

Maths Theory

= Sat Def. Plural
Sens - any data - quantified
Singular
Sens

Ch=1 Ratio & Proportion formula of all the chapters

$a:b = \frac{a}{b}$ → antecedent
b - consequent

Ratio

Proportion

1) Invertendo = $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

2) Alternendo = $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$

3) Componendo = $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$

4) Dividendo = $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$

5) Componend-Dividendo = $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

1) Inverse Ratio → $\frac{a}{b} = \frac{b}{a}$

2) duplicate Ratio → $\frac{a}{b} = \frac{a^2}{b^2}$

3) triplicate Ratio → $\frac{a}{b} = \frac{a^3}{b^3}$

4) Sub-duplicate = $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{2}{3}$

5) Sub-triplicate = $\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \frac{2}{3}$

6) compound = $\frac{a}{b} \times \frac{c}{d} = ac : bd$

6) Adendo = $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$

7) Subtrahendo = $\frac{a-c-e}{b-d-f}$

Indices

1) $a^0 = 1$

2) $\sqrt[n]{a} = a^{\frac{1}{n}}$

3) $a^m \times a^n = a^{m+n}$

4) $\frac{a^m}{a^n} = a^{m-n}$

5) $(a^m)^n = a^{m \times n}$

6) $(a \times b)^n = a^n \times b^n$

calci bi c for any root

Base $\sqrt[n]{x} = x^{\frac{1}{n}}$ - 12 Hing - 1 $\frac{1}{n}$

+1 $\times = x = 12$ Hing

More formulae

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^2 - b^2 = (a+b)(a-b)$

Log

$$a^x = n \Rightarrow \log_a n = x$$

$$1) \log_a a = 1$$

$$2) \log_a 1 = 0$$

$$3) \log_a mn = \log_a m + \log_a n$$

$$4) \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$5) \log_a m^n = n \log_a m$$

$$6) \log_b m = \frac{\log_a m}{\log_a b} = \frac{\log_a m}{\log_a b}$$

$$\log_a a \times \log_a b = 1$$

Time Value of Money

$$1) SI = \frac{P \times R \times T}{100}$$

$$A = P \left(1 + \frac{r}{100}\right)^n$$

CI

$$2) A = P(1+i)^n$$

Trick for compound interest

$$\text{eg } 1000 + 10\% + 10\% + 10\%$$

P

3 years

$$\Rightarrow CI = P(1+i)^n - 1$$

$$\text{Effect} = (1+i)^n - 1 \times 100$$

$$CI - SI = P(1+i)^n - P - P \times i \times n$$

Equation

Quadratic Equation $ax^2 + bx + c = 0$

$$1) \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2) \text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$3) \text{Product of roots} = \alpha\beta = \frac{c}{a}$$

Construction of quadratic Eq

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

to get nature

$$b^2 - 4ac = 0 \quad \text{Real \& equal}$$

$$b^2 - 4ac < 0 \quad \text{Imaginary}$$

$$b^2 - 4ac > 0 \quad \text{Real \& unequal}$$

$$b^2 - 4ac > 0 \text{ and perfect square Real \& unequal}$$

$$b^2 - 4ac > 0 \text{ not perfect square Real, unequal}$$

In Depreciation

$$S.V = P(1-i)^n \quad \text{S.V = scrap value}$$

= future value of annuity

(Regular) at end

$$= A \times \frac{(1+i)^n - 1}{i}$$

Due

$$A \times \frac{(1+i)^n - 1}{i} \times (1+i)$$

Present Regular

$$\frac{A(1+i)^n - A}{(1+i)^n - 1} \times (1+i)^n$$

Compound factor

$$A(1+i)^n$$

Measure of Central tendency and Dispersion

Mean:

Assumed mean = $A + \frac{\sum fidi}{\sum f} \times C$

AM:

Individual series = $\bar{x} = \frac{\sum x}{n}$

Discrete series = $\bar{x} = \frac{\sum fx}{N} = \frac{\sum f}{\sum f} = \frac{\sum f}{N}$

Continuous series = $\bar{x} = \frac{\sum fx}{N}$

Combined AM = $\frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$

GM = Geometric mean.

Individual series = $G = (x_1 \times x_2 \times x_3 \dots \times x_n)^{\frac{1}{n}}$

Discrete series = $G = (x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \dots x_n^{f_n})^{\frac{1}{N}}$

HM

HM = Harmonic mean.

