Exercise F – Question 2

If $2^{2x+3} - 3^2.2^x + 1 = 0$, then the values of x are:

(a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, –3

Solution (d)

Exercise F – Question 4

If α, β be the roots of the equation $2x^2-4x-3=0$, then the value of $\alpha^2+\beta^2$ is:

(a) 5 (b) 7 (c) 3 (d) –4

Solution (b)

Exercise G – Question 1

A solution of the quadratic equation $(a+b-2c)x^2+(2a-b-c)x+(c+a-2b)=0$ is:

(a) 1 (b) –1 (c) 2 (d) –2

Solution (b)

Exercise G – Question 3

The values of x in the equation $7(x+2p)^2+5p^2=35xp+117p^2$ are:

(a) $(4p, -3p)$ (b) $(4p, 3p)$ (c) $(-4p, 3p)$ (d) $(-4p, -3p)$

Solution (a)

Exercise G – Question 4

The solutions of the equation $\frac{6x}{x+1}+\frac{6(x+1)}{x}=13$ are:

(a) (2, 3) (b) (3, –2) (c) (–2, –3) (d) (2, –3)

Solution (d)

Exercise G – Question 5

The satisfying values of x for the equation $\frac{1}{x+p+q}=\frac{1}{x}+\frac{1}{p}+\frac{1}{q}$ are:

(a) (p, q) (b) $(-p, -q)$ (c) $(p, -p)$ (d) $(-p, q)$

Solution (b)

Exercise G – Question 6

The values of x for the equation $x^2+9x+18=6-4x$ are:

(a) (1, 12) (b) (–1, –12) (c) (1, –12) (d) (–1, 12)

Solution (b)

Exercise G – Question 7

The values of x satisfying the equation $\sqrt{\left(2x^2+5x-2\right)}-\sqrt{\left(2x^2+5x-9\right)}=1$ are:

(a) (2, –9/2) (b) (4, –9) (c) (2, 9/2) (d) (–2, 9/2)

Solution (a)

Exercise G – Question 8

The solution of the equation $3x^2-17x+24=0$ are:

(a) (2, 3) (b) $\left(2,\ 3\frac{2}{3}\right)$ (c) $\left(3,\ 2\frac{2}{3}\right)$ (d) $\left(3,\ \frac{2}{3}\right)$

Solution (c)

Exercise G – Question 9

The equation $\frac{3\left(3x^2+15\right)}{6}+2x^2+9=\frac{2x^2+96}{7}+6$ has got the solution as:

(a) (1, 1) (b) (1/2, –1) (c) (1, –1) (d) (2, –1)

Solution (c)

Exercise G – Question 10

The equation $\left(\frac{l-m}{2}\right)x^2-\left(\frac{l+m}{2}\right)x+m=0$ has got two values of x to satisfy the equation given as:

(a) $\left(1, \frac{2m}{l-m}\right)$ (b) $\left(1, \frac{m}{l-m}\right)$ (c) $\left(1, \frac{2l}{l-m}\right)$ (d) $\left(1, \frac{1}{l-m}\right)$

Solution (a)

Important Rules

- Sum of Roots $(\alpha+\beta)=-\frac{b}{a}$
- Product of Roots $(\alpha\beta)=\frac{c}{a}$
- If α and β are the roots of the equation, the equation is given by:
  $x^2-(\alpha+\beta)x+\alpha\beta=0$
  In other words,
  $x^2-(Sum\ of\ Roots)x+Product\ of\ Roots=0$.

Nature of Roots

We know that the quadratic formula gives us the value of x as follows:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

In this formula, the term b^2-4ac plays a very important role. The nature of the roots is dependent on b^2-4ac .

1. If $b^2-4ac=0$, the roots are real and equal.
2. If $b^2-4ac>0$, the roots are real and unequal.
   a. If b^2-4ac is a perfect square, the roots are real, rational, and unequal.
   b. If b^2-4ac is not a perfect square, the roots are real, irrational, and unequal.
3. If $b^2-4ac<0$, the roots are imaginary and unequal.

Since b^2-4ac discriminates the roots, it is known as the discriminant.

