

Dharmesh

Vijay

60 kg

50 kg

Ratio of weight
of V & D = 5:6

Ratio of
weight of D & V

$$\frac{\cancel{60} \text{ kg}}{\cancel{50} \text{ kg}} = \frac{6}{5} = 6:5$$

Ratio

- › A ratio is a comparison of the sizes of **two or more** quantities of the same kind by division.
- › If a and b are two quantities of the same kind (in same units), then the fraction a/b is called the ratio of a to b. It is written as $a : b$.
- › The quantities **a** and **b** are called the **terms** of the ratio, **a** is called the first term or antecedent and **b** is called the second term or consequent.

CA. PRANAV POPAT

3 : 4
consequent ? 4



PEN

The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is

(a) 16

(b) 60

(c) 22

☒ (d) 20



PEN

$$\frac{3}{4} = \frac{15}{x}$$

$$\frac{15}{20} = \frac{3}{4}$$

CA. PRANAV POPAT

$$3x = 60$$

$$x = \frac{60}{3} = 20$$

Features of Ratio

- Both terms of a ratio can be multiplied or divided by the same (non-zero) number. Usually a ratio is expressed in lowest terms (or simplest form).

$$\frac{\overset{2}{\cancel{60}}}{\underset{3}{\cancel{90}}} = \frac{2}{3} = 2:3$$

$$\frac{1.6 \times 5}{2.4 \times 5} = \frac{8}{12} = \frac{2}{3}$$

CA. PRANAV POPAT

$$\frac{2}{5} = 0.4$$

$$\frac{2 \times 4}{5 \times 4} = 0.4$$



PEN

Features of Ratio

The order of the terms in a ratio is important.

$$\frac{2}{3} \neq \frac{3}{2}$$

$$\left. \begin{array}{l} a:b \\ b:a \end{array} \right\} \text{different}$$

CA. PRANAV POPAT



PEN

Features of Ratio

- › Ratio exists only between quantities of the same kind.
- › Example: There is no ratio between the weight of one child and age of another

(w)
Vijay = 60 kg
Dharmesh = 168 cm
(h)

$$V:D = 60:168$$



PEN

Features of Ratio

› Quantities to be compared (by division) must be in the same units.

› Example, Find ratio between

150gm & 2 kg

150:2

25 minutes & 45 seconds

$$\begin{array}{r} 3 \\ 150 \cancel{g} \\ \hline 200 \cancel{g} \\ 40 \end{array} = 3:40$$

CA. PRANAV POPAT

do it



PEN

Features of Ratio

fractions having same denom.

- › To compare two ratios, convert them into equivalent like fractions.
- › Example, compare which of the below ratio is greater

$$2\frac{1}{3} : 3\frac{1}{3}$$

$$3.6 : 4.8$$

$$\frac{3}{7} : \frac{9}{7}$$

$$\frac{7}{3} : \frac{10}{3}$$

$$7 : 10$$

$$\frac{3.6}{4.8} = \frac{36}{48} : \frac{48}{48} = 3 : 9 = 1 : 3$$

$$= 36 : 48$$

$$= 3 : 4$$



if worker produces 10 units in 8 hours
how many units will be produced in 40 hours

$$\frac{8 \text{ hours}}{40 \text{ hours}} = \frac{10 \text{ units}}{x}$$

$$x = \frac{10 \times 40}{8} = 50 \text{ units}$$



Example Page 1.3

Illustration VII: Rounaq weighs 56.7 kg. If he reduces his weight in the ratio 7 : 6, find his new weight.

$$\frac{\text{current}}{\text{new}} = \frac{7}{6}$$

$$\frac{56.7}{\text{new}} = \frac{7}{6}$$

$$\text{new} = \frac{6}{7} \times 56.7 = 48.6 \text{ kg}$$



PEN

if height of Sanskar
increases in ratio 10:11

What is new height if original is 160 cm

$$\frac{160 \text{ cm}}{10} \times 11 = 176$$

$$160 \times \frac{11}{10} \\ = 176 \text{ cm}$$



PEN

Features of Ratio

- › If a quantity increases or decreases in the ratio $a:b$

$$\text{new quantity} = \frac{b}{a} \times \text{original quantity}$$

The fraction by which the original quantity is multiplied (i.e. b/a) to get a new quantity is called the **factor multiplying ratio**.



