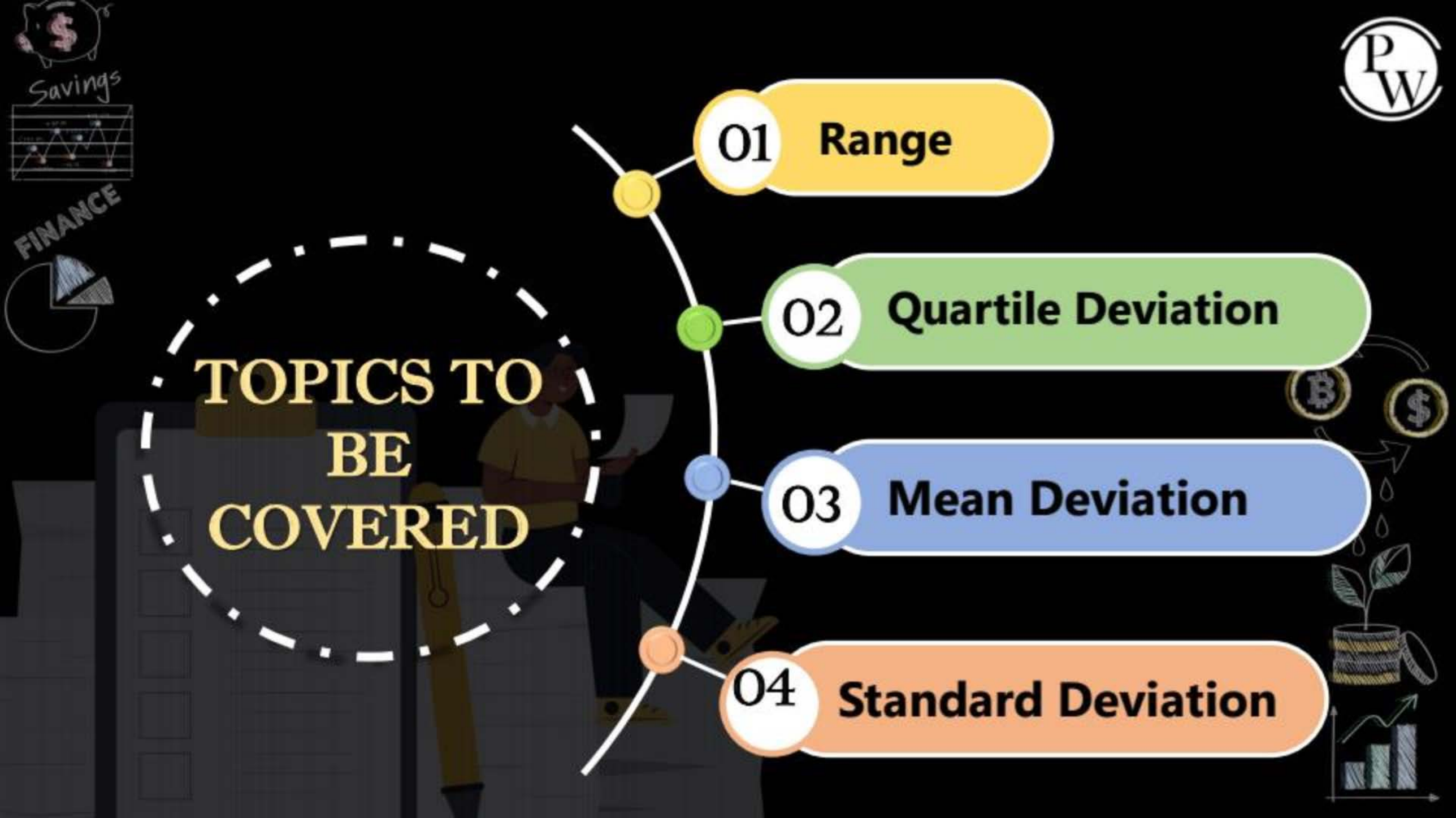
CHANAKYA 2.0

For CA Foundation

Dispersion

QUANTITATIVE APTITUDE

By Anurag Chauhan


Savings
FINANCE
TOPICS TO BE COVERED
01 Range
02 Quartile Deviation
03 Mean Deviation
04 Standard Deviation

Dispersion

"The Degree Of the scatterness or spread or variation of the variable about a central value is called dispersion"

