# RPIL



#### Ratios

• Ratio is a comparison of two similar attributes in same units.

Ratio				
+	+			
Multiplier	as a bridging element	Comparison		

- O Types of ratios: (a:b)
- 1. Duplicate a<sup>2</sup>:b<sup>2</sup>
- 2. Sub dupl. √a:√b
- 3. Triplicate a3:b3
- 4. Sub Tripl. √a:√b
- 5. Compound (a:b, c:d) → a × c:b ×d
- 6. Continued  $\rightarrow a:b:c$  (a:b,b:c)
- 7. Inverse -- b:a

### Proportion

- o If two ratios are equal they are said to be in proportion.
- Each pair of ratio should have same units.

	Proportion	
+	+	+
Mean Proportion	Third Proportion	Fourth Proportion

$$\mathbf{b} = \sqrt{\mathbf{a}\mathbf{c}} \quad \begin{vmatrix} \mathbf{a} \\ \mathbf{b} \end{vmatrix} = \frac{\mathbf{b}}{\mathbf{c}} \quad \begin{vmatrix} \mathbf{a} \\ \mathbf{b} \end{vmatrix} = \frac{\mathbf{c}}{\mathbf{d}}$$

- Product of means= Prod of extremes
- Properties of Proportion (a:b)
- 1. Invertendo
- = <del>c</del>
- 2. Alternendo
- 3. Componendo
- $\frac{a+b}{b} = \frac{c+d}{d}$   $\frac{a-b}{c} = \frac{c-d}{d}$
- 4. Dividendo
- 5 Componedo & dividendo  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

### Indices

- E.I. = {एक साल का factor 1} × 100
- It is a power game...
- Properties:

1. 
$$\sqrt[b]{a} = a^{\frac{1}{b}}$$

2. 
$$a^b a^c = a^{b+c}$$

3. 
$$\frac{a^b}{a^c} = a^{b-c}$$

4. 
$$(a \times b)^c = a^c \times b^c$$
  
 $(a+b)^c \neq a^c + b^c$ 

$$5. \left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

6. 
$$a^b = c \rightarrow a = c^b$$

- 7.  $a^b = a^c$  then Base same b = c power equate
- 8. Power same base equate

$$a^b = c^b \rightarrow a = c$$

9. 
$$\frac{1}{a^{-b}} = a^b, \frac{1}{a^b} = a^{-b}$$

$$10 \text{ (no.)}^{\circ} = 1.$$

### Logarithms

$$a^b = c$$
 $\downarrow$ 
 $log_a^c = b$ 

- Always assume base to be 10.
- O Loga + logb = logaxb
- O loga logb = loga
- $o \log 1 = 0$
- o m log n = log n"
- $\log^{\xi} = \frac{1}{\log^{\xi}}$
- o a ≈ = x
- O Shortcut:

Type no.

Type √19 times

Type -1 × 227695

# **Equations**



#### **General Form**

1 variable ax + b = 02 Variables ax + by + c = 03 Variables ax + by + cz + d = 0

> To get unique solutions No. of equations = No. of Variables

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(Infinite Solution)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

(Unique Solution)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(No Solution)

#### **Quadratic Equations**

#### General Form

- $ax^{2+}bx+c=0$
- if  $\alpha \& \beta$  are roots then,

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Sum of roots  $(\alpha + \beta) = -\frac{b}{a}$ 

Product of roots  $(\alpha \beta) = \frac{c}{a}$ 

#### The roots can be found out using,

$$x = \frac{-b \pm \sqrt{b^2 + 4ca}}{2a}$$

Nature of Roots depends on  $D = b^2 - 4ac$ 

- (a) D<0 Roots are imaginary
- (b) D=0 Real and equal
- (c) D>0 and perfect square number real, distinct (unequal) and Rational
- (d) D>0 and NOT a perfect square real distinct and Irratinal

#### **Cubic Equation**

#### General Form

$$ax^{3} + bx^{2} + cx + d = 0$$
Sum of roots  $(\alpha + \beta + \gamma) = \frac{-b}{a}$ 
Product of roots  $(\alpha\beta\gamma) = \frac{-d}{a}$ 

# Linear In-equations



#### Making the Inequation

- Use logics
- Signs of inequality
   ≥<sub>i</sub> ≤<sub>i</sub> ><sub>i</sub> <</li>
- Knock out in objective questions:

Right values - satisfy

Wrong values - not satisfy

#### **Graphical Method\*\*\***

S.01 Ignore the sign of inequality

Put x = 0 & calculate y, point (0, y)

Put y = 0 & calculate x, point(x, 0)

S.02 Plot the points from S.01 on the graph,

Draw straight lines.

**S.03** If line passes through (0,0)

Put any point in the form (x, 0)

If satisfy -Shade towards

If not -Shade against

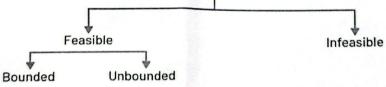
If it does not pass through (0, 0),

Put (0, 0)

If satisfy -Shade towards

If not - Shade against

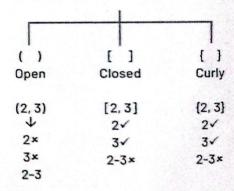
\$.04 Shade the common region; common area for all the inequations.



## Solving the inequation

#### Algebra

- Divide / Multiply with a negative no.; change sign of inequality.
- Always change both sides
- Brackets



Champions वाला Chart

# Time Value Of Money



#### Simple Interest

- $S.I. = \frac{pxrxt}{100}$
- A = P + S.1.
- S. I. is not बेवफा!
   S.I. is always calculated on principal.
- 2 S.I. is constant for every year.
- 3 If Q. is चुपकी assume it to be of S.I.
- 4 Nature of r, t should be same
- 5 Time Scale में + P होगा

#### **Compound Interest**

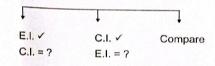
- Interest on Interest.
- O C.I. ≥ S. I.
- For the first period C.I. = S.I.
- $A = P \times (1+i)^n$
- O C.I. = A P
- Nature of r & t is always same. Always focus on factor.
- Higher the compounding higher the amount.
- Time scale में × होगा
- WDV = H.V.  $\times$  (1 i)
- Doubling Period formula  $T = 0.35 + \frac{69}{}$
- Tripling Period formula  $T = 0.35 + \frac{111.111}{5}$

### **Effective Interest**

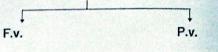
E.l. = { एक साल का factor - 1 } × 100

- Always assume t = 1 year
- Nature of r & t should be same.

#### Three types of Questions

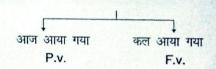


#### **Annuity**



F.V. = 
$$|\mathbf{x}| \left\{ \frac{(1+i)^n - 1}{i} \right\}$$
 | + factor = n times GT

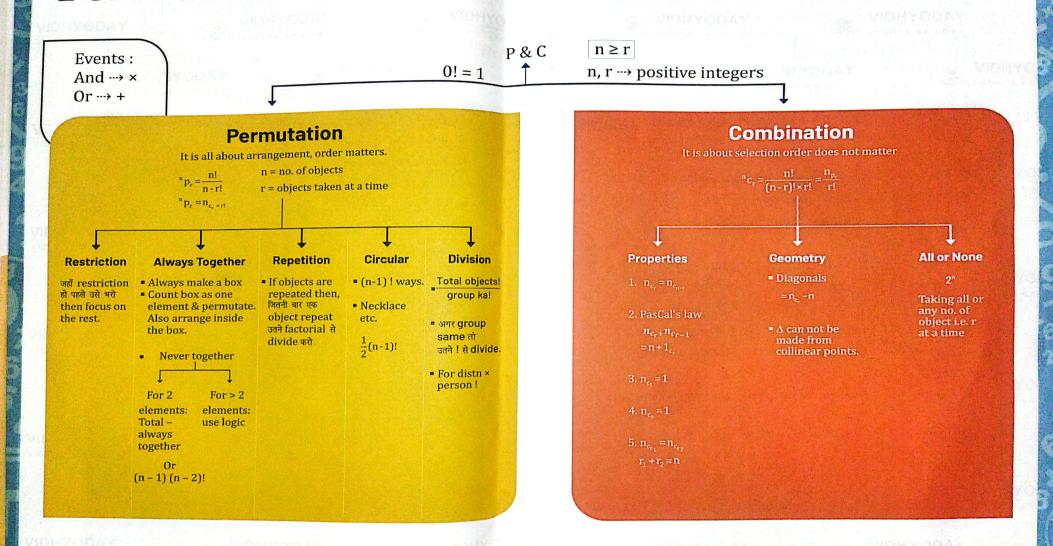
- Due: Starting from today → ans. × (1+i)
- Bulk amount



- o 3 conditions for annuity.
  - Fixed Time Interval
  - Fixed Installments
  - Regular Payments

# Permutation & Combination





# Sequence & Series



### AP / GP

### **Arithmetic Progression**

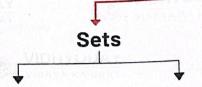
- It is about adding the constant no. to the first term & again.
- Every no. is A.M. of its previous & succeeding no.
- First Term = a common difference = d
- Variety -1 series : given value of term = ? Tn = a + (n-1) d
- Variety -2 series : given value = given n=?
- Variety 3 series : given sum =?
- Sn =  $\frac{n}{2}$ {a+r} or  $\frac{n}{2}$ {2a+(n-r)d}
- Variety -4 series : given sum = given no.?
- Variety 5 if two non consecutive term are given;  $d = \frac{Tm Tn}{m n}$
- Variety 6 Insertion of A.M. 's between two no.'s results in A.P.
- Variety 7 Sum's machine = given Term = ?

