Set, Relation & Function

↓

well Refined collection of objects.

Vowels ⇒ {a, e, i, o, u}

Natural No's less than 10 ⇒ {1, 2, 3, 4, 5, 6, 7, 8, 9}

Natural No.'s less than 100 ⇒ {1, 2, 3 ..... 99}

Natural No's ⇒ {1, 2, 3 ...} Roster Form / Tabular Form

Integers ⇒ {... -1, 0, 1 ...}

Real No's ⇒ {x : x is a real No.}
↳ Such That

↓

Set builder form / Rule method

Type 1 :-

Vowels ⇒ {x : x is a Vowel}

eg :- $\left\{ x : x = \begin{matrix} x \\ x+1 \end{matrix} \quad \begin{matrix} 1,2,3 \\ 1 \le x \le 3 \end{matrix} \quad x \in N \right\}$

$\Rightarrow \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4} \right\}$

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⇒ {x : x is a whole No.}

A = {0, 1, 2, 3 ...}

⇒ A = {a, e, i, o, u}

↗ epsilon

$a \in A$ — belongs to

$b \notin A$ — does not belong to

[member, element, belong to]

⇒ Prime No's less than 10

A = {2, 3, 5, 7} ⇒ $3 \in A$ $9 \notin A$

⇒ A = {1, 3, 5, 7, 9}

A = {x : x is odd natural No less than 10}

Types of Sets:-

{1, 2 ... 10} ⇒ finite set.

{1, 2 ... } ⇒ Infinite set.

{3} ⇒ singleton set.

{} = ϕ (Phi) ⇒ Null set Void empty.

A = {1, 2, 3} ⇒ n(A) = 3
cardinal No.

$A = \{1, 2, 3\}$
$B = \{a, b, c\}$ → equivalent set $n(A) = n(B)$

$A = \{1, 2, 3\}$
$B = \{2, 3, 1\}$ ⇒ equal set $n(A) = n(B)$ same element

Note:- order doesn't matter :-

eg:- $A = \{1, 2, 3\}$ $B = \{1, 1, 3, 2\}$
⇒ only different element consider.
$n(A) = 3$ $n(B) = 3$

Subset :-

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\}$
$A \subset B$ → subset

subset ($\subset$ is contained in)
→ $B \supseteq A$
↳ (B containing A)

Proper subset :-

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\}$
• at least one element less than other set:-
$A \subset B$

$A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$
$A \subseteq B$

Subset . Proper subset
$\subseteq$ $\subset$
$\subseteq$
{ equal = $\subseteq$ ~~$\subset$~~
Proper = $\subset$ ~~$\subseteq$~~ }

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4 mk $A = \{1, 2\}$ (2^n)

$\{\}, \{1\}, \{2\}, \{1, 2\}$

Notes:- i) empty set is subset of every set.
ii) Any set is subset of itself.

$A = \{1, 2, 3\} \Rightarrow {}^3C_0 \; {}^3C_1 \; {}^3C_2 \; {}^3C_3$

$\{\}, \{1\}, \{2\}, \{3\}$
$\{1, 2\}, \{1, 3\}, \{2, 3\} \; \{1, 2, 3\}$

$^nC_0 + {}^nC_1 + \dots + {}^nC_n = 2^n$
$^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n - 1$

4 mk Set having n elements:-
No. of subsets $\Rightarrow 2^n$
No. of Proper subsets $\Rightarrow 2^n - 1$

Set → Null set = ϕ
$\phi \subseteq \phi$ ϕ ~~$\phi \subset \phi$~~

eg:- $A = \{2, 3\}$
$P(A) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$ $\phi \in P(A)$ $\{2\} \in P(A)$

→ Power set:- set of all Possible subset of any set.

No. of elements → 2^n

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Belongs To ∈ | Subset ⊆ ⊂

⇒ A = {1, 2, 3}

- 2 ∈ A
- 1, 2 ∈ A
- {1, 2} ⊂ A
- {1, 2} ∈ A
- 3 ∈ A

eg:- A = {1, 2, {4, 5}, 7}

- 1, 2 ∈ A
- {1, 2} ⊂ A
- ~~{4, 5} ⊂ A~~
- {{4, 5}} ⊂ A
- ϕ ⊂ A
- ~~{ϕ} ⊂ A~~
- {{4, 5}, 7} ⊂ A
- {4, 7} ⊂ A Not applicable.

eg:- A {1, ϕ} = 2^{x_0} 2^{x_1} 2^{x_2}

ϕ, {1}, {ϕ}, {1, ϕ}

⇒ ϕ ⊂ A ⇒ {ϕ} ⊂ A

⇒ 1 ∈ A ⇒ ϕ ∈ A

If A = B :-

A ⊆ B B ⊆ A

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Operations :-

universal set :-

eg :- $u = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,2,3,4\}$ $B = \{3,4,5\}$ $C = \{5,6,7,8\}$

This set is universal set.