Marks
9
8
10
8
7
9
8
6
5

Marks
10
10
9
7
8
8
3
4
12

Marks Of Section(A) ⇒ Anurag

82	2
78	−2
80	0
88	8
84	4
72	−8
77	−3
79	−1

$\bar{X} = \frac{\Sigma x_i}{N} = \frac{640}{8} = 80$

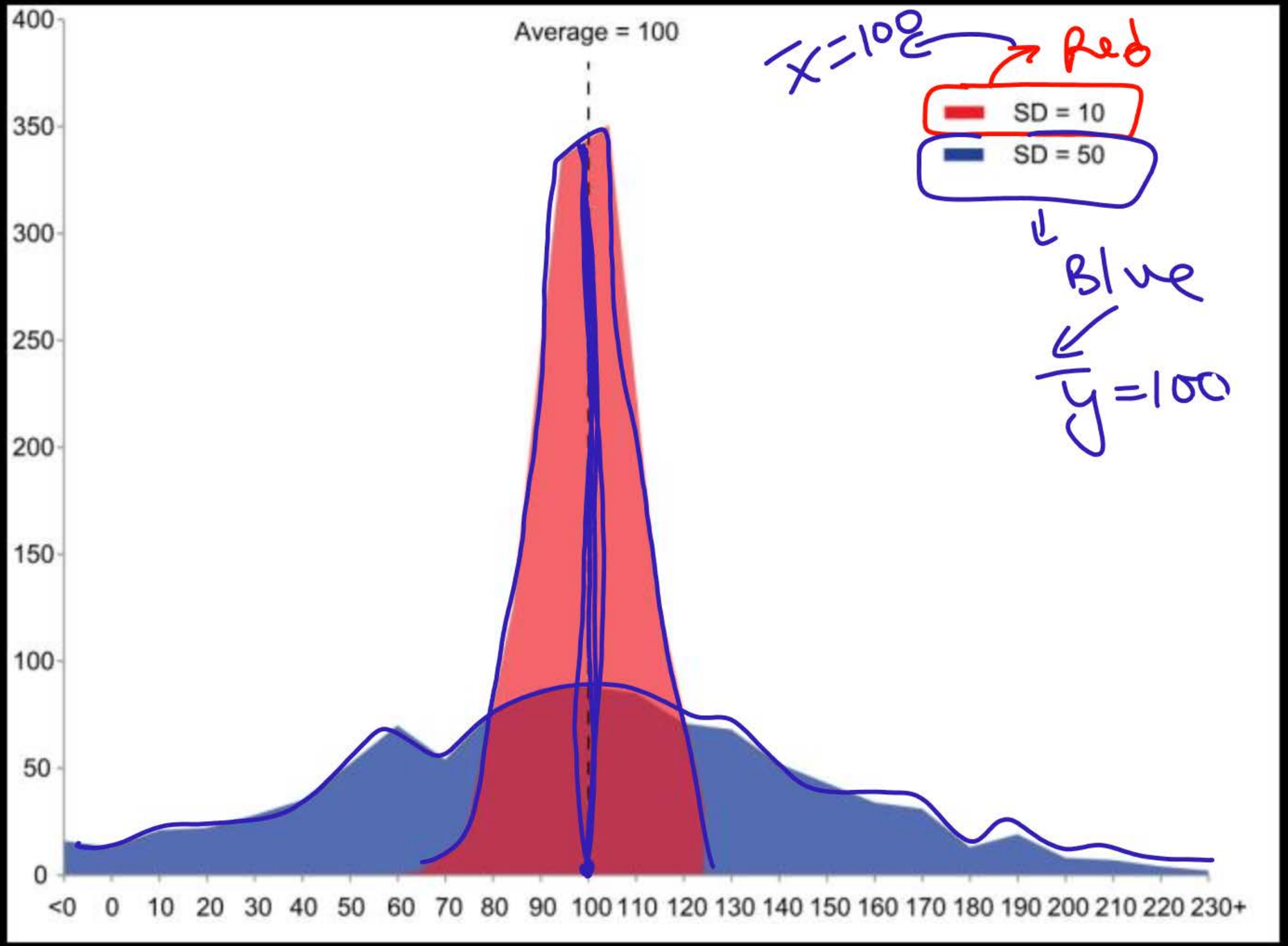
80 → central value

Marks Of Section(B) ⇒ Tone

82	2
100	20
92	12
44	−36
49	−31
82	2
100	20
91	11

$\bar{X} = \frac{640}{8} = 80$

80 → C.T Central value.


Average = 100
SD = 10
SD = 50
X=100
Red
Blue
y=100
400
350
300
250
200
150
100
50
0
<0 0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230+

Measures of Dispersion

#Absolute Measure of Dispersion

They are measured and expressed in the units of data like cm , kg , KM

- Range
- Quartile Deviation
- mean Deviation
- Standard Deviation

#Relative Measure of Dispersion

Used for comparing two variables with different units or

- coefficient of Range
- Coff of Q.D.
- Coff of MD
- Coff of Variance

A

B

$\frac{19}{20} \times 100$ → Absolute marks = 95%

$\frac{45}{50} \times 100$ → Absolute marks = 90%

Relative marks

Relative marks

Distance
3 km
2 km
1 km
4 km

Which of the following is a relative measure of dispersion?

(a) Range → Ab

(b) Mean deviation → Ab

(c) Standard deviation → Ab.

(d) Coefficient of quartile deviation → Relative

2021 - January

Range

The range is the difference between the largest and the smallest values in the distribution.

Largest = L

Smallest = S

$$\text{Range} = L - S$$

$$\text{Coefficient of Range} = \frac{L-S}{L+S} \times 100$$

Marks: 99, 92, 83, 85, 87, 75

$$\text{Range} = 99 - 75 = 24$$

$$\text{Coeff of Range} = \frac{99-75}{99+75} \times 100$$

$$= \frac{24}{174} \times 100 = 13.79\%$$

CA WALLAH

9

marks
9
10 → 2
3
5
7
2
1 → 5

Range = 2 − 5 = 10 − 1 = 9 → Absolute Value

Coeff of Range = $\frac{10-1}{10+1} \times 100$

$= \frac{9}{11} \times 100 = 81.81\%$ → Relative value

marks
98
97
85
84
83
75 →
99

Range = 99 − 75 = 24 → Absolute Value

Coeff of Range = $\frac{99-75}{99+75} \times 100$

$= 13.79\%$

Calculate the coefficient of range

Marks : 43 48 65 57 31 60 37 48 78 59

S L

H

A. 42.68

B. 43.11

C. 45.10

D. 47

$$\text{Coeff of Rng} = \frac{78-31}{78+31} \times 100$$

$$= \frac{47}{109} \times 100$$

$$= 43.11\%$$

Calculate range (Discrete)

Diameter (mm): (x_i)	130	135	140	145	143	148	149	150
No. of screws: (f_i)	3	4	6	6	3	5	2	1

A. 20

B. 30

C. -2

D. None

$$\text{Range} = L - S$$
$$= 150 - 130$$
$$= 20$$

$$\text{Coff of Rng} = \frac{150-130}{150+130} \times 100$$

Calculate coefficient of range (Exclusive) (Continous series)

Marks:	10-20	20-30	30-40	40-50	50-60
No. of Students	16	18	20	17	9

Lower limit smallest C.I
$S = 10$

Upper limit of largest C.I
$L = 60$

A. 50

B. 71.42

C. 7

D. None

$$\text{Coeff of Rng} = \frac{60-10}{60+10} \times 100$$

$$= 71.42\%$$

Calculate range — Inclusive continuous series

Marks:	10-20	21-31	32-42	43-53
No. of Students	16	18	20	17

9.5-20.5 | 20.5-31.5 | 31.5-42.5 | 42.5-53.5

A. 43

B. 44

C. 1

D. None

$L = 53.5$

$S = 9.5$

$Range = 53.5 - 9.5$

$= 44$

Properties

#1> If all the observations are same then Range = 0

Eg Marks: 9, 9, 9, 9, 9

Range = 9 − 9 = 0

#2] Change of origin

If all the observations are increased or Decreased by a fix value then Range Remains same

If x_i : 1, 2, 3, 4, 5

Range = 5 − 1

= 4

If $y_i = x_i + 10$

y_i : 11, 12, 13, 14, 15

Range = 15 − 11

= 4

#3] Change of scale

If each observation multiplied by 'K'
then New Range = $|K| \times$ [old Range]

Eg

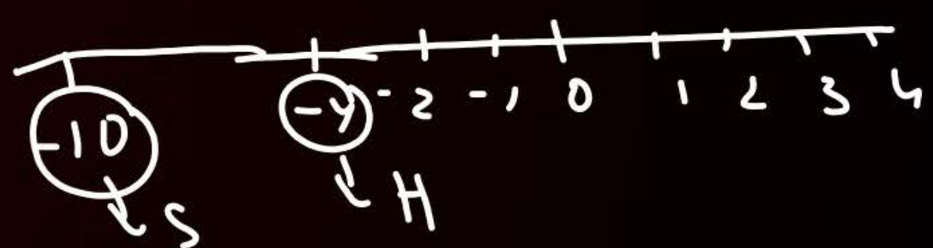
x_i
2
3
4
5

Range = 5 − 2
= 3

$y_i = 4x_i$
8
12
16
20

Range = 20 − 8
= 12

$z_i = -2x_i$
−4
−6
−8
−10



Range of z = (−4) − (−10)
= −4 + 10 = 6

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Q If Range of x is 10 & $y_i = 2(x_i + 5)$ find Range of y_i.

