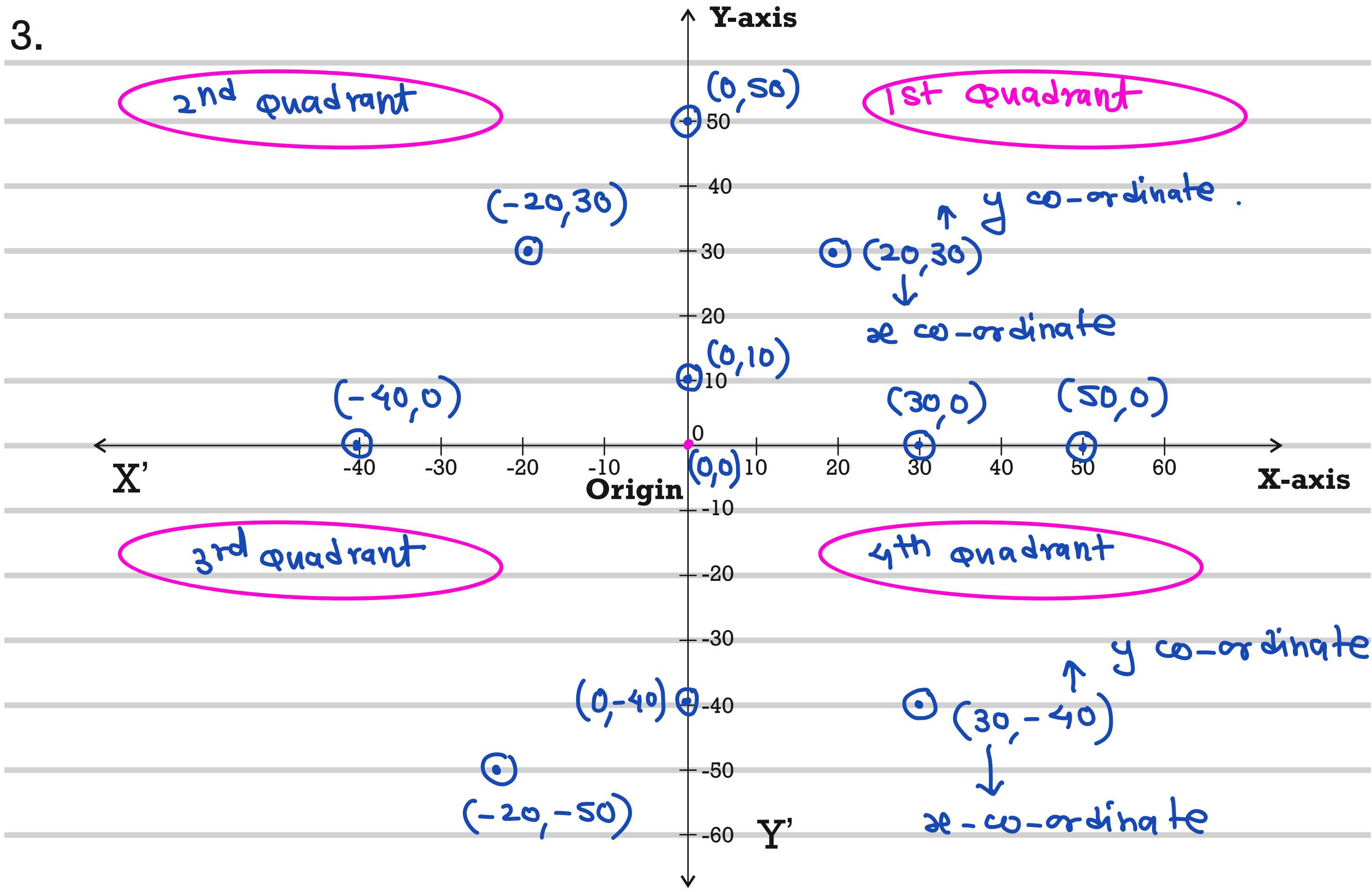
Inequalities & Equations

1. The standard format of a linear equation is : $ax + by + c = 0$
where $a, b \neq 0$ at a time.
OR $y = mx + c$ where $m =$ slope of the line

2.

Linear Equation	a	b	c
$5x + 13y + 8 = 0$	5	13	8
$133x - 18y + 2k - 18 = 0$	133	-18	$2k - 18$
$(2p+3)x - (18ky) + 339 = 93$	$(2p+3)$	$-18k$	246
$2x + 3y = 88$	2	3	-88
$17kx + 5x - 3y = 18k - 23$ $(17k+5)x - 3y - 18k + 23 = 0$	$(17k+5)$	-3	$-18k + 23$
$2x = 83$ i.e. $2x + 0y - 83 = 0$	2	0	-83
$8y = -2k - 99$ i.e. $0x + 8y + 2k + 99 = 0$	0	8	$2k + 99$
$kx - 13y - 25y - 9y + t = 0$ i.e. $kx - 47y + t = 0$	k	-47	t
$5x + 13y = 27x - 40y + 88$ i.e. $-22x + 53y - 88 = 0$	-22	53	-88
$2px - 13y + 8kx - 11py - 33$ $= 12mx - 18py - 11$ i.e. $(2p+8k-12m)x + (-13-11p+18p)y - 22 = 0$	$(2p+8k-12m)$	$(-13-11p+18p)$	-22

3.



4.

Points	Location	Equation / Inequalities
(+, +)	1st quadrant	$x > 0, \; y > 0$
(-, +)	2nd quadrant	$x < 0, \; y > 0$
(-, -)	3rd quadrant	$x < 0, \; y < 0$
(+, -)	4th quadrant	$x > 0, \; y < 0$
$(\pm, 0)$	X-axis	$y = 0$
$(0, \pm)$	Y-axis	$x = 0$
(0, 0)	origin	$x, y = 0$

- Equation of X-axis is $y = 0$
- Equation of Y-axis is $x = 0$
- (0, 0) represents origin = Point of intersection of X, Y -axis
- If x - co-ordinate of a point is 0, then that point is on : Y-axis

  examples : (0,30), (0,20), (0,-80), (0,-25), $(0, \frac{2}{3})$ are on Y-axis
- If y - co-ordinate of a point is 0, then that point is on : X-axis

  examples : (30,0), (20,0), (45,0), (-25,0), (-28.80,0) these points are on X-axis

5. Find points satisfying the linear equation $2x + 3y = 300$

If I put $x = 150, y = 0$ then $2x + 3y = 300$ is satisfied

therefore, (150, 0) is one of the point satisfying the equation $2x + 3y = 300$

Other points : (0, 100), $(10, \frac{280}{30})$, $(20, \frac{260}{30})$, (15, 90), (300, -100), (60, 60), (75, 50), $(40, \frac{220}{3})$, $(-19, \frac{338}{3})$, ...

Such infinite points can satisfy this linear equation.

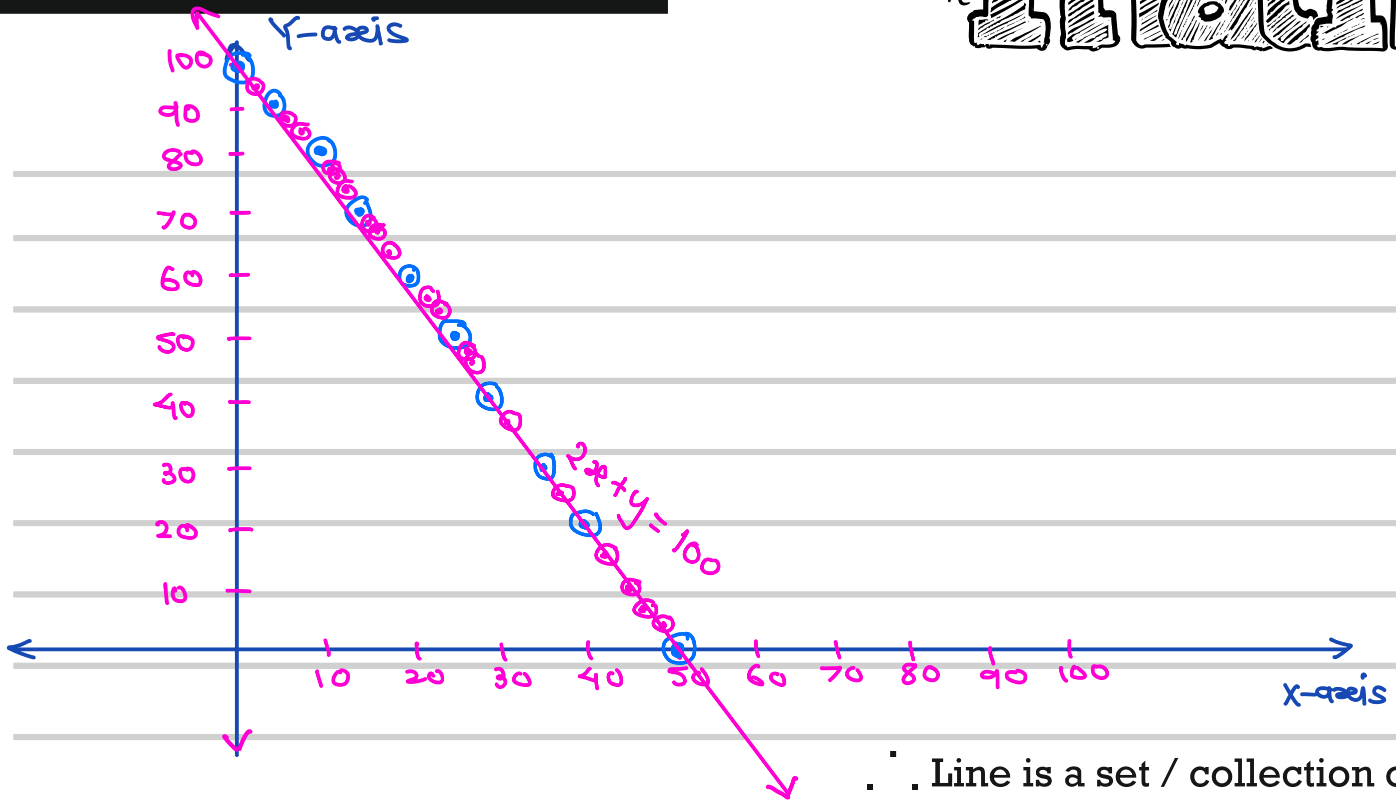
6. Find points satisfying the linear equation $x + y = 50$

→ (0,50), (50,0), (30,20), (20,30), (10,40), (25,25), (60,-10), (-20,70) (-100,150), (250,-200), (1.50,48.50), (2.85,47.15), (1,49), (15,35), (45,5), ...

such infinite points can satisfy this equation.

7. Find points satisfying the linear equation $2x + y = 100$ and plot those points on graph paper?

→ Points satisfying $2x + y = 100$ are : (0,100), (50,0), (25,50), (10,80), (20,60), (30,40), (40,20), (60,-20), (70,-40), (35,30), (15,70), (5,90), (-10,120), (-20,140), (-30,160) ....



$\therefore$ Line is a set / collection of infinite points satisfying the given <u>Linear equation</u>

- Graphical presentation of a linear equation is known as <u>Line.</u>

8. How to draw a line on a graph paper if equation of the line is given?

➡ ① Find atleast 2 points satisfying the given Linear Equation

② plot those points on Graph paper

③ Draw a straight line passing through all the points

Graphical representation of Linear equation is line
Line is a set/collection of infinite points satisfying the given linear equation.

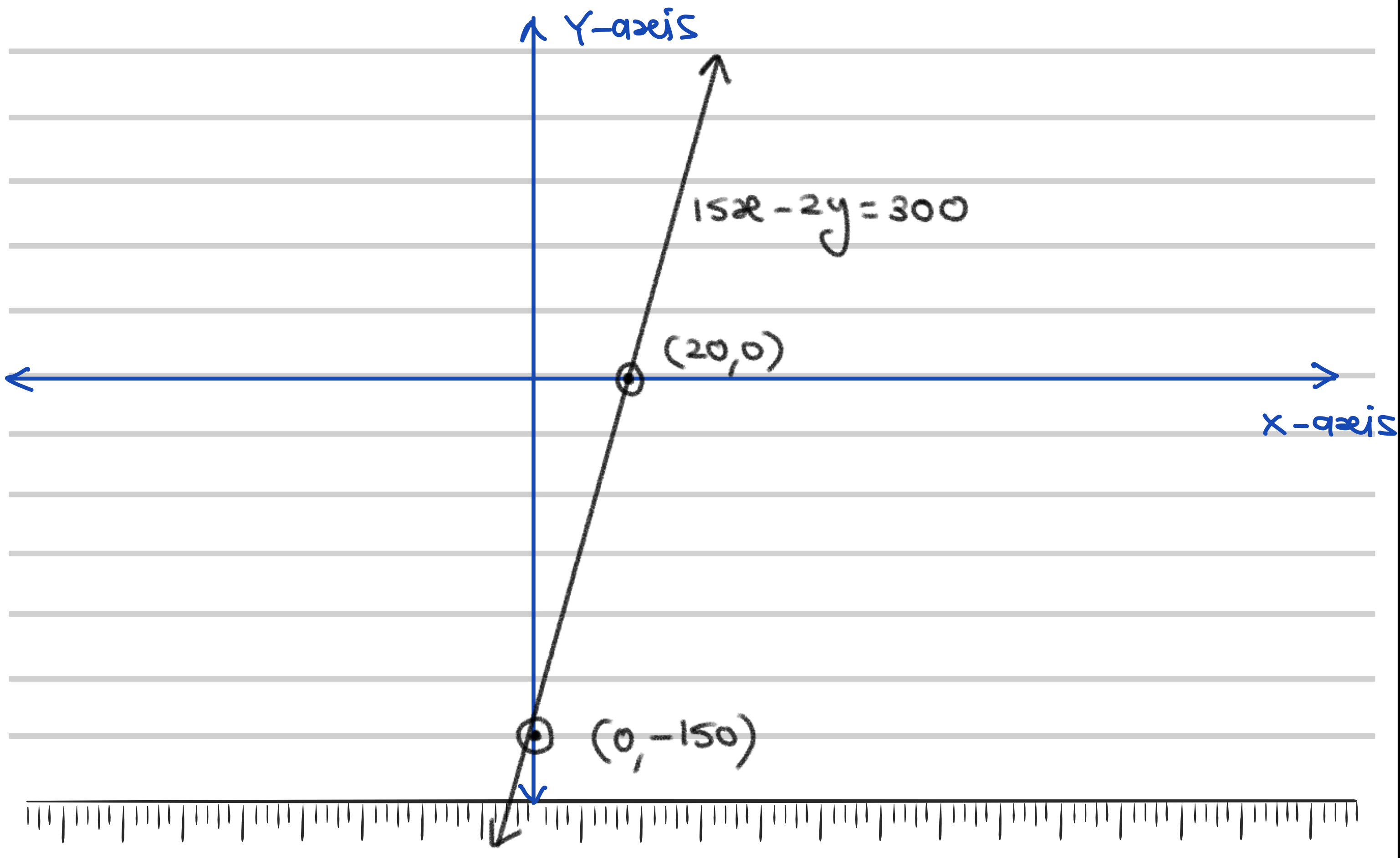
9. Draw the line 5x + 6y = 240 on graph paper

points satisfying Eq^n $5x + 6y = 240$: (0,40), (48,0)

10. Draw the line 15x - 2y = 300 on graph paper

points Satisfying the Eq^n $15x - 2y = 300$

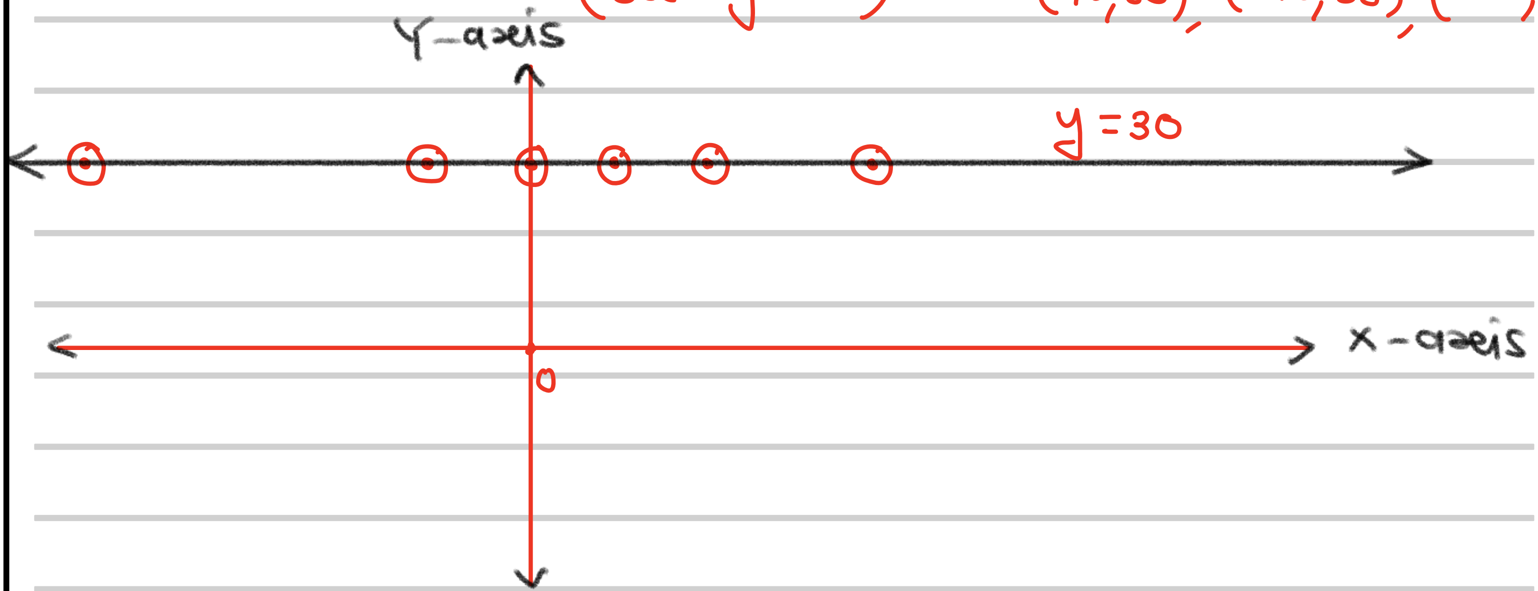
: (20,0), (0,-150)



11. Draw the line $y = 30$ on Graph paper.

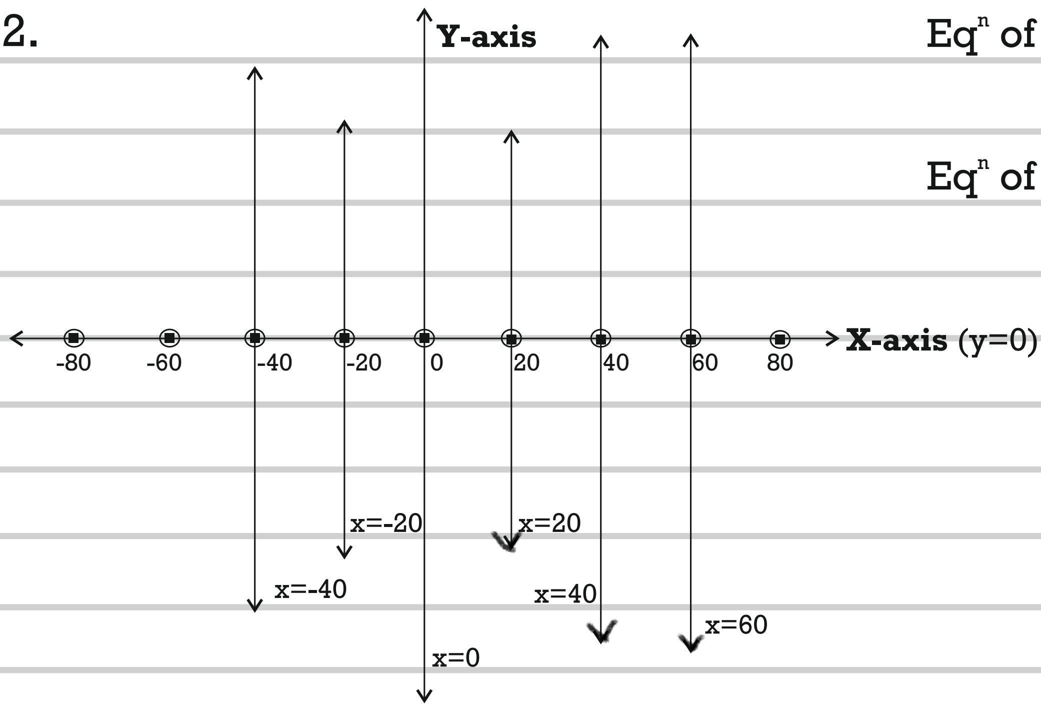
points satisfying eqn $y=30$ are $(0,30)$, $(10,30)$, $(20,30)$, $(40,30)$, $(-10,30)$, $(-100,30)$

$(0x+y=30)$



- If Eqn of the line is y = constant then that line is || to X-axis
- If Eqn of the line is x = constant then that line is || to Y-axis

12.



