

# AP & GP



## Sequence & Series

8, 88, 888, 8888, 88888

This is also a series

1. 1, 4, 9, 16, 25, 36, 49, .....

It is a series of squares of Natural Numbers.

2, 3, 5, 7, 11, 13, 17, 23 ..... It is a series of Prime Numbers.

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000 ..... Series of cubes of natural numbers

2. a, b, c, d, e, f, g, h, i are said to be in

### Arithmetic Progression

(A.P.)

$$\text{if } (b-a) = (c-b) = (d-c) = (e-d) \\ = (f-e) = (g-f) = (h-g) = (i-h)$$

If (succ. term - prec. term) is same throughout the series, then it is said to be Arithmetic progression.

$$\text{common Diff } (d) = (\text{succ. term} - \text{prec. term})$$

### Geometric Progression

(G.P.)

if

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{e}{d} = \frac{f}{e} = \frac{g}{f} = \frac{h}{g} = \frac{i}{h}$$

If  $\left(\frac{\text{succ. term}}{\text{prec. term}}\right)$  is same or constant throughout the series, then it is said that they have formed a Geometric progression.

$$\text{common Ratio } (r) = \frac{\text{succ. term}}{\text{prec. term}}$$

3. If (succeeding term - preceding term) is same throughout the series

It is called as Arithmetic progression (A.P.)

If (succeeding term / preceding term) is same throughout the series

It is called as Geometric progression (G.P.)

### Examples of A.P.

1. 5, 11, 17, 23, 29, 35, 41..... with  $d = 6$

2. 30, 27, 24, 21, 18, 15, 12, 9, 6, 3, 0, -3, -6, -9 with  $d = -3$

3.  $\frac{31}{3}, \frac{33}{3}, \frac{35}{3}, \frac{37}{3}, \frac{39}{3}, \frac{41}{3}$  with  $d = \frac{2}{3}$

4. 1.50, 3.25, 5, 6.75 with  $d = 1.75$

## Examples of G.P.

1. 10, 20, 40, 80, 160, 320 with  $r = 2$

2. 8, 12, 18, 27 with  $r = 1.50 = 3/2$

3. 10, -30, 90, -270, 810, -2430 with  $r = -3$

4.  $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$  with  $r = \frac{1}{2} = 0.50$

5.  $3, 3^2, 3^3, 3^4, 3^5, 3^6, \dots$  with  $r = 3$

4.

| Terms                                                                                | A.P. / G.P. / Both of these / None of these |
|--------------------------------------------------------------------------------------|---------------------------------------------|
| 5, 10, 20, 30, 40, 80                                                                | Neither AP nor GP                           |
| 20, 30, 45, 67.50                                                                    | G.P. with $r = 1.50$                        |
| 100, 90, 80, 70, 60, 50                                                              | A.P. with $d = -10$                         |
| $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}, \frac{8}{3}$ | A.P. with $d = \frac{1}{3}$                 |
| 300, 100, $\frac{100}{3}$                                                            | G.P. with $r = \frac{1}{3}$                 |
| 28, 33, 38, 43, 58, 63                                                               | Neither AP nor GP                           |
| $a, ax, ax^2, ax^3, ax^4$                                                            | G.P. with $r = x$                           |
| $8+m, 16+m, 24+m, 32+m$                                                              | A.P. with $d = 8$                           |
| 18, 18, 18, 18, 18, 18                                                               | A.P. as well as GP $d = 0, r = 1.00$        |
| 15, 155, 1555, 15555                                                                 | Neither AP nor GP                           |
| 1, 4, 9, 16, 25, 36, 49, 64                                                          | Neither AP nor GP                           |



|                               |                     |
|-------------------------------|---------------------|
| 5, 55, 555, 5555              | neither AP nor GP   |
| $x+3, x+8, x+13, x+18$        | A.P. with $d=5$     |
| $x, 3x, 5x, 7x, 9x, 11x$      | A.P. with $d=2x$    |
| $m, m^5, m^9, m^{13}, m^{17}$ | G.P. with $r=m^4$   |
| 18, 11, 4, -3, -10, -17       | A.P. with $d=-7$    |
| 99, 9.90, 0.99, 0.099         | G.P. with $r=0.10$  |
| 4, -8, -16, 32, -64           | neither AP nor G.P. |

5.

■ Terms can be in AP as well as GP if and only if **all the terms are same**.

■ a, b, c, d, e, f can be in AP as well as GP if and only if  **$a=b=c=d=e=f$**

6. **FEW BASIC CONCEPTS**

a.  $1 + 2 + 3 = 6$  = sum of first '3' natural numbers =  $\frac{3 \times 4}{2} = 6$

$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28 = \frac{7 \times 8}{2}$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 78 = \frac{12 \times 13}{2}$

$1 + 2 + 3 + 4 + 5 + \dots + 1008 = ? = \frac{1008 \times 1009}{2} = 5,08,536$

$1 + 2 + 3 + 4 + 5 + \dots + n = \frac{n(n+1)}{2}$

$$\text{Sum of first 'n' natural numbers} = \frac{n(n+1)}{2}$$

$$\text{b. } 1 + 4 + 9 + 16 = 30 = \frac{4 \times 5 \times 9}{6}$$

$$1^2 + 2^2 + 3^2 + 4^2 = 30$$

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = 140 = \frac{7 \times 8 \times 15}{6}$$

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + 12^2 = 650 = \frac{12 \times 13 \times 25}{6}$$

$$1^2 + 2^2 + 3^2 + \dots + 833^2 = \frac{833 \times 834 \times 1667}{6} = 193016929$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\text{Sum of squares of first 'n' natural numbers} = \frac{n(n+1)(2n+1)}{6}$$

$$\text{c. } 1^3 + 2^3 + 3^3 = 36 = \left(\frac{3 \times 4}{2}\right)^2 = 36$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 441 = \left(\frac{6 \times 7}{2}\right)^2$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3 = 3025 = \left(\frac{10 \times 11}{2}\right)^2$$

$$1^3 + 2^3 + 3^3 + \dots + 90^3 = \left(\frac{90 \times 91}{2}\right)^2 = 1,67,69,025$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Sum of cubes of first 'n' natural numbers =  $\left[ \frac{n(n+1)}{2} \right]^2$

d.  $1 + 3 + 5 = 9 =$  sum of first 3 odd natural numbers  $= 3^2$

$1 + 3 + 5 + 7 + 9 = 25 =$  sum of first 5 odd natural numbers  $= 5^2$

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 144$   
sum of first 12 odd natural numbers  $= 12^2$

$1 + 3 + 5 + 7 + 9 + \dots + 3999 = 2000^2$

$1 + 3 + 5 + 7 + \dots + n \text{ terms} = n^2$

Sum of first 'n' odd natural numbers =  $n^2$

e.  $2 + 4 + 6 + 8 =$  sum of first 4 even natural numbers  $= 20 = 4 \times 5$

$2 + 4 + 6 + 8 + 10 + 12 =$  sum of first 6 even natural numbers  $= 42$   
 $= 6 \times 7$

$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 = 110 = 10 \times 11$

Sum of first 'n' even natural numbers  $= n(n+1) = n^2 + n$

$2 + 4 + 6 + 8 + 10 + \dots + n \text{ terms} = n(n+1) = n^2 + n$

7. a.  $1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$

b.  $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

c.  $1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2$

d.  $1 + 3 + 5 + 7 + \dots + n \text{ terms} = n^2$

e.  $2 + 4 + 6 + 8 + \dots + n \text{ terms} = \frac{n(n+1)}{2}$

8.  $25^2 + 26^2 + 27^2 + \dots + 130^2 = ?$



$$= (1^2 + 2^2 + \dots + 130^2) - (1^2 + 2^2 + \dots + 24^2)$$

$$= \left( \frac{130 \times 131 \times 261}{6} \right) - \left( \frac{24 \times 25 \times 49}{6} \right)$$

$$= 7,35,905/-$$

9.  $30^3 + 31^3 + 32^3 + \dots + 98^3 = ?$



$$= (1^3 + 2^3 + 3^3 + \dots + 98^3) - (1^3 + 2^3 + 3^3 + \dots + 29^3)$$

$$= \left( \frac{98 \times 99}{2} \right)^2 - \left( \frac{29 \times 30}{2} \right)^2$$

$$= 2,33,42,976$$

Q:  $88^3 + 89^3 + 90^3 + \dots + 125^3 = ?$

$$\Rightarrow = \left( \frac{125 \times 126}{2} \right)^2 - \left( \frac{87 \times 88}{2} \right)^2 = 4,73,62,041$$

10.  $85 + 86 + 87 + \dots + 10580 = ?$



$$= (1 + 2 + 3 + \dots + 10580) - (1 + 2 + 3 + \dots + 84)$$

$$= \left( \frac{10580 \times 10581}{2} \right) - \left( \frac{84 \times 85}{2} \right) = 55969920$$

Q:  $1 + 3 + 5 + 7 + \dots + 1099 = ?$

= sum of first 550 odd natural numbers

$$= 550^2 = 3,02,500/-$$

Q:  $1 + 3 + 5 + 7 + 9 + \dots$  289 terms = ?

$$= 289^2 = 83521$$



11. 8896, 8899, 8902, 8905, .....

In this A.P. First term =  $a = 8896 = t_1$

common diff. =  $d = 3$

$$t_1 = 8896 = a$$

$$t_2 = 8896 + 3 = 8899 = a + d$$

$$t_3 = a + d + d = a + 2d$$

$$t_4 = a + 2d + d = a + 3d$$

$$t_5 = a + 3d + d = a + 4d$$

$$t_6 = a + 5d$$

$$t_{10} = a + 9d$$

$$t_{188} = a + 187d$$

$$t_n = a + (n-1)d$$

$$n^{\text{th}} \text{ term of A.P.} = t_n = a + (n-1)d$$

$$t_{100} + 3d = t_{103}$$

$$t_{203} - 4d = t_{199}$$

$$t_{86} + 5d = t_{91}$$

$$t_{10} = t_9 + d$$

$$= t_{11} - d$$

In AP: any term can be obtained by adding 'd' to its preceding term (OR)

By deducting 'd' from its succ. term

$$t_{100} = t_{99} + d$$

$$= t_{101} - d$$

$$t_n = t_{n-1} + d$$

$$t_n = t_{n+1} - d$$

12. For A.P. :  $t_m = a + (m-1)d$

$$t_k = a + (k-1)d$$

$$t_{1000} = a + (1000-1)d = a + 999d$$

$$t_{86} = a + (86-1)d = a + 85d$$

$$t_{k+1} = a + (k+1-1)d = a + kd$$

$$t_{p+10} = a + (p+10-1)d = a + (p+9)d$$

13. In AP : Any term can be obtained by adding common difference to its preceding term or by deducting common diff. from its succeeding term.

$$\therefore t_p = t_{(p-1)} + d$$

$$= t_{(p+1)} - d$$

14. 89, 93, 97, 101, 105, ..... Find  $t_{100}$ ,  $t_{205}$ ,  $t_{1083}$

→ In this A.P.  $a=89$ ,  $d=4$

$$t_{100} = a + 99d = 89 + (99 \times 4) = 485$$

$$t_{205} = a + 204d = 89 + (204 \times 4) = 905$$

$$t_{1083} = a + 1082d = 89 + (1082 \times 4) = 4417$$

15. 587, 593, 599, 605, ..... Find  $t_{190}$ ,  $t_{810}$ ,  $t_{59}$

→ In this A.P.  $a=587$ ,  $d=6$

$$t_{190} = a + 189d = 587 + (189 \times 6) = 1721$$

$$t_{810} = a + 809d = 587 + (809 \times 6) = 5441$$

$$t_{59} = a + 58d = 587 + (58 \times 6) = 935$$

16. 110, 107, 104, 101, ..... Find  $t_{40}$ ,  $t_{800}$ ,  $t_{555}$

→ In this A.P.  $a=110$ ,  $d=-3$

$$t_{40} = a + 39d = 110 + (39 \times -3) = -7$$

$$t_{800} = a + 799d = 110 + (799 \times -3) = -2287$$

$$t_{555} = a + 554d = 110 + (554 \times -3) = -1552$$

17. 1.85, 1.91, 1.97, 2.03, 2.09, ..... Find  $t_{1000}$ ,  $t_{888}$

→ In this A.P.  $a=1.85$ ,  $d=0.06$

$$t_{1000} = a + 999d = 1.85 + (999 \times 0.06) = 61.79$$

$$t_{888} = a + 887d = 1.85 + (887 \times 0.06) = 55.07$$

18. 5888, 5892, 5896, 5900, ..... 10100.

How many terms are there in above A.P. ?

→ In this A.P.  $a=5888$ ,  $d=4$ ,  $t_n=10100$





$$t_n = a + (n-1)d$$

$$10,100 = 5888 + (n-1)4$$

$$n = 1054$$

There are 1054 terms in above A.P.

19. 899, 904, 909, 914, ....., 1284344

How many terms are there in above A.P.?

$$\rightarrow t_n = a + (n-1)d$$

$$1284344 = 899 + (n-1)5$$

$n = 256690 \therefore$  There are 256690 terms in above AP

Q: 100, 96, 92, 88, ....., -81800

Find no. of terms of above AP?

$$\Rightarrow -81,800 = 100 + (n-1)(-4) \therefore n = 20476 \text{ terms}$$

20.  $89^3 + 90^3 + 91^3 + \dots + 110^3 = ?$

$$\rightarrow = (1^3 + 2^3 + 3^3 + \dots + 110^3) - (1^3 + 2^3 + 3^3 + \dots + 88^3)$$

$$= \left( \frac{110 \times 111}{2} \right)^2 - \left( \frac{88 \times 89}{2} \right)^2$$

$$= 2,19,35,969/-$$

21.  $59^2 + 60^2 + 61^2 + 62^2 + \dots + 190^2 = ?$

$$\rightarrow (1^2 + 2^2 + 3^2 + \dots + 190^2) - (1^2 + 2^2 + 3^2 + \dots + 58^2)$$

$$= \left( \frac{190 \times 191 \times 381}{6} \right) - \left( \frac{58 \times 59 \times 117}{6} \right)$$

$$= 22,37,686/-$$



22.  $588 + 593 + 598 + 603 + \dots$

Find sum of 2000 terms  $= S_{2000} = ?$



Sum of first 'n' terms of AP  
 $= [t_1 + t_2 + t_3 + \dots + t_n]$

$$= a + (a+d) + (a+2d) + \dots + [a + (n-1)d]$$

$$= [a + a + a + \dots \text{ n terms}] +$$

$$[d + 2d + 3d + 4d + \dots + (n-1)d]$$

$$= n \cdot a + d [1 + 2 + 3 + 4 + \dots + (n-1)]$$

$$= n \cdot a + d \left[ \frac{(n-1)(n-1+1)}{2} \right]$$

$$= \frac{2n \cdot a}{2} + d \times \frac{n(n-1)}{2}$$

$$S_n = \frac{n}{2} [2a + (n-1)d] = \frac{n}{2} [a + a + (n-1)d]$$

$$= \frac{n}{2} [t_1 + t_n]$$

$$588 + 593 + 598 + \dots$$

Find  $S_{2000}$

$$\Rightarrow S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{2000}$$

$$= \frac{2000}{2} [2a + 1999d]$$

$$= 1000 [1176 + (1999 \times 5)]$$

$$= 11171000/-$$

23.

For A.P.

