(A) To Find $\log_{10} m$

STEPS

- Type m (i.e. number)
- Press $\sqrt{\text{button}}$ 19 times
- Then Type '- 1'
- Then Type × 227695 = Buttons
- We get the Required value of $\log_{10} m$

(B) To find Antilog Value i.e. AL(m)

STEPS

- Type m (i.e. number)
- Then Type ÷ 227695+1= Buttons
- Then Continue Pressing [× =] Buttons in Sequence 19 times
- We get the Required value of Antilog.

(C) To Find n[th] root i.e. $\sqrt[n]{A}$

- Type A (i.e. number)
- Press $\sqrt{\text{button}}$ 12 times
- Then Type '- 1'
- Then ÷ n (i.e. n = 2, 3, 4,.....)
- Then Type '+1' = Buttons.
- Then Continue Pressing [× =] Buttons in Sequence 12 times
- We get the Required value of n[th] root i.e. $\sqrt[n]{A}$

Short-Cut Tricks Makes an Examinee to Attempt Maximum Number of Questions in Limited Time.

......Kailash Thakur

RATIO

The comparison of two or more things of same kind is called RATIO. If x and y are two values of same kind (in same units), then the ratio of x to y is written as x : y and read as x is to y.

In $\frac{x}{y}$

- Numerator "x" is called **1st term or Antecedent** and
- Denominator "y" is called **2nd term or Consequent.**
- Antecedent and Consequent must be of **same units**
- Ratio has no unit.

1. Normally a ratio is expressed in simplest form. As. 10 : 16 = 5 : 8.

2. The order of the terms in a ratio must be maintained. As. 3 : 4 is not same as 4 : 3.

3. Ratio exists only with quantities having same unit (kind).

4. (*i*) If x > y, then the ratio x : y is called of **greater inequality.**

(*ii*) If x < y, then the ratio x : y is called of **lesser inequality.**

(*iii*) If x = y, then the ratio a : b is called ratio of Equal **Equality**.

5. (*i*) **Duplicate ratio** of a : b is $a^2 : b^2$

(*ii*) **Triplicate ratio** of a : b is $a^3 : b^3$

(*iii*) **Sub-Duplicate ratio** of a : b is

$\sqrt{a} : \sqrt{b} = a^{1/2} : b^{1/2}$

(*iv*) **Sub-Triplicate ratio** of a : b is $\sqrt[3]{a} : \sqrt[3]{b} =$

$a^{1/3} : b^{1/3}$

6. **Inverse ratio** of x : y is y : x.

7. (*i*) **Commensurable :** If the terms of the ratio are **integers, the ratio** is called commensurable. *As.* 3 : 2

(*ii*) **Incommensurable :** If the terms of the ratio are not **integers, the ratio** is called Incommensurable.

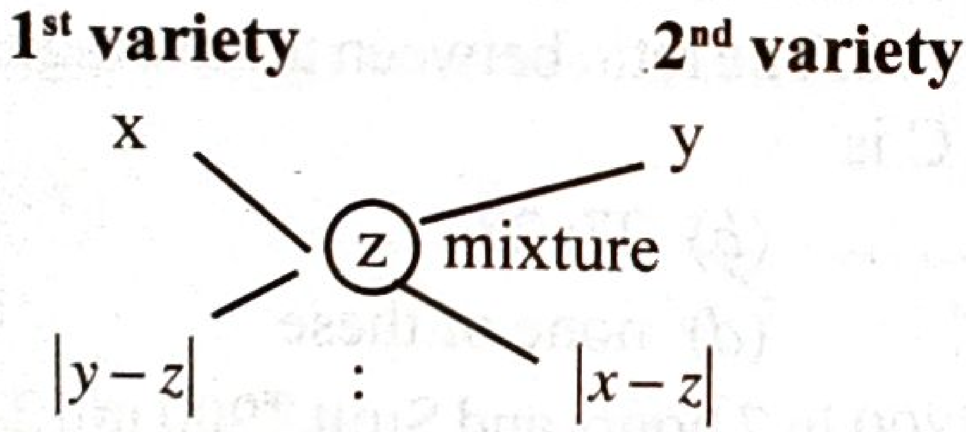
As. $\sqrt{3} : \sqrt{2}$ cannot be expressed in terms of integers. So, it is **Incommensurable.**

Rules of Alligation

It is used in mixing of two varieties of same kind. It is derived from the weighted mean method.

Rule

If two varieties of rices with rate ₹ x per kg and ₹y per kg are mixed to make a third variety of rice with rate ₹ z per kg. The ratio in which these two varieties are mixed is



Remember

(*i*) If x represents cost then y and z must be cost.

(*ii*) If x represents selling price then y and z must be selling price.

(*iii*) If x represents profit then y and z must be profit or loss.

(*iv*) If x represent milk of 1[st] mixture then y and z must represent milk of 2[nd] mixture and mixed mixture.

INDICES

If a number x is multiplied 5 times written as.

$$x. x. x. x. x. = x^5.$$

Here "x" is called BASE and 5 is called Power or INDEX or exponent.

Some Related Formulae

1. $a^m = a \times a \times a \times$ .... to m times.
2. $a^o = 1$ where $a \neq 0; \infty$
3. $a^{-1} = \frac{1}{a}$.
4. $a^{-m} = \frac{1}{a^m}$
5. (i) $a^m \times a^n = a^{m+n}$

   (ii) $a^m \times a^n \times a^x \times$ ....a^{m+n+r+} ....
6. (i) $\frac{a^m}{a^n} = a^{m-n}$.

   (ii) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$
7. (i) $(a^m)^n = a^{mn}$

   (ii) $a^{m^n} \neq a^{mn}$
8. (i) If $a^m = b^m$ Then $a = b$

   (ii) If $a^m = a^n$ Then $m = n$
9. (i) $\sqrt[m]{a^n} = a^{\frac{n}{m}}$

   (ii) $\sqrt{a} = a^{\frac{1}{2}}$

   (iii) $\sqrt[3]{a} = = a^{\frac{1}{3}}$
10. (i) If $a^m = k \Rightarrow a = k^{1/m}$

    (ii) If $a^m = k^n \Rightarrow a = k^{n/m}$

    (iii) If $a^{1/m} = k \Rightarrow a = k^m$

    (iv) If $a^{1/m} = k^n \Rightarrow a = k^{mn}$

11. (*i*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(*ii*) $(ab....)^m = a^m . b^m ....$

12. (*i*) $\sqrt[m]{ab.......} = \sqrt[m]{a} \cdot \sqrt[m]{b} ....$

(*ii*) $\sqrt{ab} = \sqrt{a}. \ \sqrt{b}.$

13. $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$

14. If $a^b = b^a$ Then

Either (*i*) $a = b$

or (*ii*) If $a = 2$

Then $b = 4$

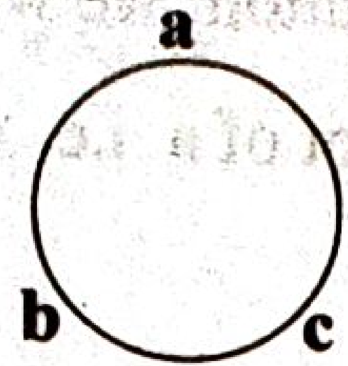
or (*iii*) If $a = 4$

Then $a = 2$

15. If $a > 1$ and $x < y$

Then $a^x < a^y$

Cyclic order Tricks



TYPE I $(a - b) + (b - c) + (c - a) = 0$

TYPE II $a(b - c) + b(c - a) + c(a - b) = 0$

TYPE III $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$

TYPE IV $(a^3 - b^3) + (b^3 - c^3) + (c^3 - a^3) = 0$

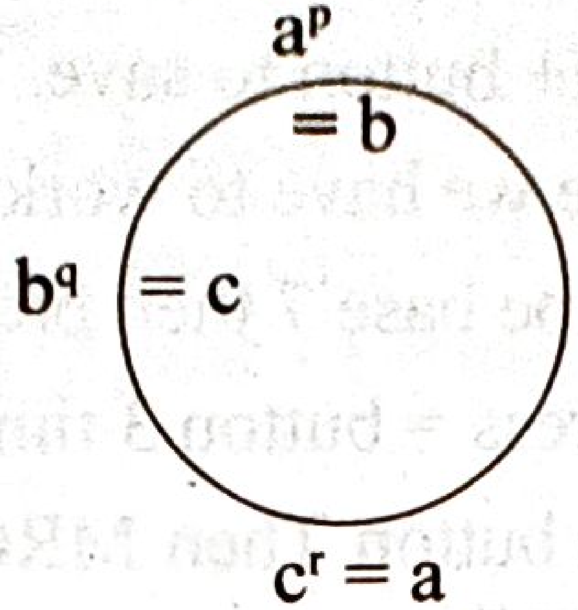
TYPE V $(b - c)(b + c - a) + (c - a)(c + a - b) + (a - b)(a + b - c) = 0$

TYPE VI

$$\frac{1}{(a-b)(b-c)} + \frac{1}{(b-c)(c-a)} + \frac{1}{(c-a)(a-b)} = 0$$

and so on ....