Example Page 1.3

Example 1: Simplify the ratio $1/3 : 1/8 : 1/6$

$$\frac{1}{3} : \frac{1}{8} : \frac{1}{6}$$

$$\frac{1}{3} \times \frac{8}{8} : \frac{1}{8} \times \frac{3}{3} : \frac{1}{6} \times \frac{4}{4}$$

$$\frac{8}{24} : \frac{3}{24} : \frac{4}{24}$$

$$= 8 : 3 : 4$$

Properties of Ratio

- › Inverse Ratio: One ratio is the inverse of another if their product is 1. Thus $b : a$ is the inverse of $a : b$ and *vice-versa*.

inv ratio of $3 : 2$ is $2 : 3$

CA. PRANAV POPAT

The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is

(a) 1 : 1

(b) 1 : 5

(c) 3 : 8

(d) none of these

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{9}^3}{\cancel{4}_2} \times \frac{\cancel{8}}{\cancel{6}} \times \frac{\cancel{8}^4}{\cancel{10}_5} = 1:1$$

CA. PRANAV POPAT

Properties of Ratio

- › A ratio compounded of itself is called its duplicate ratio.

$$a^2 : b^2$$

is the duplicate ratio of a:b
Square

dupl. ratio of 3:5

$$\frac{3}{5} \times \frac{3}{5} = \frac{9}{25} = 9:25$$

$$a^3 : b^3$$

is the triplicate ratio of a:b

CA. PRANAV POPAT

Properties of Ratio

$$\sqrt{a} : \sqrt{b}$$

is the sub-duplicate ratio of a:b

Sq root

sub dup
ratio of $16:25 = 4:5$

$$\sqrt[3]{a} : \sqrt[3]{b}$$

is the sub-triplicate ratio of a:b

CA. PRANAV POPAT

The ratio compounded of 4 : 9 and the duplicate ratio of 3 : 4 is

(a) 1 : 4

(b) 1 : 3

(c) 3 : 1

(d) none of these

$$\frac{4}{9} \times \frac{9}{16} = \frac{1}{4} = 1:4$$

CA. PRANAV POPAT

10. The ratio compounded of 4 : 9, the duplicate ratio of 3 : 4, the triplicate ratio of 2 : 3 and 9 : 7 is
(a) 2 : 7 (b) 7 : 2 (c) 2 : 21 (d) none of these

$$\frac{4}{9} \times \frac{3^2}{4^2} \times \frac{2^3}{3^3} \times \frac{9}{7} = \frac{\cancel{4}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{16}} \times \frac{\cancel{8}^2}{\cancel{27}_3} \times \frac{\cancel{9}}{\cancel{7}} = \frac{2}{21}$$

CA. PRANAV POPAT

Properties of Ratio

- › If the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are said to be commensurable.
- › Otherwise, they are said to be incommensurable

Example of incommensurable quantities

$$\sqrt{3} : \sqrt{2}$$

irrational no.

CA. PRANAV POPAT

$$a:b = 3:5 \quad b:c = 7:10$$

"continued" $a:b:c =$

$$21:35:50$$

$$a:c = 21:50$$

$$\frac{a}{b} = \frac{3}{5} \quad \frac{b}{c} = \frac{7}{10}$$

Common term should be same

$$\frac{a}{b} = \frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$$

$$\frac{b}{c} = \frac{7}{10} \times \frac{5}{5} = \frac{35}{50}$$

if $p:q=3:7$ $q:r=2:9$

$p:r=?$

$p:q:r=6:14:63$

$p:r=6:63$
 $=2:21$

$$\frac{p}{q} = \frac{3}{7}$$

$$\frac{q}{r} = \frac{2}{9}$$

$$\frac{p}{q} = \frac{3}{7} \times \frac{2}{2} = \frac{6}{14}$$

$$\frac{q}{r} = \frac{2}{9} \times \frac{7}{7} = \frac{14}{63}$$

P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is

(a) 22 : 27

(b) 27 : 22

(c) 32 : 33

(d) none of these

$$\frac{P}{Q} = \frac{11}{12}$$

$$\frac{P}{R} = \frac{9}{8}$$

$$P:Q:R = 99:108:88$$

$$Q:R = \frac{108}{88} = \frac{27}{22} = 27:22$$

$$\frac{P}{Q} = \frac{11}{12} \times \frac{9}{9}$$

$$\frac{P}{R} = \frac{9}{8} \times \frac{11}{11}$$

$$\frac{P}{Q} = \frac{99}{108}$$

$$\frac{P}{R} = \frac{99}{88}$$

Example – Page 1.4

Example 1: The monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenditures are in the ratio 7 : 9. If each saves ₹ 50 per month, find their monthly incomes.