## **Geometric Progression**

- It is about multiplying the constant no. again.
- Every no. is GM of its previous & succeeding terms.
- First term -a common ratio = r
- Variety no. 1 n = given Tn = ?  $Tn = ar^{n-1}$
- Variety 3 Series = given sum = ? =  $s_n = \frac{ax(r^n - 1)}{r - 1}$  (r > 1) =  $s_n \frac{a(1 - r^n)}{1 - r}$ If (r < 1)
- Variety 4 sum = given n=?
- Variety 5 calculation of r in two non consecutive;  $r = \left(\frac{Tm}{Tn}\right)^{\frac{1}{m-n}}$
- Variety 6 Insertion of GM's
- Variety 7 Sum of infinity series.  $S\infty = \frac{a}{1-r}$

# Sets, Function & Relations





#### **Basics**

"It is a well defined group of distinct objects."

Expression Roaster Set
(Listing) Builder

• Cardinal no. = no. of elements in a set

No. of subsets =  $2^n$ Proper subsets =  $2^n-1$ 

#### **Types**

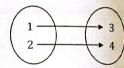
- 1. Universal Set: contains all the objects.
- 2. Subset: every element of A is in B. ACB
- 3. Superset: every element of A is in B. BOA
- 4. Null Set: 0, 0 element.
- 5. Equal Set: Every element of A is in B & vice versa.
- 6. Equivalent Set :n (A) = n (B)
- 7. Power Set : Set of all subsets.

# Relations

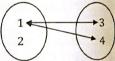
#### Basics .

Every subset of a Cartesian product of A×B is called relation

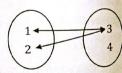
One to One



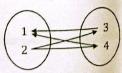
One to Many



Many to One



Many to Many



#### Types

- Reflexive A={1,2,3} R={(1,1)(2,2)(3,3)} all a,a∈R
- Symmetric
  A={1,2,3}
  R={(1,2)(2,1)
  (2,3)(3,2)}
  a,b∈ R then b,a∈ R
- Transitive

  A={1,2,3}

  R={(1,2)(2,3)(1,3)}

  a,b∈ R & b,c∈ R then,

  a,c∈ R
- S ✓ R ✓ T ✓
   = Equivalence

**Function** 

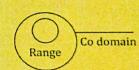
"Every R is not F but every F is a R."

**Basics** 

'No two ordered pairs should have same first element.'

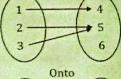
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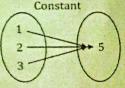


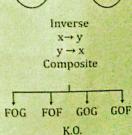
#### **Types**

Into









Champions वाला Chart

# Differential Calculus



#### Six Basic Rules of Differentiation

$$\frac{\mathrm{d}}{\mathrm{dx}}(\mathbf{x}^{\mathbf{n}}) = \mathbf{n}\mathbf{x}^{\mathbf{n}-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \log_e a$$

$$\frac{d}{dx}(a^{x}) = a^{x}\log_{e}a \qquad \qquad \frac{d}{dx}(constant) = 0$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \qquad \frac{d}{dx}(Logx) = \frac{1}{x}$$

$$\frac{d}{dx}(Logx) = \frac{1}{x}$$

Note: 
$$\frac{d}{dx} \{ cf(x) \} = cf'(x)$$
 c being constant.

#### For Two Functions

$$h(x)=f(x) \pm g(x)$$
  
(Sum/Difference of function)

$$\frac{d}{dx}\{h(x)\} = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

$$h(x)=f(x).g(x)$$
  
(Product of functions)

$$\frac{d}{dx}\{h(x)\} = f(x)\frac{d}{dx}\{g(x)\} + g(x)\frac{d}{dx}\{f(x)\}$$

$$h(x) = \frac{f(x)}{g(x)}$$
(Quotient of function)

$$\frac{d}{dx}\{h(x)\} = \frac{g(x)\frac{d}{dx}\{f(x)\} - f(x)\frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$$

#### Application of Differentiation

Average cost (AC or  $\overline{C}$ ) =  $\frac{Total\ Cost}{Out\ Put} = \frac{C(X)}{X}$ 

Average variable cost (AVC) =  $\frac{Variable\ Cost}{Out\ Put} = \frac{V(x)}{x}$ 

Average Fixed Cost (AFC) =  $\frac{Fixed\ Cost}{Out\ Put} = \frac{F(x)}{x}$ 

**Marginal Cost:** If C(x) the total cost producing x units then the increase in cost in producing one more unit is called marginal cost at an output level of x units and is given as  $\frac{dC}{dx}$ 

**Revenue Function:** Revenue, R(x), gives the total money obtained (Total turnover) by selling x units of a product. If x units are sold at 'P per unit, then R(x) = PX

Marginal Revenue: It is the rate of change in revenue per unit change in output. If R is the revenue and x is the output, then  $MR = \frac{dR}{dx}$ 

**Profit function:** Profit P(x), the difference of between total revenue R(x)and total Cost C(x). P(X) = R(x) - C(x)

Marginal Profit: It is rate of change in profit per unit change in dP output i.e. dr

**Slope of Curve:** If y is any function then  $\frac{dy}{dx}$  represent the slope of tangent to the curve.

# Intergal Calculus



$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$$\int dx = x + c, since \int 1 dx = \int x^0 dx = \frac{x^1}{1} = x$$

$$\int e^x dx = e^x + c$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + c$$

$$\int \frac{dx}{x} = \log x + c$$

$$\int a^x dx = \frac{a^x}{\log_e a} + c$$

#### Integration By parts

$$\int uvdx = u\int v\,dx - \int \left[\frac{d(u)}{dx}\int vdx\right]dx$$

$$\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$$

$$\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$$



# Definite Integration

$$\int_{a}^{b} f(x)dx = F(b) - F(a)$$

b'is called the upper limit and 'a'the lower limit of integration.

#### Important Properties of Definite Intergal

$$\int_{a}^{b} f(x)dx = \int_{b}^{a} f(x)dx$$

$$\int_{a}^{b} f(x)dx = \int_{b}^{c} f(x)dx + \int_{c}^{b} f(x) dx, a < c < b$$

$$\int_{o}^{b} f(x)dx = \int_{o}^{a} f(a - x)dx$$

$$when f(x) = f(a + x) = \int_{o}^{na} f(x)dx = n \int_{0}^{a} f(x)dx$$

$$\int_{-a}^{a} f(x)dx = 2 \int_{o}^{a} f(x)dx \quad \text{if } f(-x) = f(x)$$

$$\int_{-a}^{a} f(x)dx = 0 \quad \text{if } f(-x) = -f(x)$$

# Number Series & Coding-Decoding



#### Learn by Heart

#### Squares

$$1^2 = 1, 2^2 = 4, 3^2 = 9$$
 upto  $25^2 = 625$ 

#### Cubes

VICHYODAY

$$1^3 = 1, 2^3 = 8, 3^3 = 27$$
 upto  $15^3 = 3375$ 

#### **Alphabet Position Chart**

1	2	3	4	5	6	7	8	9	10	11	12	13
Α	В	С	D	E	F	G	Н	1	J	K	L	M
Z	Y	X	W	V	U	Т	S	R	Q	Р	0	N
26	25	24	23	22	21	20	19	18	17	16	15	14

## What is "Method of Difference"?

Questions: 2, 5, 12, 27, 54, 97,?

$$Mod \rightarrow 2_5 12_27_54_97 \overline{160}$$
  
3\_7\_15\_27\_43\_63

Ans. → 160

4 8 12 16 20



#### Thumb Rule of Position

Alphabet Forward Position + Alphabet Backward Position = 27

Example: Position of "!"

Forward Position = 9.

Backward Position = 27 - 9 = 18

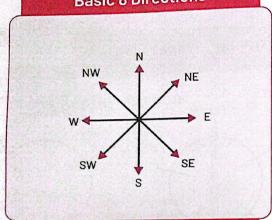
What is N Backward Position? 27 - 14 = (13)

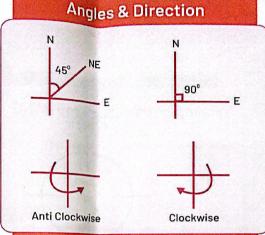
# Direction Test



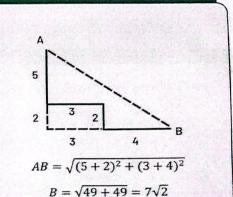
## **Basic 8 Directions**

MOHYODAY



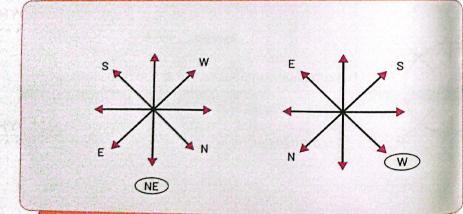


#### **How to Calculate Distance?**

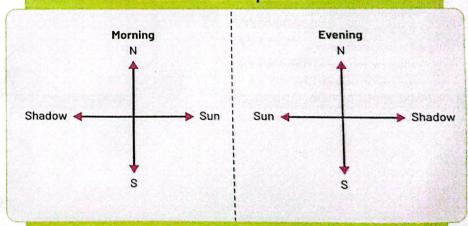


#### If South Becomes NE

If SE Becomes West



#### Shadows base questions

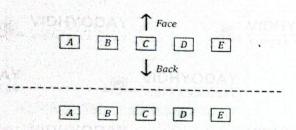


Champions वाला Chart

# Seating Arragement



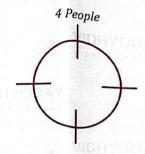
In Linear Arrangement People Sit Facing North

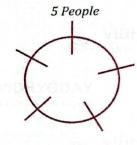


#### In the above arrangement

- → B & A are to the left of C.
- → D & E are to the right of C.
- → B is immediate left of C.
- → There are two persons between A & D.
- → D is third to the right of A.