IInd Type of Cyclic Order Tricks



Then product of powers must be equal to power of a in last term.

Example 8 : If $a^p = b$; $b^q = c$; $c^r = a$; The value of "pqr" is given by

(*a*) 0 (*b*) 1

(*c*) -1 (*d*) None

Ans. (*b*) is correct

$$\sqrt{a\sqrt{a\sqrt{a\ldots\ldots to \quad m \quad times}}} = a^{\left(\frac{2^m-1}{2^m}\right)}$$

TYPES OF LOGARITHM

(i) Natural Logarithm:

The Logarithm of a number to base "e" is called Natural Logarithm.

i.e. $Log_e x$

where x = a number

e = 2.7183

(ii) Common Logarithm:

Logarithm of a number to the base 10 is called common Logarithm.

i.e. $Log_{10} x$

where x = A number

Note: If base is not given then in arithmetical or commercial work; base is always taken as 10.

Remember Some Formulae

1. If $a^b = c \Leftrightarrow Log_a c = b$; Where $a \neq 1$.

2. $a^{x \log_a b} = b^x$

3. $\log_a a = 1$

4. $\log_a 1 = 0$

5. $\log_b a = \frac{1}{\log_a b} \Rightarrow \log_b a . \log_a b = 1$

6. (i) $\log_b a = \log_b x \log_x a = \log_x a \cdot \log_b x$

   (ii) $\log_b a = \log_x a . \log_y x . \log_z y .... \log_b k$

   $\log_b a = \log_b x . \log_x y . \log_y z .... \log_k a$

7. (*i*) $\log_b a = \dfrac{\log_x a}{\log_x b}$

(*ii*) $\log_b a = \dfrac{\log_b x}{\log_a x}$

8. If $\log_b a = x$

Then (i) $\log_{\frac{1}{b}} a = -x$

(*ii*) $\log_b \dfrac{1}{a} = -x$

(*iii*) $\log_{\frac{1}{b}} \dfrac{1}{a} = +x$

9. (*i*) $\log_a(mn) = \log_a m + \log_a n$

(*ii*) $\log_a(mnr....) = \log_a m + \log_a n + \log_a r$ +.............

10. $\log_a\left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

11. (*i*) $\log\left(a^b\right)^{(m^n)} = \dfrac{n}{b}\log a^m$.

(*ii*) $\log_a(m^n) = n \log_a m$.

(*iii*) $\log_{ab} m = \dfrac{1}{b}\log_a m$

12. (*i*) If $\log_a m = \log_b m \Rightarrow a = b$.

(*ii*) If $\log_a m = \log_a n \Rightarrow m = n$.

Illustrative Examples

$ax^2 + bx + c = 0$; where $a \neq 0$; a,b,c, are constants form equation is called Quadratic Equation or **Second degree equation.**

I. If $b = 0$ Then $ax^2 + c = 0$ is called **PURE Quadratic** Equation.

II. If $b \neq 0$ Then the equation. $ax^2 + bx + c = 0$ where $a \neq 0$ is called an **AFFECTED Quadratic** Equation.

Roots

The value of the variable "x" which satisfies the given equation is called its **Solution** or roots of the Quadratic Equation.

Discriminant

For Quad. Eqn. $ax^2 + bx + c = 0$.

Discriminant $D = b^2 - 4ac$.

Example

For Eqn. $3x^2 + 7x + 2 = 0$.

$a = 3; b = 7; c = 2$

Discriminant $D = b^2 - 4ac$

$= 7^2 - 4 . 3 . 2 = 49 - 24$

$= 25.$

III. Roots of Quad. Eqn. $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-b \pm \sqrt{D}}{2a}$

[Remember this formula, No need to prove it.]

IV. If α and β are roots of a Quadratic Equation $ax^2 + bx + c = 0$

Then $\alpha + \beta = -\dfrac{b}{a}$

$\therefore$ Sum of roots $= -\dfrac{\text{Co-efficient of } x}{\text{Co-efficient of } x^2}$

$\alpha\beta = \dfrac{c}{a}$

$\therefore$ Product of roots $= \dfrac{\text{Constant terms}}{\text{Co-efficient of } x^2}$

V. If α and β are roots of a Quadratic Eqn. Then the eqn. is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$\Rightarrow x^2$ - (sum of roots) x + Product of roots = 0.

VI. Nature of Roots

Nature of roots of a Quad. Eqn. depends upon **Discriminant $D = b^2 - 4ac$.**

(A) If $D > 0$, Roots Real & Unequal

(*i*) D a perfect square then roots are Rational & unequal

<u>*As.*</u> $\frac{2}{3}; -\frac{2}{3}$.

(*ii*) D not a perfect Square.

Then roots are irrational & unequal and Conjugate

<u>*As.*</u> $2+\sqrt{3}; \sqrt{5}$.

(B) If $D = 0$, Then Roots are Real & equal. Each root $= -\frac{b}{2a}$

(C) If $D < 0$, Then Roots are imaginary.

VII. If one root of a quadratic Eqn. is irrational then its other root is its irrational conjugate.

Cubic Equations

1. Meaning of Cubic Equation

The equation having form.

$ax^3 + bx^2 + cx + d = 0, \quad a \neq 0,$

Where a, b, c, d are real numbers, is called a **cubic equation.**

2. Relationship Between Roots and Coefficients

If α, β, γ are the roots of the cubic equation $ax^3 + bx^3 + cx + d = 0$, $a \neq 0$, then

(i) $\alpha + \beta + \gamma = \frac{-b}{a}$

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

(iii) $\alpha\beta\gamma = \frac{-d}{a}$

3. The Cubic equation having roots α, β, γ is

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

Some Important Formulae.

(*i*) $I = \frac{P.r.t}{100}$ [when r in %]

(*ii*) $I = p.r.t.$ [when r in decimal form]

(*iii*) $r = \frac{I \times 100}{p\,t}$

(*iv*) $t = \frac{I \times 100}{p\,r}$

(*v*) $P = \frac{I \times 100}{r\,t}$

(*vi*) $A = P + I$

(*vii*) $I = A - P$

(*viii*) $A = P\left(1 + \frac{rt}{100}\right)$

Where

A = Accumulated amount
[Final value of investment]

P = Principal. [Initial value of an investment]

RULE:- The simple interest S_1 on principal P in time t_1 years changes to S_2 in time t_2 years at same rate of simple interest per annum.

$$\text{Then} \quad r = \frac{(S_1 - S_2)\times 100}{(t_1 - t_2) \quad P}$$

$$r = \frac{\text{Change in S.I.}}{\text{Change in Time}} \times \frac{100}{\text{P.}}$$

Example 6 : If the simple interest on ₹ 20,000 increases by ₹ 4,000 with the increase of time by 4 Yrs. Find the rate per cent per annum.

