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CA FOUNDATION

MATHEMATICS

Head Office

Shraddha, 4th Floor, Old Nagardas Road,
Near Chinai College, Andheri (E), Mumbai - 400 069.

022 - 2683 66 66

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CAFC – BUSINESS MATHEMATICS

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01 - SURDS & INDICES

Q1

$a^m . a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

01. DEC 2022

$(2a^3b^4)^6 / (4a^3b)^2 \times (a^2b^2) =$

a. $4a^2b^3$ b $4a^6b$ c $4a^{10}b^{10}$
[d] $4a^{10}b^{20}$

$\frac{2^6.a^{18}.b^{24}}{4^2a^6b^2\ a^2b^2} = 4a^{10}.b^{20}$ option d

02. DEC 2023

$\frac{9^n \times 3^5 \times 27^5}{3 \times 81^4} = 27$, n = ?

a. 2 [b] 0 c 3 d 4

$\frac{3^{2n} \times 3^5 \times (3^3)^5}{3 \times (3^4)^4} = 27$

$3^{2n+5+15-17} = 3^3$

$2n+3 = 3$, $n = 0$ option b

03. JUNE 2022

$\sqrt{9^{-8}} \times \sqrt{3^{-5}} = 3^a$

a $\frac{2}{21}$ b $\frac{21}{2}$ [c] $\frac{-21}{2}$ d $\frac{-2}{21}$

$\sqrt{9^{-8}} \times \sqrt{3^{-5}} = 3^a$

$3^{\frac{-8-5}{2}} = 3^a$

$3^{\frac{-16-5}{2}} = 3^a$

$a = {}^{-21}/_{2}$ option c.

04. DEC 2020

$\sqrt{9^{-5}} \times \sqrt{3^{-7}} = \sqrt{3^{-a}}$

a. 13 b. 11 c. 15 [d.] 17

$\sqrt{9^{-5}} \times \sqrt{3^{-7}} = \sqrt{3^{-a}}$

$3^{-5} \times 3^{-7/2} = 3^{-a/2}$

$3^{\frac{-5-7}{2}} = 3^{-a/2}$

$3^{\frac{-10-7}{2}} = 3^{-a/2}$

$3^{-17/2} = 3^{-a/2}$

$a = 17$ option d.

05. DEC 2021

if $\left(\frac{3a}{2b}\right)^{2x-4} = \left(\frac{2a}{3b}\right)^{2x-4}$ for some a&b ,

Find x

a. 8 b 6 c 4 [d] 2

$\left(\frac{3}{2}\right)^{2x-4} \left(\frac{a}{b}\right)^{2x-4} = \left(\frac{2}{3}\right)^{2x-4} \left(\frac{a}{b}\right)^{2x-4}$

$\left(\frac{3}{2}\right)^{2x-4} = \left(\frac{2}{3}\right)^{2x-4}$

$\left(\frac{3}{2}\right)^{2x-4} = \left(\frac{3}{2}\right)^{-(2x-4)}$

Equating the powers ,

$2x-4 = -(2x-4)$

$2x-4 = -2x+4$

$4x = 8$ $x = 2$ option d

06. JUNE 2023 , JUNE 2015 , JUNE 2008

$x = y^a$, $y = z^b$, $z = x^c$

then abc = ?

[a.] 1 b 2 c 3 d 4

$x = y^a$

$x = \left[z^b\right]^a$

$x = \left[x^c\right]^{ab}$

$x = x^{abc}$, abc = 1 option a

07. NOV 2018

$$\frac{2^{m+1} \times 3^{2m-n+3} \times 5^{n+m+4} \times 6^{2n+m}}{6^{2m+n} \times 10^{n+1} \times 15^{m+3}}$$

a. 3^{2m-2n} b. 3^{2n-2m} [c.] 1 d.none

$$\frac{2^{m+1+2n+m} \times 3^{2m-n+3+2n+m} \times 5^{n+m+4}}{2^{2m+n+n+1} \times 3^{2m+n+m+3} \times 5^{n+1+m+3}}$$

$$\frac{2^{2m+2n+1} \times 3^{3m+n+3} \times 5^{n+m+4}}{2^{2m+2n+1} \times 3^{3m+n+3} \times 5^{m+n+4}}$$

1 option c

HOWEVER IN EXAM , JUST WORK ON BASE 2 , CHECK IF N & D CANCEL , WITH A FAIR AMOUNT OF RISK , YOU CAN THEN ASSUME SAME WILL HAPPEN WITH BASE 3 AND 5 .
HENCE FINAL ANS = 1

Q2

01. JUNE 2012

$$\frac{3^{n+1}+3^n}{3^{n+3}-3^{n+1}} = ?$$

a. $1/5$ b. $1/6$ c. $1/4$ d. $1/9$

PUT n = 0

$$\frac{31+30}{33-31} = \frac{3+1}{27-3} = \frac{1}{6}$$ option b

02. DEC 2009

$$\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = ?$$

a. $1/2$ [b.] $3/2$ c. $2/3$ d. $1/3$

PUT n = 0

$$\frac{2^0+2^{-1}}{2^1-2^0} = \frac{1+1/2}{2-1} = \frac{3}{2}$$ option b

03. DEC 2021

$$\frac{6^{n+4} + 3^{n+3}x2^{n+3}}{5x6^n + 6^n} = ?$$

a. 232 b. 242 [c.] 252 d. 262

$$\frac{6^{n+4} + 3^{n+3}x2^{n+3}}{5x6^n + 6^n}$$

PUT n = 0

$$\frac{6^4 + 3^3x2^3}{5x6^0 + 6^0}$$

$$\frac{1296+216}{5+1} = 252$$ option c

04. NOV 2019

$$\left(\frac{9^{n+\frac{1}{4}} \sqrt{3.3^n}}{3.\sqrt{3^{-n}}} \right)^{\frac{1}{n}}$$

a. 1 b. 3 c. 9 [d.] 27

PUT n = 1

$$9^{5/4} \cdot \left(\frac{\sqrt{3^2}}{3\sqrt{3^{-1}}} \right)$$

$$3^{2.5/4} \left(\frac{3}{3 \frac{1}{\sqrt{3}}} \right)$$

$3^{5/2} . \sqrt{3}$

$3^{5/2+1/2} = 3^3 = 27$ option d

Q3 $(a-b)(a^2+ab+b^2) = a^3-b^3$

$(a+b)(a^2-ab+b^2) = a^3+b^3$

01. JUNE 2014

$$\left[\frac{y^a}{y^b}\right]^{a^2+ab+b^2}\left[\frac{y^b}{y^c}\right]^{b^2+bc+c^2}\left[\frac{y^c}{y^a}\right]^{c^2+ca+a^2} = ?$$

a. y b. −1 [c.] 1 d. none of these

$= y^{a-b \,.\, a^2+ab+b^2} \times$ ……. PLAY SMART

$= y^{a^3-b^3 + b^3-c^3 + c^3-a^3}$

$= y^0$

$= 1$ option C

02.

$$\left[\frac{x^a}{x^{-b}}\right]^{a^2-ab+b^2}\left[\frac{x^b}{x^{-c}}\right]^{b^2-bc+c^2}\left[\frac{x^c}{x^a}\right]^{c^2-ca+a^2}$$

$= x^{a+b \,.\, a^2-ab+b^2}$ ……. PLAY SMART

$= x^{a^3+b^3}$ …….

$= x^{a^3+b^3 + b^3+c^3 + c^3+a^3}$

$= x^{2(a^3+b^3+c^3)}$

Q4 $a^{1/n} = b \Rightarrow a = b^n$

$a^n = b \Rightarrow a = b^{1/n}$

01. JUNE 2016

$3^x = 5^y = 75^z$, then

[a] $\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$

b. $\frac{2}{x} + \frac{1}{y} = \frac{1}{z}$

c. $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ d. none

$3^x = 5^y = 75^z = k$ (say)

$3 = k^{1/x}$, $5 = k^{1/y}$, $75 = k^{1/z}$

$3 \times 5 \times 5 = 75$

$k^{1/x} \,.\, k^{1/y} \,.\, k^{1/y} = k^{1/z}$

$k^{1/x + 1/y + 1/y} = k^{1/z}$

$\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$ option a

02. JUNE 2019

$2^{x^2} = 3^{y^2} = 12^{z^2}$, then

a. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$

b. $\frac{1}{x^2} + \frac{2}{y^2} = \frac{1}{z^2}$

[c.] $\frac{2}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$ d. none

$2^{x^2} = 3^{y^2} = 12^{z^2} = k$

$2 \times 2 \times 3 = 12$

$k^{1/x^2} \; k^{1/x^2} \; k^{1/y^2} = k^{1/z^2}$

$k^{1/x^2 + 1/x^2 + 1/y^2} = k^{1/z^2}$

$\frac{2}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$ option C

03. DEC 2017

if $u^{5x} = v^{5y} = w^{5z}$ and $u^2 = vw$ then

$xy + zx - 2yz =$

[a.] 0 b. 1 c. 2 d. none

$u^{5x} = v^{5y} = w^{5z}$

$u^x = v^y = w^z = k$

$u^2 = vw$

$k^{2/x} = k^{1/y}.k^{1/z}$

$k^{2/x} = k^{1/y+1/z}$

$\frac{2}{x} = \frac{1}{y} + \frac{1}{z}$

$\frac{2}{x} = \frac{y+z}{yz}$

$xy + zx - 2yz = 0$ option a

Q5

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

If $a+b+c = 0$ then

$a^3+b^3+c^3=3abc$

01. JUNE 2009

$x = 3^{1/3} + 3^{-1/3}$, $3x^3 - 9x = ?$

a. 3 b. 9 c. 12 [d.] 10

$x^3 = 3 + 3^{-1} + 3 . 3^{1/3} . 3^{-1/3} . x$

$x^3 = 3 + 3^{-1} + 3.3^0.x$

$x^3 = 3 + 3^{-1} + 3x$

$x^3 = 3 + \frac{1}{3} + 3x$

MULTIPLYING BY '3'

$3x^3 = 9 + 1 + 9x$

$3x^3 - 9x = 10$ option d

02. JUNE 2023

$\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

then $\left(\frac{a+b+c}{3}\right)^3 = ?$

[a)] abc b) 9abc c) abc/2 d) abc/9

$a^{1/3} + b^{1/3} + c^{1/3} = 0$

$x + y + z = 0 \Rightarrow x^3+y^3+z^3 = 3xyz$

APPLYING THE SAME

$a+b+c = 3\ a^{1/3}.b^{1/3}.c^{1/3}$

$\frac{a+b+c}{3} = (abc)^{1/3}$

$\left(\frac{a+b+c}{3}\right)^3 = abc$

option a

Q6 SURDS

01. DEC 2023

$x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Find the value of $\frac{1}{x^2} + \frac{1}{y^2}$

a. 63 b 61 [c] 62 d 60

$x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$x = \frac{5+3+2\sqrt{15}}{5-3}$

$x = \frac{8+2\sqrt{15}}{2}$

$x = 4+\sqrt{15}$, Sllrly $y = 4-\sqrt{15}$

$x+y = 8$

$xy = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 1$

$\frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2+y^2}{(xy)^2}$

$= \frac{(x+y)^2-2xy}{(xy)^2}$

$= \frac{8^2-2(1)}{1} = 62$ option C

02. DEC 2021

$a = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, $b = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Find the value of $a^2 + b^2$

a. 64 [b] 62 c 60 d 254

$a = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$a = \frac{5+3+2\sqrt{15}}{5-3}$

$a = \frac{8+2\sqrt{15}}{2}$

$a = 4+\sqrt{15}$, Sllrly $b = 4-\sqrt{15}$

$a+b = 8$

$ab = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 1$

$a^2+b^2 = (a+b)^2-2ab$

$= 64 - 2 = 62$ option b

03. JUNE 2017

$a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$, $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

Find the value of $\frac{1}{a^2} + \frac{1}{b^2}$

a) 486 b) 484 [c] 482 d) 500

$a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$a = \frac{6+5+2\sqrt{30}}{6-5}$

$a = 11+2\sqrt{30}$

$a = 11+2\sqrt{30}$, Sllrly $b = 11-2\sqrt{30}$

$a+b = 22$

$ab = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} = 1$

$\frac{1}{a^2} + \frac{1}{b^2} = \frac{a^2+b^2}{(ab)^2}$

$= \frac{(a+b)^2-2ab}{(ab)^2}$

$= \frac{22^2-2(1)}{1}$

$= 482$ option C

04. NOV 2019

Find the value of $x^2-10x+1$ if

$x = \frac{1}{5-2\sqrt{6}}$

a. 25 b. 1 c. 0 d. 49

$x = \frac{5+2\sqrt{6}}{5-2\sqrt{6} \cdot 5+2\sqrt{6}}$

$x = \frac{5+2\sqrt{6}}{25-24}$

$x = 5+2\sqrt{6}$

$x-5 = 2\sqrt{6}$

$x^2-10x+25 = 24$

$x^2-10x+1 = 0$ option C

KUCH HATKE

01. JUNE 2009

$\left(1-[1-(1-x^2)^{-1}]^{-1}\right)^{-1/2}$

a. 1/x b. x c. 1 d. none

$= \left\{1-\left(1-\frac{1}{1-x^2}\right)^{-1}\right\}^{-1/2}$

ON LCM

$= \left\{1-\left(\frac{-x^2}{1-x^2}\right)^{-1}\right\}^{-1/2}$

$= \left\{1-\left(\frac{1-x^2}{-x^2}\right)\right\}^{-1/2}$

$= \left(1+\frac{1-x^2}{x^2}\right)^{-1/2}$

ON LCM

$= \left(\frac{1}{x^2}\right)^{-1/2}$

$= x^{2.1/2}$

$= x$

$= x$ option b

02. DEC 2021

the value of $1 - \sqrt[3]{0.027}\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)^2$

a) $\frac{11}{16}$ b) $\frac{13}{16}$ c) $\frac{15}{16}$ d) 1

$1 - \sqrt[3]{\frac{27}{1000}} \times \frac{5}{6} \times \frac{1}{4}$

$1 - \frac{3}{10} \times \frac{5}{6} \times \frac{1}{4} = \frac{15}{16}$ option C

03. NOV 2019

$x = \sqrt{3} + \frac{1}{\sqrt{3}}$ then $\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)$

a. $^5/_6$ b. $^6/_5$ c. $^2/_3$ d. $^1/_9$

$\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)$

$\left(x-\sqrt{3}\right)\left(x-\frac{1}{x-\frac{2}{\sqrt{3}}}\right)$

$\left(\sqrt{3}+\frac{1}{\sqrt{3}}-\sqrt{3}\right)\left(\sqrt{3}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{3}+\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}}\right)$

$\left(\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{3}-\frac{1}{\sqrt{3}}}\right)$

$\left(\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}+\frac{1}{\sqrt{3}}-\frac{1}{\frac{3-1}{\sqrt{3}}}\right)$

$\left(\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}+\frac{1}{\sqrt{3}}-\frac{\sqrt{3}}{2}\right)$ option a

$1 + \frac{1}{3} - \frac{1}{2} = \frac{6+2-3}{6} = \frac{5}{6}$

02 - LOGARITHMS

Q1

$\log (ab) = \log a + \log b$

$\log(a/b) = \log a - \log b$

$\log_a a = 1$

Q2

$\log a^n = n.\log a$

$\log_a a = 1$

01. JUNE 2023

$\log_{10}x = m+n-1$, $\log_{10}y = m-n$ then express $\log_{10}(100x/y^2)$

[a.] $1-m+3n$ b. $m-1+3n$ c. $m+3n+1$ d. m^2-n^2

$\log_{10}100 + \log_{10}x - 2\log_{10}y$

$= 2\log_{10}10 + m+n-1 - 2(m-n)$

$= 2(1) + m + n - 1 - 2m + 2n$

$= 1-m+3n$ option a.

02. DEC 2021

$\log_{10}3 = x$, $\log_{10}4 = y$, then $\log_{10}120 =$

a. $x-y+1$ [b.] $x+y+1$ c. $x+y-1$ d. $2x+y-1$

$\log_{10}120$

$= \log_{10}(3.4.10)$

$= \log_{10}3 + \log_{10}4 + \log_{10}10$

$= x+y+1$ option b

03. DEC 2022

$\log_{10}2 = y$, $\log_{10}3 = x$, then $\log_{10}15 =$

[a.] $x-y+1$ b. $x+y+1$ c. $x-y-1$ d. $y-x+1$

$\log_{10}15$

$= \log_{10}(3.5)$

$= \log_{10}(3.10/2)$

$= \log_{10}3 + \log_{10}10 - \log_{10}2$

$= x+1-y$ option a

01. JUNE 2023

$[\log_{10}[5\log_{10}100]]^2 =$

[a.] 1 b. 2 c. 10 d. 25

$= [\log_{10}[5\log_{10}100]]^2$

$= [\log_{10}[5\log_{10}10^2]]^2$

$= [\log_{10}[10\log_{10}10]]^2$

$= [\log_{10}10]^2$

$= 1^2$ option a.

02. NOV2018

$\log_2 \log_2 \log_2 16$

a. 0 b. 3 [c.] 1 d. 2

$= \log_2 \log_2 \log_2 2^4$

$= \log_2 \log_2 4\log_2 2$

$= \log_2 \log_2 4$

$= \log_2 2\log_2 2$

$= \log_2 2$

$= 1$ option c.

03. SEPT 2024

$\log_{2}\log_{2}\log_{4} 256 + 2\log_{\sqrt{2}} 2$

a. 2 b. 3 [c.] 5 d. 7

$= \log_{2}\log_{2}\log_{4} 4^4 + 2\log_{\sqrt{2}} \sqrt{2}^2$

$= \log_{2}\log_{2}4\log_{4} 4 + 4\log_{\sqrt{2}} \sqrt{2}$

$= \log_{2}\log_{2} 4 + 4\log_{\sqrt{2}} \sqrt{2}$

$= \log_{2}2\log_{2} 2 + 4$

$= \log_{2} 2 + 4$

$= 1 + 4$

$= 5$ option C.

04. DEC 2021

Find $\log(x^6)$

if $\log x + 2\log(x^2) + 3\log(x^3) = 14$

a. 3 b. 4 c. 5 [d.] 6

$\log x + 2\log(x^2) + 3\log(x^3) = 14$

$\log x + 4\log x + 9\log x = 14$

$14\log x = 14$

$\log x = 1$

Finally

$\log(x^6)$

$6\log x$

6 option d.

05. JUNE 2019

$\log_{2\sqrt{2}} 512 : \log_{3\sqrt{2}} 324$

a. 128 :61 b. 2 :3 [c.] 3 :2

d. none

$\log_{2\sqrt{2}} 512$ | $\log_{3\sqrt{2}} 324$

$\log_{\sqrt{8}} 512$ | $\log_{\sqrt{18}} 324$

$\log_{\sqrt{8}} 8^3$ | $\log_{\sqrt{18}} 18^2$

3. $\log_{\sqrt{8}} 8$ | 2. $\log_{\sqrt{18}} 18$

3. $\log_{\sqrt{8}} \sqrt{8}^2$ | 2. $\log_{\sqrt{18}} \sqrt{18}^2$

6. $\log_{\sqrt{8}} \sqrt{8}$ | 4. $\log_{\sqrt{18}} \sqrt{18}$

6 | 4

3 :2 option C.

Q3

$\log a + \log b = \log (ab)$

$\log a - \log b = \log(a/b)$

$\log a^n = n.\log a$

$\log_a a = 1$

$\log 1 = 0$

01. JUNE 2022

Find the value of

$\log \frac{p^2}{qr} + \log \frac{q^2}{pr} + \log \frac{r^2}{pq}$

[a.] 0 b. 1 c. log pqr d. pqr

$\log \left(\frac{p^2}{qr} \times \frac{q^2}{pr} \times \frac{r^2}{pq}\right)$

$\log 1$

0 option a.

02. JULY 2019

$\log_5 \left[1+\frac{1}{5}\right] + \log_5 \left[1+\frac{1}{6}\right] + \ldots + \log_5 \left[1+\frac{1}{624}\right]$

a. 2 [b.] 3 c. 5 d. 0

$\log_5 \left[1+\frac{1}{5}\right] + \log_5 \left[1+\frac{1}{6}\right] + \ldots$

$\ldots\ldots + \log_5 \left[1+\frac{1}{623}\right] + \log_5 \left[1+\frac{1}{624}\right]$

$\log_5 \frac{6}{5} + \log_5 \frac{7}{6} + \log_5 \frac{8}{7} + \ldots\ldots$

$\ldots\ldots + \log_5 \frac{624}{623} + \log_5 \frac{625}{624}$

$\log_5 \left[\frac{6}{5} \frac{7}{6} \frac{8}{7} \ldots\ldots \frac{624}{623} \frac{625}{624}\right]$

$\log_5 125$

$\log_5 5^3$

$3\log_5 5$

3 option b.

Q4

01. $\log xy^2 - \log y = \log(x+y)$ then y =

a. $\frac{1}{x}$ b. $\frac{x}{x+1}$ [c.] $\frac{x}{x-1}$ d. none

NOV 2019

$\log \left(\frac{xy^2}{y}\right) = \log(x+y)$

$xy = x+y$

$y(x-1) = x$

$y = \frac{x}{x-1}$ option C

02. $\log\left[\frac{x-y}{2}\right] = \frac{1}{2}[\log x + \log y]$, then x^2+y^2

DEC 2017

[a.] 6xy b. 2xy c. $3x^2y^2$ d. $4x^2y^2$

$2\log\left[\frac{x-y}{2}\right] = \log xy$

$\log\left[\frac{x-y}{2}\right]^2 = \log xy$

$x^2-2xy+y^2 = 4xy$

$x^2+y^2 = 6xy$ option a.

03. if $x^2+y^2 = 7xy$, then $\log\left[\frac{x+y}{3}\right]$ =

a. logx+logy [b.] $\frac{1}{2}(\log x+\log y)$

c. $\frac{1}{3}(\log x+\log y)$ d. $\frac{1}{3}(\log x+\log y)$

JUNE 2014

$x^2+2xy+y^2 = 7xy+2xy$

$(x+y)^2 = 9xy$

$\left(\frac{x+y}{3}\right)^2 = xy$, TAKING LOG ON BOTH SIDES

$\log \left(\frac{x+y}{3}\right)^2 = \log (xy)$

$2\log \left(\frac{x+y}{3}\right) = \log x + \log y$

$\log \left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x+\log y)$ option b.

Q5 $\log_b a = \frac{\log a}{\log b}$, $\frac{1}{\log_b a} = \log_b a$

$a^{\log_a x} = x$

01. DEC 2013

$\left(\log_y x . \log_z y . \log_x z\right)^3$

a. 0 b. −1 [c.] 1 d. 3

$\left(\frac{\log x}{\log y} \cdot \frac{\log y}{\log z} \cdot \frac{\log z}{\log x}\right)^3 = 1$

option c.

02. DEC 2013

$\log_4 9 . \log_3 2 . =$

a. 3 b. 2 [c.] 1 d. 0

$\log_4 9 . \log_3 2$

$= \frac{\log 9}{\log 4} \cdot \frac{\log 2}{\log 3} = \frac{2\log 3}{2\log 2} \cdot \frac{\log 2}{\log 3} = 1$

option c.

03. DEC 2022

$\log_3 4 . \log_4 5 . \log_5 6 . \log_6 7 . \log_7 8 . \log_8 9 =$

a. 3 [b.] 2 c. 1 d. 0

$= \frac{\log 4}{\log 3} \cdot \frac{\log 5}{\log 4} \cdot \frac{\log 6}{\log 5} \cdot \frac{\log 7}{\log 6} \cdot \frac{\log 8}{\log 7} \cdot \frac{\log 9}{\log 8}$

$= \frac{\log 9}{\log 3}$

$= 2 \frac{\log 3}{\log 3} = 2$

option b.

04. NOV 2019

$\log_{0.01} 10000 = ?$

a. 2 [b.] −2 c. 4 d. −4

$\frac{\log 10000}{\log 0.01}$

$= \frac{\log 10000}{\log (1/100)}$

$= \frac{\log 10^4}{\log 10^{-2}} = \frac{4}{-2} = -2$

option b.

05. MAY 2018

$\log_a b . \log_b c . \log_c d . \log_d t$

a

[a.] t b. abcdt c. a+b+c+d+t d.none

$\frac{\log b}{\log a} \cdot \frac{\log c}{\log b} \cdot \frac{\log d}{\log c} \cdot \frac{\log t}{\log d}$

a

$a^{\frac{\log t}{\log a}} \quad a^{\log_a t}$

$a = a = t$ option a

06. JUNE 2016

$\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60} = ?$

a. 0 b. 1 b. 5 c. 60

$\log_{60} 3 . \log_{60} 4 . \log_{60} 5$

$\log_{60} (3.4.5)$

$\log_{60} 60 = 1$

option c.

07. JUNE 2014

$x = \log_{24} 12$, $y = \log_{36} 24$, $z = \log_{48} 36$

$xyz + 1 = ?$

a. 2xy b. 2zx [c.] 2yz d. 2

$xyz + 1$

$= \log_{24} 12 . \log_{36} 24 . \log_{48} 36 + 1$

$= \frac{\log 12}{\log 24} \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48} + 1$

$= \frac{\log 12 + \log 48}{\log 48}$

$= \log_{48} (12.48)$

$= \log_{48} 576$

$= \log_{48} 24^2$

$= 2 . \log_{48} 24 = 2yz$ option c.

08. $x = \log_a bc$, $y = \log_b ca$,

$z = \log_c ba$

Evaluate $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

a. 0 b. 1 c. −1 d. 3

LETS WORK ONLY ON ONE TERM

$$\frac{1}{\log_a bc+1} = \frac{1}{\frac{\log bc}{\log a} + 1} = \frac{\log a}{\log bc+\log a}$$

$$= \frac{\log a}{\log abc}$$

$$\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = \frac{\log a + \log b + \log c}{\log abc}$$

$$= \frac{\log abc}{\log abc} = 1$$

option **b.**

09. JAN 2021

$\log_a (ab) = x$, then $\log_b (ab) =$

a. $\frac{1}{x}$ b. $\frac{x}{x+1}$ c. $\frac{x}{x-1}$ d. none

let a = 2 , b = 4

$\log_a (ab) = x$

$x = \log_2 8 = \log_2 2^3 = 3$

LET'S SOLVE ,

$\log_b (ab)$

$\log_4 (8)$

$\frac{\log 8}{\log 4}$

$\frac{3\log 2}{2\log 2} = \frac{3}{2}$

Put x = 3 in the options ,

Option c : $^3/_2$ option **c.**

Q6 $\log_b a = n \Rightarrow a = b^n$,

01. JUNE 2022

$\log_{a^{\sqrt{3}}} = 1/6$, then find the value of a

a. 3 b. 9 c. 27 d. 81

$\frac{1}{2} \log_a 3 = \frac{1}{6}$

$\text{Log}_a 3 = 1/3$

$3 = a^{1/3}$

$a = 3^3 = 27$ option **c.**

02. JUNE 2018

$\log_x \sqrt[3]{2} = \frac{1}{15}$, x = ?

a. 2 b. 8 c. 16 d. 32

$2^{1/3} = x^{1/15}$ EXPONENTIAL FORM

$(2^{1/3})^{15} = x$

$x = 2^5 = 32$ option **d.**

03. DEC 2017

$\log_3 \left[\log_4 \left[\log_2 x\right]\right] = 0$, x = ?

a. 4 b. 8 c. 16 d. 32

$\log_4 \left[\log_2 x\right] = 3^0 = 1$

$\log_2 x = 4^1 = 4$

$x = 2^4$

$x = 16$

04. JUNE 2016

$\log_4 (x^2+x) - \log_4 (x+1) = 2$, x = ?

[a.] 16 b. 0 c. –1 d. none

$\log_4 \left[\frac{x(x+1)}{x+1}\right] = 2$

$\log_4 x = 2$

$x = 4^2$ option a.

$\log_4 x \quad \frac{25}{12} = \frac{25}{6}$

$\log_4 x = 2$

$x = 4^2 = 16$ option C.

05. JULY 2021

$\log_2 x + \log_4 x = 6$, Find x,

[a.] 16 b. 32 c. 64 d. 128

$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} = 6$

$\frac{\log x}{\log 2}\left[1 + \frac{1}{2}\right] = 6$

$\log_2 x \left[\frac{3}{2}\right] = 6$

$\log_2 x = 4$

$x = 2^4 = 16$ option a.

06. JULY 2021

$\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$

Find x,

a. 64 b. 4 [c.] 16 d. 2

$\frac{\log x}{\log 4} + \frac{\log x}{2\log 4} + \frac{\log x}{3\log 4} + \frac{\log x}{4\log 4} = \frac{25}{6}$

$\frac{\log x}{\log 4}\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right] = \frac{25}{6}$

$\log_4 x \left[\frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12}\right] = \frac{25}{6}$

03 - RATIO PROPORTION

01. DEC 2020

If a :b = 9 :4 then

$\sqrt{(a/b)} + \sqrt{(b/a)} = ?$

a. $2/3$ b. $3/2$ c. $6/13$ d. $13/6$

$\sqrt{(9/4)} + \sqrt{(4/9)}$

$= 3/2 + 2/3$

$= 13/6$ option d

02. NOV 2018

if x :y :z = 7 :4 :11 , then

$\frac{x+y+z}{z} =$

a. 2 b. 3 c. 4 d. 5

$\frac{x+y+z}{z} = \frac{7+4+11}{11} = \frac{22}{11} = 2$

option a

03. DEC 2020

If a :b = 3 :7 then 3a+2b :4a+5b

a. 27 :43 b. 23 :47 c.24 :51 d. 29 :53

$$\frac{3a+2b}{4a+5b} = \frac{3(3)+2(7)}{4(3)+5(7)} = \frac{9+14}{12+35} = \frac{23}{47}$$

option b

04. JUNE 2016

x , y , z together starts a business , if x invests 3 times as much as y invests and y invests two third of what z invest , then ratio of capitals of x , y , z

a. 3 :9 :2 b. 6 :3 :2 c. 3 :6 :2

d. 6 :2 :3

X	Y	Z
3^{x2}	1^{x2}	
	2	3
6	2	3

option d

05. JULY 2021

A :B = 5 :3 , B :C = 6 :7 , C :D = 14 :9

then A :B :C :D

a. 20 :14 :12 :9 b. 20 :9 :12 :14

c. 20 :9 :14 :12 d. 20 :12 :14 :9

A	B	C	D
5^{x4}	3^{x4}		
	6^{x2}	7^{x2}	
		14	9
20	12	14	9

option d

06. JUNE 2017

if a :b = 2 :3 , b :c = 4 :5 , c :d = 6 :7 , then a :d is

a. 24 :35 b. 8 :15 c. 16 :35 d. 7 :15

$$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{a}{d}$$

$$\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{a}{d} = \frac{16}{35}$$

option c

a , b , c are in proportion , then

$\frac{a}{b} = \frac{b}{c}$,

$b^2 = ac$, b = mean proportion

a , b , c , d are in proportion then

$\frac{a}{b} = \frac{c}{d}$, d = fourth proportion

07. SEPT 2024

The mean proprotional between $12x^2$ & $27y^2$ is

a. 18xy b. 81xy c. 8xy d.19.5xy

$12x^2$, B , $27y^2$ are in proportion

$B^2 = AC$

$= 12x^2.27y^2$

$= 324x^2.y^2$

$B = 18xy$ option a

08. DEC 2015

Find the ratio of third proportional of 12 , 30 and mean proportional of 9 ,25

a. 7 :2 b. 5 :1 c. 9 :4 d. none

12,30, c …… are in proportion

$30^2 = 12 \times c$

$c = 75$

9,b,25 ….. are in proportion

$b^2 = 9 \times 25$

$b = 15$

Required ratio = 75 :15 = 5 :1

option b

09. JUNE 2024

if the 4 nos. $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{10}$ and $\frac{1}{x}$ are proportional , then what is the value of x

a. 14 [b.] 15 c. 10 d. 1/12

$\frac{\frac{1}{4}}{\frac{1}{6}} = \frac{\frac{1}{10}}{\frac{1}{x}} \Rightarrow \frac{6}{4} = \frac{x}{10} \Rightarrow x = 15$

option b

10. DEC 2015

What must be added to each of the numbers 10 , 18 , 22 , 38 to make them proportional

a. 5 b. 2 c. 3 d. 9

$\frac{10+x}{18+x} = \frac{22+x}{38+x}$

USING OPTIONS

a. $\frac{10+5}{18+5} = \frac{22+5}{38+5}$ $\frac{15}{23} = \frac{27}{43}$ x

b $\frac{10+2}{18+2} = \frac{22+2}{38+2}$ $\frac{12}{20} = \frac{24}{40}$ ✓

option b

11. DEC 2022

Four persons A , B , C , D wish to share a sum in the ratio of 5 :2 :4 :3 . IF C gets ₹ 1000 more than D , then share of B

[a] ₹ 2000 b ₹ 1500 c ₹ 2500 d ₹ 1000

A : B : C : D

5 : 2 : 4 : 3

1

↓

1000

B's share = 2 x 1000 = 2000

option a

12. JUNE 2010

The students of the two classes are in ratio 5 :7 . If 10 students left from each class , the remaining students are in ratio 4 :6 . The number of students in each class was

a. 30,40 b. 25,24 c. 40,60 d. 50,70

$\frac{5x-10}{7x-10} = \frac{2}{3}$

$15x-30 = 14x-20$

$x = 10$

students in each class were : 50,70

option d

13. JUNE 2019

If ratio of two numbers is 7 :11 and 7 is added to each number then the new ratio will be 2 :3 . The numbers are

a. 49,77 b. 42,45 c. 43,42 d. 39,40

$\frac{7x+7}{11x+7} = \frac{2}{3}$

$21x+21 = 22x+14$

$x = 7$

numbers are : 49,77 option a

14. JUNE 2010

What must be added to each term of the ratio 49 :68 so that it becomes 3 :4

a. 3 b. 5 c. 8 d. 9

$\frac{49+x}{68+x} = \frac{3}{4} \Rightarrow 196+4x = 204+3x$

$x = 8$ option c

15. DEC 2020

The ratio of no. of boys and the no. of girls in a school is found to be 15 :32 . How many boys and equal number of girls should be added to bring the ratio 2/3

a. 20 b. 19 c. 23 d. 27

$\frac{15+x}{32+x} = \frac{2}{3} \Rightarrow 45+3x = 64+2x$

$x = 19$ option b

16. DEC 2021

In a department ,the number of males and females are in the ratio 3 :2 . If 2 males and 5 females join department , then the ratio becomes 1 :1 . Initially the number of females in the department is

a. 9 b. 6 c. 3 d. 8

$\frac{3x+2}{2x+5} = \frac{1}{1} \Rightarrow 3x+2 = 2x+5 \Rightarrow x = 3$

initial no. of females $= 2(3) = 6$

option b

17. JUNE 2018

$(a+b) :(b+c) :(c+a) = 7 :8 :9$ &

$a+b+c=18$ then a :b :c =

a. 5 :4 :3 b. 3 :4 :5 c. 4 :3 :5

d. 4 :5 :3

TRYING BY OPTIONS

	(a+b) :(b+c) :(c+a)	
a. 5 :4 :3	9 : 7 : 8	X
b. 3 :4 :5	7 : 9 : 8	X
c. 4 :3 :5	7 : 8 : 9	✓

option c

18. NOV 2019

The two numbers are in ratio 3 :4 . The difference between their squares is 28 . Find the greater number

a. 12 b. 8 c. 16 d. 10

$(4k)^2-(3k)^2 = 28$

$7k^2 = 28$

$k = 2$

$4k = 8$ option b

19. DEC 2014

The first , second and third month salaries of a person are in ratio 2 :4 :5 . The difference between the product of the salaries of first 2 months and last 2 months is ₹ 4,80,00,000 . Find the salary of 2nd month

a. 4000 b. 6000 c. 12000 d. 8000

	M1	M2	M3
Sal.	2k	4k	5k

As per the given condition

$4k.5k - 2k.4k = 480,00,000$

$12k^2 = 480,00,000$

$k^2 = 40,00,000$

$k = 2000$

$4k = 8000$ option d

20. JAN 2021 , JULY 2021

the salaries of A, B and C are in the ratio 2 :3 :5 . If increments of 15% , 10% and 20% are allowed respectively to their salary , then what will be the new ratio of their salaries

a. 3 :3 :10 b. 10 :11 :20 c. 23 :33 :60

d. can't be determined

	A		B		C	
Ratio of Salaries	2	:	3	:	5	
Salaries	200		300		500	(say)
Increment	+15%		+10%		+20%	
New Salary	230		330		600	
New Ratio	23	:	33	:	60	

option C

21. JUNE 2014

A person has assets worth ₹ 1,48,200 . he wishes to divide it amongst his wife , son & daughter in the ratio 3 :2 :1 respectively . From this assets , the share of his son will be

a. 74100 b. 37050 c. 49400 d. 24700

W	S	D	Total
3	: 2	: 1	6
	x24700		x24700
	₹ 49400		1,48,200

option C

22. DEC 2021

a bag has 105 coins containing some 50 paise and 25 paise coins . The ratio of number of coins is 4 :3 . The total value (in ₹) in the bag is

a. ₹ 43.25 b. ₹ 41.25 c. ₹ 39.25

d. ₹ 35.25

	50p	25p	TOTAL
NO.OF COINS	4	3	7
	x15	x15	x 15
	60	45	105 coins
VALUE	₹ 30	₹11.25	₹ 41.25

option b

23. JUNE 2022

a bag contains 25 paise , 10 paise and 5 paise coins in the ratio 3 :2 :1 . The total value of the bag is ₹ 40 . Find the number of 5 paise coins

a. 45 b. 48 c. 40 d. 20

	25p	10p	5p	
NO.OF COINS	3	2	1	
VALUE	75	20	5	
	15	4	1	20
			X 2	x 2
VALUE			₹ 2	₹40
			x 20	
			40 coins	

option C

24. DEC 2017

The ratio of number of 5 rupee coins to number of 10 rupee coins is 8 :15 . If the total value of 5 rupee coins is 360 , then the number of ten rupee coins is

a. 72 b. 60 c. 150 d. 135

	₹ 5	₹10
NO.OF COINS	8	15
VALUE	40	150
	4 :	15
	x 90	x 90
IN REAL	₹ 360	₹1350
		135 coins

option d

25. DEC 2016

a bag contains 23 number of coins in the form of 1 rupee , 2 rupee and 5 rupee coins . The total sum of coins is 43 . The ratio between 1 rupee and 2 rupee coins is 3 :2 . The number of 1 rupee coins is

a. 12 b. 8 c. 10 d. 16

Let ₹ 1 & ₹ 2 coins be 3x & 2x

Total coins = 23

₹ 5 coins = 23 – 5x

Total value = ₹ 43

$3x(1) + 2x(2) + (23-5x)5 = 43$

$3x+4x+115-25x = 43$

$18x = 72 \qquad x = 4$

No. of ₹ 1 coins = $3x = 12$

option a

26. SEPT 2024

The ratio of incomes of A and B is 5 :4 and their expenditure is 3 :2 . If at the end of the year each saves 1600 , then the income of A

a ₹ 3400 b ₹ 3600 c ₹ 4000 d ₹ 4400

	A	B
INCOME	5x	4x
EXP.	3y	2y
SAVINGS	1600/-	1600/-

$5x-3y = 1600 \times 2$

$4x-2y = 1600 \times 3$

$10x-6y = 3200$

$12x-6y = 4800$

$2x = 1600$

$x = 800$

$5x = 4000$

option c

27. DEC 2021

Incomes of R and S are in ratio 7 :9 and their expenditures are in the ratio 4 :5 . Their total expenditure is equal to income of R . What is the ratio of their savings

a. 23 :36 b. 28 :41 c. 31 :43 d.35 :46

	R	S
INCOME	7x	9x
EXP.	4y	5y
SAVINGS	7x–4y	9x–5y

GIVEN - total expenditure is equal to income of R

$4y+5y = 7x$

$9y = 7x \quad \Rightarrow x = {}^{9y}/_{7}$

	R	S
SAVINGS	7x–4y	9x–5y
	9y–4y	$9({}^{9y}/_{7})-5y$
	5y	$\frac{81y-35y}{7}$
	5y	$\frac{46y}{7}$
	36y	46y
	35 :	46

option d

FOR x : y

DUPLICATE RATIO = $X^2 : Y^2$

TRIPLICATE RATIO = $X^3 : Y^3$

SUB DUPLICATE RATIO = $\sqrt{X} : \sqrt{Y}$

SUB TRIPLICATE RATIO = $\sqrt[3]{X} : \sqrt[3]{Y}$

28. MAY 2018

if p :q is the sub duplicate ratio of $p-x^2 : q-x^2$, then x^2 is

a. $p/p+q$ b. $q/p+q$ c. $pq/p+q$ d. none

$$\frac{p}{q} = \frac{\sqrt{(p-x^2)}}{\sqrt{(q-x^2)}}$$

$$\frac{p^2}{q^2} = \frac{p-x^2}{q-x^2}$$

$$p^2q - p^2x^2 = pq^2 - q^2x^2$$

$$p^2q - pq^2 = p^2x^2 - q^2x^2$$

$$pq(p-q) = x^2(p^2-q^2)$$

$$x^2 = \frac{pq(p-q)}{p^2-q^2} = \frac{pq}{p+q}$$ option c

29. NOV 2018

$3x-2/5x+6$ is the duplicate ratio of 2/3 .

Find x

a. 6 b. 2 c. 5 d. 9

$$\frac{3x-2}{5x+6} = \left(\frac{2}{3}\right)^2$$

$$\frac{3x-2}{5x+6} = \frac{4}{9}$$

$$27x-18 = 20x+24$$

$$7x = 42$$

$$x = 6$$ option a

30. DEC 2022

a group of 400 soldiers [posted at the border area had provisions for 31 days . After 28 days , 280 soldiers were called back from the border . Find the number of days for which remaining ration will be sufficient ?

a) 3 b) 6 c) 8 d) 10

Let 1 soldier consume = 1 unit ration/d

400 soldiers will consume

= 400 u's ration/d

Ration left in the last 3 days

= 3x400 = 1200 u

Soldiers left in last 3 days

= 400−280 = 120

They will consume = 120u's/d

At this rate

Number of days balance ration will be sufficient = 1200/120 = 10 days

option b

31. JUNE 2015

If one type of rice of cost ₹ 13.84 is mixed with another type of rice of cost ₹ 15.54 , the mixture is sold at ₹ 17.60 with a profit of 14.6% on selling price , then in which proportion the two types of rice is mixed

a. 3:7 b. 5:7 c. 7:9 d. 9:1

CP = 17.60 − 14.6% = 15.03

	TYPE 1		TYPE II
COST/Kg	13.84		15.54
COST OF MIX		15.03	
RATIO OF MIX	0.51	:	1.19
	51	:	119
	3	:	7

option a

32. JUNE 2023

Mr Ram invested a total amount of 1,00,000 in two different banks for a fixed period . The first bank yields an interest of 9% per annum and second 11% per annum . If the total interest at the end of one year is 9.75% per annum , then the total amount invested in these banks are respectively

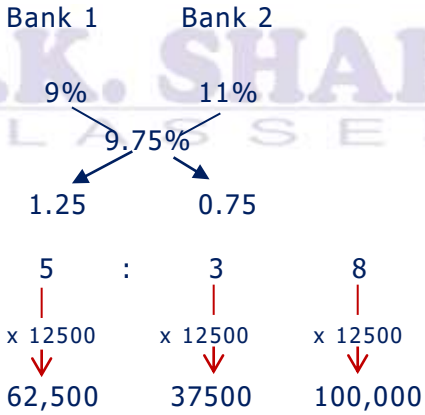
a. ₹ 52500, ₹ 47,500

b. ₹ 62,500, ₹ 37,500

c. ₹ 57500 , ₹ 42,500

d. ₹ 67500 , ₹ 32500

NOTE – This sum can be solved at ease using concepts of ALLIGATIONS & MIXTURES



option b

04 - LINEAR INEQUALITIES

Q1

01. DEC 2023 , 2020 , JUNE 2010

solution of the inequality

$$\frac{5-2x}{3} \leq \frac{x}{6} - 5$$

a) $x \geq 8$ b) $x \geq 7$ c) $x \leq 80/3$

d) $x \geq 40/3$

$$\frac{5-2x}{3} \leq \frac{x}{6} - 5$$

$$\frac{5-2x}{3} \leq \frac{x-30}{6}$$

$$5-2x \leq \frac{x-30}{2}$$

$$10-4x \leq x-30$$

$$40 \leq 5x$$

$$8 \leq x$$

$x \geq 8$ option a

02. DEC 2022

$2x+5 > 3x+2$ and $2x-3 \leq 4x-5$

then x can take from the following values

a. 3 b. −1 c. 2 d. −3

$2x+5 > 3x+2$	$2x-3 \leq 4x-5$
$5-2 > 3x-2x$	$-3+5 \leq 4x-2x$
$3 > x$	$2 \leq 2x$
$x < 3$	$1 \leq x$

$1 \leq x < 3$

From the options , option c ,

$x = 2 \in [1,3)$ option c

03. DEC 2023 _ MTP II

$$2 \leq \frac{3x-2}{5} \leq 4 \ , \ x \in N$$

Solve for x

a. {5,6,7} b. {3,4,5,6} c. {4,5,6}

d. {4,5,6,7}

$$2 \leq \frac{3x-2}{5} \leq 4 \ , \ x \in N$$

$$10 \leq 3x-2 \leq 20$$

$$10+2 \leq 3x \leq 20+2$$

$$12 \leq 3x \leq 22$$

$$4 \leq x \leq 7.3$$

$x = 4 , 5 , 6 , 7$ option d

04. NOV 2019

Solution set of the inequation $x+2>0$ and $2x-6>0$ is

a. $(-2,\infty)$ b. $(3,\infty)$ c. $(-\infty,-2)$

d. $(-\infty,-3)$

$x+2 > 0$ & $2x-6 > 0$

$x > -2$ $x > 3$

to satisfy $x > -2$ & $x > 3$

simultaneously , $x > 3$ i.e $x \in (3,\infty)$

option c

Q2

01. JUNE 2022

a labour can be paid under two methods given below

1. ₹ 600 fixed and ₹ 50 per hour
2. ₹ 170 per hour

If a labour job work takes 'x' hours to complete , find out the value of x for which the method (ii) gives the labour better wages

a. 6 b. 4 c. 3 d. 2

Scheme I : Total wages = 600 + 50x

Scheme II : Total wages = 170x

Since scheme II must offer better wages to the labour ,

$170x > 600+50x$

$120x > 600$

$x > 5$ option a

02. JUNE 2023

the largest side of a triangle is 3 times the shortest side and the third side is 4 cm shorter than the largest side . If the perimeter of the traingle is at least 59 cm , what is the length of the shortest side

a) less than 7 cm
b) greater than or equal to 7 cm
c) less than 9 cm
d) greater than or equal to 9 cm

let the shortest side = x cm
largest side = 3x
3rd side = 3x−4

Since perimeter is at least 59 cm

$x+3x+3x-4 \geq 59$

$7x-4 \geq 59$

$7x \geq 63$, $x \geq 9$ option d

03. JUNE 2019

employer recruits experienced (x) and fresh workmen(y) for his under the condition that he cannot employ more than 11 people. x and y can be related by the inequality

a. $x+y \neq 11$

b. $x+y \leq 11$, $x \geq 0, y \geq 0$

c. $x+y \geq 11$, $x \geq 0, y \geq 0$

d. none of these option b

04. DEC 2011 2012 ,JUNE 2023, 2024

in a garment factory , an average experienced tailor can stitch 5 shirts while a fresh tailor can stitch 3 shirts daily , but the employer has to maintain an output of at least 30 shirts stitched per day . This can be formulated as

a. $5x+3y \leq 30$

b. $5x+3y > 30$

c. $5x+3y \geq 30$, $x \geq 0$, $y \geq 0$

d. $5x+3y \leq 30$, $x \geq 0$, $y \geq 0$

x experienced tailors stitch @ 5 shirts/day y fresh tailors stitch @ 3 shirts/day

total shirts = 5x+3y

employer needs at least 30 shirts per day

Hence

$5x+3y \geq 30$, x , $y \geq 0$ option c

05. DEC 2023 ,

A fertilizer company produces two types of fertilizers called grade I and grade II . Each of these types is processed through a critical chemical plant unit . The plant has maximum of 180 hours available in a week . Manufacturing one bag of grade I fertilizer requires 4 hours in the plant . Manufacturing one bag of grade II fertilizer requires 10 hours in the plant . express this using linear inequalities

a. $2x_1+5x_2 \leq 180$

b. $4x_1+10x_2 > 180$

c. $2x_1+5x_2 > 180$

d. $4x_1+10x_2 \leq 180$

	Grade I x_1 units	Grade II x_2 units	Time available
Time Reqd.	4 hours	10 hours	180 hours

$4x_1+10x_2 \leq 180$ option d

06. SEPT 2024 , JUNE 2017

a dietician wishes to mix two kinds of food so that vitamin contents of the mixture is at least 45 units of carb , 25 units of protien ,15 units of fat and 15 units of fibre . content of each food is shown below

	Carb	Protien	Fat	Fibre
Food I	20	5	3	2
Food II	10	2	4	5

Assuming x units of food I is mixed with y units of food II , the situation can be expressed as

a. $20x+10y\leq45$, $5x+2y\geq25$,
$3x+4y\leq15$, $2x+5y\geq15$, $x\geq0$, $y\geq0$

b. $20x+10y\leq45$, $5x+2y\geq25$,
$3x+4y\leq15$, $2x+5y\leq15$, $x\geq0$, $y\geq0$

c. $20x+10y\geq45$, $5x+2y\geq25$
$3x+4y\geq15$, $2x+5y\geq15$, $x\geq0$, $y\geq0$

d. $20x+10y\leq45$, $5x+2y\leq25$
$3x+4y\leq15$, $2x+5y\leq15$, $x\geq0$, $y\geq0$

	Carb	Protien	Fat	Fibre
Food I (x)	20	5	3	2
Food II(y)	10	2	4	5
Min Reqd.	45	25	15	15

CONSTRAINTS

$20x+10y\geq45$, $5x+2y\geq25$
$3x+4y\geq15$, $2x+5y\geq15$, $x\geq0$, $y\geq0$

option **C**

07. JUNE 2022 – MTP I

the rules and regulations demand that the employer should employ not more than 5 experienced hands (x) to 1 fresh one (y) . This is represented by

a. $y \geq x/5$

b. $5y \leq x$

c. $y \geq 5x$

d. none

for 1 fresh employee , we can employ max 5 experienced employees

for y fresh employees , we can employ max 5y experienced hands

$\therefore x \leq 5y$

$y \geq x/5$ option a

08. DEC 2021

XYZ company has a policy for recruitment as it should not recruit more than 8 men (x) to three women (y). Express this as an inequality

a. $3y \geq 8x$ b. $3y \leq x/8$ c. $8y \geq 3x$

d. $8y \leq 3x$

for 3 women , recruiter can employ max 8 men

according to this ,

for 1 women recruiter can employ max $^8/_3$ men

Hence ,
for y women recruiter can employ max $^8/_3$ y men

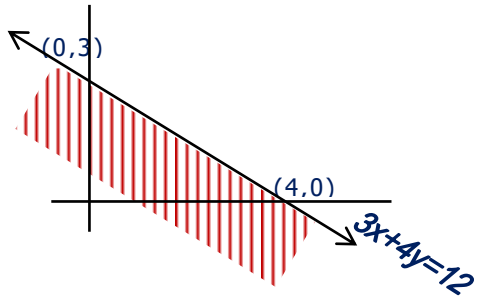
$\therefore x \leq \frac{8y}{3}$

$3x \leq 8y$ option C

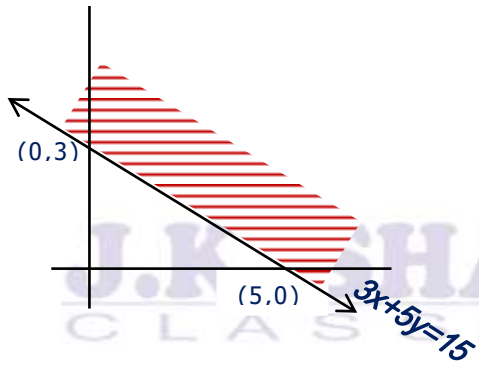
Q3

LET ME GIVE YOU FEW EXAMPLES OF HOW TO EXPRESS A LINEAR INEQUALITY GRAPHICALLY .

