

Chapter 1-4 Time value of money

or mathematics of finance

Simple Interest

Compound Interest

Annuity Interest

or Simple interest

formula = $\frac{P R N}{100}$ where i = simple interest
 P = principal sum initial deposit
 R = rate percent p.a.

$A = P + I$ $I = R(i + N)$ = number of years
 $A = P(1 + RN)$ = future value

[Amount] = principal + interest

↑ future value

$$A = P + I$$

$$A = P + \frac{PRN}{100}$$

$$A = P + \left(\frac{RN}{100} \right)$$

ST = simple interest

P = principal

T = time in years

i = rate of interest in decimal form $R/100$

$$\frac{A}{P} = (1 + i)^T$$

Note :- IF a sum invested in simple interest become K Times.

Simple interest = $(K - 1) \times 100\%$

Rate of interest = $(K - 1) \times 100/N$

Interest = $R/N \times P$

$$R(i + 1)^N = A$$

Rate of interest = $R = R/N \times 100$

IF S.I on P_1, P_2, P_3 sum

$$P_1:P_2:P_3 = \frac{1}{R_1 N_1} : \frac{1}{R_2 N_2} : \frac{1}{R_3 N_3}$$

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Note: If S.I on certain sum is k times the principal and rate percent and number of years are equal ($R=N$)

$$R = N = 100k$$

Ques Compound interest

Nothing is given then calculate yearly interest.

$$\text{Compound} = P \cdot (1+i)^n \quad \text{or} \quad P \cdot (1+\frac{R}{100})^n$$

$$\text{amount} = P + I$$

$$\text{Rate} = i$$

$$\text{Compound} = P \cdot [(1+i)^n - 1] \quad CI = A - P$$

$$\text{Interest} = P \cdot (1+i)^n - P$$

$$\text{Interest} = P \cdot [(1+i)^n - 1] \quad \text{given in } \uparrow$$

$$\text{half yearly} \rightarrow 2 \quad 1+i = A$$

$$\text{quarterly} \rightarrow 4 \quad \frac{A}{4} + i = A$$

$$\text{monthly} \rightarrow 12 \quad \frac{A}{12} + i = A$$

$$\text{daily} \rightarrow 365 \quad \frac{A}{365} + i = A$$

$$(A) = P \cdot (1+\frac{i}{m})^{mn}$$

A = compound amount

P = principal

i = rate of interest p.a decimal

m = number of conversion period in a year

n = number of years

Ques Module A = $P(1+i)^n$

i = Rate interest per conversion period

n = total conversion period

$$\text{Compound} = P (1+i_m)^{mn-1}$$

Interest

\Leftrightarrow value of machine after n years.

(1) straight line method

$$\text{to show } P_n = P_0(1+i)^n \text{ (approximating to 2nd term)} \\ \text{. pt. } A = P(1-i)$$

Note:- when no method given in question calculate WDV method on sum.

(2) written down value (WDV) method.

$$\Leftrightarrow A, P_n, P_0 = P(1-i)^n \quad (i = R/100)$$

(for 2 years) (approximate)

Note:- Difference between C.I & S.I

$$(A) \text{ for 2 years } D = pi^2$$

(approximate - does not consider)

$$(B) \text{ for 3 years } D = pi^2 (i+3)$$

\Leftrightarrow IF a sum invested in C.I become a double

$$N = \frac{72}{R} \quad \text{or} \quad R = \frac{72}{N} \quad (\text{approximately})$$

R

\Leftrightarrow Treble the sum

$$N = \frac{114}{R} \quad \text{or} \quad R = \frac{114}{N} \quad (\text{approximately})$$

R

Effective Rate $I = PET$

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↳ Effective Rate on Interest

$$Re = \{[1 + i/m]^{m-1} - 1\} \times 100 \quad e = (1 + i)^n - 1$$

↳ Annuity (future value)

A series of payments of equal size made at equal interval of time is called annuity.

↳ Recurring

↳ Sinking fund.

↳ Types of annuity

1) annuity ordinary & annuity due
(regular annuity) (annuity immediate)

Ordinary annuity :-

payments are made at the beginning of each payment interval.

2) annuity certain & perpetuity

annuity certain :-

number of payment are finite

perpetuity :-

number of payment are infinite payment are made for ever.



Ordinary annuity :-

Future value of amount of annuity

$$\begin{aligned}
 A &= a \cdot i \cdot [A_{(n,i)}] = a \cdot i \cdot \frac{(1+i)^n - 1}{i} \\
 &= \frac{a}{i} \cdot [(1+i)^n - 1]
 \end{aligned}$$

a = size of each payment

i = Rate of interest per payment interval

v = present value

N = Number of payment

A = Amount of future value.

Present value of annuity :-

$$\begin{aligned}
 v &= \frac{a}{i} \cdot [1 - (1+i)^{-n}] = \frac{a}{i} \cdot \left[1 - \frac{1}{(1+i)^n} \right] \\
 &= a \cdot P(n,i)
 \end{aligned}$$

Present value :-

→ loan repayment

→ leasing

→ capital expenditure

→ Bond

present value of = q

perpetuity (v) ~~present value of annuity~~

\Rightarrow Annuity due ~~present value of annuity~~

$$\text{amount} = \frac{q}{i} \left[\frac{(1+i)^n - 1}{(1+i)^n} \right] (1+i) = \frac{q}{i} \cdot \frac{(1+i)^n - 1}{(1+i)^n} \cdot (1+i)$$

$$P.V = \frac{q}{i} \left[1 - \frac{1}{(1+i)^n} \right] + q$$