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

OR

$$S_n = \frac{n}{2} (t_1 + t_n)$$

24. 577, 585, 593, 601, ..... Find  $t_{560}, t_{890}, S_{501}, S_{888}, S_{1000}$

→ In this A.P.  $a = 577, d = 8$

$$t_{560} = a + 559d = 577 + (559 \times 8) = 5049$$

$$t_{890} = a + 889d = 577 + (889 \times 8) = 7689$$

$$S_{501} = \frac{501}{2} [2a + 500d] = \frac{501}{2} [1154 + 500 \times 8] = 1291077$$

$$S_{888} = \frac{888}{2} [2a + 887d] = \frac{888}{2} [1154 + 887 \times 8] = 3663000$$

$$S_{1000} = \frac{1000}{2} [2a + 999d] = \frac{1000}{2} [1154 + 999 \times 8] = 4573000$$

25.  $5888 + 5892 + 5896 + \dots + 6900 = ?$

→ In this A.P.  $a = 5888, d = 4, t_n = 6900$

$$t_n = a + (n-1)d$$

$$6900 = 5888 + (n-1)4$$

$$n = 254$$

∴ There are 254 terms in above AP

$$S_{254} = \frac{254}{2} [11776 + 253 \times 4] \text{ OR } S_{254} = \frac{254}{2} (t_1 + t_{254})$$

$$= 16,24,076 \quad = 127 (5888 + 6900)$$

$$= 16,24,076$$

26.  $2887 + 2893 + 2899 + \dots + 6415 = ?$

→  $t_n = a + (n-1)d$

$$6415 = 2887 + (n-1)6$$

$$n = 589$$

$$S_{589} = \frac{589}{2} (2887 + 6415) = 27,39,439/-$$



27.  $1083 + 1090 + 1097 + 1104 + \dots + 89066 = ?$

→ In this A.P.  $a = 1083$ ,  $d = 7$ ,  $t_n = 89066$

$$t_n = a + (n-1)d$$

$$89066 = 1083 + (n-1)7 \quad \therefore S_{12570} = \frac{12570}{2} (1083 + 89066)$$

$$n = 12570 \quad = 566,586,465$$

28.  $9999 + 10004 + 10009 + \dots$  Find  $S_{80}$ ,  $S_{122}$

→ In this A.P.  $a = 9999$   $d = 5$

$$S_{80} = \frac{80}{2} [(2 \times 9999) + 79 \times 5] = 40 (19998 + 395) = 8,15,720$$

$$S_{122} = \frac{122}{2} [(2 \times 9999) + 121 \times 5] = 61 (19998 + 605) = 12,56,783$$

29. Find sum of all natural numbers divisible by 7 between 10,000 & 2,00,000

→  $10,003 + 10,010 + 10,017 + \dots + 1,99,997 = ?$

$$a = 10003, t_n = 199997$$

$$199997 = 10003 + (n-1)7$$

$$n = 27143$$

$$S_{27143} = \frac{27143}{2} (10003 + 199997)$$

$$= 2,85,00,15,000$$

30. Find sum of all 4 digit numbers divisible by 9.

→  $1008 + 1017 + 1026 + \dots + 9999 = ?$

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2} (t_1 + t_n)$$

$$9999 = 1008 + (n-1)9 \quad S_{1000} = \frac{1000}{2} (1008 + 9999)$$

$$n = 1000$$

$$= 55,03,500$$

31. Find sum of all natural numbers divisible by 13 between 1000 and 90,000

$$\longrightarrow 1001 + 1014 + 1027 + \dots + 89999 = ?$$

$$t_n = a + (n-1)d$$

$$89999 = 1001 + (n-1)13$$

$$n = 6847$$

$$S_{6847}$$

$$= \frac{6847}{2} (1001 + 89999)$$

$$= 311538500$$

32. Find sum of all natural numbers divisible by 17 between 11,000 & 1,20,000

$$\longrightarrow 11016 + 11033 + 11050 + \dots + 1,19,986 = ?$$

$$\therefore t_n = a + (n-1)d$$

$$1,19,986 = 11016 + (n-1)17$$

$$n = 6411$$

$$\therefore S_{6411} = \frac{6411}{2} (11016 + 1,19,986) = 419926911/-$$

33. Find sum of all natural numbers divisible by 5 from 100 to 900.

$$\longrightarrow 100 + 105 + 110 + \dots + 900 = ?$$

$$t_n = a + (n-1)d$$

$$900 = 100 + (n-1)5$$

$$\therefore n = 161$$

$$S_{161} = \frac{161}{2} (100 + 900)$$

$$= 80,500/-$$

34. Find sum of all natural numbers between 200 and 2500 such that on division by 12 number leaves remainder of 5. (Refer page .233)

$$\longrightarrow 209 + 221 + 233 + \dots + 2489 = ?$$

$$t_n = a + (n-1)d = 2489 = 209 + (n-1)12 \therefore n = 191$$

$$\therefore S_{191} = \frac{191}{2} (209 + 2489) = 257659/-$$

35. Find sum of all such numbers between 100 and 800 such that on division by 16 that number leaves remainder of 7.

→  $103 + 119 + 135 + \dots + 791 = ?$

$$t_n = a + (n-1)d = 791 = 103 + (n-1)16$$

$$\therefore n = 44$$

$$S_{44} = \frac{44}{2} (103 + 791) = 19,668/-$$

36. Find sum of all numbers between 1000 and 12000 such that on division by 10 that number leaves remainder of 8.

→  $1008 + 1018 + 1028 + \dots + 11998 = ?$

$$t_n = a + (n-1)d$$

$$11998 = 1008 + (n-1)10$$

$$n = 1100$$

$$S_{1100} = \frac{1100}{2} (1008 + 11998)$$

$$= 71,53,300$$

37. For A.P.  $t_8 = 238$

$t_{13} = 327$ . Find  $t_{30}, t_{90}, S_{80}$

→  $a + 7d = 238$

$$a + 12d = 327$$

$$\begin{array}{r} - \\ - \\ \hline -5d = -89 \end{array}$$

$$d = 17.80$$

$$a + (7 \times 17.80) = 238$$

$$a = 113.40$$

$$t_{30} = a + 29d$$

$$= 113.40 + (29 \times 17.80) = 629.60$$

$$t_{90} = a + 89d$$

$$= 113.40 + (89 \times 17.80) = 1697.60$$

$$S_{80} = \frac{80}{2} [226.80 + (79 \times 17.80)] = 65,320/-$$

38. For A.P.  $t_{80} = 5087$   
 $t_{92} = 5843$ . Find  $t_{200}$ ,  $S_{85}$



$$\begin{array}{r} a + 91d = 5843 \\ a + 79d = 5087 \\ \hline \end{array}$$

$$12d = 756$$

$$d = 63$$

$$a + (91 \times 63) = 5843$$

$$a = 110$$

$$\begin{aligned} t_{200} &= a + 199d \\ &= 110 + (199 \times 63) \\ &= 12647 \end{aligned}$$

$$\begin{aligned} S_{85} &= \frac{85}{2} (220 + 84 \times 63) \\ &= 234260 \end{aligned}$$

39. For A.P.  $t_{80} = 53810$   
 $t_{110} = 65850$ . Find  $a$ ,  $d$ ,  $t_{200}$ ,  $S_{250}$ ,  $S_{1000}$ ,  $t_{1250}$



$$\begin{array}{r} a + 79d = 53810 \\ a + 109d = 65850 \\ \hline \end{array}$$

$$-30d = -12040$$

$$d = 401.333333$$

$$a + (79 \times 401.3333) = 53810$$

$$a = 22104.666666$$

$$\begin{aligned} t_{200} &= 22104.66666 + \\ &\quad (199 \times 401.33333) \\ &= 1,01,970/- \end{aligned}$$

$$S_{250} = \frac{250}{2} [44209.333333 + (249 \times 401.3333)]$$

$$= 18017666.66666$$

$$S_{1000} = \frac{1000}{2} [44209.3333 + (999 \times 401.3333)]$$

$$= 222570666.66666$$

$$\begin{aligned} t_{1250} &= 22104.6666 + (1249 \times 401.33333) \\ &= 523370/- \end{aligned}$$



# AP & GP Sequence & Series



40. For AP  $t_8 = 90$ ;  $S_{20} = 1080$

Find  $a, d, t_{10}, S_{40}$



$$a + 7d = 90 \text{ ————— (1)}$$

$$S_{20} = \frac{20}{2} (2a + 19d) = 1080$$

$$2a + 19d = 108 \text{ ————— (2)}$$

$$2a + 19d = 108$$

$$2a + 14d = 180$$

$$\begin{array}{r} 2a + 19d = 108 \\ - \quad \quad \quad - \\ \hline 2a + 14d = 180 \\ - \quad \quad \quad - \\ \hline \end{array}$$

$$5d = -72$$

$$d = -14.40$$

$$a + (7 \times -14.40) = 90$$

$$a = 190.80$$

$$\begin{aligned} t_{10} &= a + 9d \\ &= 190.80 + (9 \times -14.40) \\ &= 61.20 \end{aligned}$$

$$\begin{aligned} S_{40} &= \frac{40}{2} [2a + 39d] \\ &= 20 [381.60 + (39 \times -14.40)] \\ &= -3600 \end{aligned}$$

41. For AP  $t_m = n$ ;  $t_n = m$ . Find  $t_r = ?$

a. Zero

b.  $m + n + 2r$

~~c.  $m + n - r$~~

d.  $2m + n - 2r$



$$t_m = n$$

$$a + (m-1)d = n$$

$$a + md - d = n \text{ ————— (1)}$$

$$t_n = m$$

$$a + (n-1)d = m$$

$$a + nd - d = m \text{ ————— (2)}$$

$$a + md - d = n$$

$$a + nd - d = m$$

$$\begin{array}{r} a + md - d = n \\ - \quad \quad \quad + \quad \quad - \\ \hline a + nd - d = m \\ - \quad \quad \quad - \quad \quad - \\ \hline \end{array}$$

$$md - nd = n - m$$

$$d(m-n) = -(m-n)$$

$$d = -1$$

use eqn (1)

$$a + m(-1) - (-1) = n$$

$$a - m + 1 = n$$

$$a = m + n - 1$$

$$t_r = a + (r-1)d$$

$$= m + n - 1 + (r-1)(-1)$$

$$= m + n - 1 - r + 1$$

$$= m + n - r$$

42. For AP  $S_n = (8n^2 + 10n)$ . Find  $t_{25}$



$$S_n = 8n^2 + 10n$$

$$S_{25} = 8(25)^2 + 10 \times 25$$

$$= 5250$$

$$S_{24} = 8(24)^2 + 10 \times 24$$

$$= 4848$$

$$t_{25} = S_{25} - S_{24}$$

$$= 5250 - 4848$$

$$= 402$$

$$S_n = 8n^2 + 10n$$

$$S_1 = 8(1)^2 + 10(1) = 18$$

$$\boxed{a = 18}$$

$$S_2 = 8(2)^2 + 10(2) = 52$$

$$18 + t_2 = 52$$

$$\boxed{t_2 = 34}$$

$$d = 34 - 18 = 16$$

$$\therefore t_{25}$$

$$= 18 + (24 \times 16)$$

$$= 402$$

$$S_n = 8n^2 + 10n$$

$$= n(8n + 10)$$

$$= \frac{n}{2} \times 2 \times (8n + 10)$$

$$= \frac{n}{2} (16n + 20)$$

$$= \frac{n}{2} [16(n-1) + 16 + 20]$$

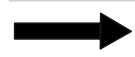
$$S_n = \frac{n}{2} [2(18) + (n-1)16]$$

$$\therefore a = 18, d = 16$$

$$t_{25} = 18 + (24 \times 16)$$

$$= 402$$

43. For AP If  $S_n = (10n^2 - 3n)$ . Find  $t_n, t_{50}$



$$S_n = 10n^2 - 3n$$

$$S_1 = 10(1)^2 - 3(1) = 7 \quad a = 7$$

$$S_2 = 10(2)^2 - 3(2) = 34 \quad t_2 = 27 \quad \therefore d = 20$$

$$\textcircled{1} t_n = a + (n-1)d$$

$$= 7 + (n-1)20 = 7 + 20n - 20$$

$$= 20n - 13$$

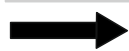
$$\textcircled{2} t_{50} = 20(50) - 13 = 987$$

OR

$$= a + 49d = 7 + 49(20) = 987$$



44. For AP  $S_n = (16n^2 - 5n)$ . Find  $t_n, t_{80}, t_{120}$



$$S_n = 16n^2 - 5n$$

$$S_1 = 16(1)^2 - 5(1) = 11$$

$$a = 11$$

$$S_2 = 16(2)^2 - 5(2) = 54$$

$$t_2 = 54 - 11 = 43$$

$$\therefore d = 43 - 11 = 32$$

$$\begin{aligned} \textcircled{1} \quad t_n &= a + (n-1)d = 11 + (n-1)32 = 11 + 32n - 32 \\ &= 32n - 21 \end{aligned}$$

$$\textcircled{2} \quad t_{80} = 32(80) - 21 = 2539$$

$$\textcircled{3} \quad t_{120} = 32(120) - 21 = 3819$$

45. For AP; If  $t_n = (16n - 9)$ . Find  $S_n, S_{20}, S_{28}$



$$t_n = 16n - 9$$

$$t_1 = 16(1) - 9 = 7 \quad \therefore \text{First term} = 7$$

$$\begin{aligned} S_n &= \frac{n}{2} [t_1 + t_n] = \frac{n}{2} [7 + 16n - 9] = \frac{n}{2} (16n - 2) \\ &= n(8n - 1) = (8n^2 - n) \end{aligned}$$

$$S_{20} = 8(20)^2 - 20 = 3180$$

$$S_{28} = 8(28)^2 - 28 = 6244$$

Question : For AP  $S_n = \left( \frac{5n^2 - 11n}{3} \right)$  Find  $t_n$ .

$$\implies S_1 = \frac{5(1)^2 - 11(1)}{3} = -\frac{6}{3} \quad a = -\frac{6}{3} \quad t_2 = -\frac{2}{3} + \frac{6}{3} = \frac{4}{3}$$

$$S_2 = \frac{5(2)^2 - 11(2)}{3} = -\frac{2}{3} \quad \therefore d = \frac{4}{3} - \left(-\frac{6}{3}\right) = \frac{10}{3} \quad \begin{matrix} = S_2 - S_1 \end{matrix}$$

$$t_n = -\frac{6}{3} + (n-1)\left(\frac{10}{3}\right) \quad 177 = -\frac{6}{3} + \frac{10n}{3} - \frac{10}{3} = \left(\frac{10n-16}{3}\right)$$

# AP & GP Sequence & Series



46. For AP if  $t_n = \left(\frac{6n+11}{2}\right)$ . Find  $S_n =$



$$t_n = \frac{6n+11}{2}$$

$$t_1 = \frac{6(1)+11}{2}$$

$$= \frac{17}{2}$$

$$S_n = \frac{n}{2} (t_1 + t_n)$$

$$= \frac{n}{2} \left( \frac{17}{2} + \frac{6n+11}{2} \right)$$

$$= \frac{n}{2} \left( \frac{6n+28}{2} \right) = \frac{6n^2+28n}{4}$$

$$= \frac{\cancel{2}(3n^2+14n)}{\cancel{2} \times 2} = \left( \frac{3n^2+14n}{2} \right)$$

47. For AP  $t_n = (6n - 7)$ . Find  $S_m$



$$t_n = 6n - 7$$

$$t_1 = 6(1) - 7 = -1$$

$$t_m = 6m - 7$$

$$S_m = \frac{m}{2} (t_1 + t_m)$$

$$= \frac{m}{2} (-1 + 6m - 7)$$

$$= \frac{m}{2} (6m - 8)$$

$$= m(3m - 4)$$

$$S_m = 3m^2 - 4m$$

48. For AP  $S_m = \left(\frac{8m^2+5m}{3}\right)$ . Find  $t_n$



$$S_m = \frac{8m^2+5m}{3}$$

$$S_1 = \frac{8(1)^2+5(1)}{3} = \frac{13}{3}$$

$$S_2 = \frac{8(2)^2+5(2)}{3} = \frac{42}{3}$$

$$a = S_1 = \frac{13}{3}$$

$$t_2 = S_2 - S_1 = \frac{42}{3} - \frac{13}{3} = \frac{29}{3}$$

$$d = \frac{29}{3} - \frac{13}{3} = t_2 - t_1 = \frac{16}{3}$$

$$t_n = a + (n-1)d$$

$$= \frac{13}{3} + (n-1)\frac{16}{3}$$

$$= \frac{13}{3} + \frac{16n}{3} - \frac{16}{3}$$

$$= \frac{13+16n-16}{3} = \left( \frac{16n-3}{3} \right)$$

## AP & GP Sequence & Series



49. For AP  $S_n = (12n^2 - 2n)$ . Find  $t_{35}$ ,  $t_p$



$$S_n = 12n^2 - 2n$$

$$S_1 = 12(1)^2 - 2(1) = 10$$

$$S_2 = 12(2)^2 - 2(2) = 44$$

$$\therefore a = 10, t_2 = 34, d = 24$$

$$\begin{aligned} t_{35} &= a + 34d \\ &= 10 + (34 \times 24) \\ &= 826 \end{aligned}$$

$$\begin{aligned} t_p &= a + (p-1)d = 10 + (p-1)24 \\ &= 10 + 24p - 24 = 24p - 14 \end{aligned}$$

50. For AP  $t_{95} = 2083.50$

$$t_{105} = 5811.25$$

Find  $a, d, t_{200}, S_{558}$



$$a + 94d = t_{95} = 2083.50 \quad \text{--- ①}$$

$$a + 104d = t_{105} = 5811.25 \quad \text{--- ②}$$

$$a + (94 \times 372.775) = 2083.50$$

$$\boxed{a = -32957.35}$$

$$-10d = -3727.75$$

$$\boxed{d = 372.775}$$

$$\begin{aligned} t_{200} &= a + 199d = -32957.35 + (199 \times 372.775) \\ &= 41,224.875 \end{aligned}$$

$$S_{558} = \frac{558}{2} [-65914.70 + (557 \times 372.775)]$$

$$= 39540152.025$$

51. 9<sup>th</sup> term of AP is 93 and 93<sup>rd</sup> term of AP is 9. Find 102<sup>nd</sup> term of AP.



