Marathon 3

CA Nishant Kumar

Measures of Central Tendency and Dispersion

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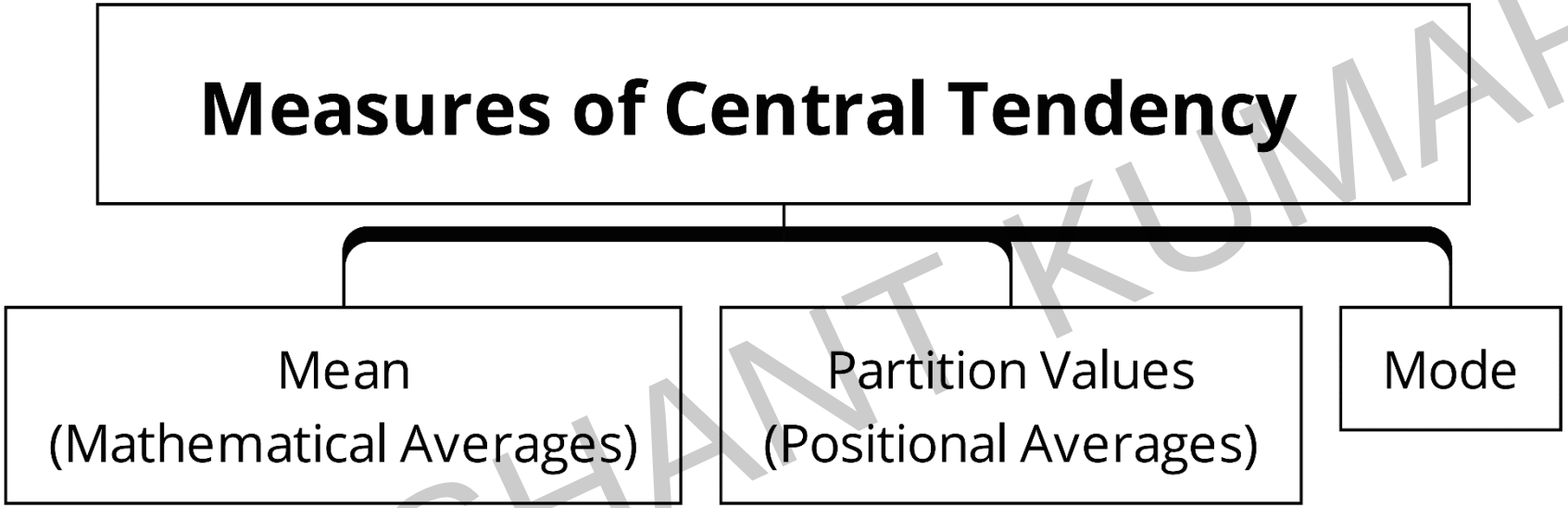
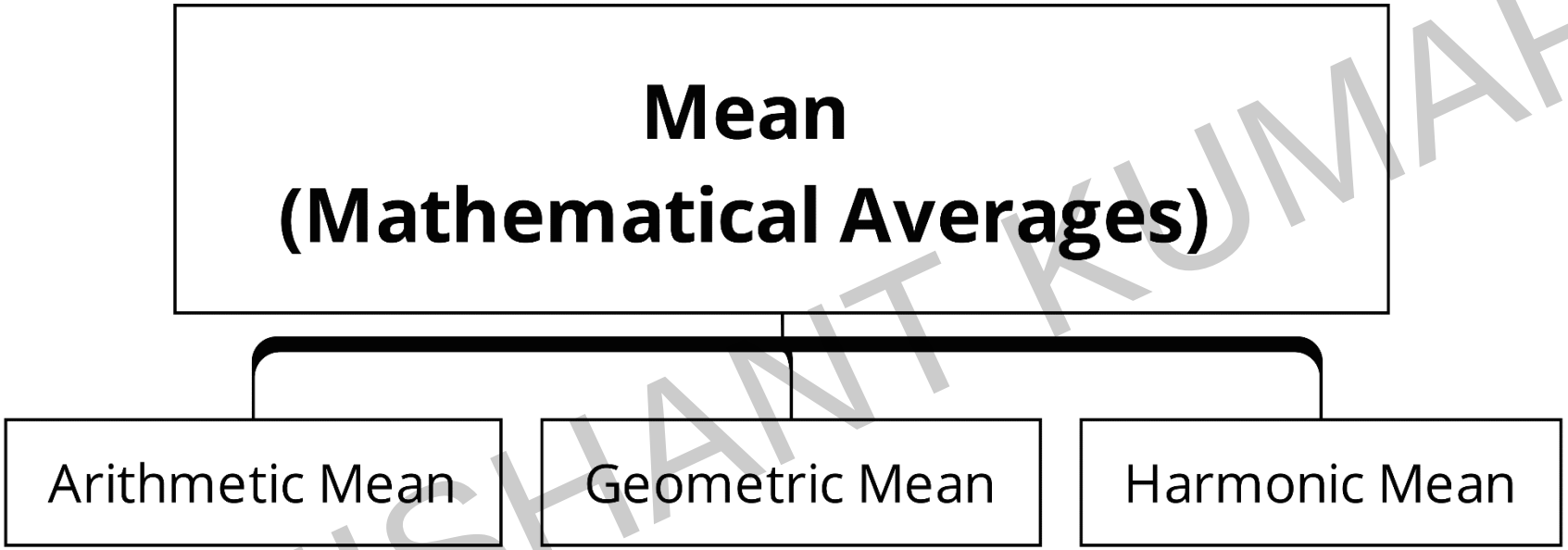
Ekagrata CA

Chapter 14 – Measures of Central Tendency and Dispersion

Measures of Central Tendency and Dispersion

- Measures of Central Tendency
- Measures of Dispersion





Arithmetic Mean

Formula

Individual Series

$$\bar{x} = \frac{\sum x}{n}$$

Discrete Series

$$\bar{x} = \frac{\sum (fx)}{N}$$

N = Total of Frequencies

Continuous Series

Same as Discrete. x is given by the mid point of the class intervals.