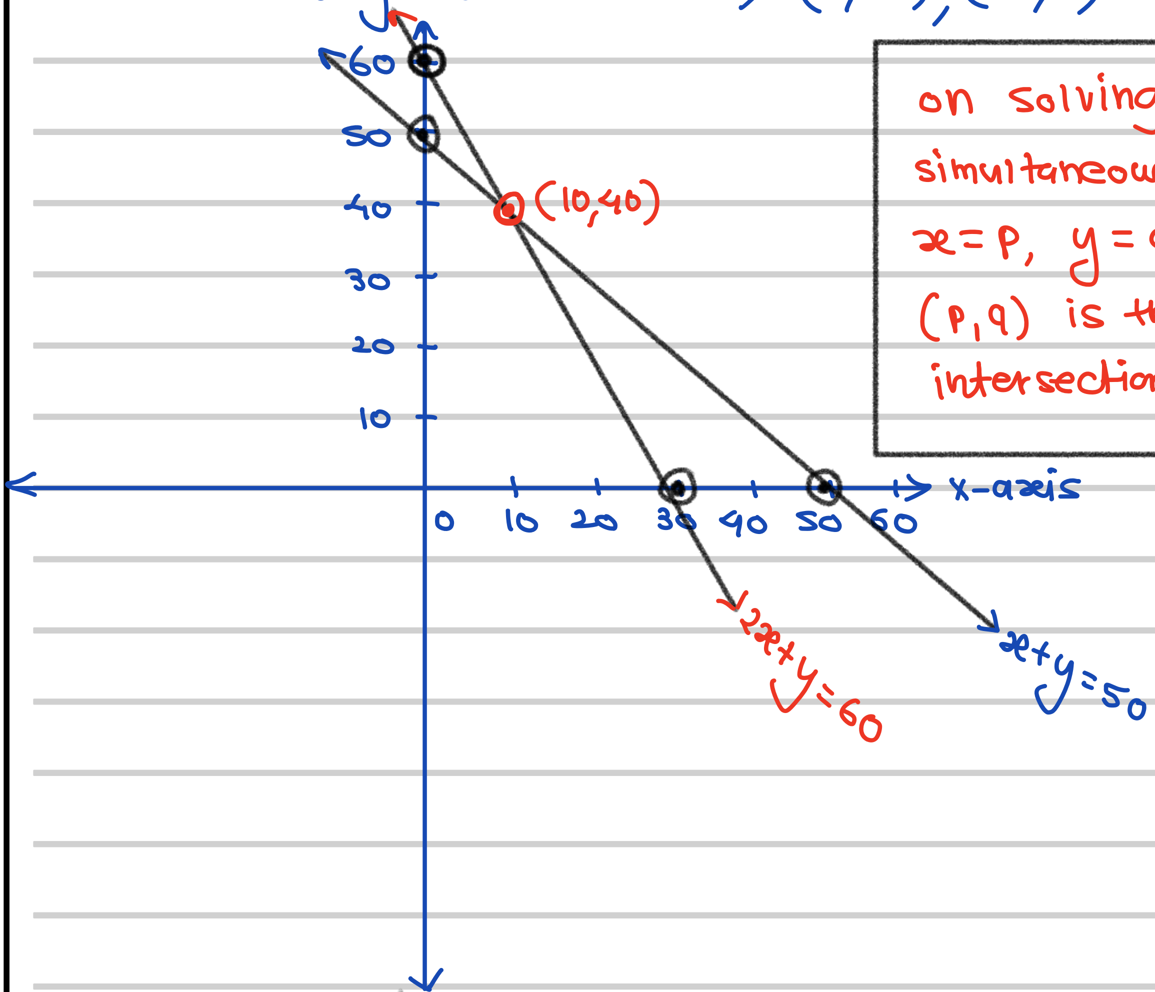
Eqn of X-axis : $y=0$

Eqn of Y-axis : $x=0$

13. Draw the lines (x + y = 50) & (2x + y = 60) on graph paper and Find point of intersection of these 2 lines.

→ $x + y = 50 \Longrightarrow (0, 50), (50, 0)$

$2x + y = 60 \Longrightarrow (0, 60), (30, 0)$



on solving 2 linear Eqns simultaneously if we get $x = p$, $y = q$ then (p, q) is the point of intersection of 2 lines

$2x + \cancel{y} = 60$ ------ ①

$\underline{-\ x \ \underset{-}{+} \ \cancel{y} = \underset{-}{50}}$ ---- ②

$x = 10$

$x + y = 50$

$10 + y = 50$

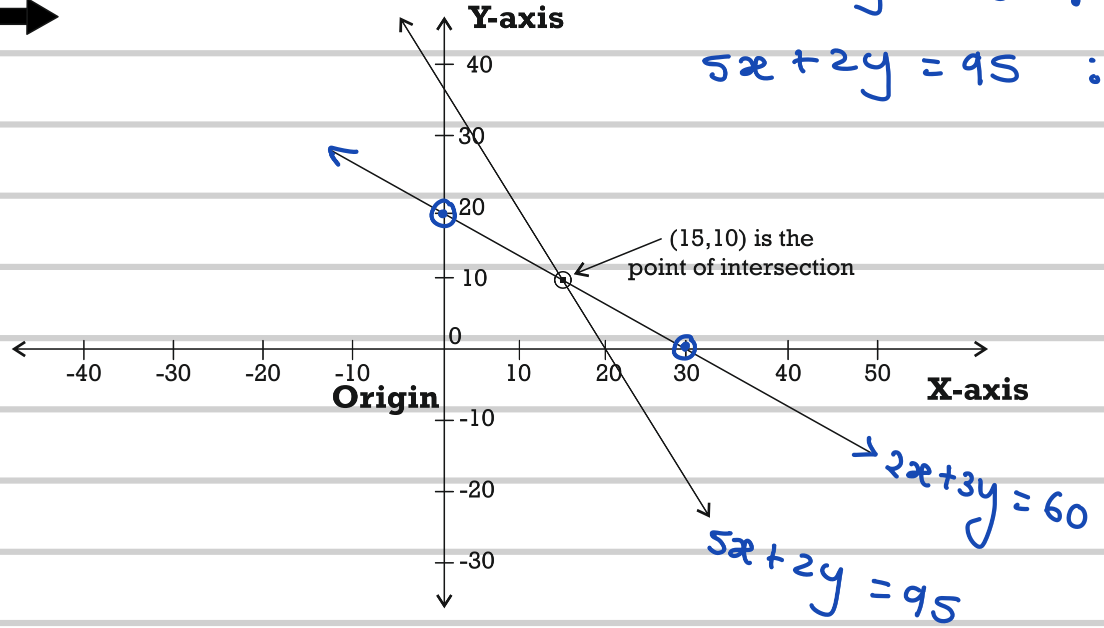
$y = 40$

(10, 40) is the point of intersection.

14. Draw the lines 2x + 3y = 60 and 5x + 2y = 95 on graph paper and Find point of intersection.

$2x + 3y = 60$: (30, 0), (0, 20)

$5x + 2y = 95$: (25, −15), (35, −40)



To get the point of intersection, Let's solve 2 linear equations simultaneously : 2x + 3y = 60 & 5x + 2y = 95

Let's multiply eqn ① by 2 on both sides, eqn ② by 3 on both sides

$$4x + \cancel{6y} = 120$$
$$\underline{-15x \mp \cancel{6y} = -285}$$
$$-11x = -165$$
$$\boxed{x = 15}$$

on solving 2 linear eqns simultaneously if we get x = m, y = n then (m, n) is the point of intersection of 2 Lines. If we put x = m, & y = n the Both eqns must be satisfied

Let's put x = 15 in one of the equation,

2x + 3y = 60

2(15) + 3y = 60

3y = 30

$\boxed{y = 10}$

∴ (15, 10) is the point of intersection of point.

15. Find point of intersection of 3x + 5y = 90 & 2x + 3y = 60

$\cancel{9x} + 15y = 270$

$10x + \cancel{15y} = 300$

$- \quad - \quad -$

$-x = -30$

$x = 30$

$3x + 5y = 90$

$3(30) + 5y = 90$

$5y = 0$

$y = 0$

$\therefore (30, 0)$ is the point of intersection.

16. Find point of intersection of lines 3x + 5y = 100 & 5x + 3y = 150

$\cancel{15x} + 25y = 500$

$15x + 9y = 450$

$- \quad - \quad -$

$16y = 50$

$\boxed{y = 3.125}$

$3x + 5(3.125) = 100$

$\boxed{x = 28.125}$

$\left(\frac{225}{8}, \frac{25}{8}\right)$

$\Downarrow$

$\therefore (28.125, 3.125)$ is the point of intersection of 2 lines.

17. Point of intersection of 7x - 3y = 20 & 5x + 13y = 200 lie in

______________ Quadrant.

$\cancel{35x} - 15y = 100$

$35x + 91y = 1400$

$- \quad - \quad -$

$-106y = -1300$

$\boxed{y = 12.264151}$

$7x - 3(12.264151) = 20$

$\boxed{x = 8.11321}$

$(8.11321, 12.264151)$ is the point of intersection. (i.e. 1st quadrant)

18. Point of intersection of lines $5x + 2y = 90$, $10x + 9y = 180$ lie in ____________ Quadrant

a. 1^{st}　　b. 2^{nd}　　c. 3^{rd}　　~~d.~~ None of these

$$\cancel{10x} + 4y = \cancel{180}$$
$$\cancel{10x} + 9y = \cancel{180}$$
$$- \quad - \quad -$$
$$-5y = 0$$
$$y = 0$$
$$\therefore \; 5x + 2(0) = 90$$
$$x = 18$$

∴ (18,0) is the point of intersection.

i.e. point of intersection is on x-axis

19. Find point of intersection of $8x - y = 90$ & $3x - 7y = 190$

➡

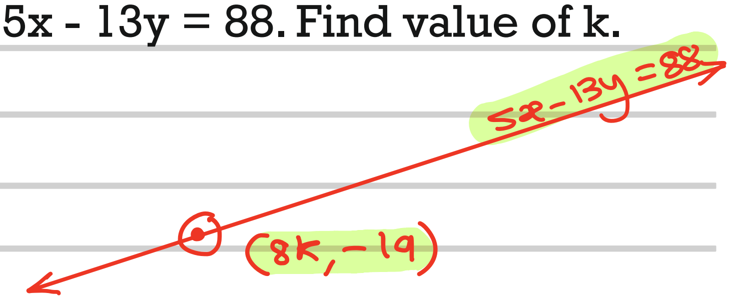
$$56x - \cancel{7y} = 630$$
$$3x - \cancel{7y} = 190$$
$$- \quad + \quad -$$
$$53x = 440$$
$$\boxed{x = 8.301887}$$
$$8(8.301887) - y = 90$$
$$\therefore \; \boxed{y = -23.58491}$$

point of intersection is (8.30, −23.58) which is in 4th quadrant

20. The point $(8k, -19)$ lie on the line $5x - 13y = 88$. Find value of k.

➡

$$5(8k) - 13(-19) = 88$$
$$40k + 247 = 88$$
$$\boxed{k = -3.975}$$

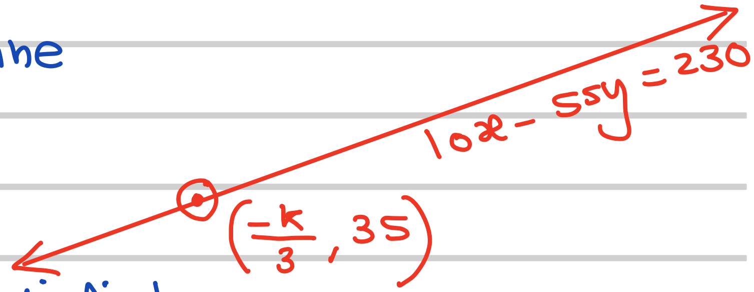


21. The point $(\frac{-k}{3}, 35)$ lie on the line $10x - 55y = 230$. Find value of k.

→

As point $\left(-\frac{k}{3}, 35\right)$ lie on the line

$10x - 55y = 230$



∴ If we put $x = -\frac{k}{3}$ & $y = 35$

then Given eqⁿ must be satisfied

$\therefore \quad 10x - 55y = 230$

$10\left(-\frac{k}{3}\right) - 55(35) = 230$

$\therefore \quad \frac{-10k}{3} = 2155 \qquad \therefore \quad k = -646.50$

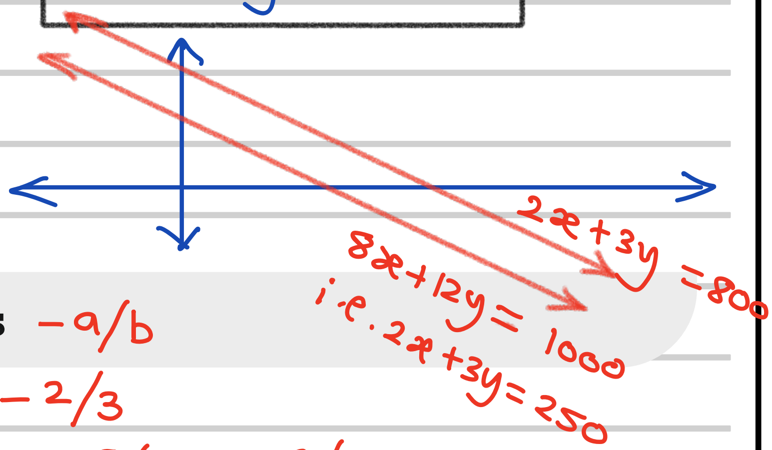
22. Find point of intersection of lines $2x + 3y = 800$ and

$8x + 12y = 1000$

→

$2x + 3y = 800$ $\qquad \boxed{8x + 12y = 1000}$

$\boxed{8x + 12y = 3{,}200}$



Slope of the line $ax + by + c = 0$ is $-a/b$

Slope of the line $2x + 3y = 800$ is $-2/3$

Slope of the line $8x + 12y = 1000$ is $-8/12 = -2/3$

As the slope of 1st line = slope of 2nd line, Lines are || to each other.

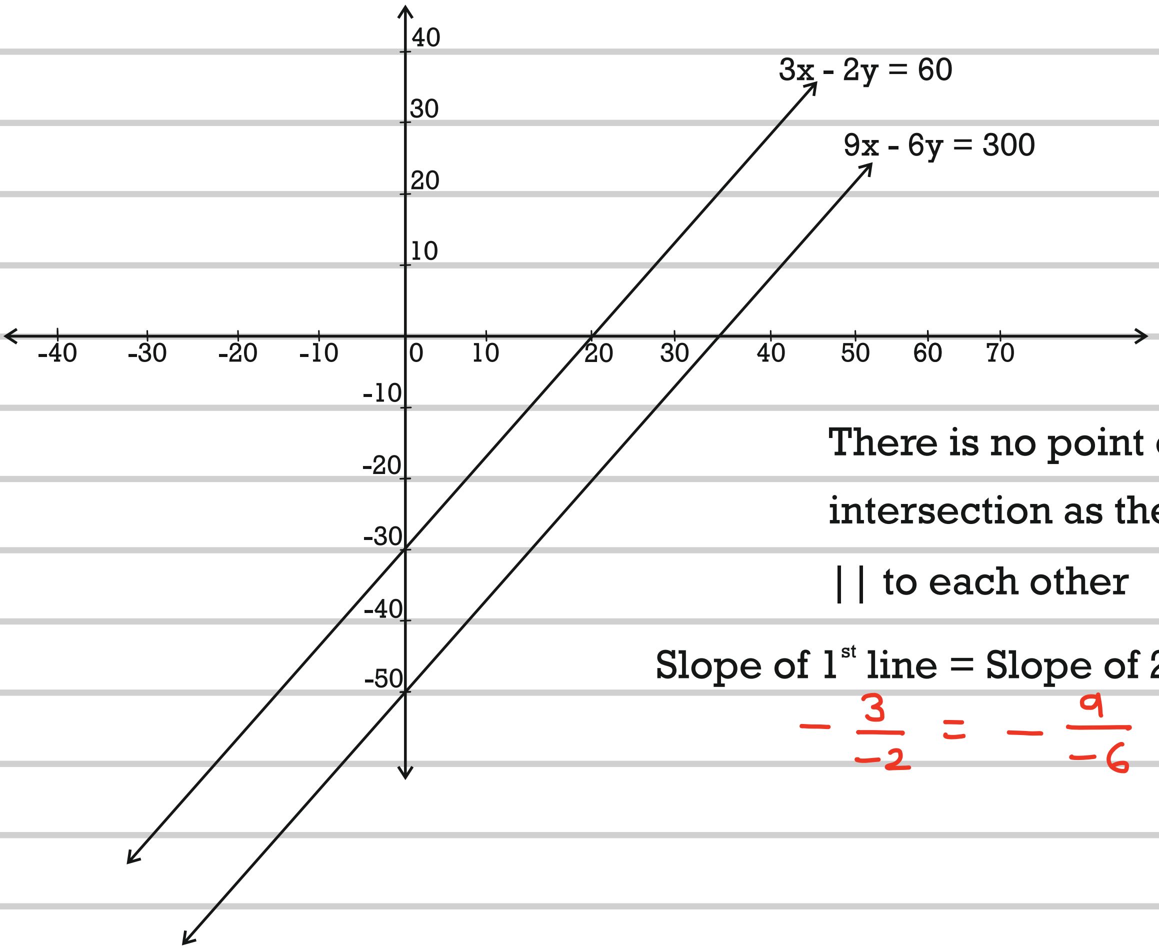
$-2/3 = -2/3$

If m_1 is slope of one line & m_2 is slope of other line then lines are said to be || to each other if $m_1 = m_2$

23. Draw the lines 3x - 2y = 60 & 9x - 6y = 300 on graph paper.
    Find point of intersection.

→ $3x - 2y = 60$ → (20, 0), (0, -30)

$9x - 6y = 300$ → (0, -50), (30, -5)



There is no point of intersection as these lines are || to each other

Slope of 1st line = Slope of 2nd line = 3/2

$-\frac{3}{-2} = -\frac{9}{-6} = 3/2$

24. The lines 5x + 11y = 22 and 8kx - 55y = -980 are || to each other.
    Find value of k.

→ As these 2 lines are || to each other,

slope of 1st line = slope of 2nd line

$\frac{-5}{11} = \frac{-8k}{-55}$ $\therefore \frac{-5}{11} = \frac{8k}{55}$

$\therefore 88k = -275 \therefore k = -3.125$

25. The lines 5x + 13y = 80 and 8mx - 22y = 810 are || to each other.
    Find value of m.

→ As these 2 lines are || to each other

Slope of 1st line = slope of 2nd line

$-\frac{5}{13} = \frac{-8m}{-22}$ $\therefore$ $104m = -110$

$\frac{-5}{13} = \frac{8m}{22}$ $m = \frac{-110}{104} = -\frac{55}{52} = -1.05769$

26.

Eqⁿ of the line	Slope of the line
$ax + by - c = 0$	$-a/b$
$3x + 5y + 30 = 0$	$-3/5$
$3x + 5y - 1000 = 0$	$-3/5$
$5x - 13y = 88$	$-a/b = -5/-13 = 5/13$
$8kx - 33py = 8k - p$	$8k/33p$
$29x - 33y = 5x - 88$	$24x - 33y + 88 = 0$ $\therefore$ Slope $= \frac{24}{33} = \frac{8}{11}$
$24x - 33y = -88$	
$13x - 2y = 88x - 130y - y + 2x$ $- 8p + 63$ i.e. $-77x + 129y = -8p + 63$	Slope $= \frac{77}{129}$
$31x - 2y = 8kx - 55y + 11$ i.e. $(31-8k)x + 53y = 11$	slope $= \frac{-(31-8k)}{53} = \frac{8k-31}{53}$
$x = 35$ i.e. $x + 0y - 35 = 0$	slope $= \frac{-1}{0}$ = Not defined = undefined
$2x = 101$ $2x + 0y - 101 = 0$	slope $= -\frac{2}{0}$ = undefined
$5y = 33$ $0x + 5y = 33$	Slope $= \frac{-0}{5} = 0$ = Zero
$0x + y = 33$	Slope $= -0/1 = 0$ = Zero
$x = 500$	i.e. $x + 0y = 500$ $\therefore$ slope $= -1/0$ = Not defined
$px + qy + r = 0$	$-p/q$
$33x + py = r$	$-33/p$

math $a^2+b^2=c^2$

27. Find slope of line x = 155 i.e. $x + 0y = 155$

Slope = $-1/0$ = Not defined

28. Find slope of line y = 30 i.e. $0x + y = 30$

Slope = $-0/1$ = zero = 0

Slope of X-axis and all the lines | | to X-axis is : Zero

Slope of Y-axis and all the lines | | to Y-axis is : undefined OR Not defined

A line	Slope	Equation
X - Axis	Zero	$y = 0$
Y - Axis	Not defined	$x = 0$
| | to X - Axis	Zero	$y =$ constant
| | to Y - Axis	Not defined	$x =$ constant

29. Standard format of a linear equation is,

$$ax + by + c = 0$$

$$by = -ax \pm c$$

dividing by b on both sides

$$\frac{by}{b} = \frac{-ax \pm c}{b}$$

$$y = \left(\frac{-a}{b}\right)x + \text{constant}$$

$$y = mx + c$$

where, m = slope of the line.