Sol:

Range of y_i = 2 × Range of x_i

= 2 × 10

= 20

x_i	$y_i = 2(x+5)$
1	12
2	14
11	32

Range of x = 11 − 1 = 10

Range of y = 32 − 12 = 20

Eg

$y_i = \underline{4}x + 5$

If Range of $x = 20$

Range of $y = ?$

Sol: Range of $y = 4 \times$ Range of x

$= 4 \times 20$

$= 80$

Q Range of $x = 50$

$y_i = -10x_i + 30$

Range of $y_i = ?$

Sol: Range of $y_i = |-10| \times$ Range of x

$= 10 \times 50$

$= 500$

If the relation between x and y is given by 2x+3y=10 & range of x is 15, what would be the range of y

A. 10
B. 22.5
C. -10
D. -20/3

$2x+3y=10$ & Range of $x=15$

$3y = 10-2x$

$y = \frac{10-2x}{3}$

$y = \frac{10}{3} - \frac{2}{3}x$

Range of $y = \frac{2}{3} \times$ Range of x

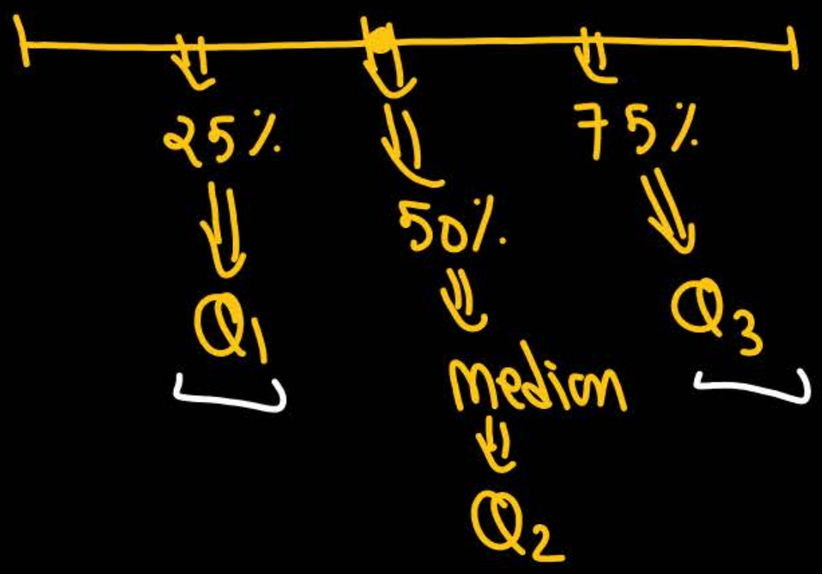
$= \frac{2}{3} \times 15 = 10$

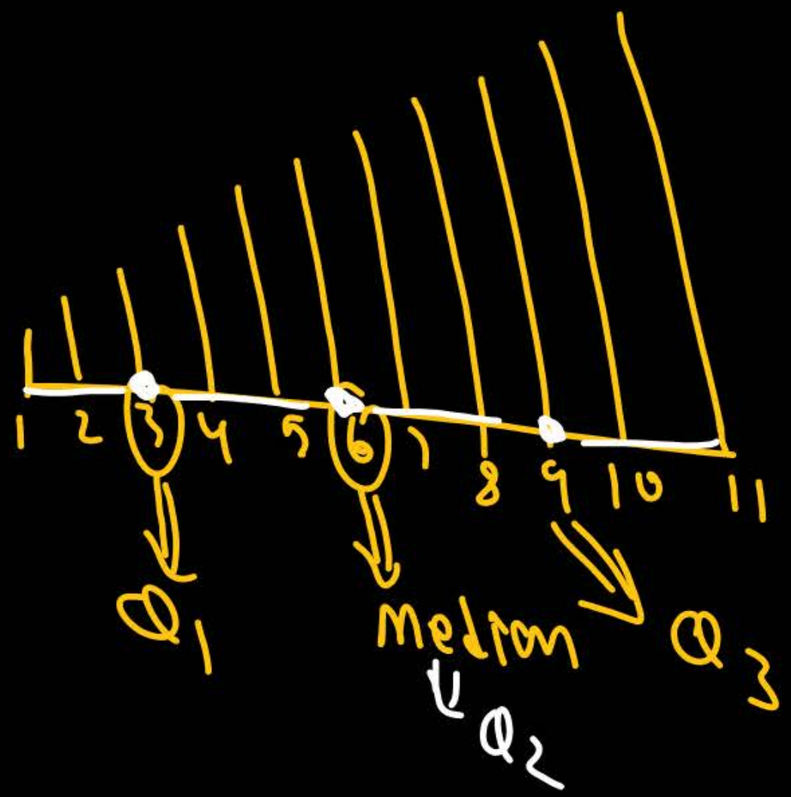
$ax+by=c$

find value of y in terms of x

Quartile Deviations

It is defined as **half** of the **difference** between the **third quartile and the first quartile** in a given data set.





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* Interquartile Range $= Q_3 - Q_1$

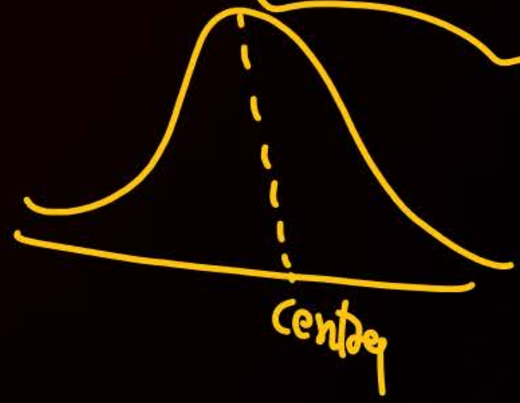
Semi Interquartile Range $= \frac{Q_3 - Q_1}{2}$
or
Quartile Deviation (QD)