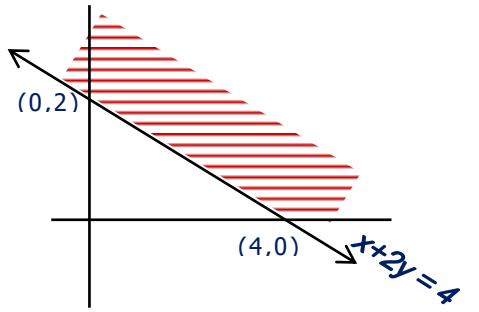
$3x+4y \le 12$, cuts axes at (4,0),(0,3)



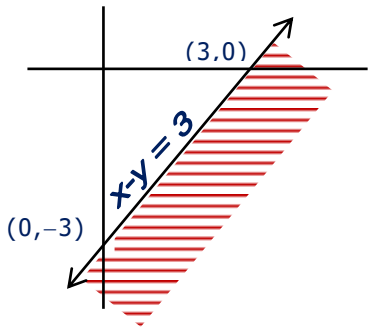
$3x+5y \ge 15$, cuts axes at(5,0),(0,3)



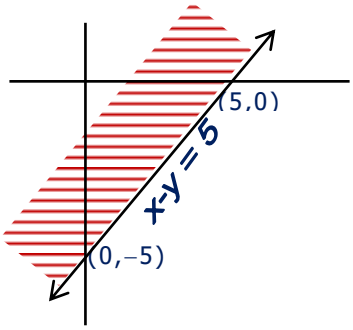
$x+2y \ge 4$, cuts axes at (4,0) ,(0,2)



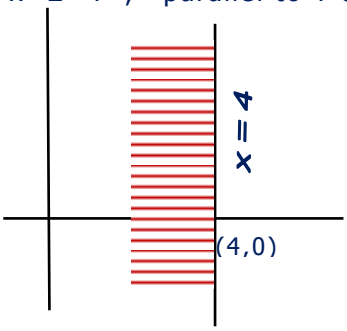
$x-y \ge 3$, cuts axes at (3,0) ,(0,−3)



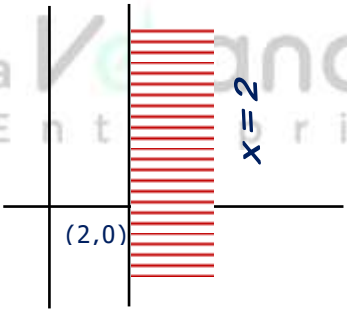
$x-y \le 5$, cuts axes at (5,0),(0,−5)



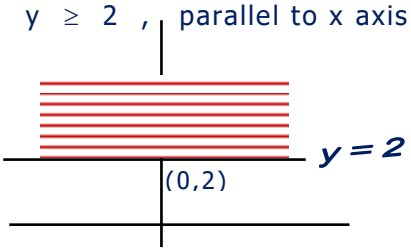
$x \le 4$, parallel to Y axis



$x \ge 2$, parallel to Y axis



$y \ge 2$, parallel to x axis



NOTE

IN THE COMING SET OF QUESTIONS , YOU WILL BE GIVEN A SET OF INEQUALITIES AND WILL BE ASKED TO FIND WHICH AREA IN THE GRAPH PROVIDED REPRESENTS THE GIVEN SET OF INEQUALITIES.

DONT'S

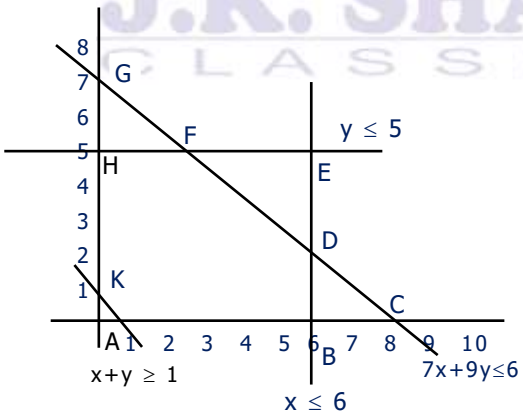
DO NOT CHECK THE LINE DETAILS IN THE GRAPH AS TO WHETHER THE LINE IS CUTTING THE X AND Y AXIS AT THE CORRECT POINTS .

DO'S

JUST MARK WHICH SIDE (ORIGIN / NON ORIGIN) THE SOLUTION SET SHOULD FALL FOR EACH OF THE INEQUALITIES AND FIND THE COMMON REGION WHICH SHOULD TAKE ONLY FEW SECONDS

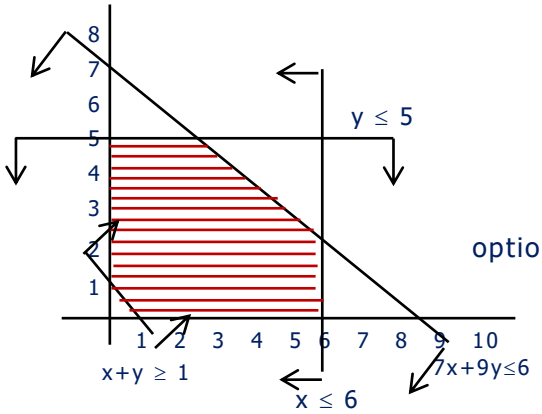
01. JUNE 2014

$x+y \geq 1$, $7x+9y \leq 63$, $0 \leq x \leq 6$, $0 \leq y \leq 5$ is represented by



a. BCDB , DEFD b. HFGH

c. ABDFHKA d. unbounded



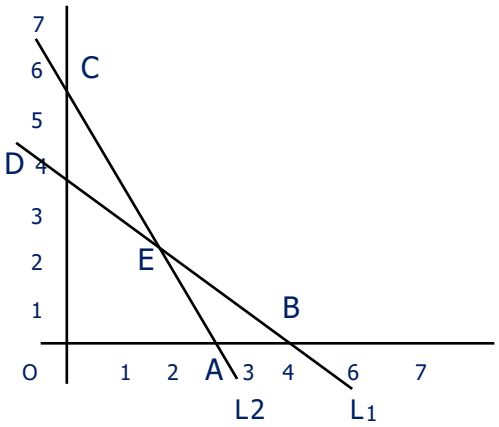
option **C**

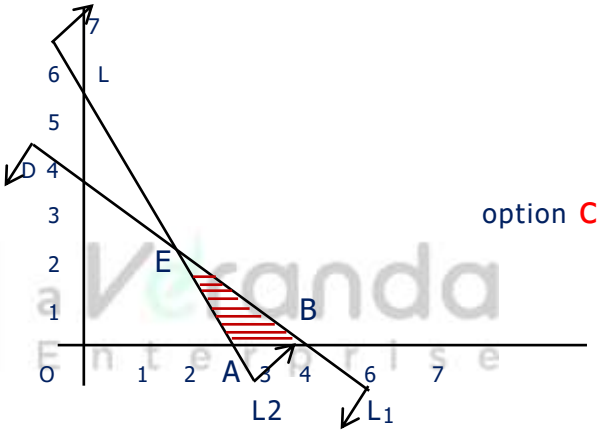
02. JUNE 2019 ,

common region represented by the following inequalities

L1 : $x_1+x_2<4$, L2 : $2x_1+x_2>6$

a. OABC b. ΔBCE c. ΔABE d. none



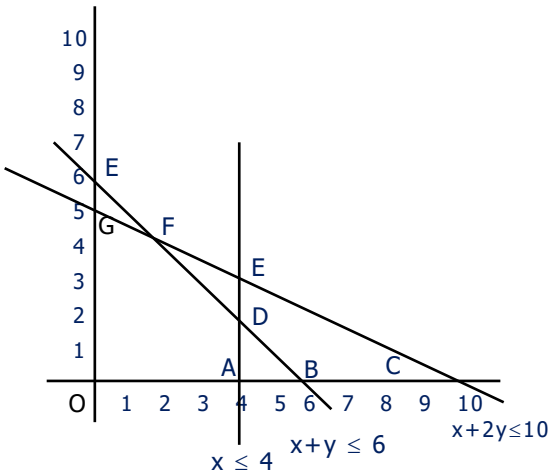


option **C**

03. JUNE 2018

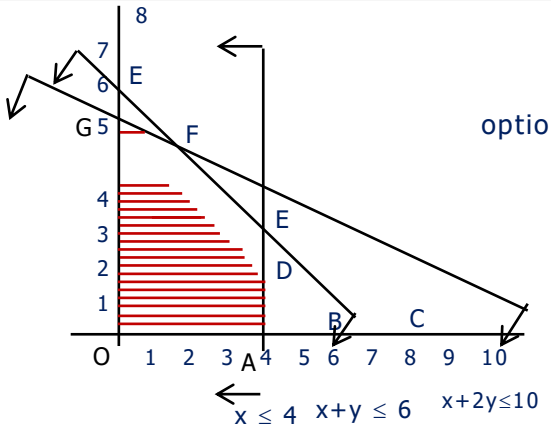
In the following diagram the region represented by the inequalities

$x+2y \leq 10$, $x+y \leq 6$, $x \leq 4$, $x,y \geq 0$



a. ADBA b. BDEC c. OADFGO

d. DEFD

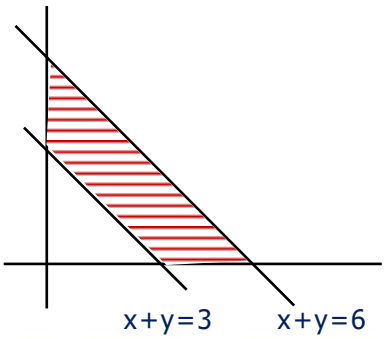


option c

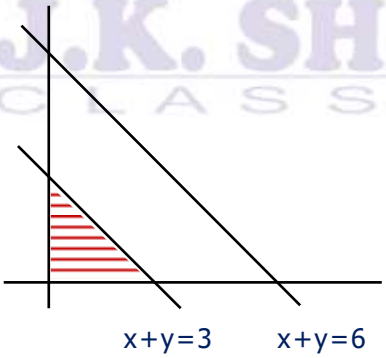
04. JUNE 2016

Common region of $x+y \leq 6$, $x+y \geq 3$, $x \geq 0$, $y \geq 0$ is shown by

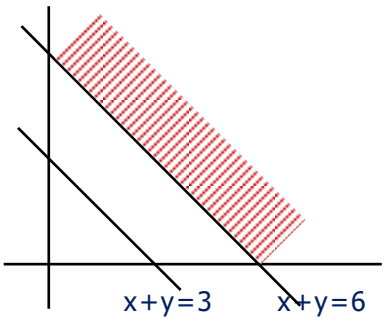
a.



b.



c.



d. none of these

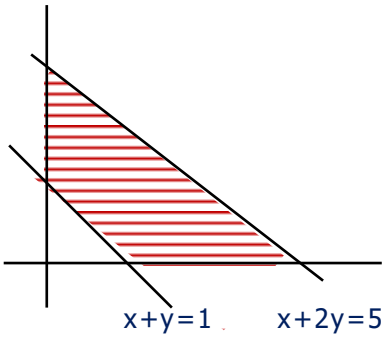
Inequality	shaded region
$x+y \leq 6$	origin side
$x+y \geq 3$	non origin side
$x,y \geq 0$	I Quadrant

option a confirms to the above

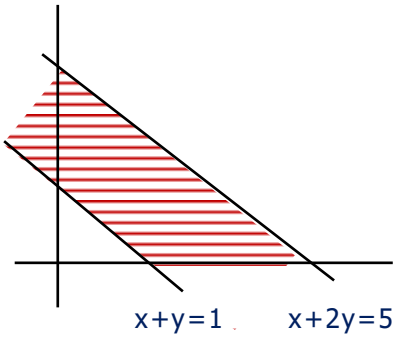
05. DEC 2016

The inequalities $x+2y \leq 5$, $x+y \geq 1$, $x \geq 0$, $y \geq 0$ represents the region

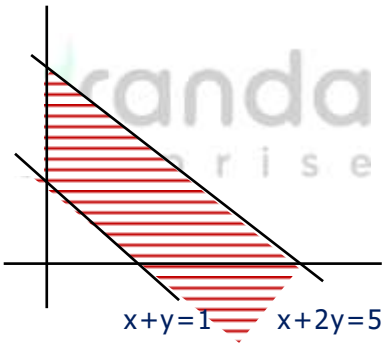
a.



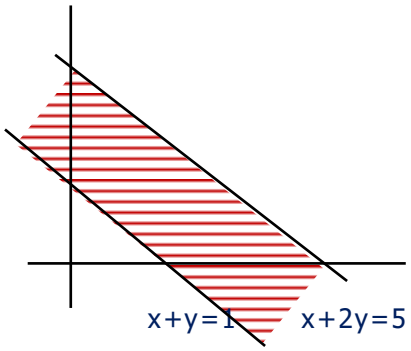
b.



c.



d.



Inequality	shaded region
$x+2y \leq 5$	origin side
$x+y \geq 1$	non origin side
$x,y \geq 0$	I Quadrant

option a confirms to the above

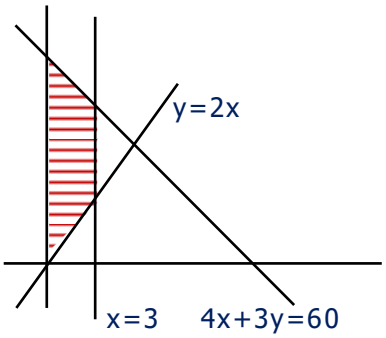
option a

06. JUNE 2017

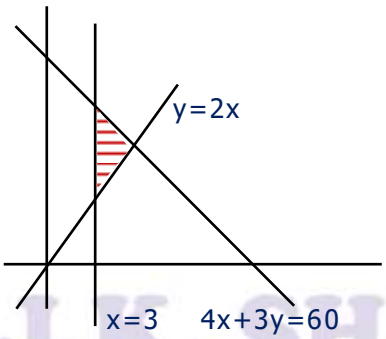
The shaded region represented by the inequalities

$4x+3y \le 60$, $y \ge 2x$, $x \ge 3$, $x \ge 0$, $y \ge 0$

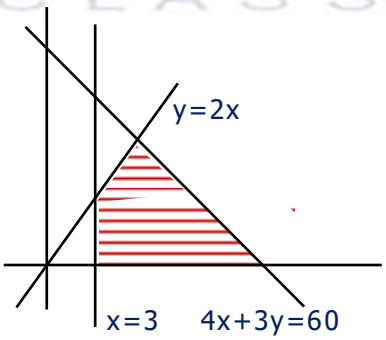
a.



b.



c.



d. none of these

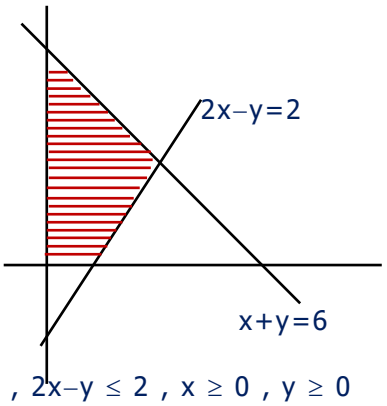
Inequality	shaded region
$4x+3y \le 60$	origin side
$y \ge 2x$	towards Y axis
$x \ge 3$	towards non origin
$x,y \ge 0$	I Quadrant

option b confirms to the above

option **b**

07. DEC 2015

By lines $x+y = 6$, $2x-y = 2$, the common region shown in the diagram refers to



a. $x+y \ge 6$, $2x-y \le 2$, $x \ge 0$, $y \ge 0$

b. $x+y \le 6$, $2x-y \le 2$, $x \ge 0$, $y \ge 0$

c. $x+y \le 6$, $2x-y \ge 2$, $x \ge 0$, $y \ge 0$

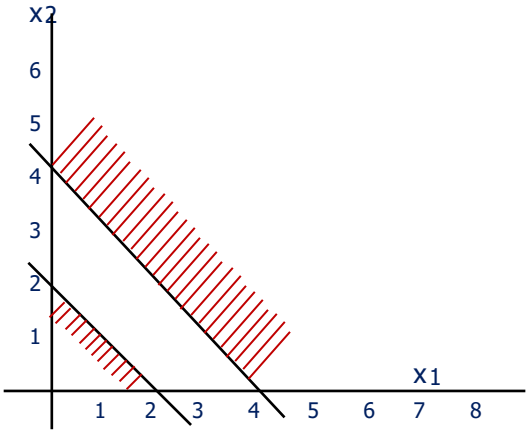
d. none of these

Since $x+y = 6$ & $2x-y = 2$ both have shaded regions towards origin ,

option **b**

08. DEC 2021

The region indicated by the shaded portion in the graph is expressed by the inequalities



a. $x_1+x_2 \le 2$, $x_1+x_2 \ge 4$, $x_1,x_2 \ge 0$

b. $x_1+x_2 \le 2$, $x_1+x_2 \le 4$, $x_1,x_2 \ge 0$

c. $x_1+x_2 \ge 2$, $x_1+x_2 \ge 4$, $x_1,x_2 \ge 0$

d. none of the above option **a**

09. DEC 2018

on solving

$5x+y \le 100$, $x+y \le 60$, $x \ge 0$, $y \ge 0$

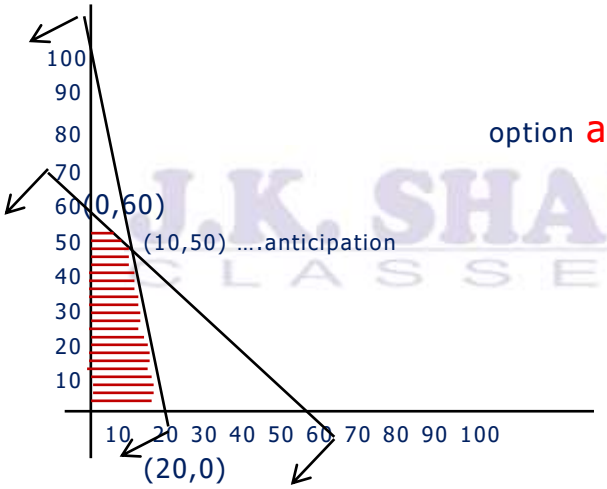
we get following situations

a. (0,0),(20,0),(10,50),(0,60)

b. (0,0),(60,0),(10,50),(0,60)

c. (0,0),(20,0),(0,100),(10,50)

d. None of these



option a

NOTE

Most of the corners except (10,50) in option a matches with corners of the common solution set in the graph .

For (10,50) , you can check , it satisfies the lines $x+y=60$ & $5x+y=100$ which probably we need to skip in real time . Hence with good amount of confirmation we say , option a

10. JUNE 2023

On solving the inequalities

$6x+y \ge 18$, $x+4y \ge 12$, $2x+y \ge 10$,

$x,y>0$, we get following solution

a. (0,18) , (12,0) , (4,2) , (2,6)

b. (3,0) , (0,3) , (4,2) , (2,6)

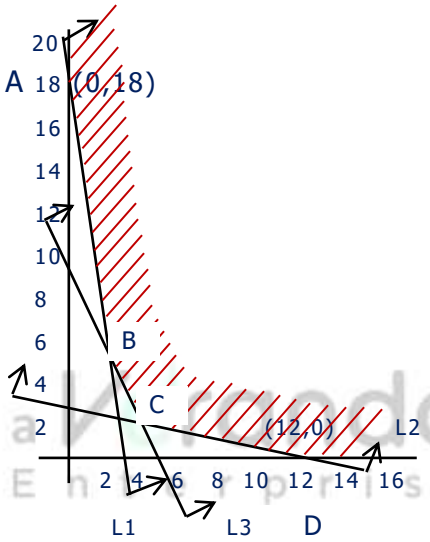
c. (5,0) , (0,10) , (4,2) , (2,6)

d. (0,18),(12,0) , (4,2) , (0,0) , (2,6)

L_1 : $6x+y \ge 18$ cuts axis at (3,0),(0,18)

L_2 : $x+4y \ge 12$ cuts axis at (12,0),(0,3)

L_3 : $2x+y \ge 10$ cuts axis at (5,0),(0,10)



NOTE Since a student must have drawn rough , corners B and C cannot be read .So then how did we arrive at option . We knew 2 corners (0,18) & (12,0) for sure . With this options b and c were OUT .Left were options a & d Since (0,0) was never one of the corners of the common solution set , option d got striked out . Finally option a

However before zeroing on option a , you can always check

(2,6) satisfies L1 : $6x+y = 18$ &

L3 : $2x+y = 10$

(4,2) satisifies L2 : $x+4y = 12$ &

L3 : $2x+y = 10$

In real time skip the above check

11. NOV 2019

The solution of the set of inequations

$2x+y \geq 12$, $5x+8y \geq 74$, $x+6y \geq 24$, $x, y \geq 0$ is given by

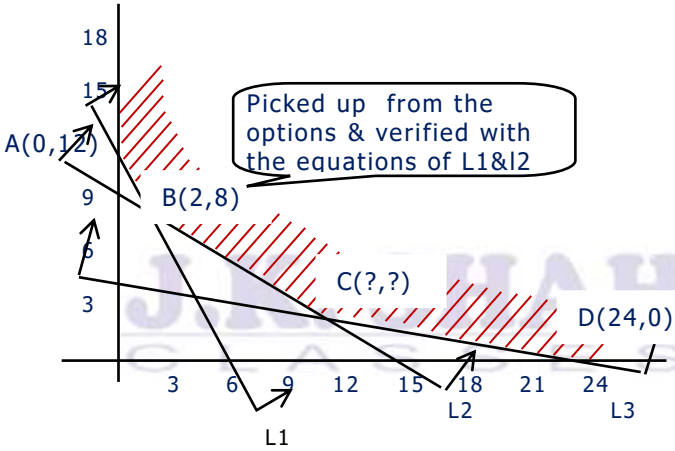
a) (24,0) (126/11,23/11) , (2,8) , (0,12)

b) (0,24) , (2,8) , (0,12) , (126/11,23/11)

c) (8,4) , (2,8) , (0,12), (0,24)

d) (8,4) , (0,0) , (0,6) , (2,0)

$L_1 : 2x+y \geq 12$	$L_2 : 5x+8y \geq 74$	$L_3 : x+6y \geq 24$
(6,0) , (0,12)	(14.8,0) , (0,9.2)	(24,0), (0,4)
NON- ORIGIN SIDE	NON- ORIGIN SIDE	NON- ORIGIN SIDE



Lets strike out the options

Option b – (0,24)

Option c – (0,24)

Option d – (0,0) , Hence option a

For more better confirmation of all corners , you can check for

(2,8) satisfies $2x+y = 12$, $5x+8y = 74$

$(^{126}/_{11},^{23}/_{11})$ satisfies $5x+8y=74$, $x+6y=24$

However in real time we need to skip this check .

12. JULY 2021

The common region in the graph of inequalities $x+y \leq 4$, $x-y \leq 4$, $x \geq 2$ is

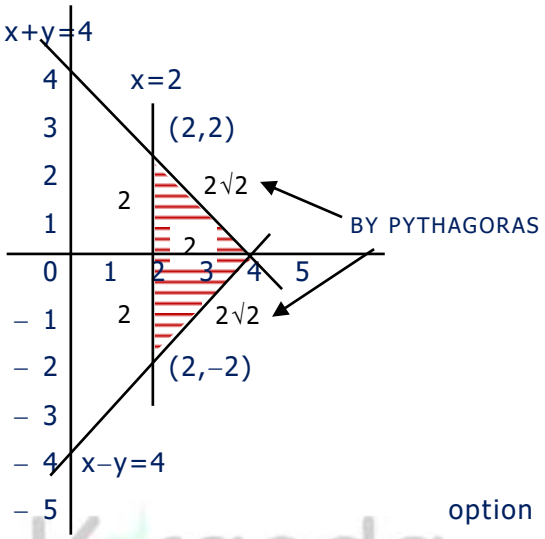
a. Equilateral Δ b. Isosceles Δ

c. Quadrilateral d. Square

$x+y = 4$ cuts axis at (4,0),(0,4)

$x-y = 4$ cuts axis at (4,0),(0,–4)

$x= 2$ // to Y axis through (2,0)



option b

05 - EQUATIONS

Q1

01. DEC 2022

the solution of the following system of linear equation $2x-5y+4=0$ & $2x+y-8=0$ is

a. (2–3) b. (1–4) [c.] (3,2) d.(–2,2)

Lets check which of the above solution satisifies both the equations

	$2x-5y+4=0$	$2x+y-8=0$
option a (2–3)	$4+15+4\neq0$	
option b (1–4)	$2+20+4\neq0$	
option c (3,2)	$6-10+4=0$	$6+2-8=0$ ✓

option c

02. DEC 2014

$x+5y = 33$, $\frac{x+y}{x-y} = \frac{13}{3}$

a. (4,8) [b.] (8,5) c. (4,16) d.(16,4)

Lets check which of the above solution satisifies both the equations

	$x+5y = 33$	$\frac{x+y}{x-y} = \frac{13}{3}$
a (4,8)	$4+20 = 33$ X	
b (8,5)	$8+25 = 33$ ✓	$\frac{8+5}{8-5} = \frac{13}{3}$ ✓

option b

03. JUNE 2017 , MAY 2018

$\frac{3}{x+y} + \frac{2}{x-y} = -1$, $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$

a. (2,1) [b.] (1,2) c. (–1,2) d.(–2,1)

	Equation1	Equation 2
a (2,1)	$\frac{3}{3}+\frac{2}{1} = 3$ X	
b (1,2)	$\frac{3}{3}+\frac{2}{-1} = -1$ ✓	$\frac{1}{3} - \frac{1}{-1} = \frac{4}{3}$ ✓

option b

04. JUNE 2022

Solve for x and y

$\left(\frac{b}{a}\right)x + \left(\frac{a}{b}\right)y = a^2 + b^2$

& $x + y = 2ab$

a. (a/b,b/a) [b.] (ab,ab)

c. 3ab , –ab d. –3ab , ab

Using options

trying option b

$ab+ab = 2ab$ ✓

$\left(\frac{b}{a}\right)ab + \left(\frac{a}{b}\right)ab = a^2 + b^2$ ✓

option b

05. MAY 2018 , JUNE 2016

if $2^{x+y} = 2^{2x-y} = \sqrt{8}$, x,y ?

[a.] 1,½ b. ½,1 c. ½,½

$2^{x+y} = 2^{2x-y} = 2^{3/2}$

$x+y = {}^3/_2$, $2x-y = {}^3/_2$

trying option a

$1+½ = {}^3/_2$

$2-½ = {}^3/_2$ ✓

option a

06. DEC 2019

Solve $4^x.8^y = 128$, $3^x/27^y = 1/3$

[a.] 2,1 b. –2,1 c. 2,–1 d. 1,2

	$4^x.8^y = 128$	$3^x/27^y = 1/3$
a. 2,1	$16.8 = 128$ ✓	$9/27 = 1/3$ ✓

option a

07. DEC 2020

SOLVE $m + \sqrt{m} = 6/25$

a. 2/25 [b.] 1/25 c. 3/25 d. 1

Since option b is a perfect square , would try with 'b' first

$m + \sqrt{m} = \frac{6}{25}$

$\frac{1}{25} + \frac{1}{5} = \frac{6}{25}$

$\frac{1+5}{25} = \frac{6}{25}$ ✓ option b

08. DEC 2023

$2^x = 4^y = 8^z$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$

then the value of z =

a) $^7/_{16}$ b. $^7/_{32}$ [c.] $^7/_{48}$ d $^7/_{64}$

$2^x = 4^y = 8^z$

$2^x = 2^{2y} = 2^{3z}$

$x = 2y = 3z$

$\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$

$\frac{1}{6z} + \frac{1}{6y} + \frac{1}{6z} = \frac{24}{7}$ since $x = 3y$, $2z = 6z$ since $2y=3z$, $4y = 6z$

$\frac{3}{6z} = \frac{24}{7}$

$z = \frac{3 \times 7}{6 \times 24} = \frac{7}{48}$ option c

09. JUNE 22 MTP I

if abc = 2 , then the value of

$$\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{b}{2}+c^{-1}} + \frac{1}{1+c+a^{-1}}$$

[a.] 1 b. 2 c. 3 d. ½

abc = 2 , assume a = 1 , b = 2 & c = 1

$$\frac{1}{1+a+\frac{2}{b}} + \frac{1}{1+\frac{b}{2}+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}}$$

$$\frac{1}{1+1+\frac{2}{2}} + \frac{1}{1+\frac{2}{2}+\frac{1}{1}} + \frac{1}{1+1+\frac{1}{1}}$$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ option a

10. JUNE 2016

$$\frac{x^2-(y-z)^2}{(x+z)^2-y^2} + \frac{y^2-(x-z)^2}{(x+y)^2-z^2} + \frac{z^2-(x-y)^2}{(y+z)^2-x^2} = ?$$

a. 0 [b.] 1 c. –1 d. none

LET's TRY THE FIRST TERM

1. $\frac{x^2-(y-z)^2}{(x+z)^2-y^2} = \frac{(x+y-z)(x-y+z)}{(x+z+y)(x+z-y)}$

$= \frac{x+y-z}{x+y+z}$

Similarly , by CYCLIC BEHAVIOUR ,

2. $\frac{y^2-(x-z)^2}{(x+y)^2-z^2} = \frac{y-x+z}{x+y+z}$

3. $\frac{z^2-(x-y)^2}{(y+z)^2-x^2} = \frac{z+x-y}{x+y+z}$

(1)+(2)+(3) $= \frac{x+y-z+y-x+z+z+x-y}{x+y+z}$

$= \frac{x+y+z}{x+y+z} = 1$ option b

11. JULY 21

if $xy + yz + zx = -1$ then the value of

$$\frac{x+y}{1+xy} + \frac{z+y}{1+zy} + \frac{x+z}{1+xz} =$$

a) xyz b $\frac{-1}{yz}$ [c] $\frac{1}{xyz}$ d $\frac{1}{x+y+z}$

$xy+yz+zx = -1$

$1+xy = -(yz+zx)$

LETS WORK ON FIRST TERM ONLY

$$\frac{x+y}{1+xy} = \frac{x+y}{-(yz+zx)} = \frac{x+y}{-z(x+y)} = \frac{-1}{z}$$

By cyclic behaviour of the terms ,

$$\frac{x+y}{1+xy} + \frac{z+y}{1+zy} + \frac{x+z}{1+xz}$$

$$= \frac{-1}{z} \frac{-1}{x} \frac{-1}{y}$$

$$= -\left(\frac{1}{z} \frac{+1}{x} \frac{+1}{y}\right)$$

$$= -\frac{xy+yz+zx}{xyz}$$

$$= -\frac{(-1)}{xyz}$$

$$= \frac{1}{xyz}$$

option c

Q2

01. DEC 2022

a man wants to cut three lengths from a single piece of board of length 91 cm . Second length is 3 cm longer than shortest and 3rd is twice as much as shortest one . Find the length of shortest piece

a. 15 b. 18. c. 20 [d.] 22

Let shortest piece = x cm
Second piece = x+3 cm
Third piece = 2x cm

$x+x+3+2x = 91$
$4x = 88$
$x = 22$

option d

02. DEC 2023

Divide 27 into 2 parts so that 5 times the first and 11 times the second together equal to 195 , then ratio of first and second part is

[a)] 17 :10 b) 15 :12 c) 14 :13 d) 16 :11

let 2 parts of 27 be x , 27−x

$5x + 11(27-x) = 195$

$5x + 297-11x = 195$

$297-6x = 195$

$102 = 6x \Rightarrow x = 17$

2 parts of 27 are 17 , 10

Ratio of 2 parts = 17 :10 option a

03. DEC 2022

if the cost of 3 bags and 4 pens is 257 where as the cost of 4 bags and 3 pens is 324 , then the cost of one bag is

a. 8 b. 24 c. 32 [d.] 75

let cost of bag = x/- , pen = y/-

$3x+4y = 257 \times 3 \quad 9x+12y = 771$
$4x+3y = 324 \times 4 \quad 16x+12y = 1296$
$7x = 525$
$x = 75$

option d

04. JULY 2021

cost of 2 oranges and 3 apples is 28 . If the cost of an apple is doubled , then the cost of 3 oranges and 5 apples is 75 . Original cost of 7 oranges and 4 apples

[a.] 59 b. 47 c. 71 d. 63

cost of orange = x/-

cost of apple = y/-

$2x+3y = 28$

$3x+5(2y) = 75 \ldots 3x+10y = 75$

Solving

$2x+3y = 28 \times 3 \quad 6x+9y = 84$

$3x+10y = 75 \times 2 \quad 6x+20y = 150$

$11y = 66$

y = 6/- , x = 5/-

cost of 7 oranges and 4 apples

$= 7x+4y$

$= 7(5)+4(6)$

$= 35+24 = \text{Rs } 59$ option a

05. SEPT 2024

A person purchased 2 apples and 5 bananas at the cost of Rs 90 . Later he visited another shop where shopkeeper told him that if you give me Rs 50 and 1 banana , I can give you 3 apples . he agreed to the deal . What is the cost of one apple and one banana

a.15,10 b. 10,15 c. 10,20 [d.] 20,10

cond 1 : $2x+5y = 90$

cond 2 : $50+1y = 3x$

$3x-y = 50$

	$2x+5y=90$	$3x-y=50$
a.15,10	$30+50=80$ ✗	
b.10,15	$20+75=95$ ✗	
c.10,20	$20+100$ ✗	
d.20,10	$40+50=90$ ✓	$60-10=50$ ✓

option d

06. JUNE 2024

fraction becomes 1 when 3 is added to numerator & 1 is added to denominator . But when numerator and denominator are decreased by 2 and 1 respectively it becomes ½ . The denominator of the fraction is

a. 5 b. 6 [c.] 7 d. 8

$\frac{x+3}{y+1} = 1 \Rightarrow x+3 = y+1$

$x-y = -2 \quad \ldots.(1)$

$\frac{x-2}{y-1} = \frac{1}{2} \Rightarrow 2x-4 = y-1$

$2x-y = 3 \quad \ldots.(2)$

solving $2x-y = 3$

$-\ x-y = -2$

$+ \quad +$

$x = 5$, $y = 7$

option C

07. JUNE 2023

the age of a persons is FOUR TIMES the sum of the ages of his two son's and after 10 years his age will be double the sum of their ages . Find his present age

A. 56 B. 45 [C.] 60 D. 64

	FATHER	SON1+SON2
PRESENT	4x	x
10 YRS ↓	4x+10	x+(10+10)
		x+20

FATHER IS THRICE THE SUM OF SON'S AGES

$4x+10 = 2(x+20)$

$x = 15$

Father's present age = $4x = 60$ YRS

option C

08. JUNE 2015

the age of a persons is 8 years more than thrice the sum of the ages of his two grandson's who were twins and after 8 years his age will 10 years more than twice the sum of their ages . Find his when twins were born

A. 86 [B.] 73 C. 68 D. 63

	PERSON	TWINS(SUM)
PRESENT	6x+8	2x
8 YRS ↓	6x+16	2x+16

PERSON IS 10 MORE THAN TWICE SUM OF AGES OF TWINS

$6x+16 = 2(2x+16)+10$

$6x+16 = 4x+42 \quad \Rightarrow x = 13$

Persons present age = $6x+8 = 86$ YRS

Person sage when twins were born

$= 86-13 = 73$ YRS

option b

09. JUNE 2019

A NUMBER CONSISTS OF TWO DIGITS . THE DIGIT IN THE ONE'S PLACE IS 3 TIMES THE DIGIT IN THE TEN'S PLACE . IF 36 IS ADDED , THE DIGITS GET REVERSED . THE NUMBER IS

A. 62 [B.] 26 C. 39 D. NONE

Option	62	26	39
Cond 1	✗	✓	✓
Cond 2		+36	
		62 ✓	

option b

10. SEPT 2024

A NUMBER CONSISTS OF TWO DIGITS . THE DIGIT IN THE TEN'S PLACE IS 3 TIMES THE DIGIT IN THE ONE'S PLACE . IF 54 IS SUBTRACTED , THE DIGITS GET REVERSED . THE NUMBER IS

A. 39 B. 62 C. 93 D. 31

Option	39	62	93
Cond 1	X	✓	✓
Cond 2		−54	−54
		8 X	39 ✓

option c

11. JUNE 2017 , MAY 2018

IF SIDES OF AN EQUILATERAL TRIANGLE ARE SHORTENED BY 3 , 4 , 5 UNITS RESPECTIVELY , A RIGHT TRIANGLE IS FORMED . THE SIDE OF EQUILATERAL TRIANGLE IS

A. 6 B. 7 [C.] 8 d. 10

Using options

a) 6–3 , 6–4 , 6–5

= 3,2,1 , NOT A RIGHT Δ

b) 7–3 , 7–4 , 7–5

= 4,3,2 , NOT A RIGHT Δ

c) 8–3 , 8–4 , 8–5

= 5,4,3 , IS A RIGHT Δ

option c

12. DEC 2021

in a MCQ type question paper of 100Q's of 1 mark each , candidate score 60% marks . Candidate attempted all Q's and there was a penalty of 0.25marks for wrong answers . The difference between the number of right answers and wrong answers will be

a. 32 [b.] 36 c. 40 d. 38

let no. of Q's answered correctly = n

no. Of Q's answered wrongly = 100–n

score earned = nx1

score lost in penalty = (100–n)x0.25

$n - 0.25(100-n) = 60$

$n - 25 + 0.25n = 60$

$1.25n = 85$

$n = 68$,

Q's answered correctly = 68

Wrongly = 32

Difference = 36

option b

13. JUNE 2015

NUMBER OF STUDENTS IN EACH SECTION OF A SCHOOL IS 36 . AFTER ADMITTING 12 NEW STUDENTS , FOUR NEW SECTIONS ARE STARTED . IF TOTAL STUDENTS IN EACH SECTION NOW IS 30 , THEN NUMBER OF SECTION INITIALLY WERE

A. 6 B. 10 C. 14 D. 18

NO. OF SECTIONS	NO OF STUDENTS PER SECTION	TOTAL STUDENTS
N	36	36N + 12
N+4	30	30.(N+4)

36N + 12 = 30(N+4)

36N + 12 = 30N + 120

N = 18 option d

14. JUNE 2015

A PERSON ON A TOUR HAS RS 9600 FOR HIS EXPENSES . BUT THE TOUR GOT EXTENDED FOR ANOTHER 16 DAYS SO HE HAS TO CUT DOWN HIS DAILY EXPENSES BY RS 20 . THE ORIGINAL DURATION OF THE TOUR IS

A. 48 B. 64 C. 80 D. 96

AMT	NO. OF DAYS	EXPENSES / DAY
9600	N	9600/N
9600	N+16	9600/N+16

$\frac{9600}{N} - \frac{9600}{N+16} = 20$

$9600 \cdot \frac{16}{N(N+16)} = 20$

N(N+16) = 7680

a. 48(48+16) = 3072 X

b. 64(64+16) = 5120 X

c. 80(80+16) = 7680 ✓

option C

15. JULY 2021

A vessel contained a solution of acid and water in which water was 64% . Four litres of the solution were taken out of the vessel and the same quantity of water was added . If the resulting solution contains 30% acid , the quantity of solution in the beginning in the solution

a. 12 b. 36 c. 24 d. 27

Initial solution = x ltrs

acid = 36% = 0.36x

4 ltrs removed

acid removed in 4 ltrs

= 0.36(4) = 1.44 ltrs

acid left = 0.36x–1.44

4 ltrs water added

Solution qty = x ltrs

acid concentration has reduced to 30%

$\frac{0.36x-1.44}{x} \times 100 = 30$

0.36x–1.44= 0.30x

0.06x = 1.44

x = 24 ltrs option C

06 - QUADRATIC & CUBIC EQUATION

Q1 Q.E.: $x^2 - Sx + P = 0$

01. DEC 2015 , MAY 2018

If α and β be the roots of Q.E. such that $\alpha+\beta = -2$ and $\alpha\beta = -3$. Find the Q.E .

a. $x^2+2x-7 = 0$ [b.] $x^2+2x-3 = 0$

c. $x^2-2x-3 = 0$ d. $x^2-2x+7 = 0$

$S = -2$, $P = -3$

REQUIRED Q.E.

$x^2 - Sx + P = 0$

$x^2 + 2x - 3 = 0$ option b

02. JUNE 2023

if α and β are roots of the equation $x^2 - 2x - 3 = 0$, then equation whose roots are $\alpha+\beta$ and $\alpha-\beta$ is

a. $x^2-6x-8=0$ [b.] $x^2-6x+8=0$

c. $x^2+6x+8=0$ d. $x^2+6x-8=0$

SOLUTION

$x^2 - 2x + 3 = 0$

$(x-3)(x+1) = 0$

$\alpha = 3$, $\beta = -1$

QE [$\alpha+\beta$, $\alpha-\beta$]

QE [$3-1$, $3+1$]

QE [2 , 4]

$S = 6$

$P = 8$

REQUIRED Q.E.

$x^2 - Sx + P = 0$

$x^2 - 6x + 8 = 0$ option b

if $\alpha = p+\sqrt{q}$, then $\beta = p-\sqrt{q}$

03. Find quadratic equation whose one root is $6+\sqrt{11}$

SOLUTION

$\alpha = 6+\sqrt{11}$, then

$\beta = 6-\sqrt{11}$

Q.E. $[6+\sqrt{11}, 6-\sqrt{11}]$

$S = 12$

$P = (6+\sqrt{11})(6-\sqrt{11}) = 36-11 = 25$

REQUIRED Q.E.

$x^2 - Sx + P = 0$

$x^2 - 12x + 25 = 0$

04. JUNE 2012

If one of the roots of the equation $x^2+px+a = 0$ is $\sqrt{3}+2$, then find the value of p and a

a. $-4,-1$ b. $4,-1$ [c.] $-4,1$ d. $4,1$

$\alpha = 2+\sqrt{3}$ then

$\beta = 2-\sqrt{3}$

Q.E. $[2+\sqrt{3}, 2-\sqrt{3}]$

$S = 4$

$P = (2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$

REQUIRED Q.E.

$x^2 - Sx + P = 0$

$x^2 - 4x + 1 = 0$

$x^2 + px + q = 0$ $p = -4$, $q=1$

option c

05. DEC 2021

When roots of quadratic equation are α , $^1/_\alpha$, then what will be the quadratic equation

a. $\alpha x^2 - (\alpha^2+1)x + \alpha = 0$

b. $\alpha x^2 - (\alpha^2+1)x + 1 = 0$

c. $\alpha x^2 - \alpha^2 x + 1 = 0$

d. None of these

Q.E. $[\alpha , {}^1/_\alpha]$

$S = \alpha + {}^1/_\alpha = \frac{\alpha^2+1}{\alpha}$

$P = \alpha \cdot {}^1/_\alpha = 1$

REQUIRED Q.E.

$x^2 - Sx + P = 0$

$x^2 - \frac{\alpha^2+1}{\alpha} x + 1 = 0$

$\alpha x^2 - (\alpha^2+1) x + \alpha = 0$ option a

06. JUNE 2012

If airthmetic mean between roots of a quadratic equation is 8 and the geometric mean between them is 5 , then the equation is

a. $x^2-16x-25 = 0$

b. $x^2-16x+25 = 0$

c. $x^2-16x+5 = 0$ d. none of these

A.M.$(\alpha,\beta) = 8$ $\frac{\alpha+\beta}{2} = 8$ $S = 16$

G.M.$(\alpha,\beta) = 5$ $\sqrt{\alpha\beta} = 5$ $P = 25$

REQUIRED Q.E.

$x^2 - Sx + P = 0$

$x^2 - 16x + 25 = 0$ option b

07. DEC 2018

if one root is half of the other of quadratic equation and difference in roots is a , then quadratic equation is

a. $x^2+ax+2a^2=0$

b. $x^2-3ax-2a^2=0$

c. $x^2-3ax+2a^2=0$

d. $x^2+3ax-2a^2=0$

$\alpha = 2\beta$, $\alpha - \beta = a$ $(\alpha > \beta)$

$2\beta-\beta = a$

$\beta = a$

$\therefore \alpha = 2a$

$S = 3a$

$P = 2a^2$

REQUIRED Q.E.

$x^2 - Sx + P = 0$

$x^2 - 3ax + 2a^2 = 0$ option a

Q2 if α , β are roots of the equation $ax^2+bx+c= 0$, $\alpha+\beta = {}^{-b}/_{a}$, $\alpha\beta = {}^{c}/_{a}$

01. JUNE 2022

if roots of $5z^2+13z+y = 0$ are reciprocals of each other , then find the positive value of y .

a. 1/5 b. −1/5 [c.] 5 d. −5

$5z^2+13z+y = 0$, $\alpha = 1/\beta$ i.e $\alpha\beta = 1$

$a = 3$, $b = -14$, $c = k$

$\alpha\beta = \frac{c}{a}$

$1 = \frac{y}{5}$ $\quad y = 5$ option C

02. JUNE 2010

if roots of $3x^2-14x-k = 0$ are reciprocals of each other , then find the value of k .

