# Chapter 1 – Ratio, Proportion, Indices, Logarithms

## Unitary Method

Suppose I tell you that 10 Apples cost ₹100 and ask you how much will 7 Apples Cost?

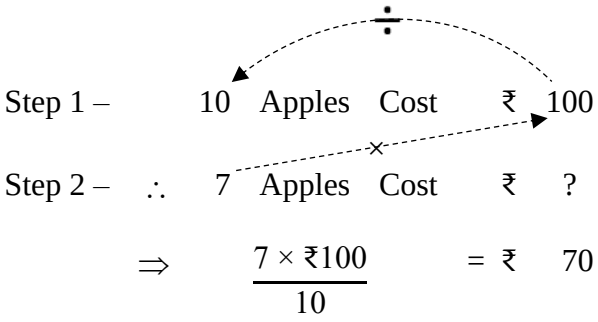
You would ordinarily do the following:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Step 1 – | | 10 | Apples | Cost | | | ₹ | 100 |
| Step 2 – | ⇒ | 1 | Apple | Costs | $\frac{₹100}{10}$ | = | ₹ | 10 |
| Step 3 – | ∴ | 7 | Apples | Cost | ₹10 × 7 | = | ₹ | 70 |

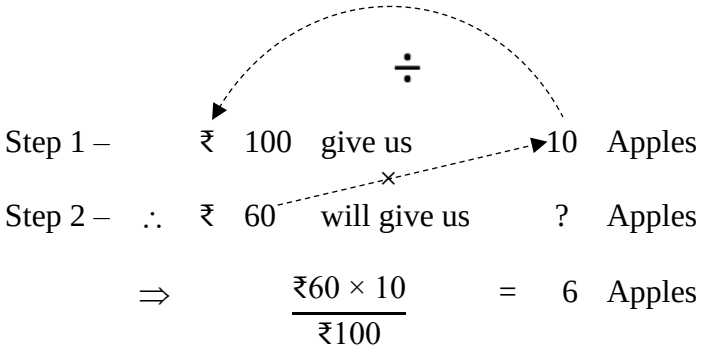
Now, let us do the same thing in just two steps:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Step 1 – | | 10 | Apples | Cost | ₹ | 100 |
| Step 2 – | ∴ | 7 | Apples | Cost | ₹ | ? |

Always proceed with the calculation as shown below:




A point to be noted here is that the **requirement** of the question should be placed on the right-hand side while forming the structure shown above. In the above example, we were supposed to calculate the amount, and therefore, we placed the amount on the right-hand side while forming the structure. If we were supposed to calculate how many apples can be bought with, say ₹60, we would place Apples on the right-hand side while forming the structure, as now the requirement of the question is no. of apples. The process is shown below:



## Ratio

A ratio is a comparison of two quantities of the same kind and of same units. If *a* and *b* are two quantities of the same kind, then the fraction *a*/*b* is called the ratio of *a* to *b*. It is written as *a* : *b*. The quantities *a* and *b* are called terms of the ratio; *a* is called the first term or the "antecedent" and *b* is called the second term or the "consequent". Usually, a ratio is expressed in its simplest form.

**Exercise 1A – Question 2**

The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is:

(a) 16 (b) 60 (c) 22 (d) 20

**Solution** (d)

## Points to Remember

1. Ratio exists only between quantities of same kind.
2. Quantities to be compared must be in the same units.

**Page 1.2 – Illustration IV (1)**

What is the ratio between 150 gm and 2 kg?

(a) 150 : 2 (b) 75 : 1 (c) 3 : 40 (d) Both (a) and (b)

**Solution** (c)

**Page 1.2 – Illustration IV (2)**

What is the ratio between 25 minutes and 45 seconds?

(a) 25 : 45 (b) 5 : 9 (c) 100 : 3 (d) Both (a) and (b)

**Solution** (c)

## Points to Remember (Contd.)

3. To compare ratios, use calculator as is explained in the class.

**Page 1.2 – Illustration VI**

Which ratio is greater: $2\frac{1}{3}:3\frac{1}{3}$, 3.6 : 4.8?

(a) $2\frac{1}{3}:3\frac{1}{3}$ (b) 3.6 : 4.8 (c) Can't Say (d) None

**Solution** (b)

## Points to Remember (Contd.)

4. If a quantity increases or decreases in the ratio $a : b$, then new quantity = $b$ of the original quantity/$a$.
   The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio. (This is basically unitary method.)

**Page 1.3 – Illustration VII**

Rounaq weighs 56.7 kg. If he reduces his weight in the ratio 7 : 6, find his new weight.

(a) 48.6 kgs (b) 49.6 kgs (c) 50 kgs (d) None

**Solution** (a)

## Inverse Ratio

The inverse ratio of $a/b$ is $b/a$.

**Exercise 1A – Question 1**

The inverse ratio of 11:15 is:

(a) 15:11 (b) $\sqrt{11} : \sqrt{15}$ (c) 121:225 (d) None

**Solution** (a)

**Exercise 1A – Question 3**

The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 5, the antecedent is:

(a) 5 (b) $\sqrt{5}$ (c) 7 (d) None

**Solution** (c)

## Duplicate Ratio

A ratio compounded of itself is called a Duplicate Ratio. The duplicate ratio of $a : b$ is $a^2 : b^2$.

**Exercise 1A – Question 5**

The duplicate ratio of 3 : 4 is:

(a) $\sqrt{3} : 2$ (b) 4 : 3 (c) 9 : 16 (d) None

**Solution** (c)

**Exercise 1A – Question 21**

If $2s:3t$ is the duplicate ratio of $2s-p:3t-p$, then:

(a) $p^2=6st$ (b) $p=6st$ (c) $2p=3st$ (d) None

**Solution** (a)

## Sub-Duplicate Ratio

The sub-duplicate ratio of $a$ : $b$ is $\sqrt{a}:\sqrt{b}$

**Exercise 1A – Question 6**

The sub-duplicate ratio of 25 : 36 is:

(a) 6 : 5 (b) 36 : 25 (c) 50 : 72 (d) 5 : 6

**Solution** (d)

**Exercise 1A – Question 20**

If $p:q$ is the sub duplicate ratio of $p-x^2:q-x^2$, then $x^2$ is:

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{p+q}$ (d) None

**Solution** (c)

## Triplicate Ratio

The triplicate ratio of $a$ : $b$ is $a^3$ : $b^3$.

