Central Tendency & Dispersion

CHEAT SHEET
(धर्रा)

Central Tendency

meaning of central tendency

→ central value of all observation
→ Representative of entire series

Characters of Good central Tendency

→ Easy To calculate → mean & mode
→ Easy To understand → mean
→ Based on all observation → mean, GM, HM
→ Rigidly Defined → mean
→ least affected by extreme values → median
→ Have mathematical properties → mean, GM & HM

Note : → AM is Best central Tendency
→ median is best for open ended series

Arithmetic mean ($\bar{x}$)

$$AM = \frac{\text{Sum of all observation}}{\text{Total no. of observations}}$$

methods

Direct method

$$\bar{x} = \frac{\Sigma f_i x_i}{N}$$

(where $N = \Sigma f_i$)

Shortcut Method

→ Assume mean method

If $x_i - A = d_i$

Then $\bar{x} = A + \frac{\Sigma f_i d_i}{N}$

→ Step Deviation method

If $\frac{x_i - A}{h} = u_i$

Then $\bar{x} = A + \frac{\Sigma f_i u_i}{N} \times h$

Properties

1> If all observations are same (let K)
then mean also K
mean of 5, 5, 5, 5, 5 will be 5

2> Sum of Deviations from their arithmetic mean is always zero
i.e. $\Sigma(x_i - \bar{x}) = 0$

3> Sum of the squares of Deviation is minimum when Deviations are taken from their arithmetic mean
i.e. $\Sigma(x_i - K)^2$ is minimum
when $K = \bar{x}$

4> Combined mean
$$\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

5> Arithmetic mean changes with change of origin & change of scale
i.e. If $y_i = ax_i + b$
Then $\bar{y} = a\bar{x} + b$

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3+2^3+3^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

A.M. of first 'n' natural numbers $= \dfrac{n+1}{2}$

Median

A number which divides entire Distribution in two parts is known as median. It represents half (50%) of Total numbers.

→ It is not based on all observations so it can be used for open ended series.

$$\boxed{\text{methods}}$$

For individual & Descrete series

median $= \left(\frac{n+1}{2}\right)^{th}$ term

Four continous series

→ make cf column

→ Locate $\frac{N}{2}$ in cf

→ select median class

→ use Formula

$$\boxed{\text{median} = l + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h}$$

→ median changes with change of origin & change in scale

i.e if $y = a + bx$

Then med. of $y = a + b(\text{med. of } x)$

→ Sum of absolute deviation is minimum when deviations are taken from the median

ie $\Sigma |x_i - K|$ is minimum when $K =$ median

Partition values (Fractiles)

Quartiles (Q_1, Q_2 & Q_3)

Divides entire series in 4 parts)

$Q_1 = 25\%$ $\left(\frac{N}{4}\right)$ | $Q_2 = 50\%$ $\left(\frac{N}{2}\right)$ | $Q_3 = 75\%$ $\left(\frac{3N}{4}\right)$

For Individual & Descrete series

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ term}$$

$$Q_2 = \text{median} = \left(\frac{n+1}{2}\right)^{th}$$

$$Q_3 = \left[3\left(\frac{n+1}{4}\right)\right]^{th} \text{ term}$$

In continous series

for Q_1

→ Locate $\frac{N}{4}$ in cf

→ select 'Q_1' class

→ $Q_1 = l + \left[\frac{\frac{N}{4} - cf}{f}\right] \times h$

for Q_3

→ Locate $\frac{3N}{4}$ in cf

→ select Q_3 class

→ $Q_3 = l + \left[\frac{\frac{3N}{4} - cf}{f}\right] \times h$

Deciles $(D_1, D_2, \ldots D_9)$

Divides entire series in 10 Parts

D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9
↓	↓	↓	↓	↓	↓	↓	↓	↓
10%	20%	30%	40%	50%	60%	70%	80%	90%

50% ⇓ 5^{th} Decile $= D_5 =$ median

For individual & Descrete series

$$D_1 = \left(\frac{n+1}{10}\right)^{th} \text{term}$$

$$D_2 = \left[2\left(\frac{n+1}{10}\right)\right]^{th}$$

$$D_K = \left[K\left(\frac{n+1}{10}\right)\right]^{th} \text{term}$$

For continous series

For D_1

→ locate $\frac{N}{10}$ in cf

→ select D_1 class

→ $D_1 = l + \left[\frac{\frac{N}{10} - cf}{f}\right] \times h$

For D_3

→ Locate $\frac{3N}{10}$ in cf

→ select D_3 class

→ $D_3 = l + \left[\frac{\frac{3N}{10} - cf}{f}\right] \times h$

$\boxed{\text{Percentiles}}$ $(P_1, P_2, \ldots\ldots P_{99})$

$\Downarrow$

Divides entire series in 100 parts

P_1 → 1% P_2 → 2%   .... P_{49} → 49% P_{50} ⇒ median → 50% P_{51} → 51%   .... P_{99} → 99%

