CHAPTER

2

Equations

If I ask you to buy some bananas and few apples such that their sum is equal to 15. How will you write mathematically?

We can represent it as:

Let the number of bananas be x and the number of apples be y having prices as a and b respectively.

Then, $ax + by = 15$

WHAT IS EQUATION?

- In simple language, equation is a mathematical statement of equality.
- When we equate '=' variable/variables with constant and find out the value of variable. That will be called an equation.

  E.g.: $2x + 3 = 0$, where x is a variable and 3 is a constant.

> In this chapter, you have to focus on understanding the questions and forming the equation, if done properly you have solved 50% of the question and 50% is just the steps which you will learn.

CONDITIONAL EQUATION

- In simple language, an equation is a mathematical statement of equality.

  E.g.: $\frac{x+2}{2} = 5$

IDENTITY

- If the equality is true for all values of the variable involved, the equation is called an identity.

  E.g.: $\sin^2 x + \cos^2 x = 1$

Solution of the Equation or Root of the Equation

The value of the variable which satisfies the equation will be the solution or root of the equation.

Equation			
Linear Equation	Quadratic Equation	Cubic Equation	Simultaneous Equation
$3x+5(x-2) = 2(3x-5)+7$	$x^2+2x+1 = 0$	$x^3+3x^2+3x = 1$	$x+3y = 1, 2x+y = 1$

LINEAR EQUATION (SIMPLE EQUATION)

- An equation in the form of $ax + b = 0$ will be called a simple equation or linear equation.
  E.g.: $2x + 3 = 0$ where $a = 2$, $b = 3$.

Example 1. The equation $2x - 1 = x + 3$ will be satisfied for x equal to

(a) -1 (b) 2 (c) 4 (d) -2

Sol. (c) Given, $2x - 1 = x + 3$

$\Rightarrow 2x - x = 3 + 1$

$\Rightarrow x = 4$

Therefore, the value of x is 4.

Hence, the correct option is (c).

Example 2. The solution of an equation $\frac{x+7}{3} = 4 + \frac{x}{2}$ is

(a) -10 (b) 20 (c) 20 (d) 4

Sol. (a) Given, $\frac{x+7}{3} = 4 + \frac{x}{2}$

$\frac{x+7}{3} - \frac{x}{2} = 4$

$\frac{2 \times (x+7) - 3 \times x}{6} = 4$

$\Rightarrow 2x + 14 - 3x = 4 \times 6$

$\Rightarrow -x = 24 - 14$

$\Rightarrow -x = 10$

$\Rightarrow x = -10$

Hence, the correct option is (a).

Example 3. The sum of two numbers is 20 and their difference is 4. The numbers are

(a) (12, 15) (b) (12, 8) (c) (2, 8) (d) None of these

Sol. (b) **Trick:** Go by choices:

For option (a): (12, 15)

Sum of numbers = $12 + 15 = 27 \neq 20$

For option (b): (12, 8)

Sum of numbers = $12 + 8 = 20$, which is true

Difference of numbers = $12 - 8 = 4$, which is also true

Therefore, the numbers are 12 and 8.

Hence, the correct option is (b).

Example 4. 8 is the solution of the equation **(ICAI)**

(a) $\frac{x+4}{4} + \frac{x-5}{3} = 11$ (b) $\frac{x+4}{2} + \frac{x+10}{9} = 8$

(c) $\frac{x+24}{5} = 4 + \frac{x}{4}$ (d) $\frac{x-15}{10} + \frac{x+5}{5} = 4$

Sol. (b) **Go by choices:** Put $x = 8$ in all the equations given in the options.

For option (a):

$$LHS = \frac{8+4}{4} + \frac{8-5}{3} = \frac{12}{4} + 1 = 3 + 1 \neq 11$$

Thus, LHS ≠ RHS

For option (b):

$$LHS \; \frac{8+4}{2} + \frac{8+10}{9} = \frac{12}{2} + \frac{18}{9} = 6 + 2 = 8$$

Thus, LHS = RHS

For option (c):

$$LHS = \frac{8+24}{5} = \frac{32}{5}$$

$$RHS = 4 + \frac{8}{4} = 4 + 2 = 6$$

Thus, LHS ≠ RHS

For option (d):

$$LHS = \frac{8-15}{10} + \frac{8+5}{5} = -\frac{7}{10} + \frac{13}{5} = \frac{-7+26}{10} = \frac{19}{10} \neq 4$$

Thus, LHS ≠ RHS

Hence, the correct option is (b).

Example 5. If we subtract twice Ram's present age from his age 15 years from the present, the result would be equal to his age after 3 years. Find Ram's present age.

(a) 10 (b) 12 (c) 15 (d) 6

Sol. (d) Let Ram's Present age be x years.

According to question,

$(x + 15) - 2x = x + 3$

$\Rightarrow x - 2x + 15 = x + 3$

$\Rightarrow -x + 15 = x + 3$

$\Rightarrow 15 - 3 = x + x$

$\Rightarrow 12 = 2x$

$\Rightarrow x = 6$

Therefore, the present age of Ram is 6 years.

Hence, the correct option is (d).

Example 6. Pick up the correct value x for which $\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} = 0$ **(ICAI)**

(a) $x = 0$ (b) $x = 1$ (c) $x = 10$ (d) None of these

Sol. (c) Given equation: $\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} = 0$

On multiplying both sides by 0.5, we get

$$0.5\times\left(\frac{x}{0.5}-\frac{1}{0.05}+\frac{x}{0.005}-\frac{1}{0.0005}\right)=0.5\times 0$$

$$\Rightarrow \frac{0.5\times x}{0.5}-\frac{0.5\times 1}{0.05}+\frac{0.5\times x}{0.005}-\frac{0.5\times 1}{0.0005}=0$$

$\Rightarrow x - 10 + 100x - 1000 = 0$

$\Rightarrow 101x - 1010 = 0$

$\Rightarrow 101x = 1010$

$$\Rightarrow x = \frac{1010}{101}$$

$\Rightarrow x = 10$

Trick:

Go by choices:

Clearly, for option (c): x = 10

$$\text{LHS} = \frac{x}{0.5}-\frac{1}{0.05}+\frac{x}{0.005}-\frac{1}{0.0005}$$

$$= \frac{10}{0.5}-\frac{1}{0.05}+\frac{10}{0.005}-\frac{1}{0.0005}$$

= 20 − 20 2000 − 2000 = 0 = RHS

Hence, the correct answer is option (c) i.e., 10.

Example 7. One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number was

(a) 320 (b) 400 (c) 480 (d) None of these

Sol. (c) Let the required number be x.

According to requirement, the number should be $\frac{1}{6}\times\frac{x}{2}+\frac{1}{4}\times\frac{x}{2}$

The number according to the student is $\frac{x}{5}$

Thus, the relation between the numbers:

$$\frac{x}{5}=\frac{x}{12}+\frac{x}{8}-4$$

$$\frac{x}{12}+\frac{x}{8}-\frac{x}{5}=4$$

LCM of 5, 8, 12 = 120

$$\Rightarrow \frac{(10x+15x-20x)}{120}=4$$

$\Rightarrow x = 120 \times 4$

$\Rightarrow x = 480$

Therefore, the required number is 480.

Hence, the correct option is (c).

Example 8. For a certain commodity, the demand equation giving demand 'd' in kg, for a price 'p' in rupees per kg is $d = 50(20 - p)$. The supply equation gives the supply 's' in kg, for a price 'p' in rupees per kg is $s = 75(p - 3)$. The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.

