Theoritical distribution

Probability distribution for discrete Random variable

Binomial distribution
poisson distribution

probability distribution for continous random variable - Normal distribution

Binomial distribution

Binomial distribution is used to calculate probability of getting x success in n trials. under following conditions.

1) Number of trials are finite
2) Results of each trials are divided into only two categories i.e sucess and failure.
3) probability of sucess in each trial is constant.

Note:- If success chance changes in every trial then you cannot used this method.

4) trial are independent

probability mass function of bienomial distribution $P(X=x) = P(x) = {}^nC_x \cdot p^x \cdot q^{n-x}$

Where n = number of trials
x = number of success = 0, 1, 2 ... n
p = Probability of success
q = Probability of failure

Note $p = q = \frac{1}{2}$ → symmetric

$p > \frac{1}{2}$ i.e $p > q$ → Negative skewness
$p < \frac{1}{2}$ i.e $p < q$ → positive skewness

probability distribution

Poisson distribution is a discrete probability distribution. it is used to find the probability of x successes in following situation.

1) As a limiting case of bionomical distribution when $n \to \infty$ (Too large) and $p \to 0$

2) When n and p are not known but mean (average number of success is known)

Probability mass function of poission distribution

$$\therefore P(x) = \frac{e^{-m} \cdot m^{x}}{x!} = \frac{e^{-\lambda} (\lambda)^{x}}{x!}$$

Where $e = 2.7183$, $m = \lambda$ = mean
x = number of success = 0, 1, 2 ... ∞

Note 1) In poission distribution

$$P(0) = \frac{e^{-m} \cdot m^0}{0!} = e^{-m}$$

2) If m a poisson distribution

$P(x=k) = P(x=k+1) \quad \therefore m = k+1$

$P(0) = P(1) \quad m=1$

$P(1) = P(2) \quad m=2$

$P(2) = P(3) \quad m=3$

Normal distribution and parent distribution

probability density function of normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (-\infty < z < \infty)$$

x = Normal variate

σ = S.D

μ = mean

$\pi = 3.1416$.

$e = 2.7183$.

standard normal variable

$$z = \frac{x-\mu}{\sigma}$$

probability density function of standard Normal variable.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \quad (\infty < z < \infty)$$

Normal distribution

Interval	Area	Percentage
$\mu \pm 2\sigma$	0.6826	68.26%
$\mu \pm \sigma$	0.9545	95.45%
$\mu \pm 3\sigma$	0.9973	99.73%
$\mu \pm 1.96\sigma$	0.95	95%
$\mu \pm 2.575\sigma$	0.99	99%

Standard normal distribution

1) mean = 0, variance = 1, SD = 1
2) $Q_1 = 0 - 0.675(1) = -0.675$
   $Q_3 = 0 + 0.675(1) = 0.675$
3) $QD = 0.675$
4) $MD = 0.8$
5) Point of inflexion are -1 & 1

z	Area
-1 to 1	0.6826
-2 to 2	0.9545
-3 to 3	0.9973
-1.96 to 1.96	0.95
-2.575 to 2.575	0.99

Theoretical probability distribution

- Discrete probability distribution
  - Binomial distribution
  - Poisson distribution
- Continuous probability distribution
  - Normal distribution

Random variable	probability function
Discrete	Probability mass function
Continuous	probability density function.

Binomial distribution

It is derived from a particular type of random experiment known as Bernoulli process named after the famous mathematician

1) each trial is associated with two mutually exclusive and exhaustive outcomes one is success and other is 'failure'

2) The trial are independent

3) The probability of success (p) and failure ($q=1-p$) remain unchanged throughout the process.

4) The number of trials is a finite positive integer.

Binomial distribution : bi-parametric discrete probability distribution

A discrete random variable x is defined to follow binomial distribution with parameters n and p

$$x \sim B(\underset{\text{trials}}{n}, \underset{\text{success}}{p})$$

Probability Mass function $= f(x) = P(X=x) = {}^nC_x \cdot p^x \cdot q^{n-x}$

for $x = 0, 1, 2, \ldots n$

Mean $= \mu = np$

Variance

The variance of the binomial distribution is given by $\sigma^2 = npq$

Variance of a binomial variable is always less than its mean $\boxed{\sigma^2 < \mu}$

Variance of x attains its maximum value at $p = q = 0.5$ and this maximum value is $n/4$

Mode

$(n+1)p$

Integer

$\mu_0 = (n+1)p$

$\mu_0 = (n+1)p - 1$

Non-integer

Non-integer

μ_0 = the largest integer contained in $(n+1)p$

uni-Modal.

