2 MARKS

CHAPTER 3

LINEAR INEQUALITIES

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INEQUALITIES

Inequalities are statements where two quantities are unequal

but a relationship exists between them.

A statement involving variable(s) and the sign of inequality

>, <, ≥ or ≤ is called an inequation or inequality

- An inequation may contain one or more variables.
- It may be linear or quadratic or cubic etc.

INEQUALITIES

Example: Following are some examples of inequations:

•
$$3x - 2 < 0$$

•
$$5x + 4y \le 3$$

Inequalities in two variables

$$5x - 3 > 0$$

$$2x + 5y \ge 4$$

•
$$x^2 + 3x + 2 < 0$$

Quadratic Inequalities

Inequalities in one variable

SOLUTION SPACE OF AN INEQUATION

• A solution Space of an inequation is the value(s) of the variable(s)

that makes it a true statement.

• Consider the inequation 30x < 200 ,x belong to whole number

• Let a be a non- zero real number and x be a variable . Then inequations of the form

•
$$ax + b < 0$$

•
$$ax + b > 0$$

Example

$$9x + 15 > 0$$

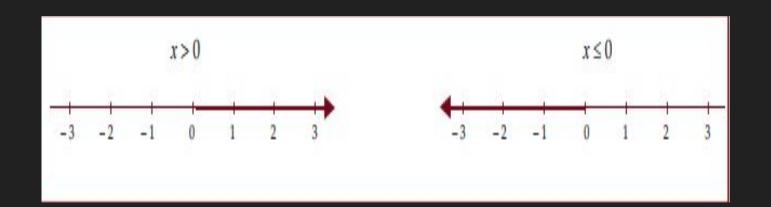
$$\bullet \ 2x - 3 \le 0$$

RULES FOR SOLVING LINEAR INEQUALITIES IN ONE VARIABLE

Rule 1: Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sign of inequality

Rule 2: Both sides of an inequality can be multiplied (or divided) by the same positive number without changing the sign of inequality

Rule 3. when both sides are multiplied or divided by a negative number, then the sign of inequality is reversed.



Example Solve the following linear inequations:

(i)
$$2x - 4 \le 0$$

Example Solve the following linear inequations:

- Let a ,b be a non zero real number and x ,y be variables . Then inequations of the form
- ax + by < c

• ax + by > c

• ax + by ≤ c

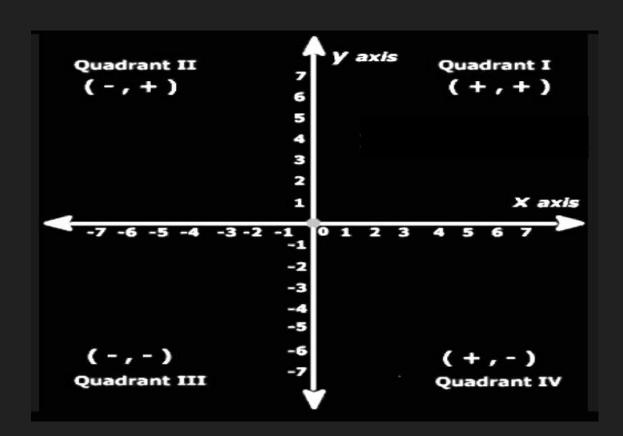
ax+by≥c

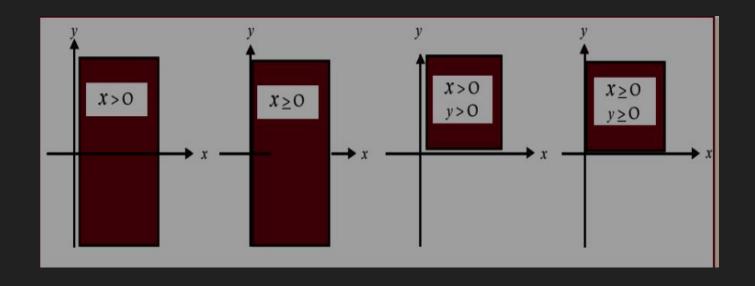
are known as linear inequations in two variables x and y

Example

$$2x + 3y > 6$$

$$2x - 39 y \le 12$$





STEPS TO SOLVE LINEAR INEQUALITIES IN TWO VARIABLES

Step 1: Convert the given inequation, say ax +by $\leq c$, into the equation which represents a straight line in ax +by =c in xy-plane.

Step 2: Put y=0 in the equation obtained in step 1 to get the point where the line meets with x-axis. Similarly, put x=0 to obtain a point where the line meets with y-axis.

Step 3: Join the points obtained in step 2 to obtain the graph of the line obtained from the given inequality of a strict inequality i.e, ax + by + < c or ax + by > c, draw the dotted line, otherwise make it thick line.