$y = 8x + 13 \Longrightarrow m = \text{slope} = 8$

$-8x + y - 13 = 0 \Longrightarrow \text{slope} = -a/b = 8$

Slope of the line $3x + 5y = 88$ is $-3/5$

$$3x + 5y = 88$$

$$5y = 88 - 3x$$

$$5y = -3x + 88$$

$$y = \left(-\frac{3}{5}\right)x + \left(\frac{88}{5}\right)$$

comparing this with $y = mx + c$

$m = -3/5$ = slope of line

30. Find slope of the line $3x + 5y = 88$

$3x + 5y - 88 = 0$

comparing this with $ax + by + c = 0$

$a = 3, b = 5$

$\therefore$ slope of the line $= -a/b$

$= -3/5$

$3x + 5y = 88$

$5y = 88 - 3x$

dividing by 5 on both sides

$y = \frac{-3}{5}x + \frac{88}{5}$

comparing this with $y = mx + c$

$m = -\frac{3}{5}$ = slope of the line

31. Find any 2 points satisfying the equation $7x - 3y = 100$

2 points satisfying the equation $7x - 3y = 100$ are : $(100, 200)$, $(10, -10)$

32. Find eq^n of the line passing through points $(100, 200)$ & $(10, -10)$

(x_1, y_1) (x_2, y_2)

$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$ ...... Eq^n of line passing through (x_1, y_1) & (x_2, y_2)

$\frac{-10 - 200}{y - 200} = \frac{10 - 100}{x - 100}$

$\frac{-210}{y - 200} = \frac{-90}{x - 100}$

$-210(x - 100) = -90(y - 200)$

$-210x + 21000 = -90y + 18{,}000$

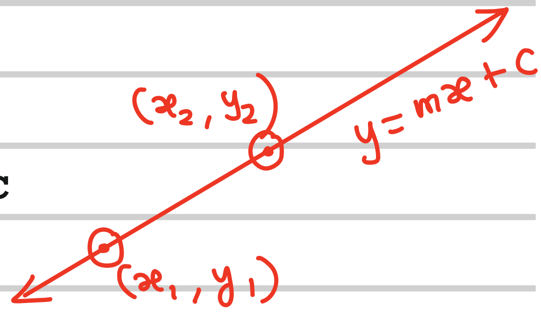
$-210x + 90y = -3000$

$210x - 90y = 3000$

$\boxed{7x - 3y = 100}$

33. Find equation of the line passing through points (x_1, y_1), (x_2, y_2)

→ Let $y = mx + c$ be the eq^n of the line passing through point (x_1, y_1) & (x_2, y_2)



$y = mx + c$ ………………………… ①

As point (x_1, y_1) is on the line $y = mx + c$

$y_1 = mx_1 + c$ ………………………… ②

Similarly, $y_2 = mx_2 + c$ ………………………… ③

eq^{n}① - eq^{n}②

$y - y_1 = mx + \cancel{c} - mx_1 - \cancel{c}$

$y - y_1 = m(x - x_1)$

$\therefore\ m = \dfrac{y - y_1}{x - x_1}$ ………………………… ④

eq^{n}③ - eq^{n}②

$y_2 - y_1 = mx_2 + \cancel{c} - mx_1 - \cancel{c}$

$y_2 - y_1 = m(x_2 - x_1)$

$\therefore\ m = \dfrac{y_2 - y_1}{x_2 - x_1}$ ………………………… ⑤

From eq^n ④ & eq^n ⑤

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\left(\frac{y_2 - y_1}{y - y_1}\right) = \left(\frac{x_2 - x_1}{x - x_1}\right) \dashrightarrow$$

This is eq^n of the line passing through points (x_1, y_1), (x_2, y_2)

34. Find Eq^n of the line passing through point (p, q), (m, n)

→ Eqn of line passing through points (x_1, y_1) & (x_2, y_2) is

$$\left(\frac{y_2 - y_1}{y - y_1}\right) = \left(\frac{x_2 - x_1}{x - x_1}\right)$$

∴ Eqn of line passing through points (p, q), (m, n) is

$$\left(\frac{n - q}{y - q}\right) = \left(\frac{m - p}{x - p}\right)$$

35. Find slope of the line $y = \frac{-8}{5}x + 33$

$y = \frac{-8}{5}x + 33$

Comparing this with y = mx + c

$m = \frac{-8}{5}$ = Slope of the line.

$y = -\frac{11}{6}x - 99$ ----- $\text{slope} = -\frac{11}{6}$

$6y = -11x - 594$

$11x + 6y + 594 = 0$ ...... $\text{slope} = -\frac{a}{b}$

$= -\frac{11}{6}$

$y = \frac{-8}{5}x + 33$

$y - 33 = \frac{-8}{5}x$

$5y - 165 = -8x$

$8x + 5y = 165$

$8x + 5y - 165 = 0$

Comparing this with ax + by + c = 0

a = 8, b = 5

$\text{Slope} = \frac{-a}{b} = \frac{-8}{5}$

36. Slope of the line $kx + 15y = 2x - 93$ is $\frac{-8}{11}$. Find k.

$kx + 15y - 2x + 93 = 0$

$(k-2)x + 15y + 93 = 0$

comparing this with $ax + by + c = 0$

$a = (k-2)$, $b = 15$, $c = 93$

$\text{slope} = \frac{-a}{b} = \frac{-(k-2)}{15} = \frac{-8}{11}$

$11(k-2) = 120$

$11k - 22 = 120$

$11k = 142$

$k = \left(\frac{142}{11}\right) = 12.90909090$

37. Slope of the line $19x - 33y + 2ky = 8x - 930$ is $\frac{11}{8}$. Find k.

$$19x - 33y + 2ky - 8x + 930 = 0$$
$$11x + (2k-33)y + 930 = 0$$

comparing this with $ax + by + c = 0$

$a = 11, \quad b = (2k-33)$

$$\text{slope} = \frac{-a}{b} = \frac{-11}{2k-33} = \frac{11}{8}$$

$$22k - 363 = -88$$

$\therefore 22k = 275$

$k = \left(\frac{275}{22}\right)$

$k = 12.50$

38. Find Eq^n of the line passing through points (8, -12), (18,33)

$(x_1, y_1), (x_2, y_2)$

Eq^n of the line Passing through (x_1, y_1) & (x_2, y_2) is

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$$

$$\frac{33-(-12)}{y-(-12)} = \frac{18-8}{x-8}$$

$$45(x-8) = 10(y+12)$$

$$45x - 360 = 10y + 120$$
$$45x - 10y = 480$$
$$45x - 10y - 480 = 0$$
$$9x - 2y - 96 = 0$$

39. Find Eq^n of the line passing through points (-30, -20), (-1.50, 80)

$(x_1, y_1) \quad (x_2, y_2)$

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$$

$$\frac{80-(-20)}{y-(-20)} = \frac{-1.50-(-30)}{x-(-30)}$$

$$100(x+30) = 28.50(y+20)$$

$$100x + 3000 = 28.50y + 570$$
$$100x - 28.50y + 2430 = 0$$
$$1000x - 285y + 24300 = 0$$
$$200x - 57y + 4860 = 0$$

40. Find Eqⁿ of the line passing through points (2, -5), (-11, 20). Also find slope of that line

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$$

$$\frac{20+5}{y+5} = \frac{-11-2}{x-2}$$

$25x - 50 = -13y - 65$

$25x + 13y + 15 = 0$

slope of the line $= -25/13$

slope of the line passing through points (x_1, y_1) & (x_2, y_2) is $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

slope of the line $= \frac{20-(-5)}{-11-2} = \frac{25}{-13}$

Eqⁿ of the line:

$25x + 13y = 25(2) + 13(-5)$

$25x + 13y + 15 = 0$

41. Find Slope, Eqⁿ of the line passing through points (20, 28), (30, 85).

Slope $= \frac{85-28}{30-20} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \left(\frac{57}{10}\right)$

Eqⁿ of the line

$57x - 10y = 57(20) - 10(28)$

$57x - 10y - 860 = 0$

OR

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$$

$$\frac{85-28}{y-28} = \frac{30-20}{x-20}$$

$57(x-20) = 10(y-28)$

$57x - 1140 = 10y - 280$

$57x - 10y = 860$

$\therefore$ slope $= \left(\frac{57}{10}\right)$

42. Find Slope, Eqⁿ of the line passing through points (1.50, 18.50), (-27, 35).

Slope $= \frac{35-18.50}{-27-1.50} = \frac{16.50}{-28.50} = \frac{165}{-285} = \frac{33}{-57} = \frac{11}{-19} = \frac{-11}{19}$

Equation of the line : $11x + 19y = 11(-27) + 19(35)$

$\therefore 11x + 19y = 368$

OR $-11x - 19y = -11(-27) - 19(35) = -368$

43. Find Slope of the line passing through points (a, b) & (c, d)

a. $\left(\frac{d-b}{c-a}\right)$ b. $\left(\frac{b-d}{a-c}\right)$ c. Both d. None

44. Slope of the line passing through (2k, 19) & (50, -8) is $\frac{-16}{3}$

Find k.

Slope of the line passing through points (2k, 19) & (50, -8) $= \left(\frac{-8-19}{50-2k}\right) = \frac{-16}{3}$

$$\frac{+27}{50-2k} = \frac{+16}{3}$$

$$16(50-2k) = 81$$

$$800 - 32k = 81$$

$$719 = 32k$$

$$\therefore k = \left(\frac{719}{32}\right) = 22.46875$$

45. The line 8x - 3y = 20 & 7kx + 55y = 250 have no solution. Find k

Means point of intersection

As these 2 lines have No solution, we can say that these 2 lines have same slope.

Slope of 1st line = Slope of 2nd line

$$\frac{8}{3} = \frac{-7k}{55}$$

$$-21k = 440$$

$$k = \left(\frac{-440}{21}\right) = -20.9523809523$$

46. The lines $5x + 11y = 29$ & $kx + 33y = 810$ have unique solution then

a. $k = 15$ ~~b.~~ $k \neq 15$ c. $k = 0$ d. Wrong qs.

→ As these 2 lines have unique solution, It means they have a point of intersection.

Slope of 1st line $\neq$ slope of 2nd line

$$\frac{-5}{11} \neq \frac{-k}{33}$$

$$-11k \neq -165$$

$$k \neq 15$$

47. If m_1 is slope of one line & m_2 is slope of other line then

Lines are said to be

$\|\|$ to each other	Oblique	$\perp$ to each other
when	when	when
$m_1 = m_2$	$m_1 \neq m_2$	$m_1 = \frac{-1}{m_2}$
		$m_1 \cdot m_2 = -1$

48. $3x - 19y = 50$ & $2kx + 51y = 200$ are $\perp$ to each other. Find value of k.

→ As these 2 lines are $\perp$ to each other,

slope of 1st line x slope of 2nd line $= -1$

$$\frac{3}{19} \times \frac{-2k}{51} = -1$$

$$\frac{-6k}{969} = -1$$

$$6k = 969 \quad \therefore k = 161.50$$

49.

Slope of the line	Slope of its || line	Slope of its $\perp$ line
$\frac{3}{5}$	$3/5$	$-5/3$
$\frac{-8}{9}$	$-8/9$	$9/8$
8	8	$-1/8$
-11	-11	$1/11$
$\frac{33}{8}$	$33/8$	$-8/33$
$\frac{-p}{q}$	$-p/q$	q/p
$\frac{p-q}{r}$	$\frac{p-q}{r}$	$\frac{r}{q-p}$
0	0	undefined
Not defined	Not defined	0
$\frac{3}{91}$	$3/91$	$-91/3$

50. The lines $18x - my = 20$ & $51x - 28y = 290$ are $\perp$ to each other.

Find the value of m.

→ As these 2 lines are $\perp$ to each other,

slope of 1st line × slope of 2nd line = −1

$$\frac{18}{m} \times \frac{51}{28} = -1$$

$$918 = -28m$$

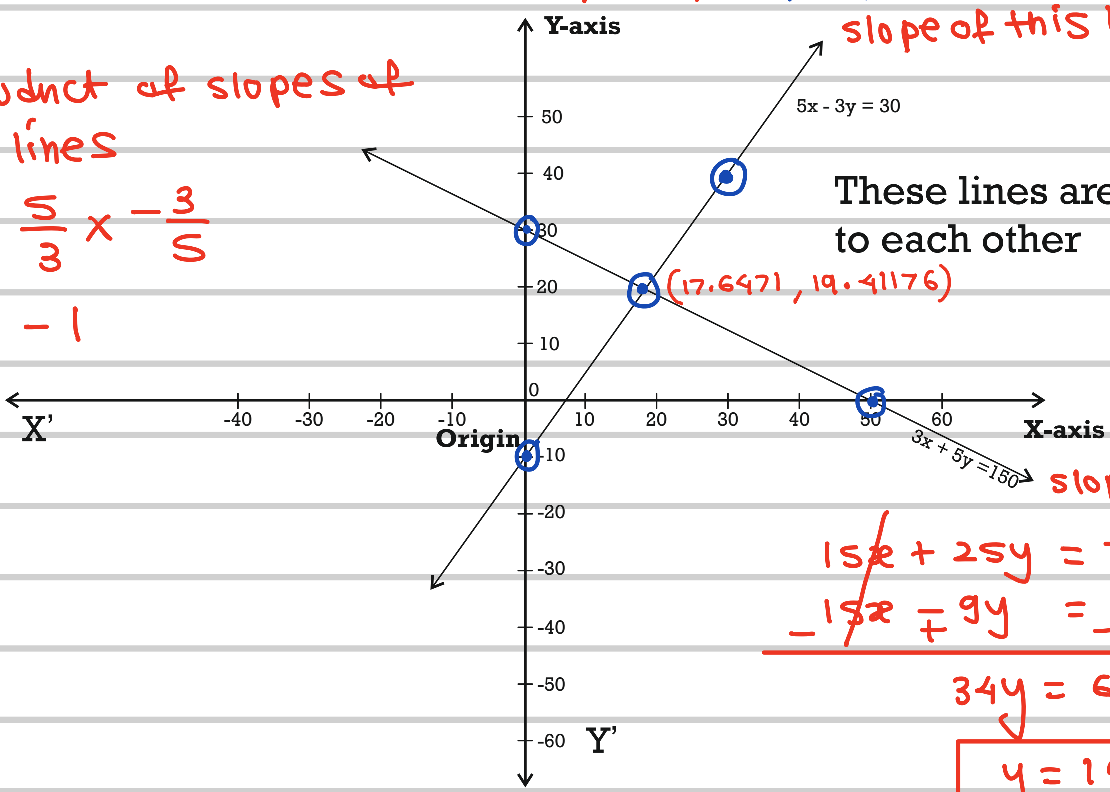
$$m = -\frac{918}{28} = -459/14$$

$$m = -32.7857142857$$

51. Draw the line 3x + 5y = 150 & 5x - 3y = 30 on graph paper

→ $3x + 5y = 150$ → $(50,0)$ $(0,30)$

$5x - 3y = 30$ → $(0,-10)$, $(30,40)$



slope of this line = $\frac{5}{3}$

product of slopes of 2 lines

$= \frac{5}{3} \times -\frac{3}{5}$

$= -1$

These lines are $\perp$ to each other as $m_1 \times m_2 = -1$

slope of this line = $-\frac{3}{5}$

$$\cancel{15x} + 25y = 750$$
$$-\cancel{15x} + 9y = -90$$
$$34y = 660$$
$$\boxed{y = 19.41176}$$

$\therefore 3x + 5(19.4117647058) = 150$

$$\boxed{x = 17.6471}$$

52. The lines 8x - 3ky + 21y = 33 & 15x - 28y = 233 are $\perp$ to each other. Find k.

→ $8x - 3ky + 21y = 33$
$8x + (-3k+21)y = 33$ } slope of the line $= -8/(-3k+21) = \left(8/(3k-21)\right)$

$15x - 28y = 233$ } slope of the line $= 15/28$

As these 2 lines are $\perp$ to each other

$$\therefore \frac{8}{3k-21} \times \frac{15}{28} = -1$$

$$120 = -28(3k-21)$$

$\therefore 120 = -84k + 588$

$k = \frac{-468}{-84} = \frac{117}{21}$

$k = 39/7$

Question $5x + 2y = 93$ and $7kx + 12x - 13y - 98y = 200$ are $\perp$ to each other. Find k.

$\Longrightarrow$ $5x + 2y = 93$ & $(7k+12)x - 111y = 200$ are $\perp$ to each other

$\therefore \left(\text{slope of 1st line} \times \text{slope of 2nd line}\right) = -1$ $\therefore\ k = \left(\frac{162}{35}\right)$

$\frac{+5}{2} \times \frac{7k+12}{111} = +1$ $k = 4.628571$

$35k + 60 = 222$

53. Find Eq^n of line passing through point (8, 20) having slope (-0.60)

➡ (8,20) slope = −0.60

Slope = (-0.60 / 1)

Eq^n of the line

0.60x + y = 0.60(8) + 20

0.60x + y = 24.80

6x + 10y = 248

$\boxed{3x + 5y = 124}$

Slope = -0.60

= -3/5

Eq^n of the line

3x + 5y = 3(8) + 5(20)

$\boxed{3x + 5y = 124}$

54. Find Eq^n of the line passing through point (8, 10) having slope of 0.70.

➡ Slope of the line $= 0.70 = \frac{7}{10}$

$\therefore$ Eq^n of the line is: $7x - 10y = 7(8) - 10(10)$

$7x - 10y = -44$

OR $-7x + 10y = -7(8) + 10(10)$

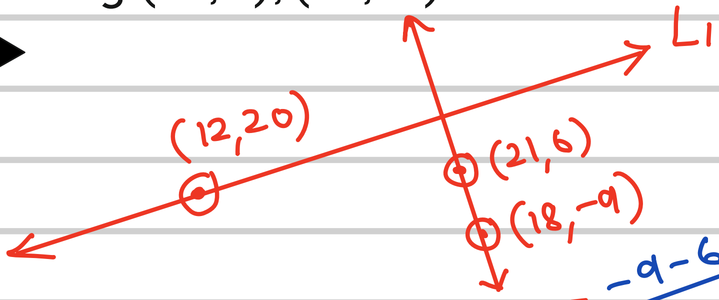
$-7x + 10y = 44$

$7x - 10y = -44$

$7x - 10y + 44 = 0$

55. Find eq^n of line passing through point (12, 20) and $\perp$ to line joining (21, 6), (18, -9)

➡



$$\text{slope} = \frac{-9-6}{18-21} = \frac{-15}{-3} = \frac{5}{1}$$

slope of $L_1 = -\frac{1}{5}$

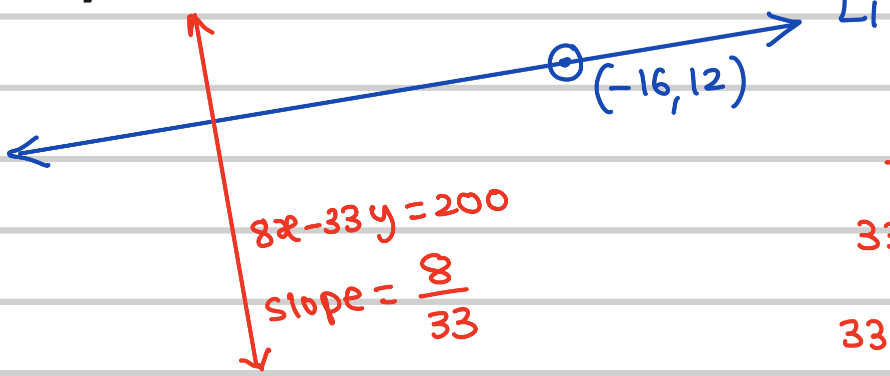
Eq^n of L_1

$$x + 5y = 12 + 5(20)$$

$$\boxed{x + 5y = 112}$$

56. Find eq^n of line passing through point (-16, 12) and $\perp$ to $8x - 33y = 200$

➡



$$\text{slope} = \frac{8}{33}$$

slope of $L_1 = \frac{-33}{8}$

Eq^n of L_1

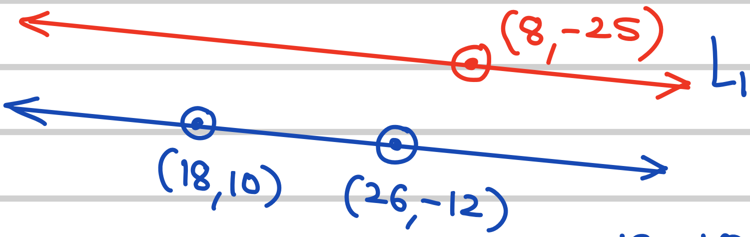
$$33x + 8y = 33(-16) + 8(12)$$

$$33x + 8y = -432$$

$$33x + 8y + 432 = 0$$

57. Find Eq^n of the line passing through point (8, -25) & || to line joining (18, 10), (26, -12)

➡



$$\text{Slope} = \frac{-12-10}{26-18} = \frac{-22}{8} = -11/4$$

slope of $L_1 = \frac{-11}{4}$

$\therefore$ Eq^n of L_1

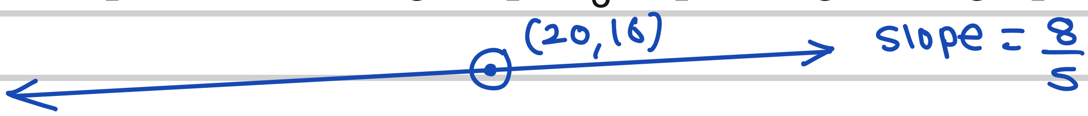
$$11x + 4y = 11(8) + 4(-25)$$

$$11x + 4y = -12$$

$$11x + 4y + 12 = 0$$

58. Find Eq^n of line having slope $\frac{8}{5}$ & passing through points (20,16)

➡



Eq^h of Line : $8x - 5y = 80$

OR

$-8x + 5y = -80$

59. Find eq^n of line passing through point (0.50, 8.75) and $\perp$ to

$17x - 20y = 88$

➡

$\therefore$ Eq^h of L_1 : $20x + 17y = 20(0.50) + 17(8.75)$

$20x + 17y = 158.75$

$\boxed{80x + 68y = 635}$

60. If slope of line is zero then that line can be

a. X-Axis b. || to X-Axis c. $\perp$ to Y-Axis ~~d.~~ All of these

61. If slope of line is Not Defined then that line can be

a. Y-Axis b. || to Y-Axis c. $\perp$ to X-Axis ~~d.~~ All of these

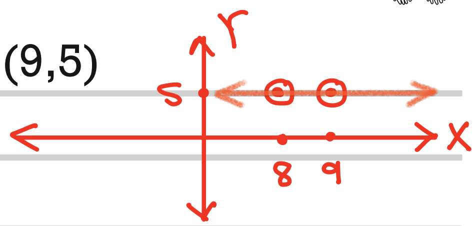
62. The line x = 25/2 is ____________

a. || to Y-Axis b. $\perp$ to X-Axis ~~c.~~ Both d. None

63. Find Eq^n of line passing through points (8, 5), (9,5)

➡ $0x + y = 5$

$y = 5$



64. Find Eq^n of line passing through points (6, 0), (19, 0)

➡ $y = 0$

65. Find Eq^n of line passing through points (0, 18), (18, 0)

➡ $x + y = 18$ slope $= \frac{0-18}{18-0} = \frac{-18}{18} = -\frac{1}{1}$

66. Find Eq^n of line passing through points (0, 19), (5, 19)

➡ $y = 19$

67. Slope of line passing through points $\left(\frac{8}{3}, \frac{7}{5}\right), \left(\frac{2k}{7}, \frac{19}{3}\right)$ is $\frac{5}{11}$. Find k.

