CHAPTER

3

Linear Inequalities

LINEAR INEQUATION

- If the equality symbol '=' in a linear equation is replaced by an inequality symbol (<, >, ≤, or ≥), then the statement is called linear inequality.

  E.g.: $x \le 3$, $x + y > 7$ etc.

- Thing to remember for the sign of the inequality:

  1. **Addition/Subtraction:** The sign of inequality doesn't change on adding or subtracting the same quantity to both sides.

     E.g.: Since, $5 < 2$

     $\Rightarrow 5 + 1 < 2 + 1 \Rightarrow 6 < 3$, which is true

     Also, $5 < 2$

     $\Rightarrow 5 - 1 < 2 - 1$

     $\Rightarrow 4 < 1$, which is true

  2. **Multiplication/Division:** The sign of inequality doesn't change while multiplying and dividing with positive integers on both sides. However, the sign changes when both sides gets multiply or divide by a negative integer.

     E.g.:

     - On multiplying and dividing with <u>positive integers</u>.

       Since, $5 < 2$

       On multiplying both the sides by 2, then

       $5 \times 2 < 2 \times 2 \Rightarrow 10 < 4$

       Also, $12 > 4$

     - On dividing both the sides by 4, then

       $12 \div 4 > 4 \div 4$

       $3 > 1$

     - On multiplying and dividing with <u>negative integers</u>.

       Since, $15 > 3$

       On multiplying both the sides by -2, then

       $15 \times (-2) < 3 \times (-2)$

       $-30 < -6$

Also, $10 > 6$

On dividing both the sides by -2, then

$\Rightarrow 10 \div (-2) > 6 \div (-2)$

$\Rightarrow -5 < -3$

WHY TO STUDY?

- The weight of all the students in your class is less than 90 kgs.
- Conference room can occupy at most 40 tables or chairs.
- Rajesh has ₹100 and wants to buy some notebooks and pens. The cost of one notebook is ₹40 and that of a pen is ₹20.
- Type of inequalities occur in business whenever there is a limit on supply, demand, sales etc.

  E.g.: Raju wants to produce sofas. But the material needed can be only get on some conditions such that he has to buy cloth of sofa at least for 40 sets of sofas and he can get wood for maximum for 100 sets.

  So in such case he needs to adjust his demands and sales accordingly and for this we need to understand the concept of inequalities.

LINEAR INEQUATION IN ONE VARIABLE

- As the heading says, when there is an inequality on one variable then it will be called as linear inequation in one variable.

  E.g.: $x \geq 5$, $y < 3$

  And the range where the equation satisfies is called as intervals solution space also abbreviated as S.S.

  Intervals (Solution space) of an inequality:

  If $a < b$, and $a < x < b$ means that $a < x$, and $x < b$. That is, x is between a and b.

  The following table will be make you understand it properly:

Interval	Inequality
$[a, b]$	$a \leq x \leq b$
$[a, b)$	$a \leq x < b$
$(a, b]$	$a < x \leq b$
(a, b)	$a < x < b$

  E.g.: $(-2, 5)$ is an open interval that includes all the real numbers greater than -2 but less than 5.

  $[-2, 5)$ is an interval that includes all the real numbers greater than and equal to -2 but less than 5.

  $(-2, 5]$ is an open interval that includes all the real numbers greater than -2 but less than and equal to 5.

  $[-2, 5]$ is closed interval that includes all the real numbers from -2 to 5 including the endpoints.

Example 1. The solution of the inequality $8x + 6 < 12x + 14$ is: **(Dec 2013)**

(a) $(-2, 2)$ (b) $(0, 2)$ (c) $(2,\infty)$ (d) $(-2,\infty)$

Sol. (d) Given: $8x + 6 < 12x + 14$

$\Rightarrow 6 - 14 < 12x - 8x$

$\Rightarrow -8 < 4x$

$\Rightarrow 4x > -8$

$\Rightarrow x > \frac{-8}{4}$

$\Rightarrow x > -2$

Solution set = $(-2, \infty)$

Hence, the correct option is (d).

Example 2. The solution set of the equations $x + 2 > 0$ and $2x - 6 > 0$ is **(June 2019)**

(a) $(-2, \infty)$ (b) $(3, \infty)$ (c) $(-\infty, -2)$ (d) $(-\infty, -3)$

Sol. (b) Since, $x + 2 > 0$

$\Rightarrow x > -2$

Also, $2x - 6 > 0$

$\Rightarrow 2x > 6$

$\Rightarrow x > 3$ ...(ii)

From (i) and (ii), the common solution set will be:

$x > 3$

i.e., $(3, \infty)$

Hence, the correct option is (b).

Example 3. Find the range of real of x satisfying the inequalities $3x - 2 > 7$ and $4x-13 > 15$. **(June 2012)**

(a) $x > 3$ (b) $x > 7$ (c) $x < 7$ (d) $x < 3$

Sol. (b) Given, $3x - 2 > 7$

$\Rightarrow 3x > 7 + 2$

$\Rightarrow 3x > 9$

$\Rightarrow x > 3$ ..... (i)

Also, $4x - 13 > 15$

$\Rightarrow 4x > 15 + 13$

$\Rightarrow 4x > 28$

$\Rightarrow x > 7$ ..... (ii)

From (i) and (ii), we get

$x > 7$

Hence, the correct answer is option (b) i.e., $x > 7$.