**Exercise 1A – Question 7**

The triplicate ratio of 2 : 3 is:

(a) 8 : 27 (b) 6 : 9 (c) 3 : 2 (d) None

**Solution** (a)

## Sub-Triplicate Ratio

The sub-triplicate ratio of $a$ : $b$ is $\sqrt[3]{a}:\sqrt[3]{b}$

**Exercise 1A – Question 8**

The sub triplicate ratio of 8 : 27 is:

(a) 27 : 8 (b) 24 : 81 (c) 2 : 3 (d) None

**Solution** (c)

## Compound Ratio

The multiplication of two or more ratios is called compound ratio. The compound ratio of $a : b$ and $c : d$ is $ac : bd$.

**Exercise 1A – Question 4**

The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is:

(a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) None

**Solution** (a)

**Exercise 1A – Question 9**

The ratio compounded of 4 : 9 and the duplicate ratio of 3 : 4 is:

(a) 1 : 4 (b) 1 : 3 (c) 3 : 1 (d) None

**Solution** (a)

**Exercise 1A – Question 10**

The ratio compounded of 4 : 9, the duplicate ratio of 3 : 4, the triplicate ratio of 2 : 3 and 9 : 7 is:

(a) 2 : 7 (b) 7 : 2 (c) 2 : 21 (d) None

**Solution** (c)

**Exercise 1A – Question 11**

The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 1 : 3, sub duplicate ratio of 81 : 256 and sub-triplicate ratio of 125 : 512 is:

(a) 4 : 512 (b) 3 : 32 (c) 1 : 12 (d) None

**Solution** (d)

**Exercise 1A – Question 12**

If $a:b=3:4$, the value of $(2a+3b):(3a+4b)$ is:

(a) 54 : 25 (b) 8 : 25 (c) 17 : 24 (d) 18 : 25

**Solution** (d)

**Exercise 1A – Question 19**

If $x:y=3:4$, the value of $x^2y+xy^2:x^3+y^3$ is:

(a) 13 : 12 (b) 12 : 13 (c) 21 : 31 (d) None

**Solution** (b)

**Exercise 1A – Question 22**

If $p:q=2:3$ and $x:y=4:5$, then the value of $5px+3qy:10px+4qy$ is:

(a) 71 : 82 (b) 27 : 28 (c) 17 : 28 (d) None

**Solution** (c)

## Word Problems

**Page 1.3 – Example 2**

The ratio of the number of boys to the number of girls in a school of 720 students is 3 : 5. If 18 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of the number of boys to the number of girls may change to 2 : 3.

(a) 24 (b) 42 (c) 36 (d) None

**Solution** (b)

**Page 1.4 – Example 1**

The monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenditures are in the ratio 7 : 9. If each saves ₹50 per month, find their monthly incomes.

(a) ₹400; ₹500 (b) ₹800; ₹1,000 (c) ₹40; ₹50 (d) None

**Solution** (a)

**Page 1.5 – Example 2**

The ratio of the prices of two houses was 16 : 23. Two years later when the price of the first has increased by 10% and that of the second by ₹477, the ratio of the prices becomes 11 : 20. Find the original prices of the two houses.

(a) ₹848, ₹1,219 (b) ₹838, ₹1,119 (c) ₹828, ₹1,219 (d) ₹848, ₹1,229

**Solution** (a)

**Page 1.5 – Example 3**

Find in what ratio will the total wages of the workers of a factory be increased or decreased if there be a reduction in the number of workers in the ratio 15 : 11 and an increment in their wages in the ratio 22 : 25.

(a) 8 : 7 (b) 7 : 6 (c) 6 : 5 (d) None

**Solution** (c)

**Exercise 1A – Question 13**

Two numbers are in the ratio 2 : 3. If 4 be subtracted from each, they are in the ratio 3 : 5. The numbers are:

(a) (16, 24) (b) (4, 6) (c) (2, 3) (d) None

**Solution** (a)

**Exercise 1A – Question 14**

The angles of a triangle are in ratio 2 : 7 : 11. The angles are:

(a) (20°, 70°, 90°) (b) (30°, 70°, 80°)
(c) (18°, 63°, 99°) (d) None

**Solution** (c)

**Exercise 1A – Question 15**

Division of ₹324 between X and Y is in the ratio 11 : 7. X & Y would get Rupees:

(a) 204, 120 (b) 200, 124 (c) 180, 144 (d) None

**Solution** (d)

**Exercise 1A – Question 16**

Anand earns ₹80 in 7 hours and Promode ₹90 in 12 hours. The ratio of their earnings is:

(a) 32 : 21 (b) 23 : 12 (c) 8 : 9 (d) None

**Solution** (a)

**Exercise 1A – Question 17**

The ratio of two numbers is 7 : 10 and their difference is 105. The numbers are:

(a) (200, 305) (b) (185, 290) (c) (245, 350) (d) None

**Solution** (c)

**Exercise 1A – Question 18**

P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is

(a) 22 : 27 (b) 27 : 22 (c) 32 : 33 (d) None

**Solution** (b)

**Exercise 1A – Question 23**

The number which when subtracted from each of the terms of the ratio 19 : 31 reducing it to 1 : 4 is

(a) 15 (b) 5 (c) 1 (d) None

**Solution** (a)

**Exercise 1A – Question 24**

Daily earnings of two persons are in the ratio 4 : 5 and their daily expenses are in the ratio 7 : 9. If each saves ₹50 per day, their daily earnings in ₹ are:

