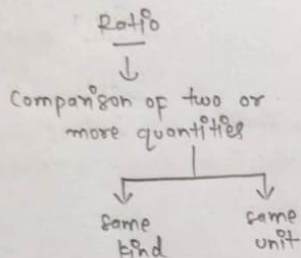


Chapter (1)
(Ratio, Proportion, Indices & Logarithm)



General form :- $a : b$ or $\frac{a}{b}$ → first term or antecedent or numerator
↓
Second term or consequence or denominator

(a and b are terms of $a : b$)

Continued ratio :- If more than two quantities of same kind and in same units are given in ratio then this is called continued ratio.

Continued ratio of a, b & c is denoted by $a : b : c$

- If two quantities are in $a : b$ then :-
first quantity = ax & second quantity = bx
- Terms of a ratio can be multiplied or divided by any non zero number.

Inverse ratio :- $a : b = b : a$

Question :- $a : b = 3 : 4$ & $b : c = 5 : 6$
find $a : b : c$?

Soln :- $a : b : c = a : b : b : c$

So, $3 : 4 : 5 : 6$

$15 : 20 : 24$ Ans =

Compounded ratio :- compounded ratio of $a : b$ & $c : d$ is $ac : bd$

(Product of first terms of the given ratio : Product of second terms of the given ratio)

Similarly with the continued ratio.

Duplicate ratio :- $a : b = a^2 : b^2$

Triplicate ratio :- $a : b = a^3 : b^3$

Sub-duplicate ratio :- $a : b = \sqrt{a} : \sqrt{b}$

Sub-triplicate ratio :- $a : b = \sqrt[3]{a} : \sqrt[3]{b}$

Quantities

Commensurable
Quantities

Incommensurable
Quantities

(Ratio of two quantities is a rational number)

(Ratio of two quantities is an irrational number)

Eg:- $2 : 3$

Eg:- $\sqrt{5} : 1$

- Ratio is unitless.
- Ratio is expressed in lowest terms or simplest form.
- The order of the terms in a ratio is important.
- To compare two ratios, convert them into equivalent like fractions.

Inequality of Ratio
($a : b$)

Greater
Inequality
ratio
($a > b$)

Less
Inequality
ratio
($a < b$)

- If a quantity increases or decreases in the ratio $a : b$.
The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio.
(New Quantity = Original Quantity $\times$ Factor Multiplying Ratio)

Question :- $a : b = 1 : 2$, $b : c = 3 : 4$ & $c : d = 5 : 6$
find $a : b : c : d$?

Soln :- $a : b : c : d = a : b : c : d$

So, $1 : 2 : 3 : 4 : 5 : 6$
 $5 : 15 : 40 : 40$
 $1 : 3 : 8 : 8$ Ans

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Proportion
↓
equality of two ratios

General form :- $a:b = c:d$ or $a:b :: c:d$ or $\frac{a}{b} = \frac{c}{d}$

- a, b, c & d are terms of the proportion.
- a & d are called extremes.
- b & c are called means (middle terms).
- a is 1st proportion, b is 2nd proportion & so on.

Cross Product Rule :- $ad = bc$ (product of extremes = product of means)

Continuous Proportion :- $a:b = b:c$ or $\frac{a}{b} = \frac{b}{c}$ i.e. $b^2 = ac$

- b is called the mean proportional.
- a is 1st proportional & c is 3rd proportional.

Continued Proportion :- $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} \dots$

Inverse Proportion :- If a ratio is equal to the reciprocal of the other, then either of them is in inverse proportion of the other.

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• Properties of Proportion :-

(a) Invertendo :- If $a:b = c:d$, then $b:a = d:c$

(b) Alternendo :- If $a:b = c:d$, then :-
 $a:c = b:d$ or $d:b = c:a$

(c) Componendo :- If $a:b = c:d$, then :-
 $(a+b):b = (c+d):d$

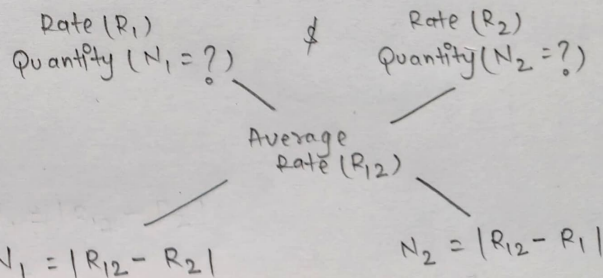
(d) Dividendo :- If $a:b = c:d$, then :-
 $(a-b):b = (c-d):d$

(e) Componendo & Dividendo :- If $a:b = c:d$, then :-
 $(a+b):(a-b) = (c+d):(c-d)$

(f) Addendo :- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \dots$, then :-
 $(a+c+e+\dots):b = (d+f+\dots):d$

(g) Subtrahendo :- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \dots$, then :-
 $(a-c-e-\dots):(b-d-f-\dots)$

• Alligation Rule :-



Indices (power / radical)

General form :- $a^m = a \times a \times a \times a \dots$ to 'm' times
 a is called as base.
 m is called as index / power / radical.