⇒ Parent set

1. Union :- $A \cup B = A \vee B = \{1,2,3,4,5\}$ (Join)

2. Intersection :- $A \cap B = A \wedge B = \{3,4\}$ (common) (meet)

3. complement :- $A^c = A' = \{5,6,7,8,9,10\}$

Imp 4. $A - B =$ A only $= A - (A \cap B) = A \cap B^c$

$A = \{1,2,3,4\}$ $B = \{3,4,5\}$

$A - B = \{1,2\}$

$A \cap B^c = \{1,2,3,4\} \cap \{1,2,3,6,8,9,10\}$

Imp $B - A =$ B only $= B - (A \cap B) = B \cap B^c$

$\{5\}$ $\{3,4,5\} \cap \{5,6,7,8,9,10\}$

Imp # $(A \cup B)^c = A^c \cap B^c$

$\{1,2,3,4,5\}^c = \{5,6,7,8,9,10\} \cap \{1,2,6,7,8,9,10\}$

$\{6,7,8,9,10\} = \{6,7,8,9,10\}$

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$(A \cap B)^c = A^c \cup B^c$

$\{3,4\}^c = \{1,2,5,6,7,8,9,10\}$

$A^c = \{5,6,7,8,9,10\}$ $B^c = \{1,2,6,7,8,9,10\}$

$A^c \cup B^c = \{1,2,5,6,7,8,9,10\}$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Distributive Property

$A = \{1,2,3,4\}$ $B = \{3,4,5\}$ $C = \{5,6,7,8\}$

$\Rightarrow a.(b+c) = a.b + a.c$

$A \cap (A \cup C) = (A \cap B) \cup (A \cap C)$

$\{3,4\} = \{3,4\}$

$A \cap C = \phi$ (empty) Disjoint set

Read

$A \Delta B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

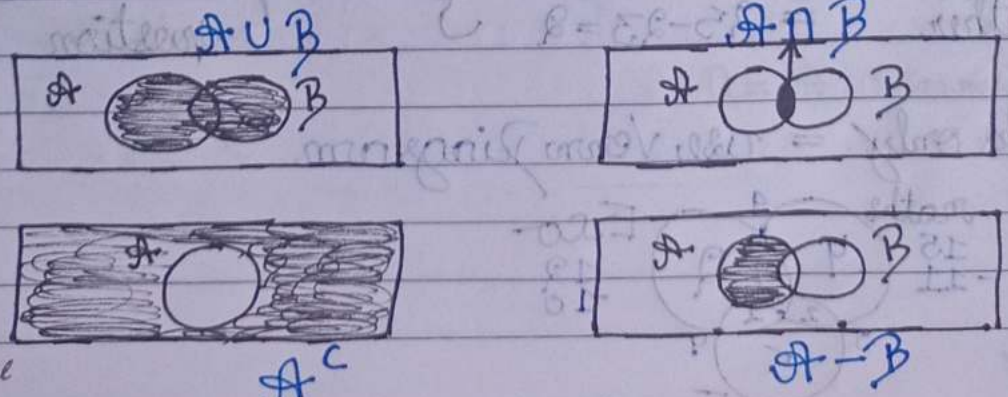
↓

symmetric Difference

$\{1,2\} \cup \{5\} = \{1,2,5\}$

$\{1,2,3,4,5\} - \{3,4\} = \{1,2,5\}$

Venn Diagram :-



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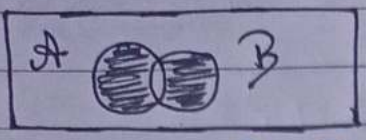


$A \cap B$

Application :-

Imp $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$n(U) = 100$ $n(A) = 50$ $n(B) = 40$ $n(A \cap B) = 20$

$50 + 40 - 20 = 70$

$100 - 70 = 30$

$A \cup B$ = A or B. (Atleast one)

$A \cap B$ = A and B.

$(A \cup B)^c$ = $1 - n(A \cup B)$

In a survey of 25 students.