Points to be noted –

1. A real number is a number which can be expressed on a number line. Therefore, every number is a real number, including negative numbers.
2. An imaginary number is a number multiplied by a unit "i", which is identified by its property $i^2=-1$.
3. An integer is a number without any fractional part. It includes positive as well as negative numbers.
4. A rational number is a number which can be expressed as a fraction of two integers. The decimal expansion of a rational number either terminates after a finite number of digits, or begins to repeat the same finite sequence of digits over and over.
   Examples:

a. 2 is a rational number as it can be expressed in the form of $\frac{2}{1}$.

b. $\frac{5}{2}$ is a rational number as its decimal expansion 2.5 terminates after a finite number of digits.

c. $\frac{2}{9}$ is a rational number as its decimal expansion comes to 0.222…, i.e. it begins to repeat itself over and over.

d. $-\frac{5}{2}$, $-\frac{2}{9}$ are also rational numbers.

5. An irrational number is a number whose decimal expansion either does not terminate after a finite number of digits or does not repeat itself over and over.
   Examples:
   a. π is an irrational number as its decimal expansion is 3.14159265359…, i.e. it neither terminates after a finite number of digits nor does it repeat itself over and over.
   b. $\sqrt{2}$ is an irrational number as its decimal expansion is 1.41421356237…, i.e. it neither terminates after a finite number of digits nor does it repeat itself over and over.
6. Irrational roots occur in conjugate pairs, i.e., if $\left(m+\sqrt{n}\right)$ is a root, then $\left(m-\sqrt{n}\right)$ is the other root of the same equation.
7. If one root is reciprocal to the other root, then their product is 1 and so $\frac{c}{a}=1$, i.e. $c=a$.
8. If one root is equal to the other root but opposite in sign, then their sum = 0, i.e. $-\frac{b}{a}=0 \Rightarrow b=0$.

Page 2.16 – Example 2 – (i)

Examine the nature of roots of the following equation: $x^2-8x+16=0$.

(a) Real and Equal
(b) Real and Unequal
(c) Imaginary and Unequal
(d) Real, Rational, Unequal

Solution (a)

Page 2.16 – Example 2 – (ii)

Examine the nature of roots of the following equation: $3x^2-8x+4=0$.

(a) Real and Equal
(b) Real and Unequal
(c) Imaginary and Unequal
(d) Real, Rational, Unequal

Solution (d)

Page 2.16 – Example 2 – (iii)

Examine the nature of roots of the following equation: $5x^2-4x+2=0$.

(a) Real and Equal
(b) Real and Unequal
(c) Imaginary and Unequal
(d) Real, Rational, Unequal

Solution (c)

Page 2.16 – Example 2 – (iv)

Examine the nature of roots of the following equation: $2x^2-6x-3=0$.

(a) Real and Equal
(b) Real, Irrational, Unequal
(c) Imaginary and Unequal
(d) Real, Rational, Unequal

Solution (b)

Exercise F – Question 1

If the roots of the equation $2x^2+8x-m^3=0$ are equal, then the value of *m* is:

(a) –3 (b) –1 (c) 1 (d) –2

Solution (d)

Exercise F – Question 6

The equation $x^2-(p+4)x+2p+5=0$ has equal roots. The value of *p* will be:

(a) ± 1 (b) 2 (c) ± 2 (d) –2

Solution (c)

Exercise F – Question 7

The roots of the equation $x^2+(2p-1)x+p^2=0$ are real if:

(a) $p\geq 1$ (b) $p\leq 4$ (c) $p\geq 1/4$ (d) $p\leq 1/4$

Solution (d)

Exercise F – Question 10

If $L+M+N=0$, and L, M, and N are rationals, the roots of the equation $(M+N-L)x^2+(N+L-M)x+(L+M-N)=0$ are:

(a) Real and Irrational
(b) Real and Rational
(c) Imaginary and Equal
(d) Real and Equal

Solution (b)

We have

$$(M+N-L)x^2+(N+L-M)x+(L+M-N)=0$$

We know that

$$L+M+N=0$$

Therefore,

$$M+N=-L;\ N+L=-M;\ L+M=-N;\ M=-N-L$$

Therefore, we have

$$(-L-L)x^2+(-M-M)x+(-N-N)=0$$

$$\Rightarrow -2Lx^2-2Mx-2N=0$$

$$\Rightarrow -2(Lx^2+Mx^2+N)=0$$

$$\Rightarrow Lx^2+Mx^2+N=0$$

Here, $a=L;\ b=M;\ c=N$

$$b^2-4ac=M^2-(4)(L)(N)$$

$$=(-N-L)^2-4LN$$

$$=\{-(N+L)\}^2-4LN$$

$$=(N+L)^2-4LN$$

$$=N^2+L^2+2LN-4LN$$

$$=N^2+L^2-2LN$$

$$=(N-L)^2$$

Therefore, D is a perfect square. Hence, the roots are rational. Also, the roots are real. This is because even if N – L comes to be a negative figure, squaring it would make it positive, and thereafter, its square root will be determined in the quadratic formula. Therefore, the roots are Real and Rational.