➡

Slope of the line passing through points $\left(\frac{8}{3}, \frac{7}{5}\right)$ & $\left(\frac{2k}{7}, \frac{19}{3}\right)$ $= \left[\dfrac{\frac{19}{3} - \frac{7}{5}}{\frac{2k}{7} - \frac{8}{3}}\right]$

$$\frac{5}{11} = \left[\dfrac{\frac{95-21}{15}}{\frac{6k-56}{21}}\right]$$

$$\therefore \frac{5}{11} = \frac{74}{15} \times \frac{21}{6k-56}$$

$$6k - 56 = \frac{74}{15} \times \frac{21}{1} \times \frac{11}{5}$$

$$\boxed{k = 47.32}$$

68. The lines $3kx - 22y = 80$ & $90x - 47y = 285$ are $\perp$ to each other.

Find k

→ As these 2 lines are $\perp$ to each other,

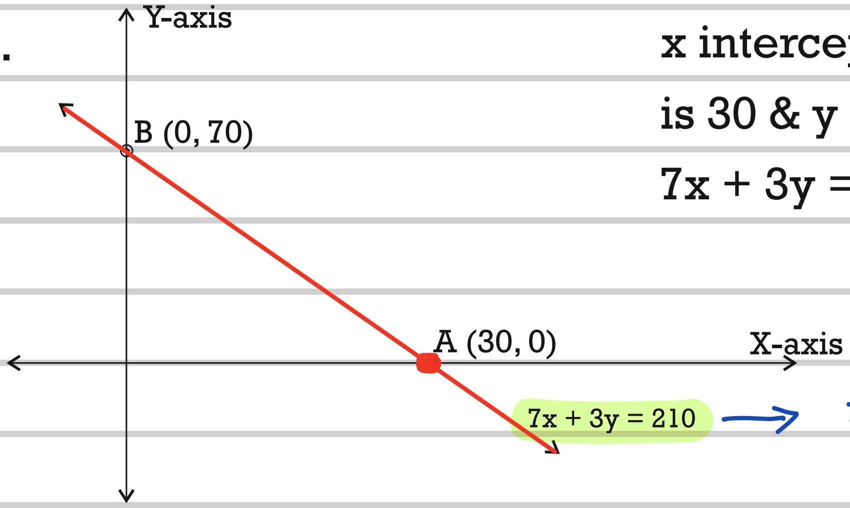
(slope of 1st line x slope of 2nd line) = −1

$$\frac{3k}{22} \times \frac{90}{47} = -1$$

$$270k = -1034$$

$$k = -3.82962962962$$

69.



x intercept of line $7x + 3y = 210$ is 30 & y intercept of line $7x + 3y = 210$ is 70

→ x intercept of this Line is 30

y intercept of this Line is 70

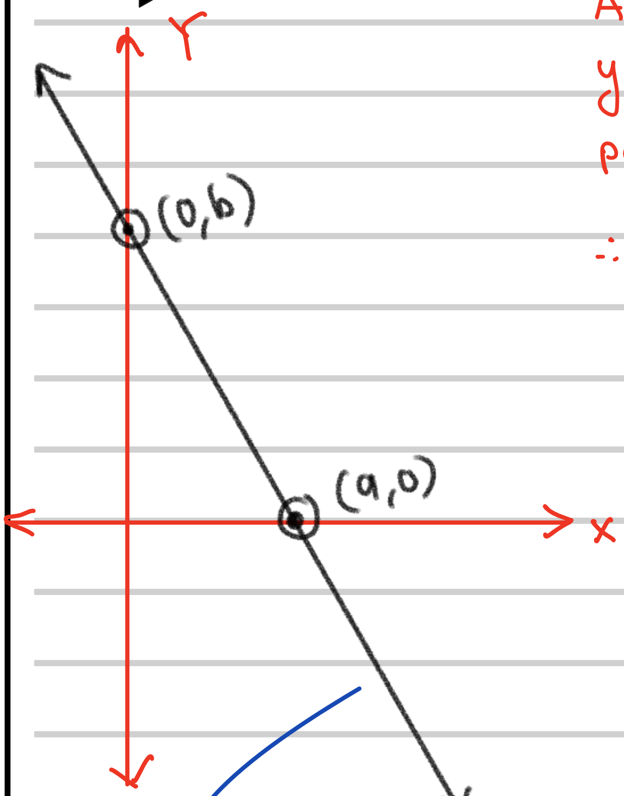
If x intercept of a line is ‘m’ & y intercept is k then that line passes through (m,0), (0, k)

If x intercept of a line is −20 & y intercept is 35 then that line passes through points :

(−20,0) & (0,35)

70. Find Eq^n of line having x, y intercepts as a, b respectively.

→



As x intercept is 'a' &
y intercept is 'b', this Line
passes through $(a,0)$ & $(0,b)$

$\therefore$ Eq^n of the line passing through $(a,0)$ & $(0,b)$ }

$$\frac{b-0}{y-0} = \frac{0-a}{x-a}$$

$$\frac{b}{y} = \frac{-a}{x-a}$$

$$b(x-a) = -ay$$

$$bx - ab = -ay$$

$$bx + ay = ab$$

$$\frac{bx}{a\cancel{b}} + \frac{\cancel{a}y}{\cancel{a}b} = \frac{ab}{ab}$$

Eq^n of this Line

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

Intercept form of Eq^n of Line

71. Find Eq^n of line passing through points (30, 0), (0, 80)

→

$(30,0)$ $(0,80)$

$$\frac{80-0}{y-0} = \frac{0-30}{x-30}$$

$$80(x-30) = -30y$$

$$80x - 2400 = -30y$$

$$80x + 30y = 2400$$

$$8x + 3y = 240$$

OR

$$\text{slope} = \frac{80-0}{0-30}$$

$$= \frac{80}{-30}$$

$$= -8/3$$

Eq^n of the line

$$8x + 3y = 240$$

OR

x intercept $= 30$

y intercept $= 80$

$$\frac{x}{30} + \frac{y}{80} = 1$$

$$\frac{80x + 30y}{2400} = 1$$

$$80x + 30y = 2400$$

$$8x + 3y = 240$$

72.

Equation of line	x-intercept	y-intercept
$3x + 5y = 90$	30	18
$5x - 2y = 200$	40	-100
$13x + 18y = k$	$\frac{k}{13}$	$\frac{k}{18}$
$20x + 13y = 500$	25	$\frac{500}{13}$
$2x - 11y = -53$	$-\frac{53}{2}$	$\frac{53}{11}$
$21x - y = 200$	$\frac{200}{21}$	-200
$x - y = 10$	10	-10
$2x + y = 58$	29	58
$x = 90$	90	No y-intercept as line is || to y-axis
$y = 65$	No x intercept	65
$kx + my = j$	j/k	j/m
$2kx + 3my = 93$	$93/2k$	$93/3m = 31/m$
$x + 2y = m$	m	$m/2$
$5x + 3y = 1500$	300	500
$x = \frac{90}{7}$	$\frac{90}{7}$	No y-intercept

73. Find eq^n of line having x intercept as 3m & y intercept as 38.

→ Intercept form of Eq^n of line is,

$\frac{x}{a} + \frac{y}{b} = 1$ where $a =$ x intercept
$b =$ y intercept

$\left(\frac{x}{3m} + \frac{y}{38}\right) = 1$

$\frac{38x + 3my}{114m} = 1$ $\therefore$ $38x + 3my = 114m$

74. Find eq^n of line having slope of $\frac{-8}{11}$ and x intercept as 12.

→ Slope of the line $= -8/11$ $\therefore$ Eq^n of Line

Lines passes through $(12, 0)$ $8x + 11y = 96$

75. Find slope of the line whose y intercept is 4 times of x intercept.

→ $a =$ x intercept

$b =$ y intercept $= 4a$

$\frac{x}{a} + \frac{y}{b} = 1$ .... Eq^n of line having x intercept as 'a' & y intercept as 'b'

$\frac{x}{a} + \frac{y}{4a} = 1$

$\frac{4x}{4a} + \frac{y}{4a} = 1$

$\frac{4x + y}{4a} = 1$

$4x + y = 4a$ .... slope of this line $= -4/1 = -4$

76. Find slope of the line whose x intercept is $(4/5)^{th}$ of y intercept.

→ x intercept $= a = \frac{4}{5}b$

y intercept $= b$

$\frac{x}{a} + \frac{y}{b} = 1$ .... Eqⁿ of line having x intercept as 'a' & y intercept as 'b'

$\frac{x}{\frac{4b}{5}} + \frac{y}{b} = 1$

$\frac{5x}{4b} + \frac{4y}{4b} = 1$

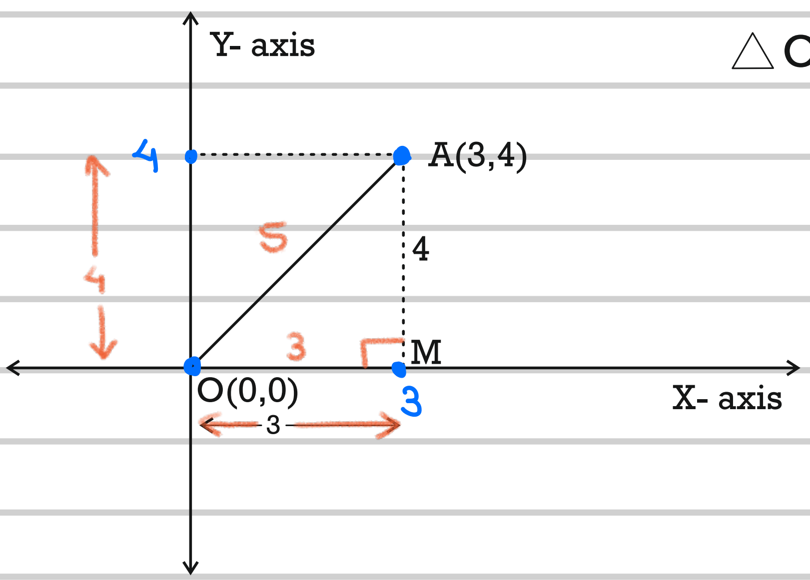
$5x + 4y = 4b$ .... Slope of this line $= -5/4$

Q. Find slope of the line whose x intercept is equal to y intercept.

$\Longrightarrow$ $\frac{x}{a} + \frac{y}{b} = 1$ $\quad \frac{x}{a} + \frac{y}{a} = 1$

$\therefore\ x + y = a = b$ $\quad \therefore$ slope of line $= -1/1 = -1$

77. If A = (3,4), O = (0,0). Find l(OA) = ?



△ OAM is a Right angle triangle

$OA^2 = OM^2 + AM^2$

$OA^2 = 3^2 + 4^2$

$l(OA) = \sqrt{3^2 + 4^2}$

$l(OA) = \sqrt{9 + 16}$

$l(OA) = \sqrt{25}$

$= 5$ units

If O = (0,0), A = (m,n)
then l (OA) = $\sqrt{m^2 + n^2}$

If $O \equiv (0,0)$, $K \equiv (12,-5)$
Find $l(OK)$

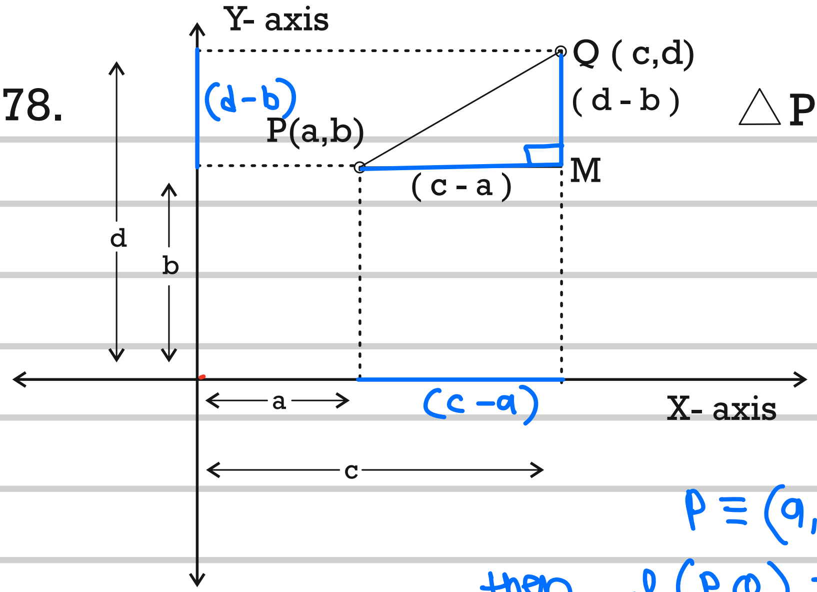
$\Longrightarrow l(OK) = \sqrt{12^2 + (-5)^2}$

$= \sqrt{144 + 25} = 13$ units

$A \equiv (0,0)$, $B \equiv (8,12)$

$l(AB) = \sqrt{8^2 + 12^2} = 14.42221$ units

78.



△PQM is a Right angle triangle

$$PQ^2 = PM^2 + QM^2$$

$$PQ^2 = (c-a)^2 + (d-b)^2$$

$$\therefore\ l(PQ) = \sqrt{(d-b)^2 + (c-a)^2}$$

$$P \equiv (a,b) \quad Q \equiv (c,d)$$

then $l(PQ) = \sqrt{(d-b)^2 + (c-a)^2}$

P(a,b), Q(c,d)
then $l\,(PQ) = \sqrt{(d-b)^2 + (c-a)^2}$

If $A = (x_1,y_1)$ & $B = (x_2,y_2)$ then

$$l(AB) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$= \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$$

If $A = (x_1,y_1)$ & $B = (0,0)$ then

$$l(AB) = \sqrt{x_1^2 + y_1^2}$$

79. A = (30,50), B = (80,-90). Find l(AB)

$$l(AB) = \sqrt{(-90-50)^2 + (80-30)^2}$$

$$= \sqrt{(-140)^2 + 50^2} = \sqrt{22100} = 148.66069 \text{ units}$$

80. P = (m,n), Q = (i,j) then l(PQ) =

$$l(PQ) = \sqrt{(j-n)^2 + (i-m)^2}$$
$$= \sqrt{(n-j)^2 + (m-i)^2}$$

81. If A = (1.50, 2.875), B = (33,81.93). Find l(AB)

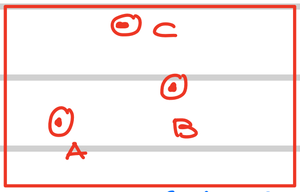
$$l(AB) = \sqrt{(81.93 - 2.875)^2 + (33 - 1.50)^2}$$
$$= \sqrt{7241.943025} = 85.09961 \text{ units}$$

82. If A = (0, 0), B = (-8.75,33.8175). Find l(AB)

$$l(AB) = \sqrt{(33.8175 - 0)^2 + (-8.75 - 0)^2}$$
$$= \sqrt{1220.18580625} = 34.9312 \text{ units}$$

83. Points (8,13), (16,19), (-2k,48) are collinear. Find k.

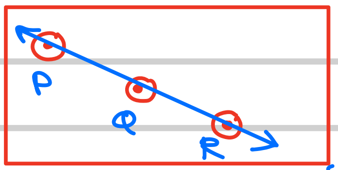
[Slope of the line passing through (8,13) & (16,19)] = [Slope of the line passing through (-2k,48), (8,13)]



Here points A, B, C are Not collinear

$$\frac{19-13}{16-8} = \frac{13-48}{8-(-2k)}$$

$$\frac{6}{8} = \frac{-35}{8+2k}$$



P, Q, R are collinear as a straight line can pass through all of them.

$$48 + 12k = -280$$
$$12k = -328 \quad \boxed{\therefore k = -27.33333}$$

If a straight line can pass through all the points then points are said to be collinear

If a straight line can Not pass through all the points then points are said to be Non collinear

84. Points A, B, C are said to be collinear if

$$\left(\begin{array}{c}\text{Slope of the line passing}\\ \text{through points A, B}\end{array}\right) = \left(\begin{array}{c}\text{Slope of the line passing}\\ \text{through C \& A or B}\end{array}\right)$$

85. Points $(16, \frac{-2k}{5})$, $(8,11)$, $(19,85)$ are collinear. Find the value of k.

→ Slope of the line passing through $(16, \frac{-2k}{5})$ & $(8,11)$ = Slope of the line passing through $(19,85)$ & $(8,11)$

$$\frac{11+\frac{2k}{5}}{8-16} = \frac{11-85}{8-19}$$

$$\frac{\frac{55+2k}{5}}{-8} = \frac{74}{11}$$

$$\therefore \frac{55+2k}{5} = \frac{-592}{11}$$

$$55+2k = \frac{-2960}{11}$$

$$\boxed{k = -162.0454545}$$

86. **QUADRATIC EQUATION**

The standard format of quadratic equation is :

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

and values of x which can satisfy the quadratic equation are known as 'roots' of quadratic equation.

$x^2 - 10x + 16 = 0$ is a quadratic eqn where $a=1$, $b=-10$, $c=16$

→ Let's put $x=8$ $8^2 - 10(8) + 16 = 0$ Now put $x=2$, $2^2 - 10(2) + 16 = 0$

$\therefore$ 8, 2 are roots of quad. eqn.

$$x^2 - 5x - 6 = 0$$ → For $x = 6$, $x = -1$ this Eqn is satisfied

In this quadratic equation a = 1, b = -5, c = -6

If we put x = 6, $6^2 - 5(6) - 6 = 36 - 30 - 6 = 0$

If we put x = -1, $(-1)^2 - 5(-1) - 6 = 1 + 5 - 6 = 0$

$\therefore$ 6, -1 are roots of quadratic equation $x^2 - 5x - 6 = 0$

87.

Equation	No. of roots
Linear	1
Quadratic	2
Cubic	3

88.

Quadratic Eq^n	a	b	c
$3x^2 + 5x - 8 = 0$	3	5	-8
$19x^2 - 55mx - 2k - 81 = 0$	19	-55m	-2k-81
$15x^2 - 21x - 8px + 39x + 88k - 93 = 18x^2$ i.e. $-3x^2 + (-21-8p+39)x + 88k - 93 = 0$	-3	(-21-8p+39) = (18-8p)	(88k-93)
$10x^2 - 2p + 63 = 0$	10	0	-2p+63
$55x^2 - kx^2 + 8px - 33mx + 18j = 63$ i.e. $(55-k)x^2 + (8p-33m)x + 18j - 63 = 0$	(55-k)	(8p-33m)	(18j-63)
$17x^2 - 3x - 93 = 0$	17	-3	-93
$x^2 - 25 = 0$	1	0	-25
$x^2 = 58$	1	0	-58
$(p+q)x^2 - p^2q^2x - 33m = 80$	(p+q)	$-p^2q^2$	(-33m-80)
$25x^2 + 18x = 0$	25	18	0

89. Find roots of quadratic equation

$$x^2 - 13x + 36 = 0$$

Formula Method

$x^2 - 13x + 36 = 0$

$a = 1, b = -13, c = 36$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-13) \pm \sqrt{-13^2-4(1)(36)}}{2 \times 1}$

$x = \frac{13 \pm \sqrt{169-144}}{2}$

$x = \frac{13 \pm 5}{2}$

$x = \frac{13+5}{2}$ OR $x = \frac{13-5}{2}$

$x = 9$ OR $x = 4$

$\therefore$ 9, 4 are the roots

Short-cut

$x^2 - 13x + 36 = 0$

First find the value of

$ac = 36$ & $b = -13$

Find 2 numbers such that their sum is 'b' & product is 'ac'

$x^2 - 9x - 4x + 36 = 0$

$x(x-9) - 4(x-9) = 0$

$(x-9)(x-4) = 0$

$x - 9 = 0$ OR $x - 4 = 0$

$x = 9$ OR $x = 4$

$\therefore$ 9, 4 are the roots of quad. eqn.