Properties

- AM for constant values is constant
- **Combined AM**

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

- Sum of Deviations from AM is always 0
- AM is affected due to change in origin as well as scale

If x and y are two variables related as $y = a + bx$, and $\bar{x}$ is given, then $\bar{y} = a + b\bar{x}$.

Questions Based on Arithmetic Mean

Question

Find the mean from the following data:

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
No. of Students	5	13	23	27	30

(ICAI SM)

(a) 19 (b) 20.45 (c) 21.12 (d) 22.33

Question

If there are 3 observations 15, 20, 25, then the sum of deviation of the observations from their AM is:

(a) 0 (b) 5 (c) –5 (d) None

(ICAI SM)

Question

If the relationship between two variables u and v are given by $2u + v + 7 = 0$ and if the AM of u is 10, then the AM of v is:

(a) 17 (b) –17 (c) –27 (d) 27

(ICAI SM)

Question

The average salary of a group of unskilled workers is ₹10,000 and that of a group of skilled workers is ₹15,000. If the combined salary is ₹12,000, then what is the percentage of skilled workers?

(a) 40% (b) 50% (c) 60% (d) None

(ICAI SM)

Question

At ABC ltd, the average age of employees is 36. Average age of male employees is 38 and that of females is 32. Find the ratio of female to male in the company.

(a) 1 : 3 (b) 2 : 1 (c) 1 : 2 (d) 3 : 1

(MTP November, 2021)

Question

If the mean of a set of observations $x_1, x_2, x_3, ..., x_n$ is $\bar{x}$, then the mean of the observations $x_i + ki$, where $i = 1, 2, 3, \ldots, n$ is:

(a) $\bar{x} + k(n+1)$ (b) $\bar{x} + kn$ (c) $\bar{x} + \frac{k}{n}$ (d) $\bar{x} + \frac{k}{2}(n+1)$

(MTP November, 2018)

Question

Two years ago, a team of four persons had an average age of 14. Now, a new member is added to the team and the average age of the team is 17. What is the age of the new member?

(a) 17 (b) 19 (c) 21 (d) 23

(December, 2021)

Question

The students of a class X^{th} have an average weight of 50 kg. The strength of the class is 49 students. On including the weight of the Principal, the average weight shoots up by 0.8 kg. Find the weight of the Principal.

(a) 75 (b) 90 (c) 85 (d) None

(MTP June, 2013)

Question

The average of $(p+q)$ consecutive numbers starting from 1 is 'r'. If 's' is added to each of the numbers then the new average will be?

(a) $r+s$ (b) $r+(s/2)$ (c) $\{r+(p+q+s)\}/(p+q)$ (d) None

(MTP June, 2013)

Question

The average weight of 40 people is increased by 2.4 kg when one man weight 73 kg is replaced by another man. Find the weight of the new man?

(a) 121 (b) 169 (c) 154 (d) 149

(MTP June, 2013)

Question

The average salary of the whole employees in a company is ₹400 per day. The average salary of officers is ₹800 per day and that of clerks is ₹320 per day. If the number of officers is 40, then find the number of clerks in the company?

(a) 50 (b) 100 (c) 150 (d) 200

(MTP June, 2013)

Question

The average of 6 numbers is 30. If the average of the first four is 25 and that of the last three is 35, the fourth number is:

(a) 25 (b) 30 (c) 35 (d) 40

(MTP June, 2013)

Question

A student's marks were wrongly entered as 85 instead of 45. Due to that, the average marks for the whole class got increased by one-fourth. The no. of students in the class is:

(a) 80 (b) 160 (c) 40 (d) 20

(MTP June, 2013)

Question

The mean of 100 observations is 50. If one of the observations which was 50 is replaced by 40, the resulting mean will be:

(a) 40 (b) 49.90 (c) 50 (d) None

(MTP June, 2013)

Question

The mean annual salary of all employees in a company is ₹25,000. The mean salary of male and female employees is ₹27,000 and ₹17,000 respectively. Find the percentage of males and females employed by the company:

(a) 60% and 40% (b) 70% and 25% (c) 70% and 30% (d) 80% and 20%

(MTP June, 2013)

Geometric Mean

Formula

Individual Series

$$\left(x_1 \times x_2 \times \ldots \times x_n\right)^{1/n}$$

Discrete Series

$$\left(x_1^{f_1} \times x_2^{f_2} \times \ldots \times x_n^{f_n}\right)^{1/N}$$

Properties

- GM for constant values is constant
- $\log G = \frac{1}{r}\sum \log x$ Here, r is the number of observations
- To calculate average RATE, use GM
- If z = xy, then GM (z) = GM (x) * GM (y)
- If z = x/y, then GM (z) = GM (x)/GM (y)

Questions Based on Geometric Mean

Question

What is the GM for the numbers 8, 24 and 40?