For Individual & Descrete series

$$P_1 = \left(\frac{n+1}{100}\right)^{th} \text{term}$$

$$P_6 = \left[6\left(\frac{n+1}{100}\right)\right]^{th} \text{term}$$

$$P_K = \left[K\left(\frac{n+1}{100}\right)\right]^{th}$$

For continous series

For P_1

→ Locate $\frac{N}{100}$ in cf

→ select P_1 class

→ $P_1 = l + \left[\dfrac{\frac{N}{100} - cf}{f}\right] \times h$

For P_7

→ Locate $\frac{7N}{100}$ in cf

→ select P_7 class

→ $P_7 = l + \left[\dfrac{\frac{7N}{100} - cf}{f}\right] \times h$

Mode

⇓

An observation with highest frequency

Individual series

eg marks: 2, 1, 3, 1, 4, 3, 2, 3, 5, 2, 3, 1, 3, 1, 3, 5, 3, 4, 3

mode = 3

Descrete series

eg

x_i	f_i
2	6
3	12
4	3
5	5

→ mode = 3

For continous series

→ Check the class interval with highest frequency (It is modal class)

→ use formula

$$\text{mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_2 - f_0} \right\} \times h$$

Note: mode also changes with change of origin or change in scale

ie. $y = a + bx$

Then mode of $y = a + b(\text{mode of } x)$

Geometric mean

n^{th} Root of the Product of n observations

used for
→ Average Dep?
→ Average of %

For individual series

$$GM = (x_1 \times x_2 \times \cdots\cdots x_n)^{1/n}$$

For Descrete & continous series

$$GM = \left[(x_1)^{f_1} \times (x_2)^{f_2} \times \cdots \times x_n^{f_n}\right]^{\frac{1}{N}}$$

Properties

1> If all items are same (Let K)
then $GM = K$

2> $$\log(GM) = \frac{\Sigma \log x_i}{N}$$

$$GM = AL\left[\frac{\Sigma \log x_i}{N}\right]$$

3) $GM(xy) = GM(x) \times GM(y)$

4) $GM\left(\frac{x}{y}\right) = \frac{GM(x)}{GM(y)}$

5) Combined Geometric mean

$$G = \left[(G_1)^{N_1} \times (G_2)^{N_2}\right]^{\frac{1}{N_1+N_2}}$$

Harmonic mean

used for
→ Avg speed
→ Avg of Rates

Reciprocal of Average of Reciprocal of all 'N' observations

→ find reciprocal of all items
→ find Average of these reciprocals
→ find Reciprocal of Average

For individual series

$$Hm = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}} = \frac{N}{\Sigma\left(\frac{1}{x_i}\right)}$$

for Descrete & continous series

$$Hm = \frac{N}{\Sigma\left(\frac{f_i}{x_i}\right)}$$

Properties

1> If all items are same (let K)
then $Hm = K$

2> Combined $Hm = \dfrac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$

Relation b/w AM GM & HM

$$AM \geq GM \geq HM$$

If all items are different

$$AM > GM > HM$$

For any two items a & b

$$AM \times HM = GM^2$$

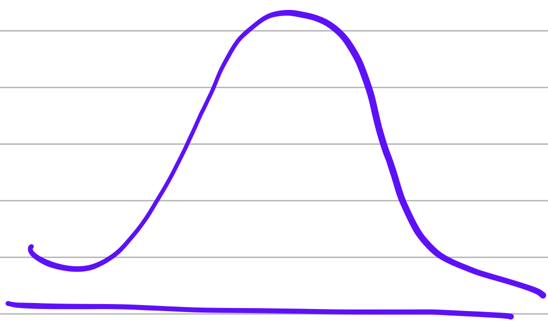
weighted $AM = \dfrac{\Sigma w_i x_i}{\Sigma w_i}$

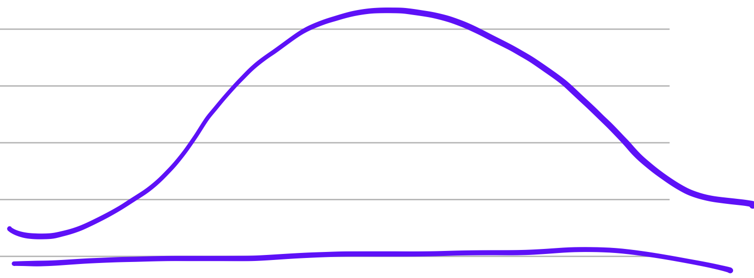
weighted $HM = \dfrac{\Sigma w_i}{\Sigma\left(\dfrac{w_i}{x_i}\right)}$

weighted $GM = AL\left[\dfrac{\Sigma w_i \log x_i}{\Sigma w_i}\right]$

Dispersion

Statistical Technique to find Degree consistency (or variability) in all the observation.