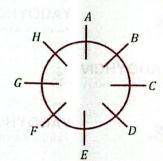
#### Circular Arrangement







Note: Spacing between any two person should be same.



A is Diametrically opposite to E, H to D ..... and so on

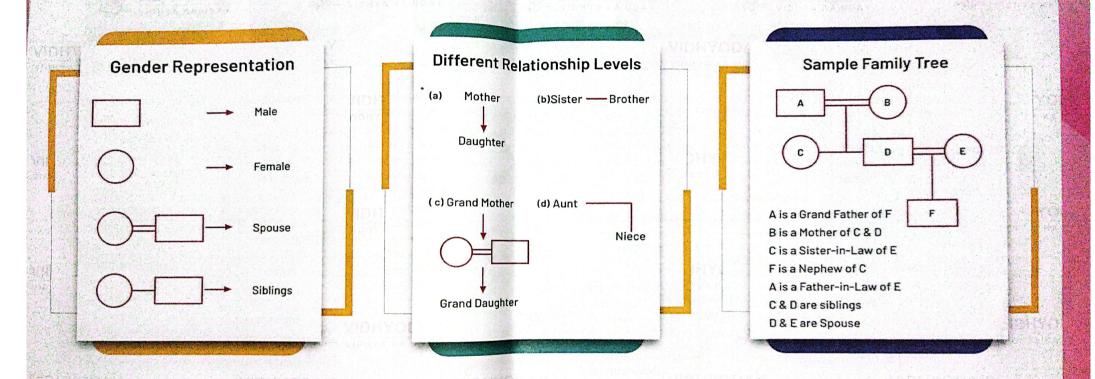
G is second to left of E.

G is third to right of B.

# **Blood Relations**

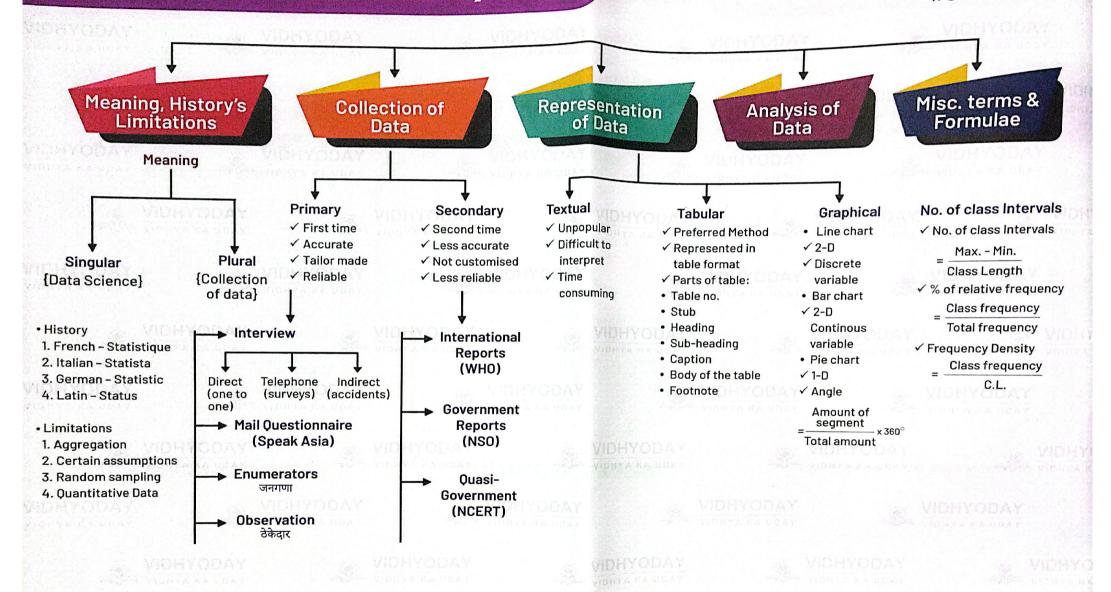
With Vancour





# Statistical Distribution of Data





# Central Tendency

लवारिस Property

- Δ of origin ✓
- Δ of scale ✓
- Δ of sign ✓

Quantitative Average • AM ≥GM≥HM • GM = √AM X HM Relationship: Mode:  $3md-2\overline{X}$  $m_0-\overline{X}=3(md-\overline{X})$ 

Positional Average

#### AM

- Average formula =  $\frac{\text{sum}}{\text{no}}$  $\frac{\sum x}{n}$ ,  $\frac{\sum fx}{\sum f}$ ,  $\frac{\sum fm}{\sum f}$ ,
- Properties
- A.M. is the most popular measure of CT.
- 2. Sum of deviations from A.M. is always 0.  $\sum X \overline{X} = 0$
- 3. Combined A.M. can be calculated.

$$\overline{X}_{12} = \frac{\overline{X}, n, +\overline{X}_2 n_2}{n + n_2}$$

 Mean can be calculated using assumed mean formula

$$\bar{X} = A + \frac{\sum d}{n}$$

- A.M. can not be represented graphically.
- 6.  $\sum (X \overline{X})^2 = \min \min$

#### GM

- GM is best measure of CT for ratios & percentages.
- Formula Individual

$$(axbac...)\frac{1}{n}$$
Discrete

 $\left(X \frac{f^1}{n} \times X \frac{f^2}{2} \dots X \frac{f^n}{n}\right) \frac{1}{\sum n}$ 

Continuos

$$(M_1^{f1} \times M_2^{f2} \dots)^{\frac{1}{\sum f}}$$

#### HM

- Used for variables having reciprocal relationship
- Formula Individual

$$HM = \frac{n}{\frac{1}{X1} + \frac{1}{X2} \cdot \dots \cdot \frac{1}{Xn}}$$

Discrete

$$HM = \frac{\sum f}{\frac{f1}{X1} + \frac{f2}{X2} \dots \frac{fn}{Xn}}$$

$$HM = \frac{\sum f}{\frac{f1}{m1} + \frac{f2}{m2} \dots \frac{fn}{m_n}}$$

- आवन जावन Q is imp
- HM is the reciprocal of AM
- Combined HM

$$=\frac{n1+n2}{\frac{n1}{HM1}+\frac{n2}{HM2}}$$

#### MEDIAN

- Individual इधर से काटो, उधर से काटो, बीच में जो बचा वो median
- Discrete

$$S.0\overline{1}\frac{N}{2}$$

 $\mathrm{S.01} rac{N}{2}$  को Locate करो in C.F.

- S.03 आगे वाला is median
- Continuos
   S.01 follow discrete

S.02 M<sub>i</sub>=1<sub>i</sub>+
$$\left\{\frac{\frac{N}{2}-C}{F}\right\}xH$$

- Md is not affected by extremities of the observations
- Sum of absolute deviation from median is minimum.

$$\sum |x-xmd| = \min \min$$

- Calculated through Ogive.
- Partition Values

Value = 
$$\left[ \text{orderX} \left\{ \frac{n+1}{4/10/100} \right\} \right]_{0}^{n}$$
 term

Ouartiles Deciles Percentiles

Best for open' end classification

#### MODE

- Individual Most repeated no.
- Discrete

No. with highest frequency

Continuos

Find out model class & use.

Formula :

$$M0 = 1_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

- · It is not uniquely defined.
- · Calculated using Histogram.
  - Mode

Unimodel Bimodel Multimodel

# Measures of Dispersion

[Measures of Dispersion] "Second order of averages"

लवारिस Property

- △ of origin ×
- A'of scale ✓
- Δ of sign ×

**Absolute MOD** 

Relationship Between MD, QD & S.D.

4 S.D. = 5 MD = 6QD {4,5,6 करके समृन्दर में कूदो}

**Relative MOD** 

$$M.D. = \frac{\sum |X - \overline{X}|}{n}$$

$$\frac{\sum |X-Xm|}{n}$$

Or  $\sum |X-Xmo|$ 

- If frequencies of all observations are same, count them only once,
- . M.D. from Median is minimum.

Range

Max Value - Min Value = Range

\* easiest & quickest

- to calculate dispersion.
- \* Also called as semi inter quartile range.