(a) (40, 50) (b) (50, 40) (c) (400, 500) (d) None

**Solution** (c)

**Exercise 1A – Question 25**

The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms. in 5 hours, the speed of the first train is

(a) 10 km/hr (b) 50 km/hr (c) 70 km/hr (d) None

**Solution** (c)

**Page 1.3 – Example 1**

Simplify the ratio 1/3 : 1/8 : 1/6

(a) 3 : 4 : 8 (b) 8 : 3 : 4 (c) 5 : 9 : 12 (d) None

**Solution** (b)

## Proportion

- An equality of two ratios is called Proportion.
- The quantities $a$, $b$, $c$, and $d$ are said to be in proportion if $a : b = c : d$.
- It is also written as $a : b :: c : d$.
- The quantities $a$, $b$, $c$, and $d$ are called the terms of the proportion; $a$, $b$, $c$, and $d$ are called the first, second, third and fourth terms respectively.
- They are also called "first proportional", "second proportional", "third proportional", and "fourth proportional" respectively.
- The terms $a$ and $d$ are called "Extremes" and the terms $b$ and $c$ are called "Means".
- Note – In a ratio $a : b$, both the quantities $a$ and $b$ should be of the same kind. However, this is not true in case of proportion. In proportion $a : b :: c : d$, the quantities $a$ and $b$ should be of the same kind, and quantities $c$ and $d$ should be of the same kind.
- Cross-Product Rule
  Clearly, if $a : b = c : d$, then by cross multiplication $ad = bc$. Therefore, product of means = product of extremes. This is called the cross-product rule.

**Page 1.9 – Example 2**

Find the value of $x$ if $10/3 : x :: 5/2 : 5/4$.

(a) 5/3 (b) 3/5 (c) 2/9 (d) None

**Solution** (a)

**Exercise 1B – Question 4**

The number which has the same ratio to 26 that 6 has to 13 is:

(a) 11 (b) 10 (c) 21 (d) None

**Solution** (d)

**Exercise 1B – Question 6**

If four numbers 1/2, 1/3, 1/5, 1/$x$ are proportional then $x$ is:

(a) 6/5 (b) 5/6 (c) 15/2 (d) None

**Solution** (c)

**Exercise 1B – Question 23**

12, 16, *, 20 are in proportion. Then * is:

(a) 25 (b) 14 (c) 15 (d) None

**Solution** (c)

**Exercise 1B – Question 24**

4, *, 9, 13½ are in proportion. Then * is:

(a) 6 (b) 8 (c) 9 (d) None

**Solution** (a)

**Page 1.9 – Example 3**

Find the fourth proportional to 2/3, 3/7, 4.

(a) 5/2 (b) 7/8 (c) 18/7 (d) None

**Solution** (c)

**Exercise 1B – Question 1**

The fourth proportional to 4, 6, 8 is:

(a) 12 (b) 32 (c) 48 (d) None

**Solution** (a)

**Exercise 1B – Question 5**

The fourth proportional to $2a$, $a^2$, $c$ is:

(a) $ac/2$ (b) $ac$ (c) $2/ac$ (d) None

**Solution** (a)

**Exercise 1B – Question 18**

The numbers 14, 16, 35, 42 are not in proportion. The fourth term for which they will be in proportion is:

(a) 45 (b) 40 (c) 32 (d) None

**Solution** (b)

## Continuous Proportion

- The quantities $a$, $b$, and $c$ are said to be in continuous proportion if $a : b = b : c$.
- In such case, the middle term $b$ is called the mean proportional.
- Obviously, $b^2 = ac$ or $b = \sqrt{ac}$.

**Page 1.9 – Example 4**

Find the third proportion to 2.4 kg, 9.6 kg.

(a) 38 (b) 38.4 (c) 39 (d) None

**Solution** (b)

**Exercise 1B – Question 2**

The third proportional to 12, 18 is:

(a) 24 (b) 27 (c) 36 (d) None

**Solution** (b)

**Exercise 1B – Question 3**

The mean proportional between 25, 81 is:

(a) 40 (b) 50 (c) 45 (d) None

**Solution** (c)

**Exercise 1B – Question 7**

The mean proportional between $12x^2$ and $27y^2$ is

(a) $18xy$ (b) $81xy$ (c) $8xy$ (d) None

**Solution** (a)

**Exercise 1B – Question 25**

The mean proportional between 1.4 gms and 5.6 gms is:

(a) 28 gms (b) 2.8 gms (c) 3.2 gms (d) None

**Solution** (b)

**Page 1.10 – Example 5**

Find the mean proportion between 1.25 and 1.8.

(a) 1.5 (b) 2.5 (c) 3.9 (d) None

**Solution** (a)

**Exercise 1B – Question 13**

If A : B = 3 : 2 and B : C = 3 : 5, then A : B : C is:

(a) 9 : 6 : 10 (b) 6 : 9 : 10 (c) 10 : 9 : 6 (d) None

**Solution** (a)

**Exercise 1B – Question 15**

If $x : y = 2 : 3$, $y : z = 4 : 3$ then $x : y : z$ is:

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 2 : 4 (d) None

**Solution** (d)

**Page 1.11 – Example 1**

If $a : b = c : d = 2.5 : 1.5$, what are the values of $ad : bc$ and $a + c : b + d$?

(a) 1:3; 5:1 (b) 1:1; 5:3 (c) 5:3; 1:1 (d) None

**Solution** (b)

**Page 1.11 – Example 2**

If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then find the value of $\frac{a+b+c}{c}$.