$$t_9 = a + 8d = 93 \quad \text{--- ①}$$

$$t_{93} = a + 92d = 9 \quad \text{--- ②}$$

$$a + 8d = 93$$

$$-a + 92d = 9$$

$$a + 8(-1) = 93$$

$$\therefore \boxed{a = 101}$$

$$-84d = 84$$

$$\boxed{d = -1}$$

$$t_{102} = a + 101d = 101 + 101(-1) = 101 - 101 = \text{Zero}$$



If  $t_m = n$  &  $t_n = m$ ; then  $t_{(m+n)} = 0$

$t_9 = 93$ ;  $t_{93} = 9$ ; then  $t_{(93+9)} = t_{102} = 0$

↑  
For AP

For AP if  $p^{\text{th}}$  term is  $q$  and  $q^{\text{th}}$  term is  $p$   
then  $t_{(p+q)} = \text{zero}$

52. For AP if  $t_{90} = 120$ ,  $t_{120} = 90$ . Find  $t_{100}$

→  $t_{90} = 120$  &  $t_{120} = 90$

∴  $t_{100} = 120 + 90 - 100$   
 $= 110$

(OR)

$$a + 89d = 120$$

$$a + 119d = 90$$

$$\underline{-30d = 30} \quad \boxed{d = -1}$$

$$a + 89(-1) = 120 \quad \therefore a = 209$$

$$t_{100} = 209 + 99(-1) = 110$$

By using If  $t_m = n$ ,  $t_n = m$   
then  $t_p = m + n - p$

If possible please remember,  
For AP if  $t_m = n$ ,  $t_n = m$  the  $t_p = m + n - p$

53. For AP if  $10 \times t_{10} = 30 \times t_{30}$ . Find  $t_{40}$



$$10 \times t_{10} = 30 \times t_{30}$$

$$10(a + 9d) = 30(a + 29d)$$

$$10a + 90d = 30a + 870d$$

$$-780d = 20a$$

$$\boxed{a = -39d}$$

$$t_{40} = ?$$

$$t_{40} = a + 39d$$

$$= -39d + 39d = \text{Zero} = 0$$

For AP If  
 $m \times t_m = n \times t_n$   
then  
 $t_{m+n} = \text{Zero}$

# AP & GP Sequence & Series



54. For AP If  $t_{60} \times 60 = 200 \times t_{200}$ . Find  $t_{260}$

→ By using

$$\text{If } t_p \times p = t_q \times q$$

$$\text{then } t_{p+q} = 0$$

$$60 \times t_{60} = 200 \times t_{200}$$

$$\therefore t_{60+200} = t_{260} = 0$$

= Zero

OR

$$(a + 59d) \times 60 = 200(a + 199d)$$

$$60a + 3540d = 200a + 39800d$$

$$-36260d = 140a$$

$$a = -259d$$

$$t_{260} = a + 259d$$

$$= -259d + 259d$$

$$= \text{Zero} = 0$$

55. For AP If  $S_{30} = S_{50}$ . Find  $S_{80}$

→  $S_{30} = S_{50}$

$$\frac{30}{2}(2a + 29d) = \frac{50}{2}(2a + 49d)$$

$$30a + 435d = 50a + 1225d$$

$$-790d = 20a$$

$$\therefore 2a = -79d$$

$$S_{80} = ?$$

$$S_{80} = \frac{80}{2}(2a + 79d)$$

$$= 40(-79d + 79d)$$

$$= 40 \times 0$$

$$= \text{Zero} = 0$$

**For AP If  $S_m = S_n$ . then  $S_{(m+n)} = 0$**

**For AP**

1. If  $t_p = q, t_q = p$  then  $t_{p+q} = 0$

2. If  $p \times t_p = q \times t_q$  then  $t_{p+q} = 0$

3. If  $S_p = S_q$  then  $S_{p+q} = 0$

**Must remember**

**3 Short-cuts for AP**

①  $t_x = y$  &  $t_y = x$  then

$$t_{(x+y)} = \text{Zero}$$

②  $x \cdot t_x = y \cdot t_y$  then

$$t_{(x+y)} = \text{Zero}$$

③  $S_x = S_y$  then

$$S_{(x+y)} = \text{Zero}$$



56.

**For AP**

1. If  $m^{\text{th}}$  term is 'n' &  $n^{\text{th}}$  term is 'm', then  
(m+n)<sup>th</sup> term must be 'ZERO'
2. If 'm' times of  $m^{\text{th}}$  term is equal to 'n' times of  $n^{\text{th}}$  term,  
then (m+n)<sup>th</sup> term must be 'ZERO'
3. If sum of first 'm' terms of AP is equal to sum of first  
'n' terms of AP then sum of first 'm+n' terms must be  
'ZERO'

57.  $(3x+7)$ ,  $(5x+8)$ ,  $(19x-17)$  are in A.P. Find x.



$$\begin{aligned}
 t_2 - t_1 &= t_3 - t_2 \\
 5x+8 - (3x+7) &= 19x-17 - (5x+8) \\
 5x+8 - 3x-7 &= 19x-17-5x-8 \\
 2x+1 &= 14x-25 \\
 26 &= 12x \\
 x &= 26/12 = 13/6 = 2.1666666 = 2\frac{1}{6}
 \end{aligned}$$

58.  $(8k+7)$ ,  $(19k-8)$ ,  $(22k+38)$  are in A.P. Find the value of k.



$$\begin{aligned}
 t_2 - t_1 &= t_3 - t_2 \\
 19k-8 - 8k-7 &= 22k+38 - 19k+8 \\
 11k-15 &= 3k+46 \\
 8k &= 61 \\
 k &= 61/8 = 7.6250
 \end{aligned}$$

59.  $(3p-19)$ ,  $(8p-55)$ ,  $(20p+281)$  are in A.P. Find the value of p.



$$\begin{aligned}
 t_2 - t_1 &= t_3 - t_2 \\
 8p-55 - 3p+19 &= 20p+281 - 8p+55 \\
 5p-36 &= 12p+336 \quad \therefore 7p = -372 \quad \therefore p = \frac{-372}{7}
 \end{aligned}$$

60.  $(2k-19), (-5k-63), (-9k+25)$  are in A.P. Find the value of  $k$ .



$$t_2 - t_1 = t_3 - t_2$$

$$-5k-63 - 2k+19 = -9k+25 + 5k+63$$

$$-7k-44 = -4k+88$$

$$-132 = 3k \quad \therefore k = \frac{-132}{3} = -44$$

61. If  $\log a, \log b, \log c$  are in A.P. then  $a, b, c$  are in \_\_\_\_\_

a. A.P.

~~b. G.P.~~

c. H.P.

d. None of these



$\log a, \log b, \log c$  are in A.P.

$$\log b - \log a = \log c - \log b$$

$$\log \left( \frac{b}{a} \right) = \log \left( \frac{c}{b} \right)$$

$$\left( \frac{b}{a} \right) = \left( \frac{c}{b} \right) \quad \therefore a, b, c \text{ are in G.P.}$$

62.  $x, y, z$  are said to be in

A.P. If

$$y - x = z - y$$

$$2y = x + z$$

$$y = \left( \frac{x+z}{2} \right)$$

$$\text{AM of } x \text{ \& } z = \left( \frac{x+z}{2} \right)$$

$$\text{AM of } 80 \text{ \& } 120 = \left( \frac{80+120}{2} \right) = 100$$

G.P.

If

$$\frac{y}{x} = \frac{z}{y} \quad \therefore y^2 = xz$$

$$y = \sqrt{xz}$$

$$\text{GM of } x \text{ \& } z = \sqrt{xz}$$

$$\text{GM of } 9 \text{ \& } 36 = \sqrt{9 \times 36} = 18$$

63. For A.P.  $a = 48, t_n = 500, n = 30$ . Find  $S_{30}$



$$S_{30} = \frac{30}{2} (t_1 + t_n) = 15 (48 + 500) = 8220$$



64.  $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + 8\frac{2}{3} + 8\frac{1}{3} + \dots$

Find  $S_{30}$ . Also Find  $S_{31}$ .



$$\frac{30}{3}, \frac{29}{3}, \frac{28}{3}, \frac{27}{3}, \frac{26}{3}, \dots$$

In this A.P.  $a = 10$ ,  $d = -\frac{1}{3}$

$$S_{30} = \frac{30}{2} \left[ 20 + (29 \times -\frac{1}{3}) \right] = 15 \left( \frac{60}{3} - \frac{29}{3} \right) = 15 \times \frac{31}{3} = 155$$

$$S_{31} = \frac{31}{2} \left[ 20 + (30 \times -\frac{1}{3}) \right] = \frac{31}{2} \times (20 - 10) = 155$$

$$S_{61} = \frac{61}{2} \left[ 20 + (60 \times -\frac{1}{3}) \right] = \frac{61}{2} (20 - 20) = \text{Zero}$$

65. Find sum of all 4 digit natural even numbers divisible by 9.



$$1008 + 1026 + 1044 + \dots + 9990 = ?$$

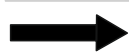
$$t_n = a + (n-1)d$$

$$9990 = 1008 + (n-1)18$$

$$n = 500$$

$$S_{500} = \frac{500}{2} (1008 + 9990) = 27,49,500/-$$

66. Find sum of all 3 digit natural numbers divisible by 7.



$$105 + 112 + 119 + \dots + 994 = ?$$

$$t_n = a + (n-1)d$$

$$994 = 105 + (n-1)7$$

$$\therefore n = 128$$

$$S_{128} = \frac{128}{2} (105 + 994) = 70,336/-$$



67.  $81^3 + 82^3 + 83^3 + \dots + 120^3 = ?$

$$\begin{aligned} \longrightarrow &= (1^3 + 2^3 + 3^3 + \dots + 120^3) - (1^3 + 2^3 + \dots + 80^3) \\ &= \left( \frac{120 \times 121}{2} \right)^2 - \left( \frac{80 \times 81}{2} \right)^2 = 42210000/- \end{aligned}$$

68.  $500^2 + 501^2 + 502^2 + \dots + 610^2 = ?$

$$\begin{aligned} \longrightarrow &= (1^2 + 2^2 + 3^2 + \dots + 610^2) - (1^2 + 2^2 + \dots + 499^2) \\ &= \left( \frac{610 \times 611 \times 1221}{6} \right) - \left( \frac{499 \times 500 \times 999}{6} \right) \\ &= 3,43,04,735 \end{aligned}$$

69.  $9999 + 10004 + 10009 + 10014 + \dots + 1192944 = ?$

$\longrightarrow$  In this A.P.  $a = 9999$ ,  $d = 5$ ,  $t_n = 1192944$

$$t_n = a + (n-1)d$$

$$1192944 = 9999 + (n-1)5$$

$$n = 2,36,590$$

$$S_{236590} = \frac{236590}{2} (9999 + 1192944) = 142302142185$$

70. For A.P.  $S_n = \frac{11n^2 - 7n}{2}$ . Find  $t_n =$

$$\longrightarrow S_n = \frac{(11n^2 - 7n)}{2}$$

$$S_1 = \frac{11(1)^2 - 7(1)}{2} = 2 \quad \therefore a = 2$$

$$S_2 = \frac{11(2)^2 - 7(2)}{2} = 15 \quad t_2 = 13$$

$$d = 11$$

$$t_n$$

$$= a + (n-1)d$$

$$= 2 + (n-1)11$$

$$= 2 + 11n - 11$$

$$= 11n - 9$$

71. If  $a, b, c$  are in A.P.

then 'b' is Arithmetic Mean (A.M.) of  $a$  &  $c$

As  $a, b, c$  are in A.P.

$$b - a = c - b$$

$$b + b = a + c$$

$$2b = a + c$$

$$b = \frac{a+c}{2}$$

$$\text{A.M. of } a \text{ \& } c = \left( \frac{a+c}{2} \right)$$

$$\text{AM of } x \text{ \& } 35 = \frac{x+35}{2}$$

$$\text{A.M. of } x \text{ \& } y = \left( \frac{x+y}{2} \right)$$

$$\text{AM of } 1 \text{ \& } 2 = \frac{1+2}{2} = 1.50$$

$$\text{A.M. of } m, n = \left( \frac{m+n}{2} \right)$$

$$\text{AM of } 1000 \text{ \& } m = \frac{1000+m}{2}$$

$$\text{A.M. of } 50 \text{ \& } 150 = \left( \frac{50+150}{2} \right) = 100$$

72.

1. If  $a, b, c$  are in A.P. then we can say that :  $b, c$  are 2  
'd

A. Mean between  $a$  &  $d$

57, 62, 67, 72 are in A.P.

$\therefore$  62, 67 are 2 A.means between 57 & 72

2. If  $p, q, r, s$  are in A.P. then we can say that  $q, r$  are 2 A.means between  $p$  &  $s$

3. If  $p, q, r, s, t$  are in A.P. then we can say that  $q, r, s$  are 3 A.means between  $p$  &  $t$

4. 90, 101, 112, 123, 134, 145 are in A.P. then

- 
- 101, 112, 123, 134 are 4 A.means bet<sup>n</sup> 90 & 145
  - 112, 123 are 2 A.means bet<sup>n</sup> 101 & 134
  - 123, 134 are 2 A.means bet<sup>n</sup> 112 & 145
  - 101, 112, 123 are 3 A.means bet<sup>n</sup> 90 & 134
  - 123 is AM bet<sup>n</sup> 101 & 145

73. 4, 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, 70, 76, 82, 88, 94, 100, 106, 112, 118, 124, 130, 136, 142, 148, 154 are in A.P. with  $d = 6$

4, 16, 28, 40, 52, 64, 76, 88, 100, 112, 124, 136, 148 are also in A.P. with  $d = 12$

4, 22, 40, 58, 76, 94, 112, 130, 148 are also in A.P. with  $d = 18$

If a, b, c, d, e, f, g, h, i, j, k are in A.P. then

|                  |   |                  |
|------------------|---|------------------|
| a, c, e, g, i, k | } | are also in A.P. |
| a, d, g, j       |   |                  |
| a, e, i          |   |                  |

74. If a, b, c, d, e are in A.P. then

1. b, c, d are 3 A.means between a & e
2. b, c are 2 A.means between a & d
3. c, d are 2 A.means between b & e
4. b is A.M. of a & c
5. c is A.M. of b & d
6. c is A.M. of a & e as a, c, e are also in A.P.

75. Find A.M. of 20 & 50

→ AM of 20 & 50 =  $\left(\frac{20+50}{2}\right) = 35$

20, 35, 50 are in A.P.

76. Insert 2 A.Means between 80 & 120

→ 80,  $93\frac{1}{3}$ ,  $106\frac{2}{3}$ , 120

$$a = 80$$

$$t_4 = 120$$

$$a + 3d = 120$$

$$80 + 3d = 120$$

$$3d = 40$$

$$d = 13.3333$$

Ans:

2 A.means are

$$93.333333, 106.6666666$$

$\therefore 93\frac{1}{3}$  &  $106\frac{2}{3}$  are 2 A.means  
bet<sup>n</sup> 80 & 120 as

80,  $93\frac{1}{3}$ ,  $106\frac{2}{3}$ , 120 are in A.P.

77. Insert 5 A.Means between 20 & 200

→ 20, 50, 80, 110, 140, 170, 200

$$a = 20$$

$$t_7 = 200$$

$$a + 6d = 200$$

$$20 + 6d = 200$$

$$6d = 180$$

$$d = 30$$

$\therefore 50, 80, 110, 140, 170$  are

5 A.means between 20 & 200

78. Insert 7 A.Means between 5000 & 80,000

→  $a = 5000$   
 $t_9 = 80,000$

$$a + 8d = 80,000$$

$$5000 + 8d = 80,000$$

$$d = 9375$$

$\therefore 7$  A.Means are :

14375, 23750, 33125, 42500, 51875, 61250, 70625.

as 5000, 14375, 23750, 33125, 42500, 51875, 61250, 70625, 80000 are in A.P.



79. Insert 4 A.Means between  $(-8256)$  and  $5820$

→  $-8256, -5440.80, -2625.60, 189.60, 3004.80, 5820$   
are in A.P.

$$t_6 = 5820$$

$$a + 5d = 5820$$

$$-8256 + 5d = 5820$$

$$d = 2815.20$$

$\therefore -5440.80, -2625.60, 189.60, 3004.80$  are 4 A.means between  $-8256$  &  $5820$

80. Insert 7 A.Means between  $80$  &  $200$

→  $a = 80$   
 $t_9 = 200$   
 $a + 8d = 200$   
 $80 + 8d = 200$   
 $8d = 120$   
 $d = 15$

$\therefore 95, 110, 125, 140, 155, 170, 185$  are 7 A.means between  $80$  &  $200$ .