(a) (9.8, 150) (b) (20, 405) (c) (15, 150) (d) None of these

Sol. (a) Given, Demand equation: $d = 50(20 - p)$

Supply equation: $s = 75(p - 3)$

Since, the market price is such at which demand equals supply thus

$d = s$

$\Rightarrow 50(20 - p) = 75(p - 3)$

$\Rightarrow 2(20 - p) = 3(p - 3)$

$\Rightarrow 40 - 2p = 3p - 9$

$\Rightarrow 3p + 2p = 40 + 9$

$\Rightarrow 5p = 49$

$\Rightarrow p = 9.8$

On substituting the value of p in any of the above equation, we get

Quantity, $d = 50(20 - 9.8) = 150$

Hence, the correct option is (a).

PRACTICE QUESTIONS (PART A)

1. The equation $-7x + 1 = 5 - 3x$ will be satisfied for x equal to **(ICAI)**

   (a) 2 (b) − 1 (c) 1 (d) None of these

2. The value of x for which $5x - 11 = 3x + 9$ is satisfied

   (a) 20 (b) − 2 (c) 10 (d) None of these

3. The value of y for which the equation $7y - 42 = 6 + 3y$ is satisfied

   (a) 5 (b) 12 (c) 16 (d) None of these

4. What will be the correct value of x for $\frac{x}{10} = \frac{3}{5}$

   (a) −5 (b) 6 (c) 4 (d) −6

5. What will be the correct value x for which $\frac{x}{0.1} - \frac{1}{0.01} + \frac{x}{0.001} - \frac{x}{0.1} = 0$

   (a) −10 (b) 2 (c) 10 (d) −5

6. If the sum of two numbers is 11 and the sum of their squares be 85, then the numbers are

   (a) 5 and 6 (b) 4 and 7 (c) 2 and 9 (d) None of these

7. Divide 65 into two parts such that two times the first part exceeds one fourth of the second by 40. The parts are.

   (a) (30, 35) (b) (20, 45) (c) (15, 50) (d) None of these

8. The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator the fraction increases by unity. The fraction is

(a) 3/5 (b) 1/3 (c) 7/9 (d) None of these

9. If a number of which the half is greater than $\frac{1}{5}$th of the number by 15 then the number is

(a) 50 (b) 40 (c) 80 (d) None of these

10. The ages of three brothers are consecutive even integers. If the sum of their ages is 78, what is the age of the youngest brother?

(a) 24 years (b) 26 years (c) 28 years (d) 30 years

Answer Key

1. (b)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (a)

SIMULTANEOUS LINEAR EQUATIONS

What will we study here?

- Simultaneous linear equation is basically a method of solving two equations given simultaneously at a time, where we need to find solution for two different variables.
- The general form of that type of equation will be: $ax + by + c = 0$,

  where, x and y are two variables.
- So, let's say two equations possibly are:

  $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

  In above equation, we need to find the value (solution or roots) of two linear equation which has two variables x and y.

Method Of Solution For Simultaneous Linear Equations

Elimination Method → Cross Product Method

ELIMINATION METHOD

- In this method two given linear equations are reduced to a linear equation in one variable by eliminating one of the variables and then solving for the other variable.

Example 9. Solve for x and y:

$2x + y = 9$ and $2x - 3y = 5$

Sol. Here, since the coefficient of x are same, thus subtracting second equation from first equation, i.e.,

$2x + y = 9$

$2x - 3y = 5$

$-\quad + \quad -$

$\Rightarrow 2x - 2x + y + 3y = 9 - 5$

$\Rightarrow y = \frac{4}{4} = 1$

Substituting the value of x in first equation, we get

$2x + 1 = 9$

$\Rightarrow 2x = 9 - 1$

$\Rightarrow x = \frac{8}{2} = 4$

Therefore, the solution of given equations is (4, 1).

Example 10. Solve for x and y:

$2x - y = -2$ and $4x - 3y = -12$

(a) (1, 2) (b) (3, 8) (c) (4, 4) (d) (8, 3)

Sol. (b) Given,

$2x - y = -2$ ...(i)

$4x - 3y = -12$ ...(ii)

Multiplying eq. (i) by 2, we get

$4x - 2y = -4$ .... (iii)

On subtracting eq. (iii) from eq. (ii), we get

$4x - 3y - 4x + 2y = -12 + 4$

$\Rightarrow -y = -8$

$\Rightarrow y = 8$

Now, put the value of y in eq. (i), we get

$2x - 8 = -2$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

Therefore, the value of x is 3 and y is 8.

Hence, the correct option is (b) i.e., (3, 8).

Example 11. The solution of the set of equations $3x + 4y = 7$, $4x - y = 3$ is

(a) (1, −1) (b) (1, 1) (c) (2, 1) (d) (1, −2)

Sol. (b) Given equations: $3x + 4y = 7$, $4x - y = 3$

We need to find the value (x, y) so that both the equations are satisfied.

$3x + 4y = 7$ ...(i)

$4x - y = 3$ ...(ii)

Multiplying equation (ii) by 4, we get

$16x - 4y = 12$ ...(iii)

On adding equations (i) and (iii), we get

$(3x + 4y) + (16x - 4y) = 7 + 12$

$\Rightarrow 19x = 19$

$\Rightarrow x = 1$

Now, putting the value of $x = 1$ in equation (i), we have

$3 \times 1 + 4y = 7$

$\Rightarrow 3 + 4y = 7$

$\Rightarrow 4y = 7 - 3$

$\Rightarrow 4y = 4$

$\Rightarrow y = 1$

Therefore, the solution is (1, 1) for the given equations.

Hence, the correct answer is option (b) i.e., (1, 1).

Example 12. The values of x and y satisfying the equations $3x - y = 3$, $x + 2y = 8$ are given by the pair:

(a) (3, 2) (b) (−2, −3) (c) (2, 3) (d) None of these

Sol. (c) Given,

$3x - y = 3$ ...(i)

$x + 2y = 8$ ...(ii)

Multiplying eq. (ii) by 3 on both sides, we get

$3x + 6y = 24$ ...(iii)

On subtracting above eq. (iii) from eq. (i), we get

$3x - y = 3$

$3x + 6y = 24$

$- \quad - \quad -$

$\Rightarrow 3x - 3x - y - 6y = 3 - 24$

$\Rightarrow -7y = -21$

$\Rightarrow y = \frac{-21}{-7} = 3$

Putting value of y in eq. (i), we get

$3x - 3 = 3$

$\Rightarrow 3x = 3 + 3 = 6$

$\Rightarrow x = \frac{6}{3} = 2$

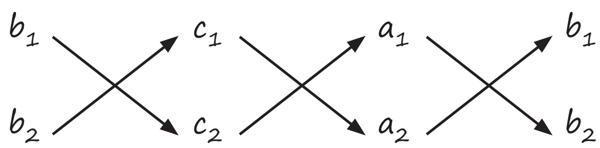
Therefore, $(x, y) = (2, 3)$

Trick:
Put all the options in given equations, you will see only option (c) will be satisfying both equations, i.e.
For option (c): (2, 3)
Substituting $x = 2$ and $y = 3$ in $3x - y = 3$,
LHS = $3(2) - 3 = 6 - 3 = 3$ = RHS
Now, substituting $x = 2$ and $y = 3$ in $x + 2y = 8$, i.e.
LHS = $2 + 2(3) = 2 + 6 = 8$ = RHS

Hence, the correct option is (c).