Maths = 15 Maths & Eng = 5 All Subject = 3

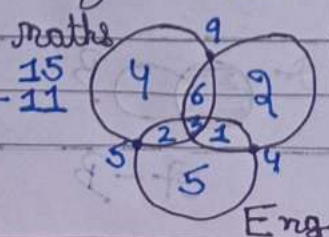
Eco = 12 Maths & Eco = 9

Eng = 11 eco & eng = 4

(ii) $n(A \cup B \cup C) = 23$ } simple use Formula

(iii) Neither = 25 - 23 = 2 } question

(iii) maths only = use Venn Diagram



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(ii) maths only ⇒ 4.
E.co only ⇒ 2
Eng only ⇒ 5

(iii) maths & eng but not E.co. ⇒ 2
(iii) maths & eng but not E.co ⇒ 4+2+5 = 11
(iv) exactly 2 sub. ⇒ 6+2+1 = 9
(v) atleast 2 sub. = ⇒ 6+3+2+1 = 12

Relation →

ordered Pair = $(x, y) \neq (y, x)$

cartesian Product = $R \subseteq A \times B$

eg :- $A = \{1, 2, 3\}$ $B = \{4, 5\}$

cross
$A \times B = \{(1,4)(1,5), (2,4)(2,5)(3,4)(3,5)\}$

Eg :- $A = \{4, 7\}$ $B = \{5, 3\}$

$A \times B = \{(4,5)(4,3)(7,5)(7,3)\}$

Relation :- a is less than b $A \times B$
Relation :- a is greater than b. ϕ empty.
a is square root of b $\{(2,4)\}$

$R = A \times B$ universal.

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Inverse $R^{-1} = \{(4,1)(5,1)(4,2)(5,2)(4,3)(5,3)\}$

$A = \{1,2,3\}$ $A = \{1,2,3\}$

$A \times A = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

R = a is equal to b

$\{(1,1)(2,2)(3,3)\}$ Identifity Relation

eg:- $A = \{1,2,3\}$ $A = \{1,2,3\}$

$A \times A$

ii Reflexive Relation :- aRa $\forall a \in A$ → For every

iii Symmetric Relation :- If aRb then bRa

iiii Transitive relation :- If aRb, bRc then aRc

$a, b, c \in A$

(A Special Type of Relation) ← Function → 2-3 Marks.

Type-1

Definition

$X = \{1,2,3\}$ $Y = \{4,5,6\}$

→ (There exist)

⇒ every element of $x \in X$ ∃ a unique $y \in Y$

$R = \{(1,4)(1,5)(2,4)(2,5)(3,4)(3,5)\}$ → No Function.

$R = \{(1,4)(2,4)(3,4)\}$ → Function.

$R = \{(1,4)(2,5)(1,6)\}$ → No Function

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Eg:- $A = \{5,7,8,9\}$ $B = \{2,4,6\}$

$R = \{(5,2)(7,2)(8,4)(8,6)(9,4)\}$

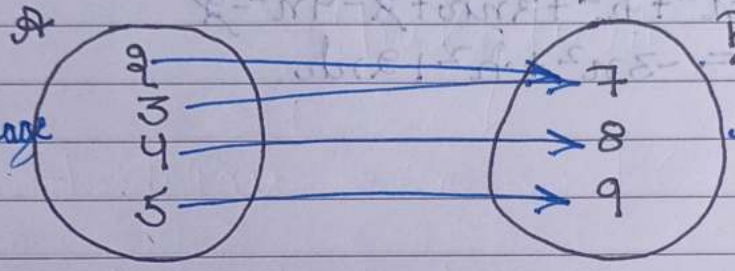
↳ No Function

Domain set — Co-Domain set

Eg:- $A = \{2,3,4,5\}$ $B = \{7,8,9\}$

$R = \{(2,7)(3,7)(4,8)(5,9)\}$ $f(R) =$

↳ Function



↙ Function X

↘ Function ✓ Ha

Imp

Type-2 Inverse Find out

~~$f(x) = y = 2x+3$~~ $f(x) = 2x+3 = y$

$2x = y-3$

$x = \frac{y-3}{2}$

$f^{-1}(x) = \frac{x-3}{2}$

Eg:- $y = 100x$ $y = 3x+7$

$x = \frac{y}{100} = \frac{x}{100}$ $\Rightarrow \frac{x=7}{3}$

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Type-3 Value Put

$f(x) = 2x+7$

$f(2) = 2(2)+7 = 4+7 = 11$

$f(f(2)) = f(11) = 2(11)+7 = 29$

$f(x) = x^2+2$

$f(3) = 3^2+2 = 9+2 = 11$

$f(x+h) = (x+h)^2+2 = x^2+h^2+2xh+2$

$f(2x) = (2x)^2+2 = 4x^2+2$

$f(x+h) - f(2x) = x^2+h^2+2xh+2-4x^2-2$

$= -3x^2+h^2+2xh.$

$f(x) = \frac{x+1}{x^2}$

$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}+1}{\left(\frac{1}{x}\right)^2} \Rightarrow \frac{\frac{1+x}{x}}{\frac{1}{x^2}} = \frac{1+x}{x} \times \frac{x^2}{1} = x(1+x)$