Example 4. Solve the inequality: $\frac{(3x-1)}{2} \le \frac{(x+2)}{4}$.

(a) $x \le 2$ (b) $x \le 0.8$ (c) $x \ge 1.5$ (d) $x \ge 2$

Sol. (b) Given, $\frac{(3x-1)}{2} \le \frac{(x+2)}{4}$

$\Rightarrow \frac{(3x-1)}{1} \le \frac{(x+2)}{2}$

$\Rightarrow 2(3x - 1) \le (x + 2)$

$\Rightarrow 6x - 2 \le x + 2$

$\Rightarrow 6x - x \le 2 + 2$

$\Rightarrow x \le \frac{4}{5}$

$\Rightarrow x \le 0.8$

Hence, the correct option is (b) i.e., $x \le 0.8$.

Example 5. If $3x + 2 < 2x + 5$ and $4x - 5 \ge 2x - 3$, then x can take the following value

(a) 3 (b) -1 (c) 2 (d) -3 **(Dec 2022)**

Sol. (c) Given, $3x + 2 < 2x + 5$

$\Rightarrow 3x - 2x < 5 - 2$

$\Rightarrow x < 3$ ...(i)

Also, $4x - 5 \ge 2x - 3$

$\Rightarrow 4x - 2x \ge -3 + 5$

$\Rightarrow 2x \ge 2$

$\Rightarrow x \ge 1$ ...(ii)

From (i) and (ii), we get

$1 \le x < 3$

Thus, out of given options, x can take up the value of 2.

PRACTICE QUESTIONS (PART A)

1. $4x > -16$ implies

(a) $x \ge -4$ (b) $x < -4$ (c) $x > -4$ (d) $x \le -4$

2. Solve for real 'x' if $5x - 2 \ge 2x + 1$ and $2x + 3 < 18 - 3x$

(a) $1 < x < 3$ (b) $-1 > x > -3$

(c) $1 \le x < 3$ (d) $x = 3$

3. Which of the following value 'x' cannot take in the inequality $4x + 3 < 2x + 5$

(a) 1 (b) 0 (c) -1 (d) -4

4. Find the range of real values of x that satisfy the system of inequalities:

$2x + 5 \ge 13$ and $3x - 4 \le 8$

(a) $x = 3$ (b) $x = 0$ (c) $x = 4$ (d) $x = 5$

5. Solve for real 'x' if $5x - 3 > 2x + 6$ and $2x - 3 < + 17$

(a) $3 \le x < 4$ (b) $3 < x < 4$ (c) $3 \le x \le 4$ (d) None of these

Answer Key

1. (c) **2.** (c) **3.** (a) **4.** (c) **5.** (b)

LINEAR INEQUATION IN TWO VARIABLES

- Again as the heading says, when there is an inequality in two variables then it will be called as linear inequation in two variables. For example: $ax + by \le c$, $x + 2y < 6$, $x + y \ge 5$, $3x + y > 9$ are some examples of the linear inequality of two variables.

  E.g.: Let's plot a graph of $x + 2y = 10$

  The line of equation corresponding to above inequality is $x + 2y = 10$

  When $x = 0$ then $y = 5$

  When $y = 0$ then $x = 10$

  Thus, the points are (0, 5) and (10, 0).

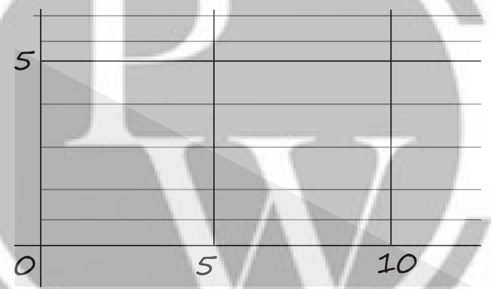
  Now, put $x = 0$ and $y = 0$ in given inequality, we get

  $0 + 0 \le 10$

  $0 \le 10$, which is true

  Thus, the shaded region will be towards the origin.

  Therefore, the required graph is:



Example 6. Rajesh has ₹100 and wants to buy some notebooks and pens. The cost of one notebook is ₹40 and that of a pen is ₹20. Draw the linear inequality

(a) $40x + 20y < 100$ (b) $40x + 20y \le 100$

(c) $40x - 20y > 100$ (d) $40x + 20y > 100$

Sol. (b) Given, Cost of 1 notebook = ₹40

Cost of 1 pen = ₹20

Let the required number of notebooks be x and the number of pens be y.

According to the question,

$40x + 20y \le 100$

Hence, the correct option is (b).

Example 7. There are 150 students in a class. If the number of boys are x and number of girls are y, then which of the following inequalities represents the situation?

(a) $x + y \le 150$ (b) $x + y \ge 150$

(c) $x + y \ne 150$ (d) None of these

Sol. (a) Given, Number of boys = x

Number of girls = y

Since, the number of students are 150, thus

$x + y \le 150$

Hence, the correct answer is option (a) i.e., $x + y \le 150$.

Example 8.