Combined HM

Individual series = $\frac{n}{\sum (\frac{1}{x_i})}$

= $\frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$

Discrete series = $\frac{N}{\sum (\frac{f_i}{x_i})}$

Weighted average

$$WAM = \frac{\sum w_i x_i}{\sum w_i}$$

$$WGM = \frac{\sum w_i \log x_i}{\sum w_i}$$

$$WHM = \frac{\sum w_i^p}{\sum \left(\frac{w_i^p}{x_i^q} \right)}$$

Median

first arrange in ascending order.

indices $= \left(\frac{n+1}{2} \right)^{th}$ ~~if u.s. = odd~~

discrete $= \left(\frac{n+1}{2} \right)^{th}$

continuous $= 1 + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$

$$\text{Percentile } \frac{1}{100}, \frac{2}{100} \dots \frac{99}{100} \\ = 1 + \left(\frac{\frac{p}{100}(n+1) - CF}{f} \right) \times c$$

$$\text{decile} = \frac{1}{10}, \frac{2}{10} \dots \frac{9}{10}$$

$$\text{Quartile} = \frac{1}{4}, \frac{2}{4} \dots \frac{3}{4}$$

Mode

$$\text{Mode} = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c$$

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

$$\text{AM} \quad \text{HM} \quad \text{GM} \\ \text{AM} = \sqrt[3]{\text{GM}^2}$$

4, 8, 10, 12, 14, 16
2, 6, 8, 10, 12, 14

CLASSTIME Pg. No.
Date / /

Properties

AM

- 1) if all observation are same
(consonant AM is also consonant)
- 2) algebraic sum of deviation of a set of observations from this AM is zero
- 3) AM is affected by both by change of origin & scale
(+, -) $(\times, \div)$

Review ?

AM is best measure of central

AM is based on all observation

AM is affected by sampling fluctuations.

AM is amenable to mathematical property

AM cannot be used in case of open end classification

Intro

for finding composite series

you have to need different series of frequency & mean given

N

$f = 50, 60, 90$ $M = 12, 15, 20$

to solve this you have need combine mean formulae

$$\frac{50 \times 12 + 60 \times 15 + 90 \times 20}{50 + 60 + 90}$$

$$15 = 9 \\ 9 = 11 \quad 760 = 2400 + 70 \times 10 \\ 6 = 12$$

$$2400 + 70 \times 10 \\ d = 242 \\ A = 419.5$$

$$135 = 99 + 6 \times M$$

A

$$419.5 \times \frac{43}{16} \times 16$$

Properties of GM

- 1) Logarithm of G for a set observation is the AM of the logarithm of the observation $\log G = \frac{1}{n} \sum \log x$

2) if all observation are same then GM is also

$$3) \text{GM of } 2 = \text{GM of } x \times \text{GM of } y$$

$$4) \text{GM of } 2 = \frac{\text{GM } x}{\text{GM } y}$$

Use of GM & HM

Both	Both are used for calculating average rate
GM	appropriate for rates having percentages
HM	appropriate for rates other than percentages

Spill also

Relation AM, GM, HM

When all the observation are same

$$AM = GM = HM$$

When observation are distinct

$$AM > GM > HM$$

In question is silent

$$AM \geq GM \geq HM$$

Median:

1) For a set of observation, the sum of absolute deviation is minimum when the deviation are taken from the median.

$$\sum (x_i - \bar{x}) = 0$$

2) Median is also affected by both change of origin & scale

Review

1) it is also called as positional average

2) Median is not based on all observations

3) it is not affected by sampling of fluctuation

4) Median is best measure of central tendency in case of open-end classification

Median is middle value

So, Middle of Quartile is Q_2

Middle of Percentile is P_{50}

Middle of Decile is D_5

Mode

* if two or more observation are having maximum frequency then there are multiple mode (multimodal distribution)

* if there are exactly two modes then distribution is called as Bimodal distribution

* if all observation are having same frequency then distribution has no mode

* we can say that mode is not rigidly defined.

A if all observation is constant that it is also mode

1 mode is affected by both due to change of origin and scale.