Page 2.19 – Illustration 6

If one root of the equation is $2-\sqrt{3}$, form the equation given that the roots are irrational.

(a) $x^2-4x+2=0$ (b) $x^2-3x+9=0$ (c) $x^2-5x+2=0$ (d) $x^2-4x+1=0$

Solution (d)

Page 2.20 – Illustration 8

If the roots of the equation $p(q-r)x^2+q(r-p)x+r(p-q)=0$ are equal, find the value of $\frac{1}{p}+\frac{1}{r}$.

(a) $\frac{2}{q}$ (b) $\frac{1}{q}$ (c) $\frac{1}{2}$ (d) None

Solution (a)

Here, $a=p(q-r)$; $b=q(r-p)$; $c=r(p-q)$

Since the roots of this equation are equal, $b^2-4ac=0$.

$$\{q(r-p)\}^2-(4)\{p(q-r)\}\{r(p-q)\}=0$$

$$q^2(r-p)^2-\left[4pr(q-r)(p-q)\right]=0$$

$$q^2\left(r^2+p^2-2rp\right)-\left[4pr\left(qp-q^2-pr+qr\right)\right]=0$$

$$q^2r^2+q^2p^2-2rpq^2-\left[4p^2qr-4pq^2r-4p^2r^2+4pqr^2\right]=0$$

$$q^2r^2+q^2p^2-2rpq^2-4p^2qr+4pq^2r+4p^2r^2-4pqr^2=0$$

$$q^2r^2+q^2p^2+4pq^2r-2rpq^2-4p^2qr+4p^2r^2-4pqr^2=0$$

$$q^2r^2+q^2p^2+2pq^2r-4p^2qr+4p^2r^2-4pqr^2=0$$

We know that $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

If we look closely at the LHS of the following equation, $q^2r^2+q^2p^2+2pq^2r-4p^2qr+4p^2r^2-4pqr^2=0$, we'll find that it is the expansion of $(qr+qp-2pr)^2$.

Therefore,

$$(qr+qp-2pr)^2=0$$

$$\Rightarrow qr+qp-2pr=0$$

$$\Rightarrow qr+qp=2pr$$

Dividing the entire equation by *pqr*, we get:

$$\frac{qr}{pqr}+\frac{qp}{pqr}=\frac{2pr}{pqr}$$

$$\Rightarrow \frac{1}{p}+\frac{1}{r}=\frac{2}{q}$$

Page 2.17 – Illustration 1

If α and β be the roots of $x^2 + 7x + 12 = 0$, find the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.

(a) $x^2 - 50x + 49 = 0$ (b) $x^2 + 50x + 49 = 0$ (c) $x^2 - 50x - 49 = 0$ (d) None

Solution (a)

Page 2.19 – Illustration 7

If α, β are the two roots of the equation $x^2 + px + q = 0$, form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(a) $qx^2-(p^2-2q)x+q=0$ (b) $px^2-(p^2-2q)x+q=0$

(c) $qx^2-(p^2-2q)x+p=0$ (d) $qx^2+(p^2-2q)x+p=0$

Solution (a)

Exercise F – Question 13

If one root of $5x^2 + 13x + p = 0$ be reciprocal of the other, then the value of *p* is:

(a) –5 (b) 5 (c) 1/5 (d) –1/5

Solution (b)

Exercise F – Question 11

If α and β are the roots of $x^2 = x + 1$, then the value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is:

(a) $2\sqrt{5}$ (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) $-2\sqrt{5}$

Solution (d)

Page 2.17 – Illustration 2

If α, β be the roots of $2x^2 - 4x - 1 = 0$, find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

(a) –22 (b) 23 (c) –23 (d) None

Solution (a)