(*a*) 0.15% (*b*) 0.5%

(*c*) 5% (*d*) None

Solution : Option (c) is correct.

$$\textbf{Tricks :-}\ r = \frac{\text{Increase S.I.}}{\text{Increase in Time}} \times \frac{100}{P}$$

$$= \frac{4{,}000\times 100}{4\times 20{,}000} = 5\%$$

TYPE III

RULE- Mr. A deposits Rs. P_1 at r_1% per annum S.I. in a bank and ₹ P_2 at r_2% per annum S.I. in another bank, then rate of interest on the whole amount =

$$r = \frac{P_1 r_1 + P_2 r_2}{P_1 + P_2} \%$$

$$\text{i.e. } r = \frac{\text{Sum of S.I. for 1 Yr.}}{\text{Sum of Principals}} \%$$

Example : Mr. X lends ₹ 2,000 at 4% per annum S.I. to Mr. Y and ₹ 3,000 at 14% per annum S.I. to Mr. Z. Find the rate of interest on the whole sum.

(*a*) 8% (*b*) 10%

(*c*) 12% (*d*) None

Solution:- Option (b) is correct

Tricks :- $r = \frac{2000 \times 4\% + 3000 \times 14\%}{2000 + 3000}$

$= 10\%$

TYPE IV

[Principal (P) Changes]

[Change In Principal : $P_1 - P_2$] [Change in S.I. : $S_1 - S_2$] [Change in rate : $r_1 - r_2$] [Change in Time $t_1 - t_2$]

Tricks:- Then

(*i*) $S_1 - S_2 = \frac{(P_1 - P_2)\, r\, t}{100}$

(*ii*) $S_1 - S_2 = \frac{P.(r_1 - r_2).t.}{100}$

(*iii*) $s_1 - s_2 = \frac{P.r.(t_1 - t_2)}{100}$

Example : If the difference between simple interest on ₹ 4,000 and on ₹ 6,500 for 5 Yrs. Be ₹ 800 at same rate of simple interest per annum. Then the rate of interest is

(*a*) 5.3% (*b*) 6.2%

(*c*) 6.4% (*d*) None

Solution:- Option (*c*) is correct.

Tricks:-

$$S_1 - S_2 = \frac{(P_1 - P_2)\, r\, t}{100}$$

or $$800 = \frac{(6500 - 4000).r \times 5}{100}$$

or $$r = \frac{800 \times 100}{2500 \times 5} = 6.4\%$$

Tricks II : Extra Simple interest per year = 800/5

I = ₹ 160

Extra interest due to extra principal = 6500 - 4000

P = ₹ 2500

So, r = (160 x 100/2500) = 6.4 %

Example : If the simple interest on ₹ 300 increases by ₹ 75, when the rate of interest % increases by 5% per annum. Find the time.

TYPE V

RULE:- If the simple interest on a certain sum of money is ' k' times of the principal and the number of

(ii)

Exa
year
is

years is equal to the rate per cent per annum,

the rate % $=\sqrt{100\times\frac{1}{k}}=\sqrt{\frac{100}{k}}\%$

Example:- The simple interest on a certain sum of money is $\frac{1}{25}$ times of principal, the rate of interest when rate of interest and time are equal is

(*a*) 2% (*b*) 3%

(*c*) 4% (*d*) None

Solution:- (*a*) is correct

Tricks : $r=\sqrt{100\times\frac{1}{25}}=2\%$.

TYPE VI

RULE:- If the simple interest on a certain sum of money P is I When rate of interest = Time, then

(*i*) the rate of interest $r=\sqrt{\frac{100\times I}{P.}}\%$

(*ii*) time $=\sqrt{\frac{100\times I}{P.}}$ Yrs.

Example : If the simple interest on a certain sum ₹ 625 is ₹ 81, the rate of interest when rate of interest and the time are equal is

RULE:- A certain sum of money becomes **m** times in t years at a certain rate r% p.a. S.I. Then

(i) $$r = \frac{(m-1)100}{t}\%$$

(ii) $$t = \frac{(m-1)100}{r} Yrs.$$

Example : A certain sum of money trebles itself in 10 years at a certain rate of S.I. p.a. then the rate of interest is

RULE:- A certain sum of money amounts to ₹ A_1 at r_1 % p.a. S.I. and to ₹ A_2 at r_2% p.a. S.I. in the same time interval. Then the sum of money invested initially

$$= \left(\frac{A_2 r_1 - A_1 r_2}{r_1 - r_2} \right)$$

or $$= \frac{A_1 - P}{A_2 - P} = \frac{r_1}{r_2}$$

Example : Mr. A invested ₹ x in an organisation, it amounts to ₹ 150 at 5% p.a. S.I. and to ₹ 100 at 3% p.a. S.I. Then the value of x is

(*a*) ₹ 70 (*b*) ₹ 40

(*c*) ₹ 25 (*d*) None

Solution:- Option (c) is correct.

Tricks :- Go by choices

For option (*a*) $\frac{150-70}{100-70} = \frac{80}{30} \neq \frac{5}{3}$

For option (*b*) $\frac{150-40}{100-40} = \frac{110}{60} \neq \frac{5}{3}$

For option (*c*) $\frac{150-25}{100-25} = \frac{125}{75} = \frac{5}{3}$

∴ Option (*c*) is correct.

Or $$P = \frac{100 \times 5 - 150 \times 3}{5-3}$$

= ₹ 25.

TYPE X

RULE- A certain sum of money amounts to A_1 in time t_1 years and amounts to A_2 in time t_2 year at same rate of S.I. p.a. Then the rate of interest

$$r = \frac{(A_1 - A_2) \times 100}{A_2 t_1 - A_1 t_2} \%$$

Example : A certain sum of money amounts to ₹ 756 in 2 years and to ₹ 873 in 3.5 years at same rate of S.I. p.a. The rate of interest is

(*a*) 12% (*b*) 13%

(*c*) 14% (*d*) None

RULE - A certain sum of money was put at S.I for t years at a certain rate of S.I. p.a. (*i*) If it had been put at x% higher rate it would have fetched ₹ 'K' more; the sum of money

$$= ₹ \frac{K \times 100}{t \times x}$$

(*ii*) If it had been put at x% lower rate it would have fetched ₹ 'K' less ; the sum of money

$$= ₹ \frac{K \times 100}{t \times x}$$

Example : A certain sum of money was put at S.I. for 2.5 years at a certain rate of S.I. p.a. Had it been put at 4% higher rate, it would have fetched ₹ 500 more.

Find the sum of money.

(*a*) ₹ 4000 (*b*) ₹ 5000

(*c*) ₹ 6000 (*d*) None

Solution:- (*b*) is correct

Let sum of money = P.

& rate of interest = r.

$$\therefore \quad \frac{P(r+4)\times 2.5}{100} - \frac{pr \times 2.5}{100} = 500$$

$$\text{or} \quad \frac{P \times 2.5}{100}(r+4-r) = 500$$

$$\text{or} \quad \frac{P \times 2.5 \times 4}{1000} = 500$$

$\therefore$ P = ₹ 5000.

Tricks :- Sum of Money $= \frac{500 \times 100}{2.5 \times 4}$

= ₹ 5,000.

RULE- ₹ A is invested in two different organizations, some amount of it at r_1% in 1[st] organisation and rest amount at r_2% in 2[nd] organisation for one year. Total interest obtained from both is ₹ K.