[a.] −3 b. 0 c. 3 d. 14

$3x^2-14x+k = 0$, $\alpha = 1/\beta$ i.e $\alpha\beta = 1$

$a = 3$, $b = -14$, $c = -k$

$\alpha\beta = \frac{c}{a}$

$1 = \frac{-k}{3}$ $\quad k = -3$ option a

Q3 if α , β are roots of the equation $ax^2+bx+c= 0$, $\alpha+\beta = {}^{-b}/_{a}$, $\alpha\beta = {}^{c}/_{a}$ $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

01. DEC 2012

If α and β are roots of $2x^2 + 3x + 7 = 0$, find $\alpha\beta^{-1}+\beta\alpha^{-1}$

a. 2 b. ${}^{3}/_{7}$ c. ${}^{7}/_{2}$ [d.] ${}^{-19}/_{14}$

$2x^2+3x+7 = 0$, $a = 2$, $b = 3$, $c = 7$

$\alpha+\beta = {}^{-b}/_{a} = -{}^{3}/_{2}$ $\quad \alpha\beta = {}^{c}/_{a} = {}^{7}/_{2}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{\frac{9}{4} - 2\frac{7}{2}}{\frac{7}{2}}$

$= \frac{9-28}{4} \times \frac{2}{7} = -\frac{19}{14}$ option d

02. JULY 2021

if α , β are the roots of $2x^2+5x+k = 0$ & $4(\alpha^2+\beta^2+\alpha\beta) = 23$ then

a. $k^2+3k-2 = 0$ b. $k^2-2k+3 = 0$

c. $k^2-2k-3 = 0$ [d.] $k^2-3k+2 = 0$

$2x^2+5x+k = 0$,

$\alpha+\beta = -5/2$, $\alpha\beta = k/2$

$4(\alpha^2+\beta^2+\alpha\beta) = 23$

$4[(\alpha+\beta)^2-2\alpha\beta + \alpha\beta] = 23$

$4[(\alpha+\beta)^2-\alpha\beta] = 23$

$4\left[\frac{25}{4} - \frac{k}{2}\right] = 23$

$4\left[\frac{25}{4} - \frac{2k}{4}\right] = 23$

$25-2k = 23$

$2k = 2$

$k = 1$ satisfies option d. option d

03. DEC 2023

IF α & β are roots of equation $x^2-(n^2+1)x+\frac{1}{2}(n^4+n^2+1)=0$ then $\alpha^2+\beta^2$ =

a. $2n$ [b.] n^2 c. $2n^2$ d. n^3

$x^2-(n^2+1)x+\frac{1}{2}(n^4+n^2+1)=0$

$a=1$, $b=-(n^2+1)$, $c=\frac{1}{2}(n^4+n^2+1)$

$\alpha+\beta = -b/a = n^2+1$

$\alpha\beta = c/a = \frac{1}{2}(n^4+n^2+1)$

$\alpha^2+\beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= (n^2+1)^2 - 2.\frac{1}{2}(n^4+n^2+1)$

$= n^4+2n^2+1-n^4-n^2-1$

$= n^2$ option b

Q4 if α , β are roots of the equation $ax^2+bx+c=0$,

$\alpha+\beta = {}^{-b}/_{a}$, $\alpha\beta = {}^{c}/_{a}$

$\alpha^3+\beta^3 = (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

01. DEC 2023

if α and β are roots of $x^2-4x+1=0$, then the value of $\alpha^3+\beta^3$

a -76 b 76 c -52 [d] 52

$x^2-4x+1=0$

$a=1$, $b=-4$, $c=1$

$\alpha+\beta = -\frac{b}{a} = 4$

$\alpha\beta = \frac{c}{a} = 1$

$\alpha^3+\beta^3 = (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$= 43-3(1)(4)$

$= 64-12 = 52$ option d

02. JUNE 2011

if α and β are roots of $x^2+x+r=0$, & $\alpha^3+\beta^3=-6$. Find r

[a] ${}^{-5}/_{3}$ b ${}^{7}/_{3}$ c ${}^{-4}/_{3}$ d 1

$x^2+x+r=0$

$a=1$, $b=1$, $c=r$

$\alpha+\beta = -\frac{b}{a} = -1$

$\alpha\beta = \frac{c}{a} = r$

$\alpha^3+\beta^3 = (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$-6 = -1-3(-1)(r)$

$-6 = -1+3r$

$r = {}^{-5}/_{3}$ option a

03. JUNE 2017 , MAY 2018

if α and β are roots of $x^2+x+5=0$, then the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

a ${}^{16}/_{5}$ b 2 c 3 [d] ${}^{14}/_{5}$

$x^2+x+5=0$

$a=1$, $b=1$, $c=5$

$\alpha+\beta = -\frac{b}{a} = -1$

$\alpha\beta = \frac{c}{a} = 5$

$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$

$= \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= \frac{-1-3(-1)(5)}{5}$

$= \frac{14}{5}$ option d

04. JUNE 2015

If α and β are roots of $2x^2 - 4x = 1$

find $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

a. -11 b. 22 [c.] -22 d. 11

$2x^2 - 4x - 1 = 0$

$a = 2$, $b = -4$, $c = -1$

$\alpha+\beta = {}^{-b}/_{a} = 2$ $\quad \alpha\beta = {}^{c}/_{a} = {}^{-1}/_{2}$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$$

$$= \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{2^3 - 3\,({}^{-1}/_{2})\,2}{{}^{-1}/_{2}}$$

$$= \frac{8+3}{{}^{-1}/_{2}}$$

$= -22$ option C

05. NOV 2018

if α and β are roots of $x^2+7x+12 = 0$, then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

a. ${}^{-12}/_{91}$ b ${}^{-12}/_{91}$ [c] ${}^{-91}/_{12}$ d ${}^{91}/_{12}$

METHOD 1

$x^2+7x+12 = 0$

$(x+4)(x+3) = 0$

$\alpha = -4$, $\beta = -3$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{-16}{3} + \frac{-9}{4} = -\frac{91}{12}$$

METHOD 2

$x^2+7x+12 = 0$

$a = 1$, $b = 7$, $c = 12$

$$\alpha+\beta = -\frac{b}{a} = -7$$

$$\alpha\beta = \frac{c}{a} = 12$$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{(-7)^3 - 3(12)(-7)}{12}$$

$$= \frac{-343+252}{12}$$

$$= -\frac{91}{12}$$

option C

Q5 if α , β are roots of the equation $ax^2+bx+c = 0$,

$\alpha+\beta = {}^{-b}/_{a}$, $\alpha\beta = {}^{c}/_{a}$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

01. DEC 2017

the difference in roots of the equation $x^2-7x-9 = 0$ is

a. 7 [b.] $\sqrt{85}$ c. 9 d. $2\sqrt{85}$

$x^2-7x-9 = 0$

$a=1$, $b = -7$, $c = -9$

$\alpha+\beta = -\frac{b}{a} = 7$ | $\alpha\beta = \frac{c}{a} = -9$ | $\alpha-\beta = ?$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= 49-4(-9)$

$= 85$

$\alpha-\beta = \sqrt{85}$ option b

02. JAN 2021

The value of p for which the difference between the roots of equation $x^2+px+8 = 0$ is 2

a. ±2 b. ±4 c. ±6 d. ±8

$x^2+px+8 = 0$

$a=1 , b = p , c = 8$

$\alpha+\beta = -\frac{b}{a} = -p$ | $\alpha\beta = \frac{c}{a} = 8$ | $\alpha-\beta = 2$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$4 = p^2-4(8)$

$4 = p^2-32$

$p^2 = 36$

$p = \pm6$ option C

03. JUNE 2016

if the difference between the roots of the equation $x^2-kx+8=0$ is 4 then the value of k is

a. 0 b. ±4 c. $\pm8\sqrt{3}$ d. $\pm4\sqrt{3}$

$x^2-kx+8=0$

$a=1 , b = -k , c = 8$

$\alpha+\beta = -\frac{b}{a} = k$ | $\alpha\beta = \frac{c}{a} = 8$ | $\alpha-\beta = 4$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$16 = k^2-4(8)$

$16 = p^2-32$

$p^2 = 48$

$p = \pm\sqrt{48}$

$p = \pm4\sqrt{3}$ option d

04. JUNE 2013

If α , β are roots of $x^2+7x+11=0$ then the equation whose roots are $(\alpha+\beta)^2,(\alpha-\beta)^2$

a. $x^2-54x+245=0$ b. $x^2-14x+49=0$

c. $x^2-24x+144=0$ d. $x^2-50x+49=0$

$x^2+7x+11 = 0 , a=1 , b = 7 , c = 11$

$\alpha+\beta = -\frac{b}{a} = -7$ | $\alpha\beta = \frac{c}{a} = 11$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= 49-4(11) = 5$

Q.E. $[(\alpha+\beta)^2,(\alpha-\beta)^2]$

Q.E. $[49 , 5]$

$S = 54 , P = 245$

$x^2 - Sx + P = 0$

$x^2 - 54x + 245 = 0$ option a

Q6 if α , β are roots of the equation $ax^2+bx+c= 0$,

$\alpha+\beta = {}^{-b}/_{a}$, $\alpha\beta = {}^{c}/_{a}$

01. JUNE 2019

If one root of $ax^2+bx+c = 0$ is twice the other then

a. $2b^2=3ac$ b. $b^2=3ac$ c. $2b^2=9ac$

d. none of these

$ax^2 + bx + c = 0$

$\alpha+\beta$	$=-\frac{b}{a}$	$\alpha\beta = \frac{c}{a}$	$\alpha = 2\beta$
$2\beta+\beta$	$=-\frac{b}{a}$	$2\beta^2 = \frac{c}{a}$	
3β	$=-\frac{b}{a}$	$\frac{2b^2}{9a^2} = \frac{c}{a}$	
β	$=-\frac{b}{3a}$	$2b^2 = 9ac$	

option C

02. DEC 2011

If roots of $4x^2-6x+p = 0$ are in ratio 1 :2 , then find the value of p

a. 1 [b.] 2 c. –2 d. –1

$4x^2-6x+p = 0$, a = 4 , b = –6 , c = p

$\alpha+\beta = \frac{-b}{a}$ $\alpha\beta = \frac{c}{a}$ $\alpha : \beta = 1 : 2$

$\beta = 2\alpha$

$\alpha+2\alpha = \frac{6}{4}$ $\alpha.2\alpha = \frac{p}{4}$

$3\alpha = \frac{3}{2}$ $2\alpha^2. = \frac{p}{4}$

$\alpha = \frac{1}{2}$ $2.\frac{1}{4} = \frac{p}{4}$, p = 2

option b

03. DEC 2010

If roots of $12x^2+kx+5 = 0$ are in ratio 3 :2 , then find the value of k

a. $^5/_{12}$ [b.] $^{12}/_5$ c. $^{5\sqrt{10}}/_2$ d. $5\sqrt{10}$

$12x^2+kx+5 = 0$, a = 12 , b = k , c = 5

$\alpha+\beta = \frac{-b}{a}$ $\alpha\beta = \frac{c}{a}$ $\frac{\alpha}{\beta} = \frac{3}{2}$

Let $\alpha = 3\alpha$, $\beta = 2\alpha$

$5\alpha = \frac{-k}{12}$ $3\alpha.2\alpha = \frac{5}{12}$

$\alpha = \frac{-k}{60}$ $6.\alpha^2 = \frac{5}{12}$

$\frac{6k^2}{3600} = \frac{5}{12}$

$k^2 = 250$

$k = 5\sqrt{10}$

option b

04. DEC 2022

if the roots of the equation $x^2-px+q=0$ are in ratio 2 :3 then

a. $p^2=25q$ b. $p^2=6q$ c. $6p^3=5q$

[d.] $6p^2=25q$

$x^2-px+q = 0$

$\alpha+\beta = \frac{-b}{a}$ $\alpha\beta = \frac{c}{a}$ $\frac{\alpha}{\beta} = \frac{2}{3}$

Let $\alpha = 2\alpha$, $\beta = 3\alpha$

$2\alpha+3\alpha = p$ $2\alpha.3\alpha^2 = q$

$5\alpha = p$ $6\alpha^2 = q$

$\alpha = \frac{p}{5}$ $\frac{6p^2}{25} = q$

$6p^2 = 25q$ option d

Q7 For equal roots , $b^2-4ac = 0$

01. DEC 2012

Find k for which roots of the equation $x^2-2kx+16 = 0$ are equal

a. ± 1 b. ± 2 c. ± 3 [d.] ± 4

$x^2-2kx+16 = 0$

a = 1 , b = –2k , c = 16

FOR EQUAL ROOTS

$b^2 - 4ac = 0$

$4k^2 - 4(1)16 = 0$

$4k^2 - 4.16 = 0$

$k^2 = 16$ $k = \pm 4$ option d

02. DEC 2015

Find k for which roots of the equation $4x^2-12x+k = 0$ are equal

a. 144 [b.] 9 c. 5 d. none

$4x^2-12x+k = 0$

a = 4 , b = –12 , c = k

FOR EQUAL ROOTS

$b^2 - 4ac = 0$

$144 - 4(4)k = 0$

$144 = 16k$

$k = 9$ option b

03. DEC 2022

What will be the value of k , if the roots of the equation

$(k-4)x^2-2kx+(k+5) = 0$ are equal

a. 18 [b.] 20 c. 19 d. 21

$(k-4)x^2-2kx+(k+5) = 0$

$a = k-4$, $b = -2k$, $c = k+5$

FOR EQUAL ROOTS

$b^2 - 4ac = 0$

$4k^2 - 4.(k-4).(k+5) = 0$

$4k^2-4(k^2+k-20) = 0$

$4k^2-4k^2-4k+80 = 0$

$-4k+80 = 0$

$k = 20$ option b

Q8

01. JAN 2021

The harmonic mean of the roots of $(5+\sqrt{2})x^2 -(4+\sqrt{5})x + 8+2\sqrt{5} = 0$ is

a. 2 [b.] 4 c. 6 d. 8

$HM(a,b)= \frac{2ab}{a+b}$

$H.M.(\alpha,\beta) = \frac{2\alpha\beta}{\alpha+\beta}$

$= \frac{2c/a}{-b/a}$

$= \frac{2c}{-b}$

$= \frac{2(8+2\sqrt{5})}{4+\sqrt{5}}$

$= \frac{2.2(4+\sqrt{5})}{4+\sqrt{5}}$

$= 4$ option b

QE: [2,3]

$S = 5$, $P = 6$

$x^2 - Sx + P = 0$

$x^2-5x+6 = 0$

QE: $[1/2, 1/3]$

$S = 5/6$, $P = 1/6$

$x^2 - Sx + P = 0$

$x^2-5/6x+1/6 = 0$

$6x^2-5x+1 = 0$

a & c have exchanged places

Conclusion :

Q.E.$[\alpha,\beta]$ $ax^2+bx+c = 0$

Q.E.$[1/\alpha,1/\beta]$ $cx^2+bx+a = 0$

02. JUNE 2024

if α and β are the roots of the equation $ax^2+bx+c = 0$, then the equation whose roots are $1/\alpha$ and $1/\beta$ is

a. $cx^2-bx+a=0$

[b.] $cx^2+bx+a=0$

c. $x^2+bx+a=0$

d. $x^2+bx-a=0$ option b

03. JUNE 2018

If the roots of $kx^2-3x-1 = 0$ are the reciprocal of the roots of $x^2+3x-4 = 0$ then k = ?

[a.] 4 b. −4 c. 3 d. −3

QE (α,β) QE.$[1/\alpha,1/\beta]$

$x^2+3x-4 = 0$ $-4x^2+3x+1=0$

$4x^2-3x-1 = 0$

$k = 4$

option a

Q9 CUBIC EQUATION

$ax^3 + bx^2 + cx + d = 0$

$\alpha+\beta+\gamma = S = {}^{-b}/_{a}$, $\alpha\beta\gamma = P = {}^{-d}/_{a}$

01. DEC 2023 , JUNE 2014

roots of the equation

$x^3 + x^2 - x - 1 = 0$

[a.] 1,–1,–1 b. 1,1,–1 c. –1,–1,–1 d. 1,1,1

$x^3 + x^2 - x - 1 = 0$

$x^2(x+1)-1(x+1) = 0$

$(x^2-1)(x+1) = 0$

$x = 1 , -1 , -1$ option a

02. DEC 2019

THE ROOTS OF THE EQUATION

$x^3 + 9x^2 - x - 9 = 0$

[a.] 1,–1,–9 b. 1,–1,9 c. 1,1,9 d. –1,–1,–9

$(x^2-1)(x+9) = 0$

$(x-1)(x+1)(x+2) = 0$

$x = 1 , -1 , -9$ option a

03. DEC 2020

RATIONAL ROOTS OF THE EQUATION

$2x^3 - x^2 - 4x + 2 = 0$

a. –2 b. 2 [c.] ½ d. –½

$x^2(2x - 1) - 2(2x - 1) = 0$

$(2x - 1)(x^2 - 2) = 0$

$x = ½ , \sqrt{2} , -\sqrt{2}$

RATIONAL ROOTS $x = ½$ option c

04. DEC 2017

THE ROOTS OF THE EQUATION

$x^3 + 7x^2 - 21x - 27 = 0$

a. –3,–9–1 [b] 3,–9,–1 c. 3,9,1 d. –3,9,1

$S = {}^{-b}/_{a} = -7$ $P = {}^{-d}/_{a} = 27$

a. –3,–9–1 x x

b. 3,–9,–1 ✓ ✓ option b

05. DEC 2023

the solution of cubic equation

$x^3-23x^2+142x-120 = 0$ is given by

[a.] 1,10,12

b. 1,–10,12

c. –1,–10,–12

d. 1,10,–12[i]

$S = {}^{-b}/_{a} = 23$ $P = {}^{-d}/_{a} = 120$

a. 1,10,12 ✓ ✓ option a

06. SEPT 2024

If one root of the cubic equation

$3x^3 - 5x^2 - 11x - 3 = 0$ is –1/3 then the other roots are

a. 1,3 [b.] –1,3 c. 1,–3 d.–1,–3

$3x^3 - 5x^2 - 11x - 3 = 0$

$a = 3 , b = -5 , c = -11 , d = -3$

$S = {}^{-b}/_{a}$ $P = {}^{-d}/_{a}$

$-\frac{1}{3}+\beta+\gamma = \frac{5}{3}$ $-\frac{1}{3}.\beta.\gamma = 1$

$\beta+\gamma = 2$ $\beta.\gamma = -3$

$\beta+\gamma = 2$ $\beta.\gamma = -3$

a. 1,3 x x

b. –1,3 ✓ ✓

option b

07. Find value of k if 2 is a root of the equation $3x^2-2kx+5 = 0$

[a.] $^{17}/_4$ b. $^{4}/_{17}$ c. $^{-17}/_4$ d. $^{-4}/_{17}$

Since '2' is one of the roots of the given equation , it must satisfy the equation .

$3.2^2-2k.2+5 = 0$

$12-4k+5 = 0$

$17-4k = 0$

$k = 17/4$ option a

08. JULY 2021

Find value of k if 2 is a root of the following cubic equation

$x^3 - (k+1)x + k = 0$

a.2 [b.] 6 c. 1 d. 4

Since '2' is one of the roots of the given equation , it must satisfy the equation .

$2^3-(k+1)2+k = 0$

$8-2k-2+ k = 0$

$6-k = 0$

$k = 6$ option b

09. JULY 2018

If roots of, $x^3 - 15x^2 + kx - 45 = 0$ are in A.P. , find the value of k

a. 56 [b.] 59 c. –56 d. –59

since roots are in AP

$(\alpha,\beta,\gamma) = (A-D , A , A+D)$

$x^3 - 15x^2 + kx - 45 = 0$

$\alpha+\beta+\gamma = -\frac{b}{a} = 15$

$A-D+A+A+D = 15$

$3A = 15 \Rightarrow A = 5 \Rightarrow \beta = 5$

$x = 5$ being a root must satisfy the eq.

$x^3 - 15x^2 + kx - 45 = 0$

$5^3 - 15.5^2 + k.5 - 45 = 0$

$125 - 375 + 5k - 45 = 0$

$5k = 295$

$k = 59$ option b

08 - TIME VALUE OF MONEY

SIMPLE INTEREST

$SI = \frac{Pnr}{100}$, $A = P + SI = P + \frac{Pnr}{100} \Rightarrow A = P\left(1 + \frac{nr}{100}\right)$

Q1

01. an amount of ₹ 4500 will become ₹ 7200in 2 years at a simple interest rate of
a. 15% b. 25% [c.] 30% d. 40% [JUNE 2024]

$A = P + {}^{Pnr}/_{100}$ $7200 = 4500 + 4500x2x\ {}^{r}/_{100}$ $\Rightarrow r = 30\%$

02. Manoj invests ₹ 12,000 at 6% per annum SIMPLE INTEREST to obtain a total amount of ₹ 14,880 . What is the time for which the amount was invested
a. 3 years [b.] 4 years c. 2 years d. 5 years [DEC 2023]

$A = P + {}^{Pnr}/_{100}$ $14880 = 12000 + 12000xnx\ {}^{6}/_{100}$ $\Rightarrow n = 4$

03. Mr. XYZ invested ₹ 60,000 in a nationalized bank in the form of fixed deposit at the rate of 7.5% p.a. SIMPLE INTEREST RATE .He recieved ₹ 73,500 after the end of the term of fixed deposit . Calculate the period for which 60,000 was invested in fixed deposit
[a.] 3 years b. 3.5 years c. 4 years d. 4.5 years [DEC 2023]

$A = P + {}^{Pnr}/_{100}$ $73500 = 60000 + 60000xnx{}^{7.5}/_{100}$ $\Rightarrow n = 3$

04. a farmer borrowed ₹ 3600 at the rate of 15% SI p.a. . At the end of 4 years he cleared this account by paying ₹ 4000 and a cow . The cost of the cow
a. ₹ 1000 b. ₹ 1200 c. ₹ 1550 [d.] ₹ 1760 [DEC 2022]

$A = P + {}^{Pnr}/_{100}$ $A = {}^{3600\ +\ 3600x4x15}/_{100} = 5760$ (amount to be repaid)

amount repaid = 4000 + cow Hence Cost of cow = ₹ 1760

05. an investor is saving to pay off an obligation of ₹ 15,250 which will be due in 7 years . If the investor is earning 7.5% S.I. rate per annum , he must deposit ₹ ____ to meet the obligation
a. ₹ 8000 b. ₹ 9000 [c.] ₹ 10000 d. ₹ 11000 [DEC 2022]

$A = P + {}^{Pnr}/_{100}$ $15250 = P\left(1 + 7x\ {}^{7.5}/_{100}\right)$, $P = ₹\ 10000$

06. a certain sum amounted to 575 at 5% in a time in which 750 amounted to 840 at 4% . If rate of interest is simple , find the sum a. ₹ 525 b. ₹ 550 c. ₹ 515 [d.] ₹ 500 [JAN 2021]

$A = P + {}^{Pnr}/_{100}$ $840 = 750 + 750xnx{}^{4}/_{100}$ $\Rightarrow n = 3$

$575 = P\left(1 + 3x\ {}^{5}/_{100}\right)$ $\Rightarrow P = 500$

07. a certain sum of money Q was deposited for 5 years and 4 months at 4.5% SI and amounted to ₹ 248 , then Q = a. ₹ 240 **b.** ₹ 200 c. ₹ 220 d. ₹ 210 [NOV 2018]

$A = P + {Pnr}/{100}$ $\quad 248 = Q\left(1 + \frac{64}{12} \times \frac{4.5}{100}\right)$, $Q = 200$

Q2 GIVEN TWO A'S , FIND P & R

01. in SI , a certain sum of money amounts to ₹ 59000 in 3 years and ₹ 62000 in 4 years . Find principal amount and rate of interest
a. ₹ 50000,6% b. ₹ 45000,4.5% c. ₹ 55000,5% d. ₹ 52000,7% [DEC 2021]

$A = P + {Pnr}/{100}$

$P + 4{Pr}/{100} = 62000$ .... (1)
$P + 3{Pr}/{100} = 59000$ .... (2)
$1{Pr}/{100} = 3000$ ..... (3) $P = 50000$, subs in (3) $r = 6$

02. a sum of money amount to ₹ 6200 in 2 years and ₹ 7400 in 3 years as per S.I. , then the principal is
a. ₹ 3000 b. ₹ 3500 **c.** ₹ 3800 d. none [JUNE 2019]

$A = P + {Pnr}/{100}$

$P + 3{Pr}/{100} = 7400$ .... (1)
$P + 2{Pr}/{100} = 6200$ .... (2)
$1{Pr}/{100} = 1200$ ..... (3) $P = 3800$

03. a sum of money amount to ₹ 692 in 2 years and ₹ 800 in 5 years as per S.I. , then the principal is
a. ₹ 520 **b.** ₹ 620 c. ₹ 720 d. ₹ 820 [JUNE 2019]

$A = P + {Pnr}/{100}$

$P + 5{Pr}/{100} = 800$ .... (1)
$P + 2{Pr}/{100} = 692$ .... (2)
${Pr}/{100} = 36$ ..... (3) $P = 620$

04. in SI , a certain sum of money amounts to ₹ 97,920 in 3 years and ₹ 1,15,200 in 5 years . Find rate of interest
a. 10% b. 11.2% **c.** 12% d. 13.6% [JUNE 2018]

$A = P + {Pnr}/{100}$

$P + 5{Pr}/{100} = 115200$ .... (1)
$P + 3{Pr}/{100} = 97920$ .... (2)
${Pr}/{100} = 8640$ ..... (3) $P = 72000$, subs in (3) $r = 12$

Q3 DIFFERENCE IN S.I.

01. A person borrows ₹ 5000 for 2 years at 4% p.a. simple interest . He immediately lends it to another person at 6 ¼% p.a. for 2 years . Find his gain in the transaction
a. ₹ 112.50 **b.** ₹ 225 c. ₹ 125 d. ₹ 107.50 [JAN 2021]

$SI_2 - SI_1 = 5000.2.(6.25)/100 - 5000.2.(4)/100 = 5000 . 2 .(6.25-4)/100 = 225$

02. Two equal sum was lent at simple interest at 15% p.a. for 3 ½ yrs and 5 yrs respectively . If the difference in interest for two periods was ₹ 144 , Find the sum
a. ₹ 620 **b. ₹ 640** c. ₹ 820 d. ₹ 840 [JAN 2021]

$SI_2 - SI_1 = 144$ $\quad \frac{P.5.15}{100} - \frac{P.(3.5).15}{100} = 144$

$\frac{P.15}{100}[5-3.5] = 144 \Rightarrow P = 640$

03. if the difference between interest received by 2 persons A and B on the same sum of ₹ 1500 for 3 years is 18 , then what is the difference between the two rates of interest
a. 1% b. 2.5% c. 3% **d. 0.4%** [DEC 2019]

$SI_2 - SI_1 = 18$ $\quad 1500.3.\frac{r_1}{100} - 1500.3.\frac{r_2}{100} = 18$

$1500.3.\frac{(r_1 - r_2}{100} = 18$

$r_1 - r_2 = 0.4$

04 Mr X invest 90,500 in post office at 7.5% p.a. SI . While calculating the rate was wrongly taken as 5.7% p.a. . The difference in amounts of maturity is 9774 . Find the period for which the sum was invested .
a. 7 years b. 5.8 years **c. 6 years** d. 8 years [DEC 2012]

$SI_2 - SI_1 = 9774$ $\quad \frac{90500.n.(7.5)}{100} - \frac{90500.n.(5.7)}{100} = 9774$

$\frac{90500.n}{100}[7.5-5.7] = 9774 \Rightarrow n = 6$

05 By mistake a clerk calculated the SI on principal for 5 months at 6.5% p.a. instead of 6 months at 5.5% p.a. If the error in calculation was 25.40 , the original sum of principal was
a. ₹ 60,690 b. ₹ 60,960 c. ₹ 90,660 d. ₹ 90,690 [JUNE 2011]

$SI_2 - SI_1 = 25.40$ $\quad \frac{P.6}{12}.\frac{(5.5)}{100} - \frac{P.5}{12}.\frac{(6.5)}{100} = 25.40$

$\frac{P}{1200}[6.(5.5) - 5.(6.5)] = 25.40 \Rightarrow P = 60,960$

06. A certain sum of money was invested at simple rate of interest for 3 years . If the same had been invested at the rate that was 7 percent higher , the interest amount would have been Rs 882 more . The amount of sum invested is
a. ₹ 12600 b. ₹ 6800 **c. ₹ 4200** d. ₹ 2800 [DEC 2014]

$SI_2 - SI_1 = 882$ $\quad \frac{P.3.r+7}{100} - \frac{P.3.r}{100} = 882$

$\frac{P.3}{100}[r+7-r] = 882 \Rightarrow P = 4200$

07. a certain sum amounts to ₹ 15748 in 3 years at SI at r% p.a. The same sum amounts to ₹ 16510 at (r+2)% p.a. SI in the same time . What is the value of r ?
a. 10% **b. 8%** c. 12% d. 6% [JULY 2021]

$\frac{P.3.r+2}{100} - \frac{P.3.r}{100} = 16510 - 15748$

$\frac{P.3}{100}[r+2-r] = 762 \Rightarrow P = 12700$

$A = P + \frac{Pnr}{100}$ $\quad 15748 = 12700 + \frac{12700 \times 3 \times r}{100} \Rightarrow r = 8\ \%$

08. a man invested $^1/_3$ of his capital at 7% , $^1/_4$ at 8% and remainder at 10% . If annual income is 561 , the capital is
a. ₹ 4400 b. ₹ 5500 **c.** ₹ 6600 d. ₹ 5800 [JAN 2021]

$P - [P/3 + P/4] = 5P/12$

$\frac{P}{3} \times \frac{7}{100} + \frac{P}{4} \times \frac{8}{100} + \frac{5P}{12} \times \frac{10}{100} = 561 \Rightarrow \frac{P}{100}\left(\frac{28}{12} + \frac{24}{12} + \frac{50}{12}\right) = 561 \Rightarrow P = 6600$

09. $^1/_7$ of a money is deposited at 4% p.a. , $^1/_2$ of money is deposited at 5% p.a. and the remaining at the rate of 6% , then the total interest gained is ₹ 730 . Find the amount
a. ₹ 14000 b. ₹ 215500 c. ₹ 212800 d. ₹ 214500 [DEC 2019]

$P - [P/7 + P/2] = 5P/14$

$\frac{P}{7} \times \frac{4}{100} + \frac{P}{2} \times \frac{5}{100} + \frac{5P}{14} \times \frac{6}{100} = 730 \Rightarrow \frac{P}{100}\left(\frac{8}{14} + \frac{35}{14} + \frac{30}{14}\right) = 730 \Rightarrow P = 14000$

COMPOUND INTEREST

$A = P(1+i)^n$, A = Future val. , P = Present value
i = rate per compounding period ,
n = no. of times the money is compounded

Q1 FIND AMOUNT/ F.V.

01. the population of a city increases at the rate of 5% every year . What will be the population of the city in the year 2023 if the population in 2021 was 1,00,000
a) 105,000 **b)** 110,250 c) 115,240 d) 120,550 [DEC 2023]

$A = P(1+i)^n$ $A = 100000(1.05)^2 = 110,250$

02. Mr X makes a deposit of ₹ 50,000 in a bank for 2 ½ years . If the rate of interest is 12% p.a. compounded HALF YEARLY , then maturity value is , Given $(1.06)^5 = 1.3382$
a) ₹ 66,910 b) ₹ 66,123 c) ₹ 67,925 d) ₹ 65,550 [DEC 2023]

$A = P(1+i)^n$ i = 6% , n = 5 , $A = 50000(1.06)^5 = 50000 \times 1.3382 =$ ₹ 66,910

03. if ₹ 10,000 is invested at 8% per year COMPOUNDED QUARTERLY , then the value of investment after 2 years is
a. ₹ 10,716.59 **b.** ₹ 11,716.59 c. ₹ 117.1659 d. none [NOV 2018]

$A = P(1+i)^n$ i = 2% , n = 8 , $A = 10000(1.02)^8 = 11,716.59$

04. a bank pays 10% interest calculated HALF YEARLY . A sum of ₹ 400 is deposited in the bank . The amount at the end of 1 year will be
a. ₹ 439 b. ₹ 440 c. ₹ 442 d. ₹ 441 [NOV 2018]

$A = P(1+i)^n$ $i = 5\%$, $n = 2$, $A = 400(1.05)^2 = ₹ 441$ OR $400+5\%+5\% = 441$

05. Mr X bought an electronic item for 1000 . What would be the future value of the same item after 2 years if the value is compounded SEMI ANNUALLY at the rate of 22% p.a.
a) ₹ 1488.40 b) ₹ 1518.07 c) ₹ 2008.07 d) ₹ 2200 [JUNE 2016]

$A= P(1+i)^n$ $i = 11\%$, $n = 4$, $A = 1000(1.11)^4 = ₹ 1518.07$

06. the present population of a town is 25000 . If it grows at the rate of 4% , 5% , 8% during 1st , 2nd & 3rd year respectively , then find the population after 3 years
a. 29484 b. 29844 c. 29448 d. 29994 [DEC 2019]

$A = P(1+i)^n$ $A = 25000(1.04)(1.05)(1.08) = 29484$

OR $A = 25000 + 4\% + 5\% + 8\% = 29484$

07. How much will ₹ 25000 amount to in 2 years at compound interest if the rates for the successive years are 4% and 5% per year
a. ₹ 27,000 b. ₹ 27,300 c. ₹ 27,500 d. ₹ 27,900 [NOV 2018]

$A = P(1+i)^n$ $A = 25000(1.04)(1.05) = ₹ 27,300$

OR $A = 25000 + 4\% + 5\% = 29484$

Q2 FIND PRESENT VALUE

01. Jonny wants to have 2,00,000 in his saving account after three years . The rate of interest offered by bank is 8% p.a. COMPOUNDED ANNUALLY . How much should he invest today to achieve his target amount . [JUNE 2023]
a. ₹ 1,47,489.10 b. ₹ 1,58,766.44 c. ₹ 1,71,035.59 d. ₹ 1,84,417.96

$A = P(1+i)^n$ $200000 = P(1.08)^3 \Rightarrow P = ₹ 1,58,766.44$

02. Calculate the present value of ₹ 2000 to be required after 10 years COMPOUNDED ANNUALLY at 5% p.a. [given $1.05^{10} = 1.62889$]
a) ₹ 1,227.82 b) ₹ 1,282.48 c) ₹ 1,328.35 d) ₹ 1,822.65 [DEC 2023]

$A = P(1+i)^n$ $2000 = P(1.05^{10}) \Rightarrow 2000 = P \times 1.62889 \Rightarrow P = ₹ 1,227.82$

03. a company needs 10,000 in five years to replace an equipment . How much must be invested now at the interest rate of 8% p.a. in order to provide for the equipment .
a) ₹ 6606 b) ₹ 6806 c) ₹ 10500 d) ₹ 11500 [DEC 2021]

$A = P(1+i)^n$ $10000 = P(1.08)^5 \Rightarrow P = ₹ 6806$

04. the present value of ₹ 2,000 after 8 years at the rate of 6% ,given $(1.06^8 = 1.59385)$
a. ₹ 1054 **b.** ₹ 1254 c. ₹ 3054 d. ₹ 2054 [JUNE 2022]

$A = P(1+i)^n$ $2000 = P(1.06)^8 \Rightarrow P = ₹ 1254.82$

05. if the desired future value after 5 years with 18% interest rate is ₹ 150,000 , then the present value is [Given $(1.18)^5 = 2.2877$]
a. ₹ 63,712 **b.** ₹ 65,568 c. ₹ 53,712 d. ₹ 41,712 [JULY 2021]

$A = P(1+i)^n$ $150000 = P(1.18)^5 \Rightarrow P = ₹ 65,568$

06. Find the present value of ₹ 1,00,000 required after 5 years if the rate of interest is 9% ,given $(1.09^5 = 1.5386)$
a. ₹ 78995.98 **b.** ₹ 64994.20 c. ₹ 88992.43 d. ₹ 93902.12 [DEC 2020]

$A = P(1+i)^n$ $100000 = P(1.09)^5 \Rightarrow P = ₹ 64,994.15$

07. A sum was invested for 3 years as per CI and the rate of interest for first year is 9% , 2nd year is 6% and 3rd year is 3% respectively . Find the sum if the amount in 3 years is 550
a) ₹ 250 b) ₹ 300 **c)** ₹ 462.16 d) ₹ 350 [JUNE 2019]

$A = P(1+i)^n$ $550 = P(1.09)(1.06)(1.03) \Rightarrow P = ₹ 462.16$

08. What is the present value of ₹ 5000 to be obtained after 6 years if the interest rate is 5% p.a.
[Use if needed : $1/(1.05)^n = 0.74621 , 0.71068 , 0.67684 , 0.64462$ for $n = 6 , 7 , 8 , 9$]
a. ₹ 3731 b. ₹ 3553 c. ₹ 3384 d. ₹ 3223 [JUNE 2024]

$A = P(1+i)^n$ $5000 = P(1.05)^6 \Rightarrow P = 5000 \times \frac{1}{(1.05)^6} = 5000 \times 0.74621 = ₹ 3731$

09. a certain sum invested at 4% p.a. COMPOUNDED SEMI ANNUALLY amounts to 1,20,000 at the end of one year . Find the sum
a. ₹ 1,10,120 **b.** ₹ 1,15,340 c. ₹ 1,21,812 d. ₹ 1,13,113 [DEC 2020]

$A = P(1+i)^n$ $i = 2\% , n = 2 , 120000 = P(1.02)^2 \Rightarrow P = ₹ 115340$

10. It needs to pay ₹ 5,00,000 after 10 years . He invested a sum in a scheme at 9 % rate of interest COMPOUNDED HALF YEARLY. How much amount be invested $(1.045^{20} = 2.41171)$
a. ₹ 397321 **b.** ₹ 207321 c. ₹ 297321 d. ₹ 340321 [DEC 2021]

$A = P(1+i)^n$ $i = 4.5\% , n = 20 , 500000 = P(1.045)^{20} \Rightarrow P = ₹ 207321$

Q3 FIND COMPOUND INTEREST $C.I. = P[(1+i)^n - 1]$

01. Ram borrowed ₹ 5000 @ 12.5% p.a compound interest . The money was repaid after 3 years .The total interest paid by him approximately is $(1+0.125)^3 = 1.4238$
a. ₹ 2119 b. ₹ 2200 c. ₹ 2000 d. ₹ 2500 [JUNE 2024]

$CI = P[(1+i)^n - 1]$ $CI = 5000 (1.125^3 - 1) = ₹ 2119$

02. Compute the compound interest on 6,000 for 1 ¼ years at 8% p.a. Interest will be COMPOUNDED QUARTERLY
a) ₹ 642 b) ₹ 630.78 c) ₹ 634.68 **d** ₹ 624.48 [DEC 2023]

$CI = P[(1+i)^n - 1]$ i = 2% , n = 5 $CI = 6000\left[1.02^5 - 1\right] = ₹\ 624.48$

03. the compound interest on ₹ 15,625 for 9 months at 16% per annum COMPOUNDED QUARTERLY is
a. ₹ 1,851 b. ₹ 1,941 **c.** ₹ 1,951 d. ₹ 1,961 [JUNE 2023]

$CI = P[(1+i)^n - 1]$ i = 4% , n = 3 $CI = 15625\left[1.04^3 - 1\right] = ₹\ 1,951$

04. what is the CI on sum of ₹ 12600 for 1 ½ year at 20% p.a. if the interest is COMPOUNDED HALF YEARLY
a. ₹ 4271 **b.** ₹ 4171 c. ₹ 4711 d. ₹ 4117 [JULY 2021]

$CI = P[(1+i)^n - 1]$ i = 10% , n = 3 $CI = 12600\left[1.10^3 - 1\right] = ₹\ 4170.60$

05. Find the amount of CI , if an amount of ₹ 50,000 is deposited in a bank for 1 year at the rate of 8% p.a. COMPOUNDED SEMIANNUALLY
a. ₹ 3,080 **b.** ₹ 4,080 c. ₹ 5,456 d. ₹ 7856 [JAN 2021]

$CI = P[(1+i)^n - 1]$ i = 4% , n = 2 $CI = 50000\left[1.04^2 - 1\right] = ₹\ 4,080$

06. there is 60% increase in an amount in 6 years at simple interest . What will be the compound interest of ₹ 12000 after 3 years at the same rate
a. ₹ 3,972 b. ₹ 2160 c. ₹ 3120 d. ₹ 3742 [JUNE 2022]

$SI = Pnr/100 \Rightarrow 60 = 100x6xr/100 \Rightarrow r = 10\%$

$CI = P[(1+i)^n - 1]$ $CI = 12000\left[1.10^3 - 1\right] = ₹\ 3972$

07. On what sum will the compound interest at 5% p.a. for 2 years compounded annually be ₹ 3280
a. ₹ 16,000 **b.** ₹ 32,000 c. ₹ 48,000 d. ₹ 64,000 [DEC 2020]

$CI = P[(1+i)^n - 1]$ $3280 = P\left[1.05^2 - 1\right] = ₹\ 32000$

08. if compound interest on a sum for 2 years at the rate 5% p.a. is 512.50 , then the principal is
a. ₹ 4,000 b. ₹ 3,000 **c.** ₹ 5,000 d. none [DEC 2017]

$CI = P[(1+i)^n - 1]$ $512.50 = P\left[1.05^2 - 1\right] = ₹\ 5000$

09. Mr Prakash invested money in two schemes A and B offering compound interest at the rate of 8% and 9% p.a respectively . If the total amount of interest accrued through two schemes together in two years was ₹ 4818.30 and total amount invested was ₹ 27000 . What was the amount invested in scheme A
a. ₹ 12000 b. ₹ 12500 c. ₹ 13000 d. ₹ 13500 [DEC 2022]

$CI_1 + CI_2 = 4818.30$

Using options $12000\left[1.08^2-1\right] + 15000\left[1.09^2-1\right] = 4818.30$, hence option a

Q4 CI - SI $= P[(1+i)^n-1-ni]$

01. the difference between the compound interest amount and the simple interest amount for a period of 2 years at a simple interest of r is
[a.] $P \times r^2$ b. $P \times r/2$ c. $P \times 2 \times r$ d. $P^2 \times r$ [JUNE 2024]

$P\left[(1+r)^2-1-2r\right] = P\left[1+2r+r^2-1-2r\right] = Pr^2$

02. the difference between SI and CI on a certain sum invested for 2 years at 5% p.a. is ₹ 30 Find principal amount is
a. ₹ 10,000 [b.] ₹ 12,000 c. ₹ 13,000 d. none [DEC 2016]

$P\left[1.05^2 - 1 - 2x0.05\right] = 30$ OR $Pi^2 = 30$

P = ₹ 12000 $P(0.05)^2 = 30 \Rightarrow P$ = ₹ 12000

03. the difference between SI and CI on a certain sum invested for 2 years at 10% p.a. is ₹ 10 Find sum
a. ₹ 1010 b. ₹ 1095 [c.] ₹ 1000 d. ₹ 990 [JUNE 2008]

$P\left[1.10^2 - 1 - 2x0.10\right] = 10$ OR $Pi^2 = 10$

P = ₹ 1,000 $P(0.10)^2 = 10 \Rightarrow P$ = ₹ 1000

04. the difference between SI and CI on a certain sum invested for 2 years at 10% p.a. is ₹ 11 Find sum
a. ₹ 1200 [b.] ₹ 1100 c. ₹ 1000 d. ₹ 990 [DEC 2008]

$P\left[1.10^2 - 1 - 2x0.10\right] = 11$ OR $Pi^2 = 11$

P = ₹ 1,100 $P(0.10)^2 = 11 \Rightarrow P$ = ₹ 1100

05. the difference between SI and CI on a certain sum invested for 2 years at 12% p.a. is ₹ 72 Find sum a. ₹ 8000 b. ₹ 6000 [c.] ₹ 5000 d. ₹ 7750 [JUNE 2011]

$P\left[1.12^2 - 1 - 2x0.12\right] = 72$ OR $Pi^2 = 72$

P = ₹ 5000 $P(0.12)^2 = 72 \Rightarrow P$ = ₹ 5000

06. the difference between SI and CI on a certain sum invested for 2 years at 7% p.a. is ₹ 29.4 Find principal sum

a. ₹ 5000 b. ₹ 5500 [c.] ₹ 6000 d. ₹ 6500 [DEC 2013]

$P\left[1.07^2 - 1 - 2x0.07\right] = 29.4$ OR $Pi^2 = 29.4$

P = ₹ 6000 $P(0.07)^2 = 29.4 \Rightarrow P$ = ₹ 6000

07. the difference between compound interest compounded annually and simple interest for 2 years at 10% p.a. is ₹ 372 Find principal amount is

a. ₹ 37,000 [b.] ₹ 37,200 c. ₹ 37,500 d. none [NOV 2018]

$P\left[1.10^2 - 1 - 2x0.10\right] = 372$ OR $Pi^2 = 372$

P = ₹ 37200 $P(0.10)^2 = 372 \Rightarrow P$ = ₹ 37200

08. if an amount is kept at S.I. , it earns an interest of ₹ 600 in first 2 years but when kept at compound interest it earns an interest of 660 . Find the rate and principal

a. 20%,1200 [b.] 20%,1500 c. 10%,1200 d. 10%,1500 [JUNE 2016 ,MAY 2018]

$CI - SI = 60$ $Pi^2 = 60$

Using options a. $1200x0.2^2 \neq 60$, b. $1500x0.2^2 = 60$ ✓

09. Difference between CI and SI on an amount of ₹ 15000 for 2 years is ₹ 96 . What is rate of interest per annum a. 9% [b.] 8% c. 11% d. 10% [DEC 2022]

$15000\left[(1+i)^2 - 1 - 2i\right] = 96$ $Pi^2 = 96$

$15000\left[1+2i+i^2-1-2i\right] = 96$ OR $15000i^2 = 96$

$15000 \times i^2 = 96 \Rightarrow i = 0.08 = 8\%$ $i = 0.08 = 8\%$

10. the difference between SI and CI on a certain sum invested for 3 years at 6% p.a. is ₹ 110.16 Find principal amount is

a. ₹ 3,000 b. ₹ 3,700 c. ₹ 12,000 [d.] ₹ 10,000 [JUNE 2023]

$P\left[1.06^3 - 1 - 3x0.06\right] = 110.16 \Rightarrow P$ = ₹ 10,000

11. the difference between SI and CI for 3 years at 4% p.a. is ₹ 912 Find principal amount is

[a.] ₹ 1,87,500 b. ₹ 1,87,000 c. ₹ 1,87,550 d. ₹ 1,85,700 [MAY 2018]

$P\left[1.04^3 - 1 - 3x0.04\right] = 912 \Rightarrow P$ = ₹ 1,87,500

12. the difference between compound interest compounded SEMI ANNUALLY and simple interest on ₹ 400 at 10% p.a. for 1 year

[a.] ₹ 1 b. ₹ 28 c. ₹ 35 d. ₹ 40 [DEC 2019]

$400\left[1.05^2 - 1 - 1x0.10\right] = 1$

13. the difference between compound interest compounded SEMI ANNUALLY and simple interest on ₹ 10000 at 10% p.a. for 4 years

a. ₹ 650 b. ₹ 640 [c.] ₹ 641 d. ₹ 600 [DEC 2019]

$10000\left[1.10^4 - 1 - 4x0.10\right] = 641$

14. what is the difference between SI and CI on a sum of ₹ 8000 for $2\ ^2/_5$ years at the rate of 10% when the interest is compounded yearly

a. ₹ 135.75 b. ₹ 129.50 c. ₹ 151.75 [d.] ₹ 147.20 [JULY 2021]

CI = 8000+10%+10%+4% (2/5x10%) − 8000 = 2067.20 , SI = 8000 x 2.4 x 10/100 = 1920

CI − SI = 147.20

Q4 DEPRECIATION

$V = P(1-i)^n$, P = initial price/cost of M/c

V = value of M/c after n years / Scrap Value

i = rate of depreciation

01. a machine costing ₹ 1,00,000 has useful life of 10 years . If the rate of depreciation is 12% , what is the scrap value of the machine at the end of life $(0.88)^{10} = 0.27850$

[a)] ₹ 25,850 b) ₹ 26,850 c) ₹ 27,850 d) ₹ 28,850 [DEC 2023]

$V = P(1-i)^n$ $V = 100000(0.88)^{10} = 100000 \times 0.27850 = ₹\ 27,850$

02. useful life of a machine whose cost is ₹ 10,000 is 10 years . if it depreciates @ 10% p.a. , then the scrap value of machine is

[a.] ₹ 3487 b. ₹ 3158 c. ₹ 3500 d. ₹ 7033 [DEC 2019]

$V = P(1-i)^n$ $V = 10000(0.90)^{10} = ₹\ 3487$

03. the cost of machinery is ₹ 1,25,000 . If its useful life is estimated to be 20 years and the rate of depreciation is 10% p.a. , then the scrap value of the machinery is $[0.9^{20} = 0.1215]$

[a.] ₹ 15,187 b. ₹ 15400 c. ₹ 15300 d. ₹ 15250 [DEC 2010]

$V = P(1-i)^n$ $V = 125000(0.90)^{20} = ₹\ 15187.50$

04. The value of furniture depreciates by 10% a year . If the present value of the furniture is ₹ 21870 , calculate the value of furniture 3 years ago

[a.] ₹ 30,000 b. ₹ 35,000 c. ₹ 40,000 d. ₹ 50000 [NOV 2018]

$V = P(1-i)^n$ $21870 = P(0.90)^3 = ₹\ 30000$

05. The present value of scooter is ₹ 7290 . The rate of depreciation is 10% . What was the value 3 years ago

[a.] ₹ 10,000 b. ₹ 10,010 c. ₹ 9,990 d. ₹ 12000 [DEC 2019]

$V = P(1-i)^n$ $7290 = P(0.90)^3 = ₹\ 10000$

06. Cost of a laptop is ₹ 1,10,000 and its value depreciates 12% annually . Its life is 6 years . Scrap value is _____ times its cost

a. 0.44 [b.] 0.46 c. 0.45 d. 0.48 [DEC 2021]

$V = P(1-i)^n$ $V = P(0.88)^6 \Rightarrow V = 0.46P$

Q5 FIND PERIOD

01. a sum of money invested of compound interest doubles itself in 4 years . In how many years it becomes 32 times of itself at the same rate of CI

a. 12 years b. 16 years [c.] 20 years d. 24 years [DEC 2014 , DEC 2022]

$A = P(1+i)^n$ $2 = 1(1+i)^4 \Rightarrow 2^5 = 1(1+i)^{4.5} \Rightarrow 32 = 1(1+i)^{20} \Rightarrow n = 20$

02. a sum of money invested of compound interest doubles itself in 4 years . In how many years it becomes 8 times of itself at the same rate of CI

[a.] 12 years b. 14 years c. 16 years d. 18 years [DEC 2022]

$A = P(1+i)^n$ $2 = 1(1+i)^4 \Rightarrow 2^3 = 1(1+i)^{4.3} \Rightarrow 8 = 1(1+i)^{12} \Rightarrow n = 12$

03. at 8% compounded annually , how long will it take ₹ 750 to double

a. 6.5 years b. 48 months [c.] 9 years d. 12 years [JUNE 2024]

$A = P(1+i)^n$ $2 = 1(1.08)^n \Rightarrow 2 = (1.08)^n \Rightarrow n = 9$ years (using calc)

04. a sum amounts to 1331 at a principal of 1000 at 10% compounded annually . Find the time

a. 3.31 years [b.] 3 years c. 4 years d. 2 years [JUNE 2009, DEC 2009]

$A = (1+i)^n$ $1331 = 1000(1.10)^n \Rightarrow 1.331 = (1.10)^n \Rightarrow n = 3$ years (using options)

05. the population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

a. 15 [b.] 17 c. 19 d. 20 [JUNE 2023]

$A = P(1+i)^n$ $140 = 100(1.02)^n \Rightarrow (1.02)^n = 1.4 \Rightarrow n = 17$ (using calc)

06. a machine depreciates at 10% of its value at the beginning of a year . The cost and scrap value realized at the time of sale being ₹ 23,240 and ₹ 9,000 respectively . For how many years the machine was put to use

a. 7 b. 8 [c.] 9 d. 10 [JUNE 2023]

$V = P(1-i)^n$ $9000 = 23240(0.90)^n \Rightarrow 0.90^n = 0.3873 \Rightarrow n = 9$ (using calc)

07. A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year . When its value will reduce to 2,00,750 [DEC 2022]

a. 5 years 5 months [b.] 5 years 6 months c. 5 years 7 months d. 5 years 8 months

$V = P(1-i)^n$ $200750 = 490740(0.85)^n \Rightarrow 0.85^n = 0.4091 \Rightarrow 5 < n < 6$

Try option b $\Rightarrow$ $0.85^{5.5} = 0.85^5 \times 0.85^{0.5} \Rightarrow 0.85^5 \times \sqrt{0.85} = 0.40907$ ✓

Q6 FIND INTEREST RATE

01. at what rate of compound interest will a sum of money become 16 times in 4 years , if interest is being calculated compounding annually

[a.] 100% b. 10% c. 20% d. 200% [JUNE 2010]

$A = P(1+i)^n$ $16 = = 1(1+i)^4 \Rightarrow (1+i) = 2 \Rightarrow i = 1 = 100\%$

02. in compound interest , if amount is 9 times of principal in 2 years , then rate of interest is

a. 300% [b.] 200% c. 150% d. 100% [JUNE 2018]

$A = P(1+i)^n$ $9 = = 1(1+i)^2 \Rightarrow (1+i) = 3 \Rightarrow i = 2 = 200\%$

03. If in 2 years 100 amounts to 121 when interest at the rate of r% is compounded annually , then the value of r will be

a. 14 b. 10.5 c. 15 [d.] 10 [NOV 2018]

$A = P(1+i)^n$ $121 = = 100(1+i)^2 \Rightarrow (1+i)^2 = 1.21 \Rightarrow 1+i = 1.1 \Rightarrow i = 0.1 = 10\%$

04. if the initial investment of ₹ 4,00,000 becomes ₹ 6,00,000 in 24 months , then the compound annual growth rate (CAGR) is

a. 30.33% [b.] 22.4% c. 19.46% d. 14.47% [DEC 2023]

$600000 = 400000(1+i)^2 \Rightarrow (1+i)^2 = 1.5 \Rightarrow 1+i = 1.2247 \Rightarrow i = 0.2247 = 22.47\%$

05. You bought a painting 10 years ago as an investment . You originally paid ₹ 85000 for it . If you sold it for ₹ 4,84,050, what was your annual return on investment

a. 47% b. 4.7% [c.] 19% d. 12.8% [JUNE 2024]

$A = P(1+i)^n$ $484050 = 85000(1+i)^{10} \Rightarrow (1+i)^{10} = 5.6947$

Using option , $(1+0.19)^{10} = 5.6947$

06. 10 years ago the earnings per share (EPS) of ABC ltd was ₹ 5 per share . If EPS for this year is ₹ 22 . Compute at what rate , EPS of the company grow annually .