Super Short-cut

is applicable only when $a = 1$

$x^2 - 13x + 36 = 0$

Find 2 numbers such that their sum is 'b' & product is 'c'

$(x-9)(x-4) = 0$

$x = 9$ / $x = 4$

$\therefore$ 9, 4 are the roots of quad. eqn.

90. Find roots of quadratic equation $5x^2 - 13x - 18 = 0$

Formula Method

$a = 5, b = -13, c = -18$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-13) \pm \sqrt{(-13)^2-4(5)(-18)}}{2 \times 5}$

$= \frac{13 \pm \sqrt{529}}{10} = \frac{13+23}{10}$ OR $\frac{13-23}{10}$

$\therefore$ $x = 18/5$ OR $x = -1$

Short-cut

$5x^2 - 13x - 18 = 0$

$ac = -90, b = -13$

$5x^2 - 18x + 5x - 18 = 0$

$x(5x-18) + 1(5x-18) = 0$

$(5x-18)(x+1) = 0$

$\therefore$ $x = 18/5$ OR $x = -1$

$\therefore$ 18/5, −1 are the roots of quad. eqn.

91. Find the roots of quadratic equation $2x^2 + 21x + 9 = 0$

$2x^2 + 21x + 9 = 0$

This question can not be solved by short-cut.

Let's use formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-21 \pm \sqrt{21^2 - 4(2)(9)}}{2 \times 2}$$

$$x = \frac{-21 \pm \sqrt{369}}{4} \quad \therefore \; x = \frac{-21 + \sqrt{369}}{4} \text{ OR } x = \frac{-21 - \sqrt{369}}{4}$$

$$x = -0.4476568 \text{ OR } x = \frac{-40.2093}{4}$$

$$x = -10.0523$$

92. Find the roots of $x^2 - 11x - 102 = 0$

$$x = \frac{-(-11) + \sqrt{121 - 4(1)(-102)}}{2 \times 1}$$

$$x = \frac{11 \pm 23}{2}$$

$$x = \frac{11 + 23}{2}, \; x = \frac{11 - 23}{2}$$

$x = 17, \; x = -6$

$x^2 - 11x - 102 = 0$

$x^2 - 17x + 6x - 102 = 0$

$x(x - 17) + 6(x - 17) = 0$

$(x - 17)(x + 6) = 0$

$x = 17, \; x = -6$

$x^2 - 11x - 102 = 0$

$(x - 17)(x + 6) = 0$

$x = 17, \; x = -6$

93. Find the roots of $10x^2 - x - 24 = 0$

$10x^2 - x - 24 = 0$

$10x^2 - 16x + 15x - 24 = 0$

$2x(5x - 8) + 3(5x - 8) = 0$

$(5x - 8)(2x + 3) = 0$

$x = \frac{8}{5}$ OR $x = -\frac{3}{2}$

Roots are : $\frac{8}{5}, -\frac{3}{2}$

$10x^2 - x - 24 = 0$

By Formula

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(10)(-24)}}{2 \times 10}$$

$$x = \frac{1 \pm \sqrt{961}}{20} \quad x = \frac{1 + 31}{20} \text{ OR } \frac{1 - 31}{20}$$

$$\therefore x = \frac{32}{20} = \frac{8}{5} \text{ OR } x = \frac{-30}{20} = -3/2$$

94. Find roots of quadratic equation $80x^2 - 138x + 13 = 0$

$80x^2 - 138x + 13 = 0$

$80x^2 - 130x - 8x + 13 = 0$

$10x(8x - 13) - 1\,(8x - 13) = 0$

$(8x - 13)\,(10x - 1) = 0$

$x = \frac{13}{8}$ or $x = \frac{1}{10}$

Roots are $\frac{13}{8}, \frac{1}{10}$

(i.e. 1.625, 0.10)

$x = \frac{-(-138) \pm \sqrt{19044 - 4(80)(13)}}{2 \times 80}$

$x = \frac{138 \pm \sqrt{14884}}{160} = \frac{138 \pm 122}{160}$

$x = \frac{138 + 122}{160}$ or $x = \frac{138 - 122}{160}$

$x = \frac{260}{160}$ or $x = \frac{16}{160}$

$x = \frac{13}{8}$ or $x = \frac{1}{10}$

95. Find roots of quadratic equation $14x^2 + 29x - 15 = 0$

Also find sum of roots, product of roots.

$14x^2 + 29x - 15 = 0$

$14x^2 + 35x - 6x - 15 = 0$

$7x\,(2x + 5) - 3\,(2x + 5) = 0$

$(2x + 5)\,(7x - 3) = 0$

$\therefore\ x = -\frac{5}{2}$ OR $x = \frac{3}{7}$

$\therefore$ Roots are $-\frac{5}{2}$ & $\frac{3}{7}$

Sum of roots

= 1st root + 2nd root

$= \frac{-5}{2} + \frac{3}{7} = \frac{-35 + 6}{7 \times 2}$

$= -\frac{29}{14}$

product of roots

= 1st root x 2nd root

$= \frac{-5}{2} \times \frac{3}{7} = \frac{-15}{14}$

$14x^2 + 29x - 15 = 0$

For a Quadratic Eq^n } Sum of roots $= \frac{-b}{a} = -\frac{29}{14}$ Product of roots $= \frac{c}{a} = \frac{-15}{14}$

96. Find roots of quadratic equation $10x^2 - 59x - 6 = 0$

Also find sum of roots, product of roots.

$10x^2 - 59x - 6 = 0$

$10x^2 - 60x + x - 6 = 0$

$10x(x-6) + 1(x-6) = 0$

$(x-6)(10x+1) = 0$

$x = 6$ OR $x = -\frac{1}{10}$

Roots are : $6, -\frac{1}{10}$

Sum of roots $= 6 + \frac{-1}{10} = \frac{60-1}{10} = \frac{59}{10}$

product of roots $= 6 \times -\frac{1}{10} = \frac{-6}{10} = -\frac{3}{5}$

Sum of roots $= \frac{-b}{a}$

$= \frac{-(-59)}{10} = \frac{59}{10}$

product of roots $= \frac{c}{a}$

$= \frac{-6}{10} = -\frac{3}{5}$

97.

Quadratic Equation	Sum of Roots	Product of Roots
$ax^2 + bx + c = 0$	$-b/a$	c/a
$8x^2 - 15x - 33 = 0$	$15/8$	$-33/8$
$2x^2 - px + mq + 93 = 0$	$p/2$	$\left(\frac{mq+93}{2}\right)$
$x^2 - 40 = 0$	Zero	$\frac{-40}{1} = -40$
$px^2 + qx + r = 0$	$-q/p$	r/p
$(3k+3)x^2 - (2p-q)x + 8j + 63 = 0$	$\frac{(2p-q)}{(3k+3)}$	$\frac{(8j+63)}{(3k+3)}$

98. In a Quadratic Equation $ax^2 + bx + c = 0$

1. Sum of roots = 1^{st} root + 2^{nd} root

$$= \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) + \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{-b+\cancel{\sqrt{b^2-4ac}} \; -b-\cancel{\sqrt{b^2-4ac}}}{2a}$$

$$= \frac{-2b}{2a}$$

$$= \left(-b/a\right)$$

2. Product of roots = 1^{st} root x 2^{nd} root

$$= \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) \times \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{\left(-b+\sqrt{b^2-4ac}\right) \times \left(-b-\sqrt{b^2-4ac}\right)}{2a \times 2a}$$

$$= \frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{2a \times 2a}$$

$$= \frac{b^2-(b^2-4ac)}{4 \times a \times a} = \frac{\cancel{b^2}-\cancel{b^2}+4ac}{4a \times a}$$

$$= \frac{\cancel{4a} \times c}{\cancel{4a} \times a} = \left(c/a\right)$$

99. If α, β are roots of quadratic equation $5x^2 - 3x - 8 = 0$.
Find the value of $(\alpha + \beta)$, $\alpha\beta$, $(\alpha + \beta)^2$, $(\alpha^2 + \beta^2)$

1. $\alpha + \beta$ = sum of roots = $3/5$
2. $\alpha\beta$ = product of roots = $-8/5$
3. $(\alpha + \beta)^2 = \left(\frac{3}{5}\right)^2 = 9/25$
4. $(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{3}{5}\right)^2 - 2\left(-\frac{8}{5}\right) = \frac{9}{25} + \frac{80}{25} = \frac{89}{25}$

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$$(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$$
$$(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$
$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

please remember

100. If p, q are roots of quadratic equation $x^2 - 3x + 20 = 0$. Find values of

a. $p + q = -(-3)/1 = 3$ = sum of roots

b. pq = product of roots $= 20/1 = 20$

c. $(p - q)^2 = (p+q)^2 - 4pq = 3^2 - 4(20) = -71$

d. $(p^2 + q^2) = (p+q)^2 - 2pq = 3^2 - 2(20) = -31$

e. $p^3 + q^3 = (p+q)^3 - 3pq(p+q) = 3^3 - 3 \times 20(3) = 27 - 180 = -153$

f. $p^2q + q^2p = pq(p+q) = 20 \times 3 = 60$

101. If α, β are roots of quadratic equation $3x^2 - 5x + 2 = 0$.

Find the values of :

1. $\alpha + \beta$ = sum of roots $= \left(5/3\right)$
2. $\alpha\beta$ = product of roots $= \left(2/3\right)$
3. $(\alpha + \beta)^3 = \left(5/3\right)^3 = \left(125/27\right)$
4. $(\alpha^2 + \beta^2) = (\alpha+\beta)^2 - 2\alpha\beta = \frac{25}{9} - 2 \times \frac{2}{3} = \frac{25}{9} - \frac{12}{9} = \left(\frac{13}{9}\right)$
5. $(\alpha^3 + \beta^3) = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

   $= \frac{125}{27} - \left(3 \times \frac{2}{3} \times \frac{5}{3}\right) = \frac{125}{27} - \frac{90}{27} = \left(\frac{35}{27}\right)$
6. $(\alpha - \beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

   $= \frac{25}{9} - 4\left(\frac{2}{3}\right) = \frac{25}{9} - \frac{24}{9} = \left(\frac{1}{9}\right)$
7. $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta)$

   $= \frac{2}{3} \times \frac{5}{3} = \left(\frac{10}{9}\right)$
8. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{13/9}{2/3} = \frac{13/9}{6/9} = \left(13/6\right)$
9. $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{35/27}{2/3} = \frac{35/27}{18/27} = \left(35/18\right)$

102. If α, β are roots of quadratic equation $x^2 + 5x + 13 = 0$.

Find the value of :

1. $\alpha + \beta = -5$
2. $\alpha\beta = 13$
3. $(\alpha^2 + \beta^2) = (\alpha+\beta)^2 - 2\alpha\beta = (-5)^2 - 2(13) = 25 - 26 = -1$
4. $(\alpha + \beta)^2 = (-5)^2 = 25$
5. $(\alpha - \beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = (-5)^2 - 4(13) = -27$
6. $\alpha^2\beta^2 = (\alpha\beta)^2 = 13^2 = 169$
7. $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta) = 13 \times -5 = -65$
8. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right) = \frac{-1}{13} = -\left(1/13\right) = \left(1/-13\right)$
9. $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{70}{13}$
10. $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = (-5)^3 - 3(13)(-5)$
$= -125 + 195$
$= 70$

103. The standard format of quadratic equation is

$ax^2 + bx + c = 0$ where $a \neq 0$

$ax^2 - (-b)x + c = 0$

dividing by 'a' on both sides

$\frac{ax^2}{a} - \left(\frac{-b}{a}\right)x + \frac{c}{a} = \frac{0}{a}$

$x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$

$x^2 - (\text{sum of roots})\,x + (\text{product of roots}) = 0$

x^2 - (Sum of roots) x + (Product of roots) = 0

math

104.

Find roots of quadratic equation

$10x^2 + 11x + 1 = 0$

→

$10x^2 + 10x + x + 1 = 0$

$10x(x+1) + 1(x+1) = 0$

$(x+1)(10x+1) = 0$

$\therefore\ x = -1,\ x = -\frac{1}{10}$

$\therefore\ -1, -\frac{1}{10}$ are the roots of quad. eqn.

Find the quadratic equation whose roots are -1 & $-\frac{1}{10}$

→

$x^2 - \left(\begin{matrix}\text{sum of}\\ \text{roots}\end{matrix}\right)x + \left(\begin{matrix}\text{product of}\\ \text{roots}\end{matrix}\right) = 0$

$x^2 - \left(-1 + \frac{-1}{10}\right)x + \left(-1 \times \frac{-1}{10}\right) = 0$

$x^2 - \left(\frac{-11}{10}\right)x + \frac{1}{10} = 0$

$x^2 + \frac{11}{10}x + \frac{1}{10} = 0$

$10x^2 + 11x + 1 = 0$

105. Find roots of quadratic equation $16x^2+36x-10 = 0$

→

$16x^2 + 36x - 10 = 0$

$8x^2 + 18x - 5 = 0$

$8x^2 + 20x - 2x - 5 = 0$

$4x(2x+5) - 1(2x+5) = 0$

$(2x+5)(4x-1) = 0$

$x = -5/2,\ x = 1/4$

Roots are : $-\frac{5}{2}$ & $\frac{1}{4}$

Find the quadratic equation whose roots are $\frac{1}{4}$ & $-\frac{5}{2}$

→

$x^2 - \left(\begin{matrix}\text{sum of}\\ \text{roots}\end{matrix}\right)x + \left(\begin{matrix}\text{product of}\\ \text{roots}\end{matrix}\right) = 0$

$x^2 - \left(\frac{1}{4} + \frac{-5}{2}\right)x + \left(\frac{1}{4} \times \frac{-5}{2}\right) = 0$

$x^2 - \frac{-18}{8}x - \frac{5}{8} = 0$

$8x^2 + 18x - 5 = 0$

106. Find roots of quadratic equation $6x^2+19x-7 = 0$

→

$6x^2 + 19x - 7 = 0$

$6x^2 + 21x - 2x - 7 = 0$

$3x(2x+7) - 1(2x+7) = 0$

$(2x+7)(3x-1) = 0$

$\therefore\ x = -7/2,\ x = 1/3$

Roots are : $\frac{1}{3}$ & $\frac{-7}{2}$

Find the quadratic equation whose roots are $\frac{1}{3}$ & $\frac{-7}{2}$

→

$x^2 - \left(\begin{matrix}\text{sum of}\\ \text{roots}\end{matrix}\right)x + \left(\begin{matrix}\text{product of}\\ \text{roots}\end{matrix}\right) = 0$

$x^2 - \left(\frac{1}{3} - \frac{7}{2}\right)x + \left(\frac{1}{3} \times \frac{-7}{2}\right) = 0$

$x^2 - \frac{-19}{6}x - \frac{7}{6} = 0$

$6x^2 + 19x - 7 = 0$

107. Find roots of quadratic equation $x^2-10x+23 = 0$

$\rightarrow$ $x^2-10x+23=0$

By Formula,

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-10) \pm \sqrt{100-4(1)(23)}}{2\times 1}$

$= \dfrac{10 \pm \sqrt{8}}{2} = \dfrac{10 \pm \sqrt{4\times 2}}{2}$

$= \dfrac{10 \pm 2\sqrt{2}}{2} = \dfrac{\cancel{2}(5 \pm \sqrt{2})}{\cancel{2}}$

$= (5 \pm \sqrt{2})$ $\therefore$ Roots are: $5+\sqrt{2}$ & $5-\sqrt{2}$

Find the quadratic equation whose roots are $(5+\sqrt{2})$ & $(5-\sqrt{2})$

$\rightarrow$

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$x^2 - [5+\cancel{\sqrt{2}}+5-\cancel{\sqrt{2}}]x + [(5+\sqrt{2})(5-\sqrt{2})] = 0$

$x^2 - 10x + (5^2 - \sqrt{2}^2) = 0$

$x^2 - 10x + 23 = 0$

108.

Roots of Quadratic Eq^n	Quadratic Equation
5, 10	$x^2 - 15x + 50 = 0$
-18, 20	$x^2 - 2x - 360 = 0$
1, -1	$x^2 - 0x - 1 = 0$ i.e. $x^2 - 1 = 0$
15, 18	$x^2 - 33x + 270 = 0$
-16, -20	$x^2 + 36x + 320 = 0$
$-\frac{5}{2}, \frac{9}{2}$	$x^2 - \left(\frac{9}{2} - \frac{5}{2}\right)x + \left(\frac{9}{2} \times \frac{-5}{2}\right) = 0$ $x^2 - 2x - \frac{45}{4} = 0$, $4x^2 - 8x - 45 = 0$
$\frac{9}{7}, \frac{8}{13}$	$x^2 - \left(\frac{9}{7} + \frac{8}{13}\right)x + \left(\frac{9}{7} \times \frac{8}{13}\right) = 0$ $x^2 - \left(\frac{173}{91}\right)x + \frac{72}{91} = 0$, $91x^2 - 173x + 72 = 0$
16, 0	$x^2 - 16x = 0$
$(8+\sqrt{3}), (8-\sqrt{3})$	$x^2 - (16)x + (61) = 0$
$(1+\sqrt{30}), (1-\sqrt{30})$	$x^2 - (2)x + (-29) = 0$ $x^2 - 2x - 29 = 0$

109. Find roots of quadratic equation $4x^2 + 12x + 9 = 0$

→

$4x^2 + 12x + 9 = 0$

$4x^2 + 6x + 6x + 9 = 0$

$2x(2x+3) + 3(2x+3) = 0$

$(2x+3)(2x+3) = 0$

$\therefore\ 2x+3 = 0$ OR $2x+3 = 0$

$x = -3/2$ OR $x = -3/2$

$\therefore$ Roots are $-\frac{3}{2}$ & $-\frac{3}{2}$

$x = \frac{-12 \pm \sqrt{12^2 - 4(4)(9)}}{2 \times 4}$

$x = \frac{-12 \pm 0}{8}$

$x = \frac{-12+0}{8}$ OR $\frac{-12-0}{8}$

$x = -3/2$ OR $x = -3/2$

$b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0$

When $b^2 - 4ac = 0$ then Roots of quadratic Equation are equal.

110. Find roots of quadratic equation $5x^2 + 15x + 91 = 0$

→ Lets solve by formula

$x = \frac{-15 \pm \sqrt{15^2 - 4(5)(91)}}{2 \times 5}$

$x = \frac{-15 \pm \sqrt{225 - 1820}}{10}$

$x = \frac{-15 \pm \sqrt{-1595}}{10}$

Roots are imaginary/unreal/complex numbers

When $(b^2 - 4ac) < 0$ then roots of quadratic equation are unreal/complex/Imaginary Numbers

111. Find roots of quadratic equation $x^2 - 14x + 46 = 0$

$\rightarrow$

$$x^2 - 14x + 46 = 0$$

Let's use formula

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(46)}}{2 \times 1} = \frac{14 \pm \sqrt{196-184}}{2}$$

$$x = \frac{14 \pm \sqrt{12}}{2} = \frac{14 \pm \sqrt{4 \times 3}}{2}$$

$$x = \frac{14 \pm 2\sqrt{3}}{2} = \frac{2(7 \pm \sqrt{3})}{2} = (7 \pm \sqrt{3})$$

Roots are : $(7+\sqrt{3})$ & $(7-\sqrt{3})$

we can clearly see that : Roots are Irrational

$$\boxed{\text{when } (b^2-4ac) > 0 \text{ \& Not a perfect square then Roots of quadratic eq}^n \text{ are Irrational}}$$

Find roots of : $3x^2 - 8x - 11 = 0$

$\Rightarrow$

$$x = \left[\frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-11)}}{2 \times 3}\right] = \frac{8 \pm \sqrt{196}}{6}$$

$$= \frac{8+14}{6} \text{ OR } \frac{8-14}{6} = \frac{11}{3} \text{ OR } -1$$

$$\boxed{\text{when } (b^2-4ac) > 0 \text{ \& a perfect square then Roots of quadratic eq}^n \text{ are Rational}}$$

112.