Then,

$$1^{st}\text{ Part} = \frac{100\,k - r_2\,A}{r_1 - r_2}$$

$$\text{and } 2^{nd}\text{ Part} = \frac{Ar_1 - 100K}{r_1 - r_2}$$

Example : Mrs. Sudha lent ₹ 4,000 in such a way that some amount to Mr. A at 3% p.a. S.I. and rest amount to B at 5% p.a. S.I., the annual interest from both is ₹ 144. Find the amount lent to Mr. A

(*a*) ₹ 2,800 (*b*) ₹ 1,200

(*c*) ₹ 2,500 (*d*) None

Solution:- (*a*) is correct

Tricks I:-

$$1^{st}\text{ Part} = \frac{100\times144 - 5\times4{,}000}{3-5}$$

$$= ₹\ 2{,}800$$

$$2^{nd}\text{ Part} = \frac{3\times4{,}000 - 100\times144}{3-5}$$

$$= ₹\ 1{,}200$$

Tricks II:-

RULE - At a certain rate of simple interest ₹ P amounts to ₹ A in t_1 years. If the rate of interest is

(*i*) decreased by r%, then after t_2 years the new

$$\text{interest} = \left[\frac{A - P}{t} - \frac{Pr}{100}\right] t_2$$

(*ii*) Increased by r %, then after t_2 years the new interest

$$= \left[\frac{A - P}{t} + \frac{rP}{100}\right] t_2$$

₹ 920 in 3 years at a

RULE - A certain sum of money "P" is lent out in "n" parts in such a way that the simple interest on 1st part at r_1% for t_1 years; the S.I. on 2nd part at r_2% for t_2 years; the S.I. on 3rd part at r_3% for t_3 years and so on, are equal; the ratio in which the sum of money was divided into n parts are

$$\frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3} : \ldots\ldots : \frac{1}{r_n t_n}$$

TYPE XV

RULE:- (*i*) A certain sum of money becomes x_1 times of r_1 % rate of S.I. p.a. and x_2 times at r_2% rate of S.I. p.a. Then

$$\frac{r_2}{r_1} = \frac{x_2 - 1}{x_1 - 1}$$

$$\Rightarrow \quad r_2 = \left(\frac{x_2 - 1}{x_1 - 1}\right) \times r_1.$$

(*ii*) A certain sum of money becomes x_1 times in t_1 years and x_2 time in t_2 years at same rate of S.I. p.a.

Then.

$$\frac{t_2}{t_1} = \frac{x_2 - 1}{x_1 - 1}$$

$$\Rightarrow \quad t_2 = \left(\frac{x_2 - 1}{x_1 - 1}\right) \times t_1.$$

$$S.I. = \frac{15000 \times 8}{100} \times \frac{8}{12} = ₹\,800$$

Difference = ₹ 1050-₹ 800 = ₹ 250

∴ option (a) is correct.

TYPE XXII

RULE:- If simple interest received from two different banks on ₹ P for t years differ by ₹ D, Then, the difference between their rates

$$(r_1 - r_2) = \frac{D \times 100}{Pt}\%$$

Example : The difference between the S.I. received from two banks S.B.I. and PNB on ₹ 750 for 2 years is ₹ 90. Find the difference between their rates.

(*a*) 5% (*b*) 6%

(*c*) 7% (*d*) None

Solution :- (*b*) is correct

Tricks I:- $r_1 - r_2 = \frac{90 \times 100}{750 \times 2} = 6\%$

Tricks II:- $S.I._1 - S.I._2 = \frac{pt(r_1 - r_2)}{100}$

or $90 = 750 \times 2 \frac{(r_1 - r_2)}{100}$

or $r_1 - r_2 = \frac{90 \times 100}{750 \times 2} = 6\%$

TYPE XXIII

Mr. X borrows ₹ P from a bank at simple interest. He paid ₹ P_1 after t_1 years and ₹ P_2 at the end of t...

$r = \frac{1{,}450 \times 100}{9{,}000 + 2\ldots}$

Tricks II:- Go by ch...

Let option (a) is co...

So, For 1st 3 yrs. A...

$= 7{,}000 \times 3 \times 5\% + \ldots$

Amount paid = 3,...

Balance = 5,050

After next 2 yrs.

Interest on ₹ (7,00...

= 4,000 = 4,000 ×...

∴ Amount due at ...

= 5,050 + 400 = 5,...

Which is equal to...

∴ Option (a) is c...

RULE:- Some an... and the remainin... interest obtained ... p.a. S.I. is

$$\frac{100\,I - P\ldots}{(r_1 - r_2\ldots}$$

and at $r_2\% = \frac{Pr_1\ldots}{(r\ldots}$

Example : Mr...

RULE:- If the simple interest on a certain sum of money "P" is I when rate of interest and time are equal then the rate per cent or time

$$= \sqrt{\frac{100 \times I}{P}}$$

Example : If S.I. on a certain sum of money ₹ 100 is ₹ 9 and the number of years and rate % are equal. Find the rate per cent.

(*a*) 3% (*b*) 4%

(*c*) 5% (*d*) None

Solution :- Option (*a*) is correct

Tricks $r = \sqrt{\frac{100 \times 9}{100}} = 3\%$

is called **Compound Interest.**

Conversion Period : The period at the end of which the interest is computed is called **Conversion peri**

Description	Conversion period	No. of Conversion periods in a year = m	Rate of interest of a conversion peri $i = \frac{r}{100m}$
10% compounded yearly	1 year	m = 1	i = 10 / 100
10% compounded half-yearly	6 months	m = 2	i = 10 / 200
10% compounded Quarterly	3 months	m = 4	i = 10 / 400
10% compounded monthly	1 month	m = 12	i = 10 / 1200

Formula : Compound Amount $= A = P\left(1+\frac{r}{100m}\right)^{mt}$

$= P(1+i)^n$

Where $i = \frac{r}{100m}$ & $n = mt$

Compound Interest $= P\left[(1+i)^n - 1\right]$

TYPE I

(To find Amount & Compound Interest)

Working Rule:

(*i*) If rate of interest compounded yearly then divide r by 100 *i.e.* $i = \frac{r}{100}$.

(*ii*) If rate of interest compounded $\frac{1}{2}$ yearly Then divide r by 200 *i.e.* $i = = \frac{r}{200}$.

(iii) If rate of interest compounded $\frac{1}{4}$ yearly the

$$i = \frac{r}{400}$$

(iv) If rate of interest compounded monthly the

$$i = \frac{r}{1200}$$ and so on.

EFFECTIVE RATE OF INTEREST

TYPE II

The equivalent annual rate of interest compounded annually if interest is compounded more than once in a year is called EFFECTIVE RATE of INTEREST. It is denoted by E or r_e.

Formula

$$r_e = E = \left[\left(1+\frac{r}{100m}\right)^m - 1\right]\times 100 = (1+i)^m - 1$$

where r = Nominal rate of interest, m = No. of conversion periods in a year.

(To find Present Value)

$$\because \quad A = P(1+i)^n$$

$$\text{or } P = \frac{A}{(1+i)^n}$$

$$\text{or } P = A(1+i)^{-n}$$

(Varying rate of interest)

RULE- If rate of interest for 1st year, 2nd year and 3rd year are $r_1\%$; $r_2\%$; $r_3\%$ respectively then the

$$\text{Amount} = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$$

Where P = Principal

RULE - If the rate of interest for Ist t_1 years next t_2 yrs and next t_3 years are r_1 % compounded m_1 times in a year r_2% compounded m_2 times in a year and r_3% compounded m_3 times in a year respectively. The amount of principal P is = A =

$$P\left(1+\frac{r_1}{100m_1}\right)^{m_1t_1}\left(1+\frac{r_2}{100m_2}\right)^{m_2t_2}\left(1+\frac{r_3}{100m_3}\right)^{m_3t_3}$$

RULE - A certain sum of money becomes m times in t_1 years and n times in t_2 years at same rate of compound interest per annums. Then the equation is

$$m^{1/t_1} = n^{1/t_2} \Rightarrow m^{t_2} = n^{t_1}.$$

RULE - If a certain sum of money becomes m times in t years, the rate of interest $r = \left(m^{1/t} - 1\right) \times 100\ \%$

Example: At what rate of compound interest a certain sum of money becomes 27 times of itself in 3 years ?