[a.] 15.97% b. 16.77% c. 18.64% d. 14.79% [DEC 2022]

$A = P(1+i)^n$ $22 = 5(1+i)^{10} \Rightarrow (1+i)^{10} \Rightarrow 4.4$

Using options a. $(1+0.1597)^{10} = 4.4$, hence option a

Q7 KUCH KATKE

01. if ₹ 64 amounts to ₹ 83.20 in 2 years , what will ₹ 86 amount to in 4 years at the same rate prevent per annum

a. ₹ 127.60 b. ₹ 147.60 [c.] ₹ 145.34 d. ₹ 117.60 [DEC 2022]

$A = P(1+i)^n$ $83.20 = 64(1+i)^2 \Rightarrow (1+i)^2 = 1.3$

$A = 86(1+i)^4 = 86(1.3)^2 = ₹ 145.34$

02. an investment is earning compound interest . ₹ 100 invested in the year 2 accumulated to ₹ 105 by year 4 . ₹ 500 invested in the year 5 will become _____ by year 10

a. ₹ 364.80 [b.] ₹ 564.80 c. ₹ 464.80 d. ₹ 664.80 [JUNE 2022]

$A = P(1+i)^n$ $105 = 100(1+i)^2 \Rightarrow (1+i)^2 = 1.05 \Rightarrow (1+i) = 1.024695$

$A = 500(1+i)^5 = 500(1.024695)^5 = ₹ 564.86$

03. if the sum of money when compounded annually becomes 1140 in 2 years and 1710 in 3 years , find the rate of interest

a. 30% b. 40% [c.] 50% d. 60% [JUNE 2013]

$A = P(1+i)^n$ $1710 = P(1+i)^3$ (1) ÷ (2)

$1140 = P(1+i)^2$ $(1+i) = 1.5 \Rightarrow i = 0.5 = 50\%$

04. What is the present value of an investment that pays ₹ 400 at the end of 3 years and ₹ 500 at the end of 6 years

[a.] ₹ 320 b. ₹ 335 c. ₹ 340 d. ₹ 280 [JUNE 2024]

$A = P(1+i)^n$ $500 = P(1+i)^6$ (1) ÷ (2)

$400 = P(1+i)^3$ $(1+i)^3 = 1.25 \Rightarrow$ subs in (2) P = ₹ 320

05. a sum of ₹ x amounts to ₹ 27900 in 3 years and ₹ 41,850 in 6 years at a certain rate percent p.a. when the interest is compounded yearly . The value of x

a. ₹ 16,080 [b.] ₹ 18600 c. ₹ 18060 d. ₹ 16800 [JULY 2021]

$A = P(1+i)^n$ $41850 = P(1+i)^6$ (1) ÷ (2)

$27900 = P(1+i)^3$ $(1+i)^3 = 1.5 \Rightarrow$ subs in (2) P = ₹ 18600

06. an amount P becomes ₹ 5100.50 and ₹ 5203 after 2nd and 4th year respectively at r% p.a. compounded annually . Find P and r

[a.] ₹ 5000 , 1% b. ₹ 4000 , 1.5% c. ₹ 6000 , 2% d. ₹ 5500 , 3% [DEC 2020]

$A = P(1+i)^n$ $5203 = P(1+i)^4$ (1) ÷ (2) $(1+i)^2 = 1.02 \Rightarrow (1+i) = 1.00999$

$5100.50 = P(1+i)^2$ $i = 0.00999 = 1\%$

$P = ₹\ 5000$

Q8 EFFECTIVE RATE $E = [(1+i)^n - 1]x100$

01. if interest rate on a loan is 1% per month , the effective annual rate of interest is

a. 12% b. 12.36% [c.] 12.68% d. 12.81% [JUNE 2024]

$E = \left[(1+i)^n - 1\right] \times 100$ $E = \left[(1.01)^{12} - 1\right] \times 100 = 12.68\%$

02. Find the effective rate of interest if an amount of ₹ 40,000 is deposited in a bank for 1 year at the rate of 10% COMPOUNDED SEMI – ANNUALLY

a. 10.20% b. 10.05% [c.] 10.25% d. 10.10% [JUNE 2024]

$E = \left[(1+i)^n - 1\right] \times 100$ $E = \left[(1.05)^2 - 1\right] \times 100 = 10.25\%$

03. What is the effective rate of interest when principal amount of ₹ 50,000 deposited at a nationalized bank for one year corresponding to a nominal rate of 8% p.a. COMPOUNDED QUARTERLY Given $1.02^4 = 1.0824$

a. 10.38% b) 8.08% c) 8.16% [d] 8.24% [DEC 2023]

$E = \left[(1.02)^4 - 1\right] \times 100 = 8.24\%$

04. the nominal rate of interest is 10% per annum . The interest is COMPOUNDED QUARTERLY . The effective rate of interest per annum will be

a. 10 % b. 10.10% c. 10.25% [d.] 10.38% [JUNE 2023]

$E = \left[(1.025)^4 - 1\right] \times 100 = 10.38\%$

05. the effective annual rate of interest corresponding to a normal rate of 6% annum payable HALF YEARLY is a. 6.06% b. 6.07% c. 6.08% [d.] 6.09% [DEC 2022]

$E = \left[(1.03)^2 - 1\right] \times 100 = 6.09\%$

06. you are considering two investments . Investment A yields 10% compounded quarterly . Investment B yields r% compounded semiannually . Both investments have equal annual yields . Find r a. 19.875% b. 10% c. 10.38% [d.] 10.125% [JUNE 2024]

$E_A = E_B$

$\left[(1.025)^4 - 1\right] \times 100 = \left[(1+R/200)^2 - 1\right] \times 100$

$(1.025)^4 = (1+ R/200)^2$

$(1.025)^2 = 1+ R/200 \Rightarrow 1+ R/200 = 1.050625$ $R = 10.125\%$

FUTURE VALUE (ACCUMULATED VALUE) OF ANNUITY

F.V. $= A\left[\frac{(1+i)^n-1}{i}\right]$, annuities happen at the END of every period

$= A\left[\frac{(1+i)^n-1}{i}\right](1+i)$, annuities happen at the START of every period

where A – instalment/annuity , i – rate of compounding per period

n – no of instalments

F.V. INTEREST FACTOR $A(n,i) = \frac{(1+i)^n-1}{i} \Rightarrow$ F.V. $= A \times A(n,i)$

Q1 FIND FUTURE VALUE OF ANNUITY (ORDINARY)

01. Find future value of an annuity of ₹ 5000 made annually for 6 years at interest rate of 12% COMPOUNDED ANNUALLY if $(1.12)^6 = 1.9738$

a. ₹ 45,375 **b.** ₹ 40,575 c. ₹ 39,465 d. ₹ 37,868 [JUNE 2024]

F.V. $= A\left[\frac{(1+i)^n-1}{i}\right]$ F.V. $= 5000\left[\frac{(1+0.12)^6-1}{0.12}\right] =$ ₹ 40,575

02. Find future value of an annuity of ₹ 1000 made annually for 7 years at interest rate of 14% COMPOUNDED ANNUALLY if $(1.14)^7 = 2.5023$

a. ₹ 10730 b. ₹ 5365 c. ₹ 8756 d. ₹ 9892 [JAN 2021,JUNE 2022]

F.V. $= A\left[\frac{(1+i)^n-1}{i}\right]$ F.V. $= 1000\left[\frac{(1.14)^7-1}{0.14}\right] =$ ₹ 10,730.71

03. Mr A invested ₹ 10,000 EVERY YEAR for next 3 years at the interest rate of 8% per annum COMPOUNDED ANNUALLY . What is future value of the annuity

a. ₹ 32644 **b.** ₹ 32464 c. ₹ 34264 d. ₹ 36442 [DEC 2022]

F.V. $= A\left[\frac{(1+i)^n-1}{i}\right]$, F.V. $= 10000\left[\frac{1.08^3-1}{0.08}\right] =$ ₹ 32464

04. Ramesh invests ₹ 20000 per year in a stock index fund which earns 9% per year for the next 10 years . What would be the accumulated value upon payment of last inv. [JUNE 2022]

[$1.09^{10} = 2.36736$] a. ₹ 3,88,764 **b.** ₹ 3,03,858 c. ₹ 2,68,728 d. ₹ 4,08,718

F.V. $= A\left[\frac{(1+i)^n-1}{i}\right]$, F.V. $= 20000\left[\frac{1.09^{10}-1}{0.09}\right] =$ ₹ 3,03,858

05. ₹ 5000 is invested EVERY MONTH in an account @ 12% p.a. compounded annually . What is the F.V. of this annuity just after making 11^{th} payment , GIVEN $1.01^{11} = 1.1156$,

[a.] ₹ 57800 b. ₹ 56100 c. ₹ 56800 d. ₹ 57100 [DEC 2022]

$F.V. = A\left[\frac{(1+i)^n-1}{i}\right]$, $F.V. = 5000\left[\frac{1.01^{11}-1}{0.01}\right] = ₹\ 57800$

06. ₹ 800 is invested at the end of EVERY MONTH in an account paying interest @ 6% p.a. compounded annually . What is the future value of this annuity just after making 10^{th} payment [GIVEN $1.005^{10} = 1.0511$, $1.005^{20} = 1.1049$]

a. ₹ 16,764 b. ₹ 8,766 c. ₹ 3,491 [d.] ₹ 8,176 [JAN 2021,JUNE 2022]

$F.V. = 5000\left[\frac{1.005^{10}-1}{0.005}\right] = ₹\ 8,176$

07. Lokesh deposits 3000 at the start of each quarter in his savings account . If the account earns interest of 5.75% p.a. COMPOUNDED QUARTERLY , how much money will he have at the end of 4 years

a. ₹ 53,624 b. ₹ 58,353 [c.] ₹ 54,308 d. ₹ 54,803 [JUNE 2022]

$i = 5.75/400 = 0.014375$, $n = 4x4 = 16$

$F.V. = A\left[\frac{(1+i)^n-1}{i}\right](1+i) = 3000\left[\frac{(1.014375)^{16}-1}{0.014375}\right](1.014375) = ₹\ 54,308.57$

Q2 FIND FUTURE VALUE OF ANNUITY (IMMEDIATE)

01. Suppose Mr X invested ₹ 5000 every year starting FROM TODAY in mutual fund for next 10 years . Assuming average return COMPOUNDED ANNUALLY is at 18% p.a. What is the future value

a. ₹ 183,677.68 b. ₹ 138,678.85 c. ₹ 1,83,776.53 [d.] ₹ 1,38,774.55 [DEC 2023]

$F.V. = A\left[\frac{(1+i)^n-1}{i}\right](1+i)$, $F.V. = 5000\left[\frac{(1+0.18)^{10}-1}{0.18}\right] 1.18 = = 138775.72$

02. suppose you have decided to make a Systematic Investment plan (SIP) in a mutual fund with ₹ 100,000 every year FROM TODAY for next 10 years where you get return at rate of 10% p.a. COMPOUNDED ANNUALLY . What is the future value of this annuity . $(1.1)^{10} = 2.59374$

a. ₹ 17,35,114 b. ₹ 17,53,411 c. ₹ 17,35,411 [d.] ₹ 17,53,114 [JUNE 2023]

$F.V. = A.\left[\frac{(1+i)^n-1}{i}\right](1+i)$, $F.V. = 100,000\left[\frac{1.1^{10}-1}{0.1}\right] x\ 1.1 = ₹\ 17,53,114$

03. Raju invests 20,000 EVERY YEAR in a deposit scheme STARTING TODAY for next 12 years . Assuming the interest rate on this deposit is 7% per annum compounded annually . What will be the future value of this annuity ? GIVEN $(1+0.07)^{12} = 2.25219150$

a. ₹ 540,576 **b.** ₹ 382,813 c. ₹ 643,483 d. ₹ 357,769 [DEC 2022]

$F.V. = A\left[\frac{(1+i)^n-1}{i}\right](1+i)$, $F.V. = 20000\left[\frac{(1.07)^{12}-1}{0.07}\right] \times 1.07 = ₹\ 3,82,812.83$

Q3 SINKING FUND - GIVEN F.V. FIND THE INSTALMENT SIZE

01. How much amount is required to be invested every year so as to accumulate ₹ 30,000 at the end of 10 years if the interest COMPOUNDED ANNUALLY at 10% Given A(10,0.1) = 15.9374

a ₹ 1882.36 b) ₹ 1828.30 c) ₹ 1832.65 d) ₹ 1853.65 [DEC 2023]

F.V. = A x A(n,i) $30000 = A \times A(10,0.1) \Rightarrow A = ₹\ 1882.36$

02. How much amount is required to be invested every year so as to accumulate ₹ 5,00,000 at the end of 12 years if interest is COMPOUNDED ANNUALLY at 10% . GIVEN A(12,0.1) = 21.384284

a. ₹ 23381.65 b. ₹ 24385.85 c. ₹ 26381.65 d. ₹ 28362.75 [DEC 2022]

F.V. = A x A(n,i) $500,000 = A \times A(12,0.1) \Rightarrow A = ₹\ 23381.65$

03. company wants to replace its existing tool machine at the end of 10 years , the expected cost of machine would be 10,00,000 . If the management of the company creates a sinking fund , how much provision needs to be made out of revenue each year which can earn at the rate of 10% COMPOUNDED ANNUALLY . A(10,0.10) = 15.937425

a. ₹ 74,625 b. ₹ 72,514 **c.** ₹ 62,745 d. ₹ 67,245 [JUNE 2023]

F.V. = A x A(n,i) $10,00,000 = A \times A(10,0.10) \Rightarrow A = ₹\ 62,745$

04. a company creates a sinking fund of ₹ 200,000 in a bank account for 15 years at interest rate of 6% p.a. . The YEARLY PAYMENT to be paid by company will be $[1.06^{14} = 2.209]$

a. ₹ 8,945 b. ₹ 8,145 c. ₹ 9,345 d. ₹ 9,645 [DEC 2022]

$F.V. = A\left[\frac{(1+i)^n-1}{i}\right]$ $2,00,000 = A \times \left[\frac{1.06^{15}-1}{0.06}\right] \Rightarrow A = ₹\ 8945$

05. Mr X wants to accumulate ₹ 50,00,000 at the end of 10 years . Then how much amount is required to be invested every year if interest is COMPOUNDED ANNUALLY at 10%
GIVEN A(10,0.10) = 15.9374298

a. ₹ 3,13,726 b. ₹ 4,12,726 c. ₹ 3,53,726 d. ₹ 4,53,726 [DEC 2021]

F.V. = A x A(n,i) $50,00,000 = A \times A(10,0.10) \Rightarrow A = ₹\ 3,13,726.87$

06. let a person invest a fixed sum at the END OF EACH MONTH in an account paying interest 12% per year compounded monthly . IF the future value of this annuity after the 12th payment is Rs 55,000 then amount invested every month is

a. ₹ 4,837 b. ₹ 4,637 **c.** ₹ 4,337 d. ₹ 3,337 [JUNE 2019]

$F.V. = A\left[\frac{(1+i)^n-1}{i}\right]$ $55,000 = A \times \left[\frac{1.01^{12}-1}{0.01}\right]$ $\Rightarrow A = ₹\ 4336.68$

07. RAJA aged 40 wishes his wife Ratna to have 40 lacs at his death . His expectation of life is another 30 years and he starts making equal annual investments COMMENCING NOW at 3% p.a . How much should he invest annually

a. ₹ 84,077 **b.** ₹ 81,628 c. ₹ 84,449 d. ₹ 84,000 [NOV 2007]

$4000000 = A\left[\frac{1.03^{30}-1}{0.03}\right](1.03)$ $A = 81,628.20$

08. Sinking fund factor is the reciprocal of

a. Present value interest factor of a single cash flow
b. Present value interest factor of an annuity
c. Future value interest factor of an annuity
d. Future value interest factor of a single cash flow

Sinking fund

F.V. = Ax An,i)

$A = \frac{F.V.}{A(n,i)}$

= F.V. x SFF , SFF = 1/A(n,i)

Q4 PRESENT VALUE OF ANNUITY

$P.V. = A\left[\frac{1-(1+i)^{-n}}{i}\right]$ ORDINARY ANNUITY → P(n,i) ← $P.V. = A\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$ ANNUITY IMMEDIATE

01. ₹ 2,500 is paid every year for 10 years to pay off a loan . What is the loan amount if interest rate be 14% per annum compounded annually

a. ₹ 13,040 b. ₹ 15,848 c. ₹ 14,674 d. ₹ 16,345 [DEC 2020]

P.V. loan $= A\left[\frac{1-(1+i)^{-n}}{i}\right]$ $= 2500\left[\frac{1-(1.14)^{-10}}{0.14}\right] = ₹\ 13040$

02. If Mrs. X invests in an annuity IMMEDIATELY that promises annual payments of ₹ 50,000 for the next 16 years . If the interest rate is 6% compounded annually then the approximate present value of this annuity is $(1.06)^{16} = 2.54035$

a) ₹ 5,51,217.75 b) ₹ 5,75,900.00 c) ₹ 5,05,288.08 **d** ₹ 5,35,612.45 [DEC 2023]

$P.V = A\left[\frac{1-(1+i)^{-n}}{i}\right](1+i) = 50000\left[\frac{1-(1.06)^{-16}}{0.06}\right](1.06) = ₹\ 5,35,612.45$

03. Govinda's mother decides to gift him ₹ 50,000 every year STARTING FROM TODAY for the next 5 years . Govinda deposits this amount in a bank as and when he receives and gets 10% per annum interest rate compounded annually . What is the present value of this annuity .

[GIVEN P(4,0.10) = 3.16987]

a. ₹ 2,80,493.5 **b.** ₹ 2,08,493.5 c. ₹ 2,08,943.5 d. ₹ 2,58,493.5 [JUNE 2023]

NOTE – IGNORE the 1st annuity of ₹ 50,000 and look at the next 4 annuities of ₹ 50,000 starting from 1st year end which can then be considered as ORDINARY ANNUITY wrt TODAY . However 1st annuity of ₹ 50,000 can then be added to the answer as present value of ₹ 50,000 paid today will be ₹ 50,000

P.V. = 50000. P(4,0.10) (Present value of 4 inst. Starting from 1st year end) + 50000 (P.V. of 1st 50000 deposited today) = ₹ 2,08,493.50

04. Suppose your mom decides to gift your 10,000 every year STARTING FROM TODAY for next 16 years . You deposit this amount in a bank as and when your receive and get 8.5% p.a. compounded annually . What is the present value of this money [P(15,0.085) = 8.304236]

a. ₹ 83,042 b. ₹ 90,100 **c.** ₹ 93,042 d. ₹ 10,100 [DEC 2015]

P.V. = 10000. P(15,0.085) (Present value of 15 inst. Starting from 1st year end) + 10000 (P.V. of 1st 10000 deposited today) = ₹ 93,042

05. The present value of an annuity immediate is the same as

a. annuity regular for (n–1) years plus the initial receipt in the beginning of the period

b. annuity regular for (n–1) years

c. annuity regular for (n+1) years

d. annuity regular for (n–1) years plus the initial receipt in the beginning of the period

REFER SUM 03 & 04 SOLVED BEFORE THIS IN THE SAME Q SET

Q5 APP OF PRESENT VALUE OF ANNUITY - GIVEN THE LOAN AMT , FIND THE INSTALMENT SIZE

01. A loan of ₹ 30,000 is to be paid in 5 annual instalments with interest rate of 14% p.a. then equal annual instament will be . P(5,0.14) = 3.43308

a. ₹ 8100 b. ₹ 7400 c. ₹ 8738 **d.** ₹ 8378 [JUNE 2024]

P.V (LOAN) = A x P(n,i) , A = instalment ⇒ 30000 = A x P(5,0.14) ⇒ A = ₹ 8378

02. a car is available for ₹ 4,98,200 cash payment or ₹ 60,000 cash down payment followed by 3 equal annual instalments . If the rate of interest charged is 14% per annum compounded yearly , then TOTAL INTEREST CHARGED in the instalment plan [Given P(3,0.14) = 2.32163]
a. ₹ 1,46,314 b. ₹ 1,46,137 c. ₹ 1,28,040 d. ₹ 1,58,040 [JUNE 2023]

Loan amount V = 4,98,200 – 60,000 = 4,38,200 ,

4,38,200 = A x P(3,0.14) $\Rightarrow$ A(instalment) = 1,88,746.70

Amount repaid = 1,88,746.70 x 3 = 5,66,240
Amount borrowed = 4,38,200
Interest charged = ₹1,28,040

03. A loan of ₹ 1,02,000 is to be paid back in 2 equal instalments . If the rate of interest is 4% p.a compounded annually , then the TOTAL INTEREST CHARGED under this instalment plan is
a. ₹ 6160 b. ₹ 8120 c. ₹ 5980 d. ₹ 7560 [JULY 2021]

P.V. (loan) $= A\left[\frac{1-(1+i)^{-n}}{i}\right]$, A = instalment $\quad 102000 = A\left[\frac{1-(1.04)^{-2}}{0.04}\right] \Rightarrow A =$ ₹ 54,080

Amount repaid = 54080 x 2 = 1,08,160
Amount borrowed = 1,02,000
Interest charged = ₹ 6160

04. Madhu takes a loan of 50,000 from XYZ bank @ 10% p.a. The first instalment will be paid at the end of year 5 . Determine the amount of equal instalments , if Madhu wishes to repay the amount in 5 instalments
a. ₹ 19,510 b. ₹ 19,430 c. ₹ 19,310 d. ₹ 19,630 [JUNE 2022]

Loan amt at the end of 5 years = $50000(1.10)^5$ = ₹ 80,525.50
At this very point , Madhu starts repaying the loan of ₹ 80,525.50 and so we need to apply P.V. OF ANNUITY IMMEDIATE KA FORMULA

$P.V = A\left[\frac{1-(1+i)^{-n}}{i}\right](1+i) \quad 80525 = A\left[\frac{1-(1.10)^{-5}}{0.10}\right](1.10) \Rightarrow A =$ ₹ 19,311

Q6 PRESENT VALUE OF ANNUITY (LEASING)

01. a person wants to open a shop . He has 2 options . He can acquire the commmercial space either by leasing for 10 years at annual rent of ₹ 2,00,000 or purchase the space for ₹ 12,00,000 . If person can borrow the money at 14% compounded per annum , which alternate is most suitable . GIVEN – P(10,0.14) = 5.21611
a) Leasing b. Purchase c. Can't say d. Data insufficient [DEC 2023]

PURCHASE PRICE = ₹ 12,00,000

COST OF LEASING (P.V. OF ALL RENTAL OUTFLOW) = 200000 x P(10,0.14) = ₹ 10,43,222

02. a company may obtain a machine either by leasing it for 4 years (USEFUL LIFE) at an annual rent of ₹ 1250 or by purchasing the machine for ₹ 4000 . If the company can borrow money at 14% p.a. , which alternative is preferable

[a)] Leasing b. Purchase c. Can't say d. Data insufficient [MAY 2007]

PURCHASE PRICE = ₹ 4000 V/S COST OF LEASING

$PV = 1250\left[\frac{1 - 1.14^{-4}}{0.14}\right] = ₹\ 3642$

P.V. OF ALL RENTAL OUTFLOW

03. a company may obtain a machine either by leasing it for 5 years (USEFUL LIFE) at an annual rent of ₹ 2000 or by purchasing the machine for ₹ 8100 . If the company can borrow money at 18% p.a. , which alternative is preferable

[a)] Leasing b. Purchase c. Can't say d. Data insufficient [MAY 2007]

PURCHASE PRICE = ₹ 8100 V/S COST OF LEASING

$PV = 2000\left[\frac{1 - 1.18^{-5}}{0.18}\right] = ₹\ 6,254$

P.V. OF ALL RENTAL OUTFLOW

04. A person wants to lease out a machine costing ₹ 5,00,000 for a 10 year period . It has a fixed rental of 51,272 per annum payable annually starting from the end of first year . Suppose the rate of interest is 10% p.a. compounded annually on which money can be invested . To whom the agreement is favorable

[a.] Lessee b. Lessor c. Not for both d. Can't be determined [FEB 2008]

CASH OUTFLOW = ₹ 5,00,000

CASH INFLOW (P.V. OF RENTAL INFLOWS) $= 51272\left[\frac{1 - 1.10^{-10}}{0.10}\right] = ₹\ 3,15,044$ (NOT FAVORABLE TO LESSOR) HENCE FAVORABLE TO LESSEE

05. A machine can be purchased for ₹ 50,000 . Machine will contribute 12000 per year for the next five years . Assume borrowing cost is 10% per annum compounded annually . Determine whether machine should be purchased or not

a. should purchase [b.] should not purchase c. can't say d. none [FEB 2007]

CASH OUTFLOW = ₹ 50,000

CASH INFLOW (P.V. OF CONTRIBUTION) $= 12000\left[\frac{1 - 1.10^{-5}}{0.10}\right] = ₹\ 45,489$ SHOULD NOT PURCHASE

Q7 PERPETUITY

PERPETUITY -

PERPETUITY IS AN ANNUITY IN WHICH THE PERIODIC PAYMENTS OR RECIEPTS BEGIN ON A FIXED DATE AND CONTINUE INDEFINITELY OR PERPETUALLY . FIXED COUPON PAYMENTS ON PERMANENTLY INVESTED SUM OF MONEY

PRESENT VALUE OF PERPETUITY (ANNUITY INFINITE)

$$PVA_\infty = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \ldots\ldots \infty = \frac{R}{i}$$

GROWING PERPETUITY

RECEIPTS / CASH INFLOWS MUST GROW AT CONSTANT RATE .

LETS UNDERSTAND

EXAMPLE – ASSUMING GROWTH RATE 5% AND FIRST RECEIPT (CASH INFLOW) R = 100 ,

CASH INFLOWS AT THE END OF 1^{ST} YEAR = 100

2^{ND} YEAR = $100(1.05)$ = 105

3^{RD} YEAR = $100(1.05)^2$ = 110.25

4^{TH} YEAR = $100(1.05)^3$ = 115.7625 ……∞

PRESENT VALUE OF GROWING PERPETUITY (ANNUITY INFINITE)

LET i = DISCOUNT RATE , g = GROWTH RATE

$$PVA_\infty = \frac{R}{1+i} + \frac{R(1+g)}{(1+i)^2} + \frac{R(1+g)^2}{(1+i)^3} + \ldots\ldots \infty = \frac{R}{i-g}$$

01. Determine the present value of perpetuity of ₹ 10 per month for infinite period at an effective rate of 14% p.a

a. ₹ 657 b. ₹ 757 **c.** ₹ 857 d. ₹ 957 [DEC 2020]

$$PVA_\infty = {}^{R}/_{i} = \frac{10}{{}^{14}/_{1200}} = \frac{10 \times 1200}{14} = ₹\ 857$$

02. Mr Sharad got his retirement benefits amounting to 50,00,000 . He wants to receive a fixed monthly sum of amount for his rest of life after one month and thereafter he wants to pass on the same to future generation . He expects to earn an interest of 9% compounded annually . Determine how much perpetuity amount he will receive every month

a. ₹ 39,500 b. ₹ 38,500 **c.** ₹ 37,500 d. ₹ 36,600 [JUNE 2023]

$$PVA_\infty = \frac{R}{I} \Rightarrow 50,00,000 = \frac{R}{{}^{9}/_{1200}}, \quad R = ₹\ 37,500$$

03. assuming that the discount rate is 7% p.a , how much would you pay to get ₹ 200 per year growing @ 5% forever

a. ₹ 2500 b. ₹ 5000 c. ₹ 7500 **d.** ₹ 10000 [JAN 21 , JUNE 22]

$$PVA_\infty = {}^{R}/_{i-g} = 200/(0.07-0.05) = ₹\ 10000$$

04. assuming that the discount rate is 14% p.a , how much sould company pay to get ₹ 280 growing @ 9% forever

[a.] ₹ 5600 b. ₹ 2800 c. ₹ 1400 d. ₹ 4200 [JULY 2021]

$PVA_\infty = {}^{R}/_{i-g} = 280/(0.14-0.09) = ₹\ 5600$

05. assuming that the discount rate is 12% p.a , how much would you pay to get ₹ 100 per year growing @ 4% forever

a. ₹ 1425 b. ₹ 1300 [c.] ₹ 1250 d. ₹ 1150

$PVA_\infty = R/i-g = 100/(0.12-0.04) =$ ₹ 1250 [JUNE 2024]

06. If a person bought a house by paying ₹ 45,00,000 down payment and ₹ 80,000 at the end of each year till the perpetuity , assuming the rate of interest as 16% , the present value of house is

a. ₹ 47,00,000 b. ₹ 45,00,000 c. ₹ 57,80,000 [d.] ₹ 50,00,000 [JULY 2021]

$P.V. = 45,00,000 + {}^{R}/_{i} = 45,00,000 + {}^{80,000}/_{0.16} = ₹\ 50,00,000$

Q8 BOND EVALUATION

01. ₹ 1000 bond paying annual dividends of 8.5% will be redeemed at par value at the end of 10 years . Find the purchase value of this bond if the investor wishes a yield rate of 8%

a ₹ 907.135 [b] ₹ 1033.54 c ₹ 945.67 d none of these [MTP DEC 2023]

MY EXPLANATION

If you need to recieve ₹ 85 yearly dividend for 10 long years , you need to pay upfront an amount equal to PRESENT VAL. OF ANNUITY OF ₹ 85 receivable annually for 10 year . This will form one part of the purchase price of the bond

If you need to redeem the bond at its par value of 1000 at the end of 10 years , again you will have to pay present value of 1000 receivable at the end of 10 years . This will form the the second part of the purchase price of the bond

PURCHASE VALUE OF BOND

= present value of annuity of ₹ 85 (dividend) receivable annually for 10 year + Present value of ₹ 1000 receivable at the end of 10 years @ 8%

$= 85\left[\frac{1-1.08^{-10}}{0.08}\right] + \frac{1000}{(1.08)^{10}}$

$= 570.36 + 463.19$

$= ₹\ 1033.55$

02. 3 year , ₹ 1000 bond paying annual nominal rate of 10% . At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%

a ₹ 904 [b] ₹ 907.125 c ₹ 905.25 d 909 [SEPT 2024]

Purchase value of bond'

= present value of annuity of ₹ 100 (interest) receivable annually for 3 year + Present value of ₹ 1000 receivable at the end of 3 years @ 14%

$$= 100\left[\frac{1-1.14^{-3}}{0.14}\right] + \frac{1000}{(1.14)^3}$$

= 232.16 + 674.97

= ₹ 907.13

09 - PERMUTATION

r items out of n can be arranged in ${}^nP_r$ ways

example 1. Arrange 3 men on to 5 chairs .... ${}^5P_3 = 5.4.3 = 60$ ways

2. 5 men , arrange any 3 out of them into 3 chairs = ${}^5P_3 = 5.4.3. = 60$ ways

Q1

01. A student has 3 books on Computer , 3 books on Economics , 5 on Commerce . If these books are to be arranged SUBJECTWISE on a shelf , in how many ways they can be arranged
a. 25290 [b.] 25920 c. 4230 d. 4320 [JUNE 2015/DEC 2017]

$\overline{3}$ C , $\overline{3}$ E , $\overline{5}$ C $= 3!.3!.3!.5! = 25920$

02. if 5 books of English , 4 books of Tamil and 3 books of Hindi are to be arranged in a single row so that books of same language come together . Find number of ways
a. 180630 b. 160830 [c.] 103680 d. 130680 [DEC 2014]

$\overline{5}$ E , $\overline{4}$ T , $\overline{3}$ H $= 3!.3!.3!.5! = 103680$

03. The number of ways of arranging 6 boys and 4 girls in a row so that all 4 girls are together
a. 6!.4! b. 2(7!.4!) [c.] 7!.4! d. 2(6!.4!) [JUNE 2010]

6 B , $\overline{4}$ G = 7!.4!

04. In how many ways can the 5 trophies be arranged on a shelf if one particular trophy must always be in the middle [a.] 24 b. 120 c. 48 d. 144 [JUNE 2024]

One particular trophy will occupy middle shelf in 1 way & remaining 4 will arrange into the remaining 4 shelves in 4! ways
Total arrangements = $4! = 24$

05. total number of arrangements of 8 persons in a row with President and Vice President occupying middle chairs
a. 6! b. 7! [c.] 6!.2! d. 7!.2! [JUNE 2022]

President and Vice President will occupy middle chairs in 2! ways & remaining 6 will arrange into the remaining 6 chairs in 6! ways
Total arrangements = 2!.6!

06. 5 boys and 4 girls are to be seated in row . If the girls occupy even places then the number of arrangements a. 288 b. 2808 c. 2008 d. 2880 [DEC 2014]

4 GIRLS 5 BOYS
2,4,6,8 1,3,5,7,9
4! x 5! = 24 x 120 = 2880

07. Eight chairs are numbered from 1 to 8 . 2 women and 3 men are to be seated by allowing one chair for each . First the women choose the chairs from the chairs numbered 1 to 4 and then men select the chairs from the remaining . The number of possible arrangements is
a. 120 b. 288 c. 32 d. 1440 [JAN 2021]

2 WOMEN 3 MEN
1 - 4 REMAINING 6 CHAIRS
$^4P_2$ x $^6P_3$ = 12 x 120 = 1440

08. Number of ways , 4 boys and 3 girls can be seated in a row so that they are alternate
a. 12 b. 288 c. 144 d. 256 [DEC 2022]

B G B G B G B = 4!.3! = 24 x 6 = 144

09. a garden having 6 tall trees in a row . In how many ways 5 children stand , one in a gap between trees in order to pose for a photograph .
a. 24 b. 120 c. 720 d. 30 [DEC 2010]

T_1 B T_2 B T_3 B T_4 B T_5 B T_6 = 5! = 120

10. 5 boys and 3 girls are to be seated together such that no two girls are together
a. 14400 b. 2400 c. 720 d. none [DEC 2019/JUNE 2024]

x B x B x B x B x

3 girls will occupy places marked x in $^6P_3$, done with that 5 boys will occupy the remaining 5 chairs in 5!

Total arrangement = $^6P_3$x 5! = 120 x 120 = 14400

11. in a class of 4 boys and 3 girls are to be seated together in a row such that no two girls are together a. 60 b. 480 c. 720 d. 1440 [SEPT 2024]

x B x B x B x

3 girls will occupy places marked x in $^5P_3$, done with that 4 boys will occupy the remaining 4 places in 4! ways . Total arrangement = $^5P_3$x 4! = 5.4.3 x 24 = 1440

12. there are 10 students in a class including 3 girls . The number of ways to arrange them in a row , when any 2 girls out of them never come together [JUNE 2016]

[a.] $^8P_3..7!$ b. $^3P_3.7!$ c. $^8P_3.10!$

x B x B x B x B x B x B x

3 girls will occupy places marked x in $^8P_3$, done with that 7 boys will occupy the remaining 7 chairs in $7!$ ways

Total arrangement = $^8P_3 x\ 7!$

13. Find the number of arrangements of 5 things taken out of 12 things , in which one particular thing always be included

a. 39000 b. 37600 [c.] 39600 d. 36000 [JUNE 2011]

One particular thing from 12 will arrange into any of the 5 places in $^5P_1$ ways . Done with that , remaining 4 things from the remaining 11 will arrange into the available places in $^{11}P_4$ ways . Hence total arrangements = $^5P_1 x\ ^{11}P_4 = 5.11.10.9.8 = 39600$

Q2

01. How many different words can be formed with letters of the word 'LIBERTY'

a. 4050 [b.] 5040 c. 5400 d. 4500 [DEC 2013]

LIBERTY = 7L , $7! = 5040$

02. In how many ways letters of the word STADIUM be arranged in such a way that the vowels occur together a. 7!/3! [b.] 5!.3! c. 5!.4! d. 7!.3! [JUNE 2024]

STADIUM = 7L , 3vowels , 4 consonants

3 V , 4 C = 5!.3!

03. How many permutation can be formed from the letters of the word 'DRAUGHT' if vowels may not be separated .

a. 720 [b.] 1440 c. 140 d. 1000 [DEC 2012]

DRAUGHT = 7L , 2vowels , 5 consonants

2 V , 5 C = 6!.2! = 720x2 = 1440

04. Number of ways the letters of the word DETAIL can be arranged in such a way that vowels occupy odd places a. 32 [b.] 36 c. 48 d. 60 [DEC 2021]

3 VOWELS A,E,I — 3 CONSONANTS D,T,L

1,3,5 — 2,4,6

$3! \times 3! = 6 \times 6 = 36$

05. In how many ways the word ARTICLE can be arranged in a row so that vowels occupy even places a. 132 [b.] 144 c. 72 d. 160 [JUNE 2013]

3 VOWELS A,I,E — 4 CONSONANTS

2,4,6 — 1,3,5,7

$3! \times 4! = 6 \times 24 = 144$

06. In how many ways the word VIOLENT can be arranged in a row so that vowels occupy even places [a.] 144 b. 120 c. 24 d. 72 [JUNE 2012]

3 VOWELS I,O,E — 4 CONSONANTS

2,4,6 — 1,3,5,7

$3! \times 4! = 6 \times 24 = 144$

07. Number of words that can be formed using the letters of the word PETROL such that words do not have P in the first place a. 720 b. 120 [c.] 600 d. 540 [DEC 2021]

PETROL – 6 L

P will be arranged into any of the remaining 5 places in ${}^5P_1$ ways . Done with that , remaining 5 letters into the remaining 5 places in $5!$ ways

Total arrangement $= {}^5P_1 \times 5! = 5 \times 120 = 600$

08. 4 letter words with or without any meaning can be formed using letters of word LOGARITHMS if repetition is not allowed [a.] 5040 b. 7020 c. 5400 d. 30240 [JUNE 2022]

${}^{10}P_4 = 10.9.8.7 = 5040$

09. Number of 4 letter words that can be formed with letters of the word DECTIONARY

[a.] 5040 b. 720 c. 90 d. 30240 [DEC 2021]

${}^{10}P_4 = 10.9.8.7 = 5040$

Q3

01. How many numbers between 1000 and 10000 can be formed with the digits 1 , 2 , 3 , 4 , 5 , 6
a. 720 [b.] 360 c. 120 d. 60 [DEC 2016]

1 , 2 , 3 , 4 , 5 , 6 = 6 d

_ _ _ _ $= {}^{6}P_4 = 6.5.4.3 = 360$

02. How many numbers not exceeding 1000 can be made from the digits 1 – 9 if repetition is not allowed
a. 364 [b.] 585 c. 728 d. 819 [JUNE 2010]

1 digit , 2 digit & 3 digit number , ${}^{9}P_1 + {}^{9}P_2 + {}^{9}P_3 = 9 + 72 + 504 = 585$

03. How many 7 digit numbers can be formed using digits 3 , 4 , 5, 6 , 7 , 8 , 9 with no digits repeated not ÷ by 5 . [a.] 4320 b. 4690 c. 3900 d. 3890 [JULY 2021]

3 , 4 , 5, 6 , 7 , 8 , 9 = 7 d , we need to make 7 digit number

Unit place – 3,4,6,7,8,9 = ${}^{6}P_1$, remaining 6 places by remaining 6 digits 6! ways

Total numbers formed = ${}^{6}P_1 \times 6! = 6 \times 720 = 4320$

04. How many 3 digit odd numbers can be formed using digits 5 , 6 , 7 , 8 , 9 if the digits can be repeated a. 55 [b.] 75 c. 65 d. 96 [DEC 2022]

5, 6 , 7 , 8 , 9 = 5 d , Requirement = 3 digit odd number

Unit place – 5,7,9 = ${}^{3}P_1$, since repeats are allowed , remaining 2 places can be filled by any of the 5 digits in 5 ways each . Total numbers formed = ${}^{3}P_1 \times 5 \times 5 = 75$

05. How many odd numbers of 4 digit can be formed with digit 0 , 1, , 2 , 3 , 4 , 7 , 8
a. 150 [b.] 300 c. 120 d. 210 [JAN 2021]

0 , 1, , 2 , 3 , 4 , 7 , 8 = 7 d , we need to make 4 odd digit number

Unit place – 1, 3 , 7 = ${}^{3}P_1$ ways ,

Left with 6 digits , First place must not contain 0 , can be filled in ${}^{5}P_1$ ways

Left with 5 digits , Remaining 2 places can be filled in ${}^{5}P_2$ ways

Total numbers formed = ${}^{3}P_1 \times {}^{5}P_1 \times {}^{5}P_2 = 3 \times 5 \times 5x4 = 300$

Q4

TOTAL NUMBER OF ARRANGEMENTS OF N DIFFERENT THINGS IN WHICH 'p items' ARE ALIKE AND OF ONE KIND , 'q items' ARE ALIKE AND OF ONE KIND WILL BE GIVEN BY $^{n!}/_{p!.q!}$

LET ME TRY TO EXPLAIN

A,B,C,D would arrange amongst themselves in $^4P_4 = 4!$ Ways

lets say A , B , C were like items (say all were A) . In that case whichever 3 places they occupy, they would NOT further arrange amongst themselves in $3!$ ways which they do happen to when distinct (A,B,C) . Hence 3! Needs to be removed from the final answer which we must have got initially , treating all items distinct

Answer would then be $^{4!}/_{3!} = 4$ …… (AAAD , AADA , ADAA , DAAA)

01. The number of arrangements that can be formed from the letters of the word ALLAHABAD

[a.] 7560 b. 3780 c. 30240 d. 15320 [JUNE 2017]

$$\text{AAAA LL HBD} = \frac{9!}{4!.2!} = 7560$$

02. In how many different ways can the letters of the word CORPORATION be arranged so that the vowels always come together a 810 b 1440 c 25200 [d] 50400 [DEC 2023]

CORPORATION = 11 L

[OOOAI], CRRPTN

$$\$V \quad 6C = \frac{7!}{2!} \times \frac{5!}{3!} = 50400$$

(RR) (OOO)

03. The number of words from the letters of word BHARAT in which B and H will never come together a. 120 b. 360 [c.] 240 d. none [NOV 2018]

no of arrangements in which B & H do not come together = (Total arrangements) − (no. of arrangements in which B,H come together)

[BH], AART

$$= \frac{6!}{2!} - \frac{5!}{2!} \times 2!$$

$$= 360 - 120 = 240$$

Q5

Lets consider arrangements of 4 items **LINEAR V/s CIRCULAR**

LINEAR ARRANGEMENT

1 2 3 4 … 3!
2 1 3 4 … 3!
3 1 2 4 … 3!
4 1 2 3 … 3!

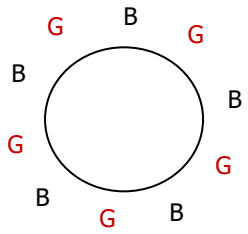
Starting with 1 , 2 , 3 , 4 the remaining gives you 3! arrangements leading to 4! = 24 arrangements . However since circle does not have start , 3! arrangements starting with 2 , 3 , 4 have to be ignored . Hence finally we are left with only 3! arrangements starting with 1

CONCLUSION

n items on a circular table can be arranged in $(n-1)!$ ways

01. In how many ways 5 boys and 5 girls are seated on a round table if no 2 boys are adjacent

a. 2550 **b. 2880** c. 625 d. 2476 [JULY 2021]

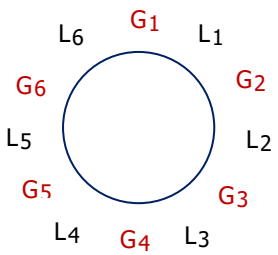


let the boys be arranged onto the round table . This will be done in $(5-1)! = 4!$ ways . Done with that girls will arrange into places marked G in 5! ways .

Total arrangements = $4!.5! = 24 \times 120 = 2880$

02 In how many ways can 6 GENTS and 5 ladies can be arranged at around table if no two ladies sit together

a. 86400 b. 14400 c. 84600 d. 34560



Let the 6 gents arrange into the seats marked G in $(6-1)! = 5!$ ways Done with that , since no two ladies must sit together , they need to be arranged into any 5 out of the 6 seats marked L .This is done in 6P5 ways .