STEPS TO SOLVE LINEAR INEQUALITIES IN TWO VARIABLES

Step 4: Choose a point, if possible (0,0), not lying on this line: Substitute its coordinates in the inequation. If the inequation is satisfied, then shade the portion of the plane which contains the chosen point; otherwise shade the portion which does not contain the chosen point.

Step 5: The shaded region obtained in step 4 represents the desired solution set.

Remark: In case of the inequalities $ax + by \le c$ and $ax + by \ge c$ points on the line are also a part of the shaded region while in the case of inequalities ax + by < c and ax + by > c points on the line ax + by = c are not in the shaded region

Solve the following:

• $3x + y \le 6$

Solve the following:

• x - y ≤ -2

Solve the following:

• y ≤ x/2

EXAMPLE: Draw graphs of the following inequalities

 $x \ge 0$, $y \ge 0$, $x \le 6$, $y \le 7$, $x + y \le 12$ and shading the common region.

EXERCISE: 3(A)

Choose the correct answer/answers

Ques 1 (i) An employer recruits experienced (x) and fresh workmen (y) for his firm under the condition that he cannot employ more than 9 people. x and y can be related by the inequality

- a. x+y≠9
- b. $x+y \le 9 \times \ge 0, y \ge 0$
- c. $x+y\geq 9 x\geq 0, y\geq 0$
- d. None of these

(ii) On the average experienced person does 5 units of work while a fresh one 3 units of work daily but the employer has to maintain an output of at least 30 units of work per day. This situation can be expressed as

a.
$$5x + 3y \le 30$$

b.
$$5x + 3y > 30$$

c.
$$5x + 3y \ge 30 \times 20, y \ge 0$$

(iii) The rules and regulations demand that the employer should employ not more than 5 experienced hands to 1 fresh one and this fact can be expressed as

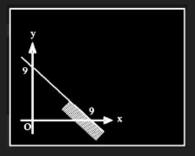
- a. $y \ge x/5$
- **b.** 5y ≤ x
- c. 5 y ≥x
- d. None of these

(iv) The union however forbids him to employ less than 2 experienced person to each fresh person. This situation can be expressed as

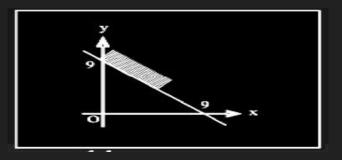
- a. $x \le y/2$
- b. y ≤ x/2
- c. y ≥ x/2
- d. x > 2y

(v) The graph to express the inequality $x + y \le 9$ is

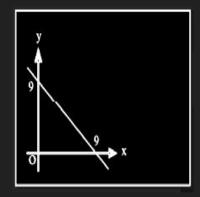
a.



b.

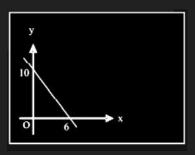


C.

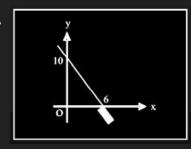


(vi) The graph to express the inequality 5x + 3y ≥ 30 is

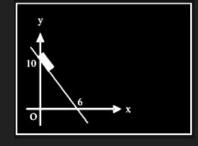
a.



b.

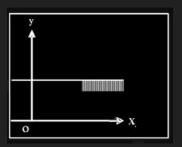


C.

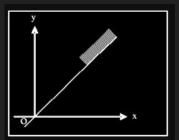


(vii) The graph to express the inequality $y \le (\frac{1}{2}) x$ is indicated by

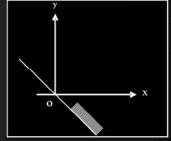
a.



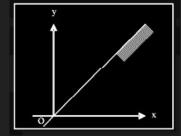
b.



c.



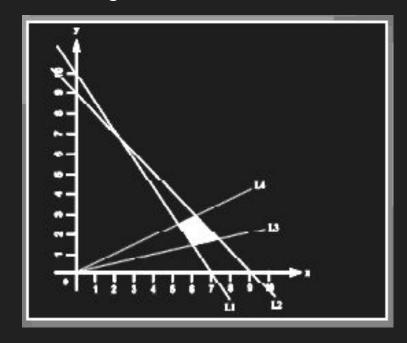
d.