(a) 1 (b) 3 (c) 2 (d) None

**Solution** (c)

**Exercise 1B – Question 8**

If A = B/2 = C/5, then A : B : C is:

(a) 3 : 5 : 2 (b) 2 : 5 : 3 (c) 1 : 2 : 5 (d) None

**Solution** (c)

**Exercise 1B – Question 9**

If $a/3 = b/4 = c/7$, then $a + b + c/c$ is

(a) 1 (b) 3 (c) 2 (d) None

**Solution** (c)

**Exercise 1B – Question 10**

If $p/q = r/s = 2.5/1.5$, the value of $ps : qr$ is

(a) 3/5 (b) 1 : 1 (c) 5/3 (d) None

**Solution** (b)

**Exercise 1B – Question 11**

If $x : y = z : w = 2.5 : 1.5$, the value of $(x + z) : (y + w)$ is:

(a) 1 (b) 3/5 (c) 5/3 (d) None

**Solution** (c)

**Exercise 1B – Question 14**

If $x / 2 = y / 3 = z / 7$, then the value of $(2x - 5y + 4z) / 2y$ is:

(a) 6/23 (b) 23/6 (c) 3/2 (d) None

**Solution** (d)

**Exercise 1B – Question 26**

If $\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$, then $\frac{a+b+c}{c}$ is:

(a) 4 (b) 2 (c) 7 (d) None

**Solution** (b)

**Exercise 1B – Question 29**

If $a:b=4:1$, then $\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}$ is:

(a) 5/2 (b) 4 (c) 5 (d) None

**Solution** (a)

**Exercise 1B – Question 16**

Division of ₹750 into 3 parts in the ratio 4 : 5 : 6 is:

(a) (200, 250, 300)
(b) (250, 250, 250)
(c) (350, 250, 150)
(d) None

**Solution** (a)

## Continued Proportion

When 3 or more numbers are related such that $a/b = b/c = c/d = d/e\ldots$ the numbers $a$, $b$, $c$, $d$, and $e$ are said to be in continued proportion.

## Properties of Proportion

1. Cross Product Rule

   If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$.
2. Invertendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$.
3. Alternendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a}{c}=\frac{b}{d}$, or, $\frac{d}{b}=\frac{c}{a}$
4. Componendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$
5. Dividendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a-b}{b}=\frac{c-d}{d}$
6. Componendo and Dividendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$
7. Addendo

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=...$, then each of these ratios is equal to $\frac{a+c+e+...}{b+d+f+...}$, i.e.,

$\frac{a}{b}=\frac{a+c+e+...}{b+d+f+...}$; $\frac{c}{d}=\frac{a+c+e+...}{b+d+f+...}$; $\frac{e}{f}=\frac{a+c+e+...}{b+d+f+...}$

8. Subtrahendo

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=...$, then each of these ratios is equal to $\frac{a-c-e-...}{b-d-f-...}$, i.e.,

$\frac{a}{b}=\frac{a-c-e-...}{b-d-f-...}$; $\frac{c}{d}=\frac{a-c-e-...}{b-d-f-...}$; $\frac{e}{f}=\frac{a-c-e-...}{b-d-f-...}$

**Exercise 1B – Question 19**

If $x/y=z/w$ implies $y/x=w/z$, this process is called:

(a) Dividendo (b) Componendo (c) Alternendo (d) None

**Solution** (d)

**Exercise 1B – Question 20**

If $p/q=r/s=p-r/q-s$, the process is called:

(a) Subtrahendo (b) Addendo (c) Invertendo (d) None

**Solution** (a)

**Exercise 1B – Question 21**

If $a/b=c/d$ implies $(a+b)/(a-b)=(c+d)/(c-d)$, the process is called:

(a) Componendo
(b) Dividendo
(c) Componendo and Dividendo
(d) None

**Solution** (c)

**Exercise 1B – Question 28**

If $\frac{a}{4}=\frac{b}{5}$, then

(a) $\frac{a+4}{a-4}=\frac{b-5}{b+5}$ (b) $\frac{a+4}{a-4}=\frac{b+5}{b-5}$ (c) $\frac{a-4}{a+4}=\frac{b+5}{b-5}$ (d) None

**Solution** (b)

**Exercise 1B – Question 22**

If $u/v = w/p$, then $(u-v)/(u+v) = (w-p)/(w+p)$. The process is called:

(a) Invertendo (b) Alternendo (c) Addendo (d) None

**Solution** (d)

**Exercise 1B – Question 17**

The sum of the ages of 3 persons is 150 years. 10 years ago, their ages were in the ratio 7 : 8 : 9. Their present ages are:

(a) (45, 50, 55) (b) (40, 60, 50) (c) (35, 45, 70) (d) None

**Solution** (a)

**Exercise 1B – Question 27**

Two numbers are in the ratio 3 : 4; if 6 be added to each terms of the ratio, then the new ratio will be 4 : 5, then the numbers are:

(a) 14, 20 (b) 17, 19 (c) 18, 24 (d) None

**Solution** (c)

**Exercise 1B – Question 12**

If $(5x-3y)/(5y-3x) = 3/4$, then the value of $x:y$ is:

(a) 2 : 9 (b) 7 : 2 (c) 7 : 9 (d) None

**Solution** (d)

## Alligation

- This rule is used in mixing of two varieties of the same kind.
- If two varieties of, say, tea with rate ₹*x* per kg and ₹*y* per kg (where $x < y$) are mixed to make a third variety of tea with rate ₹*z* per kg, the ratio in which these two varieties are mixed is: *(₹y – ₹z) : (₹z – ₹x)*.
- Remember
    - If *x* represents cost, then *y* and *z* must also be cost.

- If $x$ represents selling price, then $y$ and $z$ must also be selling price.

**Page 1.11 – Example 3**

A dealer mixes tea costing ₹6.92 per kg. with tea costing ₹7.77 per kg and sells the mixture at ₹8.80 per kg and earns a profit of 17.5% on his sale price. In what proportion does he mix them?

(a) 6 : 4 (b) 3 : 2 (c) 5 : 6 (d) Both (a) and (b)

**Solution** (d)

**Scanner – Question 80 (June, 2015)**

A dealer mixes rice costing ₹13.84 per kg. with rice costing ₹15.54 per kg. and sells the mixture at ₹17.60 per kg. So, he earns a profit of 14.6% on his sale price. The proportion in which he mixes the two qualities of rice is:

(a) 3 : 7 (b) 5 : 7 (c) 7 : 9 (d) 9 : 11

**Solution** (a)

## Indices

- The word "Indices" is the plural of "Index".
- When a number is expressed in the form of $a^n$, $a$ is called the base, and $n$ is called the index/exponent/power.