(*a*) 150%
(*b*) 200%
(*c*) 250%
(*d*) None

Solution: (b) is correct

Trick I

$$r = \left[27^{1/3} - 1\right] \times 100 = \left[(3^3)^{1/3} - 1\right] \times 100 = 200\%$$

RULE - If the compound Interest on a certain sum of money be "C" then simple Interest given

$$S.I = \frac{\text{Compound Interest}}{\text{Compound Interest on ₹ 1}} \times \frac{r \times t}{100}$$

$$\therefore \; S.I = \frac{C}{\left(1+\frac{r}{100}\right)^t - 1} \times \frac{rt}{100}$$

RULE - If the simple interest (S.I) on a certain sum of money at a certain rate of interest r% p.a. for t years be S then compound interest (C.I) at same rate and time

$$= \frac{\text{Simple Interest}}{\text{S.I, of Re 1}} \times \left[\left(1+\frac{r}{100}\right)^t - 1\right]$$

$$= \frac{S \times 100}{rt}\left[\left(1+\frac{r}{100}\right)^t - 1\right]$$

Example: If the S.I. on a certain sum of money for 3 years at 5% p.a. is ₹ 1260. Then its compound interest (C.I.) is

(*a*) ₹ 1324.05 (*b*) ₹ 1330

(*c*) ₹ 1425 (*d*) None

Solution: (a) is correct

Tricks

$$C.I = \frac{1260 \times 100}{(3 \times 5)} \times \left[\left(1+\frac{5}{100}\right)^3 - 1\right] = ₹\ 1324.05$$

Calculator Work

(To find principal if difference between C.I & S.I is given)

RULES - (*i*) If the difference between C.I and S.I. on a certain sum of money is D for time t years at r% rate of interest then Sum of Money (P) =

$$\frac{Diff.(C.I - S.I.)}{(C.I - S.I)\ on\ ₹1} = \frac{D}{\left[\left(1+\frac{r}{100}\right)^t - 1\right] - \frac{rt}{100}}$$

(*ii*) For 2 years $P = \frac{Difference \times 100^2}{r^2}$

$$P = \frac{D \times 100^2}{r^2}$$

(*iii*) For 3 years Principal= $P = \frac{(C.I - S.I) \times 100^3}{r^2(300+r)}$

Example : If the difference between C.I and S.I on a certain sum of money at 5% p.a. for 2 years is ₹ 1.50. Find the sum of money.

(*a*) ₹ 600 (*b*) ₹ 500

(c) ₹ 400 (*d*) None

Solution: (a) is correct

Trick - I

$$P = \frac{D}{r^2} \times (100)^2 = \frac{1.50}{5^2} \times (100)^2 = ₹\ 600$$

RULES - (i) If the simple interest and compound interest on a certain sum of money be ₹ S and ₹ C respectively The difference between simple interest and compound interest at the rate of r % p.a. for time "t" years is = Sum of Money × [(c-s) for ₹ 1]

$$C.I - S.I = P\left[\left\{\left(1+\frac{R}{100}\right)^{T} - 1\right\} - \frac{rt}{100}\right]$$

(ii) For 2 years $C.I - S.I = P\left(\frac{r}{100}\right)^2$

(iii) For 2 years $C.I - S.I = P\frac{r^2(300+r)}{(100)^3}$

Example : Find the difference between the C.I and S.I. for the sum of ₹ 625 at 8% p.a. for 2 years

(a) ₹ 1.5 (b) ₹ 4.5

(c) ₹ 4 (d) None

Solution. (c) is correct

Trick $C.I - S.I = 625\left(\frac{8}{100}\right)^2 = ₹\ 4$

Example : Find the difference between the S.I. and C.I. on ₹ 8000 for 3 years at 5% p.a.

(a) ₹ 65 (b) ₹ 62

(c) ₹ 61 (d) None

Solution. (c) is correct

$$C.I - S.I = \frac{p.r^2(300+r)}{(100)^3} = \frac{8000 \times 5^2 \times (300+5)}{(100)^3}$$

= ₹ 61

RULE - A certain sum of money amounts to A_1 in t_1 years at a certain rate of compound interest and A_2 in (t+1) years at same rate of compound interest. Then

$$\text{rate of interest "}r\text{"} = \frac{(A_2 - A_1) \times 100}{A_1}$$

RULE - A certain sum of money amounts to A_1, in "t" year and A_2 in (t + 1) years at same rate of compound interest then the sum of money $= \left[A_1\left(\frac{A_1}{A_2}\right)^n\right]$

Example : A certain sum of money amounts to ₹2750 in 2 years and ₹3125 in 3 years at same rate of compound interest, the sum of money is

(*a*) ₹2129.60 (*b*) ₹2210.37

(*c*) ₹2531.62 (*d*) Data inadequate.

Solution:- (a) is correct.

Tricks-

$$P = 2750\left(\frac{2750}{3125}\right)^2 = 2129.60$$

RULE - On a certain sum of money simple interest and compound interest are S and C respectively at r% for 2 years then $\frac{C}{S} = \frac{200+r}{r}$

i.e. Ratio of compound interest and simple interest

C : S = 200 + r : 200

RULE - A certain sum of money "P" amounts to "Q" in time t_1 years at certain rate of compound interest then the amount after t_2 years

$$= P\left(\frac{Q}{P}\right)^{\frac{t_2}{t_1}}$$

Example: Mohan deposited ₹ 4800 in a bank after 4 years it becomes ₹ 6000 at a certain rate of compound interest what will be his amount in the bank after 12 years.

(*a*) ₹ 9375 (*b*) ₹ 9000

(*c*) ₹ 9525 (*d*) None

Solution:- (a) is correct

Trick-I Amount $= 4800\left(\frac{6000}{4800}\right)^{\frac{12}{4}} =$ ₹ 9375.

Trick-II ₹ 4800 becomes ₹ 6000 i.e.1.25 times in 4 yrs.

∴ ₹ 6000 becomes 1.25 times i.e. ₹ 7500 in next 4 yrs and ₹ 7500 becomes 1.25 times i.e. ₹ 9375 in (4+4+4=12) yrs.

RULE - If a person borrows P at r% compound interest and pays A at the end of each year, then at the end of nth year the amount is to be paid.

$$= P\left(1+\frac{r}{100}\right)^n -$$

$$A\left[\left(1+\frac{r}{100}\right)^{n-1}+\left(1+\frac{r}{100}\right)^{n-2}+\ldots\ldots+\left(1+\frac{r}{100}\right)\right]$$

$$=\frac{100A}{r}\left(1+\frac{r}{100}\right)-\left(\frac{100A}{r}-p\right)\left(1+\frac{r}{100}\right)^n$$

Example: Mr. X borrows ₹ 3000 at 10% compound rate of interest. At the end of each year he pays back ₹ 1000. How much he should pay at the end of the 3rd year to clear his entire dues ?

(a) ₹ 1583 (b) ₹ 1683

(c) ₹ 1153 (d) None

Solution: (b) is correct

The required amount

$$= P\left(1+\frac{r}{100}\right)^3 - A\left[\left(1+\frac{r}{100}\right)^{3-1} + \left(1+\frac{r}{100}\right)^1\right]$$

$$= 3000\left(1+\frac{10}{100}\right)^3 - 1000\left[\left(1+\frac{10}{100}\right)^2 + \left(1+\frac{10}{100}\right)\right]$$

$$= 3000(1.1)^3 - 1000[(1.1)^2 + (1.1)]$$

[Use Calculator and solve it] = ₹ 1683

RULE - A is divided among "n" parts in such a way that each part for periods t_1 ; t_2 ; t_3;..... t_n; at the rate of compound interests r_1 ; r_2 ; r_3 ; ....r_n% respectively become equal then ratio of each part of the sum money is

$$\frac{1}{\left(1+\frac{r_1}{100}\right)^{t_1}}:\frac{1}{\left(1+\frac{r_2}{100}\right)^{t_2}}:\frac{1}{\left(1+\frac{r_3}{100}\right)^{t_3}}:$$

$$\ldots\ldots\ldots:\frac{1}{\left(1+\frac{rn}{100}\right)^{t_n}}$$

Example: Mr. X lent ₹ 6100 to Mr. A and Mr. B at same rate of compound interest of 20% p.a. so that A's share at the end of 3 years may equal to B's share at the end of 5 years.