I. Fisherman catches x fishes and y crabs for his family to eat on daily basis till the time it reaches a number of minimum 10. Now tell, x and y can be related by the inequality

(a) $x + y \ne 10$ (b) $x + y \le 10, x \ge 0, y \ge 0$

(c) $x + y \ge 10, x \ge 0, y \ge 0$ (d) None of these

II. An average size fish can feed 4 people while one crab can feed only one people but the employer has to maintain an output of at least 15 people per done. This situation can be expressed as

(a) $4x + y \le 15$ (b) $4x + y > 15$

(c) $4x + y \ge 15, x \ge 0, y \ge 0$ (d) None of these

III. But he makes sure that he takes home more than 2 fishes to each crab. This situation can be expressed as

(a) $x \ge y/2$ (b) $y \le x/2$ (c) $y \ge x/2$ (d) $2x > y$

Sol. I. (c) Let the number of fishes be x and the number of crabs be y

Since, the number should be minimum of 10 i.e. the sum of fish and crabs should be equal to or greater than 10.

Also, $x \ge 0, y \ge 0$

Thus, the inequality can be represented as:

$x + y \ge 10$, $x \ge 0, y \ge 0$

Hence, the correct option is (c).

II. (c) Let the number of fish be x and the number of crabs be y.

Since, the employer has to maintain an output of at least 15 people per day,

Then according to the problem, it can be represented as

$4x + y \ge 15, x \ge 0, y \ge 0$

Hence, the correct option is (c).

III. (b) Let the number of fishes be x and the number of crabs be y.

Since, for each crab he take more than 2 fishes, then

$x \ge 2y$

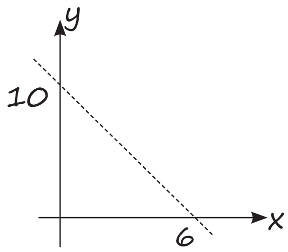
$\Rightarrow \frac{x}{2} \ge y$

or $y \le \frac{x}{2}$

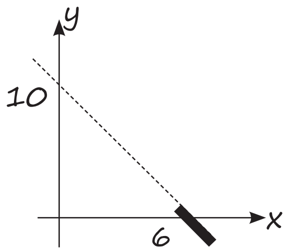
Hence, the correct option is (b).

Example 9. The graph to express the inequality $5x + 3y \geq 30$ is **(ICAI)**

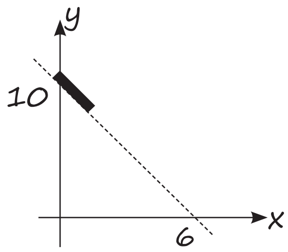
(a)



(b)



(c)



(d) None of these

Sol. (c) **Given inequality:** $5x + 3y \geq 30$ is

The line of equation corresponding to given inequality: $5x + 3y = 30$

When $x = 0$, then $y = 10$

When $y = 0$, then $x = 6$

Thus, the coordinates satisfying the equation is (0, 10) and (6, 0).

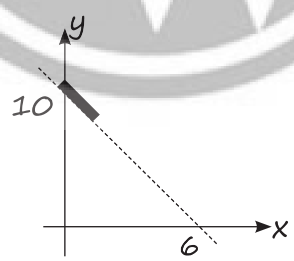
Now, on putting $x = 0$ and $y = 0$ in the above inequality, we get

$5(0) + 3(0) \geq 30$

$\Rightarrow 0 \geq 30$, which is false.

So, the shaded region will be away from the origin.

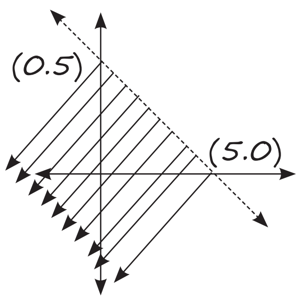
Thus, the required graph is:



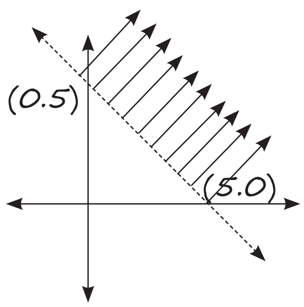
Hence, the correct answer is option (c).

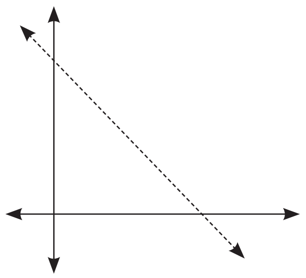
Example 10. The graph to express the inequality $x + y \leq 5$ is

(a)



(b)



(c) 

(d) None of these

Sol. (a) Given inequality $x + y \le 5$

The line of equation corresponding to given inequality: $x + y = 5$

When $x = 0$, then $y = 5$

When $y = 0$, then $x = 5$

Thus, the coordinates satisfying the equation is (0, 5) and (5, 0).

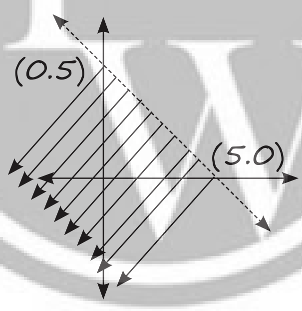
Now, on putting $x = 0$ and $y = 0$ in the above inequality, we get

$0 + 0 \le 5$

$\Rightarrow 0 \le 5$, which is true.

