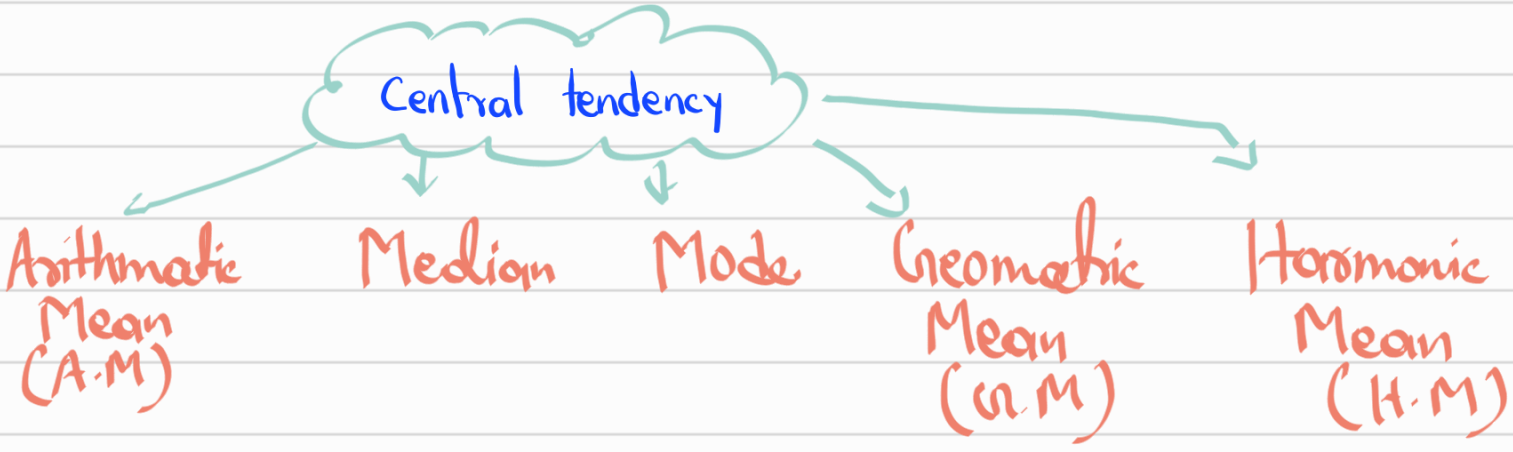
Ch-14
Central Tendency



Arithmatic Mean

↓

Average ($\bar{x}$)

* Individual
* Discrete
* Continue

↓

$\bar{x} = \frac{\Sigma x_i}{N}$

($\bar{x}$)

- Direct method: $\bar{x} = \frac{\Sigma f_i x_i}{N}$
- Short-cut method
  - Assume mean: $\bar{x} = A + \frac{\Sigma f_i d_i}{N}$, $d_i = x_i - A$
  - Step Deviation: $\bar{x} = A + \left[\frac{\Sigma f_i v_i}{N}\right] \times h$, $v_i = \frac{x_i - A}{h}$, h = class length

$\Sigma(x_i - k) = \Sigma x_i - kn$

$\Sigma(x_i - \bar{x}) = 0$ # $\Sigma(\bar{x} - x_i) = 0$

$\Sigma(x_i - k)^2$ is minimum when $k = A.M\ (\bar{x})$

$\Sigma|x_i - A|$ is minimum when $A = Median$

Mean, Mode, Median Changes with orgin & Scale

Combined mean

↳ N_1 $\bar{x}_1$ N_2 $\bar{x}_2$ $\Rightarrow \bar{x}_{12} = \dfrac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$

Median

* Individual / Descrete

N

N = Odd

↓

$\text{Median} = \left(\frac{N+1}{2}\right)^{th}$ term

N = even

$\text{Median} = \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2}$

* Continus Series

→ Calculate $\frac{N}{2}$

→ Locate $\frac{N}{2}$ in c.f

→ Select median class

→ $\text{Median} = l + \left\{\frac{\frac{N}{2} - c.f}{f}\right\} \times h$

Fractiles

- **Median** → 2
- **Quartiles** → 4
  - # Descreate / Individual
  - $Q_1 = \left(\frac{n+1}{4}\right)^{th}$
  - $Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$
  - # Continuied
  - → Locate $\frac{N}{4}$ & $\frac{3N}{4}$ in c.f
  - → Select Q_1 & Q_3 class
  - → $Q_1 = l + \left[\frac{\frac{N}{4} - c.f}{f}\right] \times h$
  - → $Q_3 = l + \left[\frac{\frac{3n}{4} - c.f}{f}\right] \times h$
- **Decilies** → 10
  - # Descreate / Individual
  - $P_1 = \left(\frac{n+1}{10}\right)^{th}$
  - $P_3 = 3\left(\frac{n+1}{10}\right)^{th}$
  - # Continuied
  - → Locate $\frac{N}{10}$ & $\frac{3N}{10}$ in c.f
  - → Select Q_1 & Q_3 class
  - → $Q_1 = l + \left[\frac{\frac{n}{10} - c.f}{f}\right] \times h$
  - → $Q_3 = l + \left[\frac{\frac{3n}{10} - c.f}{f}\right] \times h$
- **Percentilies** → 100
  - # Descreate / Individual
  - $P_1 = \left(\frac{n+1}{100}\right)^{th}$
  - $P_3 = 3\left(\frac{n+1}{100}\right)^{th}$
  - # Continuied
  - → Locate $\frac{N}{100}$ & $\frac{3N}{100}$ in c.f
  - → Select Q_1 & Q_3 class
  - → $Q_1 = l + \left[\frac{\frac{n}{100} - c.f}{f}\right] \times h$
  - → $Q_3 = l + \left[\frac{\frac{3n}{100} - c.f}{f}\right] \times h$