Mode is not based on all observation

Mode is not rigidly defined

Mode is not amenable to mathematical purpose

Measure of Dispersion

★ Dispersion for a given of observation may be defined as the amount of deviation of the observations usually, from an appropriate measure of central tendency.

Types of measure of Dispersion

Absolute measures of dispersion	• These are with units • These are not useful for comparison of two variable with different units. • Example: Range, mean Deviation, Standard Deviation, Quartile deviation
Relative measures of Dispersion.	• These are unit free measures • These are useful for comparison of two variable with different units • Example: Coefficient of Range, Coefficient of mean deviation, Coefficient of deviation, Coefficient of quartile Deviation.

Range

$$\text{Range} = L - S$$

$$\text{Coefficient of Range} = \frac{L - S}{L + S} \times 100$$

Properties

- Not affected by change of origin
- Affected by change of scale (only value)
- No impact of sign of change of scale
- Note: Measure of Dispersion can never be negative
- Not based on all observation
- Easy to compute

Mean Deviation

Meaning :- it is the arithmetic mean of the absolute deviations of the observations from an appropriate measure of central tendency.

Individual $MD_A = \frac{1}{n} \sum |x_i - A|$

Discrete $MD_A = \frac{1}{n} \sum f_i |x_i - A|$

$$\text{Coefficient of Mean} = \frac{\text{Mean Deviation about } A}{A} \times 100$$

$$MD_{ME} = \frac{1}{n} \sum |x_i - M_e|$$

$$MD_{ME} = \frac{1}{n} \sum f(x_i - M_e)$$

$$N = \sum f_i$$

coefficient = $\frac{\text{Mean deviation about median}}{M_e} \times 100$

Properties

- Mean Deviation takes its minimum value when deviation are taken from Median.
- change of origin no effect, change of scale - Affect of value not sign.
- Based on all observation
- Improvement over range
- Difficult to compute
- Not amenable to mathematical Property
- number of usage of Modulus.

Standard Deviation.

Meaning

Improvement over mean deviation.
It is defined as the root mean square deviation are taken from the AM of the observations.

$$\sigma = SD_n = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$$

$$\sigma_n = SD_n = \sqrt{\frac{1}{n} \sum f(x_i - \bar{x})^2}$$

$$\text{coefficient of variation} = \frac{SD}{\bar{x}} \times 100$$

$$SD \text{ of any two number} = SD = \text{Range}$$

$$SD \text{ of } n \text{ first } n \text{ natural number} = S = \sqrt{\frac{n^2 - 1}{12}}$$

$$\text{Combined SD} = \sqrt{n_1 S_1^2 + n_2 S_2^2 + n_1 d_1^2 + n_2 d_2^2}$$

$$d_1 = \bar{x}_1 - \bar{x}, \quad d_2 = \bar{x}_2 - \bar{x}$$

Proportional

- 1) if all the observation are constant SD is zero
- 2) No effect of change but affected by change of scale in the magnitude (ignore sign.)

Quantile deviation.

~~Qd =~~

$$Q_d = \frac{Q_3 - Q_1}{2}$$

coefficient of quantile deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

Relation between SD, MD, QD

$$4SD = 5MD = 6QD$$

or

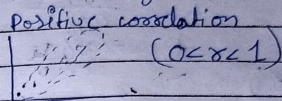
$$SD : MD : QD = 15 : 12 : 10$$

Review

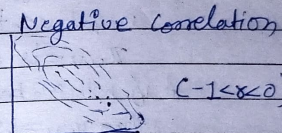
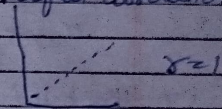
- it is the best measure of dispersion for open-end classification.
- it is also less affected due to sampling fluctuations
- like other measure of Dispersion QD is also not affected by change of origin but affected by scale ignoring sign.