(viii)
$$L_1$$
: 5x + 3y = 30 L_2 : x+y=9 L_3 : y=x/3 L_4 : y = x/2

The common region (shaded part) shown in the diagram refers to

a.
$$5x + 3y \le 30$$
 b. $5x + 3y \ge 30$
 c. $5x + 3y \ge 30$
 $x + y \le 9$
 $x + y \le 9$
 $x + y \ge 9$
 $y \le 1/5 x$
 $y \ge x/3$
 $y \le x/3$
 $y \le x/2$
 $y \ge x/2$
 $y \ge x/2$
 $x \ge 0, y \ge 0$
 $x \ge 0, y \ge 0$



Ques 2 A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per Kg. of each food is shown below:

	Α	В	c	D
Food I:	2	1	1	2
Food II:	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II the situation can be expressed as

a.
$$2x + y \le 9$$

 $x + y \le 7$
 $x + 2y \le 10$
 $2x + 3y \le 12$
 $x > 0, y > 0$

b.
$$2x + y \ge 30$$
 c. $2x + y \ge 9$
 $x + y \le 7$ $x + y \ge 7$
 $x + 2y \ge 10$ $x + y \le 10$
 $x + 3y \ge 12$ $x + 3y \ge 12$

$$. 2x + y \ge 9$$

$$x + y \ge 7$$

$$x + y \le 10$$

$$x + 3y \ge 12$$

1.
$$2x + y \ge 9$$

 $x + y \ge 7$
 $x + 2y \ge 10$
 $2x + 3y \ge 12$
 $x \ge 0, y \ge 0,$

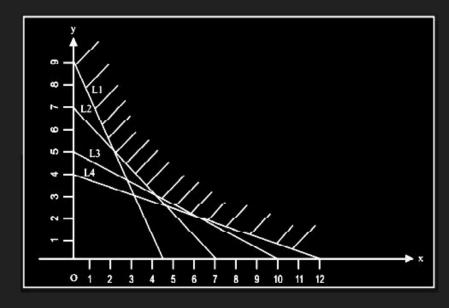
Ques 3 Graph of inequalities drawn below:

$$L_1$$
: 2x + y = 9 L_2 : x + y = 7 L_3 : x + 2y = 10 L_4 : x + 3y = 12

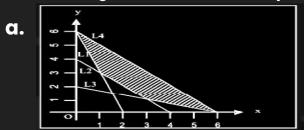
The common region (shaded part) indicated on the diagram is expressed by the set of inequalities

a.

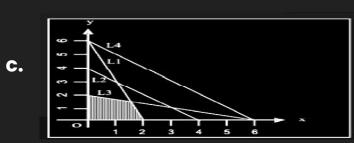
$$2x + y \le 9$$
 b.
 $2x + y \ge 9$
 c.
 $2x + y \ge 9$
 $x + y \ge 7$
 $x + y \le 7$
 $x + y \ge 7$
 $x + y \ge 7$
 $x + 2y \ge 10$
 $x + 2y \ge 10$
 $x + 3y \ge 12$
 $x + 3y \ge 12$
 $x + 3y \ge 12$

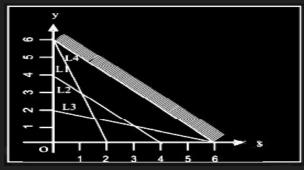


Que. 4 The common region satisfied by the inequalities L_1 : $3x + y \ge 6$, L_2 : $x + y \ge 4$, L_3 : $x + 3y \ge 6$, and L_4 : $x + y \le 6$ is indicated by



b.





Que. 5 The region indicated by the shading in the graph is expressed by inequalities

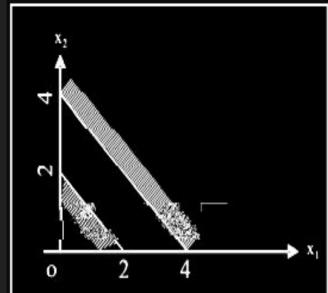
a.
$$x_1 + x_2 \le 2$$

 $2x_1 + 2x_2 \ge 8$
 $x_1 \ge 0$, $x_2 \ge 0$,

b.
$$x_1 + x_2 \le 2$$

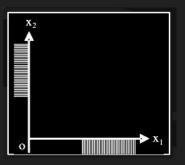
 $x_2 x_1 + x_2 \le 4$

c.
$$x_1 + x_2 \ge 2$$
 d. $x_1 + x_2 \le 2$
 $2x_1 + 2x_2 \ge 8$ $2x_1 + 2x_2 > 8$

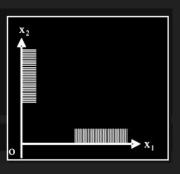


Que. 6 (i) The inequalities $x_1 \ge 0$, $x_2 \ge 0$, are: represented by one of the graphs shown below:

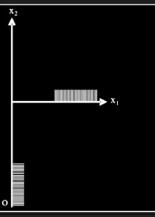
a.



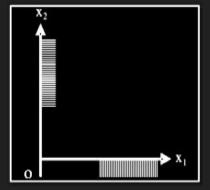
b.



C.



d.