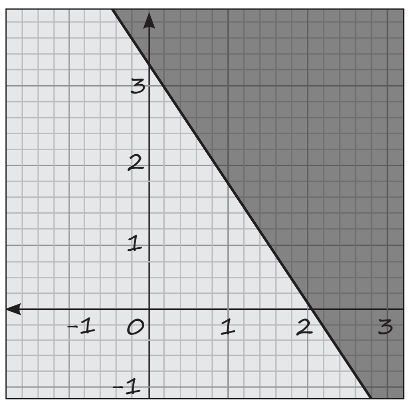
So, the shaded region will be towards the origin.

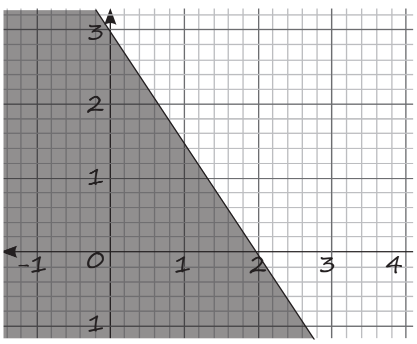
Thus, the required graph is:

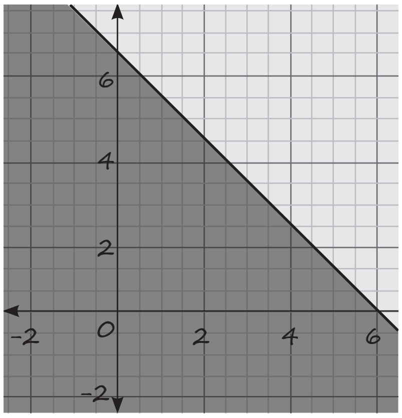


Hence, the correct option is (a).

Example 11. The graph to express the inequality $3x + 2y \le 6$ is

(a) 

(b) 

(c) 

(d) None of the above

Sol. (b) **Given inequality:** $3x + 2y \le 6$

For line of equation of above inequality: $3x + 2y = 6$

When $x = 0$ then $y = 3$

When $y = 0$ then $x = 2$

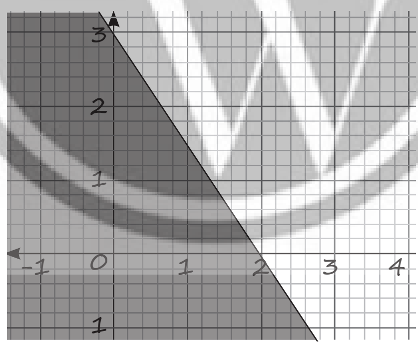
Thus, the coordinates satisfying the equation is (0, 3) and (2, 0).

Now, on putting $x = 0$ and $y = 0$ in the above inequality, we get

$3x + 2y = 3(0) + 2(0) = 0 \le 6$ which is true

So, the shaded region will be towards the origin.

Thus, the required graph is:



Hence, the correct option is (b).

Example 12. On solving the inequalities $2x + 5y \le 20, 3x + 2y \le 12, x \ge 0, y \ge 0$, we get the following situation **(ICAI)**

(a) (0, 0), (0, 4), (4, 0) and $\left(\frac{20}{11}, \frac{36}{11}\right)$

(b) (0, 0), (10, 0), (0, 6) and $\left(\frac{20}{11}, \frac{36}{11}\right)$

(c) (0, 0), (0, 4), (4, 0) and (2, 3)

(d) (0, 0), (10, 0), (0, 6) and (2, 3)

Sol. (a) Given, $2x + 5y \le 20$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$

The line of equation corresponding to given inequalities are;

$2x + 5y = 20$ ...(i)

$3x + 2y = 12$ ...(ii)

Multiplying eq. (i) by 3 and eq. (ii) by 2, we get

$6x + 15y = 60$

$6x + 4y = 24$

On subtracting, we get

$11y = 36$

$\Rightarrow y = \frac{36}{11}$

Put value of y in eq. (i), we get

$2x + \frac{180}{11} = 20$

$\Rightarrow 2x = \frac{40}{11}$

$\Rightarrow x = \frac{20}{11}$

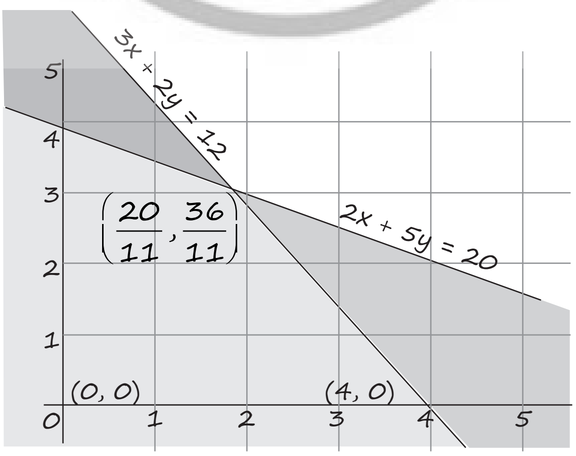
Now, for $2x + 5y = 20$, we have

x	0	10
y	4	0

For $3x + 2y = 12$, we have

x	0	4
y	6	0

Therefore, the graph can be plotted as



Thus, the grey shaded region represents the solution set and the required solution is: (0, 0), (0, 4), (4, 0) and $\left(\frac{20}{11}, \frac{36}{11}\right)$

Hence, the correct option is (a).