(a) 24 (b) 12 (c) $8.\sqrt[3]{15}$ (d) 10

(ICAI SM)

Question

If GM of x is 10 and GM of y is 15, then the GM of xy is:

(a) 150 (b) log 10 × log 15 (c) log 150 (d) None

(ICAI SM)

Question

If GM of x is 10, and GM of y is 15, then GM of x/y is:

(a) 150 (b) 2/3 (c) log 2/log 3 (d) None

Question

If the rates return from three different shares are 100%, 200% and 400% respectively. The average rate of return will be:

(a) 350% (b) 233.33% (c) 200% (d) 300%

(MTP November, 2021)

Question

The geometric mean of the series 1, k, k^2,...,k^n, where k is a constant is:

(a) $k^{(n+1)/2}$ (b) $k^{n+0.5}$ (c) k^{n+1} (d) $k^{n/2}$

(MTP November, 2018)

Harmonic Mean

Formula

Individual Series

$$HM = \frac{n}{\sum(1/x_i)}$$

Discrete Series

$$HM = \frac{N}{\sum\left(\frac{f_i}{x_i}\right)}$$

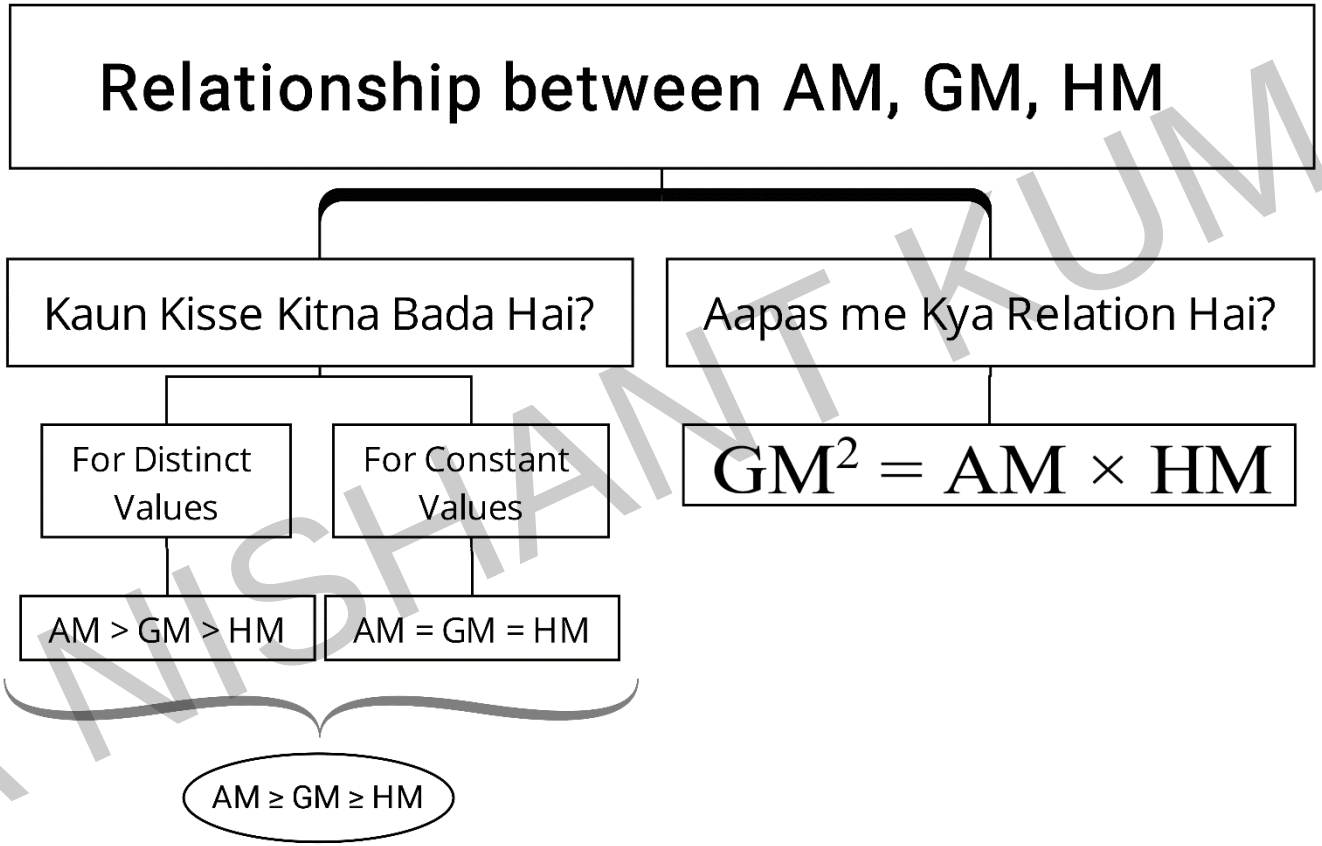
Properties

- HM for constant values is constant
- Combined HM

$$\text{HM} = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$

- The harmonic mean of $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}$ is $\frac{2}{(n+1)}$.
- Average Speed

$$\frac{2xy}{x+y}$$



Questions Based on Harmonic Mean

Question

Compute the HM for the numbers 6, 8, 12, 36.