 $Q.D. = \frac{Q_3 - Q_1}{2}$ 

\* It is best suited

for open end

classification

as it is based

values

on middle 50%

Standard Deviation

$$S.D. = \sqrt{\frac{\sum (X - \overline{X})^2}{n}}$$

$$S.D. = \sqrt{\frac{\sum X^2}{n} - (\overline{X})^2}$$

$$S.D. = \sqrt{\text{cigma}}$$

- S.D. = (sigma)  $S.D^2$  = variance
- $\sqrt{\text{variance}} = \text{S.D.}$
- For first n natural

S.D. = 
$$\sqrt{\frac{n^2 - 1}{12}}$$

If frequencies of all observations are same. count them once only.

S.D. between 2 no.'s

$$=\frac{|a-b|}{2}$$

· Combined S.D.

$$= \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

$$d_1 = \overline{X}_{12} - \overline{X}_1$$

$$\mathbf{d}_1 = \mathbf{X}_{12} - \mathbf{X}_1$$
$$\mathbf{d}_2 = \overline{\mathbf{X}}_{12} - \overline{\mathbf{X}}_2$$

M.D.

#### Coeff. Of M.D.

$$= \frac{\text{M.D.}}{\hat{x} |x_{\text{md}}| x_{\text{mo}}} \times 100$$

Coeff of Range

Range

$$= \frac{\text{Max-Min}}{\text{Max+Min}} \times 100$$

Coeff of O.D.

O.D.

Coeff of O.D.

 $=\frac{Q_3 - Q_1}{Q_2 + Q_1} \times 100$ 



Coeff of variation

Variation

 $=\frac{\text{S.D.}}{\frac{1}{2}}\times 100$ 

\* It is used to measure consistency

Decision rule: Lower the better

# **Probability**



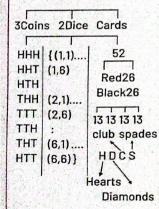
#### **Basics**

$$P(A) = \frac{Fav.}{Total}$$

Odds in favour = m:n

Odds in against = n : m

$$P(A) = \frac{m}{m+n} P(\bar{A}) = \frac{n}{m+n}$$



#### Terminology

- Exp. = कीड़े करना
- Random Exp = outcome is not know
- Exhaustive = ूपी दुनिया Union = 1
- Equally likely = सब बराबर P(A) = P(B)
- Mutually Exclusive ਸਲਕੀ = 0 P(A N B) = 0
- Sure event
   P(A)=1
- Impossible Event P(A) = 0
- Dependent = formula
   P(A∩B) = P(A) × P(B/A)
- Independent ਸਲਕੀ = P(A) × P(B)

#### P & C

- Mostly combination (balls, cards, committee, geometry figures etc.)
- Fav. Total
- Fav = with restrictions
- Total = w/o restriction

#### Exp. Value

- Nothing but weighted avg.
- $E(x) = \sum px$
- Sum of probability = 1
   ∑ p = 1
- अगर bracket में x के अलावा expression है तो change x.
- Properties:
   E(x±y) = E(x) ± E(y)
   E(Kx) = KE(x)
   E(x+y) = E(x) / E(y)
   E(x×y) = E(x) × E(y)
- Variance
   E[X-e(X)]<sup>2</sup>

#### Venn Diagram

- 1) AUB = A+B-AAB
- 2) AOB

Dependent Independent

- 3) Only A A – A∩B
- 4) Only B B - A∩B
- 5)  $\overline{A \cap B}$ = $\overline{A} \cup \overline{B} = 1 - P(A \cup B)$
- 6)  $\overline{A \cup B}$  $\overline{A} \cap \overline{B} = 1 - P(A \cup B)$
- 8) B<sup>c</sup> =

7) A<sup>c</sup> \_

1-A

#### Conditional Probability

Probability of A when B has already occurred P(A/B)

$$=\frac{P(A \cap B)}{P(B)}$$

For e.g. P(A/B)

 $=\frac{P(\bar{A}\cap \bar{B})}{P(\bar{B})}$ 

 $\frac{1 - P(A \cap B)}{1 - P(B)}$ 

# Theoretical Distribution



## Binomial

Given by James Bournouli (Discrete Distribution)

Formula

Applicability

: Future में IPM करेंग

Finite Independent Trials Events Probability

: सप्तऋषि

Mutually **Exclusive Events** 

**Properties** 

- 1. Mean = np
- 2. Variance = npg
- 3. Mean = Variance
- 4. Biparametric B(n, P)
- 5. Max. Variance =
- 6. Mode = (n+1)xp

Decimal Ignore Decimal: Unimodal Case

Integer Bimodal: Ans. Ans-1

7. Combined binomial Distribution P(x+y=r)="1"CxP'x0"1"2"

# Poission

Given by Simon Poission (Discrete Distribution)

Formula

$$P(x=r) = \frac{e^{-m}x m^r}{r!}$$

Applicability

: n→∞  $p \rightarrow 0$ 

np→infinite (Flight Example)

**Properties** 

: Six - Sense

- 1. mean = np
- 2. variance = mean
- 3. Uniparametric (only m is parameter)

4.  $mode = n \times p$ 

Decimal Ignore Decimal: Unimodal Case

Integer Bimodal: Ans, Ans-1

- 5. B.D. can be approximated with P.D.
- 6. Combined Poission Distribution

$$P(x+y=r)=e^{-(m1+m2)}(m1+m2)$$

# Normal

Given by Carl Gauss (Continuous Distribution) Also called Gaussion distribution

**Formula** 

$$o: P(x) = \frac{1}{\sigma \sqrt{2\pi}} x e^{-\frac{1}{2} \left(\frac{x-\mu^2}{\sigma}\right)^2}$$

$$z = \left(\frac{x - \mu}{\sigma}\right)^2$$

#### **Properties**

- 1. It is bi-parametric X-N (μ,σ²)
- 2. It is bell shaped curve.
- 3. It is symmetrical around mean.
- 4. Standard Results:

 $\mu + \sigma = 68.30\%$ ,  $\mu + 2\sigma = 95.50\%$ ,

 $\mu + 3\sigma = 97.70\%$ 

Fix Rate free, Shine Life Life, Shine Shine Raven

5. Points of inflexion

 $\mu + \sigma$ 

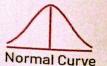
H-O

- 4, 5, 6 करके समुदंर में 'क्दो, 4.S.D. = 5 M.D. = 6 Q.D.
- 7. Standard Normal Variate
- u=0  $\sigma=1$ 8. Quartiles

Q=μ-0.675σ

Q=u

Q= +0.6750



Champions वाला Ch

# Correlation

लवारिस Property

- Δ of origin ×
- Δ of scale ×
- △ of sign ✓

# Correlation ( Measures the degree of linear relationship between two variables )

#### Scatter Diagram

- It only tell us the nature of correlation & not degree of correlation
- Five Diagrams

$$r=-1$$

- Correlation of Straight line is always +1 or -1.
   It depends upon the direction between x & y.
- Equation of a StraightLine is ax + by = c
- Proportion --> decides
   1 or not
   direction --> decides +
   or -

#### Rank Correlation

$$r_0 = 1 - \frac{6\sum d^2}{n^3 - n}$$

n= no. of observations. d = difference of ranks

- Sum of difference of ranks is always 0.
- If ranks are exactly opposite then r = -1.
- Even if ranks are reversed, it remains same.

# **Concurrent Deviation**

$$r_c = \pm \sqrt{\pm \frac{2c - m}{m}}$$

 If there is negative no inside the root, r is going to be negative.

m = no. of observations compared = n - 1.

c = no. of concurrent deviation (+'s)

#### Karl Pearson

$$r = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y}$$

- $r = \frac{n\Sigma xy \Sigma x\Sigma y}{\sqrt{n\Sigma x^2 (\Sigma x)^2} \sqrt{n\Sigma y^2 (\Sigma y)^2}}$
- Cov(x,y)

$$= \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{n}$$

- Cov (X,Y) desides the nature of correlation-
- -1srs+1
- n is a pure no. (unit free).
- Coefficient of determination = 1-r² (Unexplained variance)

• P.E. = 
$$\frac{0.675\sqrt{1 \cdot r^2}}{n}$$

- Product Moment correlation (nick name)
- Useful for variables having only linear relationship.

Champions वाला Chart

# Regression

लवारिस Property

- Δ of origin x
- △ of scale ✓
- Δ of sign ✓

### Regression

#### Regression Equations

- Unlike r, it tells us the exact increase in price of y if x is increased or vice versa.
- There are two equations:
  - 1. Y on x →to calculate y.
  - 2. X on y  $\rightarrow$  to calculate x.
- Y on  $x \rightarrow Y \overline{Y} = byx(X \overline{X})$
- X on  $y \rightarrow X \overline{X} = bxy(Y \overline{Y})$
- byx= $r\frac{6y}{6x}$  (जो पीछे है वो नीचे है।)

$$bxy = r\frac{6y}{6x}$$

#### Popular Questions:

- 1 Lines = given slope = ?
- 2 Lines = given r=?
- 3 Slopes = given r = ?
- 4 Lines = given mean = ?
- 5 पहचान कीन?