Additive property

x and y are two independent variables such that.

$x \sim B(n_1, p)$ and $y \sim B(n_2, p)$

Then $(x+y) \sim B(n_1+n_2, p)$

Question

1) An a example of bi-parametric discrete probability.

binomial distribution

2) In a binomial distribution $B(n, p)$, $n=4$

$P(X=2) = 3P(X=3)$ find p

$$P(X=x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

$$P(X=2) = 3P(x=3)$$

$${}^{4}C_{2} \cdot p^{2} \cdot q^{2} = 3 \cdot {}^{4}C_{3} \cdot p^{3} \cdot q^{1}$$

$$6 \cdot (1-p) = 3 \cdot 4 \cdot p$$

$$1-p = \frac{3 \times 4 \times p}{6}$$

$$1-p = 2p$$

$$1 = 3p$$

$$\boxed{\frac{1}{3} = p}$$

3) Mean and variance of a binomial variance are 4 and 4/3 respectively then $(x \geq 1)$ will be

$P(X=x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$	
	$npq = \frac{4}{3}$
$P(x \geq 1) = 1 - P(x < 1)$	
$= 1 - P(0)$	$(np = 4)$
$= 1 - {}^{n}C_{0} \cdot p^{0} \cdot q^{n}$	$4q = \frac{4}{3}$
$= 1 - {}^{6}C_{0} \cdot \left(\frac{2}{3}\right)^{0} \cdot \left(\frac{1}{3}\right)^{6}$	
	$q = \frac{1}{3}$ $\quad p = \frac{2}{3}$
$= 1 - 1 \times 1 \times \frac{1}{729}$	
	$np = 4$
$= \frac{728}{729}$	$n\left(\frac{2}{3}\right) = 4$
	$n = 6$

4) Find mode when $n=15$ and $p=1/4$ in Binomial distribution
(IMP)

$(n+1)p$ (Integer) $\rightarrow$ Bimode

$(15+1)\frac{1}{4} = 4$

$\boxed{4 \mid 3}$

Ans - 4, 3

5) The mean of the Binomial distribution $B(4, 1/3)$ is equal to

$B(n, p)$ Mean $= np$

$= 4 \times \frac{1}{3} = \frac{4}{3}$

6) If mean and variance are 5 and 3 respectively then relation between p and q is

$p < q$ variance is smaller than mean

7) The variance of a binomial distribution with parameters n and p is

$nq(1-q)$, npq

$p = \boxed{1-q}$

8) If x is a binomial variate with $p = 1/3$ for the experiment of 90 trials, then the standard deviation is equal

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$n = 90 \quad p = 1/3 \quad q = 2/3$

$V = npq$

$= 90 \times \frac{1}{3} \times \frac{2}{3}$

$= \frac{30}{5 \times 2 \times 2}$

$SD = 2\sqrt{5}$

9) The standard deviation of Binomial distribution is

$\sqrt{npq}$

10) If 'x' is a binomial variable with parameter 15 and 1/3 then the value of the mode of the distribution

$(n+1)\ p$

$(15+1)\ \frac{1}{3} = 5.33$

Mode - 5

poission distribution / uni-parametric (single) descrete probability distribution

poisson distribution is applied when the total number of event is pretty large but the probability of occurrence is very small.

A discrete random variable x that follows poisson distribution denoted as $x \sim P(m)$ → mean

probability Mass function $f(x) = P(X = x) = \frac{e^{-m} m^x}{x!}$

for $x = 0, 1, 2, \dots \infty$

where $e = 2.71823$

$m = np$

Mean - The mean of poisson distribution is given by $\mu = m$

variance - The variance of poisson distribution distribution is given by $\sigma^2 = m$

standard deviation - $\sqrt{m}$

Mode (M)

Integer	Non-Integer
$\mu_0 = m$	μ_0 = the largest integer contained in m
$\mu_0 = m-1$	
Bi-Modal	uni-modal

Additive property

If x and y are two independent variables such that

$x \sim P(m_1)$ and $y \sim P(m_2)$

$x + y \sim P(m_1 + m_2)$

poisson model

1. The probability of finding success in a very small time interval $(t, t+dt)$ is kt, Where $k (>0)$ is a constant.
2. The probability of having more than one success in this time interval is very low
3. The probability of having success in this time interval is independent of t as well as earlier successes.