Example 13. A firm makes two types of products: Type A and Type B. The profit on product A is ₹20 each and that on product B is ₹30 each. Both types are processed on three machines M1, M2 and M3. The time required in hours by each product and total time available in hours per week on each machine are as follows: **(ICAI)**

Machine	Product A	Product B	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

The constraints can be formulated taking x_1 = number of units A and x_2 = number of unit of B as

(a) $\begin{aligned} x_1 + x_2 &\le 12 \\ 5x_1 + 2x_2 &\le 50 \\ 2x_1 + 6x_2 &\le 60 \end{aligned}$

(b) $\begin{aligned} 3x_1 + 3x_2 &\le 36 \\ 5x_1 + 2x_2 &\le 50 \\ 2x_1 + 6x_2 &\le 60;\ x_1 \ge 0, x_2 \ge 0 \end{aligned}$

(c) $\begin{aligned} 3x_1 + 3x_2 &\le 36 \\ 2x_1 + 6x_2 &\le 60 \\ 5x_1 + 2x_2 &\le 50;\ x_1 \ge 0, x_2 \ge 0 \end{aligned}$

(d) None of these

Sol. (c) Given, x_1 = number of units of A and x_2 = number of units of B

Clearly, $x_1 \ge 0$, $x_2 \ge 0$

According to the given data,

The constraints can be formulated as:

$3x_1 + 3x_2 \le 36$

$5x_1 + 2x_2 \le 50$

$2x_1 + 6x_2 \le 60$ such that $x_1 \ge 0$, $x_2 \ge 0$

Hence, the correct answer is option (c).

Example 14. The union forbids employer to employ less than two experienced people (x) to each fresh person (y). This situation can be expressed as **(ICAI)**

(a) $x \le y/2$ (b) $y \le x/2$ (c) $y \ge x/2$ (d) None of these

Sol. (b) We know that each fresh person is denoted by y and experienced person by x.

As the union forbids employers to employ less than 2 experienced person to each fresh person. This means the number of experienced person must be equal to or greater than twice the number of fresh person.

Thus this situation can be denoted as $2y \le x$

or $y \le \frac{x}{2}$

Hence, the correct answer is option (b).

Example 15. On the average an experienced person does 7 units of work while a fresh one works 5 units of work daily but the employer has to maintain an output of at least 35 units of work per day. The situation can be expressed as:

(a) $7x + 5y < 35$ (b) $7x + 5y \le 35$

(c) $7x + 5y > 35$ (d) $7x + 5y \ge 35$

Sol. (d) Let the number of experienced people be x and the number of fresh people be y.

Work done by experienced person per day = $7x$

Work done by fresh person = $5y$

According to the question,

$7x + 5y \geq 35$

Thus, the situation can be expressed as: $7x + 5y \geq 35$

Hence, the correct option is (d).

Example 16. Solution space of the inequalities $2x + y \leq 10$ and $x - y \leq 5$:

(i) Include the origin.

(ii) Includes the point (4, 3)

Which one is correct? **(June 2011)**

(a) Only (i) (b) Only (ii)

(c) Both (i) and (ii) (d) None of these

Sol. (a) Given, $2x + y \leq 10$ and $x - y \leq 5$

(i) For the origin (0, 0):

$2x + y \leq 10$

$\Rightarrow 0 + 0 \leq 10$ or $0 \leq 10$, which is true

$\Rightarrow x - y \leq 5$

$\Rightarrow 0 + 0 \leq 5$ or $0 \leq 5$, which is also true

(ii) For the point (4, 3):

$\Rightarrow 2x + y \leq 10$

$\Rightarrow 2(4) + 3 \leq 10$

$\Rightarrow 8 + 3 \leq 10$

$\Rightarrow 11 \leq 10$, which is false

Clearly, (0, 0) satisfies both the inequations.

Hence, the correct option is (a).

Example 17. The linear relationship between two variables in an inequality: **(May 2018)**

(a) $ax + by \leq c$ (b) $axby \leq c$ (c) $axy + by \leq c$ (d) $ax + bxy \leq c$

Sol. (a) We know that,

The standard form of linear equation is $ax + by = c$.

Thus, out of the given options, the linear inequation can be represented as $ax + by \leq c$.

Hence, the correct option is (a).

Example 18. The rules and regulations demand that the employer should empty not more than 5 experienced hands to 1 fresh one and this fact is represented by: (Taking experienced person as x and fresh person as y) **(May 2007)**

(a) $y \geq \frac{x}{5}$ (b) $5y \leq x$ (c) $5y \geq x$ (d) None of these

Sol. (a) According to the question,

The employer should employ not more than 5 experienced hands to 1 fresh one.

This means that 1 fresh can have a maximum of 5 experienced hands.

So, taking experienced person as x and fresh person as y, we get,

$x \le 5y$ or $y \ge \frac{x}{5}$

Hence, the correct option is (a).

Example 19. If a > 0 and b < 0, it follows that

(a) $\frac{1}{a} > \frac{1}{b}$ (b) $\frac{1}{a} < \frac{1}{b}$ (c) $\frac{1}{a} = \frac{1}{b}$ (d) None of these

Sol. (a) Since, a > 0

$\Rightarrow \frac{1}{a} > 0$ ...(i)

Now, b < 0

$\Rightarrow \frac{1}{b} < 0$ .... (ii)

From (i) and (ii), we get

$\frac{1}{a} > \frac{1}{b}$

Hence, the correct answer is option (a).