CROSS MULTIPLICATION METHOD

- Let two equations be:
  $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
  Write coefficients of x and y, also constants in both the equations, likes:



$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

i.e., $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Example 13. The simultaneous equations $7x - 3y = 31$, $9x - 5y = 41$ have solutions given by

(a) (–4, –1) (b) (–1, 4) (c) (4, –1) (d) (3, 7)

Sol. (c) Comparing $7x - 3y - 31 = 0$ with $a_1x + b_1y + c_1 = 0$

$\Rightarrow a_1 = 7, b_1 = -3, c_1 = -31$

Now, comparing $9x - 5y - 41 = 0$ with $a_2x + b_2y + c_2 = 0$

$\Rightarrow a_2 = 9, b_2 = -5, c_2 = -41$

We know that, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$$x = \frac{(-3 \times -41) - (-5 \times -31)}{(7 \times -5) - (9 \times -3)}$$

$$\Rightarrow x = \frac{123 - (155)}{(-35) - (-27)}$$

$$\Rightarrow x = \frac{-32}{-8} = 4$$

Thus,

$$y = \frac{(-31 \times 9) - (-41 \times 7)}{(7^* - 5) - (9^* - 3)}$$

$$\Rightarrow y = \frac{-279 - (-287)}{(-35) - (-27)}$$

$$\Rightarrow y = \frac{8}{-8} = -1$$

$\Rightarrow (x, y) = (4, -1)$

Trick:
Put all the options in given equations, you will see only option (c) will be satisfying both equations.
$(7 \times 4) - (3 \times -1) = 28 + 3 = 31,$
$(4 \times 9) - (5 \times -1) = 36 + 5 = 41$

Hence, the correct option is (c).

Example 14. $1.5x + 3.6y = 2.1$, $2.5(x + 1) = 6y$ have solutions as

(a) (0.2, 0.5) (b) (0.5, 0.2) (c) (2, 5) (d) (−2, −5)

Sol. (a) Given, $1.5x + 3.6y = 2.1$

$\Rightarrow 15x + 36y = 21$

$\Rightarrow 5x + 12y = 7$ ...(i)

Also, $2.5(x + 1) = 6y$

$\Rightarrow 2.5x + 2.5 = 6y$

$\Rightarrow 25x + 25 = 60y$

$\Rightarrow 5x + 5 = 12y$ (on dividing both the sides by 5)

$\Rightarrow 5x - 12y = -5$ ...(ii)

Now, adding eq. (i) and eq. (ii), we get

$(5x + 12y) + (5x - 12y) = 7 + (-5)$

$\Rightarrow 10x = 2$

$\Rightarrow x = 0.2$

Now, putting the value of $x = 0.2$ in equation (i), we get

$5(0.2) + 12y = 7$

$\Rightarrow 1 + 12y = 7$

$\Rightarrow 12y = 7 - 1$

$\Rightarrow 12y = 6$

$$\Rightarrow y = \frac{6}{12}$$

$\Rightarrow y = 0.5$

Thus, the required solution set is $x = 0.2$ and $y = 0.5$.

Hence, the correct answer is option (a).

Example 15. A number consists of two digits. The digits in the ten's place is 3 times the digit in the unit's place. If 54 is subtracted from the number, the digits are reversed. The number is

(a) 39 (b) 92 (c) 93 (d) 94

Sol. (c) Let the two-digit number = $10x + y$

The digits in the ten's place is 3 times the digit in the unit's place

$\Rightarrow x = 3y$ ...(i)

If 54 is subtracted from the number, the digits are reversed

$\Rightarrow 10x + y - 54 = 10y + x$

$\Rightarrow 9x = 9y + 54$

$\Rightarrow x = y + 6$

$\Rightarrow 3y = y + 6$ (from i)

$\Rightarrow y = 3$

Thus, $x = 3y$

$\Rightarrow x = 9$

Therefore, the required number is 93.

Trick:
Only for option (c): 93
If 54 is subtracted from the number, the digits are reversed $93 - 54 = 39$

Hence, the correct option is (c).

PRACTICE QUESTIONS (PART B)

1. Solve for x and y: $x - 3y = 0$, $x + 2y = 20$

(a) $x = 4, y = 12$ (b) $x = 12, y = 4$
(c) $x = 5, y = 4$ (d) None of these

2. Solve for x and y: $2x + y = -4$ and $5x - 3y = 1$

(a) $x = 1, y = 2$ (b) $x = 2, y = 2$ (c) $x = -1, y = -2$ (d) None of these

3. Solve for x and y: $4x + 2y = 10$ and $5x - y = 4$

(a) $x = \frac{1}{7}, y = \frac{15}{7}$ (b) $x = \frac{1}{9}, y = \frac{17}{7}$
(c) $x = \frac{9}{7}, y = \frac{17}{7}$ (d) None of these

4. $1.5x + 2.4y = 1.8$, $2.5(x + 1) = 7y$ have solutions as **(ICAI)**

(a) (0.5, 0.4) (b) (0.4, 0.5) (c) $\left(\frac{1}{2}, \frac{2}{5}\right)$ (d) (2, 5)

5. Solve for x and y: $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$.

(a) $x = 2, y = -2$ (b) $x = 2, y = 3$ (c) $x = 1, y = 2$ (d) $x = 2, y = -3$

6. The solution to the given set of equations $\frac{x}{5}+\frac{y}{6}+1=\frac{x}{6}+\frac{y}{5}=28$ **(ICAI)**

(a) (6, 9) (b) (9, 6) (c) (60, 90) (d) (90, 60)

7. The solution for the pair of equation $\frac{1}{16x}+\frac{1}{15y}=\frac{9}{20}, \frac{1}{20x}-\frac{1}{27y}=\frac{4}{45}$ is given by,

(a) $(\frac{1}{2}, \frac{1}{3})$ (b) $(-\frac{1}{2}, \frac{1}{3})$ (c) $(\frac{1}{4}, \frac{1}{3})$ (d) $(\frac{1}{2}, \frac{1}{4})$ **(ICAI)**

8. The cost prices of 3 pens and 4 bags is ₹324 and 4 pens and 3 bags is ₹257, then the cost of 1 pen is equal to

(a) ₹16 (b) ₹8 (c) ₹50 (d) ₹75

9. The sum of the digits of a two–digit number is 8. If 13 be subtracted from it the digits in the resulting number will be equal. The number is

(a) 47 (b) 35 (c) 56 (d) None of these

Answer Key

1. (b) **2.** (c) **3.** (c) **4.** (b) **5.** (d) **6.** (c) **7.** (c) **8.** (b) **9.** (b)

METHOD OF SOLVING SIMULTANEOUS LINEAR EQUATION WITH THREE VARIABLES

$ax + by + cz = d$, where x, y, z are variable, d is a constant and a, b, c are coefficients.

Example 16. Solve for x, y and z, $2x + 3y + 4z = 0$, $x + 2y - 5z = 0$, $10x + 16y - 6z = 0$

(a) (0, 0, 0) (b) (1, –1, 1) (c) (3, 2, –1) (d) (1, 0, 2) **(ICAI)**

Sol. (a) **Detailed Method:**

Given,

$2x + 3y + 4z = 0$ ...(i)

$x + 2y - 5z = 0$ ...(ii)

$10x + 16y - 6z = 0$ ...(iii)

On multiplying equation (ii) by 2, we get

$2x + 4y - 10z = 0$

Thus, subtracting the above equation from eq. (i),

$-y + 14z = 0$

$\Rightarrow y = 14z$

Substitute y value in eq. (i), we get

$2x + 3(14z) + 4z = 0$

$\Rightarrow 2x + 42z + 4z = 0$

$\Rightarrow 2x + 46z = 0$ ...(iv)

Again, substituting y value in eq. (iii), we get

$10x + 16(14z) - 6z = 0$

$\Rightarrow 10x + 224z - 6z = 0$

$\Rightarrow 10x + 218z = 0$ ...(v)

From eq. (iv) and eq. (v), we get $z = 0$

Also, $y = 14z$

$\Rightarrow y = 0$

Substituting these values in eq. (i), we get $x = 0$

Therefore $(x, y, z) = (0, 0, 0)$

Trick:

Put all the options in given equations, you will see only option (a) will be satisfying all three equations.

$2 \times 0 + 3 \times 0 + 4 \times 0 = 0,$

$0 + 2 \times 0 - 5 \times 0 = 0,$

$10 \times 0 + 16 \times 0 - 6 \times 0 = 0$

Therefore, the required solution is (0, 0, 0).