Exercise F – Question 3

The value of $4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \ldots\infty}}}$ is:

(a) $1 \pm \sqrt{2}$ (b) $2 + \sqrt{5}$ (c) $2 \pm \sqrt{5}$ (d) None

Solution (b)

Exercise F – Question 5

If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is:

(a) 2 (b) –2 (c) 1 (d) –1

Solution (a)

Exercise F – Question 8

If $x = m$ is one of the solutions of the equations $2x^2 + 5x - m = 0$, the possible values of *m* are:

(a) (0, 2) (b) (0, –2) (c) (0, 1) (d) (1, –1)

Solution (b)

Exercise F – Question 9 – Ambiguous

If *p* and *q* are the roots of the $x^2 + 2x + 1 = 0$, then the values of $p^3 + q^3$ becomes:

(a) 2 (b) –2 (c) 4 (d) –4

Solution (b), However, ICAI has marked (a), so we'll also mark (a) in the exam.

Exercise F – Question 12

If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$, the equation having the roots as $\frac{p}{q}$ and $\frac{q}{p}$ is:

(a) $x^2 - 19x + 3 = 0$
(b) $3x^2 - 19x - 3 = 0$
(c) $3x^2 - 19x + 3 = 0$
(d) $3x^2 + 19x + 3 = 0$

Solution (c)

Exercise G – Question 2

If the root of the equation $x^2 - 8x + m = 0$ exceeds the other by 4, then the value of *m* is:

(a) 10
(b) 11
(c) 9
(d) 12

Solution (d)

Word Problems

Page 2.23 – Problem 1

Difference between a number and its positive square root is 12; find the numbers.

(a) 4, 16 (b) 16, 4 (c) 22, 35 (d) Both (a) and (b)

Solution (d)

Page 2.23 – Problem 2

A piece of iron rod costs ₹60. If the rod was 2 metre shorter and each metre costs ₹1.00 more, the cost would remain unchanged. What is the length of the rod?

(a) 10 m (b) 14 m (c) 12 m (d) None

Solution (c)

Page 2.23 – Problem 3

Divide 25 into two parts so that sum of their reciprocals is 1/6.

(a) 8 and 17 (b) 10 and 15 (c) 20 and 5 (d) None

Solution (b)

Exercise H – Question 1

The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are:

(a) (7, 10) (b) (4, 4) (c) (3, 5) (d) (2, 6)

Solution (c)

Exercise H – Question 2

The difference of two positive integers is 3 and the sum of their squares is 89. Taking the smaller integer as x form a quadratic equation and solve it to find the integers. The integers are:

(a) (7, 4) (b) (5, 8) (c) (3, 6) (d) (2, 5)

Solution (b)

Exercise H – Question 3

Five times of a positive whole number is 3 less than twice the square of the number. The number is

(a) 3 (b) 4 (c) –3 (d) 2

Solution (a)

Exercise H – Question 4

The area of a rectangular field is 2000 sq.m. and its perimeter is 180 m. Form a quadratic equation by taking the length of the field as x and solve it to find the length and breadth of the field. The length and breadth are:

(a) (205 m, 80 m) (b) (50 m, 40 m) (c) (60 m, 50 m) (d) None

Solution (b)

Exercise H – Question 5

Two squares have sides p cm and (p + 5) cms. The sum of their squares is 625 sq. cm. The sides of the squares are:

(a) (10 cm, 30 cm) (b) (12 cm, 25 cm) (c) (15 cm, 20 cm) (d) None

Solution (c)

Exercise H – Question 6

Divide 50 into two parts such that the sum of their reciprocals is 1/12. The numbers are:

(a) (24, 26) (b) (28, 22) (c) (27, 23) (d) (20, 30)

Solution (d)

Exercise H – Question 7

There are two consecutive numbers such that the difference of their reciprocals is 1/240. The numbers are:

(a) (15, 16) (b) (17, 18) (c) (13, 14) (d) (12, 13)

Solution (a)

Exercise H – Question 8

The hypotenuse of a right–angled triangle is 20 cm. The difference between its other two sides be 4 cm. The sides are:

(a) (11 cm, 15 cm) (b) (12 cm, 16 cm)
(c) (20 cm, 24 cm) (d) None

Solution (b)