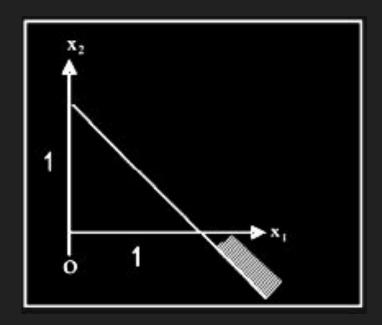


(ii) The region is expressed as

a.
$$x_1 - x_2 \ge 1$$

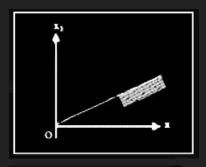
b.
$$x_1 + x_2 \le 1$$

c.
$$x_1 + x_2 \ge 1$$

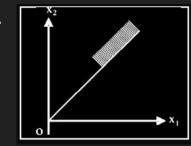


(iii) The inequality $-x_1 + 2x_2 \le 0$ is indicated on the graph as

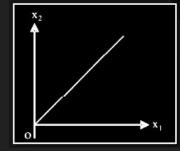
a.



b.



C.

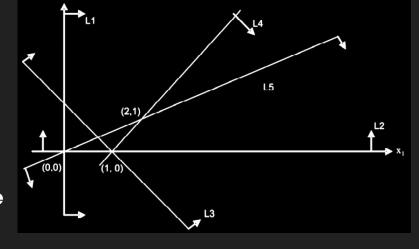


Que. 7 The common region indicated on the graph is expressed by the

set of five inequalities

a.
$$\begin{aligned} \text{L1} : & \mathbf{x}_1 \geq 0 \\ \text{L2} : & \mathbf{x}_2 \geq 0 \\ \text{L3} : & \mathbf{x}_1 + \mathbf{x}_2 \leq 1 \\ \text{L4} : & \mathbf{x}_1 - \mathbf{x}_2 \geq 1 \end{aligned}$$

$$\begin{array}{lll} \text{L1}: x_1 \geq 0 & \text{b.} & \text{L1}: x_1 \geq 0 \\ \text{L2}: x_2 \geq 0 & \text{L2}: x_2 \geq 0 \\ \text{L3}: x_1 + x_2 \leq 1 & \text{L3}: x_1 + x_2 \geq 1 \\ \text{L4}: x_1 - x_2 \geq 1 & \text{L4}: x_1 - x_2 \geq 1 \\ \text{L5}: -x_1 + 2x_2 \leq 0 & \text{L5}: -x_1 + 2x_2 \leq 0 \end{array}$$



L1:
$$x_1 \le 0$$

L2: $x_2 \le 0$
L3: $x_1 + x_2 \ge 1$
L4: $x_1 - x_2 \ge 1$
L5: $-x_1 + 2x_2 \le 0$

Que. 8 A firm makes two types of products: Type A and Type B. The profit on product A is ₹ 20 each and that on product B is ₹ 30 each. Both types are processed on three machines M₁, M₂ and M₃. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Product A	Product B	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

The constraints can be formulated taking x₁ = number of units A and x₂ = number of unit of B as

$$x_1 + x_2 \le 12$$
a. $5x_1 + 2x_2 \le 50$

$$3x_1 + 2x_2 \le 30$$

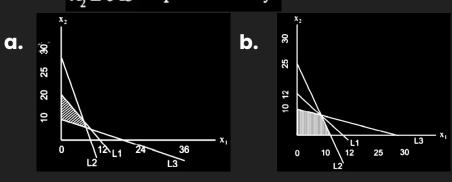
$$2x_1 + 6x_2 \le 60$$

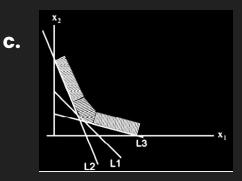
c.
$$3x_1 + 3x_2 \le 36$$
$$5x_1 + 2x_2 \le 50$$
$$2x_1 + 6x_2 \le 60$$
$$x_1 \ge 0, x_2 \ge 0$$

$3x_1 + 3x_2 \ge 36$ **b.** $5x_1 + 2x_2 \le 50$

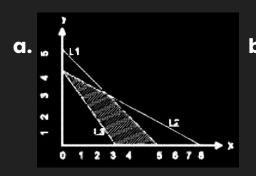
$2x_1 + 6x_2 \ge 60$ $x_1 \ge 0, x_2 \ge 0$ d. None of these

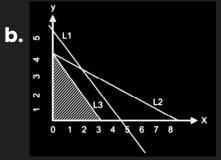
Ques. 9 The set of inequalities L1: $x_1 + x_2 \le 12$, L2: $5x_1 + 2x_2 \le 50$, L3: $x_1 + 3x_2 \le 30$, $x_1 \ge 0$, and $x_2 \ge 0$ is represented by

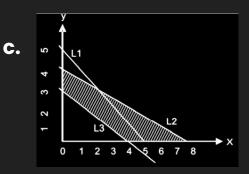




Que.10 The common region satisfying the set of inequalities $x \ge 0$, $y \ge 0$, L1: $x+y \le 5$, L2: $x + 2y \le 8$ and L3: $4x + 3y \ge 12$ is indicated by







d. None of these