When	Nature of roots
$b^2 - 4ac = 0$	Real, Rational, Equal
$b^2 - 4ac < 0$	unreal / imaginary / complex
$b^2 - 4ac > 0$ and not a perfect square	Real, Irrational, unequal
$b^2 - 4ac > 0$ and a perfect square	Real, Rational, unequal

value of $b^2 - 4ac$	Nature of roots
38	Real, Irrational, unequal
36	Real, Rational, unequal
81	Real, Rational, unequal
90	Real, Irrational, unequal
−144	Complex / imaginary / unreal
0	Real, Rational, Equal
207936	Real, Rational, unequal
810	Real, Irrational, unequal
−90	Complex / imaginary / unreal
−35	Complex / imaginary / unreal
−0	Real, Rational, Equal
905	Real, Irrational, unequal
2025	Real, Rational, unequal
86	Real, Irrational, unequal
100	Real, Rational, unequal

Quadratic Equation	b^2-4ac	Nature of roots
$3x^2-12x+1=0$	$(-12)^2-4(3)(1)$ $= 144-12 = 132$	Real, Irrational, Distinct
$5x^2-12x=0$	$(-12)^2-4(5)(0)$ $= 144-0 = 144$	Real, Rational, unequal
$4x^2+12x+9=0$	$12^2-4(4)(9)$ $= 144-144=0$	Real, Rational, Equal
$x^2-10x+23=0$	$(-10)^2-4(1)(23)$ $= 100-92=8$	Real, Irrational, Distinct
$10x^2-x-9=0$	$(-1)^2-4(10)(-9)$ $= 1+360 = 361$	Real, Rational, unequal
$8x^2+11=0$	$0^2-4(8)(11)$ $= -352$	complex / imaginary / unereal

113. Find the quadratic equation whose roots are $\frac{3}{2}, \frac{-8}{11}$

➡ $x^2-(\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2-\left(\frac{3}{2}+\frac{-8}{11}\right)x+\left(\frac{3}{2}\times\frac{-8}{11}\right)=0$$

$$x^2-\left(\frac{17}{22}\right)x+\frac{-24}{22}=0$$

$$22x^2-17x-24=0$$

114. Find the quadratic equation whose roots are $(2+\sqrt{23})$ & $(2-\sqrt{23})$

➡ $x^2-(\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2-(2+\sqrt{23}+2-\sqrt{23})x+[(2+\sqrt{23})(2-\sqrt{23})]=0$$

$$x^2-4x-19=0$$

115. Find the quadratic equation whose one root is $(15 + \sqrt{41})$

→ If one root of quad. eq^n is $(15 + \sqrt{41})$ then other root must be $(15 - \sqrt{41})$

$\therefore$ Roots are : $(15 + \sqrt{41})$ & $(15 - \sqrt{41})$

$\therefore$ quadratic Eq^n is :

$$x^2 - \left(\text{sum of roots}\right)x + \left(\text{product of roots}\right) = 0$$

$$x^2 - 30x + (225 - 41) = 0$$

$$x^2 - 30x + 184 = 0$$

116.

Quadratic Equation	$b^2 - 4ac$	Nature of Roots
$3x^2 - 5x - 8 = 0$	$(-5)^2 - 4(3)(-8)$ $= 121$	Real, Rational, unequal
$8x^2 - 13x + 200 = 0$	$(-13)^2 - 4(8)(200)$ $= 169 - 6400$ $= -6231$	unreal / complex / Imaginary
$5x^2 + 11x - 3 = 0$	$11^2 - 4(5)(-3)$ $= 121 + 60 = 181$	Real, Irrational, unequal
$4x^2 + 12x + 9 = 0$	$12^2 - 4(4)(9)$ $= 0$	Real, Rational, Equal
$x^2 - 13x + 36 = 0$	$(-13)^2 - 4(1)(36)$ $= 25$	Real, Rational, unequal
$5x^2 + 12x + 7 = 0$	$12^2 - 4(5)(7)$ $= 4$	Real, Rational, unequal
$4x^2 - 1 = 0$	$0^2 - 4(4)(-1)$ $= 16$	Real, Rational, unequal
$3x^2 + 22x = 0$	$22^2 - 4(3)(0)$ $= 484$	Real, Rational, unequal
$8x^2 - 2x + 33 = 0$	$(-2)^2 - 4(8)(33)$ $= 4 - 1056 = -1052$	unreal / complex / Imaginary

117.

Value of $b^2 - 4ac$	Nature of Roots
38	Real, Irrational, unequal
41	Real, Irrational, unequal
49	Real, Rational, unequal
-60	unreal / complex / Imaginary
0	Real, Rational, Equal
88	Real, Irrational, unequal
14641	Real, Rational, unequal
19288	Real, Irrational, unequal
3364	Real, Rational, unequal
-0	Real, Rational, Equal
380	Real, Irrational, unequal
-100	unreal / complex / Imaginary

118. Roots of quadratic equation $5x^2 - 33x + 8k + 5 = 0$ are equal. Find k.

→ As Roots of quadratic eqn are equal

$\therefore\ b^2 - 4ac = 0$

$(-33)^2 - 4(5)(8k+5) = 0$

$1089 - 20(8k+5) = 0$

$1089 - 160k - 100 = 0$

$989 = 160k$

$\therefore k = \left(\frac{989}{160}\right)$

$= 6.18125$

119. Roots of quadratic equation $5kx^2 - 3x^2 + 18x - 21 = 0$ are equal. Find k.

$(5k-3)x^2 + 18x - 21 = 0$

→ As Roots of quadratic eqn are equal

$\therefore\ b^2 - 4ac = 0$

$18^2 - 4(5k-3)(-21) = 0$

$324 + 84(5k-3)$

$\therefore 324 + 420k - 252 = 0$

$420k = -72$

$k = -\frac{72}{420} = \frac{-18}{105}$

$k = -6/35$

120. Roots of quadratic equation $5mx^2 + 33x - 28 = 0$ are equal. Find m.

→ As Roots of quadratic eqn are equal

$\therefore b^2 - 4ac = 0$

$33^2 - 4(5m)(-28) = 0$

$1089 + 560m = 0$

$m = \frac{-1089}{560}$

121. Roots of quadratic equation $5kx^2 - 33x + 8k - 19 = 0$ are reciprocals of each other. Find the value of k.

→ As Roots of quadratic eqn are reciprocals of each other,

1st root × 2nd root = 1

product of roots = 1

$\frac{c}{a} = 1$

$c = a$

$a = c$

$5k = 8k - 19$

$19 = 3k$

$\therefore k = \frac{19}{3}$

122. Roots of quadratic equation $5x^2 - 8kx + 33x - 8p - 19 = 0$ are equal but opposite in sign. Find the value of k.

→ As Roots are equal but opposite in sign for quad. eqn : $5x^2 + (-8k + 33)x - 8p - 19 = 0$

1st root + 2nd root = 0

sum of roots = 0

$$\frac{-b}{a} = 0$$

$$-b = 0$$

$$b = 0$$

$$(-8k + 33) = 0$$

$$33 = 8k$$

$$\therefore k = 33/8$$

123. If Roots of quadratic equation are	then
Equal	$b^2 - 4ac = 0$
Reciprocal of each other	$a = c$
Equal but opposite in sign	$b = 0 =$ Zero

124. Roots of quadratic equation $5x^2 + kx^2 - 19x - 33k - 93 = 0$ are reciprocal of each other. Find k.

→ $(5+k)x^2 - 19x - 33k - 93 = 0$

As Roots are Reciprocals of each other,

$$a = c$$

$$5 + k = -33k - 93$$

$$34k = -98$$

$$k = -\frac{98}{34} = -\frac{49}{17}$$

125. Roots of quadratic equation $5x^2 - 8px + 81x = 93x - 63k + 88$ are equal but opposite in sign. Find p.

→ $5x^2 + (-8p + 81 - 93)x + 63k - 88 = 0$

As Roots are Equal but opposite in sign,

$b = 0$

$-8p + 81 - 93 = 0$

$-12 = 8p$

$\therefore p = -12/8 = -3/2$

126. If p,q are roots of quadratic equation $x^2 - 11x - 28 = 0$. Find values.

1. $p + q =$ 11 = sum of roots
2. $pq =$ −28 = product of roots
3. $p^3 + q^3 = (p+q)^3 - 3pq(p+q) = 11^3 - 3(-28)(11) = 1331 + 924 = 2255$
4. $p^2 + q^2 = (p+q)^2 - 2pq = 11^2 - 2(-28) = 177$
5. $(p - q)^2 = (p+q)^2 - 4pq = 11^2 - 4(-28) = 233$
6. $\frac{p}{q} + \frac{q}{p} = \frac{p^2+q^2}{pq} = \frac{177}{-28} = -177/28$
7. $\frac{p^2}{q} + \frac{q^2}{p} = \frac{p^3+q^3}{pq} = \frac{2255}{-28} = -2255/28$
8. $p^2q + q^2p = pq(p+q) = -28 \times 11 = -308$
9. $(p - q) = \sqrt{(p-q)^2} = \sqrt{233}$
10. $p^2q^2 = (pq)^2 = (-28)^2 = 784$

127. If α, β are roots of quadratic equation $5x^2 - 2x + 3 = 0$.
Find quadratic equation whose roots are $(\alpha^2 + \beta^2)$, $(\alpha - \beta)^2$

→ $\alpha + \beta = 2/5$

$\alpha\beta = 3/5$

$(\alpha^2 + \beta^2) = (\alpha+\beta)^2 - 2\alpha\beta = \frac{4}{25} - 2 \times \frac{3}{5} = \frac{4}{25} - \frac{30}{25}$

$= \frac{-26}{25} = -1.04$

$(\alpha - \beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = \frac{4}{25} - 4 \times \frac{3}{5} = \frac{4}{25} - \frac{60}{25}$

$= -\frac{56}{25} = -2.24$

question is:
Find quad. eq^n whose roots are -1.04 & -2.24

∴ Answer is: $x^2 - (-1.04 + -2.24)x + (-1.04 \times -2.24) = 0$

$x^2 + 3.28x + 2.3296 = 0$

$10{,}000x^2 + 32800x + 23296 = 0$

$625x^2 + 2050x + 1456 = 0$

128. If α, β are roots of quadratic equation $x^2 - 12x + 17 = 0$.
Find quadratic equation whose roots are $(\alpha^3 + \beta^3)$, $(\alpha^2 + \beta^2)$

→ $\alpha + \beta = 12$, $\alpha\beta = 17$

$\alpha^3 + \beta^3 = 1728 - (3 \times 17 \times 12) = 1116$

$\alpha^2 + \beta^2 = 144 - 2 \times 17 = 110$

Find quad. eq^n whose roots are 1116 & 110

Answer: $x^2 - (1116 + 110)x + (1116 \times 110) = 0$

$x^2 - 1226x + 122760 = 0$

π math $a^2+b^2=c^2$

129. If α, β are roots of quadratic equation $5x^2 - 2x - 11 = 0$.
Find quadratic equation whose roots are (α + β), (αβ)

→ $\alpha+\beta = \frac{2}{5}$, $\alpha\beta = -\frac{11}{5}$

Question is: Find quad. eqn whose roots are $\frac{2}{5}$ & $\frac{-11}{5}$

$x^2 - \left(\frac{2}{5} - \frac{11}{5}\right)x + \left(\frac{2}{5} \times \frac{-11}{5}\right) = 0$

$x^2 - \frac{-9}{5}x - \frac{22}{25} = 0$

$25x^2 + 45x - 22 = 0$

130. If a+b = 12, ab = 60. Find $\left(\frac{1}{a}+\frac{1}{b}\right)$ = ?

→ $\frac{1}{a}+\frac{1}{b} = \frac{a+b}{ab} = \frac{12}{60} = \frac{1}{5}$

131. Find quadratic equation whose one root is $(11 + \sqrt{13})$

→ Roots are : $(11+\sqrt{13})$, $(11-\sqrt{13})$

Quad. Eqn is : $x^2 - \left[11+\sqrt{13}+11-\sqrt{13}\right]x + \left[(11+\sqrt{13})(11-\sqrt{13})\right] = 0$

$x^2 - 22x + (121-13) = 0$

$x^2 - 22x + 108 = 0$

132. Find quadratic equation whose one root is $(7 + \sqrt{230})$

→ Roots are : $(7+\sqrt{230})$ & $(7-\sqrt{230})$

Quad. Eqn is : $x^2 - 14x + (7^2 - \sqrt{230}^2) = 0$

$x^2 - 14x - 181 = 0$

133. Standard format of a quadratic equation is :

$ax^2 + bx + c = 0$ $\qquad$ x^2 - (sum of roots) x + (product of roots) = 0

Where $a \neq 0$

134.

Roots of Quadratic Eq^n	Factors of Quadratic Eq^n	Quadratic Eq^n
3, -13	$(x-3)$, $(x+13)$	$x^2+10x-39=0$
$-\frac{3}{2}$ & $\frac{1}{8}$	(2x + 3) (8x - 1)	$16x^2-2x+24x-3=0$ $16x^2+22x-3=0$
$\frac{1}{2}$, $\frac{1}{2}$	$4x^2-2x-2x+1=0$ $2x(2x-1)-1(2x-1)=0$ $(2x-1)$ & $(2x-1)$	$4x^2 - 4x + 1 = 0$
$\frac{2}{5}, \frac{9}{8}$	$(5x-2)$, $(8x-9)$	$40x^2-45x-16x+18=0$ $40x^2-61x+18=0$
$-\frac{3}{5}$, $\frac{7}{11}$	(5x+3), (11x-7)	$55x^2-35x+33x-21=0$ $55x^2-2x-21=0$
$-\frac{4}{3}$, $-\frac{1}{2}$	(3x+4), (2x+1)	$6x^2+3x+8x+4=0$ $6x^2+11x+4=0$
$\frac{7}{5}, \frac{-11}{8}$	$(5x-7)$, $(8x+11)$	$40x^2+55x-56x-77=0$ $40x^2-x-77=0$
0, 8	x, $(x-8)$	$x^2-8x=0$
1, -1	$(x-1)$, $(x+1)$	$x^2-1=0$
$\frac{5}{3}, \frac{3}{5}$	$(3x-5)$ $(5x-3)$	$15x^2-9x-25x+15=0$ $15x^2-34x+15=0$

math $a^2+b^2=c^2$

135. Standard format of a cubic eq^n $ax^3 + bx^2 + cx + d = 0$, where $a \neq 0$

3 values of x can satisfy cubic equation

∴ Cubic equation has 3 roots

136. Find cubic eq^n whose roots are 8, 3, -2.

→ cubic eq^n is,

$(x-8)(x-3)(x+2)=0$

$(x^2-11x+24)(x+2)=0$

$x^3+2x^2-11x^2-22x+24x+48 = 0$

$\therefore x^3-9x^2+2x+48=0$

OR

$x^3 - (\text{sum of roots})x^2 + \left[(1^{st}\times 2^{nd}) + (2^{nd}\times 3^{rd}) + (1^{st}\times 3^{rd})\right]x - \left(\text{product of roots}\right) = 0$

$x^3-(8+3-2)x^2+(24-16-6)x-(8\times 3\times -2)=0$

$x^3-9x^2+2x+48=0$

137. Find cubic eq^n whose roots are p, q, r

→ As Roots of cubic eq^n are p, q, r

∴ Factors must be $(x-p), (x-q), (x-r)$

∴ The cubic Eq^n is :

$(x-p)(x-q)(x-r)=0$

$(x^2-xq-px+pq)(x-r)=0$

$x^3-x^2r-x^2q+xqr-px^2+pxr+pqx-pqr=0$

$x^3-x^2r-x^2q-x^2p+pqx+qrx+prx-pqr=0$

$x^3-(p+q+r)x^2+(pq+qr+pr)x-(pqr)=0$

$x^3-(\text{sum of roots})x^2+\left[(1^{st}\times 2^{nd})+(2^{nd}\times 3^{rd})+(1^{st}\times 3^{rd})\right]x-\left(\text{product of roots}\right)=0$

compare this with $ax^3+bx^2+cx+d=0$

∴ Sum of roots $= -b/a$ & product of roots $= -d/a$

138. 1. Find cubic eq^n whose roots are 3, -11, -15.

$\rightarrow$ $x^3 - \left(\begin{matrix}\text{sum of}\\ \text{roots}\end{matrix}\right)x^2 + \left[(1^{st} \times 2^{nd}) + (2^{nd} \times 3^{rd}) + (1^{st} \times 3^{rd})\right]x - \left(\begin{matrix}\text{product}\\ \text{of roots}\end{matrix}\right) = 0$

$x^3 - (3-11-15)x^2 + [-33 + 165 + -45]x - (495) = 0$

$\boxed{x^3 + 23x^2 + 87x - 495 = 0}$

2. Find cubic eq^n whose roots are m, n, v

$x^3 - (m+n+v)x^2 + (mn+nv+mv)x - mnv = 0$

139.

	Cubic Equation	Quadratic Equation
Standard Format	$ax^3 + bx^2 + cx + d = 0$ where $a \neq 0$	$ax^2 + bx + c = 0$ where $a \neq 0$
Sum of roots	$-b/a$	$-b/a$
Product of roots	$-d/a$	c/a

Find sum of roots & product of roots for

$8x^3 - 3x^2 - 11kx^2 + 3px^3 - 22x + 18kx - 13mx - 2k + 18 = 0$

$\Longrightarrow$ cubic eq^n is :

$(8+3p)x^3 + (-3-11k)x^2 + (-22+18k-13m)x + (-2k+18) = 0$

$\therefore$ sum of roots $= -b/a = \left(\frac{3+11k}{8+3p}\right)$

product of roots $= -d/a = \left(\frac{2k-18}{8+3p}\right)$

140. Find cubic eq^n whose roots are $\left(\frac{5}{2},\frac{9}{2},\frac{-11}{2}\right)$

→ $x^3-\left(\frac{5}{2}+\frac{9}{2}-\frac{11}{2}\right)x^2+\left[\frac{45}{4}+\frac{-99}{4}+\frac{-55}{4}\right]x-\left(\frac{5}{2}\times\frac{9}{2}\times\frac{-11}{2}\right)=0$

$x^3-\left(\frac{3}{2}\right)x^2+\left(-\frac{109}{4}\right)x+\left(\frac{495}{8}\right)=0$

$8x^3-12x^2-218x+495=0$

OR

$(2x-5)(2x-9)(2x+11)=0$

$(2x-5)(4x^2+4x-99)=0$

$8x^3+8x^2-198x-20x^2-20x+495=0$

$8x^3-12x^2-218x+495=0$

141. Find quadratic eq^n whose roots are $\left(\frac{5}{2},0\right)$

→ $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$x^2-\frac{5}{2}x+0=0$

$2x^2-5x=0$

OR Factors: $(2x-5)$ & x

$x(2x-5)=0$ i.e. $2x^2-5x=0$

142. Find quadratic eq^n whose roots are (10, -10)

→ $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$x^2-0x+(-100)=0$ $\therefore$ $x^2-100=0$

OR

$(x-10)(x+10)$ are Factors $\therefore$ $(x-10)(x+10)=0$

$x^2-100=0$

143. Find quadratic eq^n whose roots are $\left(\frac{8}{9},\frac{9}{8}\right)$

→ $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$x^2-\left(\frac{8}{9}+\frac{9}{8}\right)x+\left(\frac{8}{9}\times\frac{9}{8}\right)=0$

$x^2-\frac{145}{72}x+1=0$ $\therefore$ $72x^2-145x+72=0$

144. The point (-3p, 28) lie on the line 7x + 12y = 820. Find p.

➡ If we put $x = -3p$ & $y = 28$, $7x + 12y = 820$ must be satisfied

$7(-3p) + 12(28) = 820$

$-21p = 484$

$p = \left(-\frac{484}{21}\right) = -23.047619$

145. The point of intersection of lines 7x + 3y = 90 & 8x + 7y = 210 lie in ______ Quadrant.

a. 1st b. 2nd c. 4th ~~d.~~ None of these

➡ $56x + 24y = 720$

$56x + 49y = 1470$

$- \quad - \quad -$

$-25y = -750$

$\boxed{y = 30}$

$7x + 3(30) = 90$

$7x = 0 \quad \therefore x = 0$

(0,30) is the point of intersection. It is on Y-axis

146. The lines 2x + 3y = 90 & 4x + 6y = 180 have ____________

a. No solution

b. Unique Solution

~~c.~~ Infinite No. of solutions

d. None of these

$2x + 3y = 90$ —— ①

$4x + 6y = 180$ —— ②

$4x + 6y = 180$

These 2 lines coincide

i.e. every point is point of intersection

147. Slope of the line $8x = \frac{81}{11}$ is

a. Zero b. $\frac{81}{88}$ c. $\frac{81}{-88}$ ~~d.~~ Not defined

$8x = \frac{81}{11}$

$88x = 81$

$\therefore 88x + 0y = 81$

Slope $= -a/b = -88/0 =$ Not defined

OR

$8x = \frac{81}{11}$ is a ∥ line to Y-axis

∴ It's slope is Not defined

148. Find equation of line having slope of $(\frac{8}{11})$ passing through $(\frac{3}{5}, \frac{8}{5})$

→ Slope $= \frac{8}{11}$

$\therefore$ Eqn of line $8x - 11y = 8\left(\frac{3}{5}\right) - 11\left(\frac{8}{5}\right)$

$$8x - 11y = -\frac{64}{5}$$

$$40x - 55y = -64$$

$$40x - 55y + 64 = 0$$

149. Find equation of line having x,y intercept as $\frac{8}{3}, \frac{11}{9}$ respectively.

→ Intercept form is $\frac{x}{a} + \frac{y}{b} = 1$

a = x intercept

b = y intercept

$$\left(\frac{x}{\frac{8}{3}} + \frac{y}{\frac{11}{9}}\right) = 1$$

$$\frac{3x}{8} + \frac{9y}{11} = 1$$

$$\frac{33x + 72y}{88} = 1 \quad \therefore \quad 33x + 72y = 88$$

150. Find nature of roots of $3x^2 - 14x - 31 = 0$

→ $b^2 - 4ac = (-14)^2 - 4(3)(-31)$

$= 196 + 372 = 568 > 0$ but not a perfect square

$\therefore$ Roots are : Real, Irrational, unequal

151. If α, β are roots of quadratic equation $x^2 - 5x + 9 = 0$ then Find quadratic equation whose roots are $(2\alpha + 3\beta)$ & $(3\alpha + 2\beta)$