Hence total arrangements = $5! \times {}^6P_5 = 86400$

Q6 ${}^nP_r = {}^{n!}/_{(n-r)!}$, however in practice we solve as

$${}^nP_3 = n(n-1)(n-2)$$
$${}^{n-1}P_3 = (n-1)(n-2)(n-3)$$
$${}^{n+1}P_3 = (n+1)n(n-1)$$

01. NOV 2018

$\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$, find N

$\frac{1}{7!} + \frac{1}{8.7!} = \frac{N}{9.8.7!}$

a. 81 b. 64 c. 78 d. 89

$1 + \frac{1}{8} = \frac{N}{72} \Rightarrow \frac{9}{8} = \frac{N}{72} \Rightarrow N = 81$

02. DEC 2021

${}^nP_2 = 12$. Find n

a. 2 b. 3 [c.] 4 d. 6

${}^nP_2 = 12$

$n(n-1) = 4.3$, $n = 4$

03. DEC 2020

${}^nP_4 = 20.{}^nP_2$. Find n

a.4 b. 2 c. 5 [d.] 7

$n(n-1)(n-2)(n-3)(n-4) = 20n(n-1)$

$(n-2)(n-3) = 5.4$

$n = 7$

04. JULY 2021

${}^nP_6 = 20.{}^nP_4$. Find n

5 b. 3 [c.] 9 d. 8

$n(n-1)(n-2)(n-3)(n-4)(n-5)$

$= 20n(n-1)(n-2)(n-3)$

$(n-4)(n-5) = 5.4$,

$n = 9$

05. DEC 2015

${}^nP_5 : {}^nP_3$. = 2:1 Find n

a.4 b. 7 c. 6 [d.] 5

$\frac{n(n-1)(n-2)(n-3)(n-4)}{n(n-1)(n-2)} = 2$

$(n-3)(n-4) = 2.1$

$n = 5$

06. JUNE 2014

If 6 times the no. of permutation of n items taken 3 at a time is equal to 7 times the no. of permutation of n–1 items taken 3 at a time then the value of n will be

a. 7 b. 9 c. 13 [d.] 21

${}^6{}^nP_3 = 7.{}^{n-1}P_3$

$6.n.(n-1)(n-2) = 7(n-1)(n-2)(n-3)$

$6n = 7(n-3)$

$n = 21$

07. JUNE 22

$\frac{n!}{10} = {}^{n-1}P_{n-3}$. Find n

[a.] 5 b. 6 c. 7 d. 8

using options

a. $\frac{5!}{10} = 12$, ${}^4P_2 = 12$,

LHS = RHS

08. JUNE 23

${}^6P_{2r} = 12.\ {}^6P_r$. Find r

a. 1 [b.] 2 c. 3 d. 4

Using options

a. ${}^6P_2 = 30$, $12.{}^6P_1 = 72$ ✗

b. ${}^6P_4 = 360$, $12.{}^6P_2 = 360$ ✓

09. JUNE 2019

which of the following is true

[a.] ${}^nP_n = {}^nP_{n-1}$

b. ${}^nP_n = {}^{2n}P_{n-2}$

c. ${}^nP_n = {}^{3n}P_{n-3}$

d. ${}^nP_n = {}^{n(n+1)}P_{n-1}$

Put n = 3

a. ${}^2P_2 = {}^2P_1 = 2$ ✓

b. ${}^2P_2 = {}^4P_1$ ✗

c. Put n = 3 , ${}^3P_3 = {}^9P_0$ ✗

d. ${}^2P_2 = {}^6P_1$ ✗

10. DEC 2017

if ${}^nP_{13} : {}^{n+1}P_{12} = 3:4$, then n

a. 13 [b.] 15 c. 18 d. 31

$$\frac{n!}{(n-13)!} : \frac{(n+1)!}{(n+1-12)!} = 3 : 4$$

$$\frac{n!}{(n-13)!} \times \frac{(n-11)!}{(n+1)!} = 3 : 4$$

$$\frac{n!}{(n-13)!} \times \frac{(n-11)(n-12)(n-13)!}{(n+1)n!} = 3 : 4$$

$$\frac{(n-11)(n-12)}{(n+1)} = \frac{3}{4}$$

Now using options ,
Option b , n = 15 satisfies the above condition

10 - COMBINATIONS

Q1

1. ${}^nC_r$ gives us the number of selections of r items out of the given n items

   2 out of 4 items can be selected in 4C2 ways

   3 out of 5 items can be selected in 5C3 ways

2. ${}^nC_r = {}^nC_{n-r}$

   ${}^6C_4 = {}^6C_2$

   since no.of ways of selecting 4 out of 6 = no. of ways of rejecting 2 out of 6

01. A person has 10 friends of which 6 of them are relatives . He wishes to invite 5 persons so that 3 of them are relatives . In how many ways he can invite

a. 450 b. 600 [c.] 120 d. 810 [JUNE 2015]

4 FRIENDS	6 RELATIVES
2	3

$= {}^4C_2 \times {}^6C_3 = 6 \times 20 = 120$

02. Out of 6 boys and 4 girls , find the number of ways for selecting 5 member committee in which there are exactly 2 girls [a.] 120 b. 1440 c. 720 d. 71 [DEC 2019]

6 BOYS	4 GIRLS
3	2

$= {}^6C_3 \times {}^4C_2 = 20 \times 6 = 120$

03. Out of 7 gents and 4 ladies a committee of 5 is to be formed . the number of committees such that AT LEAST 1 LADY is included a. 400 b. 440 [c.] 441 d. none

7 GENTS	4 LADIES
5 at least one lady	

all — committees with no lady JUNE 2022

$= {}^{11}C_5 - {}^7C_5$ DEC 2020

$= 462 - 21 = 441$

04. A bag contains 4 red , 3 black and 2 white balls . In how many ways 3 balls can be drawn from this bag so that they include AT LEAST ONE BLACK BALL

a. 46 [b.] 64 c. 86 d. 68 [NOV 2018]

4 R	3B	2W
3 at least one black		

all — combination with NO BLACK

$= {}^9C_3 - {}^6C_3$

$= 84 - 20 = 64$

05. How many total combinations can be formed of 8 different counters marked as 1 , 2 ,….., 8 taking 4 counters at a time and there being atleast one odd and one even counter in each combination [a.] 68 b. 66 c. 64 d. 62 [SEPT 2024]

4 EVEN	4 ODD
4 at least 1 odd & 1 even	

All combinations of 4 from 8 counters — [combination of no even i.e all 4 odd + combination of no odd i.e all 4 even]

$= {}^8C_4 - [{}^4C_4 + {}^4C_4]$

$= 70 - (1+1) = 68$

06. a selection is to be made for one post of of Principal and 2 post of Vice Principal . Amongst the 6 candidates called for the interview , only 2 are eligible for the post of Principal while they all six are eligible for the post of vice principal . The number of possible combinations

a. 4 b. 12 c. 18 **d.** 20 [SEPT 2024]

Principal can be selected in $^{2}C_{1}$ ways . Done with that , 2 vice principal can then be selected from the remaining 5 candidates in $^{5}C_{2}$ ways . Total ways $= {}^{2}C_{1} \times {}^{5}C_{2} = 2 \times 10 = 20$

07. In how many ways can a committee of 3 ladies and 4 gentlemen be appointed from a meeting consisting of 8 ladies and 7 gentle men ? What will be number of ways if Mrs X refuses to serve in a committee where Mr Y is a member

a.1530 b. 1500 c. 1520 **d.** 1540 [JUNE 2023]

ALL COMMITTIEES – COMMITTEES WHERE MRS X AND MR Y ARE TOGETHER

8L	7G
3L	4G

MRS X & MR Y ALREADY INCLUDED

7L	6G
2L	3G

$= {}^{8}C_{3} \times {}^{7}C_{4} - {}^{7}C_{2} \times {}^{6}C_{3}$

$= 1960 - 420 = 1540$

Q2

01. From a group of 8 men and 4 women , 4 persons are to be selected to form a committee so that ATLEAST 2 WOMEN are there on the committee . In how many ways can it be done

a. 201 b. 168 c. 202 d. 220 [DEC 2020]

	8 MEN 4 WOMEN	
CASE NO.	COMMITTEE CONTAINS	NO OF WAYS OF FORMING THE COMMITTEE
Case 1	2 MEN & 2 WOMEN	$^{8}C_{2} \times {}^{4}C_{2} = 28 \times 6 = 168$
Case 2	1 MAN & 3 WOMEN	$^{8}C_{1} \times {}^{4}C_{3} = {}^{8}C_{1} \times {}^{4}C_{1} = 8 \times 4 = 32$
Case 1	4 WOMEN	$^{4}C_{4} = 1 = 1$
		TOTAL COMMITTEES FORMED = 201

02. 6 gents & 4 ladies . A committee of 5 is to be formed if it includes ATLEAST 2 LADIES . How many committees can be formed a. 64 b. 162 c. 102 **d.** 186 [DEC 2015]

	6 GENTS 4 LADIES	
CASE NO.	COMMITTEE CONTAINS	NO OF WAYS OF FORMING THE COMMITTEE
Case 1	3 GENTS & 2 LADIES	$^{6}C_{3} \times {}^{4}C_{2} = 20 \times 6 = 120$
Case 2	2 GENTS & 3 LADIES	$^{6}C_{2} \times {}^{4}C_{3} = {}^{6}C_{2} \times {}^{4}C_{1} = 15 \times 4 = 60$
Case 1	1 MAN & 4 LADIES	$^{6}C_{1} \times {}^{4}C_{4} = 6 \times 1 = 1$
		TOTAL COMMITTEES FORMED = 186

03. In how many ways can a selection of 6 out of 4 teachers and 8 students be done so as to include ATLEAST 2 TEACHERS

a. 220 [b.] 672 b. 896 d. 968 [JUNE 2016]

4 T 8 S

CASE NO.	COMMITTEE CONTAINS	NO OF WAYS OF FORMING THE COMMITTEE
Case 1	2 T & 4 S	${}^4C_2 \times {}^8C_4 = 6 \times 70 = 420$
Case 2	3 T & 3 S	${}^4C_3 \times {}^8C_3 = {}^4C_1 \times {}^8C_3 = 4 \times 56 = 224$
Case 1	4 T & 2 S	${}^4C_4 \times {}^8C_2 = 1 \times 28 = 28$
		TOTAL COMMITTEES FORMED = 672

Q3

01. 6 points on a circle . The number of quadrilateral s that can be formed are

a. 30 b. 360 [c.] 15 d. none [JUNE 2010]

4 points make a quadrilateral

No of quadrilaterals that can be formed = ${}^6C_4 = {}^6C_2 = 15$

02. 20 points . Find number of triangles formed by joining 12 points if 5points are collinear

a. 550 b. 560 [c.] 1130 d. 1140 [DEC 2022]

${}^{20}C_3 - {}^5C_3 = {}^{20}C_3 - {}^5C_2 = 1140 - 10 = 1130$

03. the number of parallelograms that can be formed by a set of 6 parallel lines intersected by the another set of 4 parallel lines a. 360 [b.] 90 c. 180 d. 45 [JUNE 2017]

2 // lines from set of 6 and 2 // lines from set of 4 will make a parallelogram

No. of parallelograms formed = ${}^6C_2 \times {}^4C_2 = 15 \times 6 = 90$

04. 6 points are marked on a straight line and 5 points are marked on another line which is parallel to the first line . How many straight lines including the given 2 lines can be formed with these points a. 30 [b.] 32 c. 11 d. 2 [JUNE 2022]

1 point from a set of 6 and 1 point from a set of 5 will make a line

No. of lines formed = ${}^6C_1 \times {}^5C_1 = 6 \times 5 = 30 + 2 = 32$

(Including the 2 lines)

05. the maximum number of points of intersection of 10 circles will be

a. 2 b. 20 [c.] 90 d. 180 [JUNE 2016]

number of 2 circle intersections in 10 circles = ${}^{10}C_2 = 45$

Each 2 circle intersections results into 2 points of intersection . Hence maximum number of points of intersection = 45 x 2 = 90

06. a regular polygon has 44 diagonals , then the number of sides a. 8 b. 9 c. 10 d. 11
using options [JUNE 2013]

c) 10 – SIDED POLYGON

10 points

$^{10}C_2$ = 45 lines
– 10 Sides
= 35 Diagonals ✗

d) 11 – SIDED POLYGON

11 points

$^{11}C_2$ = 55 lines
– 11 Sides
= 44 Diagonals ✓

07. If there are 40 guests in a party , if each guest takes a shake hand with all the remaining guests , then the number of handshake is a. 780 b. 840 c. 1560 d. 1600

2 guests make a handshake , No of handshakes = $^{40}C_2 = 780$ [JUNE 2019]

08. The number of handshakes in a group of 10 persons
a. 45 b. 54 c. 90 d. 10 [JUNE 2013]

2 guests make a handshake , No of handshakes = $^{10}C_2 = 45$

09. In a party every person shakes hand with every other person . If there are 105 handshakes in total , find the number of persons in the party
a. 14 b. 15 c. 21 d. 22 [SEPT 2024]

using options , no. of guest a. 14 b. 15
no.of handshakes $^{14}C_2$ $^{15}C_2$
= 91 ✗ = 105 ✓

Q4 SELECTION OF ANY NO. OF OBJECTS FROM 'n' DISTINCT OBJECTS

Lets say , we have 5 DISTICNT items . We can select no item , 1 item , 2 items and so on uptil all 5 items . This can be done in $^5C_0$, $^5C_1$, $^5C_2$, ……., $^5C_5$ ways respectively . Hence total ways $= {}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= {}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_2 + {}^5C_1 + {}^5C_5$

$= 1 + 5 + 10 + 10 + 5 + 1 = 32$

More interesting is , we could have got the same answer by 2^5 . LETS UNDERSTAND HOW

You take a basket and go to each item ,
Each of the 5 items have 2 options SELECT/NO SELECT
Hence by FUNDAMNETAL PRINCIPLE OF COUNTING , number of ways of selecting any number of items = $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

NOTE : NUMBER OF WAYS TO SELECT ANY NUMBER OF ITEMS GIVEN N ITEMS = 2^n
NUMBER OF WAYS TO SELECT ONE OR MORE (ATLEAST ONE) ITEM = $2^n - 1$

01. The number of ways in which a man can invite one or more of his 7 friends to dinner is
a. 64 b. 128 c. 127 d. 127 [JUNE 2018]
Every friend poses 2 options – INVITE / NOT INVITE
No. of ways to invite one or more= $2^7-1 = 127$

02. a question paper consist 10 questions , 6 in math and 4 in stats . Find out the number of ways to solve question paper if atleast one question is to be attempted from each section
a. 1024 b. 950 c. 945 d. 1022 [DEC 2015]
Every Q poses 2 options – SOLVE / NOT SOLVE
No. of ways to atleast 1 Q in each section = $(2^6-1).(2^4-1) = 63x15 = 945$

03. A MCQ test has 5 Q's and each Q has 4 possible options . How many different answer keys are possible a. 512 b. 1024 c. 20 d. 625 [JUNE 2022]
Each of the 5 Q's can be answered in 4 possible ways .
No. of answer keys = $4^5 = 1024$

Q5 PERMUTATION & COMBINATION KA COMBO PACK

01. A boats crew consists of 8 men , 3 of whom can row only on one side and 2 only on the other side . The number of ways in which the crew can be arranged is
a. 1728 b. 256 c. 164 c. 126 [JUNE 2019]
Let the sides be A and B . Since 3 particular men will can row only on one side (say side A) and 2 particular men can row only on say side B ,
1 more man for side A will then be selected from the remaining 3 in $^3C_1$ ways . Done with that , remaining 2 will automatically get adjusted to other side B
Done with that , now comes the arrangement , 4 on each side will arrange in 4! ways each
Total arrangements = $^3C_1 \times (4!)^2 = 3 \times 24^2 = 1728$

02. a user wants to create a password using 4 lowercase letter (a-z) and 2 upper case letters (A-Z). No letter can be repeated in any form . In how many ways can the password be created if the password must start with an uppercase
a. 26x25x24x23x22x5x21 b. 26x25x24x23x22x2x21 c. 26x5x25x24x23x2x22x21
d. 6x26x25x24x23x22x21 [JUNE 2024]
First place can be filled by any of the 26 upper case letters in $^{26}P_1$ ways .
Done with that , left with 25 upper case letters . We need to select 1 out of these 25 and arrange it into any one of the remaining 5 places . This will be done in $^{25}C_1 \times {}^5P_1$ ways
Done with that , remaining 4 places will then be filled by remaining 24 lowercase letters (No letter can be repeated in any form) in $^{24}P_4$ ways .
Total ways = $^{26}P_1 \times {}^{25}C_1 \times {}^5P_1 \times {}^{24}P_4 = 26x25x5x24x23x22x21$

Q6 ${}^{n}P_r = {}^{n}C_r.r\ !$

01. NOV 2018

${}^{n}P_r = 720$ & ${}^{n}C_r = 120$. Find r [a.] 3 b. 4 c. 5 d. 6

${}^{n}P_r = {}^{n}C_r.\ r\ !$

$720 = 120\ .\ r!$

$r! = 6 = 3!$ $r = 3$

02. DEC 2022 , JUNE 23

${}^{n}P_r = 3024$ & ${}^{n}C_r = 126$. Find n,r [a.] 9,4 b. 10,3 c. 12.4 d. 11,4

${}^{n}P_r = {}^{n}C_r.\ r\ !$

$3024 = 126\ .\ r!$

$r! = 24$ $r = 4$

Now ; ${}^{n}P_r = 3024$

${}^{n}P_4 = 3024$

$n(n-1)(n-2)(n-3) = 9.8.7.6$

$n = 9$ $(n,r) = (9,4)$

${}^{5}C_3 = {}^{5}C_2$, note 3+2=5 , ${}^{6}C_4 = {}^{6}C_2$, note 4+2=6

03. JUNE 2019

${}^{11}C_x = {}^{11}C_{2x-4}$, $x \neq 4$ then ${}^{7}C_x = ?$ a 20 [b] 21 c 22 d 23

$x+2x-4 = 11$

$3x = 15$

$x = 5$

${}^{7}C_x = {}^{7}C_5 = {}^{7}C_2 = 21$

04. DEC 23

${}^{15}C_{3r} = {}^{15}C_{r+3}$, then r = ? a 5 b 4 [c] 3 d 2

$3r+r+3 = 15$

$4r = 12$

$r = 3$

$^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$, Eg $^{5}C_3 + {}^{5}C_2 = {}^{6}C_3$

05. JUNE 2017

$^{10}C_3 + 2.\ {}^{10}C_4 + {}^{10}C_5 = {}^{n}C_5$, n = a. 10 b. 11 c. 12 d. 13

$\left({}^{10}C_3 + {}^{10}C_4\right) + \left({}^{10}C_4 + {}^{10}C_5\right)$

$^{11}C_4 + {}^{11}C_5$

$^{12}C_5 = {}^{n}C_5$, n = 12

06. DEC 2012

$^{13}C_6 + 2.\ {}^{13}C_5 + {}^{13}C_4 = {}^{15}C_x$, x = a. 6 b. 7 c. 8 d. 5

$\left({}^{13}C_6 + {}^{13}C_5\right) + \left({}^{13}C_5 + {}^{13}C_4\right) = {}^{15}C_x$

$^{14}C_6 + {}^{14}C_5 = {}^{15}C_x$

$^{15}C_6 = {}^{15}C_x$, x = 6

07. JUNE 2018

$^{12}C_3 + 2.\ {}^{12}C_4 + {}^{12}C_5 = {}^{14}C_x$, x = a. 3,5 b. 5,9 c. 7,1 d. 9,12

$\left({}^{12}C_3 + {}^{12}C_4\right) + \left({}^{12}C_4 + {}^{12}C_5\right) = {}^{14}C_x$

$^{13}C_4 + {}^{13}C_5 = {}^{14}C_x$

$^{14}C_5 = {}^{14}C_x$

$^{14}C_5 = {}^{14}C_9 = {}^{14}C_x$, x = 5,9

08. JUNE 2014

$^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$, then x = a. 999 b. 998 c. 997 d. none

Using options

$^{999}C_{97} + {}^{999}C_{901}$

$^{999}C_{97} + {}^{999}C_{98} = {}^{1000}C_{98}$

JUNE 23

In next world cup , there will be 12 teams divided equally into 2 groups . Team of each group will play a match against other teams of the group . From each group 3 top teams will qualify for next round . In this round each team will play against each other . Four top teams of this round will qualify for semi finals and play against each other . Top 2 teams will go to final where they play best of 3 . How many MINIMUM number of matches will be held in the next world cup

a. 54 b. 53 c. 38 d. 43

	No. of Teams	No. of Matches Played
GROUP 1	6	$^6C_2 = 15$
GROUP 2	6	$^6C_2 = 15$
QUALIFYING ROUND	6	$^6C_2 = 15$
SEMI FINAL ROUND	4	$^4C_2 = 6$
FINAL	2	3

TOTAL = 54

WAIT A SECOND

What if one of the teams wins first 2 matches in the FINAL's …. GAME OVER

Hence MINIMUM number of matches that will be held in the next world cup = 54–1 = 53

11 - ARITHMETIC PROGRESSION

Q1 $t_n = a + (n-1)d$

$t_6 = a+5d$, $t_{10} = a + 9d$, $t_{20} = a+19d$

01. The 20th term of an A.P. whose 6th term is 38 and 10th term is 66 DEC 2020

[a.] 136 b. 118 c. 178 d. 210

$a+5d = 38$

$a+9d = 66 \Rightarrow a = 3 , d = 7 , \quad t_{20} = a+19d = 136$

02. 9th and 19th term of AP are 35 and 75 respectively . Find the 20th term JUNE 2023

a. 78 [b.] 79 c. 80 d. 81

$a+8d = 35$

$a+18d = 75 \Rightarrow a = 3 , d = 4 , \quad t_{20} = a+19d = 79$

03. Find the 17th term of an AP series if 15th and 21st terms are 30.5 and 39.5 repsectively

[a.] 33.5 b. 35.5 c. 36.0 d. 38.0 DEC 2023

$a+14d = 30.5$

$a+20d = 39.5 \Rightarrow a = 9.5 , d = 1.5 , \quad t_{17} = a+16d = 33.5$

04. 4th term of an A.P. is zero . Find ratio of 25th term to 11th term SEPT 2024

a. 5 b. 4 [c.] 3 d. 2

$a+3d = 0$

$a = -3d$

$\frac{t_{25}}{t_{11}} = \frac{a+24d}{a+10d} = \frac{-3d+24d}{-3d+10d} = \frac{21d}{7d} = 3$

05. pth term is q and qth term is p . Find the rth term NOV 2018

a. p+q+r b. p+q−2r c. p+q+r/2 [d.] p+q−r

p	q	r
1	2	3

1st term of an AP is 2 and 2nd term is 1 . Find the 3rd term

$t_1 = 2$

$t_2 = 1 \Rightarrow d = -1 \Rightarrow t_3 = 0 = p+q-r$

06. pth term is q and qth term is p . Find the (p+q)th term DEC 2022

a. 1 b. −1 [c.] 0 d. none of above

p	q
1	2

1st term of an AP is 2 and 2nd term is 1 . Find the 3rd term

$t_1 = 2$

$t_2 = 1 \Rightarrow d = -1 \Rightarrow t_3 = 0$

07. in an arithmetic progression , seventh term is x and $(x+7)^{th}$ term is zero , then xth term is JUNE 2024

a. 6 [b.] 7 c. 8 d. 10

x = 1 seventh term is 1 and 8^{th} term is zero , then first term is

$t_7 = 1$ $a+6d = 1$

$t_8 = 0$ $a+7d = 0 \Rightarrow d = -1 \Rightarrow a = 7$

08. nth term of 9 , 7 , 5 is same as nth term of 15 , 12 , 9 then n = JUNE 2022

a. 6 b. 9 [c.] 7 d. 11

$t_n(9,7,5 \ldots\ldots) = t_n(15,12,9,\ldots.)$

$9+(n-1)(-2) = 15+(n-1)(-3)$

$11 - 2n = 18 - 3n \qquad n = 7$

09. the third term of an AP is 7 and seventh term is 2 more than thrice of third term . The common difference is JUNE 2024

[a.] 4 b. 3 c. 5 d. 6

$t_3 = 7$ $a+2d = 7$

$t_7 = 3t_3+2 = 23$ $a+6d = 23 \Rightarrow d = 4$

10. 4^{th} term of an AP is three times the first and 7^{th} term exceeds TWICE the 3^{rd} term by 1 . Find the first term 'a' and common difference d JUNE 2012

[a.] 3,2 b. 4,3 c. 5,4 d. 6,5

$t_4 = 3a$ $a+3d = 3a \Rightarrow -2a+3d = 0$

$t_7 - 2t_3 = 1$ $a+6d-2(a+2d) = 1 \Rightarrow -a+2d = 1$ option a satisfies both cond^n

11. The number of numbers between 74 & 25556 divisible by 5 JUNE 2023

a. 5079 [b.] 5097 c. 5907 d. 5709

75 , 80 , 85 , ……., 25555

$t_n = a+(n-1)d$

$25555 = 75+(n-1)5 \qquad n = 5097$

25555 is 5097^{th} term in the sequence starting from 75 . Hence number of numbers between 74 and 25556 divisible by 5 is 5097

12. if the second and the eigth terms of an arithmetic progression are equal to constant a , then the sum of first n terms of an AP is equal to JUNE 2024

[a.] na b. a/n c. $2n+n(a-1)$ d. $n+a(n-1)$

series : a + a + ….. + a = na

Q2 3 NO'S IN A.P. = a–d , a , a+d / 5 NO'S IN A.P. = a–2d , a–d , a , a+d , a+2d

01. Divide 69 into 3 parts which are in A.P. and are such that product of first two parts is 460

a. 20,23,26 b. 21,23,25 c. 19,23,27 d. none DEC 2020

a–d , a , a+d

S = 69 P = 460

3a = 69 a(a–d) = 460

a = 23 23(23–d) = 460

d = 3 nos. are 20 , 23 , 26

METHOD 2 – Use options ,

	S = 69	$P_{\text{first two}}$ = 460
option (a) 20,23,26	✓	✓

HOLD ON – BEFORE YOU DECIDE TO AVOID LEARNING ACTUAL WAY OF SOLVING & JUST GOING BY THE OPTIONS , CHECK OUT THE NEXT SUM . YOU ALL WILL SURELY COME TO CONCLUSION – PRACTISE THE ACTUAL WAY OF SOLVING (METHOD I) IN THE SHORTEST TIME & TAKE THE BENEFIT OF METHOD 2 IF OPTIONS LIST ALL NUMBERS

02. Divide 144 into three parts which are in A.P. and such that the largest is twice the smallest . The smallest of three numbers will be MTP – MAY 2024

a. 48 b. 36 c. 13 d. 32

a–d , a , a+d

S = 144 COND 2

3a = 144 a+d = 2(a–d)

a = 48 3d = a

d = 16 a–d = 32

03. find three numbers in AP such that their sum is 18 and the product is 192

a. 4 , 6 , 8 b. –4 , –6 , –8 c. 8 , 6 , 4 d) both a and c

a–d , a , a+d

S = 18 ⇒ a=6 (6–d , 6 , 6+d)

P = 192 (6–d).6.(6+d)= 192

$36–d^2$ = 32

d = ± 2 4 , 6 , 8 OR 8 , 6 , 4

METHOD 2 – Use options ,

	S = 18	P = 192	
option (a) 4,6,8	✓	✓	
option (b) 8,6,4	✓	✓	option (d)

04. find three numbers in AP such that their sum is 27 and sum of squares is 341

a. 2 , 9 , 16 b. 16 , 9 , 2 c. –2 , –9 , –16 d. both a and b

$S = 27 \Rightarrow a=9$ $\quad 9-d , 9 , 9+d$

$SOS = 341 \quad (9-d)^2+9^2+(9+d)^2 = 341$

$243+2d^2 = 341$

$d = \pm 7$ $\quad$ 2 , 9 , 16 OR 16 , 9 , 2

METHOD 2 – Use options ,

	S	SOS	
option (a) –2,2,6	2+9+16 = 27	4+81+256 = 341	✓
option (b) 16,9,2	16+9+2 = 27	256+81+4 = 341	✓ option (d)

05. Find five numbers in AP whose sum is 20 and product of first and last is 15

$S = 20 \Rightarrow a=4$ $\quad 4-2d , 4-d , 4 , 4+d , 4+2d$

$1^{st}x4^{th} = 15 \Rightarrow (4-2d).(4+2d) = 15$

$16 - 4d^2 = 15$

$d = \pm {}^1/_2 = \pm 0.5$ $\quad$ 3,3.5,4,4.5,5 OR 4,4.5,4,3.5,3

Q3 $S_n = \frac{n}{2}[a + \ell]$

01. first and last term of AP are 5 and 905 . sum of n terms is 45955 . Find n

a. 110 b. 100 [c.] 101 d. 111 JUNE 2022

$S_n = \frac{n}{2}(a+l) \Rightarrow 45955 = \frac{n}{2}[5+905] \Rightarrow n = 101$

02. first and last term of AP are –4 and 146 . sum of n terms is 7171 . Find n

a. 99 b. 100 [c.] 101 d. 112 MTP–MAY 20,NOV 22

$S_n = \frac{n}{2}(a+l) \Rightarrow 7171 = \frac{n}{2}[-4+146] \Rightarrow n = 101$

03. if nth term of an AP series is 7n–2 , then sum of n terms is

a $0.5(7n^2+2n)$ b $0.5(7n^2-3n)$ [c] $0.5(7n^2+3n)$ d $0.5(7n^2-2n)$ DEC 2023

$t_n = 7n-2 \Rightarrow S_n = \frac{n}{2}(a+\ell) = \frac{n}{2}(⑤+7n-2) = \frac{n}{2}(7n+3) = 0.5(7n^2+3n)$

$\downarrow$

$t_1 = 5$

04. If sum of n terms of an AP be $2n^2+5n$, then its nth term is DEC 2012

a. 4n–2 b. 3n–4 [c.] 4n+3 d. 3n+4

$S_n = \frac{n}{2}(a+\ell)$ a=S1 = 2+5 = 7

$2n^2+5n = \frac{n}{2}(7+\ell)$

$\ell = t_n = 4n+10-7 =$ 4n+3

05. The sum of n terms of an AP is $3n^2+5n$, which last term is 164

a. 25 [b.] 27 c. 29 d. 31 DEC 2015

$S_n = \frac{n}{2}(a+\ell)$ a=S1 = 3+5 = 8

$3n^2+5n = \frac{n}{2}(8+\ell)$

$\Rightarrow \ell = t_n = 6n+10-8 = 6n+2 = 164 \Rightarrow$ n = 27

06. The sum of n terms of an AP is $3n^2+5n$, The series is DEC 2021

[a.] 8,14,20,26,….. b. 8,22,42,68,… c. 22,68,114,… d. 8,14,28,44

$S_n = \frac{n}{2}(a+\ell)$ a=S1 = 3+5 = 8

$3n^2+5n = \frac{n}{2}(8+\ell)$

$\ell = t_n = 6n+10-8 =$ 6n+2 $\Rightarrow$ Put n = 1 , 2 , 3 ….. $\Rightarrow$ 8 , 14 , 20 , …..

07. if the sum of n terms of an A.P. is $6n^2+6n$, then fourth term of the series

a. 120 b. 72 [c.] 48 d. 24 DEC 2014

$S_n = \frac{n}{2}(a+\ell)$ a=S1 = 6+6 =12

$6n^2+6n = \frac{n}{2}(12+\ell)$

$\Rightarrow \ell = t_n = 12n+12-12 =$ 12n $\Rightarrow$ $t_4 = 12(4) = 48$

08. if sum on n terms of an A.P. is $2n^2$, then fifth term is JULY 2021

a. 20 b. 50 [c.] 18 d. 25

$S_n = \frac{n}{2}(a+\ell)$ a=S1 = 2(1) = 2

$2n^2 = \frac{n}{2}(2+\ell)$

$\Rightarrow \ell = t_n = 4n-2 \Rightarrow t_5 = 4(5)-2 = 18$

09. if the sum of n terms of an A.P. is $2n^2+n$ then what is the difference between 10^{th} term and 1^{st} term a. 207 [b.] 36 c. 90 d. 63 JUNE 2011

$S_n = \frac{n}{2}(a+\ell)$

$a = S_1 = 2+1 = 3$

$2n^2+1n = \frac{n}{2}(3+\ell)$

$\Rightarrow \ell = t_n = 4n+2-3 = 4n-1 \Rightarrow t_{10}-t_1 = (40-1)-(4-1) = 36$

10. Sum of all natural numbers between 100 and 1000 which are divisible by 11 DEC 2017

[a.] 44,550 b. 66,770 c. 55,440 d. 33,440

$110 + 121 + \ldots\ldots + 990$

$990 = 110+(n-1)11$

$n = 81$

$S_n = \frac{n[a+\ell]}{2}$

$= \frac{81}{2}[110+990] = 44550$

Q4 $S_n = \frac{n}{2}[2a + (n-1)d]$

01. Sum of series 7+14+21+ …. To 17th term is DEC 2021

[a.] 1071 b. 971 c. 1171 d. 1271

$S_{17} = \frac{17}{2}[2(7) + (17-1)(7)] = 1071$

02. A person recieved the salary for the first year ₹ 5,00,000 and he received an increment of ₹ 15000 per year then the sum of the salary he has taken in 10 years is

[a.] ₹ 56,75,000 b. ₹ 72,75,000 c. ₹ 63,75,000 d. none of these DEC 2016

$S_{10} = \frac{10}{2}[2(500000) + (10-1)(15000)] = ₹\ 56,75,000$

03. 8th term of an A.P. is 15 then sum of its 15 terms is JUNE 2012

a. 15 b. 0 [c.] 225 d. 225/2

$a+7d = 15$, $S_{15} = \frac{15}{2}(2a+14d) = 15(a+7d) = 15(15) = 225$

04. Sum of 3^{rd} and 9^{th} terms of an AP is 8 . Find the sum of first 11 terms of the progression

[a.] 44 b. 22 c. 19 d. 11 DEC 2011

$a+2d+a+8d = 8$

$2a+10d = 8$

$S_{11} = \frac{11}{2}(2a+10d) = \frac{11}{2}(8) = 44$

05. In an A.P. if the sum of 4th and 12th term is 8 then sum of first 15 terms is

[a.] 60 b. 120 c. 110 d. 150 JUNE 2013

$a+3d+a+11d = 8$

$2a+14d = 8$

$S_{15} = \frac{15}{2}(2a+14d) = \frac{15}{2}(8) = 60$

06. Sum of five terms of A.P. is 75 , find the 3rd term

a. 20 b. 30 [c.] 15 d. none DEC 2019

$S_5 = 75 \quad \frac{5}{2}[2a+4d] = 75 \Rightarrow 5(a+2d) = 75 \Rightarrow a+2d = 15 \Rightarrow t_3 = 15$

07. an A.P. has 13 terms whose sum is 143 . The third term is 5 , then first term is

a. 4 b. 7 c. 9 [d.] 2 DEC 2013

$S_{13} = 143$

$\frac{13}{2}[2a+12d] = 143$

$a+6d = 11$

$t_3 = 5$

$a+2d = 5$

Solving $a+6d = 11$

$a+2d = 5$

$d = 1.5$, $a = 2$

08. In an A.P. , if common difference is 2 , sum of n terms is 49 , 7th term is 13 , then n =

a. 0 b. 7 [c.] 7 d. 13 DEC 2012

$d = 2$,

$t_7 = 13$

$a+6d = 13$

$a+12 = 13$

$a = 1$

$S_n = 49$

$\frac{n}{2}[2a+(n-1)d] = 49$

$\frac{n}{2}[2+(n-1)2] = 49$

$\frac{n}{2}[2+2n-2] = 49 \Rightarrow n^2 = 49 \Rightarrow n = 7$

Q5 $S_n = \frac{n}{2}[2a + (n-1)d]$

01. A person pays 975 in monthly instalments , each instalment is less than former by 5 . The amount of 1st instalment is 100 . In what time (months) will be entire amount be paid

a. 26 [b.] 15 c. a&b d. 18 MAY 2018

$a = 100$, $d = -5$, $S_n = 975$

$\frac{n}{2}[2a+(n-1)d] = 975$

$\frac{n}{2}[200+(n-1)(-5)] = 975$

$n(205-5n) = 1950$

$5n(41-n) = 1950$

$n(41-n) = 390$

using options ,

$n(41-n) = 390$

$26(15) = 390$ ✓

$15(26) = 390$ ✓

option b

Why not option a . Think for yourself , if person pays off his loan by the end of 15th instalment , why and for what he/she would continue paying upyil 26th instalment

02. The sum of the series $-8, -6, -4, \ldots$ n terms is 52 . Find n NOV 2018

a. 12 [b.] 13 c. 10 d. 9

$a = -8, d = 2, S_n = 52$

$\frac{n}{2}[2a+(n-1)d] = 52$

$\frac{n}{2}[2(-8)+(n-1)2] = 52$

$n(n-9) = 52$,

using options ,

$n(n-9) = 52$

$12(3) \neq 52$

$13(4) = 52$ ✓

option b

03. The number of terms of the series $5+7+9+\ldots\ldots$ must be taken so that the sum may be 480

[a.] 20 b. 10 c. 15 d. 25 JULY 2021

$a = 5, d = 2, S_n = 480$

$\frac{n}{2}[2a+(n-1)d] = 480$

$\frac{n}{2}[2(5)+(n-1)2] = 480$

$n(n+4) = 480$,

using options ,

$n(n+4) = 480$

$20(24) \neq 480$ ✓

option a

04. Find the value of x for the following data $1+7+13+19+\ldots\ldots+x = 225$

a. 56 b 63 [c] 49 d 42 DEC 2023

$a = 1, d = 6, S_n = 225$

$\frac{n}{2}[2a+(n-1)d] = 225$

$\frac{n}{2}[2(1)+(n-1)6] = 225$

$n(3n-2) = 225$,

$225 = 9 \times 25$

LETS TRY n = 9

$n(3n-2) = 225$

$9(25) = 225$ ✓

n = 9

$x = t_9$

$= a+8d$

$= 1+8(6)$

$= 49$

05. If $2+6+10+14+18+\ldots\ldots+x = 882$, then the value of x JUNE 2019

a. 78 b. 80 [c.] 82 d. 86

$a = 2, d = 4, S_n = 882$

$\frac{n}{2}[2a+(n-1)d] = 882$

$\frac{n}{2}[2(2)+(n-1)4] = 882$

$\frac{n(4n)}{2} = 882 \Rightarrow n^2 = 441 \Rightarrow n = 21$

$x = t_{21}$

$= a+20d$

$= 2+20(4)$

$= 82$

Q6 $S_n = \frac{n}{2}[2a + (n-1)d]$ KUCH HATKE

01. the first and fifth term of an A.P. of 40 terms are –29 and –15 respectively . Find the sum of all positive terms of this A.P. MTPI – JUNE 2022

a. 1605 **b.** 1705 c. 1805 d. 1905

$a = -29$
$t_5 = -15$
$a+4d = -15$
$d = 3.5$

$t_n > 0$
$a+(n-1)d > 0$
$-29+(n-1)(3.5)>0$
$n > 9.28$

10^{th} terms onwards , terms of these A.P. are positive . Considering 10^{th} term as the first term , we will now find sum of 40–9= 31 terms

$a_{new} = t_{10}$
$= -29+9(3.5)=2.5$

$S_{31} = \frac{31}{2}[2(2.5)+30(3.5)]$
$= 1705$

02. first term of an A.P. is 100 and sum of first 6 terms is 5 times the sum of next 6 terms . Find the common difference

a. –10 b. 10 c. –5 d. 5

sum of next 6 terms = S
sum of first 6 terms = 5S
sum of first 12 terms = 6S

$\frac{S_{12}}{S_6} = \frac{6}{5}$

$\frac{^{12}/_2[200+11d]}{^{6}/_2[200+5d]} = \frac{6}{5}$ $\quad \frac{200+11d}{200+5d} = \frac{3}{5}$

$1000+55d = 600+15d$
$d = -10$

Q7 ARITHMETIC MEANS (A.M.'s)

2+ 4 + 6 + 8 + 10 are in AP

4 , 6 , 8 are the A.M.s called as A_1 , A_2 , A_3

$A_1+A_2+A_3 = 18$
$= 3(6)$
$= 3$(mean of 2&10)

Conclusion

1. Sum of n A.M.s = n x (Mean of a & b) between a & b

2. n^{th} A.M. is $n+1^{th}$ term in the AP

01. The value of k for which the terms 7k+3 , 4k–5 , 2k+10 are in A.P. NOV 2018

a. –13 **b.** –23 c. 13 d. 23

$4k-5 = \frac{7k+3+2k+10}{2} \Rightarrow 8k-10 = 9k+13 \Rightarrow k = -23$

02. if a , –3 , b , 5 , c are in A.P. , then value of c = JUNE 2017

a. –7 b. 1 [c.] 9 d. 13

$b = \frac{-3+5}{2} = 1$

a , –3 , 1 , 5 , c are in A.P. $\Rightarrow d = 4 \Rightarrow$ c = 5+4 = 9

03. if 20 A.M.s are inserted between 3 and 51 then sum of these 20 A.M.s is

[a.] 540 b. 1080 c. 270 d. none of these MTP 1 – MAY 2023

sum of 20 A.M.s = 20x(mean of 3 & 51) = 540

04. If sum of 3 A.M.'s between 'a' and 22 is 42 , then a = DEC 2011

a. 14 b. 11 c. 10 [d.] 6

sum of 3 A.M.s = 3x(mean of a & 22) $\Rightarrow$ $42 = 3 \times \frac{a+22}{2} \Rightarrow a = 6$

05. There are n A.M.'s between 7 and 71 and the 5th A.M. is 27 . Then n

[a.] 15 b. 16 c. 17 d. 18

a = 7 5th A.M. = 27 t_m = 71

t_6 = 27 7+(m–1)(4)= 71

a+5d = 27 $\Rightarrow$ d = 4 m = 17

Hence , we have 17 terms in AP . Excluding the first and last term , we have 15 A.M.'s in between . n = 15

option a

INTERESTING OBSERVATION

CONSIDER A.P. $\underbrace{2+4}_{S_1} + \underbrace{6+8}_{S_2} + \underbrace{10+12}_{S_3}$

$S_1 = 6$, $S_2 = 14$, $S_3 = 22$ $\Rightarrow S_2 = \frac{S_1+S_3}{2}$

06. the sum of first 3 terms in an A.P. is 18 and that of last 3 terms is 28 . If an A.P. has 13 terms , what is the sum of the middle three terms

[a.] 23 b. 18 c. 19 d. none of these MTPI – NOV 2021

$S_2 = \frac{S_1+S_3}{2} = \frac{18+28}{2} = 23$

Q8

01. If a^2 , b^2 , c^2 are in AP then $a/(b+c)$, $b/(c+a)$, $c/(a+b)$ are in

a. AP b. GP c. HP d. none

a^2 , b^2 , c^2 are in AP

a^2	b^2	c^2
1	25	49

$\Rightarrow$

a	b	c
1	5	7

$^a/_{(b+c)}$, $^b/_{(c+a)}$, $^c/_{(a+b)}$

$^1/_{(5+7)}$, $^5/_{(1+7)}$, $^7/_{(1+5)}$

$^1/_{12}$, $^5/_8$, $^7/_6$

x2 x3 x4

$^2/_{24}$, $^{15}/_{24}$, $^{28}/_{24}$ …… $d = {}^{13}/_{24}$ …. AP option a

02. If $^1/_{b+c}$, $^1/_{c+a}$, $^1/_{a+b}$ are in AP then a^2 , b^2 , c^2 are in

[a.] AP b. GP c. Both AP & GP d. none of these JUNE 2016

LETS ASSUME

a^2 , b^2 , c^2 are in AP

a^2	b^2	c^2
1	25	49

$\Rightarrow$

a	b	c
1	5	7

$\frac{1}{b+c}$, $\frac{1}{c+a}$, $\frac{1}{a+b}$

$\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$

x2 x3 x4

$\frac{2}{24}$, $\frac{3}{24}$, $\frac{4}{24}$ ….. $d = \frac{1}{24}$ ….. AP

HENCE OUR ASSUMPTION IS CORRECT ,option a

03. If $^{b+c-a}/_a$, $^{c+a-b}/_b$, $^{a+b-c}/_c$ are in A.P. then a , b and c are in

a. AP b. GP [c.] HP d. none DEC 2019

$\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$, $\frac{a+b-c}{c}$ are in AP

$\frac{b+c-a}{a}+2$, $\frac{c+a-b}{b}+2$, $\frac{a+b-c}{c}+2$ are in AP

$\frac{a+b+c}{a}$, $\frac{a+b+c}{b}$, $\frac{a+b+c}{c}$ are in AP

$\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ are in AP

a , b , c are in HP

Q9

01. MTP - NOV 2022

sum of x terms of two AP's are in ratio 3x+5:5x+3 , then ratio of their 10^{th} term is

[a.] 31:49 b. 30:49 c. 28:49 d. none

FOR LANGUAGE SIMPLICITY , we KEEP 'x' as 'n'

$$\frac{\frac{n[2a+(n-1)d]}{2}}{\frac{n[2a'+(n-1)d']}{2}} = \frac{3n+5}{5n+3}$$

$$\frac{2a+(n-1)d}{2a'+(n-1)d'} = \frac{3n+5}{5n+3}$$

(18)

(18)

We want $\frac{t_{10}}{t_{10}'} = \frac{a+9d}{a'+9d'}$

PUT $n = 19$ ……$2(10)-1$

$$\frac{2a+18d}{2a'+18d'} = \frac{3(19)+5}{5(19)+3}$$

$$\frac{a+9d}{a'+9d'} = \frac{62}{98}$$

$$\frac{t_{10}}{t_{10}'} = \frac{31}{49}$$

SHORT CUT

From ratio of sum of n terms of 2 AP.s , you want to find ratio of N^{th} term of 2 AP's

Change $n = 2N-1$

you wanted to find ratio of 10^{th} term of 2 AP's

change $n = 2(10)-1 = 19$

02. JUNE 2019

the ratio of sum on n terms of two A.P.'s is n+1/n−1 , then the ratio of their mth terms is

a. m+1:2m b. m+1:m−1 c. 2m−1:m+1 [d.] m:m−1

$$\frac{S}{S'} \xrightarrow[\text{SHORT CUT}]{\text{change } n = 2N-1} \frac{t_m}{t_m'}$$

$$\frac{n+1}{n-1} \xrightarrow{\text{change } n = 2m-1} \frac{2m-1+1}{2m-1-1} = \frac{m}{m-1}$$

12 - GEOMETRIC PROGRESSION

Q1 $t_n = ar^{n-1}$

eg : $t_8 = ar^7$, $t_{10} = ar^9$ .....

01. MTP 1 – NOV 2022

In the series 2 , 4 , 8 , 16 , ...which term is 2048

a. 9 b. 10 c. 11 d. none

$a = 2$, $r = 2$

$ar^{n-1} = 2048$

$2(2)^{n-1} = 2048$ option c

$2^{n-1} = 1024 = 2^{10}$, $n = 11$

02. DEC 2019

In the series 25 , 5 , 1 , ... , $^1/_{3125}$, which term is $^1/_{3125}$

a. 8 b. 9 c. 15 d. none

$a = 25$, $r = 1/5$

$ar^{n-1} = {}^1/_{3125}$

$25\,\frac{1}{5^{n-1}} = \frac{1}{3125}$ option a

$\frac{1}{5^{n-3}} = \frac{1}{5^5}$ $n = 8$

02. JUNE 2013

in a G.P. , the 6th term is 729 and the common ratio is 3 then a is

a. 2 b. 3 c. 4 d. 7

$ar^5 = 729$

$a(3)^5 = 729$ $a = 3$ option b

03. JUNE 2022

In GP , 2nd term is 12 and 6th term is 192 . Find 11th term

a. 6411 b. 6144 c. 6414 d. none

$ar^5 = 192$

$ar^1 = 12$ (2)÷(1) $r^4 = 16$

$r = 2$, $a = 6$

$t_{11} = ar^{10} = 6(2)^{10} = 6144$

04. DEC 2022

In GP 5th term = 27 , 8th term = 729 . Find t_{11}

a. 19683 b. 16893 c. 19863 d. none

$ar^7 = 729$

$ar^4 = 27$ (2)÷(1) $r^3 = 27$

$r = 3$, $a = {}^1/_3$

$t_{11} = ar^{10} = \frac{1(3)^{10}}{3} = 3^9 = 19683$

option c

05. MTP – OCT 2021

in GP second and fifth terms are 24 and 81 respectively . The series is

a. 16 ,36 , 24 , 54

b. 24 , 36 , 53, ...

c. 16 , 24 , 36 , 54 , ... d. none

$ar^4 = 81$

$ar^1 = 24$ (1)÷(2) $r = {}^3/_2$

subs in (2)

$a = 16$

$16 , 16x^3/_2 = 24 , 24x^3/_2 = 36$

option c

06. NOV 2018

3rd term of a GP is $^2/_3$ and 6th term is $^2/_{81}$, then the 1st term is

a. 2 b. 6 c. 9 d. 1/3

$ar^5 = {}^2/_{81}$

$ar^2 = {}^2/_3$ (1)÷(2) $r = {}^1/_3$

subs in (2)

$a = 6$ option b

07. JAN 2021

3rd term of a GP is 1 and 6th term is $^{-1}/_{8}$, then the 1st term and common ratio is

a. 4, $^{1}/_{2}$ b. 4, $^{-1}/_{4}$ c.4, $^{-1}/_{2}$ d.4, $^{1}/_{4}$

$ar^5 = -^{1}/_{8}$

$ar^2 = 1$ (1)÷(2) $r = -^{1}/_{2}$

subs in (2)

$a = 4$ option c

08. DEC 2011

In a GP , if the 5th term is $\sqrt[3]{3}$, then product of first 9 terms is

a. 8 b. 27 c. 243 d. 9

$ar^4 = 3^{1/3}$

$P = a.ar^1.ar^2.ar^3 \ldots\ldots\ldots.ar^8$

$= a^9 r^{1+2+3\ldots.+8}$

$= a^9 r^{36}$

$= \left(ar^4\right)^9$

$= \left(3^{1/3}\right)^9 = 3^3 = 27$ option b

09. JUNE 2019

In a GP , if the 4th term is 3 , then product of first 7 terms is

a. 3^5 b. 3^7 c. 3^6 d. 3^8

$ar^3 = 3$

$P = a.ar^1.ar^2.ar^3 \ldots\ldots\ldots.ar^6$

$= a^7 r^{1+2+3\ldots.+6}$

$= a^7 r^{21}$

$= \left(ar^3\right)^7$

$= 3^7$ option b

a , b , c are in G.P. $\Rightarrow \frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

b = Geometric mean G.M. of a,c

1 , 2 , 4 , 8 , 16 ….. G.P.