(a) 9.93 (b) 5.77 (c) 6.77 (d) None

(ICAI SM)

Question

If there are two groups with 75 and 65 as harmonic means and containing 15 and 13 observations then the combined HM is given by:

(a) 65 (b) 70.36 (c) 70 (d) 71

(ICAI SM)

Question

An aeroplane flies from A to B at the rate of 500 km/hour and comes back from B to A at the rate of 700 km/hour. The average speed of the aeroplane is:

(a) 600 km per hour
(b) 583.33 km per hour
(c) $10\sqrt{35}$ km per hour
(d) 620 km per hour

(ICAI SM)

Question

If the AM and HM for two numbers are 5 and 3.2 respectively then the GM will be

(a) 16.00 (b) 4.10 (c) 4.05 (d) 4.00

(ICAI SM)

Question

AM and GM are both negative values, HM is equal to:

(a) $H = \frac{G}{A^2}$ (b) $H = \frac{G^2}{A}$ (c) $H = \frac{G^2}{\sqrt{A}}$ (d) None

(MTP June, 2013)

Partition Values – Summary of Formulae

Partition Value	No. of Parts	No. of Partition Values	Symbol	Rank for Individual Series	Rank for Discrete Series	Rank for Continuous Series
Median	2	1	M	$\frac{n+1}{2}$	$\frac{N+1}{2}$	$\frac{N}{2}$
Quartile	4	3	Q_1 to Q_3	$Q_1 = \frac{n+1}{4}$, $Q_3 = \frac{3(n+1)}{4}$	$Q_1 = \frac{N+1}{4}$, $Q_3 = \frac{3(N+1)}{4}$	$Q_1 = \frac{N}{4}$, $Q_3 = \frac{3N}{4}$

Decile	10	9	D_1 to D_9	$D_1 = \frac{n+1}{10}$, $D_5 = \frac{5(n+1)}{10}$ and so on…	$D_1 = \frac{N+1}{10}$, $D_5 = \frac{5(N+1)}{10}$ and so on…	$D_1 = \frac{N}{10}$, $D_5 = \frac{5N}{10}$ and so on…
Percentile	100	99	P_1 to P_{99}	$P_1 = \frac{n+1}{100}$, $P_5 = \frac{5(n+1)}{100}$ and so on…	$P_1 = \frac{N+1}{100}$, $P_5 = \frac{5(N+1)}{100}$ and so on…	$P_1 = \frac{N}{100}$, $P_5 = \frac{5N}{100}$ and so on…

The formula for any partition value of a continuous series is $l + \frac{Rank - c}{f} \times i$

Property of a Median/Quartile/Decile/Percentile

If x and y are two variables related by $y = a + bx$ for any two constants a and b, then the median of y is given by $y_{me} = a + bx_{me}$.

Questions Based on Median

Question

What is the median for the following observations?

5, 8, 6, 9, 11, 4

(a) 6 (b) 7 (c) 8 (d) None

Question

Find the median of the following data:

Variable (x)	10	50	40	30	20
Frequency (f)	50	20	30	10	40

(a) 20 (b) 30 (c) 40 (d) None

Question

What is the value of median for the following data?

Marks	5 – 14	15 – 24	25 – 34	35 – 44	45 – 54	55 – 64
No. of Students	10	18	32	26	14	10

(a) 28 (b) 30 (c) 32.94 (d) 33.18

(ICAI SM)

Question

Two variables x and y are given by $y = 2x - 3$. If the median of x is 20, what is the median of y?

(a) 20 (b) 40 (c) 37 (d) 35

(MTP May, 2020)

Questions Based on Quartiles

Question

What is the value of the first quartile for observations 15, 18, 10, 20, 23, 28, 12, 16?

(a) 17 (b) 16 (c) 12.75 (d) 12

(ICAI SM)

Question

The third quartile for the following data are:

Profits in '000 ₹	Less than 10	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No. of Firms	5	18	38	20	9	2

(a) ₹33,500 (b) ₹33,000 (c) ₹33,600 (d) ₹33,250

(ICAI SM)

Questions Based on Decile

Question

The third decile for the numbers 15, 10, 20, 25, 18, 11, 9, 12 is:

(a) 13 (b) 10.70 (c) 11 (d) 11.50

(ICAI SM)

Question

Following distribution relates to the distribution of monthly wages of 100 workers. Compute D_7.

Profits in '000 ₹	Less than 500	500 – 699	700 – 899	900 – 1099	1100 – 1499	More than 1500
No. of Firms	5	23	29	27	10	6

(a) ₹1,032.83 (b) ₹1,048.96 (c) ₹995.80 (d) None

(ICAI SM)

Questions Based on Percentile

Question

Following are the wages of the labourers: ₹82, ₹56, ₹90, ₹50, ₹120, ₹75, ₹75, ₹80, ₹130, ₹65. Find P_{82}.