1) Which one of the following is an uniparameter

poisson

2) If x is a poisson variable and $P(x=1) = P(x=2)$ then $P(x=4$

$$P(X=x) = \frac{e^{-m} m^x}{x!}$$

$$P(X=1) = P(X=2)$$

$$\frac{\cancel{e^{-m}} \cdot m^{1}}{1!} = \frac{\cancel{e^{-m}} \cdot m^{2}}{2!}$$

$$2 = m$$

$P(x=4)$

~~$P(X=4)$~~ $\frac{e^{-m} \cdot m^{x}}{x!} = \frac{e^{-2} \cdot 2^{4}}{4!}$

$\boxed{2 = M}$

$= \frac{e^{-2} \cdot 16}{24}$ (4×4 / $\cancel{24}$ 6)

$= \frac{e^{-2} \cdot 4 \cdot 2}{6 \; 3}$

$= \frac{2}{3} \cdot e^{-2}$

3) For a poisson distribution

:→ mean and variance are equal.

4) If parameters of a binomial distribution are n and p then, this distribution tends to a poission distribution when

$n \to 0,\; p \to 0,\; np = \lambda$

5) If the parameter of poisson distribution is m and (Mean + S.D) = 6/25 then find M

$$\text{mean} + S.D = 6/25$$

$$m + \sqrt{m} = 6/25$$

ans → 1/25

$$\frac{1}{25} + \frac{1}{5} = \boxed{\frac{6}{25}}$$

6) If a poisson distribution is such that $P(x=2) = P(x=3)$ then the variance of the distribution is

$$P(x=x) = \frac{e^{-m} \cdot m^x}{x!}$$

$$P(x=2) = P(x=3)$$

$$\frac{\cancel{e^{-m}} \cdot m^2}{\cancel{2}} = \frac{\cancel{e^{-m}} \cdot m^3}{\cancel{6}\ 3}$$

variance $m = 3$

7) A renowned hospital usually admits 200 patients everyday. one percent patients, on an average, require special room facilities. on one particular morning, it was found that only one special room is available. What is the probability that more than 3 patients would require special room facilities.

$n = 200 \quad p = 1/100 = 0.01 \quad q = 0.99$

$m = np$

200×0.01

$= 2$

$$\frac{e^{-m} \cdot m^x}{x!}$$

$$P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - \left[\frac{e^{-2} \cdot 2^0}{0!} + \frac{e^{-2} \cdot 2^1}{1!} + \frac{e^{-2} \cdot 2^2}{2!} + \frac{e^{-2} \cdot 2^3}{3!}\right]$$

$$= 1 - e^{-2}\left[1 + 2 + 2 + \frac{4}{3}\right]$$

$$= 1 - e^{-2}\left[\frac{3 + 6 + \cancel{12}\ 6 + \cancel{4}}{3}\right]$$

$$= 1 - e^{-2}\left[\frac{19}{3}\right]$$

$$= 1 - \frac{19}{3e^2}$$

$$= 1 - \frac{19}{3(2.7183)^2} = 0.1428$$

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Normal distribution (Bi-parametric continuous probability distribution.)

A continuous random variable x is defined to follow normal distribution with parameters μ and σ^2, to be denoted by

$$X \sim N(\mu, \sigma^2)$$

μ ↓ mean

σ^2 ↳ variance

$$\text{probability Density function} = F(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\left(\frac{x-\mu}{2\sigma}\right)^2}$$

$e = 2.7183$
x = random variable
μ = mean of normal random variable x
σ = standard deviation of the given normal distribution

Normal curve

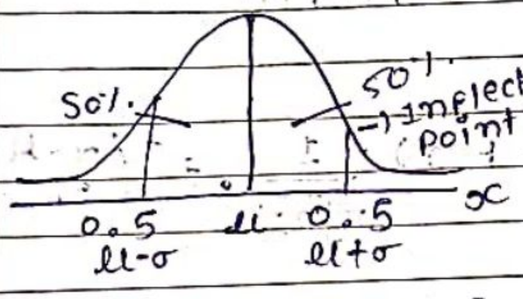
The normal curve is bell-shaped.