Example 20. A dietitian wishes to mix together two kinds of food so that the vitamin content of the mixture is at least 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D. The vitamin content per kg of each food is shown below:

	A	B	C	D
Food I:	2	1	1	2
Food II:	1	1	2	3

Assuming x units of food I is to be mixed with y units of food II, the situation can be expressed as

(a) $2x + y \le 9, x + y \le 7, x + 2y \le 10, 2x + 3y \le 12, x > 0, y > 0$

(b) $2x + y \ge 30, x + y \le 7, x + 2y \le 10, x + 3y \ge 12$

(c) $2x + y \ge 9, x + y \ge 7, x + y \le 10, x + 3y \ge 12$

(d) $2x + y \ge 9, x + y \ge 7, x + 2y \ge 10, 2x + 3y \ge 12, x \ge 0, y \ge 0$

Sol. (d) Given, Quantity of food I = x units

Quantity of food II = y units

Since, the content should be atleast 9 units of vitamin A, 7 units of vitamin B, 10 units of vitamin C and 12 units of vitamin D

$\Rightarrow 2x + y \ge 9, x + y \ge 7, x + 2y \ge 10, 2x + 3y \ge 12$

Also, the quantity of food cannot be negative.

$\Rightarrow x \ge 0, y \ge 0$

Hence, the final correct answer is option (d) i.e., $2x + y \ge 9, x + y \ge 7, x + 2y \ge 10, 2x + 3y \ge 12, x \ge 0, y \ge 0$.

Example 21. On solving the inequalities $5x + y \le 100$, $x + y \le 60$, $x \ge 0$, $y \ge 0$, we get the following situation: **(Nov 2018)**

(a) (0, 0), (20, 0), (10, 50) & (0, 60)
(b) (0, 0), (60, 0), (10, 50) & (0, 60)
(c) (0, 0), (20, 0), (0, 100) & (10, 50)
(d) None of these

Sol. (a) The inequalities are:

$5x + y \le 100$

$x + y \le 60$

The line of equation for inequality: $5x + y \le 100$

$\Rightarrow 5x + y = 100$

Reference of points:

x	0	20	10
y	100	0	50

The line of equation for inequality: $x + y \le 60$

$\Rightarrow x + y = 60$

Reference of points:

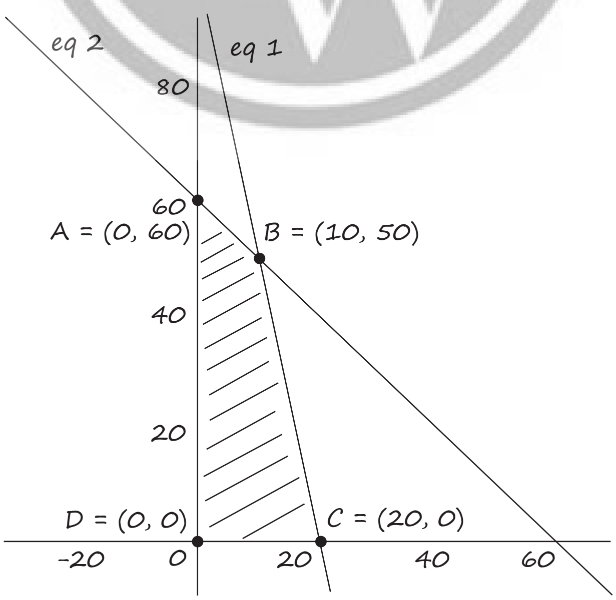
x	0	60	10
y	60	0	50

For the inequalities, the value of x and y will lie either on the lines $x + y = 60$ and $5x + y = 100$ or below them.

It is given $x \ge 0$, $y \ge 0$.

Thus the value of x and y belongs to 1st quadrant

The graph for the given set of inequalities is as follows:



Thus we get the four points

A = (0, 60); B = (10, 50); C = (20, 0); D = (0, 0)

Hence the correct option is (a).

PRACTICE QUESTIONS (PART B)

1. Raju wants to produce sofas. But the material needed can be only get on some conditions such that – he has to buy cloth of sofa at least for 40 sets of sofas (x) and he can get wood (y) for maximum for 100 sets. Form an inequality for it

(a) $x \geq 40, y \leq 100$ (b) $x \leq 40, y \leq 100$

(c) $x \geq 40, y \geq 100$ (d) None of these

2. An employer recruits experienced (x) and fresh workmen (y) under the condition that he cannot employ more than 11 people. x and y can be related by the inequality

(June 2019)

(a) $x + y = 11$

(b) $x + y \leq 11, x \geq 0, y \geq 0$

(c) $x + y > 11, x \geq 0, y \geq 0$

(d) None of these

3. A company is planning to launch a new product and decides to hire marketing executives and sales executives for the project. If the company cannot employ more than 12 executives, which of the following inequalities correctly relates the number of marketing executives (x) and sales executives (y) that the company can hire?