Coeff. of Q.D. $= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

If Distribution is symmetrical

$\Downarrow$

Median $= \frac{Q_3 + Q_1}{2}$



Individual series / Descrete series

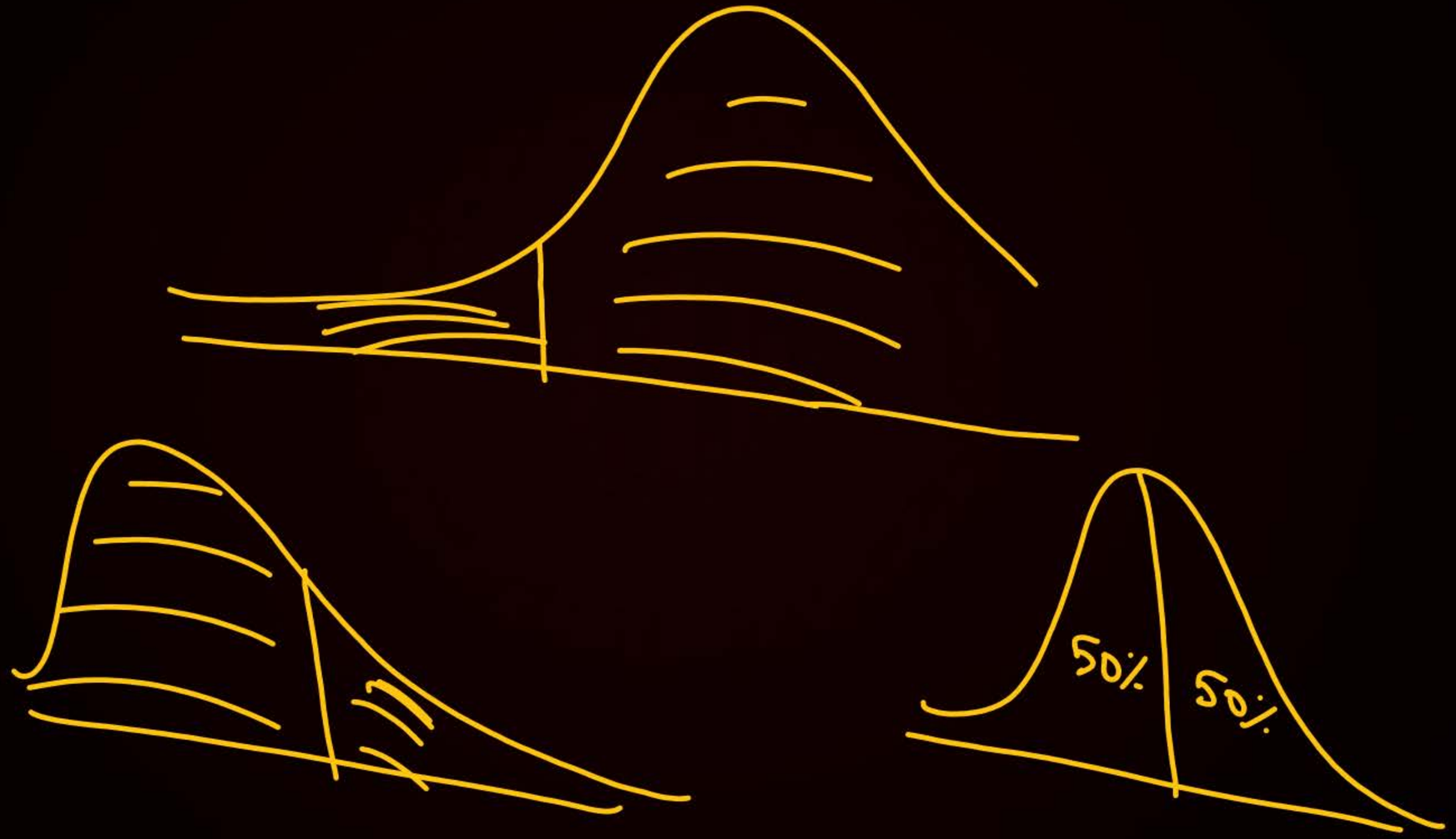
$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ \& } Q_3 = \left[3\left(\frac{n+1}{4}\right)\right]^{th}$$

Continous series

→ locate $\frac{N}{4}$ & $\frac{3N}{4}$ in cf

→ find Q_1 & Q_3 class

→ $Q_1 = l + \left\{\frac{\frac{N}{4} - cf}{f}\right\} \times h$ & $Q_3 = l + \left\{\frac{\frac{3N}{4} - cf}{f}\right\} \times h$



Marks- 35, 42, 48, 50, 55, 56, 60, 65, 75, 82

Find COEFF. OF QD

A. 18.42%

B. 15.32%

C. 19.67%

D. NONE

$N = 10$

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$

$= \left(\frac{10+1}{4}\right)^{th}$

$= (2.75)^{th}$

$= 2^{nd} + 0.75(3^{rd} - 2^{nd})$

$= 42 + 0.75[48 - 42]$

$= 42 + 4.5$

$Q_1 = 46.5$

now $Q_3 = \left[3\left(\frac{N+1}{4}\right)\right]^{th}$

$= 3\left(\frac{10+1}{4}\right)^{th}$

$= (8.25)^{th}$

$= 8^{th} + 0.25(9^{th} - 8^{th})$

$= 65 + 0.25(75 - 65)$

$= 65 + 2.5$

$Q_3 = 67.5$

now,

Coeff of Q.D.

$= \frac{67.5 - 46.5}{67.5 + 46.5} \times 100$

$= \frac{21}{114} \times 100$

$= 18.42\%$

Marks- 35,38,55,69,78,82,97

Find QD

A. 22

B. 25

C. 18

D. NONE

$n = 7$

$Q_1 = \left(\frac{n+1}{4}\right)^{th} = \left(\frac{7+1}{4}\right)^{th}$

$= 2^{nd} = 38$

$Q_3 = \left[3\left(\frac{n+1}{4}\right)\right]^{th} = \left[3\left(\frac{7+1}{4}\right)\right]^{th}$

$= 6^{th} = 82$

$Q.D. = \frac{Q_3 - Q_1}{2}$

$= \frac{82-38}{2}$

$Q.D = \frac{44}{2} = 22$

Coeff $QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

$= \frac{82-38}{82+38} \times 100$

$= \frac{44}{120} \times 100 = 36.67\%$

X:	6	12	18	24	30	36	42	
F:	4	7	9	18	15	10	5	FIND QD

$Q_3 = \left[3\left(\frac{68+1}{4}\right)\right]^{th}$

$= (51.75)^{th}$

$= 51^{th} + 0.75(52^{nd} - 51^{st})$

$= 30 + 0.75(30-30)$

$= 30$

A. 5

B. 6

C. 7

D. NONE

x_i	f_i	C.f.	Class
6	4	4	(1–4)
12	7	11	(5–11)
18	9	20	(12–20)
24	18	38	(21–38)
30	15	53	(39–53)
36	10	63	(54–63)
42	5	68	(64–68)

