PYQ Series

CA Foundation

Lecture No- 13

Sequence & Series

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TODAY'S TARGETS

1. PYQs of 5 Attempts

Q.1 The number of terms of the series: 5 + 7 + 9 +.... must be taken so that the sum may be 480 [July 2021]

A 20

B 10

C 15

D 25

$n=20$
R.H.S
$20(20+4)=480$

$5+7+9+---=480$

$a=5 \quad d=7-5=9-7=2$

$480=\frac{n}{2}(2\times5+(n-1)2)$

$480=\frac{n}{\cancel{2}}\times\cancel{2}(5+n-1)$

$\Rightarrow 480=n(n+4)$

$\Rightarrow 20\times24=n(n+4)$

$\Rightarrow n=20$

$\Rightarrow n+4=24 \Rightarrow n=20$

Q.2 If the sum of 'n' terms of an AP (Arithmetic Progression) is $2n^2$, the fifth term is_____ [July 2021]

A 20

B 50

C 18

D 25

$a + a + d - \text{---}$ $n^{th} = 2n^2$

$S_n = 2n^2$ $\quad t_n = S_n - S_{n-1}$

$t_5 = S_5 - S_4 = 2(5)^2 - 2(4)^2$

$= 50 - 32 = 18$

Q.3 The nth terms of the series $3 + 7 + 13 + 21 + 31 + \ldots\ldots$ is

[July 2021]

$n=1 \quad n=2 \quad n=3$

$4 \quad 6 \quad 8$

		$n=1 \Rightarrow 3$	$n=2 \Rightarrow 7$	$n=3 \Rightarrow 13$
A	$4n-1$	$4(1)-1=3$	$4(2)-1=7$	$4(3)-1=11 \neq 13$
B	n^2+2n	$(1)^2+2(1)=3$	$2^2+2(2)=8 \neq 7$	
C	n^2+n+1	$1^2+1+1=1$	$2^2+2+1=7$	$3^2+3+1=9+4=13$
D	n^3+2	$1^3+2=3$	$2^3+2=10 \neq 7$	

Q.4 The sum of square of any real positive quantity and its reciprocal is never less that [July 2021]

$$\left(x+\frac{1}{x}\right)^2 \geq (2)^2$$

$$\left(x+\frac{1}{x}\right)^2 \geq 4$$

A 1

B 2

C 3

D 4

$x > 0$

0.5	1	2
$\left(0.5+\frac{1}{0.5}\right)^2$	$(1+1)^2$	$\left(2+\frac{1}{2}\right)^2$
$=(0.5+2)^2$	$=4 \geq 4$	$=(2.5)^2=6.25>4$
$=(2.5)^2 > 4$		
$=6.25 > 4$		

Q.5 If the sum and products of three numbers in G.P. are 7 and 8 respectively, then 4th term of the series is

[Dec 2021]

A 6

B 4

C 8

D 16

Sum = 7

$\frac{a}{r} + a + ar = 7$

$\Rightarrow 2 + 2r^2 = 5r$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow 2r^2 - 4r - r + 2 = 0$

$\Rightarrow 2r(r-2) - 1(r-2) = 0$

$\Rightarrow (2r-1)(r-2) = 0$

$\Rightarrow r = 2$ or $r = \frac{1}{2}$

Product = 8

$\frac{a}{r}, a, ar \Rightarrow \frac{2}{r}, 2, 2r$

Fourth $\rightarrow (ar)r$

$\frac{a}{r} \times a \times ar = 8$

$a^3 = 8 = 2^3$

$\Rightarrow a = 2$

Fourth term $= ar^2$

$r = 2, \quad = 2(2 \times 2) = 8$

$r = \frac{1}{2} \quad \Rightarrow 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$

Q.6 The sum of series $7 + 14 + 21 + \cdots$. to 17th term is:

[Dec 2021]

$a = 7 \quad d = 7$

$$S_{17} = \frac{17}{2}(2 \times 7 + (17-1)7)$$

$$S_{17} = \frac{17}{2}(14 + 112)$$

$$= 1071$$

A 1071

B 971

C 1171

D 1271

Q.7 The sum of first n terms of an AP is $3n^2 + 5n$. The series is:

