chapter (1)
(Ratio, Proportion, Indices & Logarithm)

Ratio

↓

Comparison of two or more quantities

- same kind
- same unit

General form :- $a : b$ or $\frac{a}{b}$ — a → first term or antecedent or numerator; b → second term or consequence or denominator (a and b are terms of $a : b$)

Continued ratio :- If more than two quantities of same kind and in same units are given in ratio then this is called continued ratio.

Continued ratio of a, b & c is denoted by
$a : b : c$

- If two quantities are in $a : b$ then :-
  first quantity $= ax$ & second quantity $= bx$
- Terms of a ratio can be multiplied or divided by any non zero number.

Inverse ratio :- $a : b = b : c$

Question :- $a : b = 3 : 4$ & $b : c = 5 : 6$
find $a : b : c$?

Soln :-
$$a : b : c = \begin{matrix} a & : & b & & \\ & & b & : & c \end{matrix}$$

$$\text{So, } \begin{matrix} 3 & : & 4 & & \\ & & 5 & : & 6 \end{matrix}$$

$15 : 20 : 24$ Ans

Compounded ratio :- compounded ratio of $a : b$ & $c : d$ is
$ac : bd$
(Product of first terms of the given ratio : product of second terms of the given ratio)
similarly with the continued ratio.

Duplicate ratio :- $a : b = a^2 : b^2$

Triplicate ratio :- $a : b = a^3 : b^3$

Sub-duplicate ratio :- $a : b = \sqrt{a} : \sqrt{b}$

Sub-triplicate ratio :- $a : b = \sqrt[3]{a} : \sqrt[3]{b}$

Quantities

- Commensurable Quantities
  - (Ratio of two quantities is a rational number)
  - Eg :- 2 : 3
- Incommensurable Quantities
  - (Ratio of two quantities is a irrational number)
  - Eg :- $\sqrt{5} : 1$

- Ratio is unitless.
- Ratio is expressed in lowest terms or simplest form.
- The order of the terms in a ratio is important.
- To compare two ratios, convert them into equivalent like fractions.

Inequality of Ratio (a : b)

- Greater inequality ratio (a > b)
- Less inequality ratio (a < b)

- If a quantity increases or decreases in the ratio a : b. The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio.

(New Quantity = Original Quantity × Factor Multiplying Ratio)

<u>Question</u> :- a : b = 1 : 2, b : c = 3 : 4 & c : d = 5 : 6 find a : b : c : d ?

<u>Soln</u> :- a : b : c : d = a : b

b : c

c : d

So, 1 : 2

3 : 4

5 : 6

5 : 15 : 40 : 40

1 : 3 : 8 : 8 <u>Ans</u>

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Proportion

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equality of two ratios

General form :- $a:b = c:d$ or $a:b::c:d$ or $\frac{a}{b} = \frac{c}{d}$

- a, b, c & d are terms of the proportion.
- a & d are called extremes.
- b & c are called means (middle terms).
- a is 1st proportion, b is 2nd proportion & so on.

Cross Product Rule :- $ad = bc$ (product of extremes = product of means)

Continuous Proportion :- $a:b = b:c$ or $\frac{a}{b} = \frac{b}{c}$ i.e. $b^2 = ac$

- b is called the mean proportional.
- a is 1st proportional & c is 3rd proportional.

Continued Proportion :- $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} \ldots$

Inverse Proportion :- If a ratio is equal to the reciprocal of the other, then either of them is in inverse proportion of the other.

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- **Properties of Proportion** :-

(a) Invertendo :- If $a:b = c:d$, then $b:a = d:c$

(b) Alternendo :- If $a:b = c:d$, then :-
$a:c = b:d$ or $d:b = c:a$

(c) Componendo :- If $a:b = c:d$, then :-
$(a+b):b = (c+d):d$

(d) Dividendo :- If $a:b = c:d$, then :-
$(a-b):b = (c-d):d$

(e) Componendo & Dividendo :- If $a:b = c:d$, then :-
$(a+b):(a-b) = (c+d):(c-d)$

(f) Addendo :- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \ldots$, then :-
$(a+c+e+\ldots):(b+d+f+\ldots)$