$N = 68$

$Q_1 = \left(\frac{68+1}{4}\right)^{th}$

$= (17.25)^{th}$

$= 17^{th} + 0.25(18^{th} - 17^{th})$

$= 18 + 0.25(18-18)$

$= 18$

6, 6, 6, 6, 12, 12, 12, 12, 12, 12, 12, 18, 18, 18, 18, 18, 18, 18, 18, 18

NOW

$QD = \frac{Q_3 - Q_1}{2}$

$= \frac{30 - 18}{2}$

$= 6$

Class interval:	8-12	12-16	16-20	20-24	24-28
Frequency:	5	12	20	10	3

Find QD

A. 3.25

B. 6.48

C. 1.75

D. 2.85

CI	fi	cf
8-12	5	5
12-16	12	17
16-20	20	37
20-24	10	47
24-28	3	50
	N=50	

$N = 50$

$\frac{N}{4} = \frac{50}{4} = 12.5$

$\frac{3N}{4} = 3 \times \frac{50}{4} = 37.5$

$Q_1 = l + \left\{\frac{\frac{N}{4} - cf}{f}\right\} \times h$

$= 12 + \left\{\frac{12.5 - 5}{12}\right\} \times 4$

$= 12 + 2.5$

$= 14.5$

$Q_3 = l + \left\{\frac{\frac{3N}{4} - cf}{f}\right\} \times h$

$= 20 + \left\{\frac{37.5 - 37}{10}\right\} \times 4$

$= 20.2$

Now

$QD = \frac{20.2 - 14.5}{2}$

$= 2.85$

If the relation between x and y is given by Y=4X+5

& QD of x is 10, what would be the ~~range of y~~ QD of y

A. 40

B. 45

C. 15

D. None

$y = 4x + 5$

$4x$ change of scale

$+5$ change of origin

Q.D of $y = 4 \times$ Q.D of x

$= 4 \times 10$

$= 40$

If Q.D of $x = 15$

$y_i = -2x + 10$

Q.D of $y = ?$

Sol:

$$\text{Q.D of } y = |-2| \times \text{Q.D of } x$$

$$= 2 \times 15$$

$$= 30$$

Mean Deviation

→ Modulus of Deviation

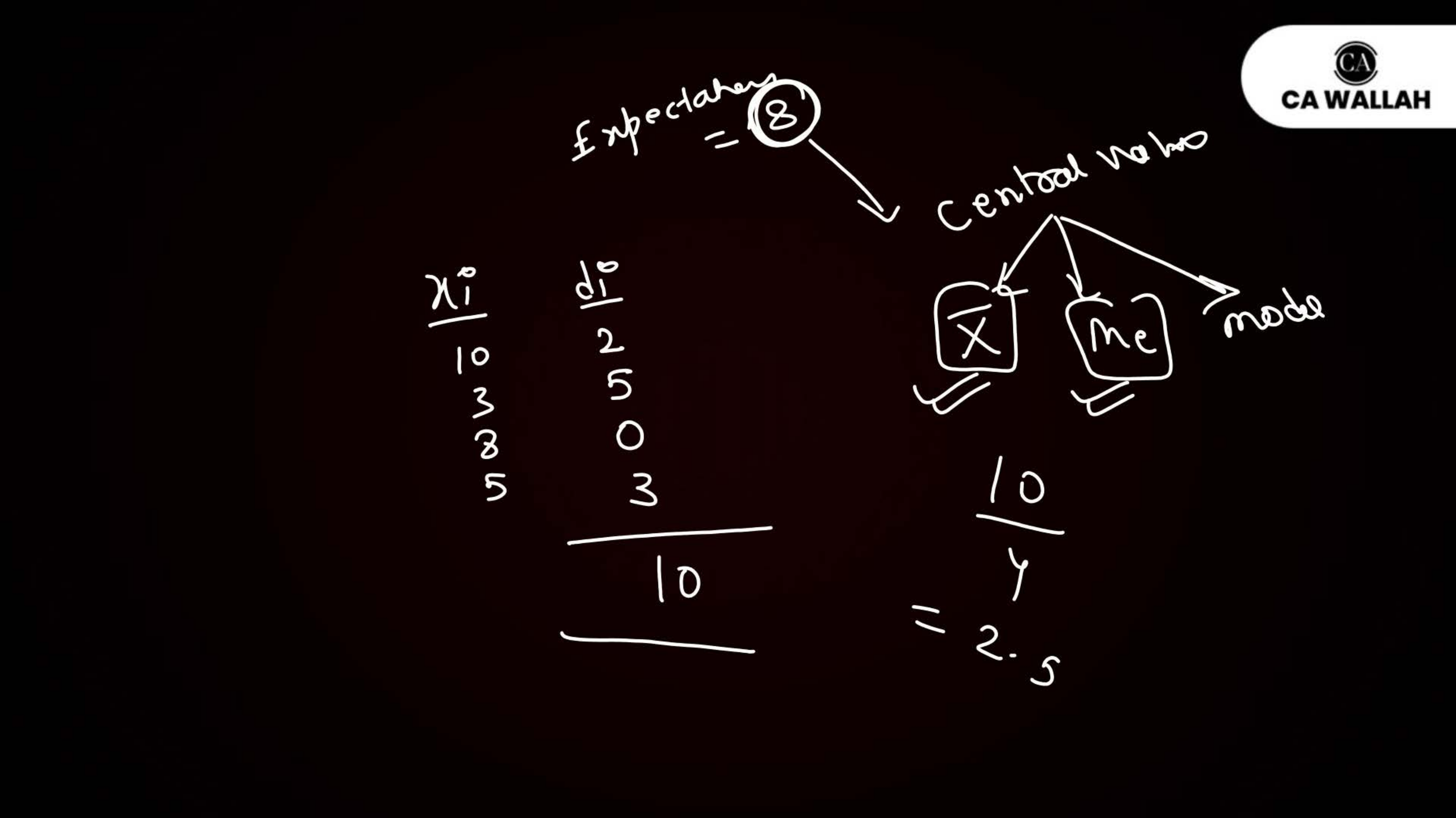
The average of the absolute deviations from a central value is called Mean deviation

$$MD_{\bar{x}} = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$MD_m = \frac{\sum f_i |x_i - m|}{N}$$

$$\text{Coeff of } MD = \frac{MD}{\bar{x}} \times 100$$

$$\text{Coeff of } MD = \frac{MD}{\text{Med.}} \times 100$$


CA WALLAH
Expectation = 8
Central Value
$\bar{X}$
Me
mode
x_i
10
3
8
5
d_i
2
5
0
3
10
10
4
= 2.5