81. Insert 9 A.Means between  $208.60$  &  $4286.20$

→  $a = 208.60$   
 $t_{11} = a + 10d = 4286.20$   
 $208.60 + 10d = 4286.20$   
 $d = 407.76$

$\therefore$  9 A.Means between  $208.60$  &  $4286.20$  are :  
 $616.36, 1024.12, 1431.88, 1839.64, 2247.40, 2655.16, 3062.92, 3470.68, 3878.44$

82. Insert 11 A.Means between 2860 and 9640



$$a = 2860$$

$$t_{13} = 9640$$

$$a + 12d = 9640$$

$$2860 + 12d = 9640$$

$$d = 565$$

$\therefore$  11 A.means between 2860 & 9640 are

3425, 3990, 4555, 5120, 5685,  
6250, 6815, 7380, 7945, 8510,  
9075

83. Insert 2 A.Means between -20 & 22



$$a = -20$$

$$t_4 = 22$$

$$a + 3d = 22$$

$$-20 + 3d = 22$$

$$3d = 42$$

$$d = 14$$

$\therefore$  2 A.means between -20 & 22 are :

-6, 8

84. 2, 8, 32, 128, 512, 2048, 8192, ..... are in G.P. Find  $t_8$

In this G.P. First Term = 2 = a

Second Term =  $t_2 = a.r$

$$t_3 = t_2.r = a.r.r = a.r^2$$

$$t_4 = t_3.r = a.r^2.r = a.r^3$$

$$t_{10} = a.r^9$$

$$t_{2000} = a.(r)^{1999}$$

$$t_n = a.(r)^{n-1}$$

$$\frac{t_2}{t_1} = r$$

$$\therefore t_2 = t_1 \times r$$

$$\frac{t_{10}}{t_9} = r$$

$$\therefore t_{10} = t_9 \times r$$

$$t_8 = a.r^7 = 2 \times (4)^7 = 32768$$

$$t_{25} = t_{24} \times r = t_{26} \div r$$

## AP & GP Sequence & Series



$$\therefore n^{\text{th}} \text{ term of G.P.} = t_n = a \times (r)^{n-1} \\ = a \cdot r^{n-1}$$

$$t_8 = a \cdot r^7 = 2 \times (4)^7 = 32,768$$

85. 5, 15, 45, 135, ..... Find  $t_{10}, t_{12}, t_{22}$

→ In this G.P.  $a = 5, r = 3$

$$\therefore t_{10} = a \cdot r^9 = 5 \times (3^9) = 98,415$$

$$t_{12} = a \cdot r^{11} = 5 \times (3^{11}) = 8,85,735$$

$$t_{22} = a \cdot r^{21} = 5 \times (3^{21}) = 52301766015$$

86. 2, 3, 4.5, 6.75, ..... Find  $t_{23}, t_{28}, t_{40}$

→ In this G.P.  $a = 2, r = 1.50$

$$t_{23} = a \cdot r^{22} = 2 \times (1.50)^{22} = 14963.6552851$$

$$t_{28} = a \cdot r^{27} = 2 \times (1.50)^{27} = 113630.25732$$

$$t_{40} = a \cdot r^{39} = 2 \times (1.50)^{39} = 14743109.7607$$

87.  $2 + 6 + 18 + 54 + 162 = 242$  → In this G.P.  
First term =  $a = 2$   
common ratio =  $r = \frac{6}{2} = 3$

by Calculator

Let's solve by formula,

$$S_n \text{ for G.P.} = \left[ \frac{a(r^n - 1)}{r - 1} \right] \text{ when } r > 1$$

$$S_5 = \left[ \frac{2(3^5 - 1)}{3 - 1} \right] = \left[ \frac{2(243 - 1)}{2} \right] = 242$$

$$S_5 = \frac{2(3^5 - 1)}{(3 - 1)} = \frac{2 \times 242}{2} \\ = 242$$

88.  $10 + 15 + 22.50 + 33.75 + \dots$  Find  $S_{20}$ ,  $S_{31}$



In this G.P.  $a = \text{First term} = 10$

$r = \text{common ratio} = 1.50$

As  $r > 1$

$$S_n = \frac{a(r^n - 1)}{(r - 1)} \quad \therefore S_{20} = \frac{10(1.50^{20} - 1)}{1.50 - 1}$$

$$= 66485.1346008$$

$$S_{31} = \frac{a(r^{31} - 1)}{r - 1} = \frac{10(1.50^{31} - 1)}{1.50 - 1} = 5752511.7768$$

$$S_{50} = \frac{10(1.50^{50} - 1)}{1.50 - 1} = 12752429983.6$$

| 89. |       | For A.P.                                                       | For G.P.                                                                               |
|-----|-------|----------------------------------------------------------------|----------------------------------------------------------------------------------------|
|     | $t_n$ | $a + (n-1)d$                                                   | $a \times (r)^{n-1}$                                                                   |
|     | $S_n$ | $\frac{n}{2} [2a + (n-1)d]$<br>OR<br>$\frac{n}{2} (t_1 + t_n)$ | $\frac{a(r^n - 1)}{(r - 1)}$ when $r > 1$<br>$\frac{a(1 - r^n)}{(1 - r)}$ when $r < 1$ |

90.  $0.60, 0.80, 1.00, 1.20, 1.40, \dots$  Find  $t_n$ ,  $S_{30}$ ,  $t_{100}$



In this A.P.  $a = 0.60$ ,  $d = 0.20$

$$t_n = a + (n-1)d = 0.60 + (n-1)0.20 = 0.60 + 0.20n - 0.20$$

$$t_n = 0.20n + 0.40$$

$$S_{30} = \frac{30}{2} [1.20 + (29 \times 0.20)] = 165$$

$$t_{100} = 0.20(100) + 0.40 = 20.40 \text{ OR } a + 99d \\ = 0.60 + 99(0.20)$$

91.  $2 + 6 + 18 + 54 + \dots$  Find  $S_{20}, t_{28}$

In this G.P.  $a = 2; r = 3$

$$\rightarrow S_{20} = \frac{a(r^{20}-1)}{(r-1)} = \frac{2(3^{20}-1)}{(3-1)} = 3486784400$$

$$t_{28} = a \times r^{27} = 2 \times (3)^{27} \\ = 2 \times 3^{24} \times 3^3 \\ = 2 \times 282429536481 \times 27 \\ = 282429536481 \times 54$$

92.  $1.20 + 1.20^2 + 1.20^3 + 1.20^4 + \dots$  Find  $S_{40}$

$\rightarrow$  In this G.P.  $a = 1.20, r = 1.20$

$$S_{40} = \frac{a(r^{40}-1)}{(r-1)} = \frac{1.20(1.20^{40}-1)}{1.20-1} \\ = 8812.62940725$$

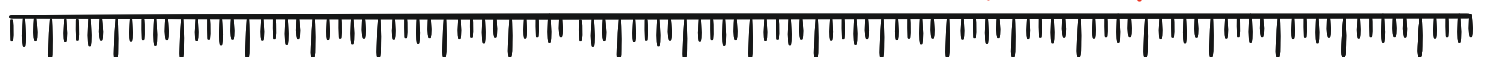
$$S_{51} = \frac{a(r^{51}-1)}{(r-1)} = \frac{1.20(1.20^{51}-1)}{1.20-1} = 65517.154675$$

93.  $100 + 80 + 64 + 51.20 + \dots$  Find  $S_{12}$

$\rightarrow$  In this G.P.  $a = 100, r = 0.80$

AS  $r < 1$

$$S_n = \frac{a(1-r^n)}{(1-r)} = \frac{100(1-0.80^{12})}{1-0.80} = \frac{100(1-0.06871947673)}{0.20} \\ = 465.640261635$$



94.  $200 + 140 + 98 + 68.60 + \dots$  Find  $S_{18}$

→ In this G.P.  $a = 200$ ,  $r = 0.70$

AS  $r < 1$

$$S_n = \frac{a(1-r^n)}{(1-r)} = \frac{200(1-0.70^{18})}{(1-0.70)} = S_{18}$$

$$= \frac{200(1-0.00162841359)}{0.30} = 665.581057606$$

95.  $4 + 6 + 9 + 13.50 + \dots$  Find  $S_{20}$ ,  $t_{25}$

→ In this G.P.  $a = 4$ ,  $r = 1.50$

$$S_{20} = \frac{4(1.50^{20}-1)}{(1.50-1)} = 26594.0538402$$

$$t_{25} = a \cdot r^{24} = 4 \times (1.50)^{24} = 67336.4487828$$

$$S_{31} = \frac{4(1.50^{31}-1)}{1.50-1} = 2301004.71072$$

96.  $5 + 10 + 20 + 40 + \dots$  Find sum of infinite terms of this G.P.  
(i.e.  $S_{\infty}$ )

→  $5 + 10 + 20 + 40 + \dots$  infinite terms

AS  $r > 1$

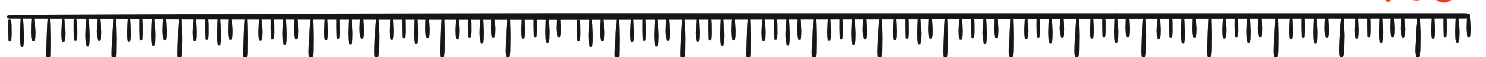
$$S_{\infty} = \frac{a(r^{\infty}-1)}{r-1} = \frac{a(2^{\infty}-1)}{(2-1)} = \infty$$

97.  $80 + 40 + 20 + 10 + 5 + \dots$  Find  $S_{\infty}$

→ In this G.P.  $a = 80$ ,  $r = 0.50$

$$S_{\infty} = \frac{a(1-r^{\infty})}{1-r} = \frac{a(1-0.50^{\infty})}{1-r} = \frac{a(1-0)}{1-r} = \frac{a}{(1-r)} = \frac{80}{1-0.50}$$

$$= 160$$



98.

Sum of Infinite terms of G.P.

When  $r > 1$

$$S_{\infty} = \infty$$

When  $0 < r < 1$

$$\begin{aligned} S_{\infty} &= \frac{a(1-r^{\infty})}{(1-r)} \\ &= \frac{a(1-0)}{(1-r)} \\ &= \left[ \frac{a}{(1-r)} \right] \end{aligned}$$

99.  $100 + 80 + 64 + 51.20 + \dots$  Find  $S_{\infty}$



In this G.P.  $a = 100$ ,  $r = 0.80$

AS  $0 < r < 1$

$$S_{\infty} = \frac{a}{1-r} = \frac{100}{1-0.80} = \frac{100}{0.20} = 500$$

100.  $500 + 200 + 80 + 32 + \dots$  Find sum of infinite terms.



In this G.P.  $a = 500$ ,  $r = 0.40$

AS  $0 < r < 1$

$$\therefore S_{\infty} = \frac{a}{1-r} = \frac{500}{1-0.40} = \frac{500}{0.60} = 833.3333$$

101.

Sum of Infinite terms of G.P. ( $S_{\infty}$ )

when  $r > 1$

$$S_{\infty} = \infty$$

when  $0 < r < 1$

$$S_{\infty} = \left( \frac{a}{1-r} \right)$$



102. If  $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p$  are in G.P.

with common ratio =  $r$

then

$a, c, e, g, i, k, m, o$  are also in G.P. with common ratio =  $r^2$

&  $a, d, g, j, m, p$  are also in G.P. with common ratio =  $r^3$

If  $a, b, c, d, e$  are in G.P. then

a.  $b, c, d$  are 3 G.Means between  $a$  &  $e$

b.  $c$  is G.M. of  $b, d$

c.  $c$  is G.M. of  $a, e$

103. Insert 3 Geometric Means between 3 and 7203

→ 3, 21, 147, 1029, 7203  
must be in G.P.

$$a = 3$$

$$t_5 = 7203$$

$$a \cdot r^4 = 7203$$

$$3 \times r^4 = 7203$$

$$r^4 = 2401$$

$$r = 7$$

$\therefore 21, 147, 1029$  are  
3 geometric means  
between 3 & 7203

104. Insert 5 G.Means between 10 & 40960

→ 10, 40, 160, 640, 2560, 10240, 40960

$$a = 10, t_7 = 40960$$

↓  
G.P.

# AP & GP Sequence & Series



$$\begin{aligned} a \cdot r^6 &= 40960 \\ 10 \times r^6 &= 40960 \\ r^6 &= 4096 \\ r^6 &= 4^6 \end{aligned}$$

$$\therefore r = 4$$

$\therefore$  5 G. means between  
10 & 40,960 are

$$40, 160, 640, 2560, 10240$$

105. Insert 3 G.Means between  $\frac{1}{9}$  & 9

→  $\frac{1}{9}, \left(\frac{1}{9} \times 3 = \frac{1}{3}\right), \left(\frac{1}{3} \times 3 = 1\right), (1 \times 3 = 3), 9$

$$a = \frac{1}{9}$$

$$t_5 = 9 \therefore r^4 = 81$$

$$a \cdot r^4 = 9 \therefore r = 3$$

$$\frac{1}{9} r^4 = 9$$

(a)  $\frac{1}{3}, 1, 3$

(b)  $-\frac{1}{3}, 1, -3$

~~(c) a OR b~~

(d) None

106.  $t_{81} = t_{80} \times r$

$$= t_{82} \div r$$

For G.P.

$$t_n \times r = t_{n+1}$$

$$t_{n+3} \div r = t_{n+2}$$

$$t_{100} \times r = t_{101}$$

$$t_{200} \times r^3 = t_{203}$$

$$t_{500} \div r^4 = t_{496}$$

In G.P  
any term  
can be  
obtained  
by  
i) multiplying  
by 'r' to its  
prec. term  
OR  
ii) Divi. by 'r'  
to its  
succ. term

$$t_{81} = t_{80} + d$$

$$= t_{82} - d$$

For A.P.

$$t_n + d = t_{n+1}$$

$$t_{n+5} - d = t_{n+4}$$

$$t_{33} + 2d = t_{35}$$

$$t_{100} - 3d = t_{97}$$

$$t_{508} - 4d = t_{504}$$

In A.P  
any term can  
be obtained by  
i) Adding 'd'  
to its prec.  
term  
OR  
ii) subtracting 'd'  
from its  
succ. term

107. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 are in Arithmetic progression.  
then  $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}, \frac{1}{18}, \frac{1}{20}, \frac{1}{22}$  are in Harmonic progression.

H.P.  $\rightarrow$  Harmonic Progression

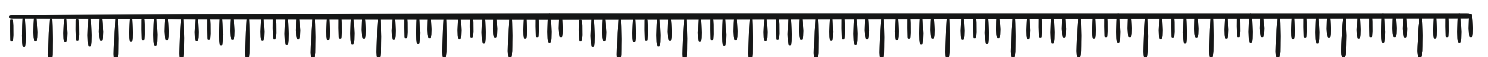
If  $\frac{p}{q}, \frac{r}{s}, \frac{m}{n}, \frac{18}{23}, \frac{x}{y}$  are in H.P. then

$\frac{q}{p}, \frac{s}{r}, \frac{n}{m}, \frac{23}{18}, \frac{y}{x}$  must be in A.P.

If  $\frac{x}{m}, \frac{n}{k}, \frac{1}{r}$  are in AP then

$\frac{m}{x}, \frac{k}{n}, \frac{r}{1}$  must be in HP

**IF FEW TERMS ARE IN A.P. THEN THEIR RECIPROCAL  
MUST BE IN H.P. AND VICE VERSA**



108.

If  $a, b, c$ , are in

A.P. then

G.P. then

H.P. then

$b$  is A.M. of  $a$  &  $c$

$b$  is G.M. of  $a$  &  $c$

$b$  is H.M. of  $a$  &  $c$

$$b - a = c - b$$

$$2b = a + c$$

$$b = \left( \frac{a+c}{2} \right)$$

$$\text{AM of } a \& c = \left( \frac{a+c}{2} \right)$$

$$\text{AM of } x \& y = \left( \frac{x+y}{2} \right)$$

$$\text{AM of } p \& q = \left( \frac{p+q}{2} \right)$$

$$\text{AM of } 10 \& 90 = \frac{10+90}{2} = 50$$

$$\text{AM of } 80, p = \frac{80+p}{2}$$

$$\frac{b}{a} = \frac{c}{b}$$

$$b^2 = ac \therefore b = \sqrt{ac}$$

$$\text{GM of } a \& c = \sqrt{ac}$$

$$\text{GM of } x \& y = \sqrt{xy}$$

$$\text{GM of } p, q = \sqrt{pq}$$

$$\text{GM of } 10 \& 90 = \sqrt{10 \times 90} = \pm 30$$

$$\text{GM of } 80 \& p = \sqrt{80p}$$

As  $a, b, c$  are in H.P.

$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  must be in A.P.

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\frac{1}{b} + \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\frac{2}{b} = \frac{a+c}{ac}$$

$$\frac{b}{2} = \frac{ac}{a+c}$$

$$b = \left( \frac{2ac}{a+c} \right)$$

$$\text{HM of } a \& c = \left( \frac{2ac}{a+c} \right)$$

$$\text{HM of } x \& y = \left( \frac{2xy}{x+y} \right)$$

$$\text{HM of } 20, 30 = \frac{2 \times 20 \times 30}{20+30} = 24.00$$

109. Find AM, GM, HM of 60, 30

$$\text{AM} = \frac{60+30}{2} = 45 \quad \text{HM} = \left( \frac{2 \times 60 \times 30}{60+30} \right) = 40$$

$$\text{GM} = \sqrt{60 \times 30} = \sqrt{1800} = \sqrt{900 \times 2} = 30\sqrt{2}$$

110. Mr. A travels Pune to Mumbai at a distance of 200 kms at a uniform speed of 100 kms / hr & returns at a uniform speed of 50 kms/hr. Find avg. speed of whole journey?