→ $\alpha + \beta = 5, \ \alpha\beta = 9$

Quad. eqn whose roots are $(2\alpha+3\beta)$ & $(3\alpha+2\beta)$ is

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$x^2 - [2\alpha+3\beta+3\alpha+2\beta]x + [(2\alpha+3\beta)(3\alpha+2\beta)] = 0$

$x^2 - (5\alpha+5\beta)x + [6\alpha^2+4\alpha\beta+9\alpha\beta+6\beta^2] = 0$

$x^2 - 5(\alpha+\beta)x + [13\alpha\beta + 6(\alpha^2+\beta^2)] = 0$

$x^2 - (5\times5)x + [13\times9 + 6(5^2 - 2\times9)] = 0$

$x^2 - 25x + (117 + 6\times7) = 0$

$x^2 - 25x + (117+42) = 0$

$x^2 - 25x + 159 = 0$

152. One root of quadratic equation $3kx^2 + 18px - 19p + 21 = 0$ is 'zero'. Find value of 'p'.

→ one root is zero, means for $x = 0$, Eqn is satisfied

$\therefore \ 3k(0)^2 + 18p(0) - 19p + 21 = 0$

$0 + 0 + 21 = 19p$

$\therefore \ p = \frac{21}{19}$

153. If α, β are roots of quadratic equation $5x^2 - 11x + 29 = 0$, Find quadratic equation whose roots are (α + 1) & (β + 1)

→ $\alpha + \beta = \frac{11}{5}$, $\alpha\beta = \frac{29}{5}$

Quad. Eqⁿ whose roots are $(\alpha+1)$ & $(\beta+1)$ is

$$x^2 - [\alpha+1+\beta+1]x + [(\alpha+1)(\beta+1)] = 0$$

$$x^2 - (\alpha+\beta+2)x + (\alpha\beta+\alpha+\beta+1) = 0$$

$$x^2 - \left(\frac{11}{5}+2\right)x + \left(\frac{29}{5}+\frac{11}{5}+\frac{5}{5}\right) = 0$$

$$x^2 - \left(\frac{21}{5}\right)x + \left(\frac{45}{5}\right) = 0$$

$$5x^2 - 21x + 45 = 0$$

154. The points (16, (-2k/9)), (18,0), (19,-23) are collinear. Find 'k'

→ [Slope of the line passing through points $(16, \frac{-2k}{9})$ & $(18,0)$] = [Slope of the line passing through points $(18,0)$ & $(19,-23)$]

$$\frac{0+\frac{2k}{9}}{18-16} = \frac{-23-0}{19-18}$$

$$\frac{\frac{2k}{9}}{2} = \frac{-23}{1}$$

$$\frac{2k}{9} = -46$$

$$2k = -414$$

$$k = -207$$

155. $\frac{x+24}{5} = 4 + \frac{x}{4}$ Find x.

$$\frac{x+24}{5} = 4 + \frac{x}{4}$$

$$\frac{x+24}{5} = \frac{16+x}{4}$$

$$4x + 96 = 80 + 5x$$

$$\boxed{16 = x} \qquad \therefore x = 16$$

156. $x + 5y = 36, \frac{x+y}{x-y} = \frac{5}{3}$ then $(x, y) = ?$

$x = 36 - 5y$, $\frac{36-5y+y}{36-5y-y} = \frac{5}{3}$

$\therefore x = 36 - 5(4)$

$= 16$

$$\frac{36-4y}{36-6y} = \frac{5}{3}$$

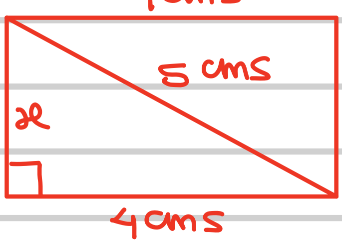
$$108 - 12y = 180 - 30y$$

$$18y = 72$$

$$y = 4$$

$$\therefore (x, y) \equiv (16, 4)$$

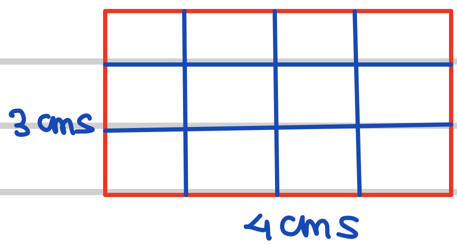
157. Diagonal of a rectangle is 5 cms and one of the side is 4 cms then Find Area of Rectangle.



$$5^2 = 4^2 + x^2$$

$$x^2 = 9$$

$$x = 3 \text{ cms}$$



Area = Length × Breadth

= 4 cms × 3 cms

= 12 sq. cms

158. If one root of quadratic equation exceeds the other by 4 in $x^2 - 8x + m = 0$. Find m.

➡

$x^2 - (\text{sum of roots})\,x + \text{product of roots} = 0$

sum of roots = 8 , product of roots = m

2   6

$\therefore$ Roots are : 2, 6

$\therefore$ product of roots $= 2 \times 6 = 12 = m$

$\boxed{\therefore\ m = 12}$

$x^2 - (2+6)x + (2 \times 6) = 0$

159. $x + y = 50, \frac{1}{x} + \frac{1}{y} = \frac{1}{8}$ then (x,y) = ?

➡

$x + y = 50$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{8}$

$\frac{x+y}{xy} = \frac{1}{8}$

$xy = 8(x+y)$

$xy = 8 \times 50 = 400$

$\boxed{x + y = 50}$ & $\boxed{x \times y = 400}$

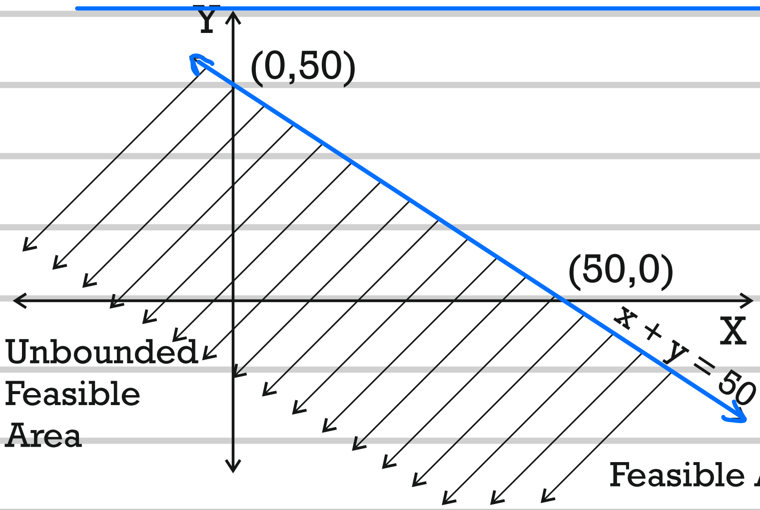
$\therefore\ x, y = (10, 40)$ OR $(40, 10)$

160. Find feasible area for $x + y \le 50$

$x + y = 50$ $2x + 3y = 90$ $3x - 5y = 60$ $x = 35$	Linear Equation Or Linear Equality	$x + y \le 50$ $2x + 3y \ge 90$ $5x - 18y < 35$ $x \le 48$ $y \ge 90$	Linear Inequations Or Linear Inequality

$x + y \le 50$ is a linear inequality.

Let's draw the line $x + y = 50$ on graph paper.



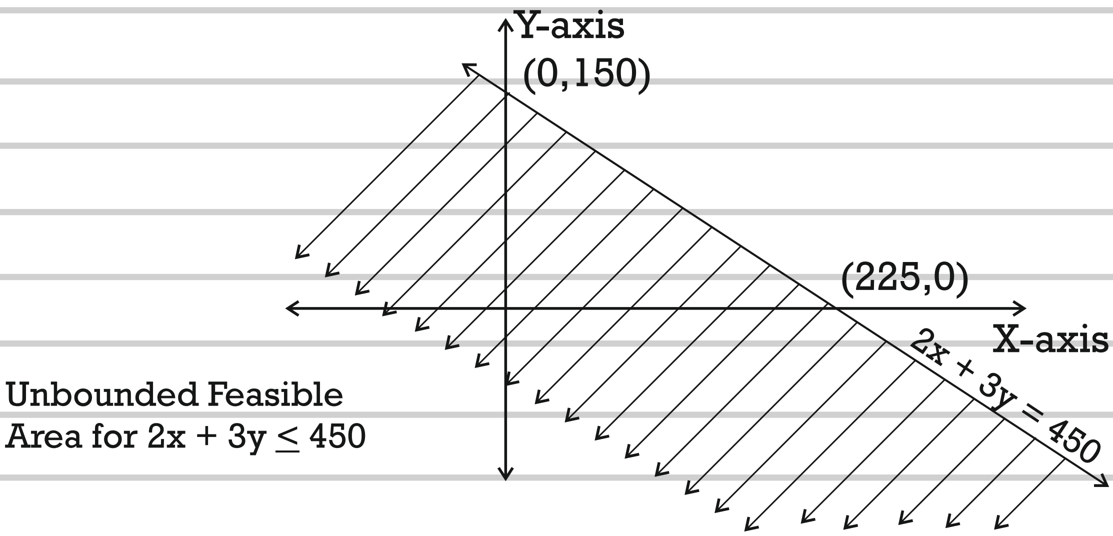
Graphical representation of a linear equation is : a Line
Graphical representation of a linear inequality is : **AN AREA**

Feasible Area or Solution Space for $x + y \le 50$

161. Find feasible area for $2x + 3y \le 450$

To find feasible area for $2x + 3y \le 450$

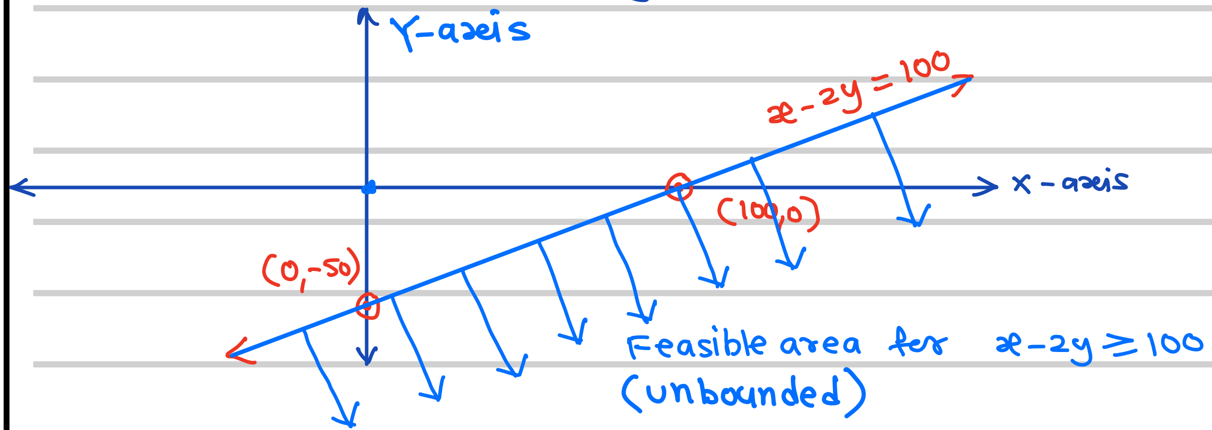
Let's draw the line $2x + 3y = 450$ by joining the points (225,0) & (0,150)



162. Find feasible area for $x - 2y \geq 100$

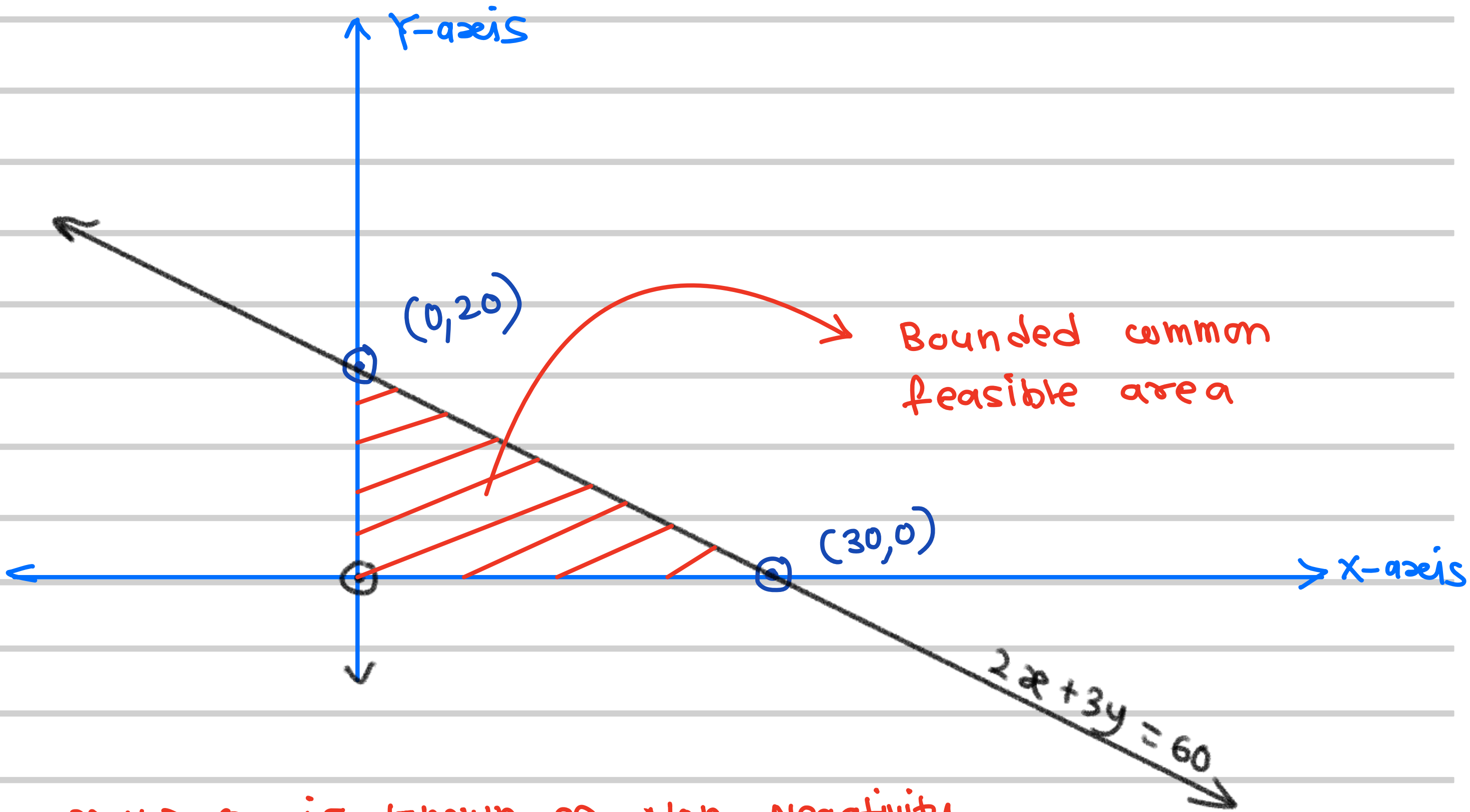
→ To Find Feasible area for $x - 2y \geq 100$,

Let's draw $x - 2y = 100$ by joining points $(100,0)$ & $(0,-50)$



163. Find common feasible area for $2x + 3y \leq 60$ & $x, y \geq 0$

→ Let's first draw $2x + 3y = 60$ by joining $(0,20)$ & $(30,0)$

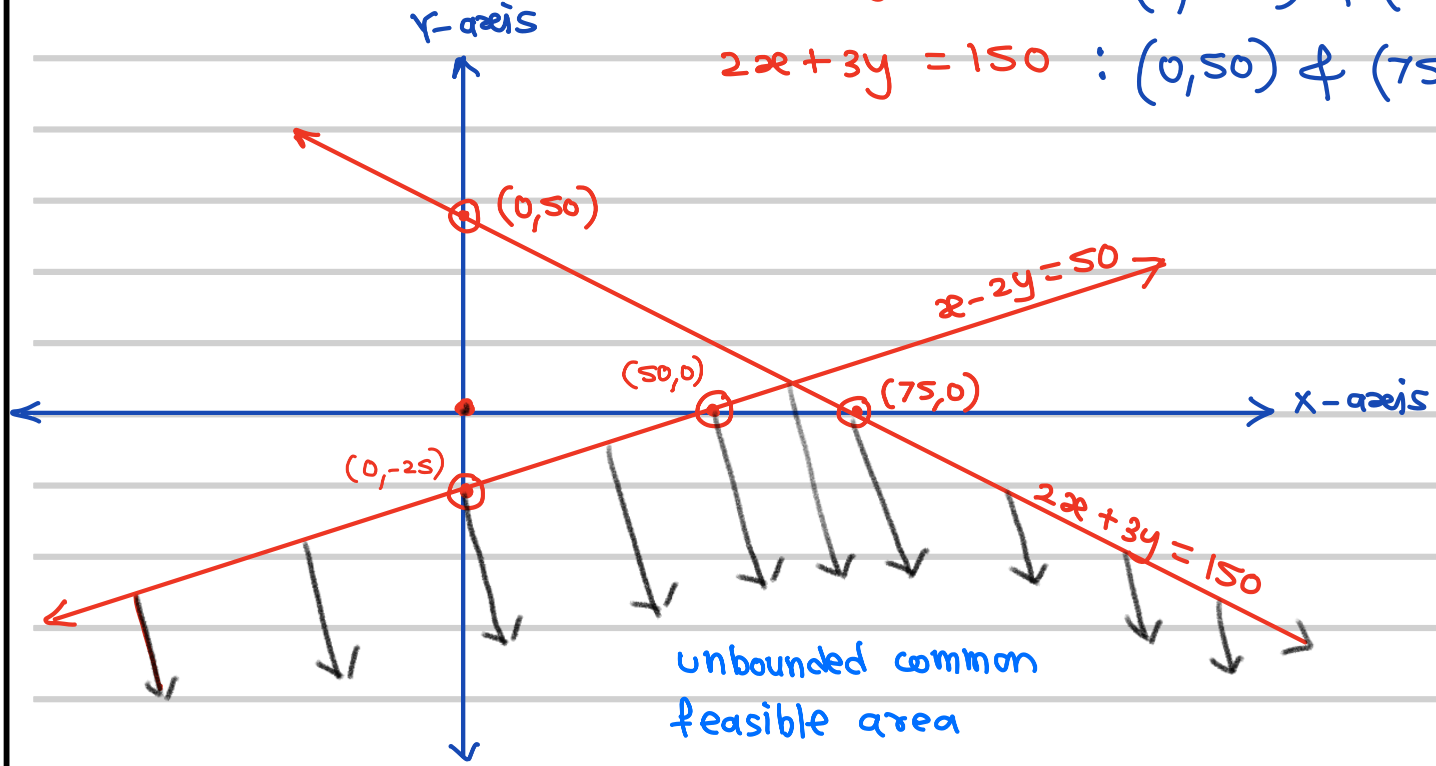


$x, y \geq 0$ is known as Non-Negativity constraint which restricts feasible area in 1st quadrant.