Type-4 fog ≠ gof composite function

$f(x) = x^2+3x \qquad g(x) = x+4$

$fog(x) = f(g(x)) = f(x+4)$

$= (x+4)^2+3(x+4) = x^2+16+8x+3x+12$

$= x^2+11x+28$

$gof(x) = g(f(x)) = g(x^2+3x)$

x^2+3x+4

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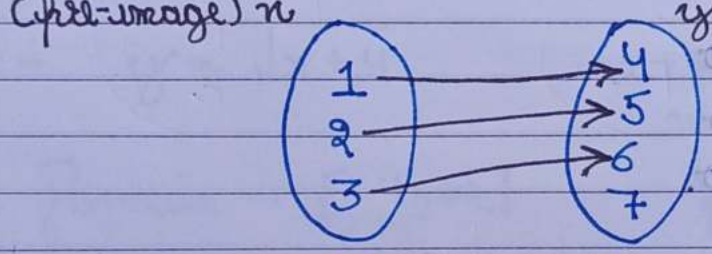
Eg:- $f(x) = x^2 + e^x$, $g(x) = 3x+2$

$fog(x) = f(g(x)) = f(3x+2) = (3x+2) + e^{3x+2}$

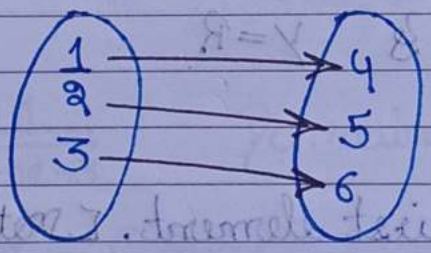
$gof(x) = g(f(x)) = g(x^2+e^x)$
$= 3(x^2+e^x)+2$

Type-5 **Types of functions**

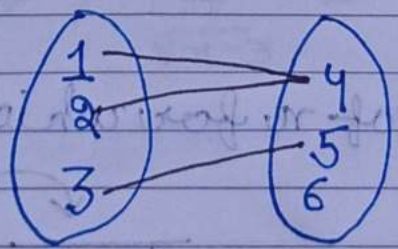
1. **one-one function** :- elements of y having unique pre-image



2. **onto function**:- every element of y have pre image.



3. **Many-one function**:- atleast one element is y having more than one pre-image.



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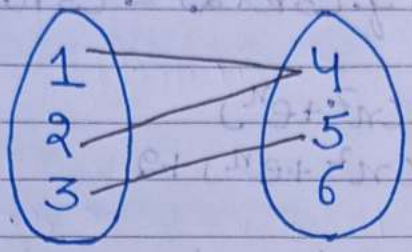
4. Into function :- (Atleast one) element in y having No. Pre-image.



⇒ one-one ≠ many one
⇒ Into ≠ onto

⇒ one-one → onto
→ Into

⇒ many one → onto
→ Into.

Type-6 Domain Range

$f = \{(1,2)(2,3)(3,4)(4,5)\}$

$X = \{1,2,3,4\}$, $Y = R$

$f(x) = x+1$

Domain ⇒ First set of First element. [Method-1]

Method-2

Domain → Value Put Kar Sako.
Range → Value Solve Kar Ke aya.

Method-3

Domain → Value of x for which y defined.

$\frac{1}{x} \quad x \neq 0$

$\sqrt{+ve}$ term

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Range → Value of y for which x Defined.

$f(x) = y$

$x = g(y)$

eg:- $f(x) = y = \frac{1}{x-2}$ $\quad x-2=0$, $x=2$

Domain = $R - \{2\}$

$f(2) = \frac{1}{3-2} = \frac{1}{0} = \infty$

eg:- $y = \sqrt{x+4}$ $\quad x+4 \geq 0$, $x \geq -4$

Domain = $[-4, \infty]$ Real No's.