➔ Average Speed =  $\left( \frac{\text{Total distance travelled}}{\text{Total time taken}} \right)$

|                | distance<br>(kms) | speed<br>(kms/hr) | Time<br>= Dist/speed |
|----------------|-------------------|-------------------|----------------------|
| pune to mumbai | 200               | 100               | 2 hrs                |
| Mumbai to pune | 200               | 50                | 4 hrs                |

$$\text{Avg. speed} = \left( \frac{\text{Total dist. traveled}}{\text{Total time taken}} \right) = \left( \frac{400 \text{ kms}}{6 \text{ hrs}} \right) = 66.66666 \text{ kms/hr}$$

$$\text{HM of } 100 \text{ \& } 50 = \frac{2 \times 100 \times 50}{100 + 50} = 66.66666666$$

Q: Avg speed of journey can be calculated with help of :

- (a) AM (b) GM (c) ~~HM~~ (d) Median

111. Find H.M of 100, 50

➔ 
$$\text{HM of } 100 \text{ \& } 50 = \frac{2 \times 100 \times 50}{100 + 50} = 66.6666$$

HM of  $x$  &  $y$

$$\begin{aligned} \Rightarrow \text{HM} &= \frac{2xy}{x+y} = \left( \frac{\frac{2xy}{xy}}{\frac{x+y}{xy}} \right) = \left( \frac{2}{\frac{x}{xy} + \frac{y}{xy}} \right) \\ &= \left( \frac{2}{\frac{1}{y} + \frac{1}{x}} \right) = \left( \frac{1+1}{\frac{1}{x} + \frac{1}{y}} \right) = \left( \frac{2}{\frac{1}{x} + \frac{1}{y}} \right) \end{aligned}$$

112. If  $S_n$  for A.P. is  $(20n^2 - 13n)$ . Find  $t_n$



$$S_n = 20n^2 - 13n$$

$$S_1 = 20(1)^2 - 13(1) = 7 \quad a = 7$$

$$S_2 = 20(2)^2 - 13(2) = 54 \quad t_2 = 54 - 7 = 47$$

$$d = 47 - 7 = 40$$

$$t_n = a + (n-1)d$$

$$= 7 + (n-1)40 = 7 + 40n - 40 = (40n - 33)$$

113. For A.P. if  $90 \times t_{90} = 100 \times t_{100}$

Find  $t_{190}$

→ ZERO

3 IMP  
SHORTCUTS OF  
A.P.

If  $m \times t_m = n \times t_n$  then  $t_{m+n} = \text{ZERO}$

If  $t_m = n$  &  $t_n = m$  then  $t_{m+n} = \text{ZERO}$

If  $S_m = S_n$  then  $S_{m+n} = \text{ZERO}$

114.  $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty$  terms

(243) = ?



$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty \text{ terms} = ?$$

In this G.P.  $a=1$ ,  $r=\frac{1}{3}$  AS  $0 < r < 1$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2} = 1.50$$

$$(243) \left( 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty \text{ terms} \right) = (243)^{\frac{3}{2}} = (3^5)^{\frac{3}{2}} = 3^{\frac{15}{2}}$$

115. If  $x = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$

$y = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \dots$

Find value of (xy)



$x = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \infty \text{ terms}$

In this G.P.  $a=1, r=\frac{1}{4}$

$x = S_{\infty} = \frac{a}{1-r}$

$\therefore x = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

$y = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \dots \infty \text{ terms}$

In this G.P.  $a=\frac{1}{5}, r=\frac{1}{5} \therefore S_{\infty} = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}}$

$y = \frac{1}{5} \times \frac{5}{4} = \frac{1}{4}$

$x \cdot y = \frac{4}{3} \times \frac{1}{4} = \frac{1}{3}$

116.

Find AM, GM, HM of p,q

AM =  $\left(\frac{p+q}{2}\right)$

GM =  $\sqrt{pq}$

HM =  $\left(\frac{2pq}{p+q}\right)$

For 2 observations (positive),  $GM^2 = AM \times HM$

..... True / False



L.H.S. =  $GM^2 = (\sqrt{pq})^2 = pq$

R.H.S. =  $AM \times HM = \frac{(p+q)}{2} \times \frac{2pq}{(p+q)}$

=  $pq$

As L.H.S. = R.H.S.  $\therefore GM^2 = AM \times HM$  is True



117. For 2 observations  $GM = 9$ ,  $AM = 10$ ,  $HM = ?$



$$GM^2 = AM \times HM$$

$$9^2 = 10 \times HM$$

$$\therefore HM = \frac{81}{10} = 8.10$$

AM    GM    HM

10 , 9 , 8.10 : GP

9 is GM of 10 & 8.10

GM is GM of AM & HM

a. If  $b^2 = ac$ ;



b is GM of a & c

b. If  $GM^2 = AM \times HM$



GM is GM of AM & HM

For 2 obs<sup>n</sup>s : AM, GM, HM will be in G.P.

For 2 observations : GM is GM of AM & HM.

Question :  $3 + 33 + 333 + 3333 + 33333 + \dots n \text{ terms} = ?$

It is neither AP nor GP

$$= 3 + 33 + 333 + 3333 + \dots n \text{ terms}$$

$$= 3 (1 + 11 + 111 + \dots n \text{ terms})$$

$$= \frac{3}{9} \times 9 \times (1 + 11 + 111 + \dots n \text{ terms})$$

$$= \frac{3}{9} \times [9 + 99 + 999 + \dots n \text{ terms}]$$

$$= \frac{3}{9} \times [(10-1) + (100-1) + (1000-1) + \dots n \text{ terms}]$$

$$= \frac{3}{9} \times [(10 + 100 + 1000 + \dots n \text{ terms}) - (1 + 1 + 1 + \dots n \text{ terms})]$$

$$= \frac{3}{9} \times \left[ \frac{10 \times (10^n - 1)}{10 - 1} - n \right] = \frac{3}{9} \left[ \frac{10(10^n - 1)}{9} - n \right]$$

$$8 + 88 + 888 + \dots n \text{ terms} = \frac{8}{9} \left[ \frac{10(10^n - 1)}{9} - n \right]$$

**QUESTIONS FOR PRACTICE :**

118.  $9x + 13, 15x - 33, 18x + 88$  are in A.P. Find value of  $x$ .



$$t_2 - t_1 = t_3 - t_2$$

$$(15x - 33) - (9x + 13) = (18x + 88) - (15x - 33)$$

$$15x - 33 - 9x - 13 = 18x + 88 - 15x + 33$$

$$6x - 46 = 3x + 121$$

$$3x = 167$$

$$x = 167/3 = 55.666666 = 55\frac{2}{3}$$

119. Insert 5 A.Means between  $(-36)$  and  $1296$



$$-36, 186, 408, 630, 852, 1074, 1296$$

$$a = -36$$

$$t_7 = 1296$$

$$a + 6d = 1296$$

$$-36 + 6d = 1296$$

$$d = 222$$

$$\therefore 186, 408, 630, 852, 1074$$

are 5 A.means between

$$-36 \& 1296.$$

120. For 2 observations  $GM = 9, AM = 12, HM = ?$



For 2 observations,

$$GM^2 = AM \times HM$$

$$9^2 = 12 \times HM$$

$$HM = \frac{81}{12} = 6.75$$

AM    GM    HM

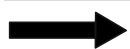
12    9    6.75

12, 9, 6.75 are in G.P.

9 is GM of 12 & 6.75

GM is GM of AM & HM

121.  $55^2 + 56^2 + 57^2 + \dots + 130^2 =$



$$(1^2 + 2^2 + 3^2 + \dots + 130^2) - (1^2 + 2^2 + 3^2 + \dots + 54^2)$$

$$= \left( \frac{130 \times 131 \times 261}{6} \right) - \left( \frac{54 \times 55 \times 109}{6} \right)$$

$$= 6,86,850/-$$

122. Sum of all terms of AP = 7171 =  $S_n$

$$t_1 = -4 \text{ \& } t_n = 146$$

Find No. of terms & common difference.

$$\begin{aligned} \rightarrow S_n &= \frac{n}{2} (t_1 + t_n) \\ 7171 &= \frac{n}{2} (-4 + 146) \\ 7171 &= n \times 71 \\ n &= 101 \end{aligned}$$

$$\begin{aligned} t_{101} &= 146 \\ a + 100d &= 146 \\ -4 + 100d &= 146 \\ 100d &= 150 \\ d &= 1.50 = \frac{3}{2} \end{aligned}$$

123. If  $a^{1/x} = b^{1/y} = c^{1/z}$  and a, b, c

are in G.P. then x, y, z are in :

- a. A.P.      b. G.P.      c. Both      d. None

$$\rightarrow a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$$

$$\begin{aligned} a^{\frac{1}{x}} &= k \\ (a^{\frac{1}{x}})^x &= k^x \\ a &= k^x \end{aligned}$$

$$\begin{aligned} \therefore b &= k^y \\ \& c &= k^z \end{aligned}$$

a, b, c are in G.P.

$$\begin{aligned} b^2 &= ac \\ (k^y)^2 &= k^x \cdot k^z \end{aligned}$$

$$k^{2y} = k^{x+z}$$

$$\therefore 2y = x + z$$

$$\begin{aligned} y+y &= x+z \\ y-x &= z-y \\ \therefore x, y, z &\text{ are in A.P.} \end{aligned}$$

124. If G is GM of a & b. Find the value of  $\frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2}$

- a.  $G^2$       b.  $3G^2$       ~~c.  $\frac{1}{G^2}$~~       d.  $\frac{2}{G^2}$

$$\rightarrow G \text{ is GM of } a \& b \therefore G^2 = ab$$

$$\begin{aligned} \frac{1}{ab - a^2} + \frac{1}{ab - b^2} &= \frac{1}{a(b-a)} - \frac{1}{b^2 - ab} \\ &= \frac{1 \times b}{ba(b-a)} - \frac{1 \times a}{ab(b-a)} = \frac{(b-a)}{ab(b-a)} = \frac{1}{G^2} \end{aligned}$$

125.  $243 \times 243^{1/6} \times 243^{1/36} \times \dots \infty$  terms = ?

- a. 3      b.  $\infty$       c. 243      d. 729

$\rightarrow 243^1 \times 243^{\frac{1}{6}} \times 243^{\frac{1}{36}} \times \dots \infty \text{ terms}$

$= (243)^{1 + \frac{1}{6} + \frac{1}{36} + \dots \infty \text{ terms}}$

$= (3^5)^{\frac{1}{1-\frac{1}{6}}} = (3^5)^{\frac{1}{1-\frac{1}{6}}} = (3^5)^{\frac{1}{5/6}} = (3^5)^{6/5} = 3^6 = 729$

126. If a, b, c are in A.P. and x, y, z are in G.P. then

$(x)^{b-c} \cdot (y)^{c-a} \cdot (z)^{a-b} = ? \rightarrow (3)^{-1} \times (6)^2 \times (12)^{-1} = \frac{1}{3} \times 36 \times \frac{1}{12} = 1$

- a. 1      b. 0      c. 2      d. None of these



$x^{-(c-b)} \cdot y^{2(b-a)} \cdot z^{-(b-a)}$

$x^{-(b-a)} \cdot (xz)^{b-a} \cdot z^{-(b-a)}$

$= x^{-(b-a)} \cdot x^{b-a} \cdot z^{b-a} \cdot z^{-(b-a)}$

$= x^{-b+a} \cdot x^{b-a} \cdot z^{b-a} \cdot z^{-b+a}$   
 $= x^{-b+a+b-a} \cdot z^{b-a-b+a}$   
 $= x^0 \cdot z^0 = 1 \times 1 = 1$

$a = 2 \quad b = 3 \quad c = 4 \quad x = 3 \quad y = 6 \quad z = 12$

127. For G.P.  $t_5 = (3)^{1/3}$ . Find product of first 9 terms of G.P.



$t_5 = a \cdot r^4 = 3^{1/3} \dots \dots \dots \textcircled{1}$

product of first 9 terms of G.P

$= a \times ar \times ar^2 \times ar^3 \times \dots \times ar^8$

$= a^9 \times r^{1+2+3+4+5+6+7+8} = a^9 \times r^{36} = a^9 \cdot (r^4)^9$

$= (a \cdot r^4)^9 = \left(3^{1/3}\right)^9 = 3^3 = 27$

128. Sum of 3<sup>rd</sup> & 9<sup>th</sup> term of AP is 8. Find  $S_{11}$

~~a. 44~~

b. 22

c. 19

d. 11



$$t_3 + t_9 = 8$$

$$a + 2d + a + 8d = 8$$

$$2a + 10d = 8 \text{ ..... ①}$$

$S_{11}$  for AP

$$= \frac{11}{2} [2a + 10d]$$

$$= \frac{11}{2} \times 8$$

$$= 44$$

129.  $1 + 3 + 5 + 7 + 9 + \dots + 513 = ?$



In this AP  $a=1, d=2$

$$t_n = a + (n-1)d$$

$$513 = 1 + (n-1)2$$

$$n = 257$$

$$S_{257}$$

$$= \frac{257}{2} (1 + 513)$$

$$= 66049$$

OR  $S_{257} = \frac{257}{2} (1+513) = 66049$  = Sum of first 257 odd natural numbers

130.  $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + \dots$  n terms = 155; then n = ?

a. 30

b. 31

~~c. a or b~~

d. None of these



$$\frac{30}{3} + \frac{29}{3} + \frac{28}{3} + \frac{27}{3} + \dots \text{ n terms} = 155$$

In this AP  $a = \frac{30}{3}, d = -\frac{1}{3}$



$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$155 = \frac{n}{2} \left[ \frac{60}{3} + (n-1) \left( -\frac{1}{3} \right) \right] = \frac{n}{2} \left[ \frac{60}{3} - \frac{n}{3} + \frac{1}{3} \right]$$

$$155 = \frac{n}{2} \left( \frac{60-n+1}{3} \right)$$

$$155 = \frac{n(61-n)}{6}$$

$$61n - n^2 = 930$$

$$n^2 - 61n + 930 = 0$$

$$(n-30)(n-31) = 0$$

$$n = 30 \text{ OR } n = 31$$

$$\therefore n(61-n) = 930$$

131. Insert 5 Geometric Means between 10 & 7290

→ 10 , 30 , 90 , 270 , 810 , 2430 , 7290

$$a = 10$$

$$t_7 = 7290$$

$$a \cdot r^6 = 7290$$

$$10 \cdot r^6 = 7290$$

$$r^6 = 729 = 3^6$$

$$\therefore r = 3$$

5 Geometric means between

10 & 7290 are :

30, 90, 270, 810, 2430

132. If a, b, c, d, e, f, g, h are in A.P. with common difference = 10;

then common diff for A.P. b, d, f, h is

a. 10

b.  $10^2$

~~c. 20~~

d. None of these

133. If a, b, c, d, e, f, g, h are in G.P. with common ratio = 4;

then common ratio for G.P. a, c, e, g is

a. 4

~~b.  $4^2$~~

c. 8

d. None of these

134. 2 A.Mean between -6 & 14 are

~~a.  $\frac{2}{3}, \frac{22}{3}$~~

b.  $\frac{2}{3}, \frac{1}{3}$

c. 0, 8

d. None of these

→ -6, 0.666666, 7.333333, 14

$a = -6$

∴ 2 A.Mean are

$t_4 = 14$  ∴  $3d = 20$

0.666666, 7.333333

$a + 3d = 14$

$d = 6.666666$

(i.e.  $\frac{2}{3}, \frac{22}{3}$ )

$-6 + 3d = 14$

135. Find the sum of all 5 digit even natural numbers divisible by 7

→  $10010 + 10024 + 10038 + \dots + 99988 = ?$

$t_n = a + (n-1)d = 99988$

$10,010 + (n-1)14 = 99988$

$n = 6428$

$S_{6428} = \frac{6428}{2} (10,010 + 99988)$

$= 35,353,3572$

136. -8, -6, -4, -2 ..... in this A.P.  $S_n = 52$  Find 'n'

a. 14

~~b. 13~~

c. 4

d. None of these

→ -8, -6, -4, ..... In this A.P.

$a = -8, d = 2, S_n = 52$

$104 = n(2n-18)$

$52 = \frac{n}{2} (-16 + (n-1)2)$

$104 = n \times 2 \times (n-9)$

$52 = n(n-9)$

$104 = n(-16 + 2n - 2)$

∴  $n = 13$



137. For G.P.  $t_3 = 12$  &  $t_5 = 48$ . Find  $t_2$

a. 2

b. 36

~~c. 6~~

d. None

→  $t_3 = a \cdot r^2 = 12$

$t_5 = a \cdot r^4 = 48$

$\frac{a \cdot r^4}{a \cdot r^2} = \frac{48}{12} = 4$

$r^2 = 4$

$r = 2 \therefore a = 3$

$t_2 = a \cdot r$

$= 3 \times 2 = 6$

common sense

$t_2 \quad t_3 \quad t_4 \quad t_5$   
6    12    24    48  
←

138. 1, x, 9 are in G.P. then  $x = ?$

a. 3

b. -3

~~c. a or b~~

d. None



1, 3, 9 are in G.P. with  $r = 3$

1, -3, 9 are also in G.P. with  $r = -3$

139. A man saved ₹ 16,500 in 10 years. In each year he saved ₹ 100 more than he saved in preceding year. How much did he save in the first year?