(a) 62.75 (b) 81.20 (c) 120.20 (d) None

(ICAI SM)

Question

The 65th percentile for the following data are:

Profits in '000 ₹	Less than 10	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No. of Firms	5	18	38	20	9	2

(a) ₹29,000 (b) ₹28,680 (c) ₹29,184 (d) ₹29,250

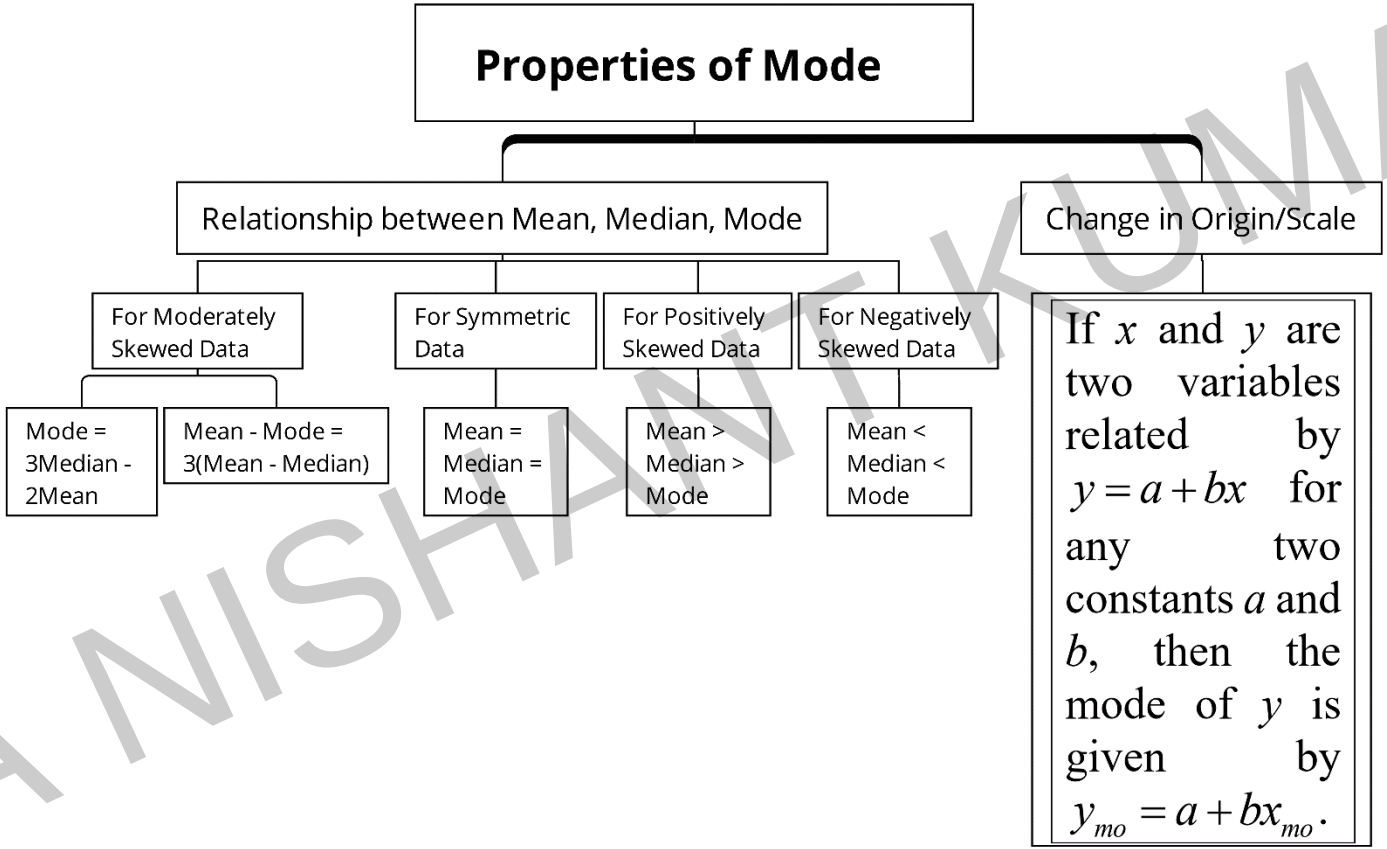
(ICAI SM)

Mode

- **Meaning**
  - Largest Occurring Observation
- **Calculation**
  - Individual Series
    - Largest Occurring Observation
  - Discrete Series
    - Observation with Highest Frequency
  - Continuous Series
    - Formula

$$l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Meaning of Symbols	
l	Lower Limit of the Modal Class Interval
f1	Frequency of the Modal Class Interval
f0	Frequency of the Preceding Class Interval
f2	Frequency of the Succeeding Class Interval
i	Class Length



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Questions Based on Mode

Question

The modal profits for the following data is:

Profit in ₹ '000	Below 5	Below 10	Below 15	Below 20	Below 25	Below 30
No. of Firms	10	25	45	55	62	65

(a) 11.50 (b) ₹11267 (c) ₹11667 (d) 11.67

(ICAI SM)

Question

If $y = 2 + 1.50x$, and mode of x is 15, what is the mode of y?

(a) ₹64.00 (b) 24.50 (c) ₹63.21 (d) ₹64.25

(ICAI SM)

Question

For a moderately skewed distribution of marks in statistics for a group of 200 students, the mean mark and median mark were found to be 55.60 and 52.40. What is the modal mark?

(a) 20 (b) 13 (c) 46 (d) 23

(ICAI SM)

Question

Given that for a distribution, the mean, median and mode are 23, 24, and 25.5. It is most likely that the distribution is _____ skewed.