[Dec 2021]

$S_n = 3n^2 + 5n$

$t_1 = S_1 = 3(1^2) + 5(1) = 8$

$t_2 = S_2 - S_1 = (3(2^2) + 5(2)) - 3(1^2 + 5(1))$

$= 22 - 8 = 14$

$t_3 = S_3 - S_2 = (3(3^2) + 5(3)) - (3(2^2) + 5(2))$

$= 27 + 15 - 22 = 20$

A 8,14,20,26....

B 8,22,42,68, ...

C 22,68,114 ....

D 8,14,28,44.....

Q.8 The largest value of n for which $\frac{1}{2}+\frac{1}{2^2}+\cdots\frac{1}{2^n}<0.998$ is.....

[Dec 2021]

$\left(\frac{1}{2}\right)^n=(0.5)^n \Rightarrow 0.002$

$(0.5)^8=0.00390002$

$(0.5)^9=0.00195.0002$

$r=\frac{1}{2}$

$a=\frac{1}{2}$

$\frac{\frac{1}{2^2}}{\frac{1}{2}}=\frac{\frac{1}{2^3}}{\frac{1}{2}}$

$r<1$

$S_n=a\left(\frac{1-r^n}{1-r}\right)$

$S_n=\frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}}\right)=1-\left(\frac{1}{2}\right)^n$

$\Rightarrow 1-\left(\frac{1}{2}\right)^n<0.998$

$\Rightarrow -\left(\frac{1}{2}\right)^n<-0.002 \rightarrow \left(\frac{1}{2}\right)^n>0.002$

$1-\left(\frac{1}{2}\right)^8<0.998$

$1-\left(\frac{1}{2}\right)^9=0.998046$

A 9

B 6

C 7

D 8

Q.9 If the nth term of the arithmetic progression 9, 7, 5..... is same as the nth term of the arithmetic progression 15, 12, 9....., then n will be [June 2022]

A 7

B 9

C 15

D 11

9, 7, 5 → $t_n = 9 + (n-1)(-2)$

$d = -2$

15, 12, 9 $\Rightarrow t_n = 15 + (n-1)(-3)$

$d = -3, -3$

$9 - 2(n-1) = 15 - 3(n-1)$

$\Rightarrow 3(n-1) - 2(n-1) = 15 - 9$

$\Rightarrow n - 1 = 6$

$\Rightarrow n = 6 + 1 = 7$

Q.10 In a geometric progression, the second term is 12 and the sixth term is 192. Find the 11^{th} term.

[June 2022]

A 3,072

B 1,536

C 12,288

D 6,144

$ar = 12 \quad -(1)$

$ar^{6-1} = 192$

$\Rightarrow ar^5 = 192 \quad -(2)$

Divide (2) by (1)

$\frac{ar^5}{ar} = \frac{192}{12} = 16$

$\Rightarrow r^{5-1} = 16$

$\Rightarrow r^4 = 2^4$

$\Rightarrow r = 2$

$a = \frac{12}{r} = \frac{12}{2} = 6$

$t_{11} = ar^{11-1} = ar^{10}$

$= 6(2^{10})$

$= 6144$

Q.11 The first and last terms of an arithmetic progression are 5 and 905. Sum of the terms is 45, 955. The number of terms is

[June 2022]

A 99

B 100

C 101

D 102

$$45955 = \frac{n}{2}(5+905)$$

$$n = \frac{45955 \times 2}{910}$$

$$n = 101$$

Q.12 The sum of first eight terms of geometric progression is five times the sum of the first four terms. The common ratio is

[June 2022]

$r > 1$

$$\frac{a(r^8-1)}{r-1} = 5\left(\frac{a(r^4-1)}{r-1}\right)$$

$r^8-1 \Rightarrow (r^4)^2-(1)^2$

$\Rightarrow (r^4-1)(r^4+1)$

$\Rightarrow (r^4-1)(r^4+1) = 5(r^4-1)$

$\Rightarrow r^4+1=5$

$\Rightarrow r^4 = 5-1 = 4$

$\Rightarrow (r^4)^{\frac{1}{4}} = (4)^{\frac{1}{4}}$

$\Rightarrow r = 2^{\frac{2}{4}} = 2^{\frac{1}{2}} = \sqrt{2}$

A $\sqrt{2}$

B $\sqrt{3}$

C 4

D 2

Q.13 If p^{th} term of an AP is q and its q^{th} term is p, when what will be the value of $(p + q)^{th}$ term ?