The line drawn through $x=\mu$ has divided the normal curve into two parts which are equal in all respect.

Normal distribution is symmetrical about $x=\mu$. As such, its skewness is zero.

The two tails of the normal curve extend indefinitely on both sides of the curve and both the left and right tails never touch the horizontal axis.

The total area of the normal curve or for that any probability curve is taken to be unity i.e one



Normal curve / probability curve.

The area between $-\infty$ to μ = the area between to μ to ∞ = 0.5

The area under this curve gives us the probability.

Normal distribution

Mean = Median = Mode = μ (symmetric dis)

variance = σ^2

standard deviation = σ

Mean deviation = 0.8σ

Quartile deviation = 0.675σ

Quartiles = $Q_1 = \mu - 0.675\sigma$

$Q_3 = \mu + 0.675\sigma$

$\mu-\sigma$ and $\mu+\sigma$

* If x and y are independent normal variables with means and standard deviation μ_1 and μ_2 and σ_1 and σ_2 respectively, then $Z = x + y$ also follows normal distribution with

mean $(\mu_1 + \mu_2)$ and $SD = \sqrt{\sigma_1^2 + \sigma_2^2}$ respectively

Q1) An example of bi-parametric continuous Probability

Normal distribution

2) Skewness of Normal distribution is → zero

3) Normal curve → symmetrical

4) In a normal distribution, variance is 16 then the value of mean deviation is

Mean deviation $= 0.8\sigma$
$= 0.8(4)$
$= 3.2$

5) In a normal distribution Mean, Median, Mode are equal

6) The quartile deviation of a normal distribution with mean 10 and standard deviation 4 is

$QD = 0.675\sigma$
$= 0.675(4) = 2.7$

7) If the points of Inflexion of a normal curve are 40 and 60 respectively, then its mean deviatio is

$$MD = 0.8\sigma$$
$$= 0.8(10)$$
$$= 8$$

$$\mu - \sigma = 40$$
$$\mu + \sigma = 60$$
$$2\mu = 100$$
$$\mu = 50$$

$$\mu + \sigma = 60$$
$$\cancel{\mu} + 50 = 60$$
$$\sigma\ \cancel{\mu} = 10$$

8) What is the mean of x having the following density function.

$$F(x) = \frac{1}{4\sqrt{2\pi}} \cdot e^{-\left(\frac{x-10}{32}\right)^2} \quad -\infty < x < +\infty$$

→ 10

9) If for a normal distribution $Q_1 = 54.52$ and $Q_3 = 78.86$ then the median of the distribution is:

$$Q_2 = \frac{Q_1 + Q_3}{2} = \frac{54.52 + 78.86}{2} = 66.69$$

$$P(\mu - \sigma < x < \mu + \sigma) = 0.6828$$

$$P(\mu - 2\sigma < x < \mu + 2\sigma) = 0.9546$$

$$P(\mu - 3\sigma < x < \mu + 3\sigma) = 0.9973$$

Standard normal distribution

If we take $\mu = 0$ and $\sigma = 1$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad \text{for } -\infty < \infty$$

The random variable z is known as standard normal variate (or variable) or standard normal deviate.

it is given by $z = \frac{x-\mu}{\sigma}$

Important results of standard ~~deviation~~ Normal distribution

Mean = Median = Mode = 0

The standard normal distribution is symmetric about z = 0

variance = 1

standard deviation = 1

point inflexion = -1 and 1

Mean deviation = 0.8

Quartile deviation = 0.675

* If the area of standard normal curve between $z=0$ to $z=1$ is 0.3412. then the value of $\phi(1) = 0.5 + 0.3412 = 0.8412$

Cumulative distribution function:

$$P(Z \le K) = \phi(K)$$

$$P(x < A) = P\left[\frac{x-\mu}{\sigma} < \frac{a-\mu}{\sigma}\right]$$

$$= P(Z = K) \quad \left(K = \frac{a-\mu}{\sigma}\right)$$

$$= \phi(K)$$

Also $P(x \le a) = P(x < a)$ as is continuous

$$\phi(-K) = 1 - \phi(K)$$

$$P(x > b) = 1 - P(x \le b)$$

$$= 1 - \phi(b - \mu/\sigma)$$

$$P(a < x < b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$$