(a) $x + y \leq 12$ (b) $2x + 3y \leq 12$

(c) $3x + 2y \leq 12$ (d) $4x + 4y \leq 12$

4. If an experienced person gets paid ₹10 per unit of work and a fresh one gets paid ₹6 per unit of work, then the inequality representing the situation where the total cost of labor per day is at most ₹180 is:

(x represents number of experienced people and y represents number of fresh people)

(a) $10x + 6y \leq 180$ (b) $10x + 6y \geq 180$

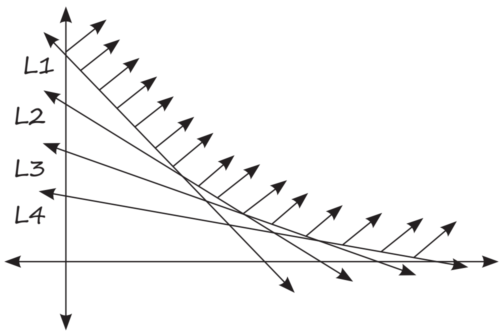
(c) $6x + 10y \leq 180$ (d) $x + y \leq 18$

5. A factory can produce two materials A and B, with machines in operation for 24 hours a day. Production of A requires 1 hour of processing in machine 1 and 2 hours in machine 2. Production of B requires 2 hours of processing in machine1 and 1 hour in machine2. The manufacturer earns a profit of ₹10 on each unit of A and ₹5 on each unit of B. How many units of each product should be produced in a day in order to achieve maximum profit?

(a) $\begin{array}{l} \text{Max } 10x + 5y \\ x + 2y \leq 24 \\ 2x + y \leq 24 \\ x, y \geq 0 \end{array}$ (b) $\begin{array}{l} \text{Max } 10x + 5y \\ x + 2y \geq 24 \\ 2x + y \leq 24 \\ x, y \geq 0 \end{array}$

(c) $\begin{array}{l} \text{Min } 10x + 5y \\ x + 2y \leq 24 \\ 2x + y \leq 24 \\ x, y \geq 0 \end{array}$ (d) None of these

6. Graphs of the inequations are drawn below:



L1: $2x + y = 2$
L2: $x + y = 2$
L3: $x + 2y = 1$
L4: $x + 3y = 4$
The common region (shaded region) shown in the graph refers to

(a) $\begin{matrix} 2x + y \le 2 \\ x + y \ge 2 \\ x + 2y \ge 1 \\ x + 3y \ge 4 \end{matrix}$ (b) $\begin{matrix} 2x + y \ge 2 \\ x + y \le 2 \\ x + 2y \ge 1 \\ x + 3y \ge 4 \end{matrix}$

(c) $\begin{matrix} 2x + y \ge 2 \\ x + y \ge 2 \\ x + 2y \ge 1 \\ x + 3y \ge 4 \end{matrix}$ (d) None of these

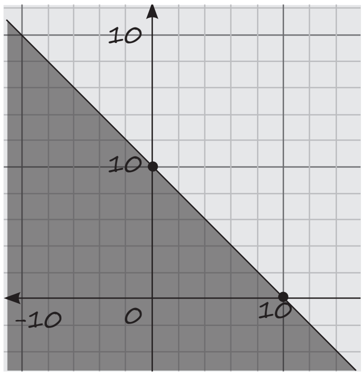
7. The solution of the inequality $\frac{(5-2x)}{3} \le \frac{x}{6} - 5$ is: **(Dec 2020, Jan 2021)**
(a) $x \ge 8$ (b) $x \le 8$ (c) $x = 8$ (d) None of these

8. Which of the following situation do we get on solving the inequalities: $x - y \le 2$, $x + y \le 4$ and $x \ge 1$
(a) (1, 3), (1, 0), (2, 0), (3, 1)
(b) (1, 1), (2, 0), (3, 1), (6, 4)
(c) (1, 0), (2, 0), (3, 1), (7, 5)
(d) None of these

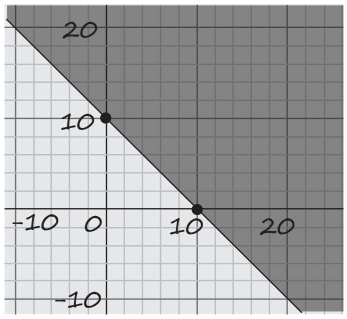
9. The solutions of the set of inequalities $2x + y \ge 12$, $5x + 8y \ge 74$, $x + 6y \ge 24$, $x \ge 0$, $y \ge 0$ are **(Nov 2019)**
(a) (24, 0), $\left(\frac{126}{11}, \frac{23}{11}\right)$, (2, 8), (0, 12)
(b) (0, 24), (2, 8), (0, 12), $\left(\frac{126}{11}, \frac{23}{11}\right)$
(c) (8, 4), (2, 8), (0, 12), (0, 24)
(d) (8, 4), (0, 0), (0, 6), (2, 0)

10. Which of the following graph represents the inequality $x + y \le 10$?

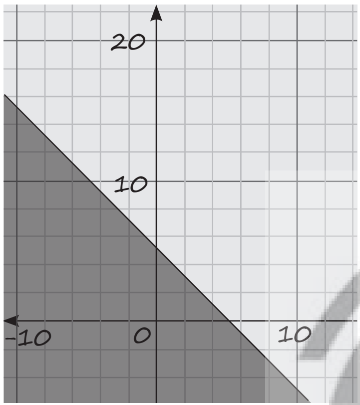
(a)



(b)



(c)



(d) None of these

11. On solving the inequalities $6x + y \ge 18$, $x + 4y \ge 12$, $2x + y \ge 10$, we get the following situation

(a) (0, 18), (12, 0), (4, 2) & (2, 6)

(b) (3, 0), (0, 3), (4, 2) & (7, 6)

(c) (5, 0), (0, 10), (4, 2) & (7, 6)

(d) (0, 18), (12, 0), (4, 2), (0, 0) & (7, 6)

12. A small manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry, and then sent to another machine for finishing. The number of man-hours for the firm available per week are as follows:

	Foundry	Machine-shop
A	10	5
B	6	4
Capacity per week (man hours)	1000	600

Let the firm manufacture x units of A and y units of B.