Correlation and Regression

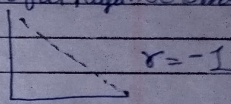
(a) Scatter Diagram.



perfect correlation



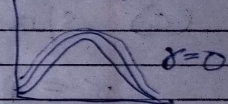
perfect negative correlation



No correlation



curvilinear correlation



(b) Karl Pearson's Product moment correlation coefficient

$$r = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}}$$

or

$$r_{xy} = \frac{\text{COV}(x, y)}{s_x s_y}$$

$$s_x = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2}$$

$$s_y = \sqrt{\frac{\sum y_i^2}{n} - \bar{y}^2}$$

$$\text{COV}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$

Properties

- (1) The coefficient of correlation is a unit-free measure.
- (2) $r_{xy} = -r_{yx}$
 $r_{xy} = \frac{bd}{|b||d|}$ $b = \frac{-\text{coeff } y}{\text{coeff } x}$
 $d = \frac{-\text{coeff } x}{\text{coeff } y}$
- (3) lies between $(-1 \leq r \leq 1)$
 - * if both sign are negative then $r_{xy} = -1$ (-ve)
 - * if both are positive then $r_{xy} = +1$ (+ve)

Examples

(C) Spearman Rank correlation

$$r = 1 - \frac{6 \sum d^2}{n(n^2 - 1)} \quad d = R_x - R_y$$

(d) COEFFICIENT of Concurrent Deviations

$$r_c = \pm \sqrt{\frac{\sum C(2C - m)}{m}}$$

if $2C - m > 0$ then $\sqrt{\frac{\sum (2C - m)}{m}}$
 if $2C - m < 0$ then $\sqrt{\frac{\sum -(2C - m)}{m}}$

$C =$ product of r_{xy} maj. & min. sign
 sign here

$m =$ no. of observations - 1

Regression Analysis

Regression analysis is a method (statistical tool) to predict the value of a variable based on the value of another variable.

These lines are derived using least square method.

It is depend on variable
 y depend on x / y on x / b_{yx}
 x depend on y / x on y / b_{xy}

Eg: $2x + 3y = 7$

b_{yx} $y = \frac{7}{3} - \frac{2}{3}x$

$b_{yx} = \frac{2}{3}$

formula -

$$b_{yx} = \frac{\sum x y - \sum x \sum y}{\sum y^2 - \frac{(\sum y)^2}{n}}$$

$$b_{xy} = \frac{\sum x y - \sum x \sum y}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

OR

$$b_{xy} = \frac{C. cov(xy)}{s_y^2}$$

Properties

★ Topic with the help of regression
find correlation
 $r = \sqrt{b_{yx} \times b_{xy}}$

★ r^2 = determination

★ $1 - r^2$ = non-determination

Properties

The regression coefficients remain unchanged due to a shift of origin but change due to a shift of scale.

$$u = \frac{x - a}{p} \quad v = \frac{y - c}{q}$$

$$b_{yx} = \frac{q}{p} \times b_{uv}$$

$$b_{xy} = \frac{p}{q} \times b_{uv}$$

Theoretical Distribution

approach of probability

- classical probability
- Set Based probab
- Modern probability (Axiomatic)

Modern probability

unlike other, here probability is not calculated by a simple ratio

probability is calculated by using a function defined on random variable.

Parameter

characteristics that are used to define a given population

Binomial Distribution

when to use it:-

when a random variable is discrete

probability is not very low. (like 1/2, 2/3, 1/4 around)

Binomial Variable

$$X \sim B(n, p)$$

Number of parameters: 2

Binomial distribution: bi-parametric discrete probability distribution

Important point

if $p = 0.5$ / $\frac{1}{2}$
distribution is symmetric

$p > 0.5$ symmetric is
toward left

$p < 0.5$ symmetric is
toward right

Probability Mass function

$$f(x) = P(X=x) = {}^n C_x p^x q^{n-x}$$

$$\text{Mean} = \mu = np$$

$$\text{Variance} = \sigma^2 = npq$$

mean is always more than variance

$$\text{maximum value of variance} = \frac{n}{4}$$

Mode = observation with highest probability

$$\text{for } \Rightarrow (n+1)p$$

integer

non-integer

two modes

one mode

$$(n+1)p \quad (n+1)p-1$$

$$\text{largest integer of } (n+1)p$$

$$n \geq 2.5 \quad \text{remove}$$

$$\text{mode} = 2 \quad \text{decimal}$$

$$\text{Ex } n=3 \quad p=\frac{1}{2}$$

$$\text{mode} = (3+1)\frac{1}{2} = 2 \rightarrow \text{integer}$$

$$u_0 = 2$$

$$u = 1$$

Additive property: if $X \sim B(n_1, p)$ and $Y \sim B(n_2, p)$
then $X+Y \sim B(n_1+n_2, p)$

Poisson Distribution

when to use

when random variable is discrete
probability is very low (like 1% or less)