#### **Properties**

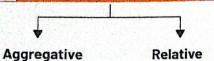
- 1) Regression lines intersect each other at  $(\overline{X} \overline{Y})i.e.$  mean. (K,0,)
- 2) Correlation coefficients is GM of regression coefficient.  $r \! = \! \sqrt{b_{\rm vx} \! \times \! b_{\rm xv}}$
- 3) The product of regression coefficients should be  $\leq$  1.  $b_{yx} \times b_{xy} \leq 1$
- If one coefficients is greater than unity the other should be less than unity.
- 5) byx, bxy & r are all of same sign.
- Regression lines are made using least squares deviation method.
- 7) नावरिस Property :  $\Delta$  of origin ×  $\Delta$  of scale  $\checkmark$   $\Delta$  of sign  $\checkmark$
- 8) r = 0 regression lines are perpendicular, if  $r \pm 1$ , lines will coincide.
- 9) पहचान कौन?
  - S.01 Calculate  $b_{yx} \& b_{xy} \le 1$ By assuming one line as Y on x & another as x on y.
  - S.02 Check if  $\sqrt{b_{yx} \times b_{xy}} \le 1$
  - S.03 Yes → assumption is true.
    No → opposite is true.

# Index No.



CY value, when B.Y value is assumed to be 100. E.g. Sensex (1978-79) Index no. is a pure no.

## Simple Method



$$P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$$

$$I = \frac{\sum I_R}{n}$$

$$I_R = \frac{P_1}{P_0} \times 100$$

# Weighted Method

$$\begin{bmatrix} \sum P_1 W \\ \sum P_0 W \end{bmatrix}$$
Laspeyer = 
$$\frac{\sum P_1 q_0}{\sum p_0 q_0}$$

(Base year Q.)

Passche = 
$$\frac{\sum P_1 q_1}{\sum p_0 q_1}$$

Fishers = 
$$\sqrt{\text{La} \times \text{Pa}}$$

Dorbish & Bowley = 
$$\frac{\text{La} \times \text{Pa}}{2}$$

Marshall Edgeworth

$$P01 = \frac{\sum P_1 \left( \frac{q_0 + q_1}{2} \right)}{\sum P_0 \left( \frac{q_0 + q_1}{2} \right)}$$

Walsh 
$$\left(\sqrt{P_{01}}\right) = \frac{\sum P_1 \sqrt{q_0 q_1}}{\sum P_0 \sqrt{q_0 q_1}}$$

Weight = 
$$\sqrt{q_0 q_1}$$

(kelly) = 
$$\frac{\sum P_i q}{\sum P_0 q}$$

### **Special Points**

1	1	J.	1
Test	Inflation	Base Shifting	Salary
	Deflation	& CBI	Q.'s
1) Unit test unit free	(Index No - 100)	СВІ	C.Y.Salary
satisfied by all.	= Inflation %	LR×PYCBI	C.Y.Index
2) Time - Reversal	D-G-4-11	= 100	B.Y.Index
Test $P_{01} \times P_{10} = 1$	Deflated value	$LR = \frac{CYPrice}{\times 100}$	= B.Y. की Salar

value

- Kelly, MEW, Fishers Simple aggregative Satisfy TRT
- 3) Circular Test
  0 2
  Extension

X	$\bigcup_{1}^{n}$		$P_{12}$	$T \\ {}_{2} \times P_{20}$	
			=1		
Fi	sher	×			
Ke	elley	/			

aggregative ✓ 4) Factor Reversal  $P_{01} \times q_{01} = V_{01}$ 

Fisher's 
$$\checkmark$$

$$\mathbf{v}_{01} = \frac{\sum \mathbf{P}_1 \mathbf{q}_1}{\sum \mathbf{P}_0 \mathbf{q}_0} \times 100$$

- Prev.Price B.Y. Salary  $= \frac{\text{C.Y.Value}}{\text{C.Y.Index}} \times 100$ B.Y.Index C.Y.Index = C.Y. की Salary
  - · Today' salary - should have been = Real gain.
  - · Should have been -Today's salary = D.A.

### DEMAND

Effective Demand



#### BASICS

	for a specific commodity
(b) Means /	to purchase the
desired comm	odity (Backed By)
(c)	to spend
(d)	of the commodity at a certain PPT
Important Terms	
	anded
3. Schedule	
4. Curve	
5. Individual	
6. Market	
7. Stock	
8. Flow	
9. Demand is a	Concept
10. Substitute Goods	
11. Complementary (	Goods
12. Inferior Goods	
3. Normal Goods	

#### FACTORS AFFECTING DEMAND

Dx =	: f (Px, Pr, Y, T, E, & N, Yd, C, G)
1,	Price of Same Good (Px)
2.	Price of Related Good (Pr)
_	Pcg
3.	Income of Consumer (Y) IG
	NG
4.	Taste & Preference (T)
5.	Future Expectations of Price, Income & Supply (E)
6.	Population (N)
7.	Income Distribution (Yd)
8.	Consumer Credit Facility & Interest Rate (C)
9.	Government Policy
Dem	onstration Effect
Veble	en Effect
land	wagon Effect
nob	Effect

#### LAW OF DEMAND

7	HAY, ILE	G IV	ST.	
_				
2.1				
Assumption	ns of Law of D	emand -		

#### Features of the Demand Curve

DX = f (D.

1.	DC slopes	from left to the right.
2.	DC is	sloped.
3.	DC is called	Curve
4.	DC may be someting	mes a straight-line or
	sometimes a free h	

- The downward sloping DC explains the Law of Demand.
- The Market Demand Curve is a \_\_\_\_\_

  summation (totalling) of Individual Demand Curves.

#### Rationale (Logic) behind the Law of Demand

## Price Effect of a fall in Price

- 1. Substitution Effect
- 2. Income Effect

#### Other Reasons

- 1. Law of Diminishing Marginal Utility
- 2. New Consumers
- 3. Different Uses

#### Exceptions to the Law of Demand

- 1. Conspicuous Goods
- 2. Giffen Goods
- 3. Ignorant/ Irrational Consumer
- 4. Basic Necessity
- 5. Speculative Goods
- 6. Expected Price Change

# CHANGE IN QUANTITY DEMANDED VS CHANGE IN DEMAND

### Change in Quantity Demanded

Extension of Q.D	Contraction of Q.D
- 1	
	the dist

#### Change in Demand

Change in Demand			
Increase in Demand	Decrease in Dema		

## **ELASTICITY OF DEMAND**



#### PRICE ELASTICITY

#### 1. Percentage Method

Ep = % Change in Quantity = \_\_\_\_x % Change in Px

#### Calculator Steps:

Case	1	2	3	4
Old Price	20	50	70	90
New Price	18	45	84	72
Old Quantity	3000	4000	?	5000
New Quantity	3900	?	4000	?
Ер	?	4	1	2
a)		5000	6000	3000
b)		5200	5000	6000
c)	<b>FEMALE</b>	5400	7000	7000
d)	The state of	5600	8000	8000

#### 2. Arc Elasticity

Arc Elasticity measures	elasticity i
case of	change in prices and quantities
(i.e. over an arc) on the De	mand Curve, rather than on a point.

Eo =		×		

#### Calculator Steps:

	Price	Quantity
Old	10	2000
New	12	1200

#### 3. Method Of Derivatives

F-	X
Ep =	^

#### 4. Point Elasticity / Graphical Method

Ep =	Lower Segment (RHS)
	Upper Segmetn (LHS)

#### 5. The Total Outlay / Expenditure / Revenue Method

Price	Total Expenditure	Elasticity
	The Line Roll	

#### CROSS ELASTICITY INCOME ELASTICITY

Ec = % Change in Quantity = x % Change in Pr			Ey = % Change in Quantity = X  % Change in Y			
Interpr	etations	Interpr	etations			
Ec	Interpretation	Fy	Interpretation			

#### Interpretation of Elasticity Values

A STATE OF THE PARTY OF THE PAR	A CONTRACTOR OF THE PROPERTY O	The second secon	THE RESERVE THE PROPERTY OF THE PERSON NAMED IN COLUMN TWO IS NOT THE PERSON NAMED IN COLUMN TWO IS NAMED IN COL
	A Commence of the Commence of		
	and the second s		

#### **Determinants of Price Elasticity**

1. Nature of Commodity	2. Availability of Substitutes	3. Different Uses
4. Postponement of Use	5. Income of Consumer	6. Habit of Consumer
7. Position in Consumer's Budget	8. Price Level	9. Time Period

# SUPPLY, ELASTICITY & PRICE DETERMINATION



# Price Elasticity 1. Stock 2. Supply 3. Quantity Supplied \_\_\_\_\_ 5. Supply is a \_\_\_\_\_ Concept Factors Determining Supply 1. Price of Same Commodity (Px) 2. Price of Substitute Good (Pr) 3. Goal of Firm (G) 4. Price of Factors of Production (Pfop) 5. Business Expectations (Be) 6. Government Policy (Gp) 7. Natural Factors 8. Nature of Competition & Size of Industry Law of Supply

Features	of the	Supply	Curve

- 1. SC slopes \_\_\_\_\_(Positive) from left to the right.
- 2. SC is called \_\_\_\_\_ Curve
- 3. SC may be a straight-line or a free hand curve.
- The Market Supply Curve is a \_\_\_\_\_
  summation (totalling) of Individual Supply
  Curves.