Hence, the correct option is (a).

Example 17. The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age. **(ICAI)**

(a) 60 years (b) 52 years (c) 51 years (d) 50 years

Sol. (d) Let the age of man be z years and their sons be x and y.

According to the question,

$z = 2(x + y)$ ...(i)

Five years ago,

Age of father = $z - 5$

Ages of their sons = $x - 5, y - 5$

Thus, the relation five years ago is:

$z - 5 = 3(x - 5 + y - 5)$

$\Rightarrow z - 5 = 3(x + y - 10)$

$\Rightarrow z = 3(x + y) - 25$ ...(ii)

From (i) and (ii), we get

$$z = \frac{3}{2}z - 25$$

$$\Rightarrow z - \frac{3}{2}z = -25$$

$$\Rightarrow -\frac{z}{2} = -25$$

$\Rightarrow z = 50$

Therefore, the age of the person is 50 years.

Hence, the correct option is (d).

Example 18. A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed then find the number. **(ICAI)**

(a) 54 (b) 53 (c) 45 (d) 55

Sol. (c) Let the two-digit number be xy.

Thus, the number be $10x + y$

Sum of its digits = $x + y$

According to the question,

$10x + y = 5(x + y)$

$\Rightarrow 10x + y = 5x + 5y$

$\Rightarrow 10x - 5x = 5y - y$

$\Rightarrow 5x - 4y = 0$ ...(i)

If 9 is added, then the digits are reversed.

$\Rightarrow 10x + y + 9 = 10y + x$

$\Rightarrow 10x - x + 9 = 10y - y$

$\Rightarrow 9x - 9y = -9$

$\Rightarrow x - y = -1$ ...(ii)

Also, $5x - 4y = 0$

Multiplying eq. (ii) with 5, thus

$\Rightarrow 5(x - y) = -5$

$\Rightarrow 5x - 5y = -5$ ...(iii)

Subtracting eq. (iii) from eq. (i), we get

$\Rightarrow (5x - 4y) = -(5x - 5y) = 0 - (-5)$

$\Rightarrow -4y + 5y = 5$

$\Rightarrow y = 5$

On substituting $y = 5$ in $x - y = -1$, we get

$\Rightarrow x = 4$

Therefore, the number is $10x + y = 10(4) + 5 = 45$

Hence, the correct option is (c).

Example 19. $\frac{xy}{x+y} = 20, \frac{yz}{y+z} = 40, \frac{zx}{z+x} = 24$ **(ICAI)**

(a) (120, 60, 30) (b) (60, 30, 120)

(c) (30, 120, 60) (d) (30, 60, 120)

Sol. (d) Given, $\frac{xy}{x+y} = 20 \Rightarrow \frac{x+y}{xy} = \frac{1}{20}$

$\Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{1}{20}$ ...(i)

Also, $\frac{yz}{(y+z)} = 40 \Rightarrow \frac{y+z}{yz} = \frac{1}{40}$

$$\Rightarrow \frac{1}{z} + \frac{1}{y} = \frac{1}{40} \quad ...(ii)$$

Now, $\frac{zx}{(z+x)} = 24 \Rightarrow \frac{z+x}{zx} = \frac{1}{24}$

$$\Rightarrow \frac{1}{x} + \frac{1}{z} = \frac{1}{24} \quad ...(iii)$$

Adding eq. (i), (ii) and (iii), we get

$$\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = \frac{1}{20} + \frac{1}{40} + \frac{1}{24}$$

$$\Rightarrow 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \frac{14}{120} \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{7}{120} \quad ...(iv)$$

From (i) and (iv), we get

$$\frac{1}{20} + \frac{1}{z} = \frac{7}{120} \Rightarrow z = 120$$

Now, from (ii) and (iv), we get

$$\frac{1}{40} + \frac{1}{x} = \frac{7}{120} \Rightarrow x = 30$$

Thus, $y = 60$

Therefore, the values of $(x, y, z) = (30, 60, 120)$.

Hence, the correct option is (d).

Example 20. The wages of 8 men and 6 boys amount to ₹33. If 4 men earn ₹4.50 more than 5 boys, determine the wages of each man and boy. **(ICAI)**

(a) (₹0.50, ₹3) (b) (₹3, ₹1.50) (c) (₹2.50, ₹2) (d) (₹2, ₹2.50)

Sol. (b) Let the wages of 1 man be ₹x and that of 1 boy be ₹y.

According to the question,

$8x + 6y = 33$ ...(i)

and $4x = 4.50 + 5y$

Or, $4x - 5y = 4.50$ ...(ii)

Multiply eq. (ii) by 2, we get

$8x - 10y = 9$

Subtract it from eq. (i), we get

$16y = 24$

$y = 1.5$

Putting value of y in $8x - 10y = 9$, we get

$x = 3$

Therefore, the wages of each man and boy are ₹3 and ₹1.50 respectively.

Hence, the correct option is (b).

Example 21. If $\frac{10}{x+y}+\frac{2}{x-y}=4$ and $\frac{15}{x+y}-\frac{5}{x-y}=-2$, then (x, y) is

(a) (3, 2) (b) (−3, 2) (c) (3, −2) (d) (−3, −2)

Sol. (a) Given, $\frac{10}{x+y}+\frac{2}{x-y}=4$ and $\frac{15}{x+y}-\frac{5}{x-y}=-2$

Trick: Go by choices

For (3, 2)

$\Rightarrow \frac{10}{x+y}+\frac{2}{x-y}=\frac{10}{3+2}+\frac{2}{3-2}=2+2=4$, which is true

and $\frac{15}{x+y}-\frac{5}{x-y}=\frac{15}{3+2}-\frac{5}{3-2}=3-5=-2$, which is also true

For the other options, the equations are not satisfied.

Therefore, (x, y) = (3, 2)

Hence, the correct answer is option (a) i.e., (3, 2).

PRACTICE QUESTIONS (PART C)

1. $\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10}$, $3xy = 10(y - x)$ **(ICAI)**

   (a) (2, 5) (b) (5, 2) (c) (2, 7) (d) (3, 4)

2. The solution for the pair of $\frac{xy}{y-x}=110, \frac{yz}{z-y}=132, \frac{zx}{z+x}=\frac{60}{11}$ is given by **(ICAI)**

   (a) (12, 11, 10) (b) (10, 11, 12) (c) (11, 10, 12) (d) (12, 10, 11)

3. A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reversed. The number is

   (a) 63 (b) 35 (c) 36 (d) 60

4. Product of the digits of a two-digit number is 20. If we add 9 to the number, the digits get reversed. Then the original two-digit number is

   (a) 54 (b) 45 (c) 20 (d) 63

5. The demand and supply equations for a certain commodity are 4q + 7p = 17 and $p=\frac{q}{3}+\frac{7}{4}$ respectively where p is the market price and q is the quantity then the equilibrium price and quantity are **(ICAI)**

   (a) $2, \frac{3}{4}$ (b) $3, \frac{3}{4}$ (c) $5, \frac{3}{4}$ (d) None of these

Answer Key

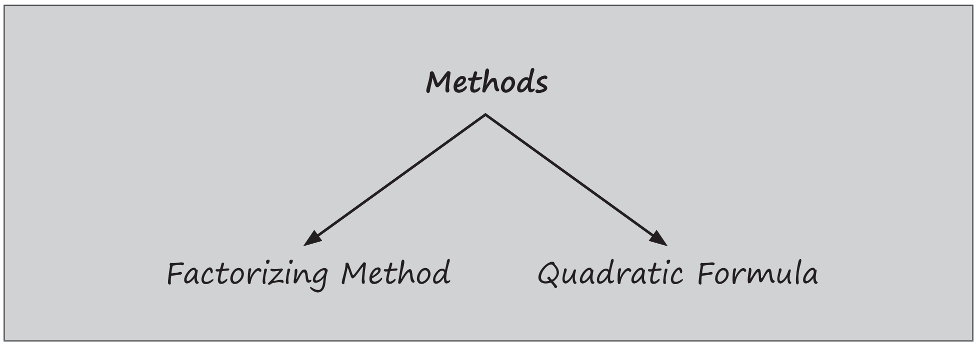
1. (d) **2.** (b) **3.** (c) **4.** (b) **5.** (a)

QUADRATIC EQUATION

- An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree.
- When $b = 0$, the equation is called a pure quadratic equation i.e., $ax^2 + c = 0$
- When $b \neq 0$, the equation is called an affected Quadratic.