Exercise H – Question 9

The sum of two numbers is 45 and the mean proportional between them is 18. The numbers are:

(a) (15, 30) (b) (32, 13) (c) (36, 9) (d) (25, 20)

Solution (c)

Exercise H – Question 10

The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right-angle triangle is formed. The side of the equilateral triangle is:

(a) 17 units (b) 16 units (c) 15 units (d) 18 units

Solution (a)

Exercise H – Question 11

A distributor of apple Juice has 5000 bottles in the store that it wishes to distribute in a month. From experience it is known that demand D (in number of bottles) is given by $D = -2000p^2 + 2000p + 17000$. The price per bottle that will result zero inventory is:

(a) ₹3 (b) ₹5 (c) ₹2 (d) None

Solution (a)

Exercise H – Question 12

The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are:

(a) $3\sqrt{2}, 2\sqrt{3}$ (b) $5\sqrt{2}, 3\sqrt{5}$ (c) $2\sqrt{2}, 5\sqrt{2}$ (d) None

Solution (c)

Cubic Equations

An equation with the highest power of the variable as 3 is known as a Cubic Equation.

Page 2.25 – Illustration 1

Solve $x^3 - 7x + 6 = 0$

(a) (1, 2, –3) (b) (2, 3, –1) (c) (1, 2, – 4) (d) None

Solution (a)

Page 2.25 – Illustration 2

Solve for real *x*: $x^3 + x + 2 = 0$

(a) $\left(-1, \frac{1+\sqrt{-7}}{2}, \frac{1-\sqrt{-7}}{2}\right)$ (b) –1 (c) 2 (d) None

Solution (b)

Exercise I – Question 1

The solution of the cubic equation $x^3 - 6x^2 + 11x - 6 = 0$ is given by the triplet:

(a) (–1, 1, –2) (b) (1, 2, 3) (c) (–2, 2, 3) (d) (0, 4, –5)

Solution (b)

Exercise I – Question 2

The cubic equation $x^3 + 2x^2 - x - 2 = 0$ has 3 roots namely:

(a) (1, –1, 2) (b) (–1, 1, –2) (c) (–1, 2, –2) (d) (1, 2, 2)

Solution (b)

Exercise I – Question 3

$x,\ x-4,\ x+5$ are the factors of the left-hand side of the equation:

(a) $x^3 + 2x^2 - x - 2 = 0$ (b) $x^3 + x^2 - 20x = 0$
(c) $x^3 - 3x^2 - 4x + 12 = 0$ (d) $x^3 - 6x^2 + 11x - 6 = 0$

Solution (b)

Exercise I – Question 4

The equation $3x^3+5x^2=3x+5$ has got 3 roots and hence the factors of the left-hand side of the equation $3x^3+5x^2-3x-5=0$ are:

(a) $x-1,\ x-2,\ x-5/3$
(b) $x-1,\ x+1,\ 3x+5$
(c) $x+1,\ x-1,\ 3x-5$
(d) $x-1,\ x+1,\ x-2$

Solution (b)

Exercise I – Question 5

The roots of the equation $x^3+7x^2-21x-27=0$ are:

(a) (–3, –9, –1) (b) (3, –9, –1) (c) (3, 9, 1) (d) (–3, 9, 1)

Solution (b)

Exercise I – Question 6

The roots of $x^3+x^2-x-1=0$ are:

(a) (–1, –1, 1) (b) (1, 1, –1) (c) (–1, –1, –1) (d) (1, 1, 1)

Solution (a)

Exercise I – Question 7

The satisfying value of $x^3+x^2-20x=0$

(a) (1, 4, –5) (b) (2, 4, –5) (c) (0, –4, 5) (d) (0, 4, –5)

Solution (d)

Exercise I – Question 8

The roots of the cubic equation $x^3+6x^2-9x+4=0$ are:

(a) (4, 1, –1) (b) (–4, –1, –1) (c) (–4, –1, 1) (d) (4, 1, 1)

Solution (b)

Exercise I – Question 9

If $4x^3+8x^2-x-2=0$, then the value of $(2x+3)$ is given by:

(a) 4, –1, 2 (b) –4, 2, 1 (c) 2, –4, –1 (d) None

Solution (a)

Exercise I – Question 10

The rational root of the equation $2x^3-x^2-4x+2=0$ is:

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) –2

Solution (a)