[Dec 2022]

A 0

B 1

C $p + q - 1$

D $2(p + q - 1)$

p^{th} term $\Rightarrow a + (p-1)d = q$ — (1)

q^{th} term $\Rightarrow a + (q-1)d = p$ — (2)

$(p+q)^{th} \Rightarrow a + (p+q-1)d$

$\Rightarrow p + q - 1 + (p+q-1)(-1)$

$\Rightarrow p + q - 1 - (p + q - 1)$

$\Rightarrow 0$

$(p-1)d - (q-1)d = q - p$

$\Rightarrow pd - d - qd + d = q - p$

$\Rightarrow d(p - q) = -(p - q)$

$\Rightarrow d = -1$

Put $d = -1$ in 1st

$\Rightarrow a + (p-1)(-1) = q \Rightarrow a = p - 1 + q = p + q - 1$

Q.14 In a G.P. = 5^{th} term is 27 and 8^{th} term is 729. Find its 11^{th} term. [Dec 2022]

A 729

B 6061

C 2187

D 19683

$t_5 \Rightarrow ar^{5-1} = ar^4 = 27$ ①

$t_8 \Rightarrow ar^{8-1} \Rightarrow ar^7 = 729$ ②

② ÷ ①

$\frac{ar^7}{ar^4} = \frac{729}{27}$

$\Rightarrow r^{7-4} = 27$

$\Rightarrow r^3 = 27 \Rightarrow r = 3$

$a(3)^4 = 27$

$a = \frac{27}{81} = \frac{1}{3}$

$t_{11} = ar^{11-1} = ar^{10}$

$= \frac{1}{3}(3)^{10}$

$= 19683$

Q.15 If 9^{th} and 19^{th} term of an Arithmetic Progression are 35 and 75 respectively then its 20^{th} term is

[June 2023]

$\Rightarrow t_{20} = t_{19} + d \quad \Rightarrow d = t_{20} - t_{19}$

$t_9 = a + (9-1)d = 35 \quad \rightarrow (1)$

$t_{19} = a + (19-1)d = 75 \quad \rightarrow (2)$

$\Rightarrow \quad a + 8d = 35$

$\quad\quad a + 18d = 75$

$\quad\quad - \quad - \quad -$

$\quad\quad -10d = -40$

$\Rightarrow d = 4$

$t_{20} = 75 + 4 = 79$

A 78

B 79

C 80

D 81

Q.16 How many numbers between 74 and 25,556 are divisible by 5 ? [June 2023]

A 5090

B 5097

C 5095

D 5075

75 , 80 , 85 ---- 25555

$n = ?$ $t_n = 25555$

$75 + (n-1)5 = 25555$

$75 + 5n - 5 = 25555$

$5n = 25485$

$n = \frac{25485}{5} = 5097$

Q.17 If 4^{th}, 7^{th} and 10^{th} terms of a Geometric Progressions are p, q and r, respectively, then

[June 2023]

$p = t_4 = ar^{4-1} = ar^3$

$q = t_7 = ar^{7-1} = ar^6$

$r = t_{10} = ar^{10-1} = ar^9$

$q^2 = (ar^6)^2 = a^2r^{12}$

$pr = (ar^3)(ar^9) = a^2r^{3+9} = a^2r^{12} = q^2$

A $p^2 = q^2 + r^2$

B $p^2 = qr$

C $q^2 = pr$

D $pqr + pq + 1 = 0$

Thank You !