(a) Positively (b) Symmetrically (c) Asymptotically (d) Negatively

(December, 2021)

Question

Which of the following is the correct relation between mean, median and mode?

(a) Median = Mode + 2/3(Mean – Mode)
(b) 2Mean = Mode – 3Median
(c) 2Mean = Mode + 3Median
(d) Mode = 3Median + 2Mean

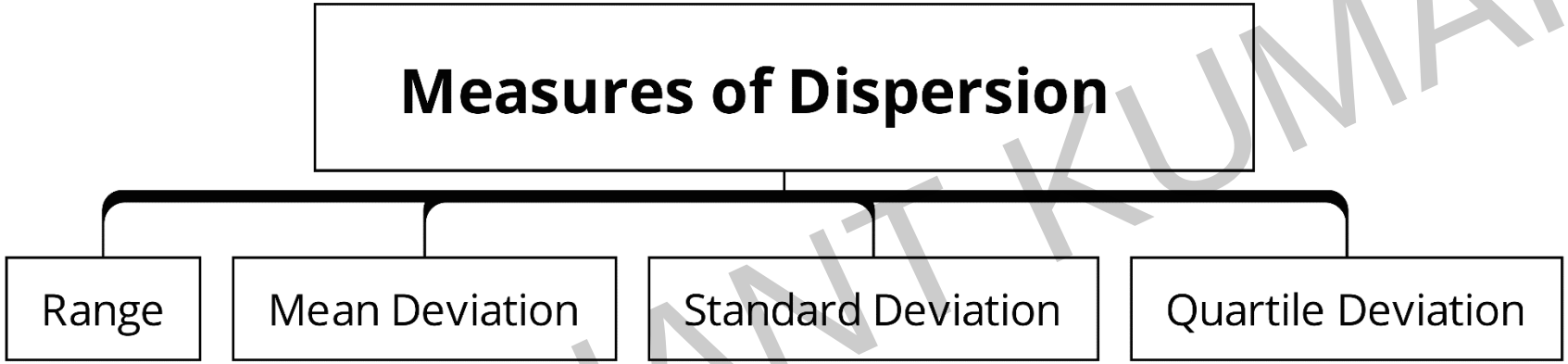
(MTP June, 2013)

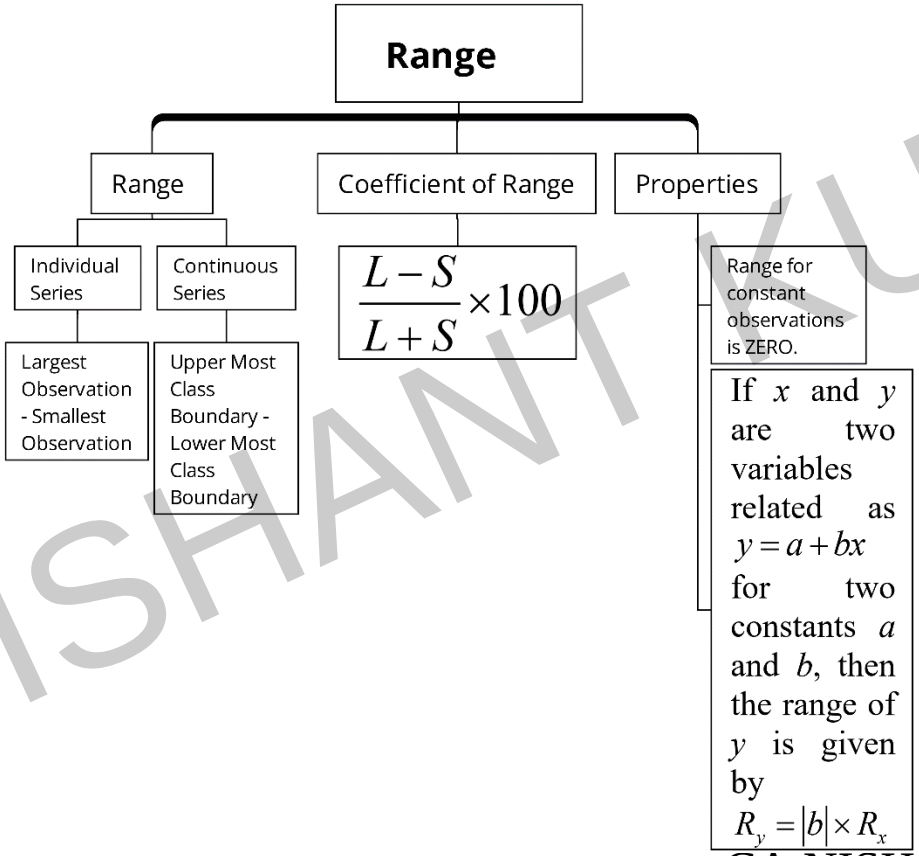
Question

If mean $(\bar{x})$ is 10, and mode (z) is 7, find out the value of median (M).

(a) 9 (b) 17 (c) 3 (d) 4.33

(MTP June, 2013)





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Questions Based on Range

Question

What is the coefficient of range for the following wages of 8 workers?

₹80, ₹65, ₹90, ₹60, ₹75, ₹70, ₹72, ₹85.

(a) ₹30 (b) ₹20 (c) 30 (d) 20

(ICAI SM)

Question

What is the coefficient of range for the following distribution?