Poisson Parameter

Poisson distribution has only one parameter
denoted by m

it can either be given in question or can
be calculated using formula

$$m = np$$

Poisson variable $X \sim P(m)$

$$\text{Probability} = \frac{e^{-m} \cdot m^x}{x!}$$

mass if

$$\text{Mean} = m$$

$$\text{Variance } \sigma^2 = m$$

$$e = 2.71828$$

find m

if result is integer

Mode = then there are

$$u_0 = m$$

$$\text{variance SD} = \sqrt{m}$$

Mode $\rightarrow$ same as in Binomial

eg

5 times in a year: Poisson variable X , $P(X=2) = 3P(X=4)$ then find

Normal Distribution

When random variable is continuous.

It is bi-parametric continuous probability distribution

$$X \sim N(\mu, \sigma^2)$$

Mean = Median = Mode = μ Symmetric distribution.

$$\text{Variance} = \sigma^2$$

$$\text{MO of Normal Distn} \rightarrow 0.8\sigma$$

$$\text{QPOf} \rightarrow 0.675\sigma$$

$$Q_1 = \mu - 0.675\sigma$$

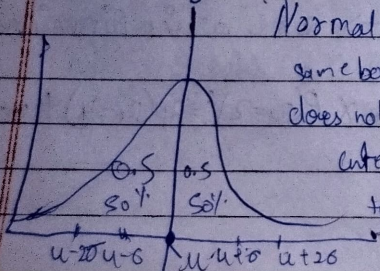
$$Q_3 = \mu + 0.675\sigma$$

$$\text{Ratio} = 10:12:15 \quad Q:D:MO:MO$$

Probability Density function

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \times \frac{1}{2}$$

Probability



Normal curve

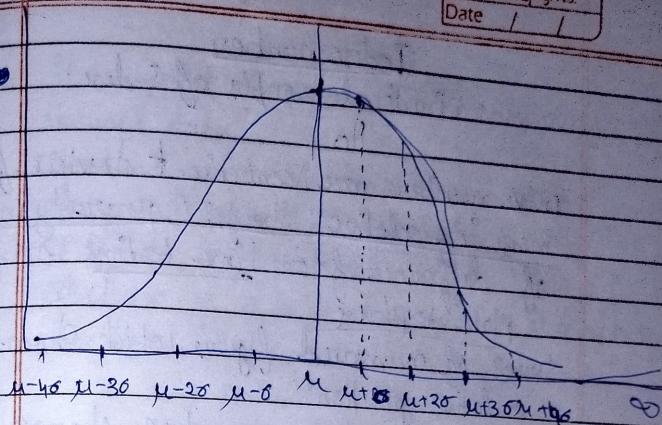
same both side

does not touch axis.

center point is mean

total area under normal curve

$$= 1$$



$$\begin{aligned} \mu - \sigma &\rightarrow \mu + \sigma = 34.135 \\ &+ 34.135 \\ &= 68.27\% \end{aligned}$$

$$\mu \rightarrow \infty = 50\%$$

$$\mu \rightarrow \mu + \sigma = 34.135\%$$

$$\mu + \sigma \rightarrow \mu + 2\sigma = 13.59\%$$

$$\mu + 2\sigma \rightarrow \mu + 3\sigma = 2.14\%$$

$$\mu + 3\sigma \rightarrow \infty = 0.135\%$$

ex:-

$$P(X \leq 50) = 50\% \quad P(X \geq 50) = 50\%$$

$$P(X \leq 60) = -\infty \text{ to } 50 + 50 \text{ to } 60$$

$$= 50\% + 34.135\%$$

$$= 84.135\%$$

$$P(X \geq 70) = 50 \text{ to } \infty - 50 \text{ to } 70$$

$$= 50\% - [34.135 + 13.59]$$

$$= 2.275\%$$

$$P(X \leq 64) = ?$$

(z table)

Index number Practical Examples of Index No.

Index numbers are convenient devices for measuring relative changes (generally in %) of difference from time to time or from place to place.

series of numerical figures which shows relative position.

Index Numbers show percentage changes rather than absolute amounts of change.

Data selection.

It depends on the purpose for which the index is used.

Index numbers are often constructed from the Sample. Random sampling, and if need be a stratified Random sampling can be used to ensure that sample is representative. Data should be comparable by ensuring consistency in selection method.