#### **Exceptions of Law Of Supply**

- Agricultural {products supply is governed by Natural Factors
- 2. Goods having Social Distinction
- 3. Perishable Goods

Change in Quantity Supplied

Extension of Q.S	Contraction of Q.S

#### Change in Supply

Decrease in Supply

### ELASTICITY OF SUPPLY

# 1. Percentage / Proportionate Method Ep = % Change in Quantity = \_\_\_\_\_ × \_\_\_\_

% Chang	e in Px			
Case	1	2	3	4
Old Price	10	80	40	90
New Price	12	84	?	72
Old Quantity	2000	3000	6000	5000
New Quantity	2800	?	6600	?
Ep	?	3	0.5	2
a)		3200	42	7000
b)		3450	43	4000
c)		3650	44	3000
d)		3850	48	2000

- 2. Arc Elasticity Same
- 3. Method of Derivatives Same

#### Determinants of Elasticity of Supply

1. Cost of Production	2. Time Period
3. Number of Producer	4. Capacity Utilization
5. Availability of RM	6. Factor Substitution
7. Mobility of FOP	8.Future Expectation



### PRICE DETERMINATION

Golden Rules

Case	EP	EQ 🤚
		The state of the s

Price Floor / Minimum Support Price



Price Ceiling

## DEMAND FORECASTING



#### BASICS

Meaning -

Usefulness	
Budgetary Pl	anning and Cost Control
	duction Planning & Scheduling
	tments & Marketing
Evaluate force	es which affect demand
Condition for	Use
No simple for	mula
Benefit > Cos	t & Time
Factor's Affecting	g Demand
1. Producer Go	oods
Rate of Profita	ability of the User Industry
Growth Prosper	ects of the User Industry
	rket of the User Industry,
Norms of Cons	sumption of Capital Goods
· Growth of Tecl	hnology in the User Industry
Increase in the Complementar	Price of a Substitutable factor & ry factor
Financing Opp	ortunities
2. Non-Durable	e-Goods
Disposable Inco	ome
• Price	
Demography	
Competition	
3. Durable Goo	ds
<ul> <li>Special Facilities</li> </ul>	s -
• Family	
· Current Holding	J-
<ul> <li>Timing of Repla</li> </ul>	cement

#### urvey of Buyer's Intention

#### Method

- · Complete Enumeration
- · Sample Survey -
- · End-Use-Method -

#### Merit

- · Market or Customer-driven approach
- Useful when bulk of sale is made to Industrial Producers who generally have definite future place

#### Demerit

- Only Useful for short-run demand forecasting.
- It is not suitable to depend wholly on the Buyers' estimates and they should be used cautiously in the light of the Seller's own judgment.
- Customers may themselves misjudge their requirements
- Not useful in case of Household Customers, due to reasons like irregularity in Customers' buying intentions

#### 2. Collective Opinion Method

#### Process

- · Estimates from Salesmen
- Reviewed to eliminate the Bias of optimism and pessimism
- The Revise Estimates for Proposed Changes
- · Estimate of Final Demand

#### Men

- Simple, First-hand information of those who are directly connected with sales.
- Useful for Firms having a wide network of Sales Personnel
- · Salesmen being closest to the Customers

#### METHOD OF FORECASTING

#### Demerit

- The demand figures may be subjective (personal opinion)
- Salesmen Unaware of the broader economic changes
- · Only useful for short-run demand forecasting.

#### 3. Expert Opinion Method

#### Delphi Technique

- Developed by Olaf Helmer @Rand Corporation,

  USA
- Firms solicit the opinion of a Specialists or Experts through a series of carefully designed Ouestionnaires.
- Experts are asked to provide forecasts and reasons for their forecasts.
- Experts are provided with information and opinion feedbacks of others at different rounds without revealing the identity of the Opinion Provider.
- These opinions are then exchanged among the various experts and the process goes on until convergence of opinions is arrived at.

#### Merit

- Accepted due to its broader applicability and ability to address complex questions. It also has the advantages of speed and cheapness.
- Best suited in circumstances where intractable changes are occurring
- No Need to depend only upon the opinions of Buyers and Salesmen.
- Informed judgment from diverse experts by avoiding the disadvantages of conventional Panel Meetings.

#### 4. Statistical Method

Superior as More Scientific, Reliable & Free from Subjectivity

Trend Projection based on Time Series Data
Trend Projection Method is also called Classical Method,
and is considered as a 'naive' approach to Demand
Forecasting.

- (i) Graphical Method (Free Hand Projection Method)
- (ii) Fitting Trend Equation or Least Squares Method.
- (iii) Regression Analysis

#### Controlled Experiments or Market Experiment Method

Conduct market studies & experiments on consumer behaviour under actual, though controlled, market conditions.

#### Consumer Clinics

Market Experiments can also be replaced by 'Controlled Laboratory Experiments' or 'Consumer Clinics' under which the Consumers are given a specified sum of money and asked to spend in a store on goods with varying prices, packages, displays, etc. The responses of the consumers are studied and used for Demand Forecasting.

#### 6. Barometric Method

#### Types of Indicators

- · Leading Indicator
- Lagging Indicators
- · Coincident Indicators

#### Merit

- Other Methods are based on past experience and trying to merely project the past into the future.
   Such projection is not effective where there are economic ups and downs. Barometric Method is most appropriate in such situations.
- Other Methods are related with the product concerned and Barometric Method is a wholistic approach

#### Demerit

- Actual quantity demanded is not forecast, only estimates
- · Expensive and time consuming.

# PRODUCTION FUNCTION THEORY



#### BASICS

#### Meaning -

- Production = Creation of Utility
- Production = Addition of the Value

Types	of	Uti	lity

- 1. Form Utility
- 2. Place Utility
- 3. Time Utility \_\_\_\_\_
- 4. Personal Utility\_

#### Land

Land refers to all free gifts of the nature. This includes soil and earth's surface, natural resources, fertility of soil, water, air, natural vegetation, etc.

#### Features

- gift of nature.
- Land is fixed in
- The Supply of Land is perfectly inelastic from the viewpoint of the entire economy. Though, it is relatively elastic from the viewpoint of an Individual Firm.
- Land is \_\_\_\_\_\_ It cannot be destroyed or lost.
- The production power of soil is indestructible since its fertility can be restored
- Land lacks \_\_\_\_\_\_in a geographical sense.
   It cannot be shifted.
- Land is a \_\_\_\_\_\_ factor of production in the sense that it does not yield any result unless human efforts are employed.
- Land varies in \_\_\_\_\_\_ and uses.

#### **FACTORS OF PRODUCTION**

#### Labour

'Labour' means\_\_\_\_\_\_ or \_\_\_\_\_exertion directed to produce goods or services, and with a view to gain an

reward.

#### Features

- · Human Efforts
- Perishable Nature
- · Weak Bargaining Power
- Self Source
- Variations
- Productivity
- · Relationship between Wage Rate & Labour Supply
- a) Direct Relation
- b) Inverse Relation at Higher Prices
- c) Inverse Relation at Lower Prices

#### Capital

Capital means that part of wealth of an individual or community, which is used for further production of wealth, or which yields an income

#### Features

- Stock Concept
- Capital ≠ Wealth
- Produced Means

### Capital Formation

### Need for Capital Formation

- Replacement and Renovation of existing machinery and equipment
- Creating Additional Productive Capacity

#### Stages in Capital Formation

Stage 1	Stage 2	Stage 3
Creation of Savings	Mobilisation of Savings	Investement of Saving in Real Capital
Individual Business Govt	Banks Financial Insti. Capital Market	Enterpreneur +ve Enviornment Inducement to Invest

- · Ability to Save
- Willingness to Save

#### Entrepreneur

#### Function of Entrepreneur

- · Initiating business & Resource Co-ordination
- · Risk Bearing
- Innovation

#### Enterprise's Objectives

- Organic \_\_\_\_\_

   Economics \_\_\_\_\_
- 3. Social-
- 4. Human
- 5. National -\_\_\_\_

## **OBJECTIVES & CONSTRAINTS**

#### Constraints in Achieving Objectives

- 1. Information
- 2. Infrastructure
- 3. Factors of Production
- 4. Economic Aspect

#### **Enterprise Problems**

- 1. Objective
- 2. Location of Plant
- 3. Size of Plant
- 4. Physical Facilities
- 5. Finance
- 6. Organizational Structure
- 7. Marketing
- 8. Legal Compliance
- 9. Industrial Relations

# PRODUCTION FUNCTION TECHINCAL



#### BASICS

Cobb-Douglas Production Function (ST) Q = K L'C<sup>(1-4)</sup> Labour 3/4 (75%), Capital ¼ (25%)

#### Short Period Vs Long Period

Basis	Short Period	Long Period
Meaning	At least One Factor cannot be Increased	All Factors can be increased.
Fixed Factor	At least One Factor	No Factor is Fixed
Variable Factor	All Except at Least One	All are Variable
Proportion Between Factors	Not Same Proportion	Same Proportion
Factor Ratio	Changes	Same

Important Terms

**Marginal Product** 

**Total Product** 

Average Product

# RETURNS TO FACTOR / SHORT RUN

Law of Variable Proportion / Proportionality / Diminishing Returns -Only One Factor is Variable. All Others Fixed.