## Integral Components of a Real Number

## Positive Integral Power

For any real number $a$ and a positive integer $n$, $a^n$ is defined as

$$a^n = a \times a \times a \times a \times \ldots \times a \ (n \text{ times})$$

## Negative Integral Power

For any real number $a$ and a negative integer $n$, $a^{-n}$ is defined as

$$a^{-n} = \frac{1}{a^n}$$

## Zero Power

For any real number $a$, $a^0$ is defined as

$$a^0 = 1$$

## Laws of Indices

1. First Law

$$a^m \times a^n = a^{m+n}$$

2. Second Law

$$\frac{a^m}{a^n} = a^{m-n}$$

3. Third Law

$$\left(a^m\right)^n = a^{mn} = \left(a^n\right)^m$$

4. Fourth Law

$$\left(ab\right)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

5. Fifth Law

$$a^{m/n} = \left(a^m\right)^{1/n}, \text{ i.e., } a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

**Page 1.16 – Example 1**

Simplify $2x^{1/2}3x^{-1}$ if $x = 4$.

(a) 3 (b) 4 (c) 5 (d) None

**Solution** (a)

**Page 1.16 – Example 2**

Simplify $6ab^2c^3 \times 4b^{-2}c^{-3}d$ .

(a) $42ad$ (b) $24ad$ (c) $52bc$ (d) None

**Solution** (b)

**Page 1.17 – Example 3**

Find the value of $\frac{4x^{-1}}{x^{-1/3}}$.

(a) $\frac{2}{x^{3/2}}$ (b) $\frac{4}{x^{3/2}}$ (c) $\frac{4}{x^{2/3}}$ (d) None

**Solution** (c)

**Page 1.18 – Example 4**

Simplify $\frac{2a^{\frac{1}{2}} \times a^{\frac{2}{3}} \times 6a^{-\frac{7}{3}}}{9a^{-\frac{5}{3}} \times a^{\frac{3}{2}}}$ if $a = 4$.

(a) 1/3 (b) ½ (c) 2/3 (d) None

**Solution** (a)

**Page 1.19 – Example 5**

Simplify $\left(x^a.y^{-b}\right)^3.\left(x^3y^2\right)^{-a}$

(a) $\frac{1}{y^{3b+2a}}$ (b) $\frac{1}{y^{3b+3a}}$ (c) $\frac{1}{y^{2b+2a}}$ (d) None

**Solution** (a)

**Page 1.19 – Example 6**

$\sqrt[6]{a^{4b}x^6}.\left(a^{2/3}x^{-1}\right)^{-b}$

(a) $x^{1+b}$ (b) $x^{c+b}$ (c) $x^{2+b}$ (d) None

**Solution** (a)

**Page 1.19 – Example 7**

Find $x$, if $x\sqrt{x}=\left(x\sqrt{x}\right)^x$

(a) 3 (b) 1 (c) 2 (d) None

**Solution** (b)

**Page 1.20 – Example 8**

Find the value of $k$ from $\left(\sqrt{9}\right)^{-7}\times\left(\sqrt{3}\right)^{-5}=3^k$

(a) 2/3 (b) 18/7 (c) –19/2 (d) None

**Solution** (c)

**Exercise 1C – Question 1**

$4x^{-1/4}$ is expressed as:

(a) $-4x^{1/4}$ (b) $x^{-1}$ (c) $4/x^{1/4}$ (d) None

**Solution** (c)

**Exercise 1C – Question 2**

The value of $8^{1/3}$ is:

(a) $\sqrt[3]{2}$ (b) 4 (c) 2 (d) None

**Solution** (c)

**Exercise 1C – Question 3**

The value of $2\times(32)^{1/5}$ is:

(a) 2 (b) 10 (c) 4 (d) None

**Solution** (c)

**Exercise 1C – Question 4**

The value of $4/(32)^{1/5}$ is:

(a) 8 (b) 2 (c) 4 (d) None

**Solution** (b)

**Exercise 1C – Question 5**

The value of $(8/27)^{1/3}$ is:

(a) 2/3 (b) 3/2 (c) 2/9 (d) None

**Solution** (a)

**Exercise 1C – Question 6**

The value of $2(256)^{-1/8}$ is:

(a) 1 (b) 2 (c) 1/2 (d) None

**Solution**(a)

**Exercise 1C – Question 7**

$2^{1/2}.4^{3/4}$ is:

(a) a fraction
(b) a positive integer
(c) a negative integer
(d) None

**Solution** (b)

**Exercise 1C – Question 8**

$\left(\frac{81x^4}{y^{-8}}\right)^{\frac{1}{4}}$ has simplified value equal to:

(a) $xy^2$ (b) $x^2y$ (c) $9xy^2$ (d) None

**Solution** (d)

**Exercise 1C – Question 9**

$x^{a-b} \times x^{b-c} \times x^{c-a}$ is equal to:

(a) $x$ (b) 1 (c) 0 (d) None

**Solution** (b)

**Exercise 1C – Question 10**

The value of $\left(\frac{2p^2q^3}{3xy}\right)^0$ where $p, q, x, y \neq 0$ is equal to:

(a) 0 (b) 2/3 (c) 1 (d) None

**Solution** (c)

**Exercise 1C – Question 11**

$\left\{\left(3^3\right)^2 \times \left(4^2\right)^3 \times \left(5^3\right)^2\right\} / \left\{\left(3^2\right)^3 \times \left(4^3\right)^2 \times \left(5^2\right)^3\right\}$ is:

(a) ¾ (b) 4/5 (c) 4/7 (d) 1

**Solution** (d)

**Exercise 1C – Question 12**

Which is true?