  E.g.: (i) $3x^2 + 2x + 1 = 0$ (ii) $x^2 - x = 0$

How to find out the roots of a quadratic equation: $ax^2 + bx + c = 0$ $(a \neq 0)$



Example 22. Solve $x^2 - 8x + 16 = 0$ and find the roots.

(a) 4, –4 (b) 4, 0 (c) 4, 4 (d) 1, 4

Sol. (c) Given, $x^2 - 8x + 16 = 0$

Method (1): By factorization method:

$\Rightarrow x^2 - 8x + 16 = 0$

$\Rightarrow x^2 - 4x - 4x + 16 = 0$

$\Rightarrow x(x - 4) - 4(x - 4) = 0$

$\Rightarrow (x - 4)(x - 4) = 0$

$\Rightarrow x - 4 = 0, x - 4 = 0$

$\Rightarrow x = 4, 4$

Method (2): By Quadratic Formula:

On comparing given equation with $ax^2 + bx + c = 0$, we get

$a = 1, b = -8, c = 16$

By Quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 * 1 * 16}}{2}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{64 - 64}}{2}$$

$$\Rightarrow x = \frac{8}{2} = 4$$

$\Rightarrow x = 4, 4$

Therefore, the roots are 4, 4.

Hence, the correct option is (c).

Example 23. The positive root of the equation $2x^2 - 5x = 3$ is

(a) 5 (b) 3 (c) 4 (d) 1

Sol. (b) Given equation: $2x^2 - 5x = 3$

$\Rightarrow 2x^2 - 5x - 3 = 0$

$\Rightarrow 2x^2 - 6x + x - 3 = 0$

$\Rightarrow 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow (x - 3)(2x + 1) = 0$

$\Rightarrow (x - 3) = 0$ or $(2x + 1) = 0$

$\Rightarrow x = 3 \text{ or } \frac{-1}{2}$

Here, $\frac{-1}{2}$ is not a positive.

$\Rightarrow x = 3$

Therefore, the required positive root is 3.

Hence, option (b) is correct i.e., 3.

SUM AND PRODUCT OF ROOTS

Let α and β be the roots of equation $ax^2 + bx + c = 0$.

Sum of roots: $\alpha + \beta = -\frac{b}{a}$

Product of roots: $\alpha\beta = \frac{c}{a}$

Example 24. If the roots of the quadratic equation $2x^2 + 5x - 3 = 0$ are α and β, what is the value of $|\alpha - \beta|$?

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{7}{2}$ (d) $\frac{5}{2}$

Sol. (c) Let α and β are roots of the quadratic equation $2x^2 + 5x - 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$

$a = 2; b = 5; c = -3$

Sum of roots $(\alpha + \beta) = \frac{-b}{a} = \frac{-5}{2}$

$\Rightarrow (\alpha + \beta) = \frac{-5}{2}$ ...(i)

And products of roots $(\alpha\beta) = \frac{c}{a} = \frac{-3}{2}$

$\Rightarrow (\alpha\beta) = \frac{-3}{2}$ ...(ii)

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$\Rightarrow (\alpha - \beta)^2 = \left(\frac{-5}{2}\right)^2 - 4\left(\frac{-3}{2}\right)$$

$$\Rightarrow (\alpha - \beta)^2 = \frac{25}{4} + 6 = \frac{25+24}{4} = \frac{49}{4}$$

$$\Rightarrow |\alpha - \beta| = \frac{7}{2}$$

Therefore, difference of roots of given equation is $\frac{7}{2}$.

Hence, the correct answer is option (c) i.e., $\frac{7}{2}$.

FOR QUADRATIC EQUATION

$ax^2 + bx + c = 0$

Nature of Roots for $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. If $b^2 - 4ac = 0$, then the roots are real and equal.
2. If $b^2 - 4ac > 0$, then the roots are real and unequal (or distinct).
3. If $b^2 - 4ac < 0$, then the roots are imaginary.
4. If $b^2 - 4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct).
5. If $b^2 - 4ac > 0$, but not a perfect square, the roots are real, irrational and unequal.

Example 25. If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal, then the value of m is

(a) -3 (b) -1 (c) 1 (d) -2

Sol. (d) We know that,

If roots of equation $ax^2 + bx + c = 0$ are equal, then $b^2 - 4ac = 0$

Given, the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal.

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow 8^2 - 4(2)(-m^3) = 0$

$\Rightarrow 64 + 8m^3 = 0$

$\Rightarrow m^3 = -\frac{64}{8} = -8$

$\Rightarrow m = -2$

Therefore, the value of m is -2.

Hence, the correct option is (d).

Example 26. For what value of k the given equation has real roots: $x^2 - 10x + k = 0$

(a) $k \leq 25$ (b) $k \geq 25$ (c) $k \leq 100$ (d) None of these

Sol. (a) Comparing the equation $x^2 - 10x + k = 0$ with $ax^2 + bx + c = 0$, we get

$a = 1, b = -10$ and $c = k$

Since, the roots are real thus

$b^2 - 4ac \geq 0$

$\Rightarrow (10)^2 - 4(1)(k) \geq 0$

$\Rightarrow 100 - 4k \geq 0$

$\Rightarrow 100 \geq 4k$

$\Rightarrow \frac{100}{4} \geq k$

$\Rightarrow 25 \geq k$ or $k \leq 25$

Hence, the correct option is (a) i.e., $k \leq 25$.

Example 27. Five times of a positive whole number is 3 less than twice the square of the number. The number is

(a) 3 (b) 4 (c) −3 (d) 2

Sol. (a) Let the positive number be x.

According to the question, we have

$2x^2 - 5x = 3$

$\Rightarrow 2x^2 - 5x - 3 = 0$

$\Rightarrow 2x^2 - 6x + x - 3 = 0$

$\Rightarrow 2x(x - 3) + (x - 3) = 0$

$\Rightarrow (x - 3)(2x + 1) = 0$

$\Rightarrow x = 3, -\frac{1}{2}$

Since, the number is positive.

Thus, $x = 3$

Hence, the correct option is (a).

Example 28. If α and β are the roots of the equation $2x^2 - 7x + 3 = 0$, then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is equal to

(a) $\frac{15}{4}$ (b) $\frac{37}{6}$ (c) $\frac{28}{5}$ (d) None of these

Sol. (b) Given, $2x^2 - 7x + 3 = 0$

We know that,

If the roots of equation are α and β, then

Sum of roots: $(\alpha + \beta) = \frac{-b}{a} = \frac{-(-7)}{2} = \frac{7}{2}$

Products of roots: $(\alpha\beta) = \frac{3}{2}$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{7}{2}\right)^2 - 2\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)} = \frac{(49-12)\times 2}{4 \times 3} = \frac{37}{6}$$

Hence, the correct option is (b) i.e. $\frac{37}{6}$.

Example 29. The sides of an equilateral triangle are shortened by 12 units, 13 units and 14 units respectively and a right–angle triangle is formed. The side of the equilateral triangle is

(a) 17 units (b) 16 units (c) 15 units (d) 18 units

Sol. (a) Let 'x' be the length of each side of the equilateral triangle.