Class Interval	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
Frequency	11	25	16	7	3

(a) 22 (b) 50 (c) 72.46 (d) 75.82

(ICAI SM)

Question

If the relationship between x and y is given by $2x+3y=10$, and the range of x is ₹15, what would be the range of y?

(a) ₹20 (b) ₹5 (c) ₹15 (d) ₹10

(ICAI SM)

Mean Deviation

Mean Deviation

Individual Series

$$\frac{1}{n}\sum|x_i - A|$$

Discrete & Continuous Series

$$\frac{f|x - Median|}{N}$$

Coefficient of Mean Deviation

$$\frac{\text{Mean Deviation about } A}{A} \times 100$$

Properties

- MD for constant observations is ZERO
- If x and y are two variables related as $y = a + bx$ for two constants a and b, then the MD of y is given by $MD_y = |b| \times MD_x$
- Sum of Absolute Deviations from Median is Minimum

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Questions Based on Mean Deviation

Question

The mean deviation about mode for the numbers

4/11, 6/11, 8/11, 9/11, 12/11, 8/11 is:

(a) 1/6 (b) 1/11 (c) 6/11 (d) 5/11

(ICAI SM)

Standard Deviation

Standard Deviation

Individual Series

$$(SD) = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

$$SD = \sqrt{\frac{\sum (x_i^2)}{n} - (\bar{x})^2}$$

Discrete & Continuous Series

$$SD = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

Coefficient of Variance

$$\frac{SD}{AM} \times 100$$

Properties of Standard Deviation

SD for constant observations is ZERO

If x and y are two variables related as $y = a + bx$ for two constants a and b, then the SD of y is given by $SD_y = |b| \times SD_x$

For any two numbers a and b, standard deviation is given by $\frac{|a-b|}{2}$

For the first n natural numbers, standard deviation is given by $\sqrt{\frac{n^2-1}{12}}$.

Combined Standard Deviation

$$SD = \sqrt{\frac{n_1 s_1^{\,2} + n_2 s_2^{\,2} + n_1 d_1^{\,2} + n_2 d_2^{\,2}}{n_1 + n_2}}$$

where,

$d_1 = \bar{x}_1 - \bar{x}$

$d_2 = \bar{x}_2 - \bar{x}$

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Questions Based on Standard Deviation

Question

What is the coefficient of variation of the following numbers?

53, 52, 61, 60, 64.

(a) 8.09 (b) 18.08 (c) 20.23 (d) 20.45

(ICAI SM)

Question

A student computes the AM and SD for a set of 100 observations as 50 and 5 respectively. Later on, she discovers that she has made a mistake in taking one observation as 60 instead of 50. What would be the correct mean and SD if the wrong observation is replaced by the correct observation?

(a) 49.90; 6.91 (b) 49.40; 4.91 (c) 49.90; 4.90 (d) None

(ICAI SM)

Question

If the SD of the first *n* natural numbers is 2, then the value of *n* must be:

(a) 2 (b) 7 (c) 6 (d) 5

(ICAI SM)

Question

If AM and coefficient of variation of x are 10 and 40 respectively, what is the variance of $15 - 2x$?

(a) 8 (b) 64 (c) 74 (d) None

(ICAI SM)

Question

If the mean and standard deviation of x are a and b respectively, then the SD of $\frac{x-a}{b}$ is:

(a) –1 (b) 1 (c) ab (d) a/b

(ICAI SM)

Question

If x and y are related by $2x + 3y + 4 = 0$ and SD of x is 6, then SD of y is:

(a) 22 (b) 4 (c) $\sqrt{5}$ (d) 9

(ICAI SM)

Question

If two samples of sizes 30 and 20 have means as 55 and 60 and variances as 16 and 25 respectively, then what would be the SD of the combined sample of size 50?

(a) 5.00 (b) 5.06 (c) 5.23 (d) 5.35

(ICAI SM)

Question

The standard deviation is independent of change of:

(a) Scale (b) Origin (c) Both (a) and (b) (d) None

(ICAI SM)

Question

The best statistical measure used for comparing two series is:

(a) Mean Deviation (b) Range (c) Coefficient of Variation (d) Standard Deviation

(ICAI SM)

Question

If sum of squares of the values = 3390, N = 30 and standard deviation = 7, find out the mean.

(a) 113 (b) 210 (c) 8 (d) None

(MTP May, 2021)

Question

The standard deviation of 10, 16, 10, 16, 10, 10, 16, 16 is:

(a) 4 (b) 6 (c) 3 (d) 0

(MTP May, 2021)

Question

The of mean and SD of a series is $a + b$, if we add 2 to each observation of the series then the sum of the mean and SD will be:

(a) $a + b + 2$ (b) $6 - a + b$ (c) $4 + a - b$ (d) $a + b + 4$

(MTP May, 2021)

Question

There are two startups in ecommerce sector struggling to acquire the market. Following data is for Mean and Standard Deviation of billing amount of bought items per month on their website:

Startup	No. of Customers/Month	Mean Billing Amount	SD of Billing Amount
A	40	₹2,500	₹10
B	30	₹2,200	₹11

Which startup has a better consistency when it comes to sales numbers?