Base period
it is point of reference in comparing various data.

Standard point of comparison.

The period should be normal.

It should be relatively recent choice of suitable base period is temporary solution.

Use of Averages

The geometric mean is better in averaging relatives,

But for most of the index's arithmetic mean is used because of its simpli-city.

Price/quantity / value Relative

for individual commodity

current period price/quantity/value

Base period price/quantity/value

Link relative

$$\frac{P_1}{P_0} \times \frac{P_2}{P_1} \times \frac{P_3}{P_2} \dots \frac{P_n}{P_{n-1}}$$

Same can be created for quantities also.

Chain relative

when the above relative are in respect to a fixed base period these are also called the chain relative

$$\frac{P_1}{P_0} \times \frac{P_2}{P_1} \times \frac{P_3}{P_2} \dots \frac{P_n}{P_{n-1}}$$

formula for chain index
(when direct data is not available)

$$\text{Link relative of current year} \times \frac{\text{chain index of previous year}}{100}$$

Limitation for index no.

- change of error due to sampling
- it gives broad trend not real picture.
- Due to many method at times it creates confusion

formula for deflated value

$$= \frac{\text{Current value}}{\text{Price index of the current year}} \times \frac{P_0}{P_n}$$

Shifted price index

$$= \frac{\text{original price index} \times 100}{\text{Price index of the year on which it has to be shifted}}$$

Teste

Unit test — formula should be independent of the unit

all formulae satisfy this test except simple aggregation index

Time Reversal test → to determine this work both ways in time backward - forward.

$$\rightarrow P_{01} \times P_{10} = 1$$

→ Laspeyres, Paasche, do not satisfy only Fisher ideal does.

Whole sale price index (WPI) is given by

~~Laspeyres~~ Laspeyres Index

Factor reversal test

$$\Rightarrow P_{01} \times Q_{01} = V_{01}$$

Price index and the quantity index should be equal to the value index.

→ Satisfy by fisher index Number

→ fisher is a ideal no. as it satisfy unit, time and factor.

Circular test

a The test of this shiftability of base is called circular test.

a This test is not met by, Laspeyres, Paasche, as the fisher.

a The weighted GM of relative simple geometric mean of price relative and the weighted aggregate with fixed weighted method test

Cost of living index (also called General index)

It is defined as the weighted AM of index number of few year groups of basic necessities.

Symbol

P_{01} is the index for time 1 to 0

P_{10} is the index from time 0 to 1

weighted ^{aggregate} formula.

1) Laspeyres' Index $\Rightarrow \frac{\sum P_1 \times Q_0}{\sum P_0 \times Q_0} \times 100$

2) Paasche's Index $\Rightarrow \frac{\sum P_1 \times Q_1}{\sum P_0 \times Q_1} \times 100$

3) Marshall edworth $= \frac{\sum P_1 (Q_1 + Q_0)}{\sum P_0 (Q_1 + Q_0)} \times 100$

4) fisher $= \sqrt{L \times P} \times 100$

5) Bowley $= \frac{L + P}{2} \times 100$

Simple aggregate method $= \frac{\sum P_1}{\sum P_0} \times 100$

Simple average method $= \frac{\sum \frac{P_1}{P_0}}{n} \times 100$

Multiplication series method $= I_1 \times I_2 \times I_3 \times I_4$

In simple average method, if the number of values is odd then we drop

Probability

Random Experiment :- An experiment may be described as performance that produces certain result

Computation of total no. of outcome when an experiment is repeated a certain number of times
Classical Definition of Probability when number of times = ab

$$P(A) = \frac{n_A}{n} = \frac{\text{total no. of events}}{\text{no. of outcomes}}$$

Probability of an event lies between:

0 - both inclusive

$$0 \leq P(A) \leq 1$$

$P=0$ A is known to be an impossible event

$P=1$ A is known to be a sure event

→ Non occurrence of event A' , $\bar{A}$ is known as complementary event.

$$P(A') = 1 - P(A) \quad P(A) + P(A') = 1$$

① add in favour of A = favourable
unfavourable

② add against A = unfavourable
favourable

Cg Box 1 = 5 Red + 7 white + 6 Blue ball = 18

Box 2 = 4 Red + 8 white + 6 Blue ball = 18

Box 3 = $\frac{3 \text{ Red}}{12} + \frac{4 \text{ white}}{12} + \frac{3 \text{ Blue ball}}{12} = \frac{9}{12}$

if one ball is drawn at random, then that is the probability that they would be of same colour?