Then, the sides of the right-angled triangle are

$(x - 12)$, $(x - 13)$ and $(x - 14)$

In the above three sides, the side represented by $(x - 12)$ is hypotenuse (Because that is the longest side).

Using Pythagorean theorem, we have:

$(x - 12)^2 = (x - 13)^2 + (x - 14)^2$

$\Rightarrow x^2 - 24x + 144 = x^2 - 26x + 169 + x^2 - 28x + 196$

$\Rightarrow x^2 - 30x + 221 = 0$

$\Rightarrow (x - 13)(x - 17) = 0$

$\Rightarrow x = 13$ or $x = 17$.

Here, $x = 13$ cannot be accepted since in that case sides would be $(12 - 13, 13 - 13, 14 - 13)$ i.e., $-1, 0, 1$ which is not possible since the side cannot be negative or zero.

Therefore, the side of the equilateral triangle is 17 units.

Hence, the correct option is (a).

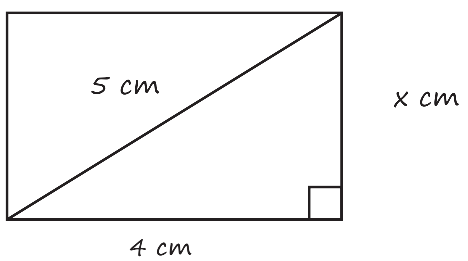
Example 30. The diagonal of a rectangle is 5 cm and one of its sides is 4 cm. Its area is ______ .

(a) 42 sq. cm (b) 12 sq. cm (c) 10 sq. cm (d) 5 sq. cm

Sol. (b) Given, Diagonal of a rectangle = 5 cm

One side of rectangle = 4 cm

Let's assume the other side of the rectangle be 'x' cm



Using Pythagoras theorem,

$4^2 + x^2 = 5^2$

$\Rightarrow 16 + x^2 = 25$

$\Rightarrow x^2 = 25 - 16$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \sqrt{9} \Rightarrow x = 3$ ($\because$ Side cannot be negative)

So, the length and breadth of the rectangle are 3 cm and 4 cm respectively. We know that,

Area = length × breadth

Area = 3 cm × 4 cm

Area = 12 sq.cm

Hence, the correct option is (b).

Example 31. A solution of the quadratic equation $(a + b - 2c)x^2 + (2a - b - c)x + (c + a - 2b) = 0$ is

(a) $x = 1$ (b) $x = -1$ (c) $x = 2$ (d) $x = -2$

Sol. (b) Given, $(a + b - 2c)x^2 + (2a - b - c)x + (c + a - 2b) = 0$

Go by choices:

For option (b): $x = -1$

Substituting $x = -1$ in the given equation, we get

LHS: $(a + b - 2c) - (2a - b - c) + (c + a - 2b)$

$= a - 2a + a - b + b - 2b - 2c + c + c = 0 =$ RHS

Therefore, -1 is the root of the given equation.

Hence, the correct option is (b).

Example 32. The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x, form an equation in x and hence find the numbers.

(a) (7, 10) (b) (4, 4) (c) (3, 5) (d) (2, 6)

Sol. (c) **Go be options:**

For option (a): (7, 10)

Here, sum of roots = $7 + 10 = 17 \neq 18$

For option (b): (4, 4)

Here, sum of roots = $4 + 4 = 8$

Sum of their squares = $4^2 + 4^2 = 16 + 16 = 32 \neq 34$

For option (c): (3, 5)

Here, sum of roots = $3 + 5 = 8$

Sum of their squares = $3^2 + 5^2 = 9 + 25 = 34$

Therefore, both conditions are satisfied.

Hence, the numbers are 3 and 5.

Detailed Method:

Let x and y be the two numbers.

Then, $x + y = 8$

$\Rightarrow y = 8 - x$

Also, $x^2 + y^2 = 34$

$\Rightarrow x^2 + (8 - x)^2 = 34$

$\Rightarrow x^2 + 8^2 + x^2 - 16x = 34$

$\Rightarrow 2x^2 + 64 - 16x = 34$

$\Rightarrow 2x^2 - 16x + 30 = 0$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 3x - 5x + 15 = 0$

$\Rightarrow x(x - 3) - 5(x - 3) = 0$

$\Rightarrow (x - 5)(x - 3) = 0$

$\Rightarrow x = 5$ or $x = 3$

Also, $y = 8 - x$

$y = 8 - 5 = 3$ or $y = 8 - 3 = 5$

Therefore, the two numbers are 3 and 5.

Hence, the correct option is (c).

Example 33. If $L + M + N = 0$ and L, M, N are rationals then the roots of the equation: $(M + N - L)x^2 + (N + L - M)x + (L + M - N) = 0$ are

(a) real and irrational (b) real and rational

(c) imaginary and equal (d) real and equal

Sol. (b) It is given that $L + M + N = 0$

$\Rightarrow M + N = -L$ ...(i)

$\Rightarrow N + L = -M$

$\Rightarrow L + M = -N$

Then equation will become

$\Rightarrow (-L - L)x^2 + (-M - M)x + (-N - N) = 0$

$\Rightarrow -2Lx^2 - 2Mx - 2N = 0$

Now, divide both sides by -2, we get

$\Rightarrow Lx^2 + Mx + N = 0$

Now compare with quadratic $ax^2 + bx + c = 0$

We get $a = L$, $b = M$, $c = N$

Now find the discriminant $D = b^2 - 4ac$

$\Rightarrow D = M^2 - 4LN$

$\Rightarrow D = [-(N + L)]^2 - 4LN$

$\Rightarrow D = N^2 + L^2 + 2LN - 4LN$

$\Rightarrow D = N^2 + L^2 - 2LN$

$\Rightarrow D = (N - L)^2$

Here, we can see that D is a perfect square.

So, the roots will be real and rational.

Hence, option (b) is correct, i.e real and rational.

Note:

(a) Irrational roots occur in conjugate pairs that is if $(m + \sqrt{n})$ is a root then $(m - \sqrt{n})$ is the other root of the same equation.

(b) If one root is reciprocal to the other root then their product is 1 and so $c/a = 1$ i.e. $c = a$

(c) If one root is equal to another root but opposite in sign then their sum = 0 and so $b/a = 0$. i.e. $b = 0$.

PRACTICE QUESTIONS (PART D)

1. The roots of the equation $x^2 + (2p - 1)x + p^2 = 0$ are real if.

(a) $p \geq 1$ (b) $p \leq 4$ (c) $p \geq \frac{1}{4}$ (d) $p \leq \frac{1}{4}$

2. Find the value of k for which the quadratic equation $2x^2 - kx + 3 = 0$ have two equal roots.

(a) ± 1 (b) ± 2 (c) $\pm 2\sqrt{6}$ (d) $2\sqrt{6}$

3. If p and q are the roots of $x^2 + 2x + 1 = 0$ then the values of $p^3 + q^3$ becomes

(a) 2 (b) -2 (c) 4 (d) -4

4. What will be the value of k, if the roots of the equation $(k - 4)x^2 - 2kx + (k + 5) = 0$ are equal? **(Dec, 2022)**

(a) 18 (b) 20 (c) 19 (d) 21

5. If the roots of the equation $x^2 - 8x + m = 0$ exceeds the other by 4, then the value of m is **(ICAI)**

(a) $m = 10$ (b) $m = 11$ (c) $m = 9$ (d) $m = 12$

6. If the sum of two numbers is 11 and the sum of their squares be 85, then the numbers are

(a) 5 and 6 (b) 4 and 7 (c) 2 and 9 (d) None of these

7. If α and β are the roots of $x^2 = x + 1$ then value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is

(a) $2\sqrt{5}$ (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) $-2\sqrt{5}$