(a) $2^0 > (1/2)^0$ (b) $2^0 < (1/2)^0$ (c) $2^0 = (1/2)^0$ (d) None

**Solution** (c)

**Exercise 1C – Question 13**

If $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$, then the value of $p+q+r$ is:

(a) 1 (b) 0 (c) 1/2 (d) None

**Solution** (b)

**Exercise 1C – Question 14**

The value of $y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$ is:

(a) $y^{a+b}$ (b) $y$ (c) 1 (d) $1/y^{a+b}$

**Solution** (d)

**Exercise 1C – Question 15**

The true option is:

(a) $x^{2/3} = \sqrt[3]{x^2}$ (b) $x^{2/3} = \sqrt{x^3}$ (c) $x^{2/3} > \sqrt[3]{x^2}$ (d) $x^{2/3} < \sqrt[3]{x^2}$

**Solution** (a)

**Exercise 1C – Question 16**

The simplified value of $16x^{-3}y^2 \times 8^{-1}x^3y^{-2}$ is:

(a) $2xy$ (b) $xy/2$ (c) 2 (d) None

**Solution** (c)

**Exercise 1C – Question 17**

The value of $(8/27)^{-1/3} \times (32/243)^{-1/5}$ is:

(a) 9/4 (b) 4/9 (c) 2/3 (d) None

**Solution** (a)

**Exercise 1C – Question 18**

The value of $\left\{(x+y)^{2/3}(x-y)^{3/2}/\sqrt{(x+y)}\times\sqrt{(x-y)^3}\right\}^6$ is:

(a) $(x+y)^2$ (b) $(x-y)$ (c) $x+y$ (d) None

**Solution** (c)

**Exercise 1C – Question 19**

Simplified value of $(125)^{2/3}\times\sqrt{25}\times\sqrt[3]{5^3}\times5^{1/2}$ is:

(a) 5 (b) 1/5 (c) 1 (d) None

**Solution** (d)

**Exercise 1C – Question 20**

$\left[\left\{(2)^{1/2}.(4)^{3/4}.(8)^{5/6}.(16)^{7/8}.(32)^{9/10}\right\}^4\right]^{3/25}$ is:

(a) a fraction (b) an integer (c) 1 (d) None

**Solution** (b)

**Exercise 1C – Question 21**

$\left[1-\left\{1-\left(1-x^2\right)^{-1}\right\}^{-1}\right]^{-1/2}$

(a) $x$ (b) $1/x$ (c) 1 (d) None

**Solution** (a)

**Exercise 1C – Question 22**

$\left[\left(x^n\right)^{n-\frac{1}{n}}\right]^{\frac{1}{n+1}}$ is equal to:

(a) $x^n$ (b) $x^{n+1}$ (c) $x^{n-1}$ (d) None

**Solution** (c)

**Exercise 1C – Question 23**

If $a^3-b^3=(a-b)(a^2+ab+b^2)$, then the simplified form of $\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2}\times\left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2}\times\left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$

(a) 0 (b) 1 (c) $x$ (d) None

**Solution** (b)

**Exercise 1C – Question 24**

Using $(a-b)^3=a^3-b^3-3ab(a-b)$, tick the correct of these when $x=p^{1/3}-p^{-1/3}$.

(a) $x^3+3x=p+1/p$ (b) $x^3+3x=p-1/p$
(c) $x^3+3x=p+1$ (d) None

**Solution** (b)

**Exercise 1C – Question 25**

On simplification, $1/(1+a^{m-n}+a^{m-p})+1/(1+a^{n-m}+a^{n-p})+1/(1+a^{p-m}+a^{p-n})$ is equal to:

(a) 0 (b) $a$ (c) 1 (d) $1/a$

**Solution** (c)

**Exercise 1C – Question 26**

The value of $\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}$

(a) 1 (b) 0 (c) 2 (d) None

**Solution** (a)

**Exercise 1C – Question 27**

If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$, then $3x^3 - 9x$ is:

(a) 15 (b) 10 (c) 12 (d) None

**Solution** (b)

**Exercise 1C – Question 28**

If $a^x = b$, $b^y = c$, $c^z = a$, then $xyz$ is:

(a) 1 (b) 2 (c) 3 (d) None

**Solution** (a)

**Exercise 1C – Question 29**

The value of $\left(\frac{x^a}{x^b}\right)^{\left(a^2+ab+b^2\right)} \times \left(\frac{x^b}{x^c}\right)^{\left(b^2+bc+c^2\right)} \times \left(\frac{x^c}{x^a}\right)^{\left(c^2+ca+a^2\right)}$

(a) 1 (b) 0 (c) –1 (d) None

**Solution** (a)

**Exercise 1C – Question 30**

If $2^x = 3^y = 6^{-z}$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is:

(a) 1 (b) 0 (c) 2 (d) None

**Solution** (b)

**Exercise 1B – Question 30**

If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$, then $(b-c)x + (c-a)y + (a-b)z$ is:

(a) 1 (b) 0 (c) 5 (d) None

**Solution** (b)

## Logarithms

- Logarithms are used to simplify huge calculations.
- $2^3 = 8$ is expressed in terms of Logarithms as $\log_2 8 = 3$.
- It is read as log 8 to the base 2 is 3.
- Notes –
  - For any positive real number, *a*, we know that $a^0 = 1$ and $a^1 = a$. Therefore, $\log_a 1 = 0$ and $\log_a a = 1$.
  - If, in a question, the base is not mentioned, it is considered to be 10.