Total no. of outcomes ${}^{18}C_1 \times {}^{18}C_1 \times {}^9C_1 = 18 \times 18 \times 9 = 2916$

$\frac{12}{3} \times \frac{12}{3} \times \frac{12}{3} = 12 \times 12 \times 12 = 35$

${}^5C_1 \times {}^4C_1 \times {}^3C_1 = 60$

${}^7C_1 \times {}^8C_1 \times {}^6C_1 = 224$

${}^6C_1 \times {}^6C_1 \times {}^2C_1 = 14$

$\frac{356}{2916} = 0.122$

Q1 if two letters are taken from the word HOME
What is the prob. of that none of the letter are vowels

T.No. of ${}^4C_2 = 6$ Probab = $\frac{1}{6}$

No. of vowels = $\frac{2}{2} = 1$

Q1 10 bulbs - 3 defective → 45 sample not more than one defective

total no. = $\frac{10}{4} = 210$

Non defective = ${}^7C_0 = 35$ defective = ${}^3C_1 = 3$ = 10.5

- A 12 balls contain numbers 1-12 a ball selected will be multiple of 5 or 6.
+ No. of 5 = $12/5 = 2$
No. of 6 = $12/6 = 2$

Types of events

★ Equally likely events = Sinki probability same or equal hote hai $P(A) = P(B)$

★ Impossible events = Sinki probability zero hote hai

★ Certain events = Sinki probability hamesha one event hote hai

★ Mutually exclusive events = use jo ek sath or incompatible events occur nahi ho sakte hai

★ Simple and composite events = Sinki break down nahi kr sakte.
 Simple = Sinki break down nahi kr sakte.
 Composite = Sinki break down kr sakte hai

★ Composite events = Sinki break down kr sakte hai

★ Exhaustive events = all the possible event of an experiment are combinedly known as exhaustive events

Formula

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- if mutually exclusive $= P(A \cup B) = P(A) + P(B)$
- ~~$P(A \cup B) = P(A) + P(B) - P(A \cap B)$~~
 $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
- if mutually exclusive $= P(A \cup B \cup C) = P(A) + P(B) + P(C)$
- two events are A and B are exhaustive $P(A \cup B) = 1$
- three events " " " $P(A \cup B \cup C) = 1$
- Three events are equally likely if $P(A) = P(B) = P(C)$
- Probability that only event A occur $P(A - B) = P(A) - P(A \cap B)$
- Probability that only event B occur $P(B - A) = P(B) - P(A \cap B)$

Example

A, B and C

$$P(A \cap B) = 0 \quad P(B \cap C) = 0 \quad P(A \cap C) = 0$$

$$P(A \cap B \cap C) = 0 \quad P(A \cup B \cup C) = 1$$

$$P(A) = P(B) = P(C) \quad P(A) = 1 - P(A)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$1 = P(A) + P(A) + P(A)$$

$$1 = 3P(A) \quad P(A) = 1 - P(A)$$

$$\frac{1}{3} = P(A) \quad \Rightarrow 1 - P(A) = \frac{1}{3}$$



Conditional probability & compound theorem

Independent & Dependent Events.

- or Mean union of sets
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- AND means intersection of sets
 $P(A \cap B) = P(A) \times P(B)$ $\rightarrow$ if all depend on ^{intersection} finding ~~that~~ are closed then use this method

Rules of Probability when events are independent

- (1) $P(A \cap B) = P(A) \times P(B)$
- (2) Probability of events A given that B ~~are~~ has already occurred is given by $P(A/B)$

$$= \frac{P(A \cap B)}{P(B)}$$
- (3) ~~$P(B/A) = P(A \cap B)$~~

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$
- (4) $P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$
- (5) ~~$P(A/B)$~~ $P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{P(B) - P(A) \times P(B)}{P(B)} = 1 - P(A)$
- (6) $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(B') - P(A \cap B)}{P(B')} = \frac{1 - P(A \cap B)}{1 - P(B)}$

7) Probability that only event A or only event B occurs. $P(A) + P(B) - 2P(A \cap B)$

Random variable

- * Expected value of Random variable
- = Expected value (u) of a Random variable (x) is given by: $u = E(x) = \sum p_i x_i$
- * Expected value of (x^2) is given by $E(x^2) = \sum [p_i x_i^2]$

X * Expected value of a monotonic function $f(x)$ is given by $E(f(x)) = \sum [p_i f(x_i)]$

5) Variance (σ^2) of a Random variable (x) is given by $V(x) = \sigma^2 = E(x - u)^2 = E(x^2) - u^2$

Sets, Relation and function

empty set: set contain no element called empty set or null set $\{\}$, $\emptyset$ or void set.