$$\text{Income} - \text{expense} = \text{Savings}$$

$$\text{Income} - \text{Savings} = \text{Expense}$$

Let income of both is $4x$ & $5x$

Income 400, 500

$$\frac{4x - 50}{5x - 50} = \frac{\text{Expense}_1}{\text{Expense}_2} = \frac{7}{9}$$

$$(4x - 50) \times 9 = (5x - 50) \times 7$$

$$36x - 450 = 35x - 350$$

$$x = 100$$

Verify

Income	Save	expd
400	50	350
500	50	450

$$\frac{7}{\frac{350}{450}} = \frac{7}{9}$$

If $a : b = 3 : 4$, the value of $(2a+3b) : (3a+4b)$ is
(a) $54 : 25$ (b) $8 : 25$

(d) $1 : 25$

subs

$$\frac{6+12}{9+16} = \frac{18}{25}$$

$$\frac{a}{b} = \frac{3}{4} = \frac{30}{40}$$

CA. PRANAV

&

13. Two numbers are in the ratio 2 : 3. If 4 be subtracted from each, they are in the ratio 3 : 5. The numbers are

(a) (16, 24)

(b) (4, 6)

(c) (2, 3)

(d) none of these

$$\frac{16}{24}$$

$$\frac{12}{20} = \frac{3}{5}$$

CA. PRANAV POPAT

14. The angles of a triangle are in ratio 2 : 7 : 11. The angles are
(a) $(20^\circ, 70^\circ, 90^\circ)$ (b) $(30^\circ, 70^\circ, 80^\circ)$ (c) $(18^\circ, 63^\circ, 99^\circ)$ (d) none of these

sum of Total angles of Triangle = 180°

$$\begin{array}{ccc} & 180^\circ & \\ \swarrow & | & \searrow \\ 180^\circ \times \frac{2}{20} & 180^\circ \times \frac{7}{20} & 180^\circ \times \frac{11}{20} \\ = 18^\circ & = 63^\circ & = 99^\circ \end{array}$$

Division of ₹ 324 between X and Y is in the ratio 11 : 7. X & Y would get Rupees

a) (204, 120)

(b) (200, 124)

(c) (180, 144)

(d) none of these

$$\begin{array}{r} 324 \\ \hline 324 \times \frac{11}{18} \\ = 198 \end{array} \quad \begin{array}{r} 324 \\ \hline 324 \times \frac{7}{18} \\ = 126 \end{array}$$

$$11 + 7 = 18$$

CA. PRANAV POPAT

Anand earns ₹ 80 in 7 hours and Promode ₹ 90 in 12 hours. The ratio of their earnings is

(a) 32 : 21

(b) 23 : 12

(c) 8 : 9

(d) none of these

$$\frac{80}{90} = \frac{8}{9}$$

X

Anand $\frac{80}{7}$

Promode $\frac{90}{12}$

$$\frac{80}{7} \div \frac{90}{12}$$

$$\frac{80}{7} \times \frac{12}{90} = \frac{32}{21}$$

CA. PRANAT POBAT

$$\frac{3^2 \times 4 + 3 \times 4^2}{3^3 + 4^3} = \frac{36 + 48}{27 + 64} = \frac{84}{91} = \frac{12}{13}$$

CA. PRANAV POPAT

If $p : q$ is the sub-duplicate ratio of $p-x^2 : q-x^2$ then x^2 is

(a) $\frac{p}{p+q}$

(b) $\frac{q}{p+q}$

(c) $\frac{pq}{p+q}$

(d) none of these

$$\frac{p}{q} = \sqrt{\left(\frac{p-x^2}{q-x^2}\right)}$$

$$\frac{p^2}{q^2} = \frac{p-x^2}{q-x^2}$$

$$p^2q - p^2x^2 = pq^2 - x^2q^2$$

$$x^2q^2 - p^2x^2 = pq^2 - p^2q$$

$$x^2(q^2 - p^2) = pq(q - p)$$

$$x^2 = \frac{pq(q/p)}{(q+p)(q/p)}$$

$$\frac{2s}{3t} = \left(\frac{2s-p}{3t-p} \right)^2$$

$$18st^2 + \underline{2sp^2} - 12spt = 12s^2t + \underline{3p^2t} - 12spt$$

$$2sp^2 - 3p^2t = 12s^2t - 18st^2$$

$$\frac{2s}{3t} = \frac{(2s)^2 + p^2 - 2(2s)(p)}{(3t)^2 + p^2 - 2(3t)(p)}$$

$$p^2(2s-3t) = 6st(2s-3t)$$

$$\underline{p^2 = 6st}$$

$$\frac{2s}{3t} = \frac{4s^2 + p^2 - 4sp}{9t^2 + p^2 - 6pt}$$

22. If $p : q = 2 : 3$ and $x : y = 4 : 5$, then the value of $5px + 3qy : 10px + 4qy$ is
- (a) $71 : 82$ (b) $27 : 28$ (c) $17 : 28$ (d) none of these

Se!

