CHAPTER : Ratio And Proportion, Indices, Logarithm

Ratios : Comparison of two or more Quantities ← (Same Kind, Same units)

$\frac{a}{b}$ is written as a:b ; Here, a = Numerator - Antecedent ; b = consequent

If more than 2 quantities of same kind are given in ratio then this is called continued ratio. [denoted as **a:b:c** of a, b & c]

If two quantities are in a:b then first quantity = aK & Second = bK

Terms of a ratio can be multiplied or divided by any non-zero number

a:b or c:d - which is greater ? } Use calculator

Ex- 4:5 & 7:9 - greater ? $\Rightarrow \frac{4}{5} = 0.8$; $\frac{7}{9} = 0.7$ $\Rightarrow \frac{4}{5} > \frac{7}{9}$

a:b = 2:3 , b:c = 5:4 then find a:b:c = ?

$\Rightarrow$ a:b → 2:3 ; b:c → 5:4 $\Rightarrow$ $= 2\times5 : 3\times5 : 3\times4 = 10:15:12$

Inverse of a:b = b:a

Ratio compounded of :-

a:b & c:d = ac:bd i.e $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ (direct multiply)

Duplicate of a:b = $a^2 : b^2$ (squaring)

Triplicate ratio of a:b = $a^3 : b^3$ (cubing)

Sub-duplicate of a:b = $a^{1/2} : b^{1/2}$ [Square root]

Sub Triplicate of a:b = $a^{1/3} : b^{1/3}$ [Cubic root]

Commensurable Quantities

If ratio of two quantities is a rational number.

Ex- 5 & 2 are commensurable as $\frac{5}{2}$ is a rational numbers

Ex- $\sqrt{3}$ & 2 are incommensurable as $\sqrt{\frac{3}{2}}$ isn't a rational number

Proportion :

Equality of two ratios make a proportion

denoted by a:b :: c:d or a:b = c:d or $\frac{a}{b} = \frac{c}{d}$

If a, b & c & d are in proportions then : a:b :: c:d

a, b, c, d = terms ; a & d = Extremes ; b & c = means

Continuous Proportion :-

For 3 terms a, b & c if $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$ then a, b & c are in continuous

where a = first proportions, b = mean proportional, c = third proportional

Continued Proportion :-

for more than 3 terms a, b, c, d, e, f ...

$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} \ldots$

Ex = 2, 4, 8 & 16 are in continued proportion as $\frac{2}{4} = \frac{4}{8} = \frac{8}{16} \ldots$

☆ Properties of Proportions

1) Cross Multiplication Rule = If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$ (cross product)

2) Inverted Rule = If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

3.) Alternendo Rule = If $a:b = c:d$ then $a:c = b:d$.

4.) Componendo Rule = If $a:b = c:d$ i.e $\frac{a}{b} = \frac{c}{d}$ then,

$\frac{a+b}{b} = \frac{c+d}{d}$. Ex- $\frac{x}{2} = \frac{y}{5} \Rightarrow \frac{x+2}{2} = \frac{y+5}{5}$

5.) Dividendo Rule = If $a:b = c:d$ i.e $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{a-b}{b} = \frac{c-d}{d} \Rightarrow$ Ex- $\frac{x}{3} = \frac{y}{5} \Rightarrow \frac{x-3}{3} = \frac{y-5}{5}$

6.) Componendo & Dividendo Rule :-

If $\frac{a}{b} = \frac{c}{d}$ then; $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Ex - $\frac{x}{4} = \frac{y}{8} \Rightarrow \frac{x-4}{x+4} = \frac{y-8}{y+8}$

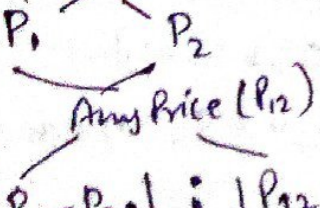
7) Addendo Rule :-

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K$; then $\frac{a+c+e}{b+d+f} = K$

8.) Subtrahendo Rule:-

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$; then $\frac{a-c-e}{b-d-f} = K$

Allegation Rule :-



$Q_1 : Q_2 = |P_{12} - P_2| : |P_{12} - P_1|$

Ex- ₹ 10/kg & ₹ 15 kg

Any Price = ₹ 12/kg.

15 - 12 = 3

12 - 10 = 2

$Q_1 : Q_2 = 3 : 2$

INDICES :-

If no. 'x' is multiplied 'n' times.

$x.x.x.x...$ n times $= x^n$ then x (Base) & n (Power, Index, exponent)

Eg:- $2^5 = 2\times2\times2\times2\times2 = 32$; 2 = Base & 5 = Index

Properties :-

1) $a^0 = 1$ where $a \neq 0$ 2) $a^{-1} = \frac{1}{a}$ 3) $a^{-n} = \frac{1}{a^n}$

Calculator Trick :- $a^n = ?$

→ 'a' on screen → Press × → Press = (n-1) times.