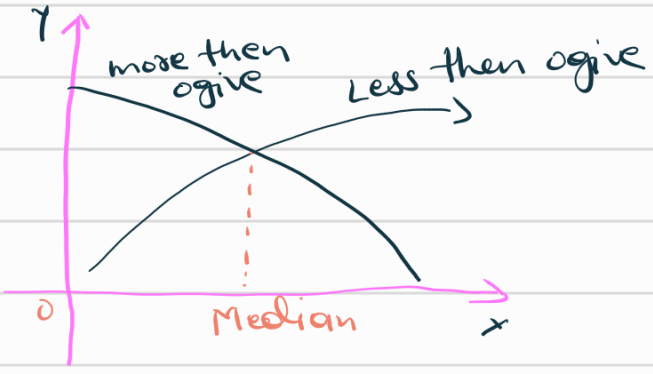
Mode

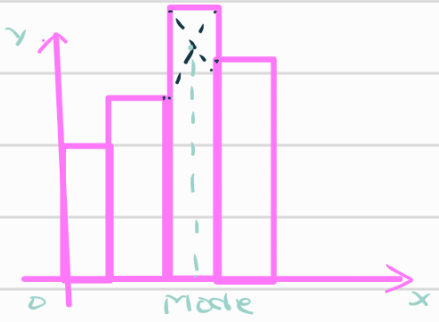
Mode → "observation with highest frequency"

Continuied Series

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$$

3 median = mode + 2 Mean





Harmonic Mean (H.M)

$$HM = \frac{N}{\Sigma\left(\frac{1}{x_i}\right)} \quad \text{or} \quad \frac{N}{\Sigma\left(\frac{f_i}{x_i}\right)}$$

when Speed use 'Harmonic Mean'

Combined H.M $= \dfrac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$

Geomatric Mean (G.M)

$$GM = (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{1/n}$$

GM of (xy) = (GM of x) × (GM of y)

GM of $\left(\frac{x}{y}\right)$ = $\frac{\text{GM of } x}{\text{GM of } y}$

$\log(GM) = \frac{\Sigma \log(x_i)}{N}$

For two items a & b

$GM^2 = AM \times HM$

For Distinct observation

$AM > GM > HM$

For Same items

$AM = GM = HM$

H.M in Case of weigh = $\frac{\Sigma w_i}{\Sigma\left(\frac{w_i}{x_i}\right)}$

Mean - Median = 3 [mean - median]

↓

Diff. of mean - median = 3 [Diff. of mean - median]

Best measure of Central tendency is 'Mean'

for open-end Series best measure Central tendency is 'median'

Dispersion

Absolute

- -> Range
- -> Quartile Deviation
- -> Mean Deviation
- -> Standard Deviation

Relative

- -> Cofficient of Range
- -> Cofficient of Quartile Deviation
- -> Cofficient of Mean Deviation
- -> Cofficient of Varriance

Range

-> R = Largest - Smallest

-> Cofficient of Range = $\frac{L-S}{L+S} \times 100$

for Range / Q.D / M.D / S.D

-> If all items are same the

Rang = 0
Q.D = 0
S.D = 0
M.D = 0

-> Sift of origin (+ or -)

Range, Q.D, M.D S.D
↓
Does not change

-> Sift of Scale (× or ÷)

Range, Q.D, MD, SD
↓
Does change

Quartile Deviation

→ Inter - Quartile Range = $Q_3 - Q_1$

→ Quartile Deviation (Semi- Quartile Range) = $\frac{Q_3 - Q_1}{2}$

→ Coefficient of Quartile Deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

Q.D is the best for open-ended Series

Mean Deviation

→ $MD = \Sigma f_i \left| \frac{x_i - \bar{x} / Median / mode}{N} \right|$

→ Coefficient $= \frac{M.D}{\bar{x} / Median / Mode} \times 100$

Standard deviation

→ $S.D \ (\sigma) = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$

→ Varraince $(\sigma)^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{N}$

OR

→ Coefficient of varraince $= \frac{\sigma}{\bar{x}} \times 100$

→ $S.D = \sqrt{\frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2}$

→ Higher C.V => More Varriable + Less consistence

→ Lower C.V => Less varriable + More consistence

S.D of two number a & b $= \left|\frac{a-b}{2}\right|$

S.D of first 'n' natural no $= \sqrt{\frac{n^2-1}{12}}$

S.D of first 'n' even number $= \sqrt{\frac{n^2-1}{3}}$

S.D of first 'n' odd number $= \sqrt{\frac{n^2-1}{3}}$

Q.D : M.D : S.D = 10 : 12 : 15

S.D > M.D > Q.D

Combined Standard deviation $= \sigma_{12}$

$$\sigma_{12} = \sqrt{\frac{N_1(d_1^2 + \sigma_1^2) + N_2(d_2^2 + \sigma_2^2)}{N_1 + N_2}}$$

$$\Downarrow$$

where $d_1 = \bar{X}_{12} - X_1$

$d_2 = \bar{X}_{12} - X_2$

$$X_{12} = \frac{N_1\bar{x}_1 \times N_2\bar{x}_2}{N_2 + N_1}$$