~~a. ₹ 1200~~

b. ₹ 1300

c. ₹ 1550

d. None of these

→ Money saved in first year = a

a, a+100, a+200, a+300 .....  $S_{10} = 16500$

$16500 = \frac{10}{2} (2a + 9 \times 100) \therefore 16500 = 5(2a + 900)$   
 $a = 1200/-$

## AP & GP Sequence & Series



140.  $120 + 60 + 30 + 15 + \dots$  Find  $S_{\infty}$

→ In this G.P.  $a = 120$ ,  $r = 0.50$

$$\text{As } 0 < r < 1 \quad S_{\infty} = \frac{a}{1-r} = \frac{120}{1-0.50} = 240$$

141.  $60 + 6 + 0.60 + 0.06 + \dots$  Find sum of infinite terms.

→ In this G.P.  $a = 60$ ,  $r = 0.10$

$$\text{As } 0 < r < 1 \quad S_{\infty} = \frac{a}{1-r} = \frac{60}{1-0.10} = 66.666666 \\ = 66\frac{2}{3}$$

142. For G.P.  $t_7 = 40,820$  and  $t_9 = 63,781.25$ . Find common ratio

~~a. -1.25~~

b. -1.30

c. 1.50

d. None

→

$$t_7 \times r \times r = t_9 \\ 40820 \times r^2 = 63781.25 \\ r^2 = 1.5625 \\ r = \pm 1.25$$

143.  $(5+x)$ ,  $(7+x)$ ,  $(18+x)$  are in A.P. Find  $x$ .

a. 2

b. 0

c. 10

~~d. Wrong data in question~~

→

$$t_2 - t_1 = (7+x) - (5+x) = 7+x-5-x = 2$$

$$t_3 - t_2 = (18+x) - (7+x) = 18+x-7-x = 11$$

Above data is wrong.  $(5+x)$ ,  $(7+x)$ ,  $(18+x)$  are NOT in A.P.

144. For A.P. (First term = Common diff.) then what is the ratio of  $m^{\text{th}}$  term to  $n^{\text{th}}$  term

~~a.  $m:n$~~

b.  $n:m$

c.  $m^2:n^2$

d. None

For AP Here  $a=d$

$$\frac{t_m}{t_n} = \frac{a + (m-1)d}{a + (n-1)d} = \frac{\cancel{d} + md - \cancel{d}}{\cancel{d} + nd - \cancel{d}} = \frac{md}{nd} = m:n$$

145. For A.P. sum of first 50 terms is equal to sum of first 60 terms then sum of first 110 terms is :

- a. 110    ~~b. zero~~    c. 250    d. None

→ By using short cut: If  $S_m = S_n$  then  $S_{m+n} = 0$   
of AP    If  $S_{50} = S_{60}$  then  $S_{50+60} = S_{110} = 0$

146.  $n^{\text{th}}$  term of the sequence 16, 8, 4, 2 is  $\left(\frac{1}{2^{17}}\right)$  Find value of 'n'

- a. 20    b. 21    ~~c. 22~~    d. None

→

16, 8, 4, 2 ..... In this G.P.  $a=16, r=\frac{1}{2}$

$$t_n = \frac{1}{2^{17}} = a \times (r)^{n-1}$$

$$\therefore \frac{1}{2^{21}} = \frac{1}{2^{n-1}}$$

$$\frac{1}{2^{17}} = 16 \times \left(\frac{1}{2}\right)^{n-1}$$

$$\therefore 2^{n-1} = 2^{21}$$

$$\frac{1}{2^{17}} \times \frac{1}{2^4} = \left(\frac{1}{2}\right)^{n-1}$$

$$\therefore n-1 = 21$$

$$n = 22$$

147. In A.P. terms increase or decrease by a fixed number.

This statement is

- ~~a. True~~    b. False

148. For A.P.  $t_9 = -6$  and common difference  $= \frac{5}{4}$

Find  $t_{25}$

a. 12

b. 14

c. 16

d. 18



$$\begin{aligned} t_9 &= a + 8d = -6 \\ a + \left(8 \times \frac{5}{4}\right) &= -6 \\ a + 10 &= -6 \\ \boxed{a} &= \boxed{-16} \end{aligned}$$

$$\begin{aligned} t_{25} &= a + 24d \\ &= -16 + \left(24 \times \frac{5}{4}\right) \\ &= -16 + 30 \\ &= 14 \end{aligned}$$

149. Ratio of 7<sup>th</sup> to 3<sup>rd</sup> term of A.P. is 12:5 then

Find ratio of 13<sup>th</sup> to 4<sup>th</sup> term?

a. 8:5

b. 9:4

c. 7:3

~~d. 10:3~~

→  $\frac{t_7}{t_3} = \frac{12}{5}$  ..... Given

$$\frac{a+6d}{a+2d} = \frac{12}{5}$$

$$5a+30d = 12a+24d$$

$$6d = 7a$$

$$3d = 3.50a$$

$$12d = 14a$$

$$t_{13} : t_4 = ?$$

$$\frac{a+12d}{a+3d} = \frac{a+14a}{a+3.50a}$$

$$= \frac{15a}{4.50a} = \frac{15}{4.50}$$

$$= \frac{150}{45} = 10:3$$

150. Sum of a series in A.P. is 525. 1<sup>st</sup> term is 3 and last term is 39.

Find common difference.

~~a.  $\frac{3}{2}$~~

b.  $\frac{2}{3}$

c.  $\frac{1}{3}$

d. None of these



$$a = t_1 = 3, \quad t_n = 39, \quad S_n = 525$$

$$S_n = \frac{n}{2} (t_1 + t_n)$$

$$525 = \frac{n}{2} (3 + 39)$$

$$525 = n \times 21$$

$$\therefore n = 25$$

$$t_{25} = 39$$

$$a + 24d = 39$$

$$3 + 24d = 39$$

$$24d = 36$$

$$\boxed{d = \frac{3}{2}}$$

151. Find common difference of A.P. whose first term is 100 and sum of whose first 6 terms is 5 times the sum of next 6 terms?

- a. -10      b. -15      c. 10      d. None of these



$$a = 100$$

$$S_6 = 5 \times (S_{12} - S_6)$$

$$\frac{3}{5}(200 + 5d) = 6(200 + 11d) - 3(200 + 5d)$$

$$120 + 3d = 1200 + 66d - 600 - 15d$$

$$120 + 3d = 600 + 51d$$

$$-480 = 48d \quad \therefore d = -10$$

152. Which term of G.P, 5, 10, 20, 40, ..... is 1280

- a. 11<sup>th</sup>      ~~b. 9<sup>th</sup>~~      c. 8<sup>th</sup>      d. 12<sup>th</sup>

5, 10, 20, 40, 80, 160, 320, 640, 1280  
 $q^{\text{th}}$  term      **OR**       $t_n = 1280 = 5 \times (2)^{n-1}$   
 $256 = 2^{n-1}$   
 $2^8 = 2^{n-1} \quad \therefore n-1=8 \quad \therefore n=9$

153. Find AM, GM, HM of 20 & 50

AM =  $\left(\frac{20+50}{2}\right) = 35$       GM =  $\sqrt{20 \times 50} = \sqrt{1000}$       HM =  $\left(\frac{2 \times 20 \times 50}{20+50}\right)$   
 GM =  $\pm 31.6227766016$       HM =  $\left(\frac{2000}{70}\right) = 28.571428$

Question:  $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$  are in

- a. A.P.      b. G.P.      ~~c. H.P.~~      d. M.P.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$  are in H.P. as 2, 4, 6, 8, 10

are in A.P.

# AP & GP Sequence & Series



154. If  $x, y, z$  are in H.P. then  $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$  are in :

- ~~a. A.P.~~      b. G.P.      c. H.P.      d. None

155. If  $a, b, c$  are in G.P. then  $a^2, b^2, c^2$  are in

- a. A.P.      ~~b. G.P.~~      c. H.P.      d. None



$2, 4, 8$  are in G.P.

$2^2, 4^2, 8^2$  i.e.  $4, 16, 64$  are also in G.P.

156. If one AM 'A' and 2 Geometric Means  $G_1$  &  $G_2$  are inserted between any 2 numbers then  $(G_1^3 + G_2^3) = ?$

- a.  $2 \cdot G_1 \cdot G_2$       ~~b.  $2A \cdot G_1 \cdot G_2$~~       c.  $2G_1/A$       d. None



Let 2 terms be  $x, y$       A is Am of  $x$  &  $y$

$x, A, y$  are in A.P.  $\therefore A = \frac{x+y}{2} \therefore x+y = 2A$  ..... ①

$x, G_1, G_2, y$  are in G.P.  $G_1^2 = x \cdot G_2$  &  $G_2^2 = y \cdot G_1$

$G_1^3 + G_2^3 = ?$

$$= G_1^2 \cdot G_1 + G_2^2 \cdot G_2 = x \cdot G_2 \cdot G_1 + y \cdot G_1 \cdot G_2 = G_1 \cdot G_2 (x+y)$$

$$= G_1 \cdot G_2 \cdot 2A = 2 \cdot A \cdot G_1 \cdot G_2$$

$$x = 2 \quad y = 16$$

$$A = 9, G_1 = 4, G_2 = 8$$

$$G_1^3 + G_2^3$$

$$= 4^3 + 8^3 = 64 + 512 = 576$$

157. AM of 2 numbers is 15 and their GM is 9, then the numbers are :

- ~~a. 27, 3~~      b. 9, 9      c. 16, 9      d. None



$$\frac{x+y}{2} = 15$$

$$\sqrt{xy} = 9$$

$$x+y = 30$$

$$xy = 81$$

$$27+3 = 30$$

$$27 \times 3 = 81$$

158. If  $a, b, c$  are in G.P.,  $a, x, b$  and  $b, y, c$  are both in A.P. then  $\left(\frac{a}{x} + \frac{c}{y}\right) = ?$

- a. 1      b. 0      ~~c. 2~~      d. None of these

$\rightarrow$ 

$$\begin{array}{ccc} a = 4 & a = 4 & b = 8 \\ b = 8 & x = 6 & y = 12 \\ c = 16 & b = 8 & c = 16 \\ \boxed{\text{G.P.}} & \boxed{\text{A.P.}} & \boxed{\text{A.P.}} \end{array}$$

$$\begin{aligned} &= \frac{a}{x} + \frac{c}{y} \\ &= \frac{4}{6} + \frac{16}{12} \\ &= \frac{8}{12} + \frac{16}{12} = \frac{24}{12} = 2 \end{aligned}$$

159. Find  $t_{25}$  for A.P. If  $t_9 = -6$  and common difference is  $\frac{9}{4}$

$\rightarrow$ 

$$\begin{aligned} t_9 &= a + 8d = -6 \\ a + \left(8 \times \frac{9}{4}\right) &= -6 \\ a + 18 &= -6 \\ a &= -24 \end{aligned}$$

$$\begin{aligned} t_{25} &= a + 24d \\ &= -24 + \left(24 \times \frac{9}{4}\right) \\ &= -24 + 54 = 30 \end{aligned}$$

160. If  $(p+1)^{\text{th}}$  term of A.P. is twice the  $(q+1)^{\text{th}}$  term then ratio of  $(p+q+1)^{\text{th}}$  term to  $(3p+1)^{\text{th}}$  term is

- ~~a. 1:2~~      b. 2:1      c. 1:3      d. None of these

$\rightarrow$ 

$$\begin{aligned} t_{p+1} &= 2 \times t_{q+1} \quad \text{..... Given} \\ a + (p+1-1)d &= 2 \times [a + (q+1-1)d] \\ a + pd &= 2 \times [a + qd] \\ a + pd &= 2a + 2qd \\ pd - 2qd &= a \quad \text{..... (1)} \end{aligned}$$

$$\begin{aligned} &\frac{t_{p+q+1}}{t_{3p+1}} \\ &= \frac{a + (p+q+1-1)d}{a + (3p+1-1)d} \\ &= \frac{a + pd + qd}{a + 3pd} \\ &= \frac{pd - 2qd + pd + qd}{pd - 2qd + 3pd} \\ &= \frac{2pd - qd}{4pd - 2qd} = \frac{(2pd - qd)}{2(2pd - qd)} \\ &= \frac{1}{2} = 1:2 \end{aligned}$$



161. 4<sup>th</sup> term of A.P. is equal to 3 times of first term and 7<sup>th</sup> term exceeds twice of third term by 1. Find first term.

~~a. 3~~

b. 5

c. 7

d. 9



$$t_4 = 3a$$

$$a + 3d = 3a$$

$$3d = 2a$$

$$t_7 = 2 \times t_3 + 1$$

$$a + 6d = 2(a + 2d) + 1$$

$$a + 6d = 2a + 4d + 1$$

$$-1 = a - 2d$$

$$-1.50 = 1.50a - 3d$$

$$-1.50 = 1.50a - 2a$$

$$-1.50 = -0.50a$$

$$a = 3$$

162. In G.P.  $S_n = 4095$ ,  $r = 2$ ,  $t_n = 2048$ . Find 'n'

a. 10

b. 11

~~c. 12~~

d. 15



$$S_n = 4095$$

$$r = 2$$

$$t_n = 2048 = a \times (2)^{n-1}$$

$$2048 = a \times \frac{2^n}{2^1}$$

$$4096 = a \times 2^n \text{ ——— ①}$$

$$4095 = \frac{a(2^n - 1)}{(2 - 1)}$$

$$4095 = a \times 2^n - a$$

$$4095 = 4096 - a$$

$$a = 1$$

Let's use Eq<sup>n</sup> ①

$$4096 = 1 \times 2^n$$

$$2^{12} = 2^n \therefore n = 12$$

163. Find 6<sup>th</sup> term from end for G.P. 8, 4, 2, 1, ...,  $\frac{1}{1024}$

a.  $\frac{1}{64}$

~~b.  $\frac{1}{32}$~~

c. 32

d. None of these



8, 4, 2, 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ ,  $\frac{1}{32}$ ,  $\frac{1}{64}$ ,  $\frac{1}{128}$ ,  $\frac{1}{256}$ ,  $\frac{1}{512}$ ,  $\frac{1}{1024}$



6<sup>th</sup> term from end



1<sup>st</sup> term from end

164. For G.P.  $t_4 = x, t_{10} = y, t_{16} = z$  then

a.  $x^2 = y.z$

b.  $y^2 = x.z$

c.  $z^2 = x.y$

d.  $xyz = 1$



$$a \cdot r^3 = x$$

$$a \cdot r^9 = y$$

$$a \cdot r^{15} = z$$

$$x \cdot z = a \cdot r^3 \times a \cdot r^{15} = a^2 \cdot r^{18}$$

$$y^2 = (a \cdot r^9)^2 = a^2 \cdot (r^9)^2 = a^2 \cdot r^{18}$$

$$\therefore y^2 = x \cdot z$$

165.  $2 + 5 + 10 + 17 + 26 + \dots$  Find sum of 100 terms of this series.

↓ This is neither AP nor GP



$$\begin{aligned} & 2 + 5 + 10 + 17 + 26 + \dots \text{100 terms} \\ &= (1^2+1) + (2^2+1) + (3^2+1) + (4^2+1) + \dots + (100^2+1) \\ &= (1^2+2^2+3^2+\dots+100^2) + (1+1+1+\dots+1 \text{ 100 terms}) \\ &= \frac{100 \times 101 \times 201}{6} + 100 = 3,38,450/- \end{aligned}$$

166.  $0 + 6 + 24 + 60 + 120 + \dots$  Find sum of 80 terms of this series.



$$\begin{aligned} & 0 + 6 + 24 + 60 + 120 + \dots \text{80 terms} = ? \\ & \text{This is neither AP nor GP} \\ &= (1^3-1) + (2^3-2) + (3^3-3) + (4^3-4) + \dots + (80^3-80) \\ &= (1^3+2^3+3^3+\dots+80^3) - (1+2+3+4+\dots+80) \\ &= \left( \frac{80 \times 81}{2} \right)^2 - \left( \frac{80 \times 81}{2} \right) = 10494360/- \end{aligned}$$

167.  $3\frac{1}{2}, 7, 10\frac{1}{2}, 14, \dots$  Find  $S_{60}$



$$\begin{aligned} & 3.50, 7, 10.50, 14, 17.50, 21, \dots \\ & \text{In this AP } a = 3.50, d = 3.50 \\ & S_{60} = \frac{60}{2} [7 + (59 \times 3.50)] = 6405/- \end{aligned}$$

# AP & GP Sequence & Series



168.  $2 + 12 + 36 + 80 + 150 + \dots$  Find sum of first 70 terms of this series.