(*a*) ₹ 3500 ; ₹ 2600 (*b*) ₹ 3600 ; ₹ 2500

(*c*) ₹ 3400 ; ₹ 2700 (*d*) None

Solution:- (b) is correct

Ratio of A's & B's Share

$$=\frac{1}{\left(1+\frac{20}{100}\right)^{3}}:\frac{1}{\left(1+\frac{20}{100}\right)^{5}}=$$

$$\left(\frac{100}{120}\right)^{3}:\left(\frac{100}{120}\right)^{5}=1:\left(\frac{5}{6}\right)^{2}=36:25.$$

$$\therefore \text{A's Share}=\frac{36}{36+25}\times 6100=₹\,3600$$

$$\text{B's Share}=\frac{25}{36+25}\times 6100=₹\,2500$$

Definition:- A sequence of payments, generally equal in size, made at equal intervals of times is called an **annuity.**

Monthly Rent; premiums of LIC; deposited into a recurring account in a bank; equal monthly payments got by a retired government servant as pension and loan instalments to houses or automobiles etc.

Some terms related with annuities

Periodic Payment:- The size of each payment of an annuity is called the periodic payment of the annuity.

Annual Rent:- The sum of all payments of an annuity made in one year is called its annual rent.

Payment Period/Interval :- The duration between two successive payments of an annuity is called the payment period (or payment interval) of the annuity

Term:- The total duration from the beginning time of the first payment period to the end of the last payment period is called the **term** of the annuity.

Amount of an Annuity:- The total value of all the payments at the maturity time of an annuity is called the amount (or future value) of the annuity.

Present Value of an Annuity:- Sum of the present values of all the payments of an annuity is called the present value or capital value of the annuity.

TYPES OF ANNUITIES

Ordinary Annuity: If the payments of an annuity are made at the end of payment interval is called An Ordinary annuity or Regular annuity.

Annuity Due: If the payments of an annuity are made at the beginning of payment interval is called An Annuity Due or Annuity Immediate.

Perpetuity: A perpetuity is an annuity whose payments continue forever.

Note. In what is to follow, it is understood that the payment interval coincides with the interest period unless statement to the contrary is made.

ORDINARY ANNUITY OR ANNUITY REGULAR

Definition:- Payments of an annuity are made at the end of payment interval.

TYPE I

(TO Find Amount)

$$S = A\left[\frac{(1+i)^n - 1}{r}\right] \times 100m.$$

Where S = Amount of an Annuity

A = Value of each instalment

r = rate of interest

m = No. of conversion periods in a year

n = m.t = No. of instalments made in t yrs.

$i = \frac{r}{100m}$ = Rate of interest of one conversion Period

Calculator Trick

Step I Find $(1 + i)^n$ by calculator i.e. Type r ÷ 100 m + 1 Then push × button then push = button (n - 1) times.

Step II Then - 1

Step III ÷ r × 100m

Step IV Then × A push = button (We get the required value of Amount)

Example 1. Find the future value of an annuity of ₹ 500 is made annually for 7 years at interest rate of 14% compounded annually. [Given that $(1.14)^7$ = 2.5023]

(*a*) ₹ 5365.25 (*b*) ₹ 5265.25

(*c*) ₹ 5465.25 (*d*) none

Solution :- Option (*a*) is correct

Calculator Trick

$$S = A\left[\frac{(1+i)^n - 1}{r}\right] \times 100m. = ₹\ 5365.25$$

Step I Find $\left(\frac{14}{100}+1\right)^7$ *As* Type 14 ÷ 100 + 1 × Push = button 6 times.

(To find Present Value for Ordinary Annuity)

$$PV = \text{Present value} = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

(To find Amount)

$$FV = \text{Amount } S = A\left[\left\{\frac{(1+i)^{n+1}-1}{r}\right\}\times 100m-1\right]$$

Calculator Trick (work as ordinary annuity)

(To Find Present Value of Annuity Due or Annuity immediate)

$$PV = P = A\left[\left\{\frac{1-(1+i)^{-(n-1)}}{i}\right\}+1\right]$$

(To Find Instalment Value if Amount is given)

$$FV = S = A\left[\left\{\frac{(1+i)^{n+1}-1}{r}\right\}\times 100m - 1\right]$$

(To find Instalment Value if **Present Value** is given)

$$PV = A\left[\frac{1-(1+i)^{-(n-1)}}{i}+1\right]$$

SINKING FUND

A **sinking fund** is a type of fund that is created and set purposely for repaying debt. The owner of the account sets aside a certain amount regularly and uses it only for a specific purpose. Interest is compounded at the end or beginning of every period.

We use formula, $FV = A\left[\frac{(1+i)^n - 1}{i}\right]$

AMORTIZATION OF LOANS

A loan is said to be **Amortized** if it can be discharged by a sequence of equal payments made over equal periods of time. Each payment can be considered as consisting of two parts:

(*i*) Interest on the outstanding loan, and

(*ii*) Repayment of part of the loan. Thus, a loan is amortized when part of each periodic payment is used to pay interest and the remaining part is used to reduce the principal.

Formula

$$P = PV = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Example 1. A loan of ₹30,000 at the interest rate of 6% compounded annually is to be amortized by equal payments at the end of each year for 5 years, find the annual payment

(*a*) ₹ 7121.89 (*b*) ₹ 7200

(*c*) ₹ 6921.89 (*d*) None

Solution :- (*a*) is correct

Here A = ?; PV = ₹ 30,000

r = % yearly ; t = 5 years ⇒ $n = 5 \times 1 = 5.$

$$PV = P = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$30{,}000 = A\left[\frac{1-\left(1+\frac{6}{100}\right)^{-5}}{0.06}\right],\ A = ₹7121.89$$

Capital Expenditure

(Investment Decision):-

Example 1. Machine A costs ₹ 10,000 and has useful life of 8 years. Machine B costs ₹ 8,000 and has useful life of 6 years. Suppose machine A generates an annual labour savings of ₹ 2,000 while machine B generates an annual labour saving of ₹ 1800. Assuming the time value of money is 10% per annum, find which machine is preferable?

(*a*) Machine A (*b*) Machine B

(*c*) Both are equivalent (*d*) None of these

Solution :- (a) is correct

For Machine A

PV of a sequence of annual savings of ₹ 2000 for

$$8 \text{ years @ } 10\% \text{ p.a.} = 2000\left[\frac{1-(1.10)^{-8}}{0.10}\right]$$

[Use Calculator Trick]

= ₹ 10669.85 = ₹ 10670

Net saving = ₹ 10,670 - ₹ 10000 = ₹ 670

For Machine B

PV of a sequence of annual savings of ₹ 1800 for 6 years @ 10% p.a.

$$= 1800\left[\frac{1-(1.10)^{-6}}{0.10}\right]$$

[Use calculator Trick]

= ₹ 7839.46

Net saving = ₹ 7839.46 - ₹ 8000 = - ₹ 160.53

Thus, machine B costs ₹ 160.53 more.

Decision:- Machine A is preferable.

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TYPE XIII

Bond Valuation

Bonds are long-term debt securities issued by companies or government entities to raise debt finance. Investors who invest in bonds receive periodic interest payments, called coupon payments, and at maturity, they receive the face value of the bond along with the last coupon payment. Each payment received from the bonds, be it coupon payment or payment at maturity, is termed as cash flow for investors.

2. The redemption price of the bond at maturity.

The purchase price of a bond is therefore equal to the present value of the annuity formed by all its future dividends plus the present value of its redemption price.

Formula for computing the purchase Price

To derive the formula for the purchase price of a bond the following notations will be used:

F = the face value

C = the redemption price

i = the yield rate per period

n = the number of periods before redemption.

R = the periodic dividend payment

V = Purchase price = present value of the bond.

$$V = C(1+i)^{-n} + R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Note:- If a bond is redeemed at par then

C = F then

$$V = F(1+i)^{-n} + R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

n = No. of ...
V = Present ...
resumed price + ...

$$= 1000\left(1+\frac{14}{100}\right) ...$$

= ₹ 907.135.

Note:- **Use calculat...**

Example 2. A ₹ 10...
at 8.5% will be rede...
Find the purchase ...
wishes a yield rate ...