(1,2,4) , (2,4,8) , (4,8,16) satisfies $B^2 = AC$

2 , 4 , 8 are called as G.M.'s

10. MTP 1 – APR 2023

insert 4 geometric means between 4 and 972

4 , __ , __ , __ , __ , 972

$t_6 = 972$

$ar^5 = 972$

$r^5 = 243$

$r = 3$

4 , 12 , 36 , 108 , 324 , 972

Q2 3 nos. in GP : $\frac{a}{r}$, a , ar

5 nos. in GP : $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2

01. DEC 2020

3 numbers in GP with their sum is 130 and their product is 27000 are

a. 90,30,10 b. 10,30,90 c. a & b

METHOD 1

P = 27000 S = 130

a^3 = 27000 $a\left(\frac{1+1+r}{r}\right)$ = 130

a = 30 $\frac{1+r}{r} = \frac{130}{30} - 1$

$\frac{1+r}{r} = \frac{10}{3}$

BY TRIAL & ERROR r = $3, {}^1/_3$

a = 30 , r = 3 $\Rightarrow$ 10,30,90

a = 30 , r = $^1/_3 \Rightarrow$ 90,30,10

METHOD 2 using options

	S=130	P= 27000
a. 10,30,90	✓	✓
b. 90,30,10	✓	✓

option d

02. DEC 2021

If the sum and product of three numbers in GP are 7 and 8 respectively , then the 4^{th} term of the series is

a. 6 b. 4 c. 8 d. 16

P = 8 S = 7

a^3 = 8 $a\left(\frac{1+1+r}{r}\right)$ = 7

a = 2 $\frac{1+r}{r} = \frac{7}{2} - 1$

$\frac{1+r}{r} = \frac{5}{2}$

BY TRIAL & ERROR r = $2, {}^1/_2$

a = 2 , r = 2 , $t_4 = ar^3 = 16$ option d

03. MTP 1 – MAR 2019

Find 3 nos. in G.P. sum is 19 and product is 216

a. 9,6,4/4,6,9 b. 9,6,3/3,6,9

c. 1,3,9/9,3,1 d. none

P = 216 S = 19

a^3 = 216 $a\left(\frac{1+1+r}{r}\right)$ = 19

a = 6 $\frac{1+r}{r} = \frac{19}{6} - 1$

$\frac{1+r}{r} = \frac{13}{6}$

r = $^3/_2, {}^2/_3$

a = 6 , r = $^3/_2 \Rightarrow$ 4,6,9

a = 6 , r = $^2/_3 \Rightarrow$ 9,6,4 option a

METHOD 2 using options

	S=19	P= 216
a. 9,6,4/4,6,9	✓	✓

04. JULY 2021

sum of 3 nos. in G.P. is 28 . When 7,2,and 1 are subtracted from first , second and third number respectively , then the resulting numbers are in A.P. Find sum of squares of original three nos.

a. 510 b. 456 c. 400 d. 336

a+b+c = 28 ....(1)

b^2 = ac ....(2)

a–7 , b–2 , c–1 ..... A.P.

b–2 = $\frac{a-7+c-1}{2}$

2b +4= a+c ....(3)

Subs (3) in (1)

3b+4 = 28 $\Rightarrow$ b = 8

subs in (1) & (2)

a+c = 20 , ac = 64

(4,16) satisfies both conditions

Finally 3 nos are 4 . 8 . 16

Sum of squares = 16+64+256 = 336

05. MTP 1 – MAY 2020

Sum of 3 nos. in G.P. is 70 . If the extremes are multiplied by 4 and the mean by 5 , they form A.P. The numbers are

a. 12,18,40 b. 10,20,40

c. 40,20,15 d. none of the above

$a+b+c = 70$ ....(1) $\Rightarrow 2a+2b+2c=140$

$b^2 = ac$ ....(2)

4a , 5b , 4c ..... A.P.

$5b = \frac{4a+4c}{2}$

$5b = 2a+2c$

$7b = 140$

$b = 20$

subs in (1) & (2)

$a+c = 50$

$ac = 400$

(10,40) satisfies both conditions

Finally 3 nos are 10 . 20 . 40

Using option would have been really quick as only option b satisfies the given conditions

1. GP ✓ 2. Sum = 70 ✓

3. 4(10),5(20),4(40) = 40,100,160 AP ✓

NOTE – STUDENTS NEED TO PRACTISE THE METHOD AS WE CAN ALL SEE JULY 2021 SUM COULD NOT HAVE BEEN SOLVED USING OPTIONS . WE NEED TO BE WARRIORS READY TO FACE THE WORST SITUTATIONS . LETS NOT TAME OURSELVES

06. MTP 1 – MAY 2020

three numbers are in AP and their sum is 21. If 1 , 5 , 15 are added to them respectively , they form GP . The numbers are

a. 5,7,9 b. 9,5,7 c. 7,5,9 d. none

METHOD 1

$a-d + a + a+d = 21$ $a = 7$

$7-d$, 7 , $7+d$ ..... AP

+1 +5 +15

$8-d$, 12 , $22+d$ ..... GP

$144 = (8-d)(22+d)$ ..... $b^2 = ac$

$144 = 176-14d-d^2$

$d^2+14d-32= 0$

$(d+16)(d-2) = 0$

$d = 2 , d = -16$

for d = 2

5 , 7 , 9 ....option a

for d = −16

23, 7 ,−9 option not available

METHOD 2 using options

option a

5 , 7 , 9 S = 21 ✓

+1 +5 +15

6 , 12 , 24..... GP ✓ ....option a

$S_n = \frac{a(1-r^n)}{1-r}$, $r < 1$

$S_n = \frac{a(r^n-1)}{r-1}$, $r > 1$

Q3

01. DEC 2021

$^1/_2 + 1/2^2 + \ldots\ldots {}^1/_{2^n} < 0.998$

Find the largest value of n

a. 9 b. 6 c. 7 d. 8

$S_n = \frac{a(1-r^n)}{1-r}$,

$\frac{^1/_2\,(1-{}^1/_{2^n})}{1-{}^1/_2} < 0.998$

$1-{}^1/_{2^n} < 0.998$

$1-0.998 < {}^1/_{2^n}$

$0.002 < {}^1/_{2^n}$ option d

$2^n < 500$

$2^8 = 256$, $2^9 = 512 > 500$ ….. largest value of n = 8

02. sum of n terms of a GP whose 1st term is 1 & r is ½ is equal to $1\ {}^{127}/_{128}$. The value of n is

a. 9 b. 6 c. 7 d. 8

$S_n = \frac{a(1-r^n)}{1-r}$,

$\frac{255}{128} = \frac{1(1-{}^1/_{2^n})}{1-{}^1/_2}$

$\frac{255}{128} = 2(1-{}^1/_{2^n})$

$\frac{255}{256} = 1 - \frac{1}{2^n}$

$\frac{1}{2^n} = \frac{1}{256} \Rightarrow 2^n = 256 \Rightarrow n = 8$

option d

03. a person borrows 8000 at 2.76% p.a. SI . The principal and interest are to be paid in 10 monthly instalments . If each instalment is double the preceding one , find the value of first and the last instalment

Total Amount to be repaid in 10 instalments

$= 8000\left(1+\frac{10}{12}\,\frac{2.76}{100}\right) = ₹\ 8184$

S = ₹ 8184 , r = 2

$\frac{a(2^{10}-1)}{2-1} = 8184$

$a(1023) = 8184$

$a = 8$

first instalment = 8

last instalment $= t_{10} = ar^9 = 8(29) = 4096$

04. SEPT 2024

a person borrows 9000 at 2.76% p.a. SI . The principal and interest are to be paid in 10 monthly instalments . If each instalment is double the preceding one , find the value of first and the last instalment

a. 1024 b. 4608 c. 9207 d. 4096

Total Amount to be repaid in 10 instalments

$= 9000\left(1+\frac{10}{12}\,\frac{2.76}{100}\right) = ₹\ 9207$

S = ₹ 9207 , r = 2

$\frac{a(2^{10}-1)}{2-1} = 9207 \Rightarrow a = 9$

last instalment = $t_{10} = ar^9 = 9 \times 2^9$

= ₹ 4608

option b

Q4

01. JUNE 22

sum of first 8 terms of a GP is 5 times the sum of first 4 terms . The common ratio is

a. ±2 b. ±3 c. ±√2 d. ±√3

$S_8 = 5.S_4$

$$\frac{a(r^8-1)}{r-1} = 5\,\frac{a(r^4-1)}{r-1}$$

$$\frac{r^8-1}{r^4-1} = 5$$

$$\frac{(r^4-1)(r^4+1)}{r^4-1} = 5$$

$$r^4 = 4$$

$r^2 = 2$, $r = \pm\sqrt{2}$ option c.

02. JUNE 2017

Sum of first 20 terms of a GP is 1025 times the sum of first 10 terms , then the common ratio is

a. 2 b. 2√2 c. ½ d. √2

$S_{20} = 1025\, S_{10}$

$$\frac{a(r^{20}-1)}{r-1} = 1025\,\frac{a(r^{10}-1)}{r-1}$$

$$\frac{r^{20}-1}{r^{10}-1} = 1025$$

$$\frac{(r^{10}-1)(r^{10}+1)}{r^{10}-1} = 1025$$

$r^{10} = 1024 = 2^{10}$,r=2 option a

Q5

01. JUNE 2014 , JUNE 2015 , MAY 2018

Sum of m terms of the series

1 + 11 + 111 + ……. m terms is

a. $^1/_{81}(10^{m+1}-9m-10)$

b. $^1/_{27}(10^{m+1}-9m-10)$

c. $10^{m+1}-9^m-10$

d. none of these

For comfort in solving , we will take n terms instead of m terms

1 + 11 + 111 + ……. n terms is

$$= \frac{1}{9}(9+99+999+\ldots\ldots)$$

$$= \frac{1}{9}\left[(10-1)+(100-1)+(1000-1)\ldots\ldots\right]$$

$$= \frac{1}{9}\left[\underset{\times 10}{(10+100+1000+\ldots..)} - \underset{\times 10}{(1+1+1+\ldots..)}\right]$$

$$= \frac{1}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$= \frac{1}{9}\left[\frac{10^{n+1}-10-9n}{9}\right]$$

$$= \frac{1}{81}\left[10^{n+1}-10-9n\right]$$

option a.

02. MTP1 – MAY 23 , MAR 21

7+ 77 + 777 + ……..

a. $(^7/_{81})(10^{n+1}+10) - (^7/_9)n$

b. $(^7/_{81})(10^{n+1}-10) + (^7/_9)n$

c. $(^7/_{81})(10^{n+1}-10) - (^7/_9)n$

d. None

$$= 7\,(7+77+777+\ldots\ldots)$$

$$= \frac{7}{9}(9+99+999+\ldots\ldots)$$

$$= \frac{1}{9}\left[(10-1)+(100-1)+(1000-1)\ldots\ldots\right]$$

$$= \frac{7}{9}\left[\underset{\times 10}{(10+100+1000+\ldots..)} - \underset{\times 10}{(1+1+1+\ldots..)}\right]$$

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$= \frac{7}{81}(10^{n+1}-10) - \frac{7n}{9}$$

option c

03. $0.5 + 0.55 + 0.555 + \ldots\ldots$

a. $(5/9)n - (5/81)(1-10^{-n})$

b. $(5/9)n + (5/81)(1-10^{-n})$

c. $(5/9)n + (5/81)(1+10^{-n})$. d. none

$= \frac{5}{9}(0.9 + 0.99 + 0.999 + \ldots\ldots)$

$= \frac{5}{9}[(1-0.1)+(1-0.01)+(1-0.001)+\ldots]$

$= \frac{5}{9}[(1+1+1+\ldots)-(0.1+0.01+0.001+\ldots)]$

$= \frac{5}{9}\left[n - \frac{0.1(1-0.1^n)}{1-0.1}\right]$

$= \frac{5}{9}\left[n - \frac{1(1-0.1^n)}{9}\right]$

$= \frac{5n}{9} - \frac{5}{81}(1-10^{-n})$

option a.

Q6 $S_\infty = \frac{a}{1-r}$, $|r| < 1$

01. JUNE 2014

$1+1/3 + 1/9 + 1/27 + \ldots\ldots$

a. 1.95 b. 1.5 c. 1.75 d. none

$S_\infty = \frac{a}{1-r} = \frac{1}{1-1/3} = \frac{1}{2/3} = \frac{3}{2} = 1.5$

option b

02. MTP 1 – MAY 2023

$1/2 + 1/6 + 1/18 + \ldots\ldots$

a. $5/4$ b. $3/4$ c. $7/3$ d. none

$S_\infty = \frac{a}{1-r} = \frac{1/2}{1-1/3} = \frac{1/2}{2/3} = \frac{3}{4}$

option b

03. MTP 1 – OCT 2021

$1+10^{-1}+10^{-2}+10^{-3}+\ldots\ldots \infty$

a. $9/10$ b. $1/10$ c. $10/9$ d. none

$S_\infty = \frac{a}{1-r} = \frac{1}{1-1/10} = \frac{1}{9/10} = \frac{10}{9}$

option c

04. DEC 2019

$1/2 + 1/3^2 + 1/2^3 + 1/3^4 + 1/2^5 + 1/3^6 \ldots\ldots\infty$

a. $25/24$ b. $19/24$ c. $1/12$ d. none

$\underset{x\,1/4}{\left(1/2 + 1/2^3 + \ldots.\infty\right)} + \underset{x\,1/9}{\left(1/3^2 + 1/3^4 + \ldots\infty\right)}$

Using $S_\infty = a/1-r$

$\frac{1/2}{1-\frac{1}{4}} + \frac{1/9}{1-\frac{1}{9}}$

$\frac{1/2}{3/4} + \frac{1/9}{8/9}$

$\frac{2}{3} + \frac{1}{8} = \frac{19}{24}$

option b

05. JUNE 2011

$243.243^{1/6}.243^{1/36}\ldots\ldots\ldots\infty = \_\_\_$

a. 1024 b. 27 c. 729 d. 246

$243^{1+1/6+1/36+\ldots.\infty}$

$243^{\frac{1}{1-1/6}} = 243^{\frac{1}{5/6}} = \left(\frac{5}{3}\right)^{6/5} = 3^6$

option c

06. JUNE 2019

If $y = 1+x+x^2+ \ldots\ldots \infty$ then $x =$

a. $y-1/y$ b. $y+1/y$ c. $y/y+1$ d. $y/y-1$

$a = 1$, $r = x$ $y = \frac{1}{1-x}$ Using $a/1-r$

$1-x = \frac{1}{y}$

$1-\frac{1}{y} = x$

$x = y-1/y$

option a

Q7

01. DEC 2012

The first term of a G.P. where second term is 2 and sum of infinite term is 8 will be

a. 6. b. 3 c. 4 d. 1

$ar = 2$, $\frac{a}{1-r} = 8 \Rightarrow a = 8(1-r)$

$8(1-r)r = 2$

$r(1-r) = {}^1/_4$

${}^1/_2 \times {}^1/_2$

TRY

$\frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$ ✓ ....r = ½ ⇒ a = 4

02. DEC 2023

Given an infinite geometric series with first term a and common ratio r . If its sum is 4 and second term is ¾ , then one of the correct option is

a) a = 1 & r = ¼ b) a = 3 & r = ¾

c) a = 3 & r = ¼ d) a = 1 & r = ½

METHOD 1

$ar = \frac{3}{4}$, $\frac{a}{1-r} = 4 \Rightarrow a = 4(1-r)$

$4(1-r)r = \frac{3}{4}$

$r(1-r) = {}^3/_{16}$

${}^1/_4 \times {}^3/_4$

TRY

$\frac{1}{4}\left(1-\frac{1}{4}\right) = \frac{3}{16}$ ✓ ....r = ¼ ⇒ a = 3

option c

$\frac{3}{4}\left(1-\frac{3}{4}\right) = \frac{3}{16}$ ✓ ....r = ¾ ⇒ a = 1

option not available

METHOD 2

$ar = \frac{3}{4}$ ..... only option C satisfies

03. SEPT 2024

infinite geometric series with first term a and common ratio r . If its sum is 8 and second term is ${}^7/_8$, then one of the correct option is

a) a = 3 & r = ${}^7/_{24}$ b) a= 4 & r = ${}^7/_{16}$

c) a = 7 & r = ${}^1/_8$ d) a= 2 & r = ${}^7/_{32}$

METHOD 1

$ar = \frac{7}{8}$, $\frac{a}{1-r} = 8 \Rightarrow a = 8(1-r)$

$8(1-r)r = \frac{7}{8}$

$r(1-r) = {}^7/_{64}$

${}^1/_8 \times {}^7/_8$

TRY

$\frac{1}{8}\left(1-\frac{1}{8}\right) = \frac{7}{64}$ ✓ ....r = ${}^1/_8$ ⇒ a = 7

option c

$\frac{7}{8}\left(1-\frac{7}{8}\right) = \frac{7}{64}$ ✓ ....r = ${}^7/_8$ ⇒ a = 1

option not available

METHOD 2

	$ar = \frac{7}{8}$	$\frac{a}{1-r} = 8$
a. 3, ${}^7/_{24}$	✓	✗
b. 4, ${}^7/_{16}$	✗	
c. 7, ${}^1/_8$	✓	✓

SUGGESTION – DEC 2012 SUM COULD NOT HAVE BEEN SOLVED USING OPTIONS SO CONCLUSION PRACTICE METHOD 1

WARNING – DO NOT FALL TRAP TO OPTIONS WHILE PRACTISING THE TOPIC

04. MTP 1 – OCT 2020

sum of first two terms of an infinite G.P. is 15 and each term is equal to the sum of all the terms following it , then the sum of series is

a. 20 b. 15 c. 25 d. 30

$a+ar = 15 \Rightarrow a(1+r) = 15$

a = sum of remaining terms

$a = S - a$

$a = a/1-r - a$

$2a = a/1-r$

$2 = 1/1-r$

$1-r = 1/2 \Rightarrow r = 1/2$, $a = 10$

$S = a+a = 20$ option a

Sum of infinite terms in G.P.

$S\infty = a+ar+ar^2+\ldots\ldots \infty = a/1-r$

Sum of squares of the same infinite terms

$S\infty = a^2+a^2r^2+a^2r^4+\ldots\infty = a^2/1-r^2$ x r^2

05. in a GP sum of infinite terms is 5 and sum of squares of these infinite terms is 15 . Find the series

a. $\frac{15}{4}, \frac{15}{16}, \frac{15}{64} \ldots\ldots \infty$

b. $\frac{5}{4}, \frac{5}{16}, \frac{5}{64} \ldots\ldots \infty$

c. $\frac{3}{4}, \frac{9}{16}, \frac{27}{64} \ldots\ldots \infty$

$a + ar + ar^2 + \ldots\ldots \infty = 5$

$\frac{a}{1-r} = 2 \qquad \frac{a^2}{(1-r)^2} = 25 \quad \ldots\ldots\ldots(1)$

$a^2 + a^2r^2 + \ldots\ldots \infty = 15$

$\frac{a^2}{1-r^2} = 15 \quad \ldots\ldots\ldots(2)$

(1)÷(2)

$\frac{1-r^2}{(1-r)^2} = \frac{25}{15}$

$\frac{1-r.1+r}{(1-r)^2} = \frac{5}{3}$

$\frac{1+r}{1-r} = \frac{5}{3}$

$3+3r = 5-5r$

$8r = 2 \qquad \therefore r = 1/4$

subs in (1)

$\frac{a}{1-\frac{1}{4}} = 5$

$\frac{4a}{3} = 5$

$\therefore a = \frac{15}{4}, r = \frac{1}{4}$

$\therefore$ series : $a , ar , ar^2 , \ldots\ldots \infty$

$\frac{15}{4}, \frac{15}{16}, \frac{15}{64} \ldots\ldots \infty$

METHOD 2 Using options

Lets TRY Option a

$\frac{15}{4}, \frac{15}{16}, \frac{15}{64} \ldots\ldots \infty$

sum of infinite terms is 5

$S\infty = \frac{a}{1-r} = \frac{15/4}{1-1/4} = \frac{15/4}{3/4} = 5$ ✓

sum of squares of these infinite terms is 15

$S\infty = \frac{a^2}{1-r^2} = \frac{225/16}{1-1/16} = \frac{225/16}{15/16} = 15$ ✓

option a

Q8 A.M. of $(a,b) = \frac{a+b}{2}$

G.M. of $(a,b) = \sqrt{ab}$

01. DEC 2022

AM and GM of 2 numbers is 5 and 4 respectively . The numbers are

a. 2,3 b. 2,8 c. 4,6 d. 1,16

$AM(a,b) = 5 \Rightarrow \frac{a+b}{2} = 5 \Rightarrow a+b = 10$

$GM(a,b) = 4 \Rightarrow \sqrt{ab} = 4 \Rightarrow ab = 16$

2,8 satisfies both the conditions

02. 2011

AM and GM of 2 numbers is 12.5 and 10 respectively . The numbers are

a. 20,5 b. 10,5 c. 5,4 d. none

$AM(a,b) = 12.5 \Rightarrow \frac{a+b}{2} = 12.5 \Rightarrow a+b = 25$

$GM(a,b) = 10 \Rightarrow \sqrt{ab} = 10 \Rightarrow ab = 100$

20,5 satisfies both the conditions

03. DEC 2015

Find nos whose G.M. is 5 and A.M. is 7.5

a. 12,13 b. 13.09,1.91 c. 14,11

d. 17,19

$AM(a,b) = 7.5 \Rightarrow \frac{a+b}{2} = 7.5 \Rightarrow a+b = 15$

$GM(a,b) = 5 \Rightarrow \sqrt{ab} = 5 \Rightarrow ab = 25$

13.09,1.91 satisfies both the conditions

$1 + 2 + 4 + \mathbf{8} + 16 + 32 + 64$

(1,8,64) , (2,8,32) , (4,8,16) …

satisfies $B^2 = AC$

8 = G.M.OF (1,64) , (2,32) , (4,16)

NOTE

8 is GM of any 2 terms in G.P. which stand equidistant on either side of 8.

04. JUN 2023

4^{th} , 7^{th} and 10^{th} term of GP are p,q,r then

a. $p^2=q^2+r^2$ b. $p^2 = qr$ **c. $q^2 = pr$**

d. pqr+pq+1 = 0

t_4 & t_{10} are equidistant from t_7 .

Hence t_7 = G.M. of (t_4,t_{10})

$t_{10}^2 = t_4.t_{10}$ ….$B^2 = AC$

$q^2 = p.r$ option C

G.M $(x_1,x_2,x_3,\ldots\ldots x_n)$

$= (x_1.x_2.x_3\ldots\ldots x_n)^{1/n}$

05. DEC 2013

GM of a,b,c,d is 3 , then GM of $^1/a, ^1/b, ^1/c, ^1/d$ is

a. $^1/3$ b. 3 c. $^1/81$ d. 81

GM of a,b,c,d = 3

$(a.b.c.d)^{1/4} = 3$

$a.b.c.d = 3^4$

GM of $(^1/a, ^1/b, ^1/c, ^1/d) = (^1/abcd)^{1/4}$

$= (^1/3^4)^{1/4}$

$= ^1/3$

G.M $(x_1,x_2,x_3,\ldots\ldots x_n) = k$

G.M $(1/x_1,1/x_2,1/x_3,\ldots\ldots 1/x_n) = ^1/k$

Q9

01. DEC 2014

If x, y, z are in GP then,

x^2+y^2, $xy+yz$, y^2+z^2 are in

a. AP b. GP c. HP d. none

x	y	z
1	2	4

x^2+y^2 , $xy+yz$, y^2+z^2

1+4 , 2+8 , 4+16

1 , 10 , 20 ....GP

option b.

02. JUNE 2018

if a, b, c, d are in GP then the value of

$(b-c)^2 + (c-a)^2 + (d-b)^2$ =

a. $(a-b)^2$ b. $(a-d)^2$ c. $(c-d)^2$

d. none

a	b	c	d
1	2	4	8

$(b-c)^2 + (c-a)^2 + (d-b)^2$

$= (2-4)^2 + (4-1)^2 + (8-2)^2 - (1-8)^2$

$= 4 + 9 + 36$

$= 49$

$= (a-d)^2$ option b

03. a, b, c are in AP and x, y, z are in GP, then value of $x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$ is

a. 1 b. 0 c. b(c–a) d. none

a	b	c
1	2	3

AP

x	y	z
1	2	4

GP

$x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$

$1^{2-3} \cdot 2^{3-1} \cdot 4^{1-2}$

$1^{-1} \cdot 2^2 \cdot 4^{-1}$

$\frac{1}{1} \cdot 4 \cdot \frac{1}{4} = 1$

option a

04. JUNE 2018

If the p^{th}, q^{th} and r^{th} terms of a GP be a, b, c respectively then

$(q-r)\log a + (r-p)\log b + (p-q)\log c$ =

a. 0 b. 1 c. 2 d. none

CONSIDER G.P. 1 + 2 + 4

p	q	r
1	2	3

$\Rightarrow$

a	b	c
1	2	4

$(q-r)\log a + (r-p)\log b + (p-q)\log c$

$= -1\log 1 + 2\log 2 + -1\log 4$

$= 0 + 2\log 2 - \log 2^2$

$= 2\log 2 - 2\log 2 = 0$

05. SEPT 2024

the numbers x, 8, y are in GP and the numbers x, y, –8 are in AP. The value of x and y are

a. –8,–8 b. 16,4 c. 8,8 d.none

AT THE FIRST GLANCE INTO THE OPTIONS, OPTION b. SEEMS TO CATCH MY SIGHT MORE THAN THE OTHERS. SO LETS TRY

16 , 8 , 4 ....GP ✓

x1/2 x1/2

16 , 4 , –8 ....AP ✓

–12 –12

option b.

06. JUNE 2016

A GP consists of 2n terms. If the sum of terms occupying the odd places is S_1 and that of the terms in even places is S_2, then the common ratio of the progression is

a. n b. $2S_1$ c. S_2/S_1 d. S_1/S_2

Lets consider GP of 4 terms

1 + 2 + 4 + 8, r = 2

$S_1 = 1+4 = 5$

$S_2 = 2+8 = 10$ $S_2/S_1 = 2 = r$

option c

07. JUNE 2015

Let S be the sum , P be the product and R be the sum of reciprocals of n terms of a GP then P^2R^n =

a. S^{2n} b. S^{-n} c. S^n d. S^{-2n}

Consider this GP : 1 + 2 + 4

$n = 3$, $S = 7$, $P = 8$

$R = 1+\frac{1}{2} + \frac{1}{4} = \frac{7}{4}$

$P^2R^n = P^2R^3 = 8^2 . \left(\frac{7}{4}\right)^3 = 64 . \frac{7^3}{64}$

$= 7^3$

$= S^n$

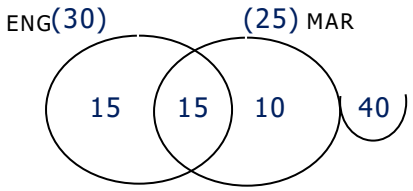
13 - SETS - RELATION - FUNCTION

Q1 SETS - VENN DIAGRAM

01. JUNE 2011

in a class of 40 students , 30 passed in English , 25 of them passed in Math and 15 passed in both . Every student has passed in atleast one subject . How many have passed in English but not in Math

[a.] 15 b. 20 c. 10 d. 25



number of students who have passed in English but not in Math = 15

02. JUNE 2014

in a class of 50 students , 35 students have taken mathematics , 37 have taken commerce . Find the number of the students who have taken both .

a. 13 b. 15 [c.] 22 d. 28

$n(A \cup B) = n(A)+n(B) - n(A \cap B)$

$50 = 35 + 37 - n(A \cap B)$

assume

$n(A \cap B) = 22$

03. JUNE 2017

in a class of 35 students , 24 play cricket and 16 play football . Also each student likes to play at least one game . How many students like to play both cricket and foot ball

[a.] 5 b. 11 c. 12 d. 17

$n(A \cup B) = n(A)+n(B) - n(A \cap B)$

$35 = 24 + 16 - n(A \cap B)$

$n(A \cap B) = 5$

04. JUNE 2017

in a class , 80 students speak Hindi , 60 speak English and 40 speak both Hindi & English . Find no. of students in the class

[a.] 100 b. 120 c. 140 d. 180

$n(A \cup B) = n(A)+n(B) - n(A \cap B)$

$= 80 + 60 - 40$

$= 100$

05. JUN '23

Survey shows 74% of Indians like grapes where as 68% like bananas What % of Indians like both if everybody likes either of the two

a. 32 b. 26 c. 6 [d.] 42

$n(A \cup B) = n(A)+n(B) - n(A \cap B)$

$100 = 74 + 68 - n(A \cap B)$

$n(A \cap B) = 42$

06. JULY 2021

$n(U) = 650$, $n(A) = 310$, $n(A \cap B) = 95$, $n(B) = 190$, find $n(\overline{A} \cap \overline{B})$

a. 400 b. 200 c. 300 [d.] 245

$n(A \cup B) = n(A)+n(B) - n(A \cap B)$

$= 310+190-95$

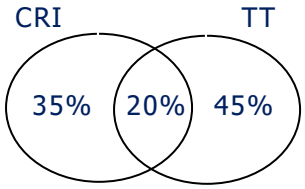
$n(A \cup B) = 405$

$n(\overline{A} \cap \overline{B}) = 650-405 = 245$

07. DEC 2015

in a class of 80 students , 35% students can play only cricket , 45% play only table tennis and the remaining students can play both the games . In all how many students can play cricket

a. 86 b. 54 c. 36 d. 44



Number of students who play cricket

= 35 + 20 = 55%(80)

= 44

Q2 SETS - VENN DIAGRAM

01. DEC 2013

out of 200 candidates who were interviewed for a position at call centre , 100 had a two wheeler , 70 had a credit card and 140 had a mobile phone . 40 of them had both two wheeler and a credit card , 30 had both a credit card and mobile phone , 60 had both a two wheeler and mobile phone . 10 had all three . How many candidates had none of the three

a. 0 b.20 c. 10 d. 18

n(AUBUC)

$$= \begin{pmatrix} n(A) \\ + n(B) \\ + n(C) \end{pmatrix} - \begin{pmatrix} n(A\cap B) \\ + n(A\cap B) \\ + n(A\cap B) \end{pmatrix} + n(A\cap B\cap C)$$

$$= \begin{pmatrix} 100 \\ + 70 \\ + 140 \end{pmatrix} - \begin{pmatrix} 40 \\ + 30 \\ + 60 \end{pmatrix} + 10$$

= 310 – 130 + 10 = 190

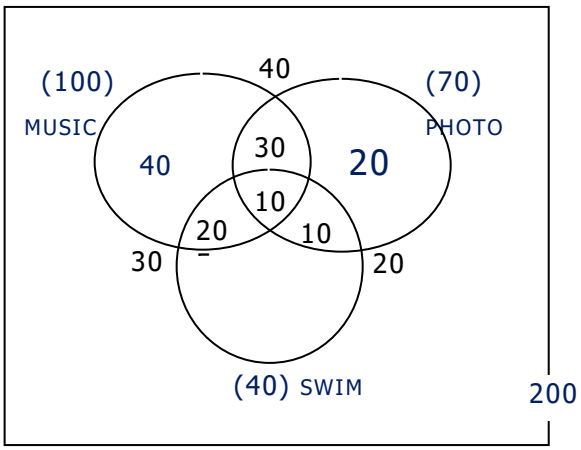
candidates having none of the three

= 200 – 190 = 10

02. DEC 2012

For a group of 200 persons , 100 are interested in music , 70 in phototgraphy and 40 in swimming . Further more 40 are interested in both music and photography . 30 in both music and swimming , 20 in photography and swimming and 10 in all three . How many are interested in photography but not music and swimming

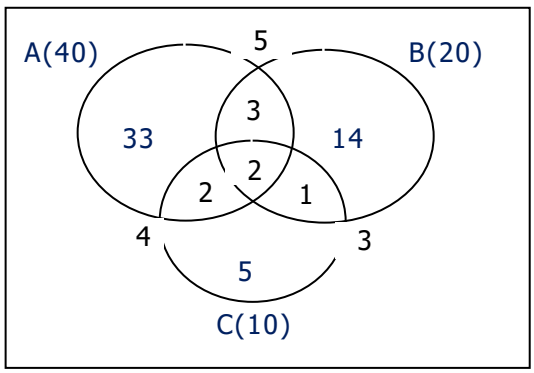
a. 30 b. 15 c. 25 d. 20



03. MAY 2018

in a town of 20,000 families , 40% buy newspaper A , 20% buy B , 10% buy C , 5% buy A and B , 3% buy B and C and 4% buy A and C . If 2% buy all three newspaper then number of families which buy only A

a. 6600 b. 6300 c. 5600 d. 600



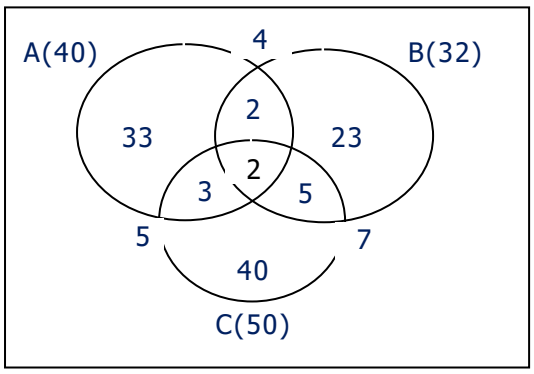
number of families which buy only A

= 33%(20000)

= 6600

04. DEC 2020

n(A) = 40 , n(B) = 32 , n(C) = 50 , n(A∩B) = 4 , n(A∩C) = 5 , n(B∩C) = 7 , n(A∩B∩C) = 2 . How many are in only one set a. 65 b. 110 [c.] 96 d. 84



In only one set = 33 + 23 + 40 = 96

05. DEC 2023

In a survey of 100 boys , it was found that 50 used white shirts , 40 red shirts and 30 blue shirts , 20 were using white and red shirts , 15 were using both red and blue shirts and 10 were using both blue and white shirts . Find number of boys using all colours

a) 20 b) 25 c) 30 d) 35

$$n(WURUB) = \left[n(W)+n(R)+n(B)\right] - \left[n(W\cap R)+n(R\cap B)+n(B\cap W)\right] + n(W\cap R\cap B)$$

$$100 = (50+40+30) - (20+15+10) + n(W\cap R\cap B)$$

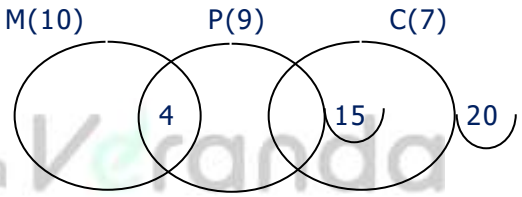
$$100 = 120 - 45 + n(W\cap R\cap B)$$

$$n(W\cap R\cap B) = 25$$ option b

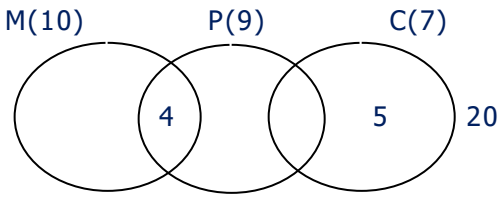
06. DEC 2021

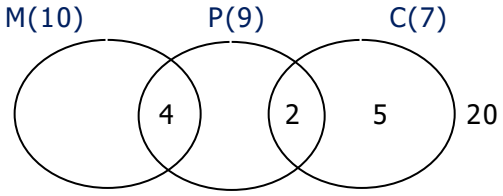
Out of a group of 20 teachers in a school , 10 teach Mathematics , 9 teach Physics & 7 teach Chemistry . 4 teach Math & Physics but none teach both Math and Chemistry . How many teach Chemistry & Physics . How many teach only Physics

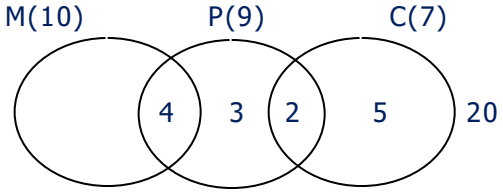
[a.] 2,3 b. 3,2 c. 4,6 d. 6,4



n(MUP) = 10 + 9 – 4 = 15







No. of teachers teaching Chemistry & Physics = 2

Only Physics = 3

Q3 SETS

01. JUNE 2019

A = {1,2,3,4,5,6,7,8,9}

B = {1,3,4,5,7,8}

C = {2,6,8}

Find $(A-B) \cup C$

$(A-B) \cup C$

$= \{\cancel{1},2,\cancel{3},\cancel{4},\cancel{5},6,\cancel{7},\cancel{8},9\} \cup \{2,6,8\}$

$= \{2,6,8,9\}$

02. JUNE 2016

$A = \{x : {}^{x}/_{2} \in Z , 0 \le x \le 10\}$

B = {x : x is one digit prime number}

$C = \{x : {}^{x}/_{3} \in N , x \le 12\}$

$A \cap (B \cap C)$

[a.] ϕ b. A b. B c. C

A ≡ {0,2,4,6,8,10}

B ≡ {2,3,5,7}

C ≡ {3,6,9,12}

$A \cap (B \cap C) \equiv \phi$ option a

03. JUNE 2018

$A = \{x/x=3^n-2n-1, n \in N\}$

$B = \{x/x=4(n-1) , n \in N\}$

[a.] $A \subset B$ b. $B \subset A$ c. A=B d. none

put n = 1,2,3,....

in 3^n-2n-1	in $4(n-1)$
$A = \{0,4,20,.....\}$	$B=\{0,4,8,12,20,...\}$

$A \subset B$

Q4

SUBSETS , NON EMPTY SUBSETS & PROPER SUBSETS

A = {1,2,3}

Write the possible subsets of A

{} , {1} , {2} , {3} , {1,2} , {1,3} , {2,3}, {1,2,3}

PROPER SUBSETS

NOTE 1 : an empty set and the set itself are too subsets of any set

NOTE 2 :

n(A) = 3

No. of subsets formed = $2^3 = 8$ ..... (2^n)

No. of NON EMPTY SUBSETS

{1} , {2} , {3} , {1,2} , {1,3} , {2,3} , {1,2,3}

$= 7 = (2^n - 1)$

No. of NON EMPTY PROPER SUBSETS

{1} , {2} , {3} , {1,2} , {1,3} , {2,3}

$= 6 = (2^n - 2)$

01. DEC 2022

No. of subsets of {0,1,2,3}

a. 2 b. 4 c. 8 [d.] 16

No. of subsets $= 2^4$

$= 16$

02. JUNE 2019

No. of subsets of {3,4,5}

a. 2 b. 4 [c.] 8 d. 16

No. of subsets $= 2^3$

$= 8$

03. DEC 2023 , MAY 2018

B = {1,2,3,4,5} , then the number of proper subsets of B is

a) 120 b) 30 [c)] 31 d) 32

n(B) = 5

number of proper subsets $= 2^5 - 1$

$= 31$

04. DEC 2022

A = {1,2,3,4,5,7,8,9}

B = {2,4,6,7,9}

How many proper subsets of A∩B can be created

a. 16 [b.] 15 c. 32 d. 31

A∩B = {2,4,7,9}

No. of proper subsets $= 2^4 - 1 = 15$

05. JUNE 2016

the number of subsets of the word formed from the letters of the word ALLAHABAD

a. 128 b. 16 [c.] 32 d. 64

X = { A , L , H , B , D}

n(X) = 5

No of subsets = 2^5 = 32 option c

06. JUNE 2022 , DEC 2020

Two finite sets with and a and b elements . The total number of subsets of the first set is 56 more than the total numbers subsets of the 2nd set . Find a , b

[a.] 6,3 b. 3,6 c. 8,4 d. 6,4

No. of subsets of

First set = 2^a

Second set = 2^b

$2^a - 2^b = 56$

Trying options

$2^6 - 2^3$ = 64−8 = 56 ✓. option a

07. Two finite sets with p and q as elements . The total number of subsets of the first set is 64 more than the total number of subsets of the second . The values of p and q are `

a. 5,7 [b.] 7,6 c. 8,7 d. 9,7

$2^p = 2^q + 64$

USING OPTIONS

b. (7,6) $2^7 = 2^6 + 64$

128 = 64+64 ✓ option b

Q5 CARTESIAN PRODUCT OF 2 SETS

ALL ELEMENTS OF A ARE MAPPED TO ALL ELEMENTS OF B

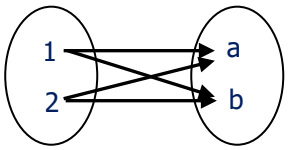
Let A and B be two non empty sets then the cartesian product of A and B is defined as a set of ordered pairs (a,b) such that a ∈ A , b ∈ B

Its denoted as AxB , read as 'A cross B'

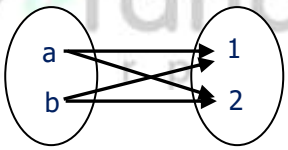
AxB = {(a,b) / a ∈ A , b ∈ B }

01. A = {1,2} , B = {a,b}

AxB = {(1,a),(1,b),(2,a),(2,b)}



BxA = {(a,1),(a,2),(b,1),(b,2)}



NOTE : 1. AXB ≠ BXA

2. n(AXB) = n(BXA)

3. n(A) = 2 , n(B) = 3 , n(AxB) = n(BxA) = 2x3 = 6

02. DEC 2011

A = {1,2,3,4,5} , B = {2,4} , C={1,3,5} then (A−C) x B

a. {(2,2),(2,4),(4,2),(4,4),(5,2),(5,4)}

b. {(1,2),(1,4),(3,2),(3,4),(5,2),(5,4)}

c. {(2,2),(4,2),(4,4),(4,5)}

[d.] {(2,2),(2,4),(4,2),(4,4)}

(A−C) x B

{1,2,3,4,5} x {2,4}

{(2,2) , (2,4) , (4,2) , (4,4)} option d

03. JUNE 2023

A = {a,b,c} ,B= {b,c,d}, C= {a,d,c} then (A–B)x(B∩C) is

a. {(a,d),(c,d)}

b. {(a,c),(a,d)}

c. {(c,a),(d,a)}

d. {(a,c),(a,d),(b,d)}

(A–B) x (B∩C)

{a} x {c,d}

≡ {(a,c) , (a,d)}

04. DEC 2023

If A = {1,2} , B = {3,4} , C = {5,6}

then A x (BUC)

a) {(1,2),(3,4),(5,6)}

b) {(1,3),(2,3),(1,4),(2,4),(2,5),(1,5), (1,6),(2,6)}

c) {(1,3),(2,3),(1,4),(2,4),(2,5),(1,5)}

d) {(3,1),(2,3),(4,1),(2,4),(2,5),(1,5), (1,6),(2,6)}

A x (BUC)

= {1,2} x {3,4,5,6}

= {(1,3),(2,3),(1,4),(2,4),(2,5),(1,5), (1,6),(2,6)} option b

05. DEC 2023

A = {2,4} , B = {1,2,3} then

(AUB) x (A∩B)

a) {(1,2),(2,2),(3,2)}

b) ((1,2),(2,2),(2,3),(2,4)}

c) {(2,1),(2,2),(2,4)}

d) {(1,2),(2,2),(3,2),(4,2)}

(AUB) x (A∩B)

= {1,2,3,4} x {2}

= {(1,2),(2,2),(3,2),(4,2)} option d

Q6 INVERSE RELATION

INVERSE RELATION (R^{-1})

If R:A→B then inverse relation R^{-1}:B→A

$R \subseteq AxB$ then $R^{-1} \subseteq BxA$

R = {(1,a) , (2,b) , (3,c)}

then

R^{-1} = {(a,1) , (b,2) , (c,3)}

domain of R = range of R^{-1}

range of R = domain of R^{-1}

01. A = {2,3,5,7} B = {4,6,9,10,11}

R is a relation defined as 'is a divisor of' from A to B . find the domain and range of R^{-1}

R ={(2,4) , (2,6) , (2,10) , (3,6) , (3,9) , (5,10)}

Domain (R^{-1})= Range (R) = {4,6,9,10}

Range (R^{-1}) = Domain (R) = {2,3,5}

02. JUNE 2019

A = {1,2,3,4,…..,10}

R is a relation on A

R = {(x,y)/x+y=10 , x∈A , y∈ A, x ≥ y }

Find domain of R^{-1}

A = {1,2,3,4,…..,10}

R : A→A

R = {(x,y)/x+y=10 , x∈A , y∈ A, x ≥ y }

= {(5,5),(6,4),(7,3),(8,2),(9,1)}

Domain of R^{-1} =Range (R) = {1,2,3,4,5}

Q7 REFLEXIVE - SYMMETRIC & TRANSITIVE RELATION

A = {1,2,3} , R:A→A

Is REFLEXIVE if (a,a)∈R for all a∈A

R = {(1,1),(2,2),(3,3)}

IS SYMMETRIC if (a,b)∈R then (b,a)∈R

R = {(1,2),(2,1),(2,3),(3,2)}

IS TRANSITIVE

if (a,b),(b,c) ∈ R then (a,c) ∈ R

R = {(1,2) , (2,3) , (1,3)}

If R is REFLEXIVE , SYMMETRIC & TRANSITIVE the R has attained EQUIVALENCE

01. DEC 2022 , SEPT 2024

A = {1,2,3}

R : A→A

R = {(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)}

R is

a. Symmetric & Transitive

b. Reflexive but not Transitive

c. Reflexive but not Symmetric

d. Neither Symmetric not Transitive

Since R contains (1,1) , (2,2) & (3,3) , its REFLEXIVE

Since R contains (1,2) BUT NOT (2,1) , R is NOT SYMMETRIC

Since R contains (1,2) ,(2,3) & (1,3) , R is TRANSITIVE

option c

02. JUNE 2022

R={(3,3),(6,6),(9,9),(12,12),(6,12), (3,9),(3,12),(3,6)}

defined on A = {3,6,9,12} then R is

a. Equivalence

b. Reflexive & Transitive only

c. Reflexive

d. Reflexive & Symmetric only

Since R contains (3,3),(6,6),(9,9),(12,12) its REFLEXIVE

Since R contains (6,12) BUT NOT (12,6) , R is NOT SYMMETRIC

CHECKING TRANSITIVITY

(6,12) does not have any (HETRO) ordered pair starting with 12

Same with (3,9) & (3,12) . So these pairs do not pose any threat to TRANSITIVITY

NEXT COMES (3,6)

(3,6),(6,12) ∈ R , (3,12) ∈ R , TRANSITIVE

Finally

REFLEXIVE / SYMMETRIC / TRANSITIVE

✓ ✗ ✓

option b

03. JUNE 2023

R = {(1,2),(2,3)} defined on set A = {1,2,3} . Find minimum no. of ordered pairs which when added to R will make it equivalence relation

a. 5 b. 7 c. 6 d. 8

For REFLEXIVE , we will have to add (1,1) , (2,2) , (3,3)

Since R already contains (1,2) ,(2,3), To make R SYMMETRIC we will have to add (2,1) , (3,2)

Since R already contains (1,2) ,(2,3), To make R TRANSITIVE we will have to add (1,3) .

HOLD – DON'T arrive at the answer '6'

On adding (1,3) to make the relation TRANSITIVE , we have broken the the SYMMETRIC nature of relation . So go back and add (3,1) to make relation once again SYMMETRIC .