→  $2 + 12 + 36 + 80 + 150 + \dots$  70 terms = ?

This is neither AP nor GP

$$= (1^2 + 1^3) + (2^2 + 2^3) + (3^2 + 3^3) + (4^2 + 4^3) + \dots + (70^2 + 70^3)$$

$$= (1^2 + 2^2 + 3^2 + \dots + 70^2) + (1^3 + 2^3 + 3^3 + \dots + 70^3)$$

$$= \left( \frac{70 \times 71 \times 141}{6} \right) + \left( \frac{70 \times 71}{2} \right)^2 = 6292020$$

169. For G.P.  $a = 729$ ,  $t_7 = 64$ . Find  $S_7$ .

- a. 2259      b. 3059      ~~c. 2059~~      d. 2459

→ For GP  $a = 729$ ,  $t_7 = a \cdot r^6 = 64$

$$729 \times r^6 = 64$$

$$r^6 = \frac{64}{729}$$

$$r^6 = \left( \frac{2}{3} \right)^6$$

$$\therefore r = \frac{2}{3}$$

$$S_7 = \frac{a(1-r^n)}{(1-r)}$$

$$= \frac{729 \left[ 1 - \left( \frac{2}{3} \right)^7 \right]}{1 - \frac{2}{3}} = \frac{729 \left( 1 - \frac{128}{2187} \right)}{\frac{1}{3}}$$

$$= \frac{729 \times \left( \frac{2187 - 128}{2187} \right)}{\frac{1}{3}} = 729 \times \frac{2059}{2187} \times \frac{3}{1} = \cancel{2187} \times \frac{2059}{\cancel{2187}} = 2059$$

170. 1.  $a, b, c$  are in A.P. as well as G.P. then

- ~~a.  $a = b = c$~~       b.  $a > b < c$       c.  $a = b \neq c$       d. None

Terms can be in AP as well as GP if and only if all the terms are Equal

2. Sum of first '2n' terms of A.P.  $2, 5, 8, \dots$  is equal to sum of first 'n' terms of A.P.  $57, 59, 61, \dots$  then  $n = ?$

- a. 10      b. 12      c. 11      d. 13

$2, 5, 8, \dots$        $57, 59, 61, \dots$

↘  $S_{2n} = S_n$  ↙

$$\frac{2n}{2} [4 + (2n-1)3] = \frac{n}{2} [114 + (n-1)2]$$

$$4 + 6n - 3 = 57 + n - 1$$

$$6n + 1 = 56 + n$$

$$\therefore n = \frac{11 \times 1}{1} = \frac{11}{1} \therefore n = 11 \quad 5n = 55 \quad \therefore n = \frac{55}{5} = \frac{11 \times 5}{5}$$

$$171. 2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots = ?$$

$$\rightarrow 2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots \infty \text{ terms} = ?$$

$$\text{In this G.P. } a=2, r=\frac{1}{2} \text{ AS } 0 < r < 1$$

$$S_{\infty} = \frac{a}{1-r} = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = \frac{2}{1} \times \frac{2}{1} = \frac{4}{1} = 4$$

$$172. 4, x, 36 \text{ are in G.P. then } x = ?$$

~~a. 12~~      b. 24      c. 20      d. None

$$\rightarrow 4, x, 36 \text{ are in G.P.}$$

$$\frac{x}{4} = \frac{36}{x} \quad \therefore x = \sqrt{144}$$

$$x^2 = 144 \quad x = \pm 12$$

$$173. 3 \text{ numbers are in G.P. If we double the middle term, we get A.P.}$$

$$\text{then common ratio of G.P. is equal to -}$$

~~a.  $2 \pm \sqrt{3}$~~       b.  $3 \pm \sqrt{2}$       c.  $3 \pm \sqrt{5}$       d.  $5 \pm \sqrt{3}$

$$\rightarrow \frac{a}{r}, a, ar \text{ are in G.P.}$$

$$\frac{a}{r}, 2a, ar \text{ are in A.P.}$$

$$2a - \frac{a}{r} = ar - 2a$$

$$\cancel{a} \left( 2 - \frac{1}{r} \right) = \cancel{a} (r - 2)$$

$$2 - \frac{1}{r} = r - 2$$

For a quadratic Eqn

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# AP & GP Sequence & Series



$$2x - 1 = x^2 - 2x$$

$$0 = x^2 - 4x + 1$$

$$\therefore x^2 - 4x + 1 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 3}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = \frac{2(2 \pm \sqrt{3})}{2}$$

174.  $50 + 45 + 40 + \dots$  n terms = zero; then  $n = ?$   $= 2 \pm \sqrt{3}$

→ In this A.P.  $a = 50, d = -5, S_n = 0$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$0 = 100 + (n-1)(-5)$$

$$-100 = (n-1)(-5)$$

$$20 = n-1$$

$$\therefore n = 21$$

$$0 = \frac{n}{2} [100 + (n-1)(-5)]$$

175. If you save 1 paise today, 2 paise the next day, 4 paise the succeeding day and so on, then your total savings in two weeks will be  
(Please Note : ₹1 = 100 paise)

a. ₹ 163

b. ₹ 183

~~c. ₹ 163.83~~

d. None of these

→  $1 + 2 + 4 + 8 + \dots$  14 terms = ?

$$S_{14} = \frac{1(2^{14}-1)}{(2-1)} = \frac{1(2^{14}-1)}{1} = 16383 \text{ paise} = ₹ \frac{16383}{100} = ₹ 163.83$$

176. The sum of the first 20 terms of G.P. is 244 times the sum of its first 10 terms. The common ratio is

a.  $\pm\sqrt{3}$

b.  $\pm 3$

~~c.  $\sqrt{3}$~~

d. None of these

→  $S_{20} = 244 \times S_{10}$

$$\frac{a(r^{20}-1)}{(r-1)} = \frac{244 \times a(r^{10}-1)}{(r-1)}$$

$$\begin{aligned} (r^{10})^2 - 1^2 &= 244 \times (r^{10} - 1) \\ (r^{10}-1)(r^{10}+1) &= 244(r^{10}-1) \\ r^{10} &= 243 = 3^5 = [\sqrt{3}^2]^5 \\ r^{10} &= \sqrt{3}^{10} \therefore r = \sqrt{3} \end{aligned}$$

177. The product of 3 numbers of G.P. is 729 & sum of squares is 819.  
The numbers are :

a. 9, 3, 27

b. 27, 3, 9

~~c. 3, 9, 27~~

d. None of these

$$3 \times 9 \times 27 = 729$$

$$3^2 + 9^2 + 27^2 = 819$$

178. The number of terms to be taken so that  $1 + 2 + 4 + 8 + \dots$  will be 8191 is

a. 10

~~b. 13~~

c. 12

d. None of these

$$\Rightarrow 8191 = \frac{1(2^n - 1)}{(2 - 1)} \therefore 2^n - 1 = 8191 \therefore 2^n = 8192 = 2^{13}$$

• Four geometric means between 4 and 972 are

a. 12, 36, 108, 324

b. 12, 24, 108, 320

c. 10, 36, 108, 320

d. None of these



AS 4, 12, 36, 108, 324, 972 are in G.P.

we can say that :

12, 36, 108, 324 are

4 Geometric means between  
4 & 972.

179.  $4 + 0.40 + 0.04 + 0.004 + \dots \infty$  terms = ?



In this G.P.  $a = 4$ ,  $r = 0.10$

AS  $0 < r < 1$

$$S_{\infty} = \frac{a}{1-r} = \frac{4}{1-0.10} = \frac{4}{0.90} = 4.4444444 = 4\frac{4}{9}$$

$2 + 3 + 4.50 + 6.75 + \dots \infty$  terms = ?



In this G.P.  $a = 2$ ,  $r = 1.50$

AS  $r > 1$ ,  $S_{\infty} = \infty$



180. 3, -9, 27, -81, 243, ..... Find  $t_{10}$ ,  $S_{12}$



In this G.P.  $a=3$ ,  $r=-3$

$$t_{10} = a \times (r)^9 = 3 \times (-3)^9 = 3 \times -19683 = -59049$$

$$S_{12} = \frac{a(1-r^{12})}{(1-r)} = \frac{3[1-(-3)^{12}]}{1-(-3)} = \frac{3(1-531441)}{4} = -398580$$

181. If a, b, c are in G.P. then Log a, Log b, Log c are in :

~~a. A.P.~~

b. G.P.

c. H.P

d. None of these



a, b, c are in G.P.

$$\therefore b^2 = ac$$

Taking Log on both sides

$$\text{Log } b^2 = \text{Log } ac$$

$$2 \cdot \text{Log } b = \text{Log } a + \text{Log } c$$

$$\text{Log } b + \text{Log } b = \text{Log } a + \text{Log } c$$

$$\text{Log } b - \text{Log } a = \text{Log } c - \text{Log } b$$

$\therefore \text{Log } a, \text{Log } b, \text{Log } c$   
are in A.P.

182. If a, b, c, d, e, f, g, h, i are in A.P. then (c-a) = ?

a. d-b

b. g-e

c. i-g

~~d. All of these~~

Question

$$\sum_{i=3}^8 (2i+3) = ?$$



$$= [2(3)+3] + [2(4)+3] + [2(5)+3] + [2(6)+3] + [2(7)+3] + [2(8)+3]$$

$$= 9 + 11 + 13 + 15 + 17 + 19 = 84$$



183. For A.P.  $S_p = 8p^2 - 50p$ . Find  $t_p$



$$S_p = 8p^2 - 50p$$

$$S_1 = 8(1)^2 - 50(1) = -42$$

$$a = -42$$

$$S_2 = 8(2)^2 - 50(2) = -68 \quad t_2 = S_2 - S_1 = -68 - (-42) = -26$$

$$d = t_2 - t_1 = -26 - (-42) = 16$$

$$\begin{aligned} \therefore t_p &= a + (p-1)d \\ &= -42 + (p-1)16 = -42 + 16p - 16 \end{aligned}$$

$$t_p = 16p - 58$$

184. For A.P.  $a = 30, d = -\frac{1}{5}$

Find  $S_{150}, t_{200}$



$$S_{150} = \frac{150}{2} \left[ 2(30) + (149 \times -\frac{1}{5}) \right]$$

$$= 75 \left( 60 - \frac{149}{5} \right) = 75 \times \frac{151}{5} = 2265$$

$$t_{200} = a + 199d$$

$$= 30 + (199 \times -\frac{1}{5})$$

$$= 30 - 39.80 = -9.80 = -49/5$$

# Mixed Questions



1. Two numbers are in the ratio 2:3 and the difference of their squares is 320. The numbers are :

- a. 12, 18      ~~b. 16, 24~~      c. 14, 21      d. None

By options :  $12:18 = 2:3$        $18^2 - 12^2 \neq 320$   
 $16:24 = 2:3$        $24^2 - 16^2 = 576 - 256 = 320$

2. An alloy is to contain copper and zinc in the ratio 9:4. The zinc required to melt with 24 kg of copper is :

- ~~a.  $10 \frac{2}{3}$  kg~~      b.  $10 \frac{1}{3}$  kg      c.  $9 \frac{2}{3}$  kg      d. 9 kg

copper : Zinc  
 $9 : 4$

$\frac{9}{4} = \frac{24 \text{ kgs}}{?}$

$24 \text{ kgs} : ?$

$? = \frac{4 \times 24}{9} = 10.66666 \text{ kgs}$

3.  $7 \log\left(\frac{16}{15}\right) + 5 \log\left(\frac{25}{24}\right) + 3 \log\left(\frac{81}{80}\right)$  is equal to :

- a. 0      b. 1      ~~c. Log 2~~      d. Log 3

$= 7 (\log 16 - \log 15) + 5 (\log 25 - \log 24) + 3 (\log 81 - \log 80)$

$= 7 (4 \cdot \log 2 - \log 5 - \log 3) + 5 (2 \cdot \log 5 - 3 \cdot \log 2 - \log 3) + 3 (4 \cdot \log 3 - \log 10 - 3 \cdot \log 2)$

$= 28 \log 2 - 7 \log 5 - 7 \log 3 + 10 \log 5 - 15 \log 2$

$- 5 \log 3 + 12 \log 3 - 3 - 9 \log 2 = 4 \cdot \log 2 + 3 \log 5 - 3 = \log (16 \times 125)$   
 $= \log 2000 - \log 1000 = \log 2$

4. A box contains ₹ 56 in the form of coins of one rupee, 50 paise and 25 paise.

The number of 50 paise coin is double the number of 25 paise coins and four times the number of one rupee coins. The number of 50 paise coins in the box is:

- ~~a. 64~~      b. 32      c. 16      d. 14

$100 \text{ paise} \times x = 100x$        $350x = 5600$

$50 \text{ paise} \times 4x = 200x$

$x = 16$

$25 \text{ paise} \times 2x = 50x$

No. of 50 paise coins  $= 4x = 64$

# Mixed Questions



5. If  $\log(2a - 3b) = \log a - \log b$ , then  $a = ?$

~~a.  $\frac{3b^2}{2b-1}$~~

b.  $\frac{3b}{2b-1}$

a.  $\frac{b^2}{2b+1}$

a.  $\frac{3b^2}{2b+1}$

$$\begin{aligned}\log(2a-3b) &= \log a - \log b \\ \log(2a-3b) &= \log\left(\frac{a}{b}\right) \\ \text{Taking A.log on both sides}\end{aligned}$$

$$\begin{aligned}2a-3b &= \frac{a}{b} \\ 2ab-3b^2 &= a \\ 2ab-a &= 3b^2 \\ a(2b-1) &= 3b^2 \\ a &= \frac{3b^2}{(2b-1)}\end{aligned}$$

6. The difference between the simple and compound interest on a certain sum for 3 years at 5% p.a. is ₹ 228.75. The compound interest on the sum for 2 years @5% p.a. is

a. ₹ 3,175

~~b. ₹ 3,075~~

c. ₹ 3,275

d. ₹ 2,975

$$\text{Diff} = P[(1+r)^n - 1] - P \cdot n \cdot r$$

$$\therefore P = 30,000$$

$$228.75 = P[1.05^3 - 1] - P \times 3 \times 5\%$$

$$C.I. = 30,000[1.05^2 - 1]$$

$$228.75 = 0.157625P - 0.15P$$

$$= ₹ 3075/-$$

$$228.75 = 0.007625P$$

7. In what time will ₹ 3,90,625 amount to ₹ 4,56,976 at 8% p.a, when the interest is compounded semi-annually? [Given :  $(1.04)^4 = 1.16986$ ]

~~a. 2 years~~

b. 4 years

c. 5 years

d. 7 years

$$A = P(1+r)^n$$

$$2 \times n \text{ years}$$

$$4,56,976 = 3,90,625 \times \left(1 + \frac{0.08}{2}\right)^{2n}$$

$$\therefore 2n = 4$$

$$1.16985856 = (1.04)^{2n}$$

$$n = 2 \text{ years}$$

$$(1.04)^4 = (1.04)^{2n}$$

8. The annual birth and death rates per 1000 are 39.4 & 19.4 resp. The number of years in which the population will be doubled assuming there is no immigration or emigration is :

~~a. 35 years~~

b. 30 years

c. 25 years

d. None of these

$$\text{opening population of a year} = 1000$$

$$\text{closing population} = 1000 + 39.4 - 19.4 = 1020$$

$$\text{Growth rate} = 2\%$$

$$A = P(1+r)^n$$

$$2P = P(1.02)^n$$

$$(1.02)^n = 2 = (1.02)^{35}$$

# Mixed Questions



9. The effective rate equivalent to nominal rate of 6% compounded monthly is :

- a. 6.05      ~~b. 6.16~~      c. 6.26      d. 6.07

$$\text{Effective rate of interest} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$= \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 1.005^{12} - 1 = 6.167781182\% \text{ p.a.c.a.}$$

10. If the difference between simple interest and compound interest is ₹ 11 at the rate of 10% for two years, then find the sum.

- a. ₹ 1,200      ~~b. ₹ 1,100~~      c. ₹ 1,000      d. None of these

$$\text{Difference} = \text{compound interest} - \text{simple interest}$$

$$11 = P [1.10^2 - 1] - P \times 2 \times 10\%$$

$$11 = 0.21P - 0.20P$$

$$0.01P = 11 \quad \therefore P = ₹ 1100/-$$

11. At what % rate of compound interest (C.I) will a sum of money become 16 times in four years, If interest is being calculated compounding annually.