**Practice Question 1**

$2^3 = 8$ is written as:

(a) $\log_2 8 = 3$ (b) $\log_8 2 = 3$ (c) $\log_3 2 = 8$ (d) None

**Solution** (a)

**Practice Question 2**

$2^4 = 16$ is written as

(a) $\log_4 16 = 2$ (b) $\log_2 16 = 4$ (c) $\log_{16} 4 = 2$ (d) None

**Solution** (b)

**Practice Question 3**

$x^y = z$ is written as

(a) $\log_z y = x$ (b) $\log_x y = z$ (c) $\log_x z = y$ (d) None

**Solution** (c)

**Practice Question 4**

$5^2 = 25$ is written as

(a) $\log_{25} 5 = 2$ (b) $\log_5 25 = 2$ (c) $\log_2 5 = 25$ (d) None

**Solution** (b)

**Practice Question 5**

$\log_2 256 = 8$ is written as

(a) $8^2 = 256$ (b) $2^{256} = 8$ (c) $2^8 = 256$ (d) None

**Solution** (c)

**Practice Question 6**

$\log_3 81 = 4$ is written as

(a) $3^4 = 81$ (b) $2^4 = 81$ (c) $\sqrt{81} = 3^3$ (d) None

**Solution** (a)

**Practice Question 7**

$\log_a c = b$ is written as

(a) $b^c = a$ (b) $a^c = b$ (c) $a^b = c$ (d) None

**Solution** (c)

**Page 1.24 – Illustration 1**

If $\log_a \sqrt{2} = \frac{1}{6}$, find the value of $a$.

(a) 8 (b) 9 (c) 10 (d) None

**Solution** (a)

**Page 1.24 – Illustration 2**

Find the logarithm of 5832 to the base $3\sqrt{2}$.

(a) 6 (b) 7 (c) 8 (d) None

**Solution** (a)

**Page 1.25 – Illustration I**

log ½ can be written as:

(a) log 1 (b) log 2 (c) –log 2 (d) None

**Solution** (c)

**Page 1.26 – Illustration II – 1 (a)**

Find the logarithm of 1728 to the base $2\sqrt{3}$.

(a) 5 (b) 6 (c) 7 (d) None

**Solution** (b)

**Page 1.29 – Example 1**

Find the value of log 5 if log 2 is equal to .3010.

(a) .6990 (b) .7890 (c) .7489 (d) None

**Solution** (a)

**Page 1.31 – Example 1**

Find the logarithm of 64 to the base $2\sqrt{2}$.

(a) 4 (b) 5 (c) 6 (d) None

**Solution** (a)

**Exercise 1D – Question 2**

$\log_2 8$ is equal to

(a) 2 (b) 8 (c) 3 (d) None

**Solution** (c)

**Exercise 1D – Question 5**

The value of log 0.0001 to the base 0.1 is:

(a) –4 (b) 4 (c) 1/4 (d) None

**Solution** (b)

**Exercise 1D – Question 7**

$\log_{\sqrt{2}} 64$ is equal to:

(a) 12 (b) 6 (c) 1 (d) None

**Solution** (a)

**Exercise 1D – Question 8**

$\log_{2\sqrt{3}} 1728$ is equal to:

(a) $2\sqrt{3}$ (b) 2 (c) 6 (d) None

**Solution** (c)

**Exercise 1D – Question 9**

log (1/81) to the base 9 is equal to:

(a) 2 (b) ½ (c) –2 (d) None

**Solution** (c)

**Exercise 1D – Question 10**

log 0.0625 to the base 2 is equal to:

(a) 4 (b) 5 (c) 1 (d) None

**Solution** (d)

**Exercise 1D – Question 13**

The value of log 1/3 to the base 9 is:

(a) –½ (b) ½ (c) 1 (d) None

**Solution** (a)

**Exercise 1D – Question 24**

The logarithm of 64 to the base $2\sqrt{2}$ is:

(a) 2 (b) $\sqrt{2}$ (c) ½ (d) None

**Solution** (d)

## Laws of Logarithms

### First Law

$\log_a (mn) = \log_a m + \log_a n$

**Exercise 1D – Question 1**

log 6 + log 5 is expressed as:

(a) log 11 (b) log 30 (c) log 5/6 (d) None

**Solution** (b)

**Exercise 1D – Question 4**

log (1 × 2 × 3) is equal to:

(a) log 1 + log 2 + log 3 (b) log 3 (c) log 2 (d) None

**Solution** (a)

**Exercise 1D – Question 11**

Given log 2 = 0.3010 and log 3 = 0.4771 the value of log 6 is:

(a) 0.9030 (b) 0.9542 (c) 0.7781 (d) None

**Solution** (c)

**Exercise 1D – Question 14**

If log $x$ + log $y$ = log ($x$ + $y$), $y$ can be expressed as:

(a) $x$ (b) $x$ (c) $x/x - 1$ (d) None

**Solution** (c)

**Exercise 1D – Question 17**

Given that $\log_{10} 2 = x$ and $\log_{10} 3 = y$, the value of $\log_{10} 60$ is expressed as:

(a) $x - y + 1$ (b) $x + y + 1$ (c) $x - y - 1$ (d) None

**Solution** (b)

## Second Law

$$\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$$

**Practice Question 8**

log 3 is written as

(a) log 12 – log 4 (b) log 4 – log 12 (c) log 4 – log 8 (d) None

**Solution** (a)

**Practice Question 9**

log 1.5 is written as

(a) log 6 – log 9
(b) log 6 + log 6
(c) log 9 – log 6
(d) None

**Solution** (c)

**Exercise 1D – Question 3**

log 32/4 is equal to:

(a) log 32/log 4 (b) log 32 – log 4 (c) $2^3$ (d) None

**Solution** (b)

**Exercise 1D – Question 18**

Given that $\log_{10} 2 = x$ and $\log_{10} 3 = y$, the value of $\log_{10} 1.2$ is expressed in terms of $x$ and $y$ as:

(a) $x + 2y - 1$ (b) $x + y - 1$ (c) $2x + y - 1$ (d) None

**Solution** (c)

## Third Law

$\log_a \left(m^n\right) = n \log_a m$

**Practice Question 10**

4log 2 is written as:

(a) $\log 2^4$ (b) $\log 4^2$ (c) $2^3 \log 4$ (d) None

**Solution** (a)

**Practice Question 11**

$\log a^b$ is written as

(a) $a\log b$ (b) $b^c \log a^b$ (c) $b\log a$ (d) None

**Solution** (c)

**Page 1.26 – Illustration II – 1 (b)**

Solve $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$.