Marks- 31 35 29 63 55 72 37

Find MD about mean

A. 14.86

B. 14.40

C. 15.25

D. 16.24

x_i	$\lvert x_i - \bar{x} \rvert$
31	15
35	11
29	17
63	17
55	9
72	26
37	9
322	104

$$\bar{x} = \frac{\Sigma x_i}{N} = \frac{322}{7} = 46$$

$$MD_{\bar{x}} = \frac{\Sigma \lvert x_i - \bar{x} \rvert}{N}$$

$$= \frac{104}{7}$$

$$= 14.85$$

X:	5	15	25	35	45	55	65
F:	8	12	10	8	3	2	7

Find MD about Mean

A. 15

B. 16

C. 17

D. NONE

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
5	8	40	24	192
15	12	180	14	168
25	10	250	4	40
35	8	280	6	48
45	3	135	16	48
55	2	110	26	52
65	7	455	36	252
	50	1450		800

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{1450}{50}$$

$$\bar{x} = 29$$

$$MD = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{N} = \frac{800}{50} = 16$$

Class interval:	0-10	10-20	20-30	30-40	40-50
Frequency:	5	8	15	16	6

Find MD about mean

A. 9.88

B. 8.45

C. 10.66

D. 9.44

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
5	5	25	22	110
15	8	120	12	96
25	15	375	2	30
35	16	560	8	128
45	6	270	18	108
	50	1350		472

$\bar{x} = \frac{\Sigma f_i x_i}{N}$

$= \frac{1350}{50}$

$\bar{x} = 27$

$MD_{\bar{x}} = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{N}$

$= \frac{472}{50}$

$= 9.44$

Marks- 31 35 29 63 55 72 37

Find MD about median

A. 14.86

B. 14.40

C. 15.25

D. None

x_i	$\lvert x_i - m \rvert$
29	8
31	6
35	2
37	0
55	18
63	26
72	35
	95

$$\text{median} = \left(\frac{7+1}{2}\right)^{th}$$

$$= 4^{th}$$

$$= 37$$

$$MD_m = \frac{\sum \lvert x_i - m \rvert}{N}$$

$$= \frac{95}{7} = 13.57$$

x_i	f_i	cf	$\lvert x_i - m \rvert$	$f_i \lvert x_i - m \rvert$
5	8	8	20	160
15	12	20	10	120
(21-30) 25	10	30	00	00
35	8	38	10	80
45	3	41	20	60
55	2	43	30	60
65	7	50	40	280
	N=50			760

median

$$= \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$$

$$= \frac{\left(\frac{50}{2}\right)^{th} + \left(\frac{50}{2}+1\right)^{th}}{2}$$

$$= \frac{25^{th} + 26^{th}}{2}$$

$$= \frac{25+25}{2} = 25$$

$$MD = \frac{\Sigma f_i \lvert x_i - m \rvert}{N}$$

$$= \frac{760}{50}$$

$$= 15.2$$

Class interval:	0-10	10-20	20-30	30-40	40-50
Frequency:	5	8	15	16	6

Find MD about Median

A. 9.88

B. 8.45

C. 10.66

D. None

CI	fi	cf	xi	$\lvert x_i - m \rvert$	$f_i \lvert x_i - m \rvert$
0-10	5	5	5	23	115
10-20	8	13	15	13	104
20-30	15	28	25	3	45
30-40	16	44	35	7	112
40-50	6	50	45	17	102
	N = 50				478

$\text{Median} = l + \left\{ \frac{\frac{N}{2} - cf}{f} \right\} \times h$

$\frac{N}{2} = 25$

$= 20 + \left\{ \frac{25 - 13}{15} \right\} \times 10$

$= 20 + \frac{120}{15}$

$= 20 + 8$

$= 28$

$MD = \frac{\sum f_i \lvert x_i - m \rvert}{N} = \frac{478}{50} = 9.56$

Which of the following measure of dispersion is based on absolute deviations?

(a) Range
(b) S. D
(c) Mean Deviation
(d) Quartile Deviation

$$\frac{\Sigma |x_i - \bar{x}|}{n}$$

Standard Deviation

Square root of the average of the square of deviations of all observations from arithmetic mean

$$S.D.\ (\sigma) = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{N}}$$

$$= \sqrt{\frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2} = \sqrt{\frac{\Sigma x_i^2}{N} - (\bar{x})^2}$$

$$S.D = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$$

$$\Rightarrow SD = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2}$$

If $x_i - A = d_i$

$$SD = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$u_i = \frac{x_i - A}{h}$$

$$SD = \sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2} \times h$$

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Marks- 1 3 6 7 8

Find Standard Deviation & ~~Coefficient of Variance~~

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	−4	16
3	−2	4
6	1	1
7	2	4
8	3	9
25		34

now $\bar{x} = \frac{\Sigma x_i}{N} = \frac{25}{5} = 5$

$$SD = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{N}}$$

$$= \sqrt{\frac{34}{5}}$$

$$S.D. = \sqrt{6.8} = 2.60$$

Marks- 1 3 6 7 8

Find Standard Deviation & coeff of variance

x_i	x_i^2
1	1
3	9
6	36
7	49
8	64
25	159

$$\bar{x} = \frac{\Sigma x_i}{N} = \frac{25}{5} = 5$$

$$SD = \sqrt{\frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2}$$

$$= \sqrt{\frac{159}{5} - \left(\frac{25}{5}\right)^2}$$

$$= \sqrt{31.8 - 25}$$

$$= \sqrt{6.8}$$

$$= 2.60$$

X:	1	2	3	4	5
F:	4	3	2	6	5

Find SD

$$\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{65}{20} = 3.25$$

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})$	$f_i (x_i - \bar{x})^2$
1	4	4	−2.25	20.25
2	3	6	−1.25	4.6875
3	2	6	−0.25	0.125
4	6	24	0.75	3.375
5	5	25	1.75	15.3125
	20	65		43.75

$$SD = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$$

$$= \sqrt{\frac{43.75}{20}}$$

$$= \sqrt{2.1875}$$

$$= 1.4790$$

∴

X:	1	2	3	4	5
F:	4	3	2	6	5

Find SD

$\bar{X} = \frac{\Sigma f_i x_i}{N} = \frac{65}{20} = 3.25$

$f_i x_i \times x_i$

x_i	f_i	$f_i x_i$	$f_i x_i^2$
1	4	4	4
2	3	6	12
3	2	6	18
4	6	24	96
5	5	25	125
	$N=26$	65	255