- a.  $r = 100\%$       b.  $r = 10\%$       c.  $r = 200\%$       d.  $r = 20\%$

$$A = P(1+r)^n$$

$$16P = P(1+r)^4$$

$$(1+r)^4 = 2^4$$

$$\therefore 1+r = 2$$

$$r = 1.00$$

$$r = 100\% \text{ P.a.C.I.}$$

12. The future value of an annuity of ₹ 5,000 is made annually for 8 years at interest rate of 9% compounded annually [ Given that  $(1.09)^8 = 1.99256$  ] is \_\_\_\_\_

- ~~a. ₹ 55,142.22~~      b. ₹ 65,142.22      c. ₹ 65,532.22      d. ₹ 57,425.22

$$\text{Future value of annuity regular} = \text{Periodical amt} \times \left[ \frac{(1+r)^n - 1}{r} \right]$$

$$= ₹ 5000 \times \left[ \frac{1.09^8 - 1}{0.09} \right] = ₹ 55,142$$

# Mixed Questions



13. The incomes of A and B are in the ratio 3 : 2 and their expenditure in the ratio 5 : 3. If each saves ₹ 1,500, then B's income is :

~~a. ₹ 6,000~~

b. ₹ 4,500

c. ₹ 3,000

d. ₹ 7,500

Income saving exp

A 3x 1500 3x-1500

B 2x 1500 2x-1500

$$\frac{3x-1500}{2x-1500} = \frac{5}{3}$$

$$9x - 4500 = 10x - 7500$$

$$\therefore 3000 = x$$

$\therefore$  Incomes are :  
₹ 9000, ₹ 6000

14. In 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3:1. The quantity of water added in the mixture in order to make this ratio 2:1 is :

a. 15 litres

b. 10 litres

c. 8 litres

~~d. 5 litres~~

Glycerine = 30 litres  
water = 10 litres

$$\frac{30}{10+x} = \frac{2}{1}$$

$$30 = 20 + 2x$$

$$\therefore 2x = 10 \quad \therefore x = 5$$

15. Fourth proportional to x, 2x, (x+1) is :

a. (x+2)

b. (x-2)

~~c. (2x+2)~~

d. (2x-2)

Let 4th proportional be 'm'

$\therefore x, 2x, (x+1), m$  are in proportion

$$\therefore mx = 2x(x+1)$$

$$\therefore m = 2x + 2$$

16. The recurring decimal 2.7777..... can be expressed as :

a. 24/9

b. 22/9

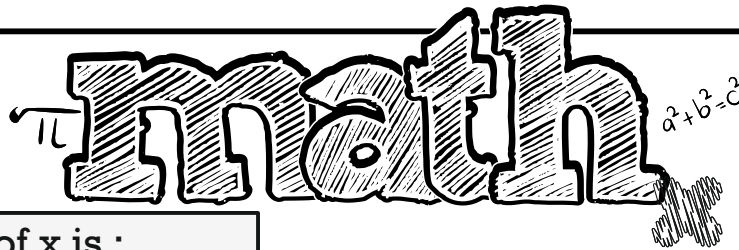
c. 26/9

~~d. 25/9~~

$$\frac{25}{9} = 2.7777$$

By using  
calculator

# Mixed Questions



17. If  $\log_2 x + \log_4 x = 6$ , then the value of  $x$  is :

- ~~a. 16~~      b. 32      c. 64      d. 128

$$\begin{aligned} \log_2 x + \log_4 x &= 6 \quad \therefore \frac{3 \log x}{2 \log 2} = 6 \\ \frac{2 \log x}{2 \log 2} + \frac{\log x}{2 \cdot \log 2} &= 6 \quad \cancel{3} \log x = \frac{12}{4} \cdot \log 2 \\ \log x &= \log 2^4 = \log 16 \\ x &= 16 \end{aligned}$$

18. Which of the numbers are not in proportion?

- ~~a. 6, 8, 5, 7~~      b. 7, 3, 14, 6      c. 18, 27, 12, 18      d. 8, 6, 12, 9

$$\begin{aligned} \Downarrow \quad 6 \times 7 &\neq 8 \times 5 & \Downarrow \quad 7 \times 6 &= 3 \times 14 & \Downarrow \quad 18 \times 18 &= 27 \times 12 & \Downarrow \quad 8 \times 9 &= 6 \times 12 \end{aligned}$$

19. If  $x : y = 2 : 3$ , then  $(5x + 2y) : (3x - y) =$  \_\_\_\_\_

- a. 19 : 3      ~~b. 16 : 3~~      c. 7 : 2      d. 7 : 3

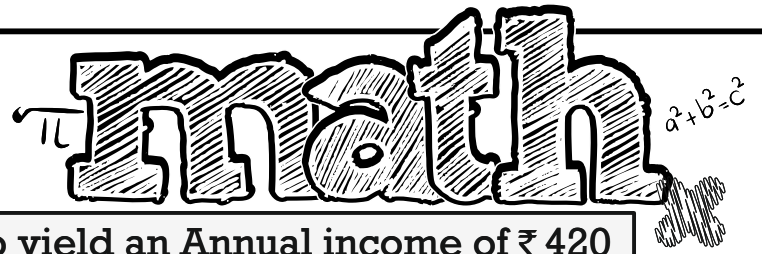
$$\begin{aligned} x &= 2m & \frac{5x + 2y}{3x - y} &= \frac{5(2m) + 2(3m)}{3(2m) - 3m} = \frac{16\cancel{m}}{3\cancel{m}} \\ y &= 3m & &= 16 : 3 \end{aligned}$$

20. By mistake a clerk, calculated the simple interest on principal for 5 months @ 6.5% p.a. instead of 6 months @ 5.5% p.a. If the error in calculation was ₹ 25.40. The original sum of principal was \_\_\_\_\_

- a. ₹ 60,690      ~~b. ₹ 60,960~~      c. ₹ 90,660      d. ₹ 90,690

| By mistake                                 | Actual                                    |                         |
|--------------------------------------------|-------------------------------------------|-------------------------|
| $SI = P \times \frac{5}{12} \times 6.50\%$ | $SI = P \times \frac{6}{12} \times 5.5\%$ | $0.0275P - 0.02708333P$ |
| $= 0.02708333333P$                         | $= 0.0275P$                               | $= 25.40$               |
|                                            |                                           | $0.0004166666P = 25.40$ |
|                                            |                                           | $P = 60,960/-$          |

# Mixed Questions



21. How much investment is required to yield an Annual income of ₹ 420 at 7% p.a. Simple Interest.

- a. ₹ 6,000      b. ₹ 6,420      c. ₹ 5,580      d. ₹ 5,000

$$P = ? , SI \text{ for } 1 \text{ year} = 420 , r = 7\%$$

$$SI = P \cdot n \cdot r = P \times 1 \times 0.07 = 420$$

$$P = ₹ 6000/-$$

22. A sum of money compounded annually becomes ₹ 1,140 in 2 years & ₹ 1,710 in three years. Find the rate of interest per annum.

- a. 30%      b. 40%      ~~c. 50%~~      d. 60%

$$1140 \times (1+r) = 1710$$

$$1+r = 1.50$$

$$r = 0.50 = 50\% \text{ P.A.C.I.}$$

23. A person lends ₹ 6,000 for 4 years and ₹ 8,000 for 3 years at simple interest. If he gets ₹ 2,400 as total interest, the rate of interest is :

- ~~a. 5%~~      b. 4%      c. 6%      d. 7%

$$6000 \times 4 \times r = 24000r$$

$$8000 \times 3 \times r = 24000r$$

$$48,000r = 2400$$

$$r = 0.05 = 5\%$$

24. How much amount is required to be invested every year so as to accumulate ₹ 3,00,000 at the end of 10 years, if interest is compounded annually at 10%?

- ~~a. ₹ 18,823.65~~      b. ₹ 18      c. ₹ 18,828.65      d. ₹ 18,882.65

$$\text{Future value of annuity Regular} = P.A. \times \frac{(1+r)^n - 1}{r}$$

$$3,00,000 = P.A. \times \frac{(1.10^{10} - 1)}{0.10}$$

$$\text{Periodical amt} = ₹ 18,823.618464$$



# Mixed Questions



25. A certain sum of money Q was deposited for 5 years and 4 months at 4.5% simple interest & amounted to ₹ 248, then the value of Q is

~~a. ₹ 200~~

b. ₹ 210

c. ₹ 220

d. ₹ 240

$$A = Q \times (1 + nr)$$

$$248 = Q \times [1 + (5.333333333 \times 0.0450)]$$

$$248 = Q \times 1.24 \quad \therefore Q = ₹ 200$$

26. A dealer mixes rice costing ₹ 13.84 per kg with rice costing ₹ 15.54 and sells the mixture at ₹ 17.60 per kg. So, he earns a profit of 14.6% on his sale price. The proportion in which he mixes the two qualities of rice is :

~~a. 3 : 7~~

b. 5 : 7

c. 7 : 9

d. 9 : 11

selling price per kg = 17.60

$$13.84x + 15.54y = 15.0304(x+y)$$

profit per kg =  $17.60 \times 14.60\%$   
= 2.5696

$$13.84x + 15.54y = 15.0304x + 15.0304y$$

$$\text{cost per kg} = 17.60 - 2.5696 = 15.0304 \text{ per kg} \quad 0.51y = 1.19x$$

27. What number must be added to each of the numbers 10, 18, 22, 38 to make the numbers in proportion?

~~a. 2~~

b. 4

c. 8

d. None of these

$$(10+2), (18+2), (22+2), (38+2)$$

i.e. 12, 20, 24, 40 are in proportion.

$$\frac{0.51}{1.19} = \frac{x}{y}$$

$$\therefore x = \frac{51}{119} = \frac{3}{7}$$

28. If  $\log 2 = 0.3010$  and  $\log 3 = 0.4771$ , then the value of  $\log 24$  is

a. 1.0791

b. 1.7323

~~c. 1.3801~~

d. 1.8301

$$\log(24) = \log(3 \times 2 \times 2 \times 2)$$

$$= \log 3 + \log 2 + \log 2 + \log 2$$

$$= \log 3 + 3 \cdot \log 2$$

$$= 0.4771 + 3(0.3010) = 1.3801$$

# Mixed Questions



29. There are total 23 coins of ₹ 1, ₹ 2 and ₹ 5 in a bag. If their value is ₹ 43 and the ratio of coins of ₹ 1 and ₹ 2 is 3 : 2. Then the number of coins of ₹ 1 is :

~~a. 12~~

b. 5

c. 10

d. 14

|     | No. of coins                    | value (₹)   |
|-----|---------------------------------|-------------|
| ₹ 1 | $3x$                            | $3x$        |
| ₹ 2 | $2x$                            | $4x$        |
| ₹ 5 | $23 - 3x - 2x$<br>$= (23 - 5x)$ | $115 - 25x$ |
|     |                                 | $115 - 18x$ |

$$115 - 18x = 43$$

$$72 = 18x$$

$$x = 4$$

$$\begin{aligned} \text{No. of coins of ₹ 1} &= 3x \\ &= 12 \end{aligned}$$

30. The mean proportional between 24 and 54 is :

a. 33

b. 34

c. 35

~~d. 36~~

Mean proportional between 24 & 54 be  $m$

$\therefore$  24,  $m$ , 54 must be in proportion

$$\therefore \frac{24}{m} = \frac{m}{54}$$

$$m^2 = 1296$$

$$m = 36$$

THE ONLY PLACE WHERE SUCCESS COMES  
BEFORE WORK IS IN THE  
DICTIONARY !

①

$$\begin{array}{r}
 9 \rightarrow \text{Quotient} \\
 \text{Divisor} \rightarrow 5 \overline{) 49} \rightarrow \text{Dividend} \\
 \underline{- 45} \\
 04 \rightarrow \text{Remainder}
 \end{array}$$

$$49 \div 5 = 9 \frac{4}{5}$$

②

$$\begin{array}{r}
 36 \\
 3 \overline{) 110} \\
 \underline{09} \\
 20 \\
 \underline{18} \\
 02
 \end{array}$$

$$\frac{110}{3} = 36 \frac{2}{3}$$

③

200

2500

$$\begin{array}{r}
 17 \\
 12 \overline{) 209} \\
 \underline{204} \\
 5
 \end{array}$$

$$\begin{array}{r}
 207 \\
 12 \overline{) 2489} \\
 \underline{24} \\
 0089 \\
 \underline{84} \\
 5
 \end{array}$$

④ For AP If  $t_{100} = 120$ ,  $t_{120} = 100$   
Find  $t_{220}$ ,  $t_{50}$



$$a + 119d = 100$$

$$a + 99d = 120$$

$$20d = -20$$

$$d = -1$$

$$a + 119(-1) = 100$$

$$\therefore a = 219$$

$$t_{220} = a + 219d$$

$$= 219 + 219(-1) = 0$$

$$t_{50} = a + 49d$$

$$= 219 + 49(-1)$$

$$= 219 - 49 = 170$$

$$t_{220} = 0$$

$$t_{50} = 170$$

Short-cut For AP

$$\text{If } t_p = q \text{ \& } t_q = p$$

then

$$t_{(p+q)} = 0$$

$$t_m = p + q - m$$

⑤ For AP If  $t_{80} = 35$ ,  $t_{35} = 80$   
Find  $t_{10}$ ,  $t_{15}$ ,  $t_{115}$ ,  $t_{200}$



$$t_{80} = 35, t_{35} = 80$$

$$t_{10} = 35 + 80 - 10 = 105$$

$$t_{15} = 35 + 80 - 15 = 100$$

$$t_{115} = 35 + 80 - 115 = 0$$

$$t_{200} = 35 + 80 - 200 = -85$$

⑥ Find AM & GM of  $x$  &  $y$

Let 'm' be AM of  $x$  &  $y$

$\therefore x, m, y$  must be A.P.

$$m - x = y - m$$

$$2m = x + y$$

$$m = \frac{x+y}{2}$$

$$\text{AM of } x \& y = \left( \frac{x+y}{2} \right)$$

Let  $k$  be GM of  $x$  &  $y$

$\therefore x, k, y$  must be in G.P.

$$\therefore \frac{k}{x} = \frac{y}{k}$$

$$\therefore k^2 = xy$$

$$\therefore k = \sqrt{xy}$$

$$\text{GM of } x \& y = \sqrt{xy}$$

⑦ Find AM & GM of 9, 36

$$\Rightarrow \text{AM} = \frac{9+36}{2} = 22.50$$

$$\text{GM} = \sqrt{9 \times 36} = \sqrt{324} = \pm 18$$

9, 18, 36 are in G.P.

9, -18, 36 are in G.P.

⑧ Find GM of 3 & 27

$\Rightarrow$  (a) 9 (b) -9 (c) ~~a or b~~ (d) None of these

⑨ Insert 3 G. means between  
10 & 50.625

$$\Rightarrow 10, \quad \boxed{10 \times 1.50 = 15}, \quad \boxed{15 \times 1.50 = 22.50}, \quad \boxed{22.50 \times 1.50 = 33.75}, \quad 50.625$$

$$a = 10$$

$$t_5 = 50.625$$

$$a \cdot r^4 = 50.625$$

$$r^4 = \frac{50.625}{10} = 5.0625$$

$$r = 1.50$$

$$\therefore 15, 22.50, 33.75$$

are 3 G. means

bet<sup>n</sup> 10 & 50.625

⑩ 3, 5, 7, 9, 11, 13, 15 are in A.P.

then  $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{15}$  are said to be

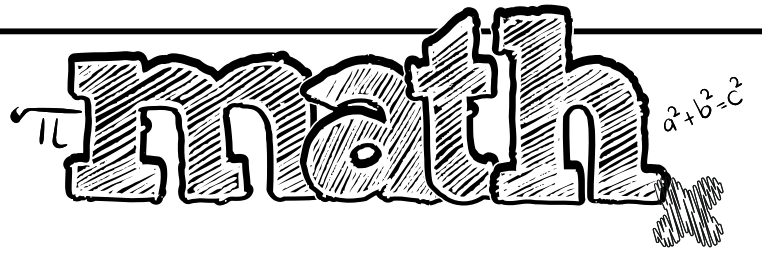
Harmonic progression

10, 13, 16, 19, 22, 25 are in A.P.

$\therefore \frac{1}{10}, \frac{1}{13}, \frac{1}{16}, \frac{1}{19}, \frac{1}{22}, \frac{1}{25}$  are in H.P.

⑪ If  $\frac{a}{b}, \frac{x}{y}, \frac{5}{13}, \frac{p}{q}$  are in H.P.

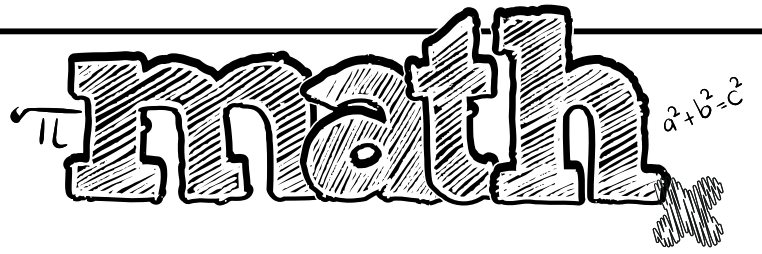
then  $\frac{b}{a}, \frac{y}{x}, \frac{13}{5}, \frac{q}{p}$  must be in A.P.



Handwriting practice lines consisting of 20 horizontal lines.







Handwriting practice lines consisting of 20 horizontal lines.