Laws :-

$$(i) a^m \times a^n = a^{m+n}$$

$$(ii) \frac{a^m}{a^n} = a^{m-n}$$

$$(iii) (a^m)^n = a^{m \times n}$$

$$(iv) a^{-1} = \frac{1}{a}$$

$$(v) a^{-m} = \frac{1}{a^m}$$

$$(vi) a^0 = 1 (a \neq 0)$$

$$(vii) \sqrt[n]{a} = (a)^{\frac{1}{n}} \text{ (n}^{\text{th}} \text{ root of 'a')}$$

$$(viii) a^m \times a^{-m} = 1$$

$$(ix) (ab)^m = a^m b^m$$

$$(x) a^m = a^n, \text{ then } m = n \text{ (but } a \neq 1)$$

$$(xi) a^m = b^m, \text{ then } a = b \text{ (but } m \neq 0)$$

$$(xii) a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m \text{ or } (a^{\frac{1}{m}})^n$$

$$(xiii) \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$(xiv) \text{ If } a^{\frac{1}{n}} = b, \text{ then } a = b^n$$

Identities :-

$$(a) (a+b)^2 = a^2 + b^2 + 2ab$$

$$(b) (a-b)^2 = a^2 + b^2 - 2ab$$

$$(c) a^2 + b^2 = (a+b)^2 - 2ab \text{ or } (a^2 + b^2) + 2ab$$

$$(d) a^2 - b^2 = (a-b)(a+b)$$

$$(e) (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$(f) (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$(g) a^3 + b^3 = (a+b)^3 - 3ab(a+b) \text{ or } (a+b)(a^2 + b^2 - ab)$$

$$(h) a^3 - b^3 = (a-b)^3 + 3ab(a-b) \text{ or } (a-b)(a^2 + b^2 - ab)$$

$$\bullet \text{ If } a = u^{1/3} + u^{-1/3}, \text{ then } a^3 - 3a = u + \frac{1}{u}$$

$$\bullet \text{ If } a = u^{1/3} - u^{-1/3}, \text{ then } a^3 + 3a = u - \frac{1}{u}$$

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logarithm
 $\downarrow$
 inverse operation of an exponential equation
 $\rightarrow$ (index/power)
 $a^y = x \rightarrow$ (value)
 $\downarrow$
 (base)

$y = \log_a x \rightarrow$ (value)
 $\downarrow$
 (index/power)
 $\rightarrow$ (base)

(where, base (a) :- positive & $a \neq 1$
 and value (x) :- positive & $x \neq 0$)

Types of logarithm

Common logarithm
 (Base = 10)

$\downarrow$
 used in arithmetic
 or algebra

Natural logarithm
 (Base = e)

(e :- Euler's constant)

(e = 2.7183)

$\downarrow$
 used in calculus

Properties of logarithm :-

(a) $\log_a a = 1$

(b) $\log_a x = \frac{1}{n} (\log_a x)$ i.e. $\log_a (x)^{1/n}$

(c) $\log(10) = 1$

(d) $\log(1) = 0$

(e) $a^{\log_a x} = x$

(f) $\log\left(\frac{1}{x}\right) = -\log x$

(g) $\log(xy) = \log x + \log y$

(h) $\log\left(\frac{x}{y}\right) = \log x - \log y$

(i) $\log(x)^n = n \log x$

(j) $\frac{1}{\log_a x} = \log_x a$

(k) $\log_a x = \frac{\log_{10} x}{\log_{10} a}$ (a is always 10)

Antilogarithm :- Inverse / opposite of logarithm.

If $\log_a x = y$, then $x = \text{antilog of } y$

Character &
 Mantissa

Example (1) :- $\log_{10}(20) = 1.301$

$= 1 + 0.301 \rightarrow$

$\downarrow$ (mantissa)
 (character) (never negative)
 (always between 0 & 1)

Example (2) :- $\log(x) = -2.345$

$= -2 - 0.345$ (but mantissa
can never be
negative)

So, Add one & subtract one :-

$$= -1 - 2 - 0.345 + 1$$

$$= -3 + 0.655$$

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