- → low Dispersion
- → concentrated Data
- → less variability
- → more consistency

- → High Dispersion
- → scattered Data
- → more variability
- → less consistency

Measures

Absolute

- → Range
- → Q.D
- → M.D
- → S.D.

Relative

- → coefficient of Range
- → coefficient of Q.D.
- → coefficient of MD
- → coefficient of variance

Range

Difference b/w Largest & smallest observation

$$\boxed{R_x = L - S}$$

$$\boxed{\text{Coefficient of Range} = \frac{L-S}{L+S} \times 100}$$

→ Range Does not change with origin

→ Range changes with scale

$$\boxed{\begin{array}{ll} \text{If} & y_i = a + b x \\ \text{then} & R_y = |b| \times R_x \end{array}}$$

Quartile Deviations

$\Rightarrow$ Interquartile Range $= Q_3 - Q_1$

$\Rightarrow$ Semi Quartile Range (Quartile Deviation) $= \dfrac{Q_3 - Q_1}{2}$

$\Rightarrow$ coefficient of Q.D. $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

or

$\dfrac{QD}{\text{median}} \times 100$

$\Rightarrow$ only for symmetrical Distribution

$\rightarrow$ QD does not change with origin

$\rightarrow$ QD changes with scale

$$\boxed{\begin{gathered} y_i = a + bx \\ QD \text{ of } y = |b| \times QD \text{ of } x \end{gathered}}$$

Mean Deviation (MD)

Average of Absolute Deviations taken from mean, median or mode

MD

About mean | About median

$$MD_{\bar{x}} = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$MD_m = \frac{\sum f_i |x_i - m|}{N}$$

coefficient of M.D. $= \frac{MD}{mean} \times 100$

or

$\frac{MD}{median} \times 100$

=> MD does not change with origin

=> MD changes with scale

$$\boxed{\begin{array}{l} \text{If } y = a + bx \\ \text{then } MD_y = |b| \times MD_x \end{array}}$$

Standard Deviation & variance

$$\text{variance} = \sigma^2$$

$$S.D. = \sigma$$

$$S.D. = \sqrt{\text{variance}}$$

$$\text{variance} = (S.D.)^2$$

$$S.D.\ (\sigma) = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$$

$$S.D. = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2}$$

$$S.D. = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

where $d_i = x_i - A$

$$S.D. = \sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2} \times h$$

where $\frac{x_i - A}{h} = u_i$

⇒ SD Does not change with origin

⇒ S.D. change with change of scale

$$\boxed{\begin{array}{l} \text{If } \quad y_i = a + bx \\ \text{S.D of } y_i = |b| \times \text{S.D of } x \\ \text{variance of } y_i = b^2 \times \text{variance of } x_i \end{array}}$$

$\boxed{\text{coefficient of variance (CV)} = \frac{\sigma}{\bar{X}} \times 100}$

⇓

will be used for consistency & variability

Higher CV ⇒ Higher variablity

lesser CV ⇒ Higher consistency

S.D. of first 'n' natural number $= \sqrt{\dfrac{n^2-1}{12}}$

S.D of first 'n' even natural no. $= \sqrt{\dfrac{n^2-1}{3}}$

S.D of first 'n' odd natural no. $= \sqrt{\dfrac{n^2-1}{3}}$

S.D. of two numbers a & b $= \dfrac{|a-b|}{2}$

Q.D : MD : SD = 10 : 12 : 15

$\dfrac{QD}{MD} = \dfrac{10}{12}$ | $\dfrac{MD}{SD} = \dfrac{12}{15}$ | $\dfrac{QD}{SD} = \dfrac{10}{15}$

Combined S.D.

$$= \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

where $d_1 = \overline{X_{12}} - \overline{X_1}$

$d_2 = \overline{X_{12}} - \overline{X_2}$

& $\overline{X_{12}} = \dfrac{N_1\overline{X_1} + N_2\overline{X_2}}{N_1 + N_2}$