CA. PRANAV POPAT

The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms. in 5 hours, the speed of the first train is

(a) 10 Km/hr

(b) 50 Km/hr

(c) 70 Km/hr

(d) none of these

$$\frac{\text{first train speed}}{\text{second train speed}} = \frac{7}{8}$$

$$\frac{\text{first}}{80} = \frac{7}{8}$$

CA. PRANAV POPAT

Example - Page 1.3

Example 2: The ratio of the number of boys to the number of girls in a school of 720 students is 3 : 5. If 18 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of the number of boys to the number of girls may change to 2 : 3.

current: Boys = $720 \times \frac{3}{8} = 270$

Girls = $720 \times \frac{5}{8} = 450$

new boys

$= 312 - 270$

$= 42$

new: Girls = $450 + 18 = 468$

$$\frac{B}{468} = \frac{2}{3} \Rightarrow B = \frac{2 \times 468}{3}$$

$$B = 312$$

department, the number of males and females are in the ratio 3:2. If 2 males and 5 females
the department, then the ratio becomes 1:1. Initially, the number of females in the
ment is

(b) 6

(c) 3

(d) 8

Let no. of males and females be $3x$ and $2x$ resp.

$$\frac{(3x+2)}{(2x+5)} = \frac{1}{1}$$

$$3x+2=2x+5$$

$$\underline{x=3}$$

$3x$ and $2x$ resp.
↓ ↓
9 2×3
 = 6



00:00:06



A bag contains 105 coins containing some 50 paise, and 25 paise coins in the ratio 4:3. The total value (in Rs.) in the bag is

(a) 41.25

(b) 43.25

(c) 105

(d) 52.5

105 coins

$$\begin{array}{r} \frac{105 \times \frac{4}{7}}{4:3} \quad \frac{105 \times \frac{3}{7}}{7} \\ = 60 \text{ coins} \quad = 45 \text{ coins} \end{array}$$
$$\begin{array}{r} (60 \times 0.5) \\ + (45 \times 0.25) \\ \hline 41.25/- \end{array}$$



Incomes of R and S are in the ratio 7:9 and their expenditures are in the ratio 4:5. Their total expenditure is equal to income of R. What is the ratio of their saving?

(a) 23:36

(b) 28:41

(c) 3:4

(d) 35:46

Let Income of R & S be $7x$ and $9x$

Let expd. of R & S be $4y$ and $5y$

$$(4y + 5y) = 7x, \quad 9y = 7x, \quad x = \frac{9}{7}y$$

$$\begin{aligned} \frac{\text{saving of R}}{\text{saving of S}} &= \frac{7x - 4y}{9x - 5y} = \frac{9y - 4y}{9\left(\frac{9}{7}y\right) - 5y} = \frac{y[9-4]}{y\left[\frac{81}{7} - 5\right]} \\ &= \frac{5}{\frac{81-35}{7}} = \frac{5}{\frac{46}{7}} = \frac{5 \times 7}{46} = \frac{35}{46} \end{aligned}$$



00:00:00



The salaries of A, B and C are in the ratio 2:3:5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(a) 23:33:60

(b) 33:23:60

(c) 23:60:33

(d) 33:60:23

$$2 : 3 : 5$$

$$2 \cdot 3 : 3 \cdot 3 : 5 \cdot 6$$

multiply by 10

$$23 : 33 : 60$$



00:01:00



The ratio of number of boys and the number of girls in a school is found to be 15:32. How many boys and equal number of girls should be added to bring the ratio to 2:3?

19

(b) 20

(c) 23

(d) 27

$$\frac{15+x}{32+x} = \frac{2}{3}$$

$$45 + 3x = 64 + 2x$$

$$\underline{x = 19}$$



00:01:00



The price of scooter and moped are in the ratio 7:9. The price of moped is ₹1600 more than that of scooter. Then the price of moped is

(a) ~~₹7200~~

(b) ₹5600

(c) ₹800

(d) ₹700

Scooter $7x$

$$9x = 1600 + 7x$$

Moped $9x$

$$2x = 1600$$

$$\underline{x = 800}$$

$$\begin{aligned} &9 \times 800 \\ &= 7200 \end{aligned}$$



00:01:00



$$\frac{A}{B}$$

$$= \frac{3}{4} = \frac{7}{9}$$

$$= \frac{21}{28} = \frac{28}{36}$$

$$\frac{B}{C}$$

$$\frac{C}{D} = \frac{2}{3}$$

$$\frac{C}{D} = \frac{36}{54}$$

$$\frac{D}{E} = \frac{54}{36}$$

$$E = \frac{54}{1.5} = 36$$

$$21 : 36$$

$$\underline{\underline{7 : 12}}$$



00:01:00