FINALLY

REFLEXIVE	SYMMETRIC	TRANSITIVE
(1,1)	(2,1)	(1,3)
(2,2)	(3,2)	
(3,3)	(3,1)	

Hence number of ordered pairs to be added to make R , a equivalence relation = 7

04. DEC 2021

'aRb' , (a–b) is an even integer , then relation R is

a. Symmetric, reflexive but not transitive

b. symmetric , transitive but not reflexive

c. transitive , reflexive but not symmetric

d. Equivalence relation

a–a=0 is an even integer

$\therefore (a,a) \in R$, HENCE REFLEXIVE

$(a,b) \in R \Rightarrow a-b$ is even

$\Rightarrow b-a$ is even

$\Rightarrow (b,a) \in R$

HENCE SYMMETRIC

$(a,b), (b,c) \in R \Rightarrow a-b, b-c$ is even

$\Rightarrow a-c$ is even

$\Rightarrow (a,c) \in R$

That makes R , TRANSITIVE

R has attained EQUIVALENCE

option d

05. JUN 2023

'xRy' , (x–y) is ÷ by 5 . x,y ∈ N , then relation R is

a. Equivalence

b. Not Symmetric

c. Symmetric but not Transitive

d. Symmetric but not Reflexive'

For any a ∈ N , a–a=0 is ÷by 5

$\therefore (a,a) \in R$, HENCE REFLEXIVE

For any a,b ∈ N ,

$(a,b) \in R \Rightarrow a-b \div 5 \Rightarrow b-a \div 5$

$\Rightarrow (b,a) \in R$

HENCE SYMMETRIC

For any a,b,c ∈ N

$(a,b), (b,c) \in R \Rightarrow a-b, b-c \div 5$

$\Rightarrow a-c \div 5$

$\Rightarrow (a,c) \in R$

That makes R , TRANSITIVE

R has attained EQUIVALENCE

option a

06. JAN 2021

In the set of all straight lines on a plane which of the following is not TRUE

a. 'Parallel to' is an equivalence relation

b. 'Perpendicular to' is a symmetric relation

c. 'Perpendicular to' is an equivalence relation

d. 'Parallel to' is a reflexive relation

a. ' x Parallel to y' , xRy , x//y

since a // a , $(a,a) \in R$

hence REFLEXIVE .

$(a,b) \in R \Rightarrow a // b \Rightarrow b // a$

$\Rightarrow (b,a) \in R$

Hence SYMMETRIC

$(a,b) , (b,c) \in R \Rightarrow a // b , b // c$

$\Rightarrow a // c$

$\Rightarrow (a,c) \in R$

Hence TRANSITIVE

Finally , 'Parallel to' is an EQUIVALENCE relation

b. xRy , $x \perp y$

$(a,b) \in R \Rightarrow a \perp b \Rightarrow b \perp a$

$\Rightarrow (b,a) \in R$

Hence SYMMETRIC

c. xRy , $x \perp y$

$(a,a) \notin R$, a cannot be $\perp$ to a

$(a,b) , (b,c) \in R \Rightarrow a \perp b , b \perp c$

$\Rightarrow a // c$

$\Rightarrow (a,c) \notin R$

R is neither reflexive nor transitive

Hence R is NOT an equivalence relation

option c.

07. JUNE 2024

Consider the following relations defined on A = {1,2,3}

R = { (1,1) , (1,2) , (1,3) , (3,3)}

S = { (1,1) , (1,2) , (2,1) , (2,2) , (3,3)}

T = { (1,1) , (1,2) , (2,3)}

ϕ = empty set

Which one of these forms an equivalence relation

a. R b. S c. T d. ϕ

R is neither Reflexive nor Symmetric but is Transitive

S is reflexive , symmetric & transitive . Hence Equivalence

T is non reflexive , non symmetric & non transitive

ϕ is non reflexive but is Symmetric & Transitive

option b

FUNCTION

Q8

IF EVERY ELEMENT OF SET A IS RELATED / MAPPED TO ONE AND ONLY ONE ELEMENT OF B , THEN
R:A→B IS CALLED f:A→B

01. DEC 2015

Which of these is a function from A→B

A = {x,y,z} , B = {a,b,c,d}

a. {(x,a),(x,b),(y,c)}

b. {(x,a),(x,b),(y,c),(z,d)}

c. {(x,a),(y,b),(z,d)}

d. {(a,x),(b,z),(c,y)}

a. {(x,a),(x,b),(y,c)}

b. {(x,a),(x,b),(y,c),(z,d)}

x is mapped to more that one element

ONE TO MANY , NOT a $f : A \to B$

c. {(x,a),(y,b),(z,d)}

every element of A is related to one and and only one element of B .

Hence it is $f : A \to B$

option c

02. JUNE 2019

which R:A→B is function

A = {a,b,c,d} , B = {p,q,r,s}

a) $R_1 \equiv \{(a,p) , (b,q) , (c,s)\}$

b) $R_2 \equiv \{(b,p) , (c,s) , (b,r)\}$

c) $R_3 \equiv \{(a,p) , (b,r) , (c,q) , (d,s)\}$

d) all the above

a) $R_1 \equiv \{(a,p) , (b,q) , (c,s)\}$

$d \in A$ is not mapped to any element of B .

hence R_1 is NOT a $f : A \to B$

b) $R_2 \equiv \{(b,p) , (c,s) , (b,r)\}$

ONE TO MANY , since $b \in A$ is mapped to more than one element in B .

Also a , $d \in A$ are not mapped to any element of B .

hence R_2 is NOT a $f : A \to B$

c) $R_3 \equiv \{(a,p) , (b,r) , (c,q) , (d,s)\}$

every element of A is related to one and and only one element of B . Hence R_3 is $f : A \to B$

option c

03. DEC 2018

which R is function

a) $R_1 \equiv \{(1,1),(1,2),(1,3)\}$

b) $R_2 \equiv \{(1,1),(2,1),(2,3)\}$

c) $R_3 \equiv \{(1,2),(2,2),(3,2),(4,2)\}$

d) none

a. $R_1 \equiv \{(1,1),(1,2),(1,3)\}$

x = 1 is mapped to more than one element , y = 2 ,3 .

hence R_1 is NOT a $f : A \to B$

b. $R_2 \equiv \{(1,1),(2,1),(2,3)\}$

x = 2 is mapped to more than one element , y = 1 ,3 .

hence R_2 is NOT a $f : A \to B$

c) $R_3 \equiv \{(1,2),(2,2),(3,2),(4,2)\}$

x = 1, 2 , 3 & 4 are mapped to one and only element y .

Hence R_3 is $f : A \to B$

option c

Consider $f:A \to B$,

If every element in B has a pre image in A , then f is ONTO ….. (RANGE = DOMAIN)

If every element in B does not have a pre image in A then f is INTO

… (RANGE $\subset$ DOMAIN)

04. JUNE 2011

A = {±2 , ±3} , B = {1,4,9} and

f = {(2,4),(−2,4),(3,9),(−3,9)}

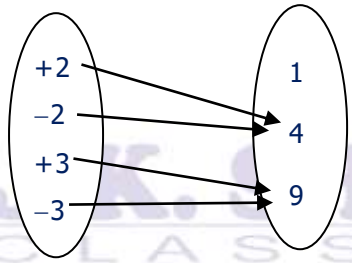
then f is defined as

a. one to one function from A into B

b. one to one function from A onto B

[c.] many to one function from A into B

d. many to one function from A onto B



MANY TO ONE , INTO FUNCTION

option c

05. $f(x) = 2x$, $f : N \to E$,

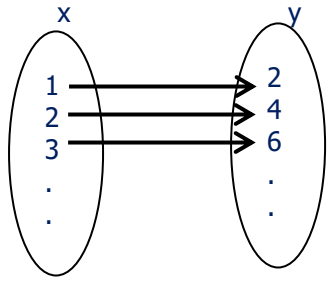
N set of all natural numbers , E : set of all even natural numbers . The function is

a. one to one , into

b. many to one , into

c. one to one , onto

d. many to one , onto



ONE TO ONE , ONTO FUNCTION

option c

Q9

01. JAN 2021

$$f(x) = \begin{cases} 2x & , \ x > 3 \\ x^2 & , \ 1 < x \le 3 \\ 3x & , \ x \le 1 \end{cases}$$

Find $f(-1) + f(2) + f(4)$

[a.] 9 b. 14 c. 5 d. 6

$f(-1) + f(2) + f(4)$

$3(-1) + 2^2 + 2(4)$

$-3+4+8$

9

option a

02. DEC 2010

$f : R \to R$, $f(x) = x+1$,

$g : R \to R$, $g(x) = x^2+1$, then $fog(-2)$

a. 6 [b.] 5 c. −2 d. none

$fog(-2)$

$f[g(-2)]$

$f[(-2)^2+1]$

$f[5]$

$5+1 = 6$

option a

03. JULY 21

$f(x) = x^2-1$, $g(x) = |2x+3|$

then $fog(3) - gof(-3) = ?$

a. 71 [b.] 61 c. 41 d. 51

$fog(3) - gof(-3)$

$f[g(3)] - g[f(-3)]$

$f(|6+3|) - g(9-1)$

$f(9) - g(8)$

$(81-1) - |16+3|$

$80 - 19$

61

option b.

04. JUNE 2019

$f(x) = x^2$, $g(x) = \sqrt{x}$, then

a. gof(3) = 3 b. gof(−3)=9

c. gof(9) = 3 d. gof(−9)=3

TRYING OUT OPTIONS ,

Option a.

gof(3) = g[f(3)] = g(9) = $\sqrt{9}$ = 3

satisfied

Hence option a

05. SEPT 2024

$f(x) = x^2+x-1$, $4f(x) = f(2x)$, then x =

a. 3/2 b. 2/3 c. 3/4 d. 4/3

$f(x) = x^2+x-1$

$4f(x) = f(2x)$

$4(x^2+x-1) = (2x)^2+2x-1$

$4x^2+4x-4 = 4x^2+2x-1$

$4x-4 = 2x-1$

$2x = 3$

$x = 3/2$ option a

Q10 COMPOSITE FUNCTIONS

01. JUNE 2017

$f(x) = \frac{1}{1-x}$ $g(x) = \frac{x-1}{x}$

Find gof(x)

a. x−1 **b.** x c. 1−x d. −x

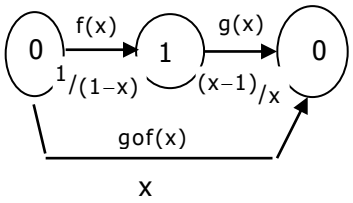
$= g[f(x)]$

$= \frac{f(x)-1}{f(x)}$

$= \frac{\frac{1}{1-x}-1}{\frac{1}{1-x}}$

$= 1-1+x$

$= x$ option b



02. JUNE 2011

$f(x) = \frac{x}{\sqrt{(1+x^2)}}$ $g(x) = \frac{x}{\sqrt{(1-x^2)}}$

fog(x) = ?

a. x b. $1/x$ c. $x/\sqrt{(1-x^2)}$

d. x. $\sqrt{(1-x^2)}$

$= f[g(x)]$

$= \frac{g(x)}{\sqrt{(1+g(x)^2)}}$

$= \frac{\frac{x}{\sqrt{(1-x^2)}}}{\sqrt{1+\frac{x^2}{1-x^2}}}$

$= \frac{\frac{x}{\sqrt{(1-x^2)}}}{\sqrt{\frac{1-x^2+x^2}{1-x^2}}}$

$= \frac{\frac{x}{\sqrt{(1-x^2)}}}{\frac{1}{\sqrt{(1-x^2)}}} = x$ option a

03. DEC 2017

$f(x) = \frac{x+1}{x+2}$. $f[f(1/x)]$

a. $\frac{2x+3}{3x+2}$ b. $\frac{2x+5}{3x+2}$ [c.] $\frac{3x+2}{5x+3}$ d. $\frac{5x+2}{2x+3}$

$f[f(1/x)]$

$= f\left[\frac{1/x + 1}{1/x + 2}\right]$

$= f\left[\frac{1+x}{1+2x}\right]$

$= \frac{\frac{1+x}{1+2x} + 1}{\frac{1+x}{1+2x} + 2}$

$= \frac{1+x+1+2x}{1+x+2+4x}$

$= \frac{3x+2}{5x+3}$ option c

Q11 RANGE OF FUNCTION

01. DEC 2018

A = {1,2,3,4}

B = {1,4,9,16,25}

f:A→B , $f(x) = x^2$. Find the range of the f(x)

a. {1,2,3,4} [b.] {1,4,9,16}

c. {1,4,9,16,25} d. none

$y = x^2 = 1,4,9,16$ when $x = 1,2,3,4$

Range of f = {1,4,9,16}

option b.

02. JULY 2021

The range of the function f defined as $f(x) = \sqrt{16-x^2}$

a. [−4,0] b. [−4,4] [c.] [0,4]

d. (−4,4)

$16 - x^2 \geq 0$

$\Rightarrow 0 \leq x^2 \leq 16$

$\Rightarrow 0 \leq 16-x^2 \leq 16$

$\Rightarrow 0 \leq \sqrt{16-x^2} \leq 16$

$\Rightarrow 0 \leq y \leq 4$

Range of f = [0,4] option c

03. JUNE 2016

The domain D and the range R of the function ,

f(x) = 2−|x+1| is

a. D = Real number , R = [2,∞)

b. D = Integers , R = (0,2)

c. D = Integers , R = (−∞,∞)

d. D = Real numbers , R = (−∞,2]

domain D = Real number ,

$|x+1| \geq 0$

$-|x+1| \leq 0$

$2-|x+1| \leq 2$

$f(x) \leq 2$

Range R = (−∞,2] option d

04. JUNE 2017

Find range of $f(x) = \frac{x}{x^2+1}$

a. $(-1/2, 1/2)$

b. $[-1/2, 1/2)$

c. $[-1/2, 1/2]$ d. none

$f(x) = \frac{x}{x^2+1}$

$y = \frac{x}{x^2+1}$

$yx^2+1y = x$

$yx^2 - x + y = 0$

Discriminant ≥ 0

$A = y , B = -1 , C = y$

$B^2-4AC \geq 0$

$1-4y^2 \geq 0$

$1 \geq 4y^2$

$y^2 \leq 1/4$

$y \in [-1/2, 1/2]$

05. JUNE 2018

Find range of $f(x) = \frac{x^6}{x^{12}+1}$

a. $(0,\infty)$

b. $[-1/2, 1/2)$

c. $[0, 1/2]$

d. $(0, 1/2)$

$f(x) = \frac{x^6}{x^{12}+1}$

$y = \frac{x^6}{x^{12}+1}$

$yx^{12}+ y = x^6$

$yx^{12} - x^6 + y = 0$

$yz^2 - z + y = 0$

Discriminant ≥ 0

$A = y , B = -1 , C = y$

$B^2-4AC \geq 0$

$1-4y^2 \geq 0$

$1 \geq 4y^2$

$y^2 \leq 1/4$

$y \in [-1/2, 1/2]$

as per the configuration of y , y cannot be negative

$y \in [0, 1/2]$ option c

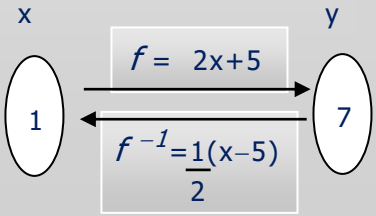
Q12 INVERSE OF A FUNCTION

$f(x) = 2x+5$

$y = 2x+5$

$x = \frac{y-5}{2}$

$f^{-1}(x) = \frac{x-5}{2}$



01. JUNE 2023

$f(x) = 4x+3$, $\forall x \in N$, then $f^{-1}(x)$ is

a. $4+^{(x+3)}/4$ b. $^{(x+3)}/4$ [c.] $^{(x-3)}/4$

d. $^{(3x+4)}/4$

$y = 4x+3$

$y-3 = 4x$

$x = \frac{y-4}{3}$

$f^{-1}(x) = \frac{x-4}{3}$ option c

02. JUNE 2022

$f(p) = \frac{1}{1-p}$, $f^{-1}(p) =$

a. $\frac{1}{p-1}$ b. $1-p$ c. $\frac{1-p}{p}$ [d.] $\frac{p-1}{p}$

$f(p) = \frac{1}{1-p}$

$y = \frac{1}{1-p}$

$1-p = \frac{1}{y}$

$1-\frac{1}{y} = p$

$p = \frac{y-1}{y}$

$f^{-1}(p) = \frac{p-1}{p}$ option d.

03. DEC 2022

$u(x) = \frac{1}{1-x}$, $u^{-1}(x) =$

a. $\frac{1}{x-1}$ b. $1-x$ [c.] $1-\frac{1}{x}$ d. $\frac{1-1}{x}$

$u(x) = \frac{1}{1-x}$

$y = \frac{1}{1-x}$

$1-x = \frac{1}{y}$

$1-\frac{1}{y} = x$

$u^{-1}(x) = 1-\frac{1}{x}$ option c.

04. DEC 2019

$f(x) = 2x^3+1$, $f^{-1}(x)$

a. $\frac{1}{2}(x-1)^{1/3}$

[b.] $(^{x-1}/2)^{1/3}$

c. $(^{x-1}/2)^{1/2}$ d. none of these

$y = 2x^3+1$

$y-1 = 2x^3$

$^{y-1}/2 = x^3$

$x = [^{y-1}/2]^{1/3}$

$f^{-1}(x) = [^{x-1}/2]^{1/3}$ option b

05. JULY 2021

$A = R-\{3\}$, $B = R-\{1\}$

$f : A \to B$, $f(x) = ^{x-2}/_{x-3}$. Find $f^{-1}(^1/2)$

a. 2/3 b. 3/4 [c.] 1 d. −1

$f^{-1}(^1/2) = x$

$^1/2 = f(x)$

$\frac{1}{2} = \frac{x-2}{x-3}$

$x-3 = 2x-4$

$-3+4 = 2x-x$

$x = 1$ option c.

RECENTLY ADDED TO THE REVISED SYLLABUS

Consider $f(x) = \frac{x^2-4}{x-2}$

Let's find $f(2) = \frac{2^2-4}{2-2} = \frac{0}{0}$ OOP's

f(x) says he can't give us value at x = 2 . Hence in that case we then try to find the limiting value of the f(x) at $x \to 2$

How ?

$$\lim_{x\to 2} f(x)$$

$$\lim_{x\to 2} \frac{x^2-4}{x-2}$$

$$\lim_{x\to 2} \frac{(x-2)(x+2)}{x-2}$$

$$\lim_{x\to 2} x+2$$

$$2+2 = 4$$

Finally we come to conclusion ,

$f(x)\Big|_{x=2} = \frac{x^2-4}{x-2} = \frac{0}{0}$ not defined

However

$\lim_{x\to 2} f(x) = 4$

Also I would like to highlight an interesting point in polynomials . If any polynomial P(x) becomes zero by putting x = a , it must contain one of the factors x–a . Once we know one of the factors of the polynomial , it then becomes very easy to find the other factors .

Let me throw some more light on this

$f(x) = \frac{x^2-4x+3}{x^2-2x-3}$

$f(x)\Big|_{x=3} = \frac{9-12+3}{9-6-3} = \frac{0}{0}$

Hence we then find ,

$$\lim_{x\to 3} \frac{x^2-4x+3}{x^2-2x-3}$$

TRICK FOR QUICK FACTORISATION

We just observed that both the polynomial x^2-4x+3 & x^2-2x-3 become 0 when we put x = 3 . Hence we can say one of the factors have to be (x–3) . That makes our job of finding the other factor very EASY . How , let me explain

$x^2-4x+3 = (x-3)(\ ?\ )$

Product of first terms in the factors have to make x^2. Hence

$x^2-4x+3 = (x-3)(x \pm ?)$

Product of the last terms in the factor have to make +3 . Hence

$x^2-4x+3 = (x-3)(x-1)$

Similarly ,

$x^2-2x-3 = (x-3)(\ ?\ )$

$x^2-2x-3 = (x-3)(x \pm ?)$

$x^2-2x-3 = (x-3)(x+1)$

GOING BACK TO SOLN ,

$\frac{(x-3)(x-1)}{(x-3)(x+1)}$ as $x \to 3$, $x-3 \neq 0$

$\frac{x-1}{x+1}$

$\frac{3-1}{3+1}$

$\frac{1}{2}$

Q1

01. $\lim_{x\to 5} \frac{x^2-9x+20}{x^2-6x+5}$

$\frac{(x-5)(x-4)}{(x-5)(x-1)}$, as $x\to 5$, $x-5 \neq 0$

$\frac{x-4}{x-1}$

$\frac{5-4}{5-1}$

$\frac{1}{4}$

02. $\lim_{x\to \sqrt{2}} \frac{x^2-2}{x^2+\sqrt{2}x-4}$

$\frac{(x-\sqrt{2})(x+\sqrt{2})}{(x-\sqrt{2})(x+2\sqrt{2})}$, as $x\to\sqrt{2}$, $x-\sqrt{2} \neq 0$

$\frac{x+\sqrt{2}}{x+2\sqrt{2}}$

$\frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}+2\sqrt{2}}$

$\frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$

03. $\lim_{x\to \sqrt{3}} \frac{x^2-3}{x^2+3\sqrt{3}x-12}$

$\frac{(x-\sqrt{3})(x+\sqrt{3})}{(x-\sqrt{3})(x+4\sqrt{3})}$, as $x\to\sqrt{3}$, $x-\sqrt{3} \neq 0$

$\frac{x+\sqrt{3}}{x+4\sqrt{3}}$

$\frac{\sqrt{3}+\sqrt{3}}{\sqrt{3}+4\sqrt{3}}$

$\frac{2\sqrt{3}}{5\sqrt{3}} = \frac{2}{5}$

Q2

01. $\lim_{x\to 0} \frac{\sqrt{1+x+x^2}-1}{x}$

$\frac{\sqrt{1+x+x^2}-1}{x} \cdot \frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1}$

$\frac{1+x+x^2-1}{x} \cdot \frac{1}{\sqrt{1+x+x^2}+1}$

$\frac{x(1+x)}{x} \cdot \frac{1}{\sqrt{1+x+x^2}+1}$ as $x\to 0$, $x \neq 0$

$\frac{1+x}{\sqrt{1+x+x^2}+1}$

$\frac{1+0}{\sqrt{1+0+0^2}+1}$

$\frac{1}{1+1} = \frac{1}{2}$

02. $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

$\frac{\sqrt{1+x}-\sqrt{1-x}}{2x} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$\frac{1+x-1+x}{2x} \cdot \frac{1}{\sqrt{1+x}+\sqrt{1-x}}$

$\frac{2x}{2x} \cdot \frac{1}{\sqrt{1+x}+\sqrt{1-x}}$ as $x\to 0$, $x \neq 0$

$\frac{1}{\sqrt{1+0}+\sqrt{1-0}}$

$\frac{1}{1+1} = \frac{1}{2}$

03. $\lim_{x\to 3} \frac{\sqrt{x+3}-\sqrt{6}}{x^2-9}$

$\frac{\sqrt{x+3}-\sqrt{6}}{x^2-9} \cdot \frac{\sqrt{x+3}+\sqrt{6}}{\sqrt{x+3}+\sqrt{6}}$

$\frac{x+3-6}{(x-3)(x+3)} \cdot \frac{1}{\sqrt{x+3}+\sqrt{6}}$

$\frac{x-3}{(x-3)(x+3)} \cdot \frac{1}{\sqrt{x+3}+\sqrt{6}}$

$\frac{1}{3+3} \quad \frac{1}{\sqrt{3+3}+\sqrt{6}}$

$\frac{1}{6} \quad \frac{1}{2\sqrt{6}} = \frac{1}{12\sqrt{6}}$

Q3 Using $\lim_{x\to\infty} \frac{1}{x^n} = 0$

01. $\lim_{x\to\infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7}$

Divide N & D by x^3

$\lim_{x\to\infty} \frac{\frac{3x^3}{x^3} - \frac{4x^2}{x^3} + \frac{6x}{x^3} - \frac{1}{x^3}}{\frac{2x^3}{x^3} + \frac{x^2}{x^3} - \frac{5x}{x^3} + \frac{7}{x^3}}$

$\lim_{x\to\infty} \frac{3 - \frac{4}{x} + \frac{6}{x^2} - \frac{1}{x^3}}{2 + \frac{1}{x^3} - \frac{5}{x^2} + \frac{7}{x^3}}$

$\frac{3-0+0-0}{2+0-0+0}$

$\frac{3}{2}$

02. $\lim_{x\to\infty} \frac{(1+2x^2)(3-x^4)}{(1+x^2)(5+x^4)}$

$\frac{\left(\frac{1}{x^2}+\frac{2x^2}{x^2}\right)\left(\frac{3}{x^4}-\frac{x^4}{x^4}\right)}{\left(\frac{1}{x^2}+\frac{x^2}{x^2}\right)\left(\frac{5}{x^4}+\frac{x^4}{x^4}\right)}$

$\frac{\left(\frac{1}{x^2}+2\right)\left(\frac{3}{x^4}-1\right)}{\left(\frac{1}{x^2}+1\right)\left(\frac{5}{x^4}+1\right)}$

$\frac{(0+2)(0-1)}{(0+1)(0+1)}$

-2

03. $\lim_{x\to\infty} \sqrt{x^2+7x} - x$

$\sqrt{x^2+7x} - x \cdot \frac{\sqrt{x^2+7x}+x}{\sqrt{x^2+7x}+x}$

$\frac{x^2+7x-x^2}{\sqrt{x^2+7x}+x}$

$\frac{\frac{7x}{x}}{\frac{\sqrt{x^2+7x}}{x}+\frac{x}{x}}$

$\frac{7}{\sqrt{\frac{x^2+7x}{x^2}}+1}$

$\frac{7}{\sqrt{1+\frac{7}{x}}+1} = \frac{7}{\sqrt{1+0}+1} = \frac{7}{2}$

04. $\lim_{x\to\infty} \dfrac{\sqrt{3x^2-1}-\sqrt{2x^2-1}}{4x+3}$

$$\frac{\frac{\sqrt{3x^2-1}}{x}-\frac{\sqrt{2x^2-1}}{x}}{\frac{4x+3}{x}}$$

$$\frac{\sqrt{\frac{3x^2-1}{x^2}}-\sqrt{\frac{2x^2-1}{x^2}}}{\frac{4x}{x}+\frac{3}{x}}$$

$$\frac{\sqrt{3-\frac{1}{x^2}}-\sqrt{2-\frac{1}{x^2}}}{4+\frac{3}{x}}$$

$$\frac{\sqrt{3-0}-\sqrt{2-0}}{4+0} = \frac{\sqrt{3}-\sqrt{2}}{4}$$

05. $\lim_{n\to\infty} \dfrac{1+2+3+\ldots\ldots.n}{n^2}$

$$\frac{\frac{n(n+1)}{2}}{n^2}$$

$$\frac{1}{2}\ \frac{n(n+1)}{n^2}$$

$$\frac{1}{2}\ \frac{n}{n}\left(\frac{n+1}{n}\right)$$

$$\frac{1}{2}\left(1+\frac{1}{n}\right)$$

$$\frac{1}{2}\ (1+0)$$

$$\frac{1}{2}$$

06. $\lim_{n\to\infty} \dfrac{1^2+2^2+3^2+\ldots\ldots.n^2}{n^3}$

$$\frac{\frac{n(n+1)(2n+1)}{6}}{n^3}$$

$$\frac{1}{6}\ \frac{n(n+1)(2n+1)}{n^3}$$

$$\frac{1}{6}\ \frac{n}{n}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$\frac{1}{6}\ (1+0)(2+0)$$

$$\frac{1}{3}$$

07. $\lim_{n\to\infty} \dfrac{1}{3}+\dfrac{1}{3^2}+\dfrac{1}{3^3}+\ldots..+\dfrac{1}{3^n}$

Sn of GP : $a = {}^1/_3$, $r = {}^1/_3$

$$\frac{a(1-r^n)}{1-r}$$

$$\frac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}}$$

$$\frac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}}$$

$$\frac{1}{2}\left(1-\frac{1}{3^n}\right)$$

$$\frac{1}{2}\left(1-0\right) = \frac{1}{2}$$

Q4 $\lim_{x\to 0} \frac{a^x-1}{x} = \log_e a$

01. $\lim_{x\to 0} \frac{4^x-2^x}{x}$

$\frac{4^x-1-2^x+1}{x}$

$\frac{(4^x-1)-(2^x-1)}{x}$

$\frac{4^x-1}{x} - \frac{2^x-1}{x}$

$\log 4 - \log 2 = \log\left(\frac{4}{2}\right) = \log 2$

02. $\lim_{x\to 0} \frac{6^x - 3^x}{4^x - 1}$

$\frac{6^x - 1 - 3^x + 1}{4^x - 1}$

$\frac{6^x - 1 - (3^x - 1)}{4^x - 1}$

$\frac{\frac{6^x - 1}{x} - \frac{3^x - 1}{x}}{\frac{4^x - 1}{x}}$

$\frac{\log 6 - \log 3}{\log 4}$

$\frac{\log\left(\frac{6}{3}\right)}{\log 4}$

$\frac{\log 2}{\log 2^2}$

$= \frac{\log 2}{2\log 2} = \frac{1}{2}$

03. $\lim_{x\to 0} \frac{7^{11x}-1}{x}$

$\frac{(7^{11})^x-1}{x}$

$\log 7^{11} = 11.\log_e 7$

04. $\lim_{x\to 0} \frac{e^{4x}-1}{x}$

$\frac{(e^4)^x-1}{x}$

$\log_e e^4$

$4.\log_e e = 4(1) = 4$

05. $\lim_{x\to 0} \frac{e^{\alpha x} - e^{\beta x}}{x}$

$\frac{e^{\alpha x}-1 - e^{\beta x}+1}{x}$

$\frac{e^{\alpha x}-1}{x} - \frac{e^{\beta x}-1}{x}$

$\frac{(e^{\alpha})^x-1}{x} - \frac{(e^{\beta})^x-1}{x}$

$\log_e e^{\alpha} - \log_e e^{\beta}$

$\alpha.\log_e e - \beta.\log_e e$

$\alpha(1) - \beta(1)$

$\alpha-\beta$

06. $\lim_{x\to 0} \frac{6^x-3^x-2^x+1}{x^2}$

$\frac{3^x2^x-3^x-2^x+1}{x^2}$

$\frac{3^x(2^x-)-1(2^x-1)}{x^2}$

$\frac{(2^x-1)(3^x-1)}{x^2}$

$\frac{2^x-1}{x} \cdot \frac{3^x-1}{x}$

log 2 . log 3

07. $\lim_{x\to 0} \frac{e^{5x}-e^{3x}-e^{2x}+1}{x^2}$

$\frac{e^{3x}e^{2x}-e^{3x}-e^{2x}+1}{x^2}$

$\frac{e^{3x}(e^{2x}-1)-1(e^{2x}-1)}{x^2}$

$\frac{(e^{3x}-1)(e^{2x}-1)}{x^2}$

$\frac{(e^3)^x-1}{x} \cdot \frac{(e^2)^x-1}{x}$

log e^3 . log e^2

3log e . 2log e

6

Q5

$\lim_{x\to\infty} \left[1+\frac{1}{x}\right]^x = e$

OR

$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

01. $\lim_{x\to 0} (1+ax)^{\frac{1}{x}}$

$\left[(1+ax)^{\frac{1}{ax}}\right]^a$

e^a

02. $\lim_{x\to\infty} \left[1+\frac{a}{x}\right]^x$

$\left\{\left[1+\frac{a}{x}\right]^{\frac{x}{a}}\right\}^a$

e^a

03. $\lim_{x\to\infty} \left[1+\frac{9}{x}\right]^x$

$\left\{\left[1+\frac{9}{x}\right]^{\frac{x}{9}}\right\}^9$

e^9

04. $\lim_{x\to\infty} \left(\frac{x+6}{x+1}\right)^{x+6}$

$= \lim_{x\to\infty} \left(\frac{1+\frac{6}{x}}{1+\frac{1}{x}}\right)^{x+6}$

$= \lim_{x\to\infty} \left(\frac{1+\frac{6}{x}}{1+\frac{1}{x}}\right)^{x} \left(\frac{1+\frac{6}{x}}{1+\frac{1}{x}}\right)^{6}$

$= \lim_{x\to\infty} \frac{\left(1+\frac{6}{x}\right)^{x}}{\left(1+\frac{1}{x}\right)^{x}} \left(\frac{1+0}{1+0}\right)^{6}$

$= \lim_{x\to\infty} \frac{\left\{\left(1+\frac{6}{x}\right)^{\frac{x}{6}}\right\}^{6}}{\left(1+\frac{1}{x}\right)^{x}}$

$= \frac{e^6}{e}$

$= e^5$

CONTINUITY

EXAMPLE 1

$f(x) = 2x+3 , \quad x < 1$

$= 3x+2 , \quad x > 1$

$= 5 , \quad x = 1$

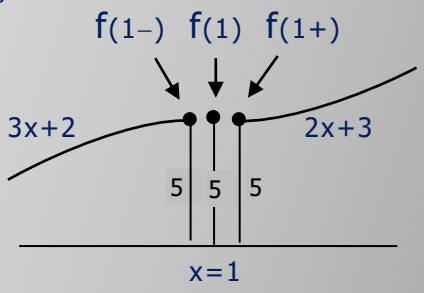
LEFT HAND LIMIT (LHL) , $f(1-)$

$\lim_{x \to 1-} f(x) = 2x+3 = 2(1)+3 = 5$

RIGHT HAND LIMIT (RHL) , $f(1+)$

$\lim_{x \to 1+} f(x) = 3x+2 = 3(1)+2 = 5$

$f(1) = 5$



f is CONTINUOUS at $x = 1$

EXAMPLE 2

$f(x) = 2x+3 , \quad x < 1$

$= 3x+5 , \quad x > 1$

$= 5 , \quad x = 1$

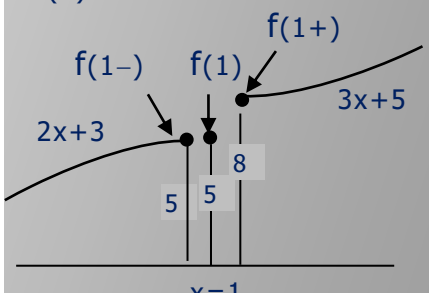
LEFT HAND LIMIT (LHL) , $f(1-)$

$\lim_{x \to 1-} f(x) = 2x+3 = 2(1)+3 = 5$

RIGHT HAND LIMIT (RHL) , $f(1+)$

$\lim_{x \to 1+} f(x) = 3x+5 = 3(1)+5 = 8$

$f(1) = 5$



f is DISCONTINUOUS at $x = 1$

01. $f(x) = 5x - 3 , \quad 0 \le x < 1$

$= x^2 + 1 , \quad 1 \le x \le 2$

Discuss continuity at $x = 1$

$f(1-) = 5x - 3 = 5(1)-3 = 2$

$f(1+) = x^2 + 1 = 1^2 + 1 = 2$

$f(2) = x^2 + 1 = 1^2 + 1 = 2$

f is CONTINUOUS at $x = 1$

02. $f(x) = 4x + 3 , \quad x \le 2$

$= 2x^2 + 3 , \quad x > 2$

Discuss continuity at $x = 2$

$f(2-) = 4x + 3 = 4(2) + 3 = 11$

$f(2+) = 2x^2 + 3 = 2(2)^2 + 3 = 11$

$f(2) = 4x + 3 = 4(2) + 3 = 11$

f is CONTINUOUS at $x = 2$

03. $f(x) = x^2 - x + 9 , \quad x \le 3$

$= 4x + 3 , \quad x > 3$

Discuss continuity at $x = 3$

$f(3-) = x^2-x+9 = 3^2-3+9 = 15$

$f(3+) = 4x + 3 = 4(3) + 3 = 15$

$f(3) = x^2-x+9 = 3^2-3+9 = 15$

f is CONTINUOUS at $x = 3$

04. $f(x) = x^3 - 2x + 1 \quad , \quad x \le 2$

$= 3x - 2 \quad , \quad x > 2$

Discuss continuity at x = 2

$f(2-) = x^3-2x+1 = 2^3-2(2)+1 = 5$

$f(2+) = 3x - 2 = 3(2)-2 = 4$

LHL $\neq$ RHL

f is DISCONTINUOUS at x = 2

05. $f(x) = 3+2x \quad , \ ^{-2}/_3 \le x < 0$

$= 3-2x \quad , \quad 0 \le x < {}^3/_2$

$= -3+2x \quad , \quad x \ge {}^3/_2$

Discuss continuity at x = 0 & x= $^3/_2$

at x = 0

$f(0-) = 3+2x = 3+2(0) = 3$

$f(0+) = 3-2x = 3-2(0) = 3$

$f(0) = 3-2x = 3-2(0) = 3$

f is CONTINUOUS at x = 0

at x = 3/2

$f({}^3/_2-) = 3-2x = 3-2({}^3/_2) = 0$

$f({}^3/_2+) = -3+2x = -3+2({}^3/_2) = 0$

$f({}^3/_2) = -3+2x = -3+2({}^3/_2) = 0$

f is CONTINUOUS at x = $^3/_2$

f is CONTINUOUS in $[^{-2}/_3, \infty]$

06. $f(x) = x+1 \quad , \quad x \le 1$

$= 3-ax^2 \quad , \quad x > 1$

Find 'a' if f is CONTINUOUS at x = 1

$f(1-) = x+1 = 1+1 = 2$

$f(1+) = 3-ax^2 = 3-a(1)^2 = 3-a$

Since f is CONTINUOUS at x = 1

$2 = 3-a \Rightarrow a = 1$

NOTE $|x| = x$ for $x \ge 0$

$= -x$ for $x < 0$

$|x-3| = x-3 \quad , \quad x \ge 3$

$= -(x-3) \quad , \quad x < 3$

07. $f(x) = \dfrac{|x|}{x} \quad , \quad x \neq 0$

$= 0 \quad , \quad x = 0$

Discuss continuity at x = 0

$f(0-) = \dfrac{-x}{x} = -1$

$f(0+) = \dfrac{+x}{x} = 1$

LHL $\neq$ RHL

f is DISCONTINUOUS at x = 0

08. $f(x) = \dfrac{3x+|x|}{7x-5|x|} \quad , \ x \neq 0$

Does $\lim\limits_{x\to 0} f(x)$ exists ?

$f(0-) = \dfrac{3x-x}{7x-5(-x)} = \dfrac{2x}{12x} = \dfrac{1}{6}$

$f(0+) = \dfrac{3x+x}{7x-5x} = \dfrac{4x}{2x} = 2$

LHL $\neq$ RHL

$\lim\limits_{x\to 0} f(x)$ DOES NOT EXIST

09. $f(x) = \dfrac{3x}{|x|+2x}$, $x \neq 0$

$= 0$, $x = 0$

Discuss continuity at $x = 0$

$f(0-) = \dfrac{3(-x)}{-x+2x} = \dfrac{-3x}{x} = -3$

$f(0+) = \dfrac{3x}{x+2x} = \dfrac{3x}{3x} = 1$

LHL $\neq$ RHL

f is DISCONTINUOUS at $x = 0$

10. $f(x) = \dfrac{|x-3|}{x-3}$, $x \neq 3$

$= 1$, $x = 3$

Discuss continuity at $x = 3$

$f(3-) = \dfrac{-(x-3)}{x-3} = -1$

$f(3+) = \dfrac{x-3}{x-3} = 1$

LHL $\neq$ RHL

f is DISCONTINUOUS at $x = 3$

15 - DIFFERENTIAL CALCULUS

DERIVATIVES OF STANDARD FUNCTIONS

$\frac{d}{dx}k = 0$

$\frac{d}{dx}x = 1$

$\frac{d}{dx}x^2 = 2x$

$\frac{d}{dx}x^3 = 3x^2$

$\frac{d}{dx}x^4 = 4x^3$

$\frac{d}{dx}x^5 = 5x^4$

$\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\log x = \frac{1}{x}$

$\frac{d}{dx}e^x = e^x$

$\frac{d}{dx}a^x = a^x.\log a$

$\frac{d}{dx}3^x = 3^x.\log 3$

$\frac{d}{dx}4^x = 4^x.\log 4$

$\frac{d}{dx}x^x = x^x.(1+\log x)$

$\frac{d}{dx}\log\left(x+\sqrt{x^2 \pm a^2}\right) = \frac{1}{\sqrt{x^2 \pm a^2}}$

01. JUNE 2012

$y = e^{a\log x} + e^{x\log k}$. Find dy/dx.

a. $x^a + a^x$

b. $ax^{a-1} + a^x \log a$

c. $ax^{a-1} + xa^{x-1}$

d. none of these

$y = e^{\log_e x^k} + e^{\log_e k^x}$

$y = x^k + k^x$

$y' = kx^{k-1} + k^x.\log k$ option b.

02. DEC 2011

$\frac{d}{dx}2^{\log_2 x}$

a. 1 b. 0 c. 1/2 d. $2^x \log 2$

RECALL $a^{\log_a x} = x$

$\frac{d}{dx}x = 1$

option a.

03. DEC 2011

$f(x) = {}^xC_3$, then $f'(1) =$

a. 1/6 b. −1/6 c. 1 d. 0

$f(x) = {}^xC_3$

$= \frac{x(x-1)(x-2)}{3!}$

$= \frac{x(x^2-3x+2)}{6}$

$= \frac{x^3-3x^2+2x}{6}$

$f'(x) = \frac{3x^2-6x+2}{6}$

$f'(1) = \frac{3-6+2}{6} = \frac{-1}{6}$

option b.

04. DEC 2011 , DEC 2020

$y = x(x-1)(x-2)$, then dy/dx =

a. $-6x$ b. $3x^2-6x+2$ c. $6x+4$

d. $3x^2-6x$

$y = x(x-1)(x-2)$

$= x^3-3x^2+2x$

$y' = 3x^2-6x+2$ option b.

05. JUNE 2014 , DEC 2017

$y = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots+\frac{x^n}{n!}+\ldots.$

then $\frac{dy}{dx} - y = ?$

a. 1 b. 0 c. -1 d. none

$y = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}\ldots\ldots. + \infty$

$\frac{dy}{dx} = 0+1+\frac{2x}{2!}+\frac{3x^2}{3!}+\frac{4x^3}{4!}+\ldots..+\infty$

$\frac{dy}{dx} = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots.+\infty$

$\frac{dy}{dx} = y$

$\frac{dy}{dx} - y = 0$

option b.

Q2 SUMS ON PRODUCT & QUOTIENT RULE

$y = uv$, then $y' = u\,d/dx\,v + v\,d/dx\,u$

$y = u/v$, $\frac{dy}{dx} = \frac{v\,d/dx\,u - u\,d/dx\,v}{v^2}$

01. DEC 2021

$y = \frac{x^4}{e^x}$. then dy/dx =

a. $x^3(4-x)/(e^x)^2$ b. $x^3(4-x)/e^x$

c. $x^2(4-x)/e^x$ d. $x^3(4x-1)/e^x$

$\frac{dy}{dx} = \frac{e^x.d/dx\,x^4 - x^4.d/dx\,e^x}{(e^x)^2}$

$= \frac{e^x.4x^3-x^4.e^x}{(e^x)^2}$

$= \frac{e^x(4x^3-x^4)}{(e^x)^2}$

$= \frac{x^3(4-x)}{e^x}$ option b.

02. JUNE 2022

which of the following is the differentiation of $e^t.\log et$ wrt t

a. $e^t(t.\log t)$ b. $e^t(1+t\log et)/t$

c. e^t/t d. $e^t(1-\log et)$

$e^t\,d/dt \log t + \log t\,d/dt\,e^t$

$e^t.1/t + \log t\,.\,e^t$

$et.(1/t + \log t)$

$e^t\,(1+t\log t)/t$ option b

Q3 COMPOSITE FUNCTIONS

$y = f(u)$, $u = f(x)$, then

$\frac{dy}{dx} = \frac{dy/du}{du/dx}$

01. JULY 21

$f(x) = 3.e^{x^4}$

$f'(x) - 4x^3f(x) + \frac{1}{3}f(0) - f'(0) = ?$

a. 0 b. e c. 1 d. -1

$f'(x) - 4x^3f(x) + \frac{1}{3}f(0) - f'(0) = ?$

$3e^{x^4}4x^3 - 4x^3 3e^{x^4} + \frac{1}{3}.3e^0 - 3e^0.4.0^3$

$12\,x^3e^{x^4} - 12x^3e^{x^4} + 1.$

1 option c

02. DEC 2019

if $f(x) = a(x^2+x+1)^2$, $f'(-1) = -6$, then find a

a. 1 b. 2 c. 3 d. 4

$f'(x) = a.2(x^2+x+1)\ {}^{d}/_{dx}(x^2+x+1)$

$= 2a(x^2+x+1)(2x+1)$

$f'(-1) = -6$

$2a(1-1+1)(-2+1) = -6$

$2a(1)(-1) = -6$

$-2a = -6$

$a = 3$ option c

Q4 LOGARITHMIC DIFFERENTIATION

01. DEC 2017

$y = \log x^x$. Find ${}^{dy}/_{dx}$

a. logex b. log(e/x) c. log(x/e)

d. 1

$y = x.\log x$

$y' = x.{}^{d}/_{dx}\log x + \log x\ {}^{d}/_{dx}\ x$

$= x.{}^{1}/_{x} + \log x$

$= 1+\log x$

$= \log e + \log x$

$= \log ex$ option a

02. DEC 2012

$y = \log\left(\frac{5-4x^2}{3+5x^2}\right)$, find $\frac{dy}{dx}$

a. $\frac{8}{4x-5} - \frac{10}{3+5x}$

b. $(4x^2-5)-(3+5x^2)$

c. $\frac{8x}{4x^2-5} - \frac{10x}{3+5x^2}$

d. $8x-10$

$y = \log(5-4x^2) - \log(3+5x^2)$

$\frac{dy}{dx} = \frac{1}{5-4x^2}\frac{d}{dx}(5-4x^2) - \frac{1}{3+5x^2}\frac{d}{dx}(3+5x^2)$

$= \frac{-8x}{5-4x^2} - \frac{10x}{3+5x^2}$

HIDE '–' IN THE DENOMINATOR

$= \frac{8x}{4x^2-5} - \frac{10x}{3+5x^2}$ option a

03. DEC 2019

$y = x^x$. Find $\frac{dy}{dx}$

DEC 2022

$y = x^x$. Find $\frac{dy}{dx}$ at $x = 1$

a. 0 b. 1 c. –1 d. 2

$\frac{dy}{dx} = x^x(1+\log x) = x^x(\log e+\log x)$

$= x^x(\log ex)$

$\left.\frac{dy}{dx}\right|_{x=1} = 1^1(\log e) = 1$ option b

04. DEC 2011

$y = x^x$, $\frac{d^2y}{dx^2} = ?$

a. $\frac{dy}{dx}(1+\log x) + y\frac{d}{dx}(1+\log x)$

b. $\frac{dy}{dx}(1+\log x) + \frac{d}{dx}(1+\log x)$

c. $\frac{dy}{dx}(1+\log x) - y\frac{d}{dx}(1+\log x)$

$y = x^x$

$\frac{dy}{dx} = x^x.(1+\log x)$ YOU HAVE MEMORISED THIS

$\frac{dy}{dx} = y.(1+\log x)$, DIFF. ONCE AGAIN WRT x

$\frac{d^2y}{dx^2} = y\frac{d}{dx}(1+\log x) + (1+\log x).\frac{dy}{dx}$

option a

05. DEC 2016

$f(x) = \log_e\left(\frac{x-1}{x+1}\right)$, $f'(x) = 1$, $x=?$

a. 1 b. 0 c. $\pm\sqrt{3}$ d. $\pm\sqrt{2}$

$f(x) = \log(x-1) - \log(x+1)$

$f'(x) = \frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$

$f'(x) = 1$ ..... GIVEN

$\frac{2}{x^2-1} = 1$

$x^2-1 = 2 \quad x = \pm\sqrt{3}$ option C

06. JUNE 2016

$y = \sqrt{\frac{1-x}{1+x}}$, then $\frac{dy}{dx} =$

a. $\frac{y}{x^2-1}$ b. $\frac{y}{1-x^2}$ c. $\frac{y}{1+x^2}$ d. $\frac{y}{y^2-1}$

$\log y = \frac{1}{2}\log(1-x) - \log(1+x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1-x}(-1) - \frac{1}{1+x}\right]$

$\frac{dy}{dx} = -\frac{y}{2}\left[\frac{1}{1-x} + \frac{1}{1+x}\right]$ option b.

$= -\frac{y}{2}\frac{2}{(1-x)(1+x)} = -\frac{y}{x^2-1}$

07. DEC 2016

$y = \log_e\left(\sqrt{x-1} + \sqrt{x+1}\right)$. Find dy/dx

a. $\frac{1}{2\sqrt{x^2-1}}$ b. $\frac{1}{2\sqrt{x^2+1}}$ c. $\frac{1}{\sqrt{x-1}+\sqrt{x+1}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{\ }+\sqrt{\ }}\frac{d}{dx}\left(\sqrt{\ }+\sqrt{\ }\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{\ }+\sqrt{\ }}\left(\frac{1}{2}\frac{1}{\sqrt{\ }} + \frac{1}{2}\frac{1}{\sqrt{\ }}\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{\ }+\sqrt{\ }}\frac{1}{2}\left(\frac{1}{\sqrt{\ }} + \frac{1}{\sqrt{\ }}\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{\ }+\sqrt{\ }}\frac{1}{2}\left(\frac{\sqrt{\ }+\sqrt{\ }}{\sqrt{x-1}.\sqrt{x+1}}\right)$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x^2-1}}$ option a.

Q5 IMPLICIT FUNCTIONS f(x,y)

01. DEC 2022

$x^5+y^5-5xy = 0$, find $\frac{dy}{dx}$

a. $\frac{y+x^4}{x+y}$ b. $\frac{y-x^4}{y^4-x}$ c. $\frac{x-y^4}{x-y}$ d. $\frac{x+y^4}{x^4+y}$

$x^5+y^5-5xy = 0$

$5x^4 + 5y^4\frac{dy}{dx} - 5\left(x\frac{dy}{dx}+y\right) = 0$

$(y^4-x)\frac{dy}{dx} = -(x^4-y)$ option b.