164. Find common feasible area for x - 2y ≥ 50 & 2x + 3y ≤ 150

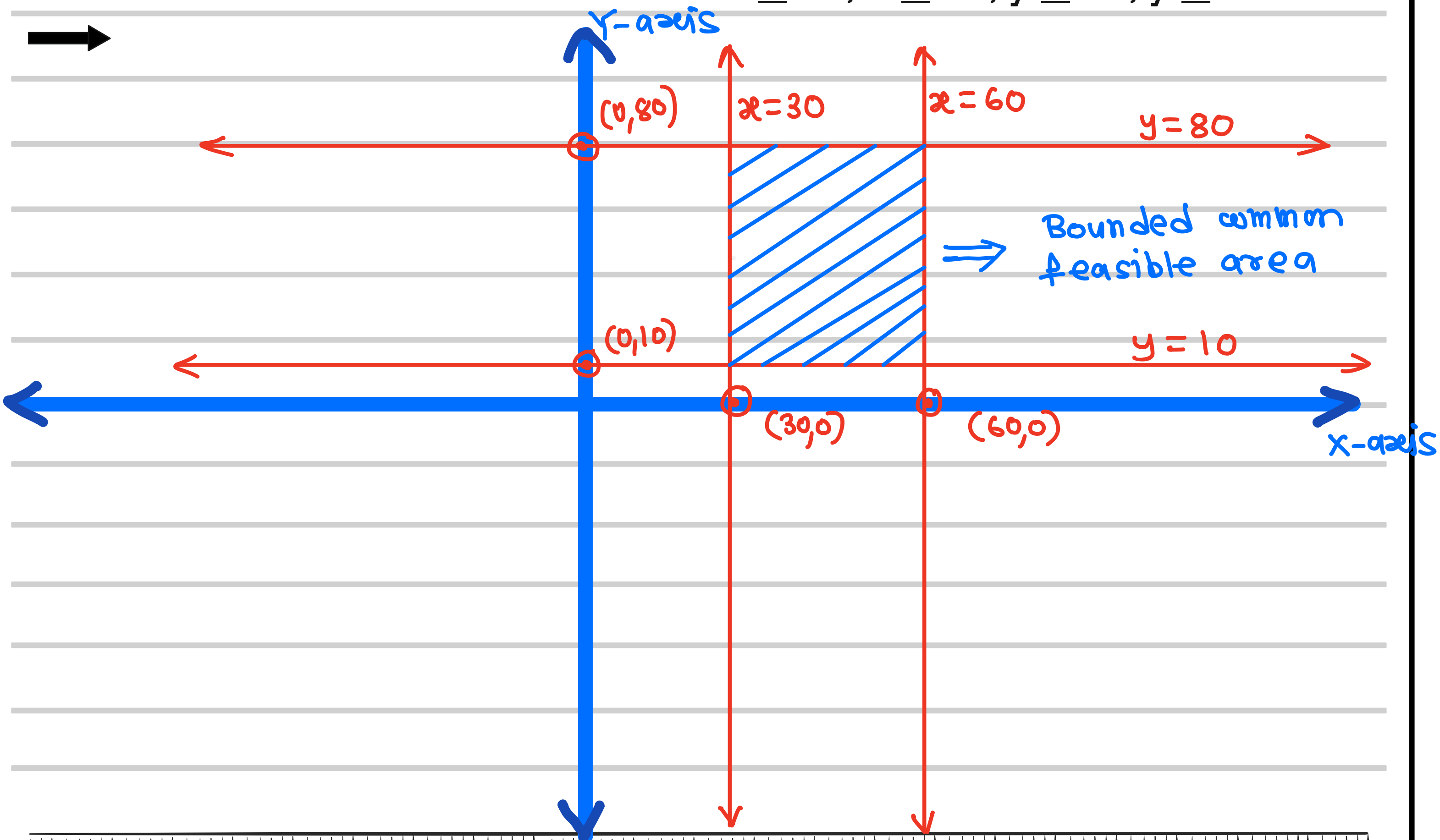
→ Lets draw Lines $x - 2y = 50$: (0,-25) & (50,0)

$2x + 3y = 150$: (0,50) & (75,0)



165. Find common feasible area for x ≥ 30, x ≤ 60, y ≥ 10, y ≤ 80

→



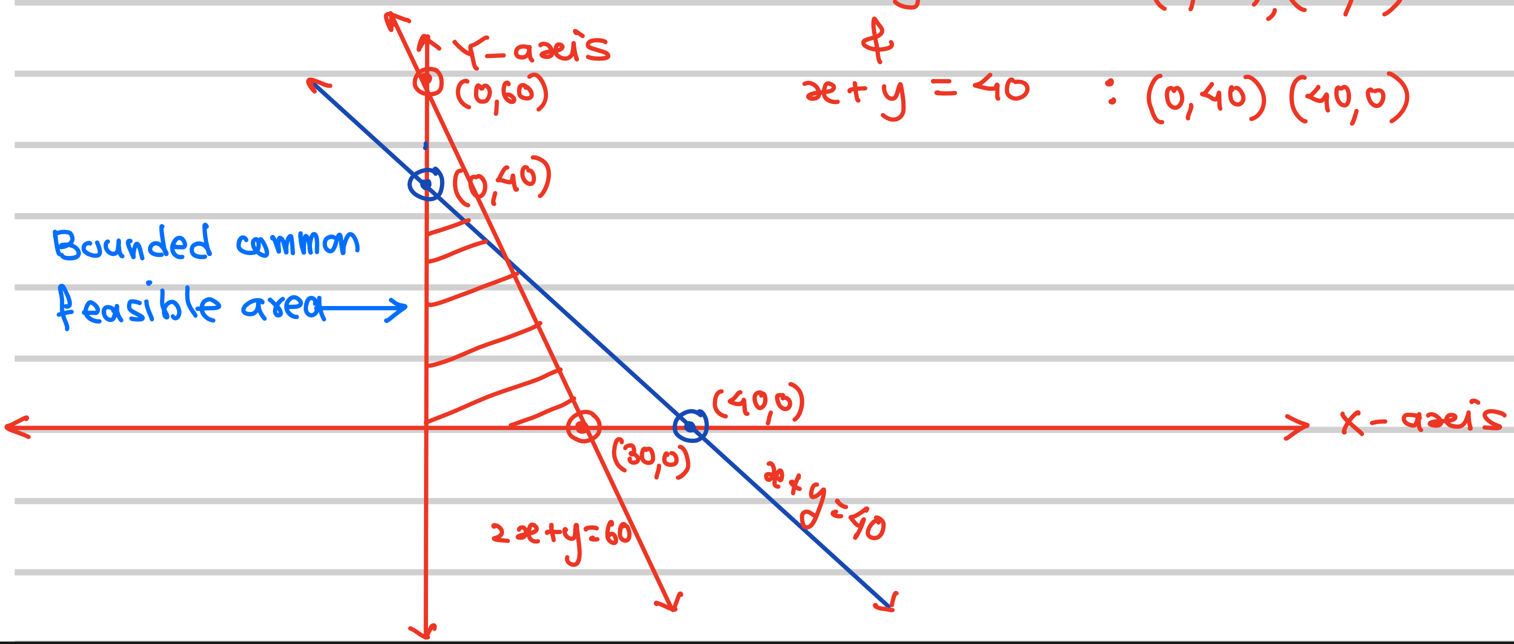
Non-negativity unstraint

166. Find common feasible area for $2x + y \leq 60$, $x + y \leq 40$, $\boxed{x, y \geq 0}$

→ Let's first draw Lines $2x+y=60$: (0,60), (30,0)

&

$x+y=40$: (0,40) (40,0)



167. Which of the following point lie in common feasible area of

$5x - 3y \leq 80$, $2x + 7y \geq 40$

a. (5,2) b. (20,3) ~~c.~~ (30,30) d. None

→

$5(5)-3(2) \leq 80$ ✓ $5(20)-3(3) \leq 80$ ✗ $5(30)-3(30) \leq 80$ ✓

$2(5)+7(2) \geq 40$ ✗ $2(30)+7(30) \geq 40$ ✓

↓

Point (30,30) satisfies both the inequalities

Q: which of the following point lie in common feasible area of $5x+2y \leq 100$, $8x-3y \geq 20$, $x+2y \leq 90$

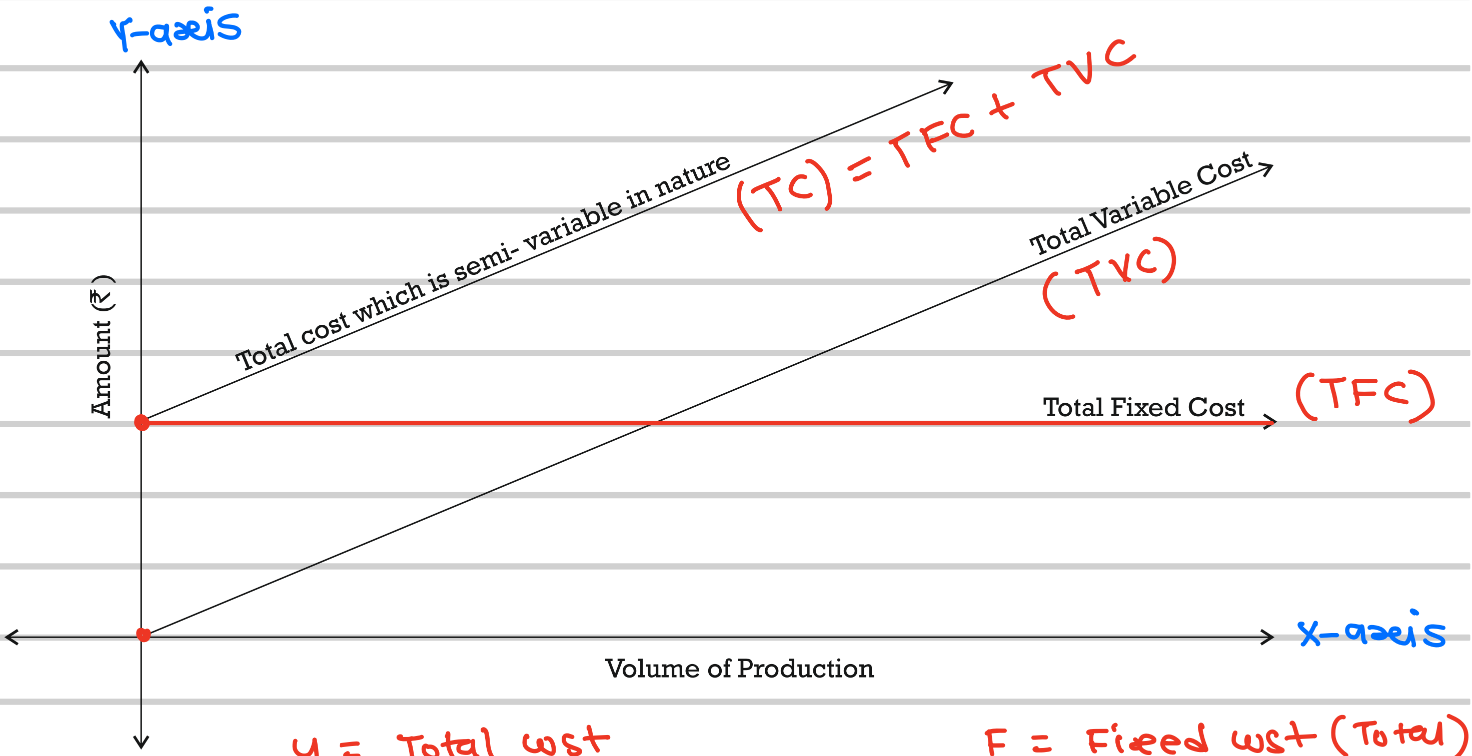
~~(a)~~ (10,20) (b) (80,60) (c) Both (d) None

168. Total cost = Fixed cost + Variable cost

Fixed cost : The cost which does not change with change in volume of production is known as Fixed Cost.

Variable cost : The cost which changes in same proportion with change in volume of production is known as Variable Cost.

Semi - Variable cost : If portion of the cost is fixed and portion is variable then cost is said to be semi-variable or semi-fixed cost.



y = Total cost

x = No. of units of produced

F = Fixed cost (Total)

V = vari. cost p.u.

$$y = F + Vx$$

Total cost = Total Fixed cost + (vari. cost p.u. × No. of units produced)

169. If $b^2 - 4ab = 0$ then roots of quadratic equation are

a. Equal b. Equal but opposite in sign

c. Reciprocals of each other ~~d.~~ Can't say

Roots are equal when $b^2 - 4ac = 0$

we can't say anything on the basis of value of $b^2 - 4ab$

170. Sum of 2 numbers is 88 and diff bet^n first number and half of second number is 10. Find the numbers.

a. 32,56 b. 44,44 ~~c.~~ 36,52 d. 30,58

$x + y = 88$

$x - \frac{y}{2} = 10$

$2x - y = 20$

$2(88-y) - y = 20$

$176 - 2y - y = 20$

$156 = 3y$

$\boxed{y = 52}$ $\boxed{\therefore x = 36}$

171. If p, q are roots of $3x^2 - 3x - 1 = 0$. Find value of (p^3+q^3), (p^2+q^2)

① $p + q = 1$

② $pq = -1/3$

③ $(p^3+q^3) = (p+q)^3 - 3pq(p+q)$

$= 1^3 - 3\left(-\frac{1}{3}\right)(1) = 1 + 1 = 2$

④ $(p^2+q^2) = (p+q)^2 - 2pq = 1^2 - 2 \times -\frac{1}{3} = 1 + \frac{2}{3} = \frac{5}{3}$

⑤ $(p-q)^2 = (p+q)^2 - 4pq = 1^2 - 4 \times -\frac{1}{3} = 1 + \frac{4}{3} = \frac{7}{3}$

⑥ $p^2q + q^2p = pq(p+q) = -\frac{1}{3} \times 1 = -1/3$

⑦ $\frac{p}{q} + \frac{q}{p} = \frac{p^2+q^2}{pq} = \frac{5/3}{-1/3} = -5$

⑧ $\frac{p^2}{q} + \frac{q^2}{p} = \frac{p^3+q^3}{pq} = \frac{2}{-1/3} = -6$

172. If p, q are roots of $3x^2 - 19x - 1 = 0$, whose roots are $\frac{p}{q}$ & $\frac{q}{p}$.
Find quadratic equation.

$$p + q = 19/3$$

$$pq = -1/3$$

Quad. eqn whose roots are $\frac{p}{q}$ & $\frac{q}{p}$ is,

$$x^2 - \left(\frac{p}{q} + \frac{q}{p}\right)x + \left(\frac{p}{q} \times \frac{q}{p}\right) = 0$$

$$x^2 - \left(\frac{p^2 + q^2}{pq}\right)x + 1 = 0$$

$$x^2 - \left(\frac{\frac{361}{9} - 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}}\right)x + 1 = 0$$

$$x^2 + \left(\frac{\frac{361}{9} + \frac{6}{9}}{\frac{3}{9}}\right)x + 1 = 0$$

$$x^2 + \frac{367}{3}x + 1 = 0$$

$$3x^2 + 367x + 3 = 0$$

173. The cubic eqn whose roots are m, n, q is :

$$x^3 - \left(\begin{matrix}\text{sum of} \\ \text{roots}\end{matrix}\right)x^2 + \left[(1^{st} \times 2^{nd}) + (2^{nd} \times 3^{rd}) + (1^{st} \times 3^{rd})\right]x - \left(\begin{matrix}\text{product} \\ \text{of roots}\end{matrix}\right) = 0$$

$$x^3 - (m + n + q)x^2 + (mn + nq + mq)x - mnq = 0$$

174. If x = No. of units produced
Fixed Cost = ₹ 3,80,000; Variable Cost p.u. = ₹ 28
then y = Total Cost = ________________

Total cost = Total Fixed cost + Total variable cost

TC = Total Fixed cost + (vari. cost p.u. × No. of units produced)

$$\boxed{y = 3,80,000 + 28x}$$

175. If (p+2) (p-3) + (p+3)(p-4) = p(2p-5), then p = ?

➞

$$(p+2)(p-3) + (p+3)(p-4) = p(2p-5)$$

$$p^2 - 3p + 2p - 6 + p^2 - 4p + 3p - 12 = 2p^2 - 5p$$

$$2p^2 - p - 6 - p - 12 = 2p^2 - 5p$$

$$2p^2 - 2p - 18 = 2p^2 - 5p$$

$$\cancel{2p^2} - 2p - 18 - \cancel{2p^2} + 5p = 0$$

$$3p = 18$$

$$\boxed{p = 6}$$

176. 15x + 23y = -10 & 3x + 4y = -2 then 3x + 2y + 2 = ?

➞

$$\cancel{15x} + 23y = \cancel{-10}$$

$$\cancel{15x} + 20y = \cancel{-10}$$

$$- \quad - \quad +$$

$$3y = 0$$

$$y = \frac{0}{3}$$

$$y = 0$$ ………………… By using calculator.

Let's put $y = 0$ in $15x + 23y = -10$

$$15x + 23(0) = -10$$

$$15x = -10$$

$$x = -\frac{10}{15} = -\frac{2}{3}$$

$\therefore$ $3x + 2y + 2 = ?$

$$= 3\left(-\frac{2}{3}\right) + 2(0) + 2$$

$$= -2 + 0 + 2$$

$$= 0$$

177. Find value of k, if $9x^2 - 24x + k = 0$ has equal roots.

→ $b^2 - 4ac = 0$

$(-24)^2 - 4(9)(k) = 0$

$576 - 36k = 0$

$576 = 36k$

$\therefore k = 16$

Roots are equal	$b^2-4ac=0$
Roots are reciprocals of each other	$a=c$
Roots are equal but opposite in sign	$b=0$

178. Calculate the number such that it is equal to 3 times of its diff from 56.

→ Let that Number be x

$x = 3 \times (56 - x)$

$x = 168 - 3x$

$4x = 168$

$x = 42$

Roots are	b^2-4ac
Real, irrational, unequal	$b^2-4ac > 0$ & Not a perfect square
Real, Rational, Equal	$b^2-4ac = 0$
Real, Rational, unequal	$b^2-4ac > 0$ & perfect square
complex/imagi.	$b^2-4ac < 0$

179. $2x + 3y = 5$ & $3x - 4y = 2$ then $5xy = ?$

→ $6x + 9y = 15$

$\underline{-6x \mp 8y = \mp 4}$

$17y = 11 \qquad \therefore y = \frac{11}{17}$

$2x + \left(3 \times \frac{11}{17}\right) = 5$

$2x = 5 - \frac{33}{17} = \frac{52}{17} \qquad \therefore x = \frac{26}{17}$

$5xy = 5 \times \frac{26}{17} \times \frac{11}{17} = \left(\frac{1430}{289}\right)$

For a cubic eqn

$ax^3 + bx^2 + cx + d = 0$

$\boxed{\text{sum of roots} = -b/a}$ $\boxed{\text{product of roots} = -d/a}$

math

180. $a^2 + b^2 = 45$ then $\frac{1}{a}+\frac{1}{b} = ?$
$ab = 18$

➡ $(a+b)^2 = (a^2+b^2) + 2ab$
$= 45 + 2(18)$
$(a+b)^2 = 81$
$(a+b) = 9$

$\frac{1}{a}+\frac{1}{b}$
$= \left(\frac{a+b}{ab}\right)$
$= \frac{9}{18} = \frac{1}{2}$

181. If roots of quadratic equation are (2m) & (-2n) then factors are :

➡ Roots are : 2m, −2n
∴ Factors are : $(x-2m)$ & $(x+2n)$

182. If roots of quadratic equation are $(\frac{3}{5})$ & $(\frac{-8}{11})$ then factors are :

➡ Factors are $(5x-3)$ & $(11x+8)$

Roots of quad. eqn	quad. eqn
8, 3	$x^2-11x+24=0$
11, 9	$x^2-20x+99=0$
−6, 88	$x^2-82x-528=0$
10, 0	$x^2-10x=0$
$2+\sqrt{20}$, $2-\sqrt{20}$	$x^2-4x-16=0$
$8+\sqrt{11}$, $8-\sqrt{11}$	$x^2-16x+53=0$
0.50, 2.50	$x^2-3x+1.25=0$, $4x^2-12x+5=0$
−6, −9	$x^2+15x+54=0$
28, −28	$x^2-0x-784=0$

183. If quadratic equation $x^2 - (p+4)x + 2p + 5 = 0$ has equal roots. Find p

→ As Roots are equal, $b^2 - 4ac = 0$

$[-(p+4)]^2 - 4(1)(2p+5) = 0$

$p^2 + \cancel{8p} + 16 - \cancel{8p} - 20 = 0$

$p^2 - 4 = 0$

$p^2 = 4$

$p = \pm 2$

184. If $4x^3 + 8x^2 - x - 2 = 0$ then $(2x + 3) = ?$

~~a.~~ 4, -1, 2 b. -4, 2, 1 c. 2, -4, -1 d. None

→ $4x^3 + 8x^2 - x - 2 = 0$

$4x^2(x+2) - 1(x+2) = 0$

$(x+2)(4x^2 - 1) = 0$

$(x+2)[(2x)^2 - 1^2] = 0$

$(x+2)(2x-1)(2x+1) = 0$

$x = -2, \quad x = \frac{1}{2}, \quad x = -\frac{1}{2}$

$(2x+3)$: $2(-2)+3 = -1$ | $2 \times \frac{1}{2} + 3 = 4$ | $2 \times -\frac{1}{2} + 3 = 2$

185. Sum of 2 numbers is 15 & their product is 50 then sum of their reciprocal is :

$x + y = 15$

$xy = 50$

$\frac{1}{x} + \frac{1}{y}$

$= \frac{x+y}{xy} = \frac{15}{50} = \frac{3}{10} = 0.30$

186. Out of 3 numbers, sum of first and second is 24, sum of 2nd & 3rd is 30, sum of first & third is 26. The smallest number is :

a. 18 b. 14 c. 16 ~~d.~~ 10

1st Number : x

2nd Number : y

3rd Number : z

$x + y = 24$

$y + z = 30$

$x + z = 26$

$x + y = 24$

$x + 30 - z = 24$

$x + 30 - (26 - x) = 24$

$x + 30 - 26 + x = 24$

$2x + 4 = 24$

$2x = 20$

$\boxed{x = 10}$

$\therefore \boxed{y = 14}$ $\boxed{z = 16}$

FORGET THE MISTAKE
REMEMBER THE LESSON !