Interval

1.- open = $(3,4)$
2- closed = $[3,4]$
3.- Semiopen = $[3,4)$
4.- Semiclosed = $(3,4]$

$(-\infty, \infty)$

eg:- $y = \frac{1}{\sqrt{x-3}}$ Domain & Range

Domain = $(3, \infty)$ $\quad x-3>0$, $x>3$

Range = $(0, \infty)$

$\Rightarrow y = \frac{1}{\sqrt{x-3}} = y^2 = \frac{1}{x-3} = x-3 = \frac{1}{y^2}$

$x = \frac{1}{y^2} + 3$

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eg:- $y = \sqrt{x^2-4}$ Domain/Range

$x^2-4 \geq 0$

$(x+2)(x-2) \geq 0$

$[2, \infty) \,/\, (-\infty, -2]$

Domain → $\Rightarrow (-\infty, -2] \cup [2, \infty)$

$= R - (-2, 2)$

Range → $y = \sqrt{x^2-4}$

$= y^2 = x^2 - 4$

$= x^2 = y^2 + 4$

$= x = \sqrt{y^2+4}$

$y^2+4 \geq 0$

$y^2 \geq 4$

MIND MAP

SET RELATIONS AND FUNCTION

RITU JINDAL

SET

Well defined Collection of objects

Method of representing a set

- Roster form or Tabular form
  - Eg. $\{a, e, i, o, u\}$
- Set Builder form or Rule method
  - Eg. $\{x / x \text{ is a vowel}\}$

Types of Set

- Unit Set
  - $\{5\}$
- Null Set
  - ϕ or $\{\}$
- Equivalent Set
  - $\{1,2,3\} = \{a,b,c\}$
- Equal Set
  - $\{1,2,3\} = \{2,3,1\}$
- Finite Set
  - Vowels
- Infinite Set
  - Natural No's

Subset

- Subset
  - $A \subseteq B$
  - Every element of A also an element of B
- Proper Set
  - $A \subset B$
  - $A \neq B$
- Power Subset
  - A set of All possible Subset of a set

Fundamental Operations on Set

- Union
  - A ∪ B
- Intersection
  - A and B
- Compliment
  - $A^c = A' =$ All Element Except A
- Difference
  - A − B
    - A only
  - B − A
    - B only
- Symmetric Difference
  - A − B ∪ (B − A)

Application Based on Set

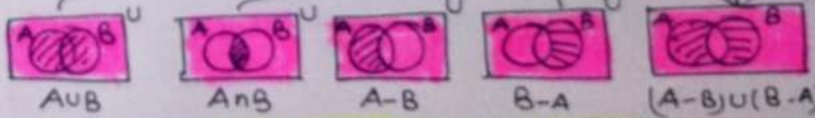
2 Values

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

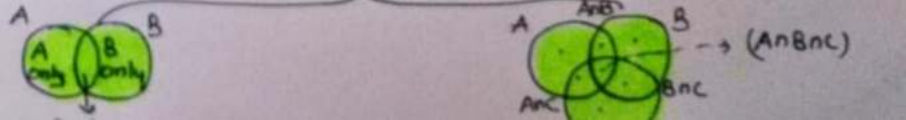
3 Values

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Venn Diagram



To Solve Numerical Problems



MIND MAP

SET RELATIONS AND FUNCTIONS

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Relation

Cartesian Product

$A=\{1,2\}$ $B=\{3,4\}$

$A\times B=\{(1,3)(1,4)(2,3)(2,4)\}$

$(1,3)\neq(3,1)$

Types of Relations

- Universal
- Void
- Identity
- Inverse

Types of Relations

- Symmetric — If aRb then bRa
- Reflexive — $aRa\ \forall a\in A$
- Transitive — If aRb, bRc then aRc
- Anti-Symmetric — aRb then bRa iff $a=b$

- Equivalence
- P.o Relations

Domain — First element

Range — Second element

Function

Definition

$x=\{1,2,3\}$ $y=\{4,5\}$

Every Element $x\in x$ ∃ a unique $y\in y$

Inverse find

$y=f(x)$

$x=f^{-1}(y)$

↓

$y=f^{-1}(x)$

Value Put

Eg. $f(x)=x^2+1$

$f(3)=3^2+1$

$f(f(x))=f(x^2+1)$

$(x^2+1)^2+1$

Composite f°

Eg. $f(x)=x^2+1$

$g(x)=x+1$

$fog(x)=f(g(x))$

$gof(x)=gf(x)$

Types

One One

Onto

Domain Range

D- first element

R — Second

Types of function

- One-One
  - Into
  - Onto
- Onto
- Many One
  - Into
  - onto
- Into

Domain

- # first element
- # observation
- # Value of x for which y defined

Range

- # Second element
- # observation
- # Value of y for which x defined

y undefined

- $\frac{N_R}{D_R}$ $D_R=0$
- $\sqrt{\ }$ term −ve value

Same.

$f(x)=y$

$x=f(y)$

Now check x defined or Not