(a) 5 (b) 6 (c) 1 (d) None

**Solution** (c)

**Exercise 1D – Question 6**

If 2log $x$ = 4log 3, then $x$ is equal to:

(a) 3 (b) 9 (c) 2 (d) None

**Solution** (b)

**Exercise 1D – Question 12**

The value of $\log_2 \log_2 \log_2 16$ is:

(a) 0 (b) 2 (c) 1 (d) None

**Solution** (c)

**Exercise 1D – Question 15**

The value of $\log_2 \left[ \log_2 \left\{ \log_3 \left( \log_3 27^3 \right) \right\} \right]$ is equal to:

(a) 1 (b) 2 (c) 0 (d) None

**Solution** (c)

**Exercise 1D – Question 19**

Given that $\log x = m + n$ and $\log y = m - n$, the value of $\log 10x/y^2$ is expressed in terms of $m$ and $n$ as:

(a) $1 - m + 3n$ (b) $m - 1 + 3n$ (c) $m + 3n + 1$ (d) None

**Solution** (a)

**Exercise 1D – Question 20**

The simplified value of $2\log_{10} 5 + \log_{10} 8 - \frac{1}{2}\log_{10} 4$ is:

(a) ½ (b) 4 (c) 2 (d) None

**Solution** (c)

## Fourth Law (Base Change Formula)

$$\log_a m = \frac{\log_b m}{\log_b a}$$

**Practice Question 12**

$\log_2 4$ is written as

(a) $\frac{\log 2}{\log 4}$ (b) $\frac{\log_{10} 4}{\log_{10} 2}$ (c) $\frac{\log_{10} 2}{\log_{10} 4}$ (d) None

**Solution** (b)

**Practice Question 13**

$\log_4 10$ is written as

(a) $\frac{\log 4}{\log 10}$ (b) $\frac{\log 2^2}{\log 10}$ (c) $\frac{\log 10}{\log 4}$ (d) None

**Solution** (c)

**Practice Question 14**

$\log_4 2 \times \log_2 4 =$

(a) 1 (b) 2 (c) 3 (d) None

**Solution** (a)

**Practice Question 15**

$\log_b a \times \log_a b =$

(a) 2 (b) 1 (c) 3 (d) None

**Solution** (b)

**Page 1.26 – Example 1**

Change the base of $\log_5 31$ into the common logarithmic base.

(a) $\frac{\log_{31} 10}{\log_{10} 5}$ (b) $\frac{\log_{10} 31}{\log_{10} 5}$ (c) $\frac{\log_{10} 31}{\log_5 10}$ (d) None

**Solution** (b)

**Page 1.31 – Example 3**

If $a=\log_{24}12$, $b=\log_{36}24$, $c=\log_{48}36$, then find the value of $1+abc$

(a) $2ac$ (b) $2ab$ (c) $2bc$ (d) None

**Solution** (c)

**Exercise 1D – Question 23**

The value of $\left(\log_b a\times\log_c b\times\log_a c\right)^3$ is equal to:

(a) 3 (b) 0 (c) 1 (d) None

**Solution** (c)

## Fifth Law

$$\frac{1}{\log_a m}=\log_m a$$

**Practice Question 16**

$$\frac{1}{\log_{57}49}=$$

(a) $1\times\log_{57}49$ (b) $\log_{57}49$ (c) $\log_{49}57$ (d) None

**Solution** (c)

**Practice Question 17**

$\log_{99}67=$

(a) $\frac{1}{\log_{99}67}$ (b) $\frac{1}{\log_{67}99}$ (c) $\frac{1}{\log_{9}67}$ (d) None

**Solution** (b)

**Page 1.31 – Example 2**

If $\log_a bc=x$, $\log_b ca=y$, $\log_c ab=z$, then $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=?$

(a) 1 (b) 2 (c) 3 (d) None

**Solution** (a)

## Sixth Law

$a^{\log_a n} = n$

**Practice Question 18**

$4^{\log_4 20} =$

(a) 20 (b) 4 (c) 24 (d) None

**Solution** (a)

**Practice Question 19**

$10^{\log 49} =$

(a) 10 (b) 49 (c) 49$x$ (d) None

**Solution** (b)

## Seventh Law

$\log_{a^q} n^p = \frac{p}{q} \log_a n$

**Practice Question 20**

$\log_{4^2} 5^9 =$

(a) $\frac{9}{2} \log_4 5$ (b) $\frac{5}{4} \log_2 9$ (c) $\frac{5}{2} \log_4 9$ (d) None

**Solution** (a)

**Practice Question 21**

$\log_{x^2} y^z =$

(a) $\frac{x}{2} \log_y z$ (b) $\frac{z}{2} \log_y x$ (c) $\frac{z}{2} \log_x y$ (d) None

**Solution** (c)

**Page 1.27 – Example 2**

Solve: $\dfrac{\log_3 8}{\log_9 16 \log_4 10}$

(a) $2\log_3 10$ (b) $2\log_{10} 3$ (c) $3\log_{10} 2$ (d) None

**Solution** (c)

**Exercise 1D – Question 16**

If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, then $x$ is equal to:

(a) 8 (b) 4 (c) 16 (d) None

**Solution** (a)

**Exercise 1D – Question 21**

$\log\left[1-\left\{1-\left(1-x^2\right)^{-1}\right\}^{-1}\right]^{-\frac{1}{2}}$ can be written as:

(a) $\log x^2$ (b) $\log x$ (c) $\log 1/x$ (d) None

**Solution** (b)

**Exercise 1D – Question 22**

The simplified value of $\log\sqrt[4]{729.\sqrt[3]{9^{-1}.27^{-4/3}}}$ is:

(a) log 3 (b) log 2 (c) log ½ (d) None

**Solution** (a)

**Exercise 1D – Question 25**

The value of $\log_8 25$ given log 2 = 0.3010 is:

(a) 1 (b) 2 (c) 1.5482 (d) None

**Solution** (c)