(*a*) ₹ 907.135

(*c*) ₹ 945.67

Solution :- (*b*) is c...

$$V = PV = 1000 ...$$

$$\left[\frac{1-(1.08)^{-10}}{0.08}\right]$$

= ₹ 1033.55.

PERPETUAL ANNUITY (OR PERPETUITY)

The sequence of payments continuing forever (i.e., the payments continue for infinite number of periods) is called **Perpetuity (or perpetually annuity)**. Here beginning date is known but its terminal date i.e. end date is not known ; So, we cannot find amount of a perpetuity, but its present value can be determined.

There are three types of perpetual annuities.

Type I. Present Value of Immediate perpetuity

Present value (P) of immediate (or ordinary) annuity consisting n payments of ₹ (R) each, paid at the end of each period at the rate i per period is given by

$$\Rightarrow \quad P = PV = \frac{R}{i}$$

Type II. Present Value of Perpetuity Due

Here, each payment of ₹ R is payable at the be-ginning of each period, the first payment is due now.

∴ The present value of perpetuity due

$$PV = P = R + \frac{R}{i}$$

Where R = Value of one instalment;

i = rate of interest per period.

Example 1. If money is worth 6 % per annum, find the present value of a perpetuity of ₹ 3300 payable annually.

Solution : It is an immediate perpetuity.

Here, $R = 3300, \; i = \frac{6}{100} = 0.06$

∴ The present value of the perpetuity is given by

$$P_{\infty} = \frac{R}{i} = \frac{3300}{0.06} = ₹\ 55{,}000.$$

Example 2. At 8% converted quarterly, find the

Nominal Interest Rate = Real Interest Rate + Inflation

COMPOUND ANNUAL GROWTH RATE (CAGR)

The compound annual growth rate (CAGR) is the rate of return that would be required for an investment to grow from its beginning balance to its ending balance assuming the profits were reinvested at the end of each year of the investment's lifespan.

To calculate the compound annual growth rate, divide the value of an investment at the end of the period by its value at the beginning of that period, raise the result to an exponent of one divided by the number of years, and subtract one from the subsequent result.

$$CAGR(t_n, t_0) = \left(\frac{V(t_n)}{V(t_0)}\right)^{\frac{1}{t_n - t_0}} - 1$$

selected from ... letters selected ...
letters A, B and C. If both the letters ... order of selection are considered, then the following 6 outcomes are possible:

They are AB ; AC ; BC; BA; CA ; CB

Each of these 6 different possible selections is called a permutation. In particular, they are called the **permu-** **permutation**. In particular, they are called the **permutations of three objects taken two at a time,** and the number of such permutations possible is denoted by the symbol $^3\mathbf{P}_2$, read "3 permute 2." In general, if there are n objects available from which to select, and permutations (P) are to be formed using r of the objects at a time, the number of different permutations possible is denoted by the symbol ^nP_r & formulated as

$$^{n}p_{r}=\frac{n!}{(n-r)!}$$

$$^{3}p_{2}=\frac{3!}{(3-2)!}=3!=3.2.1=6$$

For **Combinations,** r objects are selected from a set of n objects to produce subsets without ordering. Contrasting the previous permutation example with the corresponding combination, the AB and BA subsets are no longer distinct selections; by eliminating such cases there remain only 3 different possible subsets—AB, AC, BC. The number of such subsets is denoted by ^nC_r, read "n choose r." & is formulated as

$$^{n}C_{r}=\frac{n!}{r!.(n-r)!}$$

PERMUTATION OF N DIFFERENT THINGS

Theorem 1: The number of permutations of n different things taken r at a time, denoted by

"^nP_r" and is given by P (n , r) or $^n p_r = \frac{n!}{(n-r)!}$

where $r \leq n$

Note - Proof not required

Corollary:

(*i*) $P(n, n)$ or ${}^nP_n = n!$

(*ii*) $P(n, n) = P(n, n-1)$

(*iii*) $P(n, r) = n\,P(n-1, r-1)$

(*iv*) $P(n, r) = P(n-1, r) + r \cdot P(n-1, r-1)$

∴ Perms. of 2nd prize = 5.

Similarly perms. of 3rd prize = 5.

Therefore, Total No. of ways to distribute 3 prizes among 5 students = 5.5.5 = 125.

TYPE V

Formula

(*i*) Total No. of permutations of "n" different things taking r at a time (When repetitions not allowed) $= {}^nP_r = \dfrac{n!}{(n-r)!}$

Formula

(*i*) Total number of arrangements of "n" different things taking all at a time = n!.

(*ii*) Total No. of arrangements of "n" different things taking all at a time so that "p" particular things are always together = (n - p + 1)!. P!.

Example 1 : How many words can be made by using all letters of the word "FAILURE" so that vowels are always coming together is

(*a*) 576 (*b*) 575

(*c*) 570 (*d*) None

Solution : (*a*) is correct

Tricks

No. of words $= (7 - 4 + 1)!.\ 4! = 4!.\ 4! = 24 \times 24$

$= 576.$

TYPE VII

GAP Rule

Suppose 5 males A,B,C,D,E are arranged in a row as $\times A \times B \times C \times D \times E \times$. There will be six gaps between these five. Four in between and two at either end. Now if three females P, Q, R are to be arranged so that they are never together. We shall use GAP method i.e. arrange them in between these 6 GAPS. Hence the answer will be ${}^6P_3 . 5!$.

...ple 1 : A family of 4 brothers and 3 si...

STRING Rule

Example 1 : In how many ways 6 persons can be arranged in a row so that 2 particular persons can never sit together.

(*a*) 720 (*b*) 480

(*c*) 360 (*d*) None.

Solution : (*b*) is correct.

Without any conditions, No. of arrangements of 6 persons = 6! = 720. But if two particular persons are to be together always then we tie these two particular parsons with a string. Thus we have 6 - 2 + 1 (1 unit corresponding to these two particular persons together) = 4 + 1 = 5 units, which can be arranged in 5! = 120 ways. Those two particular persons themselves can be arranged in 2! ways.

∴ Total no. of ways to arrange 6 persons in which 2 particular persons are always together = 5!. 2! = 120 × 2 = 240

∴ Total no. of ways in which 2 particular persons never together = Total - Together 720 - 240 = 480

Note:- If can also easily be solved by GAP Method

Some units are alike

Total no. of permutations of "n" things taking all at a time when "P" of them are alike of one kind.

"Q" are alike of another kind and rest "n-(P+Q)" things are different $= \dfrac{n!}{P!Q!}$.

Example 1 : How many different words can be made from the letters of the word CALCULUS?

(*a*) 5040 (*b*) 7050

(*c*) 2040 (*d*) None

Solution : (*a*) is correct

Total no. of letters = 8

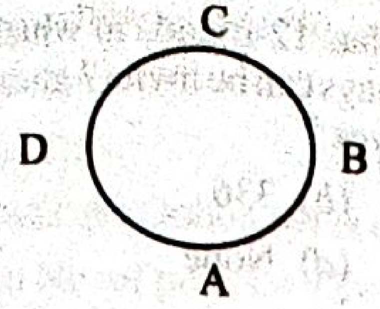
Alike letters are "C" 2 times ; "U" 2 times "L" 2 times.

$\therefore$ Total no. of words =

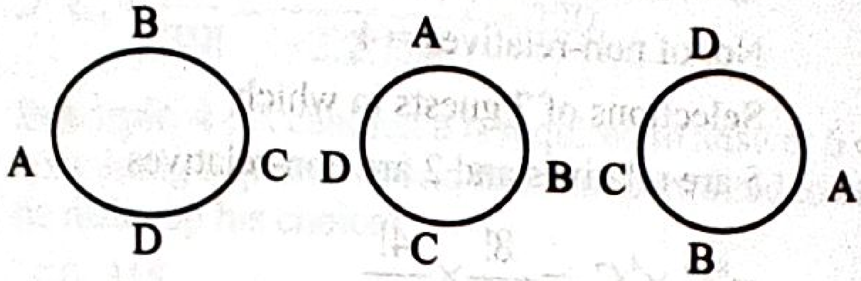
$$\frac{8!}{2!2!2!} = \frac{8.7.6.5.4.3.2!}{2!.2.1.2.1} = 8.7.6.5.3 = 5040.$$

Circular Permutations

Suppose, we have four A, B, C, D to be seated in circle. Let us look at one such circular arrangemer shown alongside.