Ex- 2^{10} = 2 on screen → Pressed × → Press = (9 times) ⇒ 1024

Calculator trick :- $a^{-n} = ?$

'0' on screen → Press ÷ → Press = (n) times.

Eg:- 2^{-9} ⇒ 2 on screen → Press ÷ → Press = 9 times ⇒ 0.001953

4) $\boxed{a^m . a^n = a^{m+n}}$ (Same Base and multiplication then Power end

Eg:- $x^2 . x^{\frac{1}{3}} = x^{2+\frac{1}{3}} = x^{\frac{6+1}{3}} = x^{7/3}$

5) $\boxed{a^m / a^n = a^{m-n}}$ (Same base & division then Power subtract)

Eg:- $x^2 / x^4 = x^{2-4} = x^{-2}$

6.) $\boxed{(a^m)^n = a^{mn}}$

Eg:- $(x^3)^2 = x^6$. [× ⇒ $a^{m^n} \neq a^{mn}$]

7.) $\boxed{\sqrt[n]{x} = x^{1/n}}$ (n^{th} root of a number)

⇒ √ 12 times → (-)1 → (÷) by n → (+)1 → (x)(=) 12 times.

Eg:- $\sqrt[5]{32} = 32^{1/5}$

⇒ $(32)^{1/5} = (2^5)^{1/5} = 2$. or by using cal. trick but ans will not exa

8.) If $a^m = a^n$ then $m = n$

9.) If $a^m = k$ then $a = k^{1/m}$.

10.) $(abcd)^m = a^m b^m c^m d^m$

Ex :- If $x = a^{1/3} + a^{-1/3}$ then $x^3 - 3x = a + \frac{1}{a}$
& If $x = a^{1/3} - a^{-1/3}$ then $x^3 + 3x = a - \frac{1}{a}$.

⇒ (i) $x = 5^{1/3} + 5^{-1/3}$ then $x^3 - 3x$

⇒ $x^3 - 3x = 5 + \frac{1}{5} = \frac{26}{5}$

Some basic Identity

$(a+b)^2 = a^2 + b^2 + 2ab$

$a^2 + b^2 = (a+b)^2 - 2ab$

$(a-b)^2 = a^2 + b^2 - 2ab$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$(a-b) + (b-c) + (c-a) = 0$

$(a)(b-c) + b(-a+c) + c(a-b) = 0$

$(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$

$(a^3 - b^3) + (b^3 - a^3) + (c^3 - a^3) = 0$

Eg:- $x^{a-b} \cdot x^{b-c} \cdot x^{c-a} \Rightarrow x^{a-b+b-c+c-a} = x^0 = 1$

LOGARITHM

If $\log_{\text{Base}}{}^{a}(\underset{\text{value}}{x}) = y$ — Power then $x = (a)^y$ where $x>0, a>0, a \neq 1$

a की power क्या रखे जिससे value x मिले

Eg:- $\log_2(8) = ?$ — यहाँ 2 की power me kya rakhein jo 8 mile.

$(2)^3 = 8$ so $\Rightarrow y = 3$. Or $(8) = (2)^y = (2)^3 = 2^y \Rightarrow y = 3$.

Common log

$\Rightarrow y = \log_{10}(x)$

Natural log

$\Rightarrow y = \log_e(x)$ — Used in Calculus.

Calculator Trick :- $\sqrt{}$ 19 times → [−] 1 → [×] 227695. (Many other tricks also)

(i) $\log 200 = 2.3010 = \underset{\text{Character}}{1} + \underset{\text{Mantissa always positive (+ btw 0 & 1)}}{0.3010}$

$\log(20) = 1.3010$

$= 1 + 0.3010$

* $\log(0.2) = -0.699$ (Special case when log is neg (-))

$\Rightarrow \log(0.2) = \underset{(+1)}{0} - 0.699 \rightarrow (+1) \Rightarrow -1 + 0 - 1 + 0.699 \Rightarrow -1 + 0.3010$

$\Rightarrow \bar{1} + 0.3010$

$\Rightarrow \log(0.02) = -1.699 \Rightarrow -1 - 0.699 = -1 - 1 - 0.699 + 1 = -2 + 0.3010$

$\Rightarrow \log(0.02) = \bar{2} + 0.3010 \Rightarrow \bar{2}.3010$

Properties of Log

1) $\log_a(1) = 0$
2) $\log_a(a) = 1$
3) $\log(xy) = \log x + \log y$
4) $\log\left(\frac{x}{y}\right) = \log x - \log y$.
5) $\log(x)^n = n \log x$.
6) $[\log(x)]^n \neq n \log x$
7) $\log_a(x) = \log_b(x) / \log_b a$
8) ~~$\log_{a^b}(x)$~~ 8) $\log_x(y) \times \log_y(x) = 1$
9) $\log_{a^b}(x)^y = \frac{y}{b} \log_a(x)$
10) $a^{\log_a(x)} = x$