$SD = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2}$

$= \sqrt{\frac{255}{20} - \left(\frac{65}{20}\right)^2}$

$= \sqrt{12.75 - 10.5625}$

$= \sqrt{2.1875}$

$= 1.4790$

Class interval:	0-10	10-20	20-30	30-40
Frequency:	3	2	4	1

C.I	f_i	x_i	$f_i x_i$	$f_i x_i^2$
0-10	3	5	15	75
10-20	2	15	30	450
20-30	4	25	100	2500
30-40	1	35	35	1225
	10		180	4250

$$SD = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2}$$

$$= \sqrt{\frac{4250}{10} - \left(\frac{180}{10}\right)^2}$$

$$= \sqrt{425 - 324}$$

$$= \sqrt{101}$$

$$= 10.0498$$

Class interval:	0-10	10-20	20-30	30-40
Frequency:	3	2	4	1

ξ	x_i	f_i	u_i	f_iu_i	$f_iu_i^2$
	5	3	−1	−3	3
A ←	15	2	0	0	0
	25	4	1	4	4
	35	1	2	2	4
		10		3	11

$$SD = \sqrt{\frac{\Sigma f_iu_i^2}{N} - \left(\frac{\Sigma f_iu_i}{N}\right)^2} \times h$$

$$= \sqrt{\frac{11}{10} - \left(\frac{3}{10}\right)^2} \times 10$$

$$= \sqrt{1.01} \times 10$$

$$= 1.004988 \times 10$$

$$= 10.0498$$

X:	1.6	2.6	3.6	4.6	5.6
F:	4	3	2	6	5

Find SD

$$\bar{X} = A + \frac{\Sigma f_i d_i}{N}$$

$$= 3.6 + \frac{5}{20} = 3.85$$

$A = 3.6$

x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
1.6	4	-2	-8	16
2.6	3	-1	-3	03
3.6	2	0	0	00
4.6	6	1	6	06
5.6	5	2	10	20
	20		5	45

$$SD = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{45}{20} - \left(\frac{5}{20}\right)^2}$$

$$= \sqrt{2.25 - 0.0625}$$

$$= \sqrt{2.1875}$$

$$= 1.47902$$

X:	1.6	5.6	9.6	13.6	17.6
F:	4	3	2	6	5

Find SD

x_i	f_i	$d_i = x_i - A$	$u_i = \frac{x_i - 9.6}{4}$	$f_i u_i$	$f_i u_i^2$
1.6	4	−8	−2	−8	16
5.6	3	−4	−1	−3	3
9.6	2	0	0	0	0
13.6	6	4	1	6	6
17.6	5	8	2	10	20
	$N = 20$			5	45

$\Rightarrow A = 9.6$

$$SD = \sqrt{\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2} \times h$$

$$= \sqrt{\frac{45}{20} - \left(\frac{5}{20}\right)^2} \times 4$$

$$= \sqrt{2.25 - 0.0625} \times 4$$

$$= 1.4790 \times 4$$

$$= 5.916$$

CA WALLAH

$$S.D. \ (\sigma) = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{N}}$$

$$\text{Variance} \ (\sigma^2) = \frac{\Sigma (x_i - \bar{x})^2}{N}$$

$$S.D. = \sqrt{\text{Variance}}$$

$$\text{Variance} = (S.D.)^2$$

CA WALLAH

Q S.D = 5

variance = ?

Sol: $\text{variance} = (5)^2$

$= 25$

Q variance = 80

find S.D.

Sol: $S.D = \sqrt{\text{variance}}$

$= \sqrt{80} = 8.9442$

9 $\sigma_x = 4$

If 5 is added to every observation. Then what is new S.D & New variance.

Sol: $\sigma_x = 4$

If all items are increased by 5 S.D. will Remain same

New S.D = 4

New variance = $4^2 = 16$

If $\sigma_x = 5$

If every element is multiplied by 3

find new S.D & variance.

Sol:

$$\text{new SD} = |3| \times \text{old SD}$$

$$= 3 \times 5$$

$$\text{New SD} = 15$$

$$\text{New variance} = (15)^2 = 225$$

old variance $= 5^2 = 25$

New variance $= 225$

$= 3^2 \times$ old vare

$$y = kx + m$$

New S.D = $|k|$ × old S.D.

New variance = $(k)^2$ × old variance

SD Deviation of 5 & 25 is

A. 11

B. 10

C. 9

D. None

x_i	x_i^2
5	25
25	625
30	650

$$SD = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$= \sqrt{\frac{650}{2} - \left(\frac{30}{2}\right)^2}$$

$$SD = \sqrt{325 - 225}$$

$$= \sqrt{100}$$

$$= 10$$

OR

SD of 5 & 25

$$= \frac{|5-25|}{2}$$

$$= \frac{20}{2}$$

$$= 10$$

A.M. of a & b $= \frac{a+b}{2}$

S.D. of a & b $= \frac{|a-b|}{2}$

A.M. of first 'n' natural no $= \frac{n+1}{2}$

S.D of first 'n' natural no $= \sqrt{\frac{n^2-1}{12}}$

CA WALLAH

Eg

x_i	x_i^2
1	1
2	4
3	9
4	16
$\Sigma x_i = 10$	$\Sigma x_i^2 = 30$

$$AM = \frac{\Sigma x_i}{N} = \frac{10}{4} = 2.5$$

or

$$AM = \frac{n+1}{2} = \frac{4+1}{2} = 2.5$$

$$SD = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$= \sqrt{\frac{30}{4} - \left(\frac{10}{4}\right)^2}$$

$$= \sqrt{7.5 - 6.25}$$

$$= \sqrt{1.25}$$

$$= 1.1180$$

or

$$S.D = \sqrt{\frac{n^2-1}{12}}$$

$$= \sqrt{\frac{4^2-1}{12}}$$

$$SD = \sqrt{\frac{15}{12}}$$

$$= \sqrt{1.25}$$

$$= 1.1180$$

N_1	N_2
$\bar{X}_1$	$\bar{X}_2$
σ_1	σ_2

$$\text{Combined A.M.} = \bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

$$\text{Combined S.D. (Pooled S.D.)} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

where $d_1 = \bar{X}_{12} - \bar{X}_1$

& $d_2 = \bar{X}_{12} - \bar{X}_2$

Q $N_1 = 3$ & $N_2 = 7$

$\bar{X}_1 = 5$ & $\bar{X}_2 = 15$

$\sigma_1 = 1$, $\sigma_2 = 2$

Find Combind AM & SD.

Sol: $\bar{X}_{12} = \dfrac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$

$= \dfrac{3(5) + 7(15)}{3+7}$

$= \dfrac{120}{10}$

$\boxed{\bar{X}_{12} = 12}$

Now $\vec{d_1} = \bar{X}_{12} - \bar{X}_1 = 12 - 5 = 7$

$\vec{d_2} = \bar{X}_{12} - \bar{X}_2 = 12 - 15 = -3$

Combined S.D. $= \sqrt{\dfrac{N_1(d_1^2 + \sigma_1^2) + N_2(d_2^2 + \sigma_2^2)}{N_1 + N_2}}$

$= \sqrt{\dfrac{3(7^2 + 1^2) + 7[(-3)^2 + 2^2]}{3+7}}$

$= \sqrt{\dfrac{150 + 91}{10}}$

$= \sqrt{\dfrac{241}{10}}$

Combined S.D. $= 4.909$

Coefficient of variance $= \frac{S.D}{mean} \times 100$

$C.V. = \frac{\sigma}{\bar{x}} \times 100$

more CV ⇒ more variable (less consistent)

less CV ⇒ less variable (more consistent)

Eg

Group-A	Group-B
$\bar{x} = 3$	$\bar{x} = 5$
$\sigma_x = 2$	$\sigma_y = 3$

Which group is more variable & which is more consistent?