(g) Subtrahendo :- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \ldots$, then :-
$(a-c-e-\ldots):(b-d-f-\ldots)$

- **Alligation Rule** :-

Rate (R_1) & Rate (R_2)

Quantity ($N_1 = ?$) Quantity ($N_2 = ?$)

Average Rate (R_{12})

$N_1 = |R_{12} - R_2|$ $N_2 = |R_{12} - R_1|$

Indices
(power / radical)

General form :- $a^n = a \times a \times a \times a \ldots$ to 'n' times

a is called as base.

n is called as index / power / radical.

laws :-

(i) $a^m \times a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{m \times n}$

(iv) $a^{-1} = \frac{1}{a}$

(v) $a^{-m} = \frac{1}{a^m}$

(vi) $a^0 = 1 \ (a \neq 0)$

(vii) $\sqrt[n]{a} = (a)^{\frac{1}{n}}$ (n^{th} root of 'a')

(viii) $a^m \times a^{-m} = 1$

(ix) $(ab)^n = a^n b^n$

(x) $a^m = a^n$, then $m = n$ (but $a \neq 1$)

(xi) $a^m = b^m$, then $a = b$ (but $m \neq 0$)

(xii) $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$ or $(a^{\frac{1}{n}})^m$

(xiii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(xiv) If $a^{\frac{1}{n}} = b$, then $a = b^n$

Identities :-

(a) $(a+b)^2 = a^2 + b^2 + 2ab$

(b) $(a-b)^2 = a^2 + b^2 - 2ab$

(c) $a^2 + b^2 = (a+b)^2 - 2ab$ or $(a^2+b^2) + 2ab$

(d) $a^2 - b^2 = (a-b)(a+b)$

(e) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(f) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(g) $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ or $(a+b)(a^2+b^2-ab)$

(h) $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$ or $(a-b)(a^2+b^2-ab)$

- If $a = u^{1/3} + u^{-1/3}$, then $a^3 - 3a = u + \frac{1}{u}$
- If $a = u^{1/3} - u^{-1/3}$, then $a^3 + 3a = u - \frac{1}{u}$

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logarithm
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inverse operation of an exponential equation

$$a^{y} = x$$

$y \rightarrow$ (index / power)
$x \rightarrow$ (value)
$a \rightarrow$ (base)

↓

$$y = \log_a x$$

$x \rightarrow$ (value)
$a \rightarrow$ (base)
$y \rightarrow$ (index / power)

(where, base (a) :- positive & $a \neq 1$
and value (x) :- positive & $x \neq 0$)

Types of logarithm

Common logarithm
(Base = 10)
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used in arithmetic or algebra

Natural logarithm
(Base = e)
(e :- Euler's constant)
(e = 2.7183)
↓
used in calculus

Properties of logarithm :-

(a) $\log_a a = 1$

(b) $\log_{a^m} x = \frac{1}{m}(\log_a x)$ i.e $\log_a (x)^{1/m}$

(c) $\log(10) = 1$

(d) $\log(1) = 0$

(e) $a^{\log_a x} = x$

(f) $\log\left(\frac{1}{x}\right) = -\log x$

(g) $\log(xy) = \log x + \log y$

(h) $\log\left(\frac{x}{y}\right) = \log x - \log y$

(i) $\log(x)^{n} = n \log x$

(j) $\frac{1}{\log_a x} = \log_x a$

(k) $\log_a x = \frac{\log_{10} x}{\log_{10} a}$ (a is alway 10)

Antilogarithm :- Inverse / opposite of logarithm.

If $\log_a x = y$, then x = antilog of y

Character & Mantissa

Example (1) :- $\log_{10}(20) = 1.301$
$= 1 + 0.301$

$1 \rightarrow$ (character)
$0.301 \rightarrow$ (mantissa) (never negative) (always between 0 & 1)

Example (2) :- $\log(x) = -2.345$

$= -2 - 0.345$ (but mantissa can never be negative)

So, Add one & subtract one :-

$= -1 - 2 - 0.345 + 1$

$= -3 + 0.655$

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