8. Solve the equation if $2^{2x+3} - 3^2 . 2^x + 1 = 0$, then the values of x are

(a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, -3

9. The values of $2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\cdots}}}}$

(a) $1 \pm \sqrt{2}$ (b) $2 \pm \sqrt{5}$ (c) $1 \pm \sqrt{2}$ (d) None of these

10. If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$ the equation having roots as p and q is

(a) $x^2 - 19x + 3 = 0$ (b) $3x^2 - 19x - 3 = 0$

(c) $3x^2 - 19x + 3 = 0$ (d) None of these

11. The solutions of the equation $\frac{6x}{x+1} + \frac{6(x+1)}{x} = 13$ are

(a) (2, 3) (b) (3, −2) (c) (−2, −3) (d) (2, −3)

12. The satisfying values of x for the equation, $\frac{1}{x+p+q} = \frac{1}{x} + \frac{1}{p} + \frac{1}{q}$ are

(a) (p, q) (b) (−p, −q) (c) (p, −p) (d) (−p, q)

13. The equation $\left(\frac{l-m}{2}\right)x^2 - \left(\frac{l+m}{2}\right)x + m = 0$ has got two values of x to satisfy the equation given as

(a) $\left(1, \frac{2m}{l-m}\right)$ (b) $\left(1, \frac{m}{l-m}\right)$ (c) $\left(1, \frac{2l}{l-m}\right)$ (d) $\left(1, \frac{l}{l-m}\right)$

14. There are two consecutive numbers such that the difference of their reciprocals is $\frac{1}{240}$.
The numbers are

(a) (15, 16) (b) (17, 18) (c) (13, 14) (d) (12, 13)

15. A distributor of apple Juice has 5000 bottles in the store that it wishes to distribute in a month. From experience it is known that demand D (in number of bottles) is given by $D = -2000p^2 + 2000p + 17000$.
The price per bottle that will result zero inventory is

(a) ₹3 (b) ₹5 (c) ₹2 (d) None of these

16. The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are

(a) $3\sqrt{2}, 2\sqrt{3}$ (b) $5\sqrt{2}, 3\sqrt{5}$ (c) $2\sqrt{2}, 5\sqrt{2}$ (d) None of these

Answer Key

1. (d)	2. (c)	3. (b)	4. (b)	5. (d)	6. (c)	7. (d)	8. (d)	9. (c)	10. (c)
11. (d)	12. (b)	13. (a)	14. (a)	15. (a)	16. (c)				

CUBIC EQUATION

An equation of 3rd degree or where the maximum power of x is 3 is called as cubic equation. Cubic Equation will look like: $ax^3 + bx^2 + cx + d = 0$ where $a \neq 0$ will be called as cubic equation.

E.g.: $x^3 - 6x^2 + 11x - 6 = 0$

For cubic equation, $ax^3 + bx^2 + c + d = 0$

Sum of roots $= -\frac{b}{a}$

Sum of roots taken two at a time $= \frac{c}{a}$

Product of roots $= -\frac{d}{a}$

Example 34. The roots of $x^3 + x^2 - x - 1 = 0$ are

(a) (−1, −1, 1) (b) (1, 1, −1) (c) (−1, −1, −1) (d) (1, 1, 1)

Sol. (a) Given, $x^3 + x^2 - x - 1 = 0$

$\Rightarrow x^2(x + 1) - x - 1 = 0$

$\Rightarrow x^2(x + 1) - (x + 1) = 0$

$\Rightarrow (x^2 - 1)(x + 1+ = 0$ $\{a^2 - b^2 = (a - b)(a + b)\}$

$\Rightarrow (x + 1)(x - 1)(x + 1) = 0$

$\Rightarrow x + 1 = 0, x - 1 = 0, x + 1 = 0$

$\Rightarrow x = -1, x = 1, x = -1$

Therefore, the roots of the equation are −1, −1, 1.

Hence, the correct option is (a).

Example 35. For what value of 'k', 1 is a root of the following cubic equation $kx^3 - 4x^2 + 3x + 2 = 0$?

(a) 1 (b) −1 (c) 2 (d) $\frac{1}{3}$

Sol. (b) Given cubic equation: $kx^3 - 4x^2 + 3x + 2 = 0$

Since, 1 is a root of the equation, thus it will satisfy the given equation.

$\Rightarrow k(1)^3 - 4(1)^2 + 3(1) + 2 = 0$

$\Rightarrow k - 4 + 3 + 2 = 0$

$\Rightarrow k + 1 = 0$

$\Rightarrow k = -1$

Hence, the correct answer is option (b) i.e., −1.

Example 36. Solve $x^3 - 6x^2 + 5x + 12 = 0$

(a) 1, 3, 4 (b) −1, 3, 4 (c) 1, 6, 2 (d) 1, −6, −2

Sol. (b) Given equation: $x^3 - 6x^2 + 5x + 12 = 0$

We know that, for cubic equation $ax^3 + bx^2 + c + d = 0$

Sum of roots $= -\frac{b}{a}$

Thus, for $x^3 - 6x^2 + 5x + 12 = 0$

Sum of roots $= -\frac{b}{a} = -\frac{-6}{1} = 6$

For option (a): 1, 3, 4

Sum of roots = 1 + 3 + 4 = 7 which is not true

For option (b): −1, 3, 4

Sum of roots = −1 + 3 + 4 = 6 which is true

Also, for other options, it does not hold.

Therefore, x = −1, 3, 4

Hence, the correct option is (b) i.e., −1, 3, 4.

Example 37. If $4x^3 + 8x^2 - x - 2 = 0$, then the value of $(2x + 3)$ is given by

(a) 4, −1, 2 (b) −4, 2, 1 (c) 2, −4, −1 (d) None of these

Sol. (a) Given cubic equation: $4x^3 + 8x^2 - x - 2 = 0$

$\Rightarrow 4x^2(x + 2) - 1(x + 2) = 0$

$\Rightarrow (x + 2)(4x^2 - 1) = 0$

$\Rightarrow (x + 2)[(2x)^2 - 1^2) = 0$

$\Rightarrow (x + 2)(2x + 1)(2x - 1) = 0$ $[\because a^2 - b^2 = (a - b)(a + b)]$

$\Rightarrow (x + 2) = 0$ or $2x + 1 = 0$ or $2x - 1 = 0$

$\Rightarrow x = -2$ or $2x = -1$ or $2x = 1$

$\Rightarrow x = -2, \frac{-1}{2}, \frac{1}{2}$

(i) When $x = -2$, then

$(2x + 3) = 2 \times (-2) + 3 = -4 + 3 = -1$

(ii) When $x = \frac{-1}{2}$, then

$$(2x + 3) = 2 \times \left(\frac{-1}{2}\right) + 3 = -1 + 3 = 2$$

(iii) When $x = \frac{1}{2}$, then

$$(2x + 3) = 2 \times \frac{1}{2} + 3 = 1 + 3 = 4$$

Therefore, the required values of $(2x + 3)$ are −1, 2, 4.

Hence, the correct answer is option (a).

PRACTICE QUESTIONS (PART E)

1. The roots of the cubic equation $x^3 + 7x^2 - 21x - 27 = 0$ are

(a) (−3, −9, −1) (b) (3, −9, −1)

(c) (3, 9, 1) (d) (−3, 9, 1)

2. The roots of equation $y^3 - 4y^2 - 9x + 36 = 0$ are

(a) 1, 3, 4 (b) 3, 3, 4

(c) −3, −3, 4 (d) 3, −3, 4

3. x, x − 4, x + 5 are the factors of the left-hand side of the equation, find the equation.