Causes of Increasing Rt.	Causes of Decreasing Rt.
Fuller Utilisation of Fixed Factor	<ol> <li>Inadequacy / Fixity of Factor</li> </ol>
2. Increased Efficiency	<ol><li>Imperfect Factor substitutability</li></ol>
Better Co-ordination between Factors (Right Combination)	<ol> <li>Poor Co-ordination between Factor (Wrong Combination)</li> </ol>

#### Relation Between MP & TP

1.	
2.	
3	

#### Relation Between MP & AP

	Stage 1	Stage 2	Stage 3
MP			
AP			
TP		factors and	

#### Area of Economics Operation

#### Assumptions of LOVP

- 1. Technology Constant
- 2. Only One Factor Variable
- 3. Factors Can be Used in Any Proportion 4. Factor Ratio Can Change
- 5. Fixed Factors are Scarce

#### **RETURNS TO SCALE / LONG RUN**

#### Returns to Scale

All the Factors of Production are increased or decreased in the \_\_\_\_\_\_proportion. The Law of Returns to Scale analyses the changes in output, due to changes in scale in the long- run, i.e. quantities of resources, keeping proportion constant.

#### **Internal Economies & Diseconomies**

Aspect	Economics	Diseconomics
Technical		
Managerial	* (NO)	
Commercial		
Financial		
Risk Bearing		

### Cobb-Douglas Production Function (LT)

a +b >1	
a+b=1	
a+b<1	

#### **Economies & Diseconomies**

	Internal	Extrnal
Depends On		
Impact On		

#### **External Economies & Diseconomies**

Economies		
Cheaper RM		
Technological		
Development of Skilled Labour		
Ancillary Industry		
Transport & Marketing	and the second s	
Disecor	nomies	
Factor Price		
Govt Restriction		

# COST FUNCTION - THEORY



Basis	Explicit Cost	Implicit Cost
Cash Outflow	Yes	No
Resources	3rd Party	Self-Owned Resource
Other Name	Out-of-Pocket Cost Outlay Cost	Notional Imputed Opportunity Cost
Measurement	Objective Accurate	Subjective Estimated
Recording In A/C	Yes	No
Purpose	Accounting, Reporting, Cost Control & Decision Making	Decision Making

Basis	Direct Cost	Indirect Cost
Identified or Traceable	To a Particular  • Product  • Service	<ul><li>No Tracking</li><li>Common Expenses</li></ul>
	• Operation • Plant	Not Quantifiable, But May Vary with Output
Relationship	Quantifiable Per Unit	Apportioned or Absorbed
Accounting	Directly Charged to a Product	(Distributed) on Appropriate Basis

Basis	Fixed Cost	Implicit Cost
Cost of	Fixed Factors of Production	Variable Factors
Change	Does not Change	Changes With Output
Relationship	Period Related	Product Related
When Incurred	Even at Zero Unit	After Start of Production
Avoidable	Unavoidable	Avoidable
Cost Per Unit	Changes	Same
Total Cost	Same	Changes

Basis	Committed Cost	Discretional Cost
Cost of	Fixed Factors of Production	Variable Factors of Production
Change	Does not Change	Changes With Output
Relationship	Period Related	Product Related
When Incurred	Even at Zero Unit	After Start of Production
Avoidable	Unavoidable	Avoidable
Cost Per Unit	Changes	Same
Total Cost	Same	Changes

Incremental Cost	Sunk Cost
Additional Cost Incurred	Already Incurred in Past
Due to Any Reason	Cannot be Recovered, Revised or reversed
Relevant for Decision Making	Act as Barrier to Entry of New Firm

Historical Cost	Replacement Cost
Acquisition/Purchase/	Cash Outflow for
Orignal Price of Asset	Replacing an Old Asset
Already incurred in Past	To be Incurred in Present
Not Relevant for	Relevant for Current
Decision making	Decision Making

Private Cost	Social Cost
Cost of Production	Cost of Disutility
Incurred by Firm	Incurred By 3rd Party
Private Cost = Explicit Cost & Implicit Cost	Social Cost = Private Csot + External Cost
Private Profit = TR – Private Cost	Social Profit = TR - Social Cost

#### Opportunity Cost

- 1. Value of sacrifice made
- 2. Cost of opportunity foregone
- 3. Arises only when alternatives are available.
- 4. Do not involve any cash payment
- 5. Not recorded in books of accounts.
- 6. Considered only for Decision-Making & Analytical purposes

#### Dependent Variable

- 1. Total Cost
- 2. Cost Per Unit

## COST FUNCTION - TECHNICAL



#### SHORT PERIOD ANALYSIS

### LONG PERIOD ANALYSIS

#### **Cost of Production**

Expenditure incurred by a firm on the factor inputs (Land, Labour, Capital & Entrepreneurship) as well as non-factor inputs (Raw material) for the production of a commodity.

C = f(Q)

#### Types of Cost

Туре	Meaning	Graph
Marginal Cost	Additional Cost due to Production of One More Unit	U Shaped
Average Variable Cost	Variable Cost Per Unit	U Shaped
Average Cost	Total Cost Per Unit	U Shaped
Average Fixed Cost	Fixed Cost Per Unit	Rectangular Hyperbola
Total Variable Cost	Increases at Decreasing     Increases at Increasing	
Total Fixed Cost	Constant     Horizontal     Parallel to X-Axis	
Total Cost	TC = TFC+ TVC	

#### Formula

MCn	•
TVCn	•
TC	
AVC	
AFC	
AC	•
The state of the s	

### 

#### Relationship

1. MC&TC/TVS  2. MC&AC&AVC  4.			
2. MC & AC & AVC	1.	MC & TC / TVS	
• 2. MC & AC & AVC • •			
2. MC & AC & AVC • •			
2. MC & AC & AVC •			
	2.	MC & AC & AVC	
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	•		

2 21			
2. Plant Curve	 	 	
3			
4	 		

# **REVENUE & FORMS OF MARKET**



#### REVENUE

Revenue – Amount Received from sale of Goods & Service

#### Cost + Profit = Revenue

-	
Total	Sum Total of Revenue from Sale of All Units
Revenue	TRn = ∑MR n OR PxQ
Marginal	Additional Revenue from sale of One More Unit
Revenue	MRn = TRn - TR(n-1) OR ATR/AN
Average	Revenue per unit of good sold
Revenue	ARn = TR/n

Perfect Competition	Monopoly/Monopolistic
Any Quantity can be sold at Same Price	Price needs to be Decreased to Sell More Quantity

	$MR = AR \times \frac{E - 1}{E}$
E < 1	MR will be Negative
E=1	MR = 0
E>1	MR will be Positive

- · P = AR, Always
- . Demand Curve = Price Line = AR Curve

#### FORMS OF MARKET

#### **Perfect Competition**

Large No. Of Buyers & Sellers	One Buyer or Seller cannot influence market demand or supply
Homogenous Product	Similar or Identical Products
Free Entry & Exit	No Barrier
Perfect Knowledge	Buyer Know Price, Quality , Quantity
Transportation	No Extra Cost, Adequate Facility
Uniform Market Price	Price Taker, All Sellers sale at
Indiffernce / Lack of Preference	Buyers- Indifferent Seller – Indifferent
Mobility of Factors	Perfect Mobility

Pure/Free Competition - SHE

#### Monopoly

Single Seller	Alone to Sell , One Seller Only
Firm = Industry	One Seller Constitute Entire Industry
No Close Substitute	Cross Elasticity is Zero or Very Small
Price Maker	Determine Output & Price
Restricted Entry	Legal/Financial/Natural Barrier

Pure Monopoly is never found in practice, with the exception of public utilities like Railways, Water and Electricity, etc.

#### Price Discrimination (CQL)

- Charging Different Price from Different Customers
   Objectives
  - a) To earn Maximum Profit
- b) To Dispose of Surplus stock
- c) To enjoy Economies of Scale
- d) To capture the foreign market
- e) To secure equity thorough pricing.
- Pre-Conditions for Price Discrimination
- a) Seller's Control
- b) Market Segmentation
- c) Differing Elasticity e>1,e<1
- d) No Scope for Resale

#### **Monopolistic Competition**

Many Seller	Many Seller with Small Share
Product Differentiation	Product of Every seller has different Features
Control Over Price	Due to Different Features, Can Charge Different Prices
Non-Price Competition	Feature War, Not Price War
Free Entry & Exit	No Barrier
Brand Loyalty	Each Seller tries to develop the Brand Loyalty for his product
Advertising Cost	Very Important due to product differentiation

#### Oligopoly

Few Seller	3-15 Seller
Interdependence	Change in Price, Output, Product by One Firm will impact rival firm who will retaliate by changing theirs.
	Firm must take consideration of reactions of competitors when taking decisions
Non-Price- Competition	Feature War, Not Price War
Advertising Cost	Very Important due to product differentiation
Group Behavior	High Probability to form Cartel

#### Kinked Shaped Demand Curve

Types of Oligopoly
--------------------

1. Pure/Perfect	
2. Impure/Imperfect	
3. Open	
4. Close	
5. Collusive -	
6. Non-Collusive -	
7. Partial -	
8. Full	
9. Syndicated-	
10 Organised -	