(a) Startup A (b) Startup B (c) Both A and B (d) Need More Information

(MTP June, 2013)

Question

If the coefficient of variation and standard deviation are 60 and 12 respectively, then the arithmetic mean of the distribution is:

(a) 40 (b) 36 (c) 20 (d) 19

(MTP June, 2013)

Question

If the sum of square of the value equals to 3390, number of observations are 30 and Standard deviation is 7, what is the mean value of the above observation?

(a) 14 (b) 11 (c) 8 (d) 5

(MTP June, 2013)

Question

If the variance of random variable x is 18, then what is variance of $y = 2x + 5$?

(a) 34 (b) 39 (c) 68 (d) 72

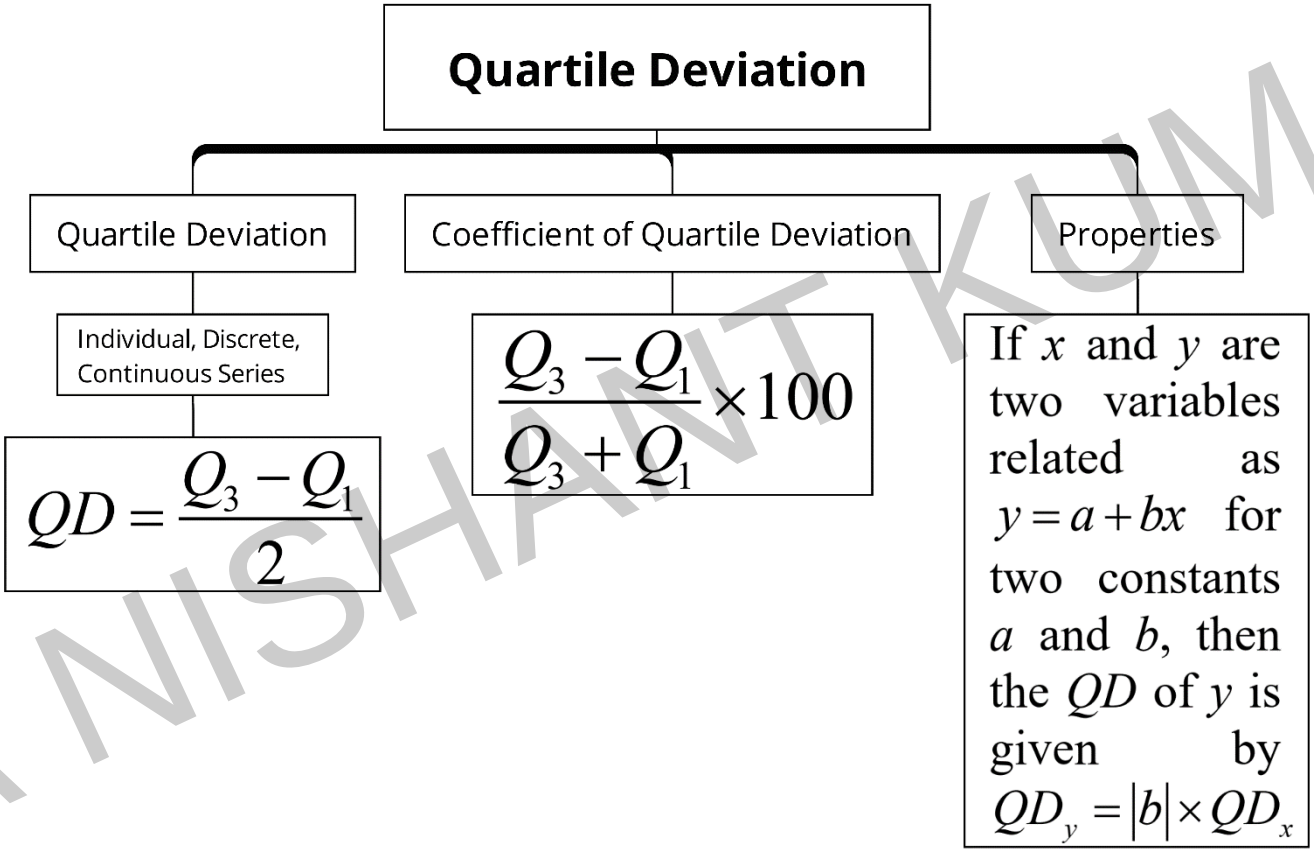
(MTP June, 2013)

Question

In a given set, if all data are of same value, then variance would be:

(a) 0 (b) 1 (c) –1 (d) 0.5

(MTP June, 2013)



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Questions Based on Quartile Deviation

Question

The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is:

(a) 10 (b) 20 (c) 25 (d) 8.30

(ICAI SM)

Question

In the equation $4x+2y=3$, quartile deviation for y is 3. Find the quartile deviation for x.

(a) 4.5 (b) 6 (c) 1.5 (d) None

(MTP June, 2013)

Question

The mean and SD for a, b, and 2 are 3 and $\frac{2}{\sqrt{3}}$ respectively. The value of ab would be?

(a) 5 (b) 6 (c) 11 (d) 3

(ICAI SM)

Question

Which one is an absolute measure of dispersion?

(a) Range (b) Mean Deviation (c) Standard Deviation (d) All these measures

(ICAI SM)

Question

If Quartile deviation is 7, find the value of *x* from the arranged series: 2, *x*, 6, 7, 9, 16, 18.

(a) 5 (b) 2 (c) 8 (d) 6

(MTP June, 2013)