Singleton set: set contain exactly one element is called singleton.

Finite set: an empty set or non-empty set in which element can count denoted as $n(A)$.
eg $A = \{2, 4, 6, 8, 10, 12\}$ $n(A) = 6$.

Infinite set: a set which contain infinite element
eg: set of all natural numbers.

Equal set: Two set which have exactly equal same elements and we write $A = B$

Equivalent set: Two finite sets A & B are said to be equivalent if $n(A) = n(B)$
eg: $A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$
 $n(A) = n(B) = 3$.

- no of possible subset of set containing n element 2^n
- The Empty set is a subset of every set
- Every set is a subset of itself.
- no of proper subset of set containing n elements $2^n - 1$
- Subset = $[C]$ all set lies in another set

Cardinal number = $n(A)$

operation sets

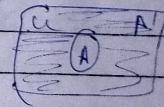
* Union (U) $(A \cup B)$ = all elements

* Intersection $(\cap)$ $(A \cap B)$ = only common elements

Difference sets = $A - B$ = only A
 $B - A$ = only B

Complement of a set

A', A^c or $U - A$
universal set



~~$A = U - A$~~

De Morgan's Law

for any two sets A & B

(i) $(A \cup B)' = (A' \cap B')$

(ii) $(A \cap B)' = (A' \cup B')$

Two Set Based

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Three set Basic

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Relation

Domain, Range, Co-Domain of a Relation

Reflexive: $A = \{1, 2, 3\}$

$(a, a) \in R$

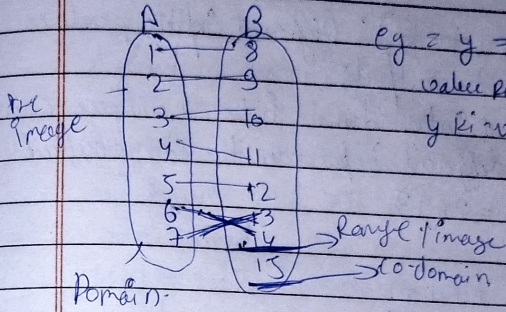
$A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$A \in R$

Symmetrical $= (a, b) \in R$
 $(b, a) \in R$

Transitive $(a, b) \in R$ when all the
 $(b, c) \in R$ conditions are met
 $(c, a) \in R$ or

function



eg: $y = 2x + 3$
value put / given - Domain
y ki value - Range

function $\rightarrow$ Every domain have an image
no one left then its a function.

$\rightarrow$ Every polynomial equation function.

Δ Inverse of function
 $\Rightarrow$ let $f: A \rightarrow B$ be a bijective to find the
inverse of the function.
these steps

- 1) Put $E(x) = y$
- 2) Solve $E(x) = y$ to obtain x in terms of y
- 3) In relation obtained in Step 2. replace x by $f^{-1}(y)$
to obtain the required inverse

one-to-one (Injective)

- 1) Injective (one-to-one): Different inputs produce different outputs.
 * each element in the domain has a unique co-domain.
 no two different inputs have the same output.
 ex $f(x) = 2x + 3$

2) Many-one function.

- * two or more elements in the domain have the same output.

a) not injective

$$f(x) = x^2 \quad 2^2 = 4 \quad (-2)^2 = 4$$

(B) onto (Surjective) function

- = every element in co-domain has at least one pre-image (domain)

the function covers the entire co-domain

$$f(x) = x^3$$

(C) into function

Some element in the co-domain are left unmapped.

$$\text{eg } f(x) = x^2$$

(V) one-one and onto (Bijective function)

A function that is both injective and surjective

every element has unique elements in the co-domain and all co-domain elements are covered.

$$f(x) = 2x + 2 \quad \text{where domain}$$