$\frac{dy}{dx} = -\frac{5x^4-5y}{5y^4-5x} = -\frac{x^4-y}{y^4-x} = \frac{y-x^4}{y^4-x}$

02. DEC 2023

$x^3+y^3-3axy = 0$. Find dy/dx

a) $\frac{ay-x^2}{y^2-ax}$ b. $\frac{x^2-ay}{y^2-ax}$ c. $\frac{ay-x^2}{ax-y^2}$ d. $\frac{x^2-ay}{ax-y^2}$

$x^3+y^3-3axy = 0$

$3x^2+3y^2\frac{dy}{dx} - 3a\left(x\frac{dy}{dx} + y.1\right) = 0$

$(y^2-ax)\frac{dy}{dx} + (x^2-ay) = 0$

$\frac{dy}{dx} = -\frac{x^2-ay}{y^2-ax} = \frac{ay-x^2}{y^2-ax}$ option a

03. JUNE 2015

$e^{xy} - 4xy = 0$. find dy/dx

a. y/x b. –y/x c. x/y d. –x/y

$e^{xy}\left(x\frac{dy}{dx}+y\right) - 4\left(x\frac{dy}{dx}+y\right) = 0$

$(xe^{xy}-4x)\frac{dy}{dx} + (ye^{xy}-4y) = 0$

$\frac{dy}{dx} = \frac{y(4-e^{xy})}{x(e^{xy}-4)}$ option b

$\frac{dy}{dx} = -\frac{y(e^{xy}-4)}{x(e^{xy}-4)} = -\frac{y}{x}$

04. JUNE 2019

$2^x - 2^y = 2^{x-y}$, then dy/dx at x = y = 2

a. 1 b. 2 c. 4 d. 5

$2^x \log 2 - 2^y \log 2 \frac{dy}{dx} = 2^{x-y}\log 2 \frac{d}{dx}(x-y)$

$2^x - 2^y\frac{dy}{dx} = 2^{x-y}\left(1-\frac{dy}{dx}\right)$ Put x = y = 2

$2^2 - 2^2\frac{dy}{dx} = 2^{2-2}\left(1-\frac{dy}{dx}\right)$

$4 - 4\frac{dy}{dx} = 1-\frac{dy}{dx}$

$3 = 3\frac{dy}{dx}$ $\frac{dy}{dx} = 1$ option a

05. JUNE 2015

$x^p.y^q = (x+y)^{p+q}$, then dy/dx

a. y/x b. –y/x c. p/q d. –p/q

option a , check the note towards the end of this solution .

$x^p.y^q = (x + y)^{p+q}$

$p.\log x + q.\log y = 12.\log(x + y)$

$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\frac{d}{dx}(x + y)$

$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\left(\frac{q}{y} - \frac{p+q}{x+y}\right)\frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$

$\frac{qx+qy-py-qy}{y(x+y)}\frac{dy}{dx} = \frac{px+qx-px-py}{x(x+y)}$

$\frac{qx - py}{y(x + y)}\frac{dy}{dx} = \frac{qx - py}{x(x + y)}$

$\frac{dy}{dx} = \frac{y}{x}$ option a

MEMORISE

$(x+y)^{m+n} = x^m y^n$ then $\frac{dy}{dx} = \frac{y}{x}$

06. JUNE 2013

$y = \log_y x$. find dy/dx

a. $\frac{1}{x.\log y}$ b. $\frac{1}{x(1+\log y)}$ c. $\frac{1}{1+x\log y}$

$y = \frac{\log x}{\log y}$

$y.\log y = \log x$

$y.\frac{1}{y}\frac{dy}{dx} + \log y\frac{dy}{dx} = \frac{1}{x}$

$(1+\log y)\frac{dy}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{x(1+\log y)}$ option c

Q6 PARAMETRIC FUNCTIONS

x = f(t) , y = f(t) , then

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

01. DEC 2017

$y = 2at$, $x = at^2$, $\frac{dy}{dx}$ at t = 2

a. 2 b. 4 c. 1/2 d. 1/4

$\frac{dy}{dx}_{t=2} = \frac{dy/dt}{dx/dt} = \frac{2a}{a.2t} = \frac{1}{t} = \frac{1}{2}$

02. JUNE 2012

$x = ct$, $y = \frac{c}{t}$, Find $\frac{dy}{dx}$

a. 1/t b. $t.e^t$ c. $-1/t^2$ d. none

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-c/t^2}{c.1} = \frac{-1}{t^2}$

option c

03. NOV 2018

$x = at^3$, $t = a/t^3$, then dy/dx

a. $\frac{-3a}{t^6}$ b. $\frac{-1}{t^6}$ c. $\frac{1}{3at^2}$ d. none

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a(-3t^{-4})}{a3t^2} = -\frac{1}{t^6}$

option b

04. DEC 2023

$x = 5t^2+3$, $y = 2t^2+1$, dy/dx =

a) 2 b) 1 c) 2t d) $^2/_5$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t}{10t} = \frac{2}{5}$ option d

05. JUNE 2013

$x = \log t$, $y = e^t$, Find $^{dy}/_{dx}$

a. 1/t b. t.et c. $^{-1}/t^2$ d. none

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^t}{1/t} = t.e^t$ option d

Q7 MAKE IMPLICIT TO EXPLICIT

06. JUNE 2023 , NOV 2018

$xy = 1$, $y^2 + \frac{dy}{dx} = ?$

a. 1 b. 0 c. −1 d. ½

$y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -1x^{-2} = -\frac{1}{x^2}$

$y^2 + \frac{dy}{dx} = \frac{1}{x^2} - \frac{1}{x^2} = 0$ option b.

07. DEC 2017

$x^y = e^{x-y}$. Find $^{dy}/_{dx}$

a. $\frac{2\log x}{(1+\log x)^2}$ b. $\frac{\log x}{1+\log x}$ c. $\frac{\log x}{(1+\log x)^2}$

$x^y = e^{x-y}$, taking log ,

$y.\log x = (x - y) . \log e$

$y.\log x = x - y$

$y.\log x + y = x$

$y = \frac{x}{1+\log x}$ APPLY QUOTIENT RULE

$\frac{dy}{dx} = \frac{(1+\log x).1 - x.\frac{1}{x}}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$ option b

08. JUN 2023

$y = \frac{x}{x+5}$ Find $\frac{dx}{dy}$

a. $\frac{5}{(1-y)^2}$ b. $\frac{5}{(1+y)^2}$ c. $\frac{3}{(1-y)^2}$ d.none

NOTE since answers given in the option are in the form f'(y) , we need to make the function x = f(y)

$xy+5y = x$

$xy - x = -5y$

$x(y-1) = -5y$

$x = \frac{5y}{1-y}$

$\frac{dx}{dy} = \frac{(1-y)^d/_{dy}5y - 5y\ ^d/_{dy}(1-y)}{(1-y)^2}$

$= \frac{(1-y).5 - 5y(0-1)}{(1-y)^2}$

$= \frac{5-5y+5y}{(1-y)^2}$

$= \frac{5}{(1-y)^2}$ option a.

Q8 APP - 1 MAXIMA / MINIMA

- ✓ At points of maxima & minima say x = a ,

  x = b , $f'(x) = 0$

- ✓ If $f''(x) \mid x = a < 0$, then f(x) is maximum at x = a

- ✓ If $f''(x) \mid x = b > 0$, then f(x) is minimum at x = b

  OR VICE VERSA

- ✓ NOTE – SAVE TIME

  on solving $f'(x) = 0$, we get only one point x = a (say) and if the Q asked was find where the f(x) is minimum or maximum , we say it's at x = a without doing $f''(x)$

01. DEC 2022

$y = 2x^3-15x^2+36x+10$

maxima & minima occurs respectively at

a. x=2,x=3 b. x=1,x=3

c. x=3,x=2 d. x=3,x=1

$f(x) = 2x^3-15x^2+36x+10$

$f'(x) = 6x^2-30x+36$

$f''(x) = 12x-30$

$f'(x) = 0$

$6x^2-30x+36 = 0$

$x^2-5x+6 = 0$

$(x-2)(x-3) = 0$, x = 2 , x = 3

$f''(x)\big|_{x=2} = 12(2)-30 = -6 < 0$

f is maximum at x=2

$f''(x)\big|_{x=3} = 12(3)-30 = 6 > 0$

f is minimum at x=3

option a

02. MAY 2018 the cost function for the production of x units of a commodity is given by $C(x) = 2x^3-15x^2+36x+15$.

The cost will be minimum when x

a. 3 b. 2 c. 1 d. 4

$C(x) = 2x^3-15x^2+36x+15$

$C'(x) = 6x^2-30x+36$

$C''(x) = 12x-30$

$C'(x) = 0$ $6x^2-30x+36 = 0$

$x^2-5x+6 = 0$

$(x-2)(x-3) = 0$

x = 2 , 3

$C''(x)\big|_{x=2} = 12(2)-30 < 0$, IGNORE

$C''(x)\big|_{x=3} = 12(3)-30 > 0$

cost is minimum at x = 3 option a

03. JUNE 2019

the total cost function of a firm is

$C(x) = 150x - 5x^2 + \frac{x^3}{6}$

Find output ,x at which marginal cost is equal to average cost

a. 5 b. 10 c. 15 d. 20

$C_A = C_M$

$C/x = dC/dx$

$150-5x+\frac{x^2}{6} = 150-10x+\frac{x^2}{2}$

$-5x+10x = \frac{x^2}{2} - \frac{x^2}{6}$

$5x = \frac{3x^2}{6} - \frac{x^2}{6}$

$5x = \frac{2x^2}{6}$

15 = x option c.

04. DEC 2021 the cost function for the production of x units of a commodity is given by $C(x) = 500-20x^2+x^3/3$

The marginal cost is minimum when x

a. 5 b. 10 c. 20 d. 50

$C = 500-20x^2+x^3/3$

$C_M = {}^{dC}/_{dx}$

$= -40x + \frac{3x^2}{3} = -40x+x^2$

$C_M' = -40+2x = 0$

$x = 20$ option C

05. JUNE 2021

the cost function of production is given

$C(x) = \frac{x^3}{2} - 15x^2+36x$

Where x denotes the number of items produced . The level of output for which marginal cost and average cost are minimum

a. 10,15 b. 10,12 c. 12,15 d. 15,10

$C_M = {}^{dC}/_{dx}$

$= \frac{3x^2-30x+36}{2}$

$C_M' = \frac{6x-30}{2} = 0 \quad x = 10$

$C_A = {}^{C}/_{x}$

$= \frac{x^2-15x+36}{2}$

$C_A' = \frac{2x-15}{2} = 0 \quad x = 15$

option a

06. JULY 2021

the cost function

$C(x) = 125 + 500x - x^2 + x^3/3$, and the demand function for the items is given by $p(x) = 1500-x$, then the marginal profit when 18 items are sold is

a. 751 b. 571 c. 676 d. 875

REVENUE

$R = p.x = 1500x-x^2$

PROFIT

$P = R-C$

$= 1500x-x^2 -\left[125+500x-x^2+\frac{x^3}{3}\right]$

$= 1500x-x^2 - 125-500x+x^2-\frac{x^3}{3}$

$= 1000x -125-\frac{x^3}{3}$

Marginal Profit $|x=18$

$= \frac{dP}{dx}\Big|_{x = 18}$

$= 1000-0-\frac{3x^2}{3} = 1000-x^2 = 1000-18^2$

$= 676$ option C

APP 2 - SLOPE OF TANGENT / GRADIENT OF CURVE)

Slope of tangent (Gradient of curve) at the point $P(x_1,y_1)$ on the curve $y = f(x)$

$= \frac{dy}{dx}\Big|_{P(x_1,y_1)}$

Eq. of tangent : $y- y_1 = m(x- x_1)$

01. find the equation of tangent to curve $y = x^3 -2x+3$ at the point (2,7)

a. y=2x–13 b. y = 10x c. y=10x–13

d. y = 10

$\frac{dy}{dx} = 3x^2-2$

Slope of tangent at (2,7)

$= \frac{dy}{dx}\Big|_{(2,7)} = 3(2^2)-2 = 10$

Equation of tangent at (2,7)

$y - 7 = 10(x-2)$

$y = 10x-13$

option C

02. JUNE 2017

the equation if the curve which passes through the point (1,2) and has slope 3x–4 at any point (x,y) is

a. $2y = 3x^2-8x+9$ b. $y = 6x^2-8x+9$

c. $y = x^2-8x+9$ d. $2y = 3x^2-8x+c$

Slope of tangent to the curve at (x,y)

$= \left.\frac{dy}{dx}\right|_{(x,y)} = 3x-4$ … given

Trying options

a. $2y = 3x^2-8x+9$

$2\frac{dy}{dx} = 6x-8$ $\frac{dy}{dx} = 3x-4$ ✓

lets check (1,2) satisfies the equation of the curve

$2(2) = 3-8+9$ ✓ option a

16 - INTEGRAL CALCULUS

LIST OF FORMULAE

$\int 1\ dx = x + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\int \frac{1}{x} dx = \log x + C$

$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$

$\int e^x dx = e^x + C$

$\int a^x dx = \frac{a^x}{\log a} + C$

$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left|\frac{x-a}{x+a}\right| + c$

$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + c$

$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log \left|x + \sqrt{x^2+a^2}\right| + C$

$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left|x + \sqrt{x^2-a^2}\right| + C$

$\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \log \left|x + \sqrt{x^2+a^2}\right|$

$\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \log \left|x + \sqrt{x^2-a^2}\right|$

$\int \frac{f'(x)}{f(x)} dx = \log f(x) + k$

$\int e^x [f(x)+f'(x)]dx = e^x [f(x) + k$

$\int uv\ dx = u\int v dx - \int \left[\frac{d}{dx}u \int v dx\right] dx$

Q1

01. JUNE 2023 – MTP I

$\int_0^2 3x^2 dx$

a. 7 b. –8 c. 8 d. –7

$\int_0^2 3x^2 dx = \left(3x^3/3\right)_0^2 = \left(x^3\right)_0^2 = 8$

option C

02. NOV 2019

$\int_{-1}^{1} (2x^2 - x^3)\ dx$

a. 14 b. 104 c. $\frac{2x^3}{3} - \frac{x^4}{4}$ d. 4/3

$= \left(\frac{2x^3}{3} - \frac{x^4}{4}\right)_{-1}^{1}$

$= \left(\frac{2}{3} - \frac{1}{4}\right) - \left(\frac{-2}{3} - \frac{1}{4}\right)$

$= \frac{5}{12} - \left(\frac{-11}{12}\right) = \frac{16}{12} = \frac{4}{3}$

option d

03. JUNE 2023 – MTP I

$\int_0^1 3x^2 + 2x + k\ dx = 0$

a. 0 b. –1 c. –2 d. 1

$\left(\frac{3x^3}{3} + \frac{2x^2}{2} + kx\right)_0^1 = 0$

$\left(x^3 + x^2 + kx\right)_0^1 = 0$

$(1+1+k) - 0 = 0$

$k = -2$

option C

04. DEC 2022

Find area under curve $f(x) = x^2+5x+2$ with limits 0 to 1

a. 3.833 b. 4.388 c. 4.833 d. none

$$= \int_0^1 (x^2+5x+2)\,dx$$

$$= \left(\frac{x^3}{3} + \frac{5x^2}{2} + 2x\right)_0^1$$

$$= \frac{1}{3} + \frac{5}{2} + 2$$

$$= 0.333+2.5+2 = 4.833$$

option a

05. NOV 2018

$$\int_{-1}^{3} (1+3x-x^3)\,dx$$

a. −3 b. −4 c. 3 d. 4

$$= \left(x + \frac{3x^2}{2} - \frac{x^4}{4}\right)_{-1}^{3}$$

$$= \left(3 + \frac{27}{2} - \frac{81}{4}\right) - \left(-1+\frac{3}{2}-\frac{1}{4}\right)$$

$$= 3+1+\frac{27}{2} - \frac{3}{2} - \frac{81}{4} + \frac{1}{4}$$

$$= 4+\frac{24}{2} - \frac{80}{4}$$

$$= 4 + 12 - 20 = -4$$

option b

06. DEC 2022 – MTP II

$$\int_0^1 (e^x+e^{-x})dx$$

a. $e-e^{-1}$ b. $e^{-1}-e$ c. $e+e^{-1}$

d. none of these

$$\left(e^x+\frac{e^{-x}}{-1}\right)_0^1 = [e^x-e^{-x}]_0^1$$

$$= [e^1-e^{-1}] - [e^0-e^0]$$

$$= e-e^{-1}$$

option a

07. DEC 2023

$$\int_1^3 (e^x - 1/x^2)\,dx$$

a. $e^3 - e - \frac{2}{3}$ b. $e^2 - e - \frac{1}{3}$

c. $e^3 + e + \frac{2}{3}$ d. $e^3 - e + \frac{2}{3}$

$$\int (e^x - x^{-2})\,dx$$

$$ex - \frac{x^{-1}}{-1}$$

$$\left(e^x + \frac{1}{x}\right)_1^3$$

$$\left(e^3 + \frac{1}{3}\right) - \left(e^1 + 1\right)$$

$$e^3 - e - \frac{2}{3}$$

option a

08. JUNE 2019

$$\int \log_e a^x\,dx$$

a. $\log_e a^{\frac{x^2}{2}} + k$ b. $\log_e a^{\frac{x}{2}} + k$

c. $x.\log a^x - x + k$ d. $x.\log a^x + k$

$$\int \log_e a^x\,dx$$

$$\int x\log_e a\,dx$$

$$\log_e a \quad \frac{x^2}{2}$$

$$\frac{x^2}{2}\log_e a$$

$$\log_e a^{\frac{x^2}{2}}$$

option a

09. DEC 2012 , JUNE 2013

$\int 2^{3x}.3^{2x}.5^{x}.dx$

a. $\frac{2^{3x}.3^{2x}.5^{x}}{\log 720} + c$ b. $\frac{2^{3x}.3^{2x}.5^{x}}{\log 360} + c$

c. $\frac{2^{3x}.3^{2x}.5^{x}}{\log 180} + c$ d. $\frac{2^{3x}.3^{2x}.5^{x}}{\log 90} + c$

$\int 2^{3x}.3^{2x}.5^{x}.dx$

$= \int 8^{x}.9^{x}.5^{x} \ dx$

$= \int (8.9.5)^{x}.dx$

$= \int 360^{x}.dx$

$= 360^{x}/\log 360 + c$

$= \frac{2^{3x}.3^{2x}.5^{x}}{\log 360} + c$ option b

if $\int f(x)dx = g(x) + K$, then

$\int f(ax+b)dx = \frac{g(ax+b)}{a} + k$

10. DEC 2022 , JUNE 2023 – MTP I

$\int (2x-3)^{5}dx$

a. $\frac{(2x-3)^{6}}{6} + c$ b $\frac{(2x-3)^{6}}{6} + c$

c $\frac{(2x-3)^{6}}{12} + c$ d $\frac{(2x-3)^{6}}{5} + c$

$\int (2x-3)^{5}dx = \frac{(2x-3)^{6}}{2.6} + c = \frac{(2x-3)^{6}}{12} + c$

coeff of x

option C

11. NOV 2019

$\int (4x+5)^{6}dx$

a. $\frac{(4x+5)^{7}}{7} + c$ b $\frac{(4x+5)^{7}}{28} + c$

c $\frac{(4x+5)^{7}}{4} + c$ d none of these

$\int (4x+5)^{6}dx = \frac{(4x+5)^{7}}{4.7} + c = \frac{(4x+5)^{7}}{28} + c$

coeff of x option b

12. DEC 2023 – MTP II

$\int_{1}^{4} (2x+5) \ dx$

a. 10 b. 15 c. 30 d. 8

$\int_{1}^{4} (2x+5)dx = \left[\frac{(2x+5)^{2}}{2.2}\right]_{1}^{4}$

$= \frac{(8+5)^{2} - (2+5)^{2}}{4}$

$= \frac{169 - 49}{4}$

$= 30$ option C

13. JUNE 2010

$$\int \frac{dx}{\sqrt{3x+4}-\sqrt{3x+1}}$$

a. $^2/_{27}\left[(3x+4)^{3/2} - (3x+1)^{3/2}\right] + C$

b. $^2/_{27}\left[(3x+4)^{3/2} + (3x+1)^{3/2}\right] + C$

c. $^2/_3\left[(3x+4)^{3/2} - (3x+1)^{3/2}\right] + C$

d. None of these

$$= \int \frac{1}{\sqrt{3x+4}-\sqrt{3x+1}} \cdot \frac{\sqrt{3x+4}+\sqrt{3x+1}}{\sqrt{3x+4}+\sqrt{3x+1}}\, dx$$

$$= \int \frac{1}{3x+4-3x+1} \cdot \frac{\sqrt{3x+4}+\sqrt{3x+1}}{1}\, dx$$

$$= \int \frac{1}{3} \cdot \frac{\sqrt{3x+4}+\sqrt{3x+1}}{1}\, dx$$

$$= {}^1/_3 \int \left[(3x+4)^{1/2} + (3x+1)^{1/2}\right] dx$$

$$= \frac{1}{3}\left[\frac{(3x+4)^{3/2}}{3.\,{}^3/_2} + \frac{(3x+1)^{3/2}}{3.\,{}^3/_2}\right] + C$$

$$= \frac{2}{27}\left[(3x+4)^{3/2} + (3x+1)^{3/2}\right] + C$$

option b

Q2 $\int \frac{f'(x)}{f(x)}\, dx = \log f(x) + k$

01. JULY 2021 – MTP I

$$\int \frac{2x+1}{x(x+1)}\, dx$$

a. $\log(x^2-x)+k$ b. $\log(x^2+x)+k$

c. $\log(x^2+1)+k$ c. none of these

$$\int \frac{2x+1}{x^2+x}\, dx = \int \frac{f'(x)}{f(x)}\, dx$$

$\log(x^2+x) + k$ option b

02. JUNE 2010

$$\int_1^2 \frac{x}{x^2+2}\, dx$$

a. $\log \sqrt{2}$ b. $\log \sqrt{3}$

c. $\log(1/\sqrt{2})$ d. none

$$\frac{1}{2}\int_1^2 \frac{2x}{x^2+2}\, dx$$

$$\int \frac{f'(x)}{f(x)}\, dx = \log f(x) + k$$

$$\frac{1}{2}\left[\log(x^2+2)\right]_1^2$$

$$\frac{1}{2}\left[\log(4+2) - \log(1+2)\right]$$

$$\frac{1}{2}\log({}^6/_3)$$

$\log \sqrt{2}$ option a

03. DEC 2022 , JUNE 2017

$$\int_2^4 \frac{x}{x^2+1}\, dx$$

a. $\frac{1}{2}\log\left(\frac{17}{5}\right)$ b. $2\log\left(\frac{17}{5}\right)$ c. $\frac{1}{2}\log\left(\frac{5}{17}\right)$

d. $2\log\left(\frac{5}{17}\right)$

$$\frac{1}{2}\int_2^4 \frac{2x}{x^2+1}\, dx$$

$$\int \frac{f'(x)}{f(x)}\, dx = \log f(x) + k$$

$$\frac{1}{2}\left[\log(x^2+1)\right]_2^4$$

$$\frac{1}{2}\left[\log(16+1) - \log(4+1)\right]$$

$$\frac{1}{2}\log({}^{17}/_5)$$

option a

Q3 SUMS ON PARTIAL FRACTION

01. JUNE 2023

$$\int \frac{1}{(x-1)(x-2)}\,dx$$

a. $\log[(x-2)/(x-1)] + c$

b. $\log[(x-2)(x-1)] + c$

c. $\log[(x-1)/(x-2)] + c$

d. $\log[(x-2)(x+1)] + c$

$$\frac{1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$1 = A(x-2) + B(x-1)$

Put $x = 1$, $B = 1$

Put $x = 1$, $A = -1$

$$\int \left(\frac{1}{x-2} - \frac{1}{x-1}\right) dx$$

$\log[(x-2)/(x-1)] + c$ option a

02. DEC 2010

$$\int \frac{6x+4}{(x-2)(x-3)}\,dx$$

a. $22\log(x-3) + 16\log(x-2) + C$

a. $11\log(x-3) - 8\log(x-2) + C$

b. $22\log(x-3) - 16\log(x-2) + C$

c. $16\log(x-3) - 22\log(x-2) + C$

$$\frac{6x+4}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$6x+4 = A(x-3) + B(x-2)$

$x = 3,\quad 18+4 = B(3-2) \quad B=22$

$x = 2,\quad 12+4 = A(2-3) \quad A=-16$

$$= \int \left(\frac{22}{x-3} - \frac{16}{x-2}\right) dx$$

$= 22\log(x-3) - 16\log(x-2) + C$

option c

03. $$\int \frac{1}{x(x^3+1)}\,dx$$

$$= \int \frac{x^3 + 1 - x^3}{x(x^3+1)}\,dx$$

$$= \int \left(\frac{1}{x} - \frac{x^2}{x^3+1}\right) dx$$

$$= \int \left(\frac{1}{x} - \frac{1}{3}\frac{3x^2}{x^3+1}\right) dx \qquad \frac{f'(x)}{f(x)}$$

$$= \log x - \frac{1}{3}\log(x^3+1) + k$$

$$= \frac{\log x^3 - \log(x^3+1)}{3} + k$$

$$= \frac{1}{3}\log\left(\frac{x^3}{x^3+1}\right) + k$$

04. SEPT 2024

$$\int \frac{1}{x(x^5+1)}\,dx$$

a. $\log\left(\frac{x^5}{x^5+1}\right) + k$ b. $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right) + k$

c. $\log\left(\frac{x^5+1}{x^5}\right) + k$ b. $\frac{1}{5}\log\left(\frac{x^5+1}{x^5}\right) + k$

$$= \int \frac{x^5 + 1 - x^5}{x(x^5+1)}\,dx$$

$$= \int \left(\frac{1}{x} - \frac{x^4}{x^5+1}\right) dx$$

$$= \int \left(\frac{1}{x} - \frac{1}{5}\frac{5x^4}{x^5+1}\right) dx$$

$$= \log x - \frac{1}{5}\log(x^5+1) + c$$

$$= \frac{\log x^5 - \log(x^5+1)}{5} + c$$

$$= \frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right) + c$$

option b

MEMORISE

$$\int \frac{1}{x(x^n+1)}\,dx = \frac{1}{n}\log\left[\frac{x^n}{x^n+1}\right] + k$$

OR

$$\int \frac{1}{x(1-x^n)}\,dx = \frac{1}{n}\log\left[\frac{x^n}{1-x^n}\right] + k$$

05. $\int (x-x^3)^{-1}\,dx$

a) $^1/_2 \log [x^2/(1-x^2)] + k$

b) $^1/_2 \log [x^2/(1-x)^2] + k$

c) $^1/_2 \log [x^2/(1+x)^2] + k$ d) none

$$\int \frac{1}{x-x^3}\,dx$$

$$\int \frac{1}{x(1-x^2)}\,dx$$

Using shortcut (CHECK THE ABOVE BOX)

$$\frac{1}{2}\log\left[\frac{x^2}{1-x^2}\right] + k$$ option a

06. $\int_0^2 \frac{x+2}{x+1}\,dx$

$$= \int_0^2 \frac{x+1+1}{x+1}\,dx$$

$$= \int_0^2 1 + \frac{1}{x+1}\,dx$$

$$= \left[x + \log(x+1)\right]_0^2$$

$$= [2+\log 3] - [0+\log 1]$$

$$= 2+\log 3 - 0$$

$$= 2+\log 3$$

07. DEC 2015, DEC 2020 , MAY 2018

$$\int_1^2 \frac{1-x}{1+x}\,dx$$

a $2\log(3/2) - 1$ b. $2\log 3 + 1$

c. $\log(3/2) - 1$ d. $2\log 2 - 1$

$$= \int_1^2 \frac{2-1-x}{1+x}\,dx$$

$$= \int_1^2 \frac{2-(1+x)}{1+x}\,dx$$

$$= \int_1^2 \left[\frac{2}{x+1} - 1\right]dx$$

$$= \left[2\log(x+1) - x\right]_1^2$$

$$= [2\log 3 - 2] - [2\log 2 - 1]$$

$$= 2\log(3/2) - 1$$ option a

Q4 INTEGRATION BY PARTS

$$\int uv\,dx = u\int v dx - \int\left[^d/_{dx}\,u.\int v dx\right]dx$$

01. SEPT 2024

$\int \log_e x\,dx$

a. $x.\log_e(ex) + k$ b. $x\log_e(x/e)+k$

c. $x.\log_e(e/x) + k$ d. $\log_e(x/e) + k$

$$= \int \log x.1\,dx$$

$$= \log x \int 1dx - \int\left[^d/_{dx}\,\log x.\int 1dx\right]dx$$

$$= x\log x - \int {}^1/_x .x\,dx$$

$$= x.\log x - x + k$$

$$= x(\log x - \log e) + k$$

$$= x.\log(x/e) + k$$ option b

02. JUNE 2022 ,DEC 2019

$\int_0^1 x.e^x.dx$ a. −1 b. 1 c. e^1 d. $1/e$

$\int x.e^x.dx$

$\left[x\int e^x dx - \int {}^d/_{dx}\, x \int e^x dx \; dx\right]_0^1$

$\left(x.e^x - e^x\right)_0^1$

$\left(1.e^1 - e^1\right) - \left(0.e^0 - e^0\right) = (0) - (0-1) = 1$

option b

03. $\int_1^2 x.\log x\, dx$

$= \int \log x.x dx$

$= \log x \int x dx - \int [{}^d/_{dx}\, \log x . \int x dx\,]\, dx$

$= \frac{x^2}{2}\log x - \int \frac{1}{x}.\frac{x^2}{2}\, dx$

$= \frac{x^2}{2}\log x - \frac{1}{2}\int x^2\, dx$

$= \left(\frac{x^2}{2}\log x - \frac{x^2}{4}\right)_1^2$

$= \left(\frac{4}{2}\log 2 - \frac{4}{4}\right) - \left(\frac{1}{2}\log 1 - \frac{1}{4}\right)$

$= 2\log 2 - 1 + \frac{1}{4} = 2\log 2 - \frac{3}{4}$

NOTE THE END ANS

$\frac{x^2}{2}\log x - \frac{x^2}{4}$ CAN BE PUT UP AS

$= \frac{x^2}{4}[2.\log x - 1]$

$= \frac{x^2}{4}[\log x^2 - \log e]$

$= \frac{x^2}{4}[\log(x^2/e)] + k$

04. $\int x^2.e^{3x} dx$

$= x^2.\int e^{3x} dx - \int [{}^d/_{dx}\, x^2.\int e^{3x} dx\,]dx$

$= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\int x.e^{3x}\, dx$

$= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left[x\int e^{3x}dx - \int [{}^d/_{dx}\, x \int e^{3x}dx]dx\right]$

$= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left(\frac{x.e^{3x}}{3} - \int \frac{e^{3x}}{3} dx\right)$

$= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left(\frac{x.e^{3x}}{3} - \frac{e^{3x}}{9} dx\right)$

$= \frac{x^2 e^{3x}}{2} - \frac{2x.e^{3x}}{9} + \frac{2e^{3x}}{27} + k$

SHORTCUT

d/dx

$x^2.\frac{e^{3x}}{2} - 2x.\frac{e^{3x}}{9} + 2.\frac{e^{3x}}{27} + k$

$\int$

05. JUNE 2022 – MTP II

$\int x^2.2^x dx$

a. $\frac{2^x.x^2}{\log 2} - x.\frac{2^{x+1}}{(\log 2)^2} + \frac{2^{x+1}}{(\log 2)^3} + c$

b. $\frac{2^x.x^3}{3} - \frac{x^2.2^{x+1}}{(\log 2)^2} + \frac{2^{x+1}}{(\log 3)^2} + c$

c. $\frac{2^x.x^2}{3} - \frac{x^3.2^x}{3} + \frac{2^{x+1}}{(\log 2)^3} + c$

SHORTCUT

d/dx

$x^2.\frac{2^x}{\log 2} - 2x.\frac{2^x}{(\log 2)^2} + 2.\frac{2^x}{(\log 2)^3} + k$

$\int$

$\frac{2^x.x^2}{\log 2} - \frac{x.2^{x+1}}{(\log 2)^2} + \frac{2^{x+1}}{(\log 2)^3} + c$

option a

Q5 $\int e^x [f(x)+f'(x)]dx = e^x [f(x) + k$

01. JUNE 2022 – MTP I

$\int e^x(x^2+2x)dx$

a. $x^2.e^x+c$ b. $x.e^x+c$ c. $-x.e^x+c$
d. $e^{-x}+c$

$\int e^x [f(x)+f'(x)]dx = e^x [f(x) + c$

$= e^x x^2 + c$

option a

02. DEC 2016

$\int_1^e e^x (x \log x + 1)x^{-1}.dx$

a. e^{-1} b. e^e c. e^e-1 d. none

$= \int e^x \frac{x \log x + 1}{x} dx$

$= \int e^x \left[\log x + \frac{1}{x}\right] dx$

$= \int e^x \left[f(x) + f'(x)\right]dx = e^x f(x) + c$

$= [e^x.\log x]_1^e$

$= [e^e.\log e] - [e^1.\log 1]$, $\log 1 = 0$

$= e^e$ option b

03. DEC 2016, JAN 2021

$\int_1^2 e^x (x^{-1} - x^{-2}).dx$

a. $e(e/2-1)$ b. $e(e-1)$ c. $e^2(e-1)$ d. none

$\int e^x \left[f(x) + f'(x)\right]dx = e^x f(x) + c$

$[e^x.x^{-1}]_1^2$

$[e^2.2^{-1}] - [e.1]$

$e[e/2-1]$ option a

04. $\int e^x \frac{x}{(x+1)^2} dx$ OR $\int e^x x(x+1)^{-2}dx$

$= \int e^x \frac{x+1-1}{(x+1)^2} dx$

$= \int e^x \left[(x+1)^{-1} - 1(x+1)^{-2}\right] dx$

$= \int e^x \left[f(x) + f'(x)\right] dx$

$= e^x.f(x) + k$

$= e^x(x+1)^{-1} + k$

SHORT CUT $\int e^x \frac{x}{(x+1)^2} dx = e^x (x+1)^{-1}$

05. $\int e^x \frac{x+2}{(x+3)^2} dx$

$= \int e^x \frac{x+3-1}{(x+3)^2} dx$

$= \int e^x \left[(x+3)^{-1} - 1(x+3)^{-2}\right] dx$

$= \int e^x \left[f(x) + f'(x)\right] dx$

$= e^x.f(x) + k$

$= e^x(x+3)^{-1} + k$

SHORT CUT $\int e^x \frac{x+2}{(x+3)^2} dx = e^x (x+3)^{-1}$

06. DEC 2011

$\int \frac{e^x}{(1+x)^3} dx - \int \frac{e^x}{2(1+x)^2} dx$

a. 0 b. $\frac{e^x}{2(1+x)^2} + k$ c. $\frac{-e^x}{2(1+x)^2} + k$
d. $\frac{e^x}{(1+x)^2} + k$

$= \int e^x \left[(1+x)^{-3} - \frac{(1+x)^{-2}}{2} \right] dx$

$= -\int e^x \left[\frac{(1+x)^{-2}}{2} - (1+x)^{-3} \right] dx$

$= -\frac{1}{2} \int e^x \left[(1+x)^{-2} - 2(1+x)^{-3} \right] dx$

$= \int e^x [f(x)+f'(x)]dx$

$= -\frac{1}{2} e^x (1+x)^{-2} + k$

option C

Q6 SUMS ON SUBSTITUTION

01. NOV 2018

$\int x(x^2+4)^5 \, dx$

a. $\frac{1}{12}(x^2+4)^6+c$ b. $(x^2+4)^6+c$

c. $\frac{1}{6}(x^2+4)^6+c$ d. none

$= {}^1/_2 \int (x^2+4)^5 \, 2x dx \quad t = x^2+4$

$dt = 2x.dx$

$= {}^1/_2 \int t^5 \, dt$

$= t^6/12 + k$

$= (x^2+4)^6/12 + k$ option a

02. JUNE 2015 , DEC 2017

$\int_0^1 e^{x^2} \, x.dx$

a. 1 b. e c. ${}^e/_2 - 1$ d. ${}^1/_2 (e-1)$

$= {}^1/_2 \int e^{x^2} \, 2x.dx$

$= {}^1/_2 \int e^t.dt$

$= {}^1/_2 \, e^t$

$= {}^1/_2 \left[e^{x^2} \right]_0^1 = {}^1/_2 (e-1)$ option d

03. JUNE 2016 , MAY 2018

$\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} .dx$

a. $\frac{3\sqrt{2}}{\log 3}$ b. 0 c. $\frac{2}{\log 3}[3^{\sqrt{2}}-1]$

d. none

$= 2 \int 3^{\sqrt{x}} \, {}^1/_{2\sqrt{x}} .dx$

$= 2 \int 3^t.dt$

$= {}^2/_{\log 3} \left[3^{\sqrt{x}} \right]_0^2$

$= {}^2/_{\log 3} [3^{\sqrt{2}} - 1]$ option C

04. DEC 2023

$\int_0^1 x\sqrt{x^2+4} \, dx$

a. $\frac{1}{3}(5\sqrt{5}-8)$ b. $\frac{1}{3}(5\sqrt{5}+8)$

c. $\frac{1}{3}(5\sqrt{5}-4)$ d. $\frac{1}{3}(5\sqrt{5}+4)$

$= \frac{1}{2} \int \sqrt{x^2+4} \, 2x dx$

$= \frac{1}{2} \int t^{1/2} \, dt$

$= \frac{1}{2} \left(\frac{t^{3/2}}{\frac{3}{2}} \right)$

$= \frac{1}{3} \left[(x^2+4)^{3/2} \right]_0^1$

$= \frac{1}{3} \left[5^{3/2} - 4^{3/2} \right]$

$= \frac{1}{3} \left[5\sqrt{5} - 2^{2\,3/2} \right] = \frac{1}{3} \left[5\sqrt{5} - 8 \right]$

option a

05. DEC 2010

$$\int \frac{(\log x^x)^2}{x^3}\,dx$$

a. $\frac{3}{2}(\log x)^3+c$ b. $\frac{1}{3}(\log x)^3+c$

c. $\frac{1}{6}(\log x)^3+c$ d. $\frac{3}{7}(\log x)^3+c$

$= \int \frac{(x.\log x)^2}{x^3}\,dx$

$= \int \frac{x^2.(\log x)^2}{x^3}\,dx$

$= \int \frac{(\log x)^2}{x}\,dx$ $\quad t = \log x$, $dt = {}^1/_x.dx$

$= \int t^2\,dt$

$= t^3/3 + k$

$= (\log x)^3/3 + k$ option b

06. DEC 2021

$$\int \frac{dx}{x(\log x)^2}$$

a. ${}^{-1}/_{\log x}+k$ b. ${}^{1}/_{\log x}+k$

c. $\log x$ d. x

$= \int \frac{1}{t^2}\,dt$ $\quad t = \log x$, $dt = {}^1/_x.dx$

$= \frac{-1}{t} + k$

$= \frac{-1}{\log x} + k$ option a

07. DEC 2010

$$\int \frac{dx}{x(1+\log x)^2}$$

a. $-\frac{1}{2(1+\log x)^2}+c$ b. $\frac{1}{1+\log x}+k$

c. $-\frac{1}{1+\log x}+c$ d. none of these

$1+\log x = t$ $\quad {}^{dt}/_{dx} = {}^1/_x$, $dx = {}^1/_x.dx$

$= \int \frac{1}{t^2}\,dt$

$= \frac{-1}{t} + k$

$= \frac{-1}{1+\log x} + k$ option c

08. JUNE 2013

$$\int_1^2 \frac{[\log(ex)]^n}{x}\,dx$$

a. $\frac{[\log 2e]^{n+1}}{n+1}$ b. $\frac{[\log 2e]^{n+1}}{n+1} - \frac{\log e}{n+1}$

c. $\frac{[\log 2e - \log 2]^{n+1}}{n+1}$ d. none

$= \int t^n\,dt$ $\quad t = \log ex$, $dt = \frac{1.e}{ex}\,dx = \frac{dx}{x}$

$= \frac{t^{n+1}}{n+1}$

$= \left(\frac{[\log(ex)]^{n+1}}{n+1}\right)_1^2$

$= \frac{[\log(2e)]^{n+1} - [\log e]^{n+1}}{n+1}$

$= \frac{[\log(2e)]^{n+1} - [1]^{n+1}}{n+1}$

$= \frac{[\log(2e)]^{n+1} - 1}{n+1}$

$= \frac{[\log(2e)]^{n+1} - \log e}{n+1}$

$= \frac{[\log(2e)]^{n+1}}{n+1} - \frac{\log e}{n+1}$

option b

09. JUNE 2016

$$\int \frac{x}{(x^2+1)(x^2+2)}\,dx$$

a. $\log\left[\frac{x^2+1}{x^2+2}\right] + c$

b. $\frac{1}{2}\log\left[\frac{x^2+1}{x^2+2}\right] + c$

c. $\frac{1}{2}\log\left[\frac{x^2+2}{x^2+1}\right] + c$ d. none

$$\frac{1}{2}\int \frac{2x}{(x^2+1)(x^2+2)}\,dx$$

$$\frac{1}{2}\int \frac{1}{(t+1)(t+2)}\,dt$$

$$\frac{1}{2}\int\left[\frac{1}{t+1} - \frac{1}{t+2}\right]dt$$

$$\frac{1}{2}\log\left(\frac{t+1}{t+2}\right) + k$$

$$\frac{1}{2}\log\left(\frac{x^2+1}{x^2+2}\right) + k$$

option b

Q7 $\int_a^b f(x).dx = \int_a^b f(a+b-x).dx$

01. JUNE 2014 , JUNE 2018

$$\int_0^5 \frac{x^2}{x^2 + (5-x)^2}\,dx$$

a. 1 b. 5/2 c. 3/2 d. 2

USING $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_0^5 \frac{(5-x)^2}{(5-x)^2 + x^2}\,dx \quad \ldots\ldots\ldots(2)$$

0 (1) + (2)

$$2I = \int_0^5 \frac{x^2 + (5-x)^2}{x^2 + (5-x)^2}\,dx$$

$2I = \left[x\right]_0^5 = 5$, $I = 5/2$ option b

SHORT CUT $b-a/2 = 5-0/2 = 5/2$

02. MAY 2018 , DEC 2020

$$\int_0^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2-x}}\,dx$$

a. 1 b. 2 c. –2 d. –1

SHORT CUT $b-a/2 = 2-0/2 = 1$

option a

03. JUNE 2019

$$\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}}\,dx$$

ans 1/2

a. 1 b. 1/2 c. 3/2 d. 5/2

SHORT CUT $b-a/2 = 3-2/2 = 1/2$

option b

04. JUNE 2018

$$\int_0^1 \log(1/x - 1)dx$$

a. 0 b. −1 c. 1 d. none

$$I = \int_0^1 \log(1-x/x)dx$$

USING $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_0^1 \log(x/1-x)dx$$

$$2I = \int_0^1 \log(1-x/x) + \log(x/1-x)dx$$

$$2I = \int_0^1 \log\left\{(1-x/x).(x/1-x)\right\}dx$$

$$2I = \int_0^1 \log 1\,dx$$

$I = 0$ option a

Q8 $\int_{-a}^{a} f(x).dx = 0$, if f(x) is ODD

What is ODD function ?

If $f(-x) = -f(x)$, then the f(x) is called ODD function

01. $\int_{-3}^{3} (x^3+x)dx$

$f(x) = x^3+x$

$f(-x) = (-x)^3+(-x)$

$= -x^3-x = -(x^3+x)$

$= -f(x)$, f(x) is ODD

Hence

$I = 0$

02. DEC 2020 , DEC 2010

$$\int_{-1}^{1} (e^x - e^{-x})dx$$

a. 0 b. 1 c. e^2 d. none

$f(x) = e^x - e^{-x}$

$f(-x) = e^{-x} - e^x$

$= -(e^x - e^{-x})$

$= -f(x)$, f(x) is ODD

Hence , 0 option a

03. DEC 2011

$$\int_{-1}^{1} \frac{|x|}{x}dx$$

a. −1 b. 0 c. 1 d. none

$f(x) = \frac{|x|}{x}$

$f(-x) = \frac{|-x|}{-x} = \frac{|x|}{-x} = -\frac{|x|}{x}$

$= -f(x)$, f(x) is ODD

Hence , 0 option b

Q9 $\int_a^b f(x).dx = \int_a^c f(x).dx + \int_c^b f(x).dx$

01. JULY 2021

$\int_{-2}^{2} f(x)\,dx$, $f(x) = 1+x$, $x \le 0$
$= 1-2x$, $x > 0$

a. 20 b. −2 c. −4 d. 0

I_1

$\int_{-2}^{0} f(x)\,dx$

$= \int_{-2}^{0} 1+x\,dx$

$= [x+x^2/2]_{-2}^{0}$

$= [0+0] - [-2+2] = 0$

I_2

$\int_{0}^{2} f(x)\,dx$

$= \int_{0}^{2} 1-2x\,dx$

$= [x-x^2]_{0}^{2}$

$= [2-4] - [0+0]$

$= -2$

$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$

$= -2$ option b

02. JULY 2021

$\int_0^2 |1-x|\,dx$,

a. 3/2 b. 1/2 c. 0 d. 1

$|1-x| = 1-x$, $x \le 1$
$= -(1-x)$, $x > 1$

$I_1 = \int_0^1 (1-x)\,dx$

$= [x-x^2/2]_0^1$

$= [1-1/2] - [0-0]$

$= 1/2$

$I_2 = \int_1^2 -(1-x)\,dx$

$= \int_1^2 x-1\,dx$

$= [x^2/2 - x]_1^2$

$= [2-2] - [1/2-1]$

$= -[-1/2] = 1/2$

$\int_0^2 |1-x|\,dx = \int_0^1 |1-x|\,dx + \int_0^2 |1-x|\,dx$

$= 1/2 + 1/2$

$= 1$ option d

Q10 APP OF INTEGRATION

01. JUNE 2022,

Determine f(x) if ,

$f'(x) = 12x^2-4x$ & $f(-3) = 17$

a. $f(x) = 4x^3-2x^2+143$

b. $f(x) = 6x^3-x^4+137$

c. $f(x) = 3x^4-x^3-137$

d. $f(x) = 4x^3-2x^2-143$

$f(x) = \int(12x^2-4x)\,dx$

$f(x) = \frac{12x^3}{3} - \frac{4x^2}{2} + k$

$f(x) = 4x^3-2x^2+k$

$f(-3) = 17$

$4(-3)^3-2(-3)^2+k = 17$

$-108-18+k = 17$

$-126+k = 17,$

$k = 143$

$f(x) = 4x^3-2x^2+143$ option a

02. DEC 2023 MTP I

equation of the curve which passes through (1,2) and has slope 3x–4 at point (x,y)

a $2y = 3x^2-8x+9$

b $y = 6x^2-8x+9$

c $y = x^2-8x+9$

d $2y = 3x^2-8x+c$

Slope of curve = 3x–4

$\frac{dy}{dx} = 3x-4$

$y = \int(3x-4)dx$

$y = \frac{3x^2-4x+K}{2}$ ….. equation of curve

curve passes through (1,2)

$2 = \frac{3(1)^2-4(1)+k}{2}$

$2 = \frac{3}{2} - 4 + k$

$6-\frac{3}{2} = k$

$k = \frac{9}{2}$, subs in the equation

$y = \frac{3x^2-4x+9}{2}$

$2y = 3x^2-8x+9$ option a

03. AUG 2018 – MTP

If MC = $10-0.01x+0.009x^2$ where x is the quantity of production and total fixed cost = 100 , then the total cost is

a. $100+10x-0.05x^2+0.0009x^3$

b. $100+10x-0.005x^2+0.003x^3$

c. $100+10x-0.05x^2+0.009x^3$

d. none of these

$MC = 10-0.01x+0.009x^2$

$\frac{dC}{dx} = 10-0.01x+0.009x^2$

$C = \int(10-0.01x+0.009x^2)dx$

$= 10x-\frac{0.01x^2}{2}+\frac{0.009x^3}{3} + k$

$C = 10x-0.005x^2+0.003x^3 +k$

Fixed cost , k = 100 given

$C = 10x-0.005x^2+0.003x^3 +100$

$= 100+10x-0.005x^2+0.003x^3$

option b

04. DEC 2021 – MTP II

Marginal cost and marginal revenue of a commodity is $C'(x) = 8+6x$ and $R'(x) = 30$. Fixed cost = 0 Find the total profit

a. $22x+3x^2$ b. $22x-3x^2$ c. $22x-x^2$

d. none of these

$C'(x) = 8+6x$

$= \int(8+6x)dx$

$= 8x+3x^2 + k$, k = fixed cost

$= 8x+3x^2 + 0$

$C(x) = 8x+3x^2$

$R'(x) = 30.$

$R(x) = \int 30dx$

$= 30x$

Profit $P(x) = R(x)-C(x)$

$= 30x - (8x+3x^2)$

$= 22x-3x^2$ option b