(a) $x^3 + 2x^2 - x - 2 = 0$ (b) $x^3 + x^2 - 20x = 0$

(c) $x^3 - 3x^2 - 4x + 12 = 0$ (d) $x^3 - 6x^2 + 11x - 6 = 0$

4. The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of the left–hand side of the equation are

(a) $x - 1, x - 2, x - \frac{5}{3}$ (b) $x - 1, x + 1, 3x + 5$

(c) $x + 1, x - 1, 3x - 5$ (d) $x - 1, x + 1, x - 2$

5. The rational root of the equation $2x^3 - x^2 - 4x + 2 = 0$ is

(a) 1 (b) −1 (c) 2 (d) −2

6. If $2x - 5y = -1, x + 2y = 13$, then

(a) $x = 7, y = -3$ (b) $x = -7, y = -3$

(c) $x = -7, y = 3$ (d) $x = 7, y = 3$

7. If $\frac{3}{x+y} + \frac{2}{x-y} = -1$ and $\frac{3}{x+y} + \frac{2}{x-y} = -1$ then (x, y) is

(a) (2, 1) (b) (1, 2) (c) (−1, 2) (d) (−2, 1)

8. The root of given equation $x^2 + 36 - 12x = 0$ is

(a) $x = 6, 6$ (b) $x = -6, 6$ (c) $x = 0, 6$ (d) None of these

9. If $2^{x+y} = 2^{2x-y} = \sqrt{8}$, then the respective values of x and y are ______.

(a) $1, \frac{1}{2}$ (b) $\frac{1}{2}, 1$ (c) $\frac{1}{2}, \frac{1}{2}$ (d) None of these

10. The roots of cubic equation $x^3 - 6x^2 + 11x - 6 = 0$ is

(a) $x = 1, 2, 3$ (b) $x = 1, 2, 7$ (c) $x = 9, 0, -9$ (d) $x = 1, 5, 9$

11. The age of a person is 8 years more than thrice the age of the sum of his two grandsons who were twins. After 8 years his age will be 10 years more than twice the sum of the ages of his grandsons. Then the age of the person when the twins were born is

(a) 86 years (b) 73 years (c) 68 years (d) None of these

12. If $u^{5x} = v^{5y} = w^{5z}$ and $u^2 = vw$, then the value of $xy + xz - 2yz$ will be

(a) 5 (b) 2 (c) 1 (d) 0

13. If the length of a rectangle is 5 cm more than the breadth and if the perimeter of the rectangle is 40 cm, then the length & breadth of the rectangle will be

(a) 7.5 cm, 2.5 cm (b) 10cm, 5cm (c) 2.5 cm, 7.5cm (d) 15.5cm, 10.5cm

14. If α & β are the roots of the equation $x^2 + x + 5 = 0$, then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is equal to

(a) 16/5 (b) 2 (c) 3 (d) 14/5

15. If the sides of an equilateral triangle are shortened by 3 units ,4 units and 5units respectively and a right triangle is formed, then the side of an equilateral triangle is

(a) 6 units (b) 7 units (c) 8 units (d) 10 units

16. If difference between the roots of the equation $x^2 - kx + 8 = 0$ is 4, then the value of k is

(a) 0 (b) +4 (c) $\pm 8\sqrt{3}$ (d) $\pm 4\sqrt{3}$

17. The roots of the cubic equation $x^3 - 7x + 6 = 0$ are

(a) 1, 2 and 3 (b) 1, −2 and 3 (c) 1, 2 and −3 (d) 1, −2 and −3

18. If α and β be the roots of the quadratic $2x^2 - 4x = 1$, then value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is ______.

(a) −11 (b) 22 (c) −22 (d) 11

19. The number of students in each section of a school is 36. After admitting 12 new students, four new sections were started. If the total number of students in each section now is 30, then the number of sections initially were

(a) 6 (b) 10 (c) 14 (d) 18

20. The equations $x + 5y = 33$; $\frac{x + y}{x - y} = \frac{13}{5}$ has the solution (x, y) as

(a) (4, 8) (b) (8, 5) (c) (4, 16) (d) (16, 4)

21. A number consists of two digits such that the digit in one's place is thrice the digit at ten's place. If 36 be added then the digits are reversed. Find the number **(June 2019)**

(a) 62 (b) 26 (c) 39 (d) None

22. The value of p for which the difference between the root of equation $x^2 + px + 8 = 0$ is 2 is **(June 2019)**

(a) +2 (b) +4 (c) +6 (d) +8

23. If the quadratic equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root then $p + q = ?$ **(Jan. 2021)**

(a) 0 (b) 1 (c) −1 (d) 2

24. If a and b are the roots of the equation $2x^2 + 5x + k = 0$, and $4(a^2 + b^2 + ab) = 23$, then which of the following is true? **[July 2021]**

(a) $k^2 - 3k + 2 = 0$ (b) $k^2 - 2k - 2 = 0$

(c) $k^2 - 2k - 3 = 0$ (d) $k^2 + 3k - 2 = 0$

25. The cost of 2 oranges and 3 apples is ₹ 28. If the cost of an apple is doubled then the cost of 3 oranges and 5 apples is ₹75. The original cost of 7 oranges and 4 apples (in ₹) is **(July 2021)**

(a) 59 (b) 47 (c) 71 (d) 63

26. The value of 'k' is ... If 2 is a root of the following cubic equation: $x^3 - (k + 1)x + k = 0$

(a) 2 (b) 6 (c) 1 (d) 4 **(July 2021)**

27. If the quadratic equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root then $p + q = ?$ **(Jan. 2021)**

(a) 0 (b) 1 (c) −1 (d) 2

Answer Key

1. (b)	2. (d)	3. (b)	4. (b)	5. (a)	6. (d)	7. (b)	8. (a)	9. (c)	10. (a)
11. (b)	12. (d)	13. (c)	14. (d)	15. (c)	16. (d)	17. (c)	18. (c)	19. (d)	20. (b)
21. (b)	22. (c)	23. (c)	24. (a)	25. (a)	26. (b)	27. (c)			

SUMMARY

- A simple equation in one variable x is in the form $ax + b = 0$, where a, b are known constants and $a \neq 0$.
- The general form of a linear equations in two variables, x and y is $ax + by + c = 0$, where a, b are non-zero coefficients and c is a constant.
- Two such equations $a_1x + b_1y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ form a pair of simultaneous equations in x and y.
- A value for each unknown which satisfies simultaneously both the equations will give the roots of the equations.
- **Elimination Method**: In this method two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknowns and then solving for the other unknown.
- **Cross Multiplication Method**: For two equations:

  $a_1x + b_1y + c_1 = 0$

  $a_2x + b_2y + c_2 = 0$

  The solution of the equation are:

  $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

QUADRATIC EQUATION

- An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree.
- When $b = 0$, the equation is called a pure quadratic equation; and when $b \neq 0$ the equation is called an affected quadratic.

  The value of x that satisfies the given quadratic equation is called as the roots of the equation.

  The roots of a quadratic equation $ax^2 + bx + c = 0$ are

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Sum and Product of the roots** of quadratic equation: $ax^2 + bx + c = 0$

  Sum of roots $= -\frac{b}{a} = -\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

  Product of the roots $= \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

- To construct a quadratic equation for the equation $ax^2 + bx + c = 0$ we have

  $x^2 +$ (Sum of the roots)x + Product of the roots = 0

  Nature of the roots: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

  (i) If $b^2 - 4ac = 0$, then the roots are real and equal;

(ii) If $b^2 - 4ac > 0$, then the roots are real and unequal (or distinct);

(iii) If $b^2 - 4ac < 0$, then the roots are imaginary;

(iv) If $b^2 - 4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct);

(v) If $b^2 - 4ac > 0$, but not a perfect square the rots are real, irrational and unequal.

Cubic Equation: An equation of 3rd degree is called as a cubic equation

i.e., $ax^3 + bx^2 + cx + d = 0$

For cubic equation, $ax^3 + bx^2 + cx + d = 0$

Sum of roots $= -\frac{b}{a}$

Sum of roots taken two at a time $= \frac{c}{a}$

Product of roots $= -\frac{d}{a}$

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