The constraints are:

(a) $10x + 6y \le 1000$, $5x + 4y \ge 600$, $x \ge 0$; $y \le 0$

(b) $10x + 6y \le 1000$, $5x + 4y \le 600$, $x \ge 0$; $y \ge 0$

(c) $10x + 6y \ge 1000$, $5x + 4y \le 600$, $x \le 0$; $y \ge 0$

(d) $10x + 6y \ge 1000$, $5x + 4y \ge 600$, $x \le 0$; $y \le 0$

13. Vitamin A and B are found in food F_1 and F_2. One unit of F_1 contains 20 units of vitamin A and 30 units of vitamin B. One unit of F_2 contains 60 units of vitamin A and 40 units of vitamin B. Cost per unit of F_1 and F_2 are ₹3 and ₹4 respectively. The minimum daily requirement of vitamin A and B is 80 and 100 units respectively.

Problem is to determine the mixture of food F_1 and F_2, which meets the requirement at minimum cost by assuming that x_1 units of food F_1 and x_2 units of food F_2 are required to fulfill the need of vitamins. The constraints are

(a) $20x_1 + 60x_2 \le 80, 30x_1 + 40x_2 \le 40, x_1 \le 0; x_2 \le 0$

(b) $20x_1 + 60x_2 \ge 80, 30x_1 + 40x_2 \le 40, x_1 \ge 0; x_2 \le 0$

(c) $20x_1 + 60x_2 \ge 80, 30x_1 + 40x_2 \ge 40, x_1 \ge 0; x_2 \ge 0$

(d) $20x_1 + 60x_2 \le 80, 30x_1 + 40x_2 \ge 40, x_1 \le 0; x_2 \ge 0$

14. A fertilizer company produces two types of fertilizers called grade I (x) and grade II (y). Each of these types is processed through two critical chemical plant units. Plant A has a maximum of 120 hours available in a week and plant B has a maximum of 180 hours available in a week. Manufacturing one bag of grade I fertilizer requires 6 hours in plant A and 4 hours in plant B. Manufacturing one bag of grade II fertilizer requires 3 hours in plant A and 10 hours in plant B. Express this using linear inequalities.

(a) $6x + 3y \le 120, 4x + 10 = 180$ (b) $6x + 3y = 120, 4x + 10y > 180$

(c) $6x + 3y \le 120, 4x + 10y \le 180$ (d) $6x + 3y < 120, 4x + 10y < 180$

15. A car manufacturing company manufacturers cars of two types A and B. Model A requires 150 man-hours for assembling, 50 man-hours for painting and 10 man-hours for checking and testing. Model B requires 60 man-hours for assembling, 40 man-hours for painting and 20 man-hours for checking and testing. There are available 30 thousand man-hours for assembling,13 thousand man-hours for painting and 5 thousand man-hours for checking and testing. Express the above situation using linear inequalities. Let the company manufacture x units of type A model of car and y units of type B model of car.

Then the inequalities are: **(May 2007)**

(a) $5x + 2y \ge 1000, 5x + 4y \ge 1300, x + 2y \le 500, x \ge 0, y \ge 0$ ≤≥

(b) $5x + 2y \le 1000, 5x + 4y \le 1300, x + 2y \le 500, x \ge 0, y \ge 0$

(c) $5x + 2y \le 1000, 5x + 4y \le 1300, x + 2y \le 500, x \ge 0, y \ge 0$

(d) $5x + 2y = 1000, 5x + 4y \ge 1300, x + 2y = 500, x \ge 0, y \ge 0$

Answer Key

1. (a)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (a)
11. (a)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (b)					

SUMMARY

- The chapter on inequalities explores mathematical relationships between expressions or numbers that are not equal. Inequalities are represented using symbols such as " < " (less than), " >" (greater than), "≤" (less than or equal to), and "≥" (greater than or equal to).

- **Inequality Symbols:** *Inequalities are expressed using symbols such as " < ", " > ", "≤" and "≥" to compare two quantities or expressions.*
- **Solving Linear Inequalities:** *Solving linear inequalities involves finding the range of values for two variables that satisfy the given inequality. This often includes algebraic manipulation, such as adding, subtracting, multiplying, or dividing both sides of the inequality by constants.*
- **Graphical Representation:** *One of the fundamental aspects of this chapter is the graphical representation of linear inequalities. These inequalities can be visualized on a coordinate plane as shaded regions. The region below or above a line (depending on the inequality symbol) represents the set of points that satisfy the inequality.*
- **Word Problems:** *Inequality concepts are applied to real-world situations in word problems. These problems require translating verbal descriptions into mathematical inequalities and solving for the unknown quantities.*
- *Overall, understanding linear inequalities and their graphical representation is crucial in mathematics and real-life applications. It helps describe relationships where quantities are not necessarily equal but can fall within a range of values. Solving linear inequalities is a fundamental skill in algebra and has numerous applications in fields such as economics, science, and engineering.*

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