## Miscellaneous Topics - For General Reading Only Reasons for Monopoly

- Strategic Control over scarce resources, inputs or technology by a Single Firm
- Developing or acquiring control over a unique product that is difficult or costly for other Companies to copy.
- Patents and Copyrights given by Government to protect Intellectual Property Rights and to encourage innovation,
- 4. Governments granting the exclusive rights
- 5. Substantial Goodwill enjoyed by a Firm
- Natural Monopoly due to very large economies of scale
- 7. Stringent Legal and Regulatory Requirements
- 8. Very high initial start-up costs
- Use of Anti-Competitive Practices or Predatory Tactics, (Like Limit Pricing or Predatory Pricing)

#### Negatives of Monopoly

	<b>。在1997年,1997年,1997年,1997年</b>
Higher Prices	Loss of Consumer Surplus
Consumer Sovereignty Falls	Lack of Innovation,
Lower Payment for FOP	Lack in Efficiency
Influence political process	Misuse of Resources
Scope for X-Inefficiency	Higher Costs of Output

# PRODUCER EQUILIBRIUM & CONSUMER EQUILIBRIUM



#### PRODUCER EQUILIBRIUM

## Producer Equilibrium / Profit Maximisation Conditions

- 1. MR = MC

AR > AC	
AR = AC	
AR < AC	

Short Period	Long Period
No Entry & No Exit	Entry & Exit Possible
Possible Profit	Possible Profit

Long Period Exception -

Supply Curve of Perfect Competition Firm -

Long Run = Equilibrium = Perfect Competition SMC = SAC = LAC = LMC = LMR = LAR = PRICE

Shut Down Point

Condition for Production – TR>TC or AR>AC
When to Shut Down Production - TR<TC or AR<AC
If Firm Shut down production, Its Loss will be equal
to Fixed Cost Only

# CONSUMER EQUILIBRIUM

Utility - \_\_\_\_\_

#### Types of Utility

Cardinal Utility	Ordinal Utility
	7.4

#### CARDINAL UTILITY ANALYSIS

Marginal Utility -

Total Utility -

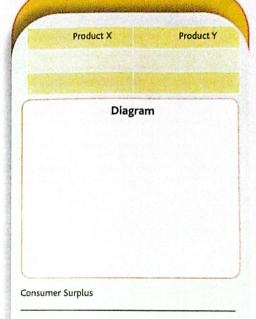
Law of Diminishing Marginal Utility -

#### Relation Between MU & TU

- 1.
- 2.
- 3,

How Does A Consumer Decide How Much Quantity He Want To Purchase

Px	
MUx	
MUm	



CARDINAL	THREE MUSKETEERS	ORDINAL
MUx = MUy Px Py		MRS = Px
MUx > MUy Px Py		MRS > Px
MUx < MUy Px Py		MRS < Px

#### ORDINAL APPROACH ANALYSIS

Consumer Equilibrium	

Indifference Curve	Isoquant Curve

# operties of IC

3. 4.

> 5. MRS =

#### MRS Variation

MRS Variation

MRS ↓

MRS Constant

MRS ↑

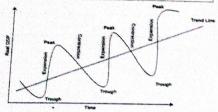
Budget Line	Iso-Cost Curve

Diagram

#### **BUSINESS CYCLE**

#### Business Cycle - Meaning

Fluctuations in	Aggregate Economic Activity
Expansion & Contraction	Of Business Activity
Trade Cycle	Good Trade & Bad Trade



#### Features of Business Cycle

- 1. Phases do not display smoothness & regularity.
- 2. BC occur periodically, But do not exhibit same regularity.
- 3. The duration of Business Cycles vary.
- 4. The intensity of fluctuations also varies.
- 5. Difficult to predict the Turning Points of BC.
- 6. BC in general originate in free market economies.
- 7. Although all sectors are adversely affected by Business Cycles

Capital Goods Industries	More Affected
Durable Consumer Goods	Disproportinately
Industrial Goods	
Agricultural Sector	Less Affected

- 8. Business Cycles are exceedingly complex phenomena
- 9. Repercussions of Business Cycles get simultaneously felt on nearly all economic variables, viz. O/Y/I/E
- 10. BC are contagious & are international in character.
- 11, Business Cycles may occur due to External Causes (known as Exogenous Factors), or Internal Causes (called Endogenous Factors), or a combination of both.
- 12. BC have serious consequences on the well-being of the society.

#### Phases of Business Cycle

#### 1. Expansion

- a) Increase in National Output, Employment, Aggregate Demand, Capital and Consumer Expenditure, Sales, Profits, rising Stock Prices and Bank Credit.
- b) Continues till there is full employment of resources
- c) Involuntary Unemployment is almost zero. Only Frictional Unemployment (i.e. due to change of jobs, or suspended work due to strikes or due to imperfect mobility of labour) or Structural Unemployment (means unemployment caused due to structural changes in the economy), exists.
- d) Prices and Costs tend to rise faster. Net Investment also occurs at a faster pace.
- e) There is increasing prosperity and standard of living
- f) Growth Rate eventually slows down & reaches at its peak.

#### 2. Peak

- a) Highest point of the Business Cycle.
- b) Now, Inputs are difficult to obtain, so Input Prices increase.
- c) Output Prices rise rapidly, leading to increased cost of living. This causes greater strain on Fixed Income earners.
- d) Consumers begin to review their Consumption Expenditure on housing, durable goods, etc.
- e) Actual demand thus stagnates. This marks the end of Expansion Stage.

#### 3. Contraction

- a) Once Peak is reached, increase in demand is halted and starts decreasing in certain sectors.
- b) Hence, there is a mismatch between Demand and Supply,
- c) Producers, holds back investment, cancellation of orders for equipment and all types of inputs including Labour.
- d) Chain of reactions in the Input Markets & the Producers of Capital Goods & Raw Materials in turn respond by cancelling their orders. This is the turning point.
- e) Decrease in Input Demand pulls Input Prices down Incomes of Wage and Interest Earners decline resulting in decreased demand.
- f) Producers lower their prices to dispose off their inventories
- g) Business Firms become Pessimistic
- h) Decrease in Bank Credit, Investor Confidence, Stock Prices. Employment, Wage Rates, Investments, Production. Demand

#### 4. Trough. Depression

- a) Lowest turning point i.e. 'Trough'.
- b) When the process of recession is complete, the severe contraction in the economic activities Pushes the economy into the phase of Depression.
- c) Depression is the severe form of recession & is characterized by the extremely sluggish economic activities.
- d) Growth Rate becomes negative and the level of National Income and Expenditure declines rapidly.
- e) Demand decreases, Prices are at their lowest, Forcing Firms to shut down. There is more bankruptcy.
- f) Capital & Consumer Durable Goods Industry, suffer from excess capacity.
- g) Unemployment increases, Very little Disposable Income.
- h) There is fall in the Interest Rate, and people's demand for holding liquid money (i.e. in Cash)
- i) Despite lower interest rates, the demand for credit declines due to Pessimism of Business.
- j) At the depth of depression, all the economic activities touch the bottom and the phase of Trough is reached.

Indicators

#### 1. Leading Indicators

- a) It is a measurable economic factor that changes before the economy starts to follow a particular pattern or
- b) It represents Variables that change before the Real Output changes, i.e. prior to large economic adjustments.

#### c) Examples:

- · Changes in Stock Prices, Profit Margins and Profits, Indices like Housing, Interest Rates and Prices. etc. are generally seen as precursors of upturns or downturns.
- · Value of New Orders for Consumer Goods, Capital Goods, Building Permits for Private Houses, fraction of Companies reporting slower deliveries, Index of Consumer Confidence and Money Growth Rate are also used for tracking and forecasting the changes in Rusiness Cycles.

#### 2. Lagging Indicators

- a) It reflects the economy's historical performance and changes in these indicators are observable only after an economic trend or pattern has already occurred.
- b) It represents variables that change after the Real Output changes, means measures that change after an economy has entered a period of fluctuation c) If Leading Indicators signal the onset of Business
- Cycles, Lagging Indicators confirm these trends.
- d) Examples: Unemployment, Corporate Profits, Labour Cost per unit of Output, Interest Rates, Consumer Price Index, Commercial Lending Activity, etc.

#### 3. Coincident Indicators

- a) It coincides or occurs simultaneously with the business-cycle movements.
- b) It gives information about the rate of change of the expansion or contraction of an economy more or less at the same point of time when it happens.
- c) It coincides closely with changes in the cycle of economic activity, & describes the current state of the BC.
- d) Examples: Gross Domestic Product, Industrial Production, Inflation, Personal Income, Retail Sales and Financial Market Trends like Stock Market Prices, etc

#### **External Causes of Business Cycle**

Population	Natural Factors	
Technology	Wars	
Post War Construction	International Trade	

#### Internal Causes of Business Cycle

Price Fluctuations	Innovations
Fluctuations in Effective Demand	Fluctuations in Investment
Fluctuations in Govt Spend	Macro-Economic Policy
Money Supply	Psychological Factors

#### Role of Business Cycle in Business Decision Making

Demand Impact	sion Production Aspects
Expansion Decision	Production Aspects
Cyclical Business	Market Entry/Product Launch



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