$CV_A = \frac{\sigma_x}{\bar{x}} \times 100$

$= \frac{2}{3} \times 100$

$= 66.67\%$

$CV_B = \frac{\sigma_y}{\bar{y}} \times 100$

$= \frac{3}{5} \times 100$

$= 60\%$

$CV_A > CV_B$

So A is more variable

CA WALLAH

$QD : MD : SD = 10 : 12 : 15$

$6\,QD = 5\,MD = 4\,SD$

$$\frac{QD}{MD} = \frac{10}{12} = \frac{5}{6}$$

$$\frac{MD}{SD} = \frac{12}{15} = \frac{4}{5}$$

$$\frac{QD}{SD} = \frac{10}{15} = \frac{2}{3}$$

Q: If $MD = 10$, find $SD = ?$

Sol: $\frac{MD}{SD} = \frac{12}{15}$

$\frac{10}{SD} = \frac{4}{5} \Rightarrow SD = \frac{50}{4} = 12.5$

or

$5\,MD = 4\,SD$

$\frac{5 \times 10}{4} = SD$

$SD = 12.5$

SD Deviation of first 5 positive natural numbers

A. $\sqrt{2}$

B. 2

C. 3

D. None

$$SD = \sqrt{\frac{n^2-1}{12}}$$

$$= \sqrt{\frac{5^2-1}{12}}$$

$$= \sqrt{2}$$

$$=$$

The standard deviation of a set of 50 items is 6.5. Find the standard deviation, if every item is increased by 5.

$N = 50$

$SD = 6.5$

A. 6.5

B. 11.5

C. 325

D. None

New SD = old SD

$= 6.5$

The standard deviation of a set of 50 item is 8. Find the standard deviation, if each item is multiplied by 2.

A. 8

B. 16

C. 400

D. None

$N = 50$

$SD = 8$

New SD = 2 × old SD

$= 2 \times 8$

$= 16$

The standard deviation of a set of 50 item is 8. Find the standard deviation, if each item is multiplied by 2.

A. 8

B. 16

C. 400

D. None

If the relation between x and y is given by Y=2-3x

& SD of x is 10, what would be the ~~range of y~~ SD of y

A. 10

B. -30

C. 30

D. -50

$y = 2 - 3x$

SD of $x = 10$

SD of $y = |-3| \times$ SD of x

$= 3 \times 10$

$= 30$

$N_1 = 50$, $Mean = 60$, $SD = 8$

$N_2 = 50$, $Mean = 55$, $SD = 7$

Calculate Combined SD

A. 8.92

B. 7.24

C. 7.92

D. None

$$\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2} = \frac{50\times 55 + 50\times 60}{50+50} = \frac{5750}{100}$$

$$\bar{X}_{12} = 57.50$$

Now

$d_1 = \bar{X}_{12} - \bar{X}_1 = 57.5 - 60 = -2.5$

$d_2 = \bar{X}_2 - \bar{X}_2 = 57.50 - 55 = 2.5$

$$SD = \sqrt{\frac{N_1(d_1^2 + \sigma_1^2) + N_2(d_2^2 + \sigma_2^2)}{N_1 + N_2}}$$

$$= \sqrt{\frac{50[2.5^2 + 8^2] + 50[2.5^2 + 7^2]}{50+50}}$$

$$= \sqrt{\frac{3512.5 + 2762.5}{100}}$$

$$= 7.9214$$

The Mean and variance of 5 items are 4.8 and 6.16.

If three items are 2,3 and 6 . Find other two items

A. 4,9

B. 3,7 ✗

C. 5,6 ✗

D. None

$\bar{x} = 4.8$

$\sigma_1^2 = 6.16$

$N = 5$

$\bar{x} = \frac{\Sigma x_i}{N}$

$4.8 = \frac{11+a+b}{5}$

$24 = 11+a+b$

$13 = a+b$

$\text{Varm} = \frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2$

$= \frac{146}{5} - (\bar{x})^2$

$= 29.2 - (4.8)^2$

$= 29.2 - 23.04$

$= 6.16$

x_i	x_i^2
2	4
3	9
6	36
$a=4$	$a^2=16$
$b=9$	$b^2=81$
$11+a+b$	146

What is ratio of QD :MD:SD ?

= 10 : 12 : 15

A. 10:12:17

B. 10:12:15

C. 10:15:12

D. None

Mean =10 & SD=2 . If Each item multiplied by 3 and then 10 is added to every item . Find New Coeff of Variance

A. 10%

B. 15%

C. 12%

D. None

$\bar{x} = 10$

$\sigma = 2$

$y_i = 3x_i + 10$

find CV of y_i

New mean $= 3(10) + 10 = 40$

New SD $= 3(2) = 6$

$$C.V. = \frac{SD}{\bar{x}} \times 100$$

$$= \frac{6}{40} \times 100$$

$$= 15\%$$

	$x_i + k$ Change of origin	$k\,x_i$ Change of scale
Mean	$= \bar{x} + k$	$= k \times \bar{x}$
Median	$=$ Median $+ k$	$=$ $k \times$ Median
Mode	$=$ Mode $+ k$	$=$ $k \times$ Mode
Range	$\times$	$= \lvert k \rvert \times$ Range
Q.D.	$\times$	$= \lvert k \rvert \times$ Q.D.
M.D.	$\times$	$= \lvert k \rvert \times$ M.D.
S.D.	$\times$	$= \lvert k \rvert \times$ S.D.

CA WALLAH

In a given set if all data are of same value then variance would be:

(a) 0

(b) 1

(c) –1

(d) 0.5

x_i : 5, 5, 5, 5

2022 - DECEMBER

If the variance of random variable 'x' is 17, then what is variance of y = 2x + 5?

(a) 34
(b) 39
(c) 68
(d) 78

Variance = 17

$y = 2x + 5$

2022 - DECEMBER

S.D of $y = 2 \times$ S.D of x

Variance of $y = (2)^2 \times$ variance of x

$= 4 \times 17$

$= 68$

$\sigma_x^2 = 17$

$\sigma_x = \sqrt{17}$

$\sigma_y = 2 \times \sqrt{17}$

$\sigma_y^2 = 4 \times 17 = 68$

Which of the following is based on absolute deviation?

(a) Standard deviation

(b) Mean deviation → $\frac{\Sigma |x_i - \bar{x}|}{N}$

(c) Range

(d) Quartile deviation

2022 - June


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