Now, if A, B, C, D are shifted by one position in any or particular direction, say, in the clock-wise sense, w get the following arrangement.



We can see that all the above four arrangements ar identical, since the relative position of A, B, C, D is th same. But in case the four persons were to be seated i a row, then above four arrangements would have bee

ABCD BCDA CDAB DABC

Thus, it is clear that corresponding to four differer linear arrangement there will be only one circula arrangement. Hence, the total number of circular ar rangement in the above case

$$= \frac{4!}{4} = 3! = (4-1)!$$

In general, the number of circular permutation of different things, is $= (n - 1)!$

When clockwise and anticlockwise arrangement ar not different, the number of circular permutation of

different things $= \dfrac{(n-1)!}{2}$

Example 1 : In how many ways can 5 men and women be seated at a round table if :

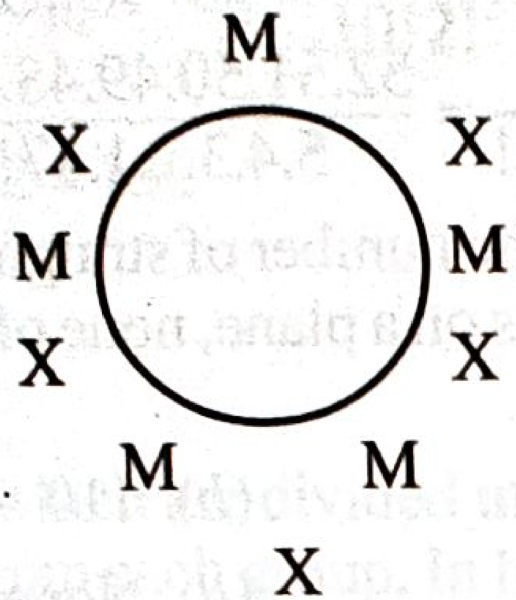
(*a*) there is no restriction.

(*b*) all the five women sit together

(*c*) no two women sit together.

(*d*) not more than four women sit to...

In this case, five women sitting together is prohibited. Hence, the required number of ways = number of seating ways without restriction - number of seating ways in which the five women sit together = $9! - (5!)^2$.



mple 2 : In how many ways can we place apples circle?

) 720 (*b*) 360

) 240 (*d*) None

lution : (*b*) is correct.

tal no. of ways to place apples in a circle

$\frac{1}{2}.6! = \frac{720}{2} = 360$.

TYPE IV

[Some units always included or excluded]

Formula.

A. (*i*) Total no. of combinations of "n" different things taking "r" at a time $= {}^nC_r = \frac{n!}{r!(n-r)!}$,

Where $0 \le r \le n$

(*ii*) Total No. of permutations of "n" different things taken "r" at a time $= {}^nP_r = {}^nC_r . r!$

B. (*i*) Total No. of combinations of "n" different things taken "r" at a time so that "P" particular things are always included $= {}^{n-p}C_{r-p}$

(*ii*) Total No. of permutations of "n" different things taken "r" at a time in which "P" particular things are always included $= {}^{n-p}C_{r-p} . r!$

C. (*i*) Total No. of combinations of "n" different things taken "r" at a time in which "p" particular things are never included $= {}^{n-p}C_r$.

TYPE VII

(Formula)

1. (*i*) Total no. of ways of selecting zero or more things from "n" identical things = n + 1

(*ii*) Total no. of ways of selecting one or more things from "n" identical things = n.

2. (*i*) Total no. of ways of selecting zero or more things from "m" identical things and "n" identical things = (m+1) (n+1)

(*ii*) Total no. of ways of selecting atleast one thing from "m" identical things and "n" identical things =(m + 1) (n + 1) - 1

(*iii*) Total no. of ways of selecting atleast one from each groups of "m" identical things and "n" identical things = m.n.

Example 1 : In how many ways can zero or more letters be selected from the letters AAAAA.

(*a*) 4 (*b*) 5

(*c*) 6 (*d*) None

Solution : (*c*) is correct

Total no. of ways of selecting letters = 5 + 1 = 6 ways.

Example 2 : From 5 apples, 4 oranges and 3 mangoes how many selections of fruits can be made ?

(*a*) 120 (*b*) 119

(*c*) 118 (*d*) None

Solution : (*b*) is correct

Here, we assume all 5 apples alike, all 4 oranges alike and all 3 mangoes alike.

∴ Total no. of ways of selecting one or more fruits

$= (5 + 1) \times (4 + 1) \times (3 + 1) - 1 = 6 \times 5 \times 4 - 1 = 119.$

Example 3 : Find the number of divisors of 21600

(*a*) 72 (*b*) 76

(*c*) 71 (*d*) None

Solution : (*a*) is correct

(Division into Groups)

(*i*) The number of ways in which m + n things can be divided into two groups containing m and n things respectively = $= \frac{(m+n)!}{m!n!}$

(*ii*) If m = n; the groups are equal and in this case the number of different ways of sub-division $= \frac{2m!}{m!.m!.2!}$ for any one way it is possible to interchange the two groups without obtaining new division.

(*iii*) If 2m things are to be divided equally between two persons then the number of divisions = $= \frac{2m!}{m!.m!}$

(*iv*) The number of divisions of (m + n + p) things into their own groups respectively $= \frac{(m+n+p)!}{m!.n!.p!}$

(*v*) If 3m things are divided equally among 3 equal groups, then the number of divisions $= \frac{3m!}{m!.m!.m!3!}$

(*vi*) If 3m things are to be divided among 3 persons equally; then the number of divisions $= \frac{3m!}{m!.m!.m!}$

Example 1 : In how many ways 12 different books can be distributed equally among 4 persons?

(*a*) $\frac{12!}{(3!)^4}$ (*b*) $\frac{12!}{(3!)^4.4!}$

(*c*) $\frac{12!}{(3!)^3}$ (*d*) None.

Formula

1. ${}^nC_r = c(n;r) = \dfrac{n!}{r!(n-r)!}$, where $0 \le r \le n$.

2. ${}^nC_r = \dfrac{1}{r!} \cdot {}^nP_r$

3. ${}^nP_r = r!\, {}^nC_r$.

4. ${}^nC_r = {}^nC_{n-r}$

5. If ${}^nC_r = {}^nC_k$ Then either

   (i) $r = k$

   or (ii) $r + k = n$.

6. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

7. ${}^nC_o = {}^nC_n = 1$

8. ${}^nC_1 = {}^nC_{n-1} = n$.

Example 1 : The value of ${}^{12}C_9$ is

(*a*) 220 (*b*) 400

(c) 505 (*d*) None.

(Sum of Numbers)

Tricks:-

(*i*) The sum of the digits in the units place of all numbers from the digits d_1, d_2, d_3 ;.......;d_n, taken all at a time and without repetition is

$$= (n-1)!.(d_1 + d_2 + d_3 + \ldots\ldots\ldots\ldots + d_n)$$

(*ii*) The sum of all the numbers that can be formed using the digits d_1; d_2; d_3; ....; d_n without repetition is $= (n-1)!.(d_1 + d_2 + d_3 + \ldots\ldots\ldots + d_n)$

$$= \left(\frac{10^n - 1}{9}\right)$$

(*iii*) Sum of digits d_1 only in all numbers at unit place

or tens place or any place $= (n-1)!d_1.\left(\frac{10^n - 1}{9}\right)$

Example 1. 2, 4, 6, 8,..........

Example 2. 5, 10, 20, 40,..........

Example 3. 256, 128, 64, 32,............

Note:- In sequence all terms are separated by comma (,).

Series:- Summation of all terms of the sequence is called **series.**

Example 1. S = 2 + 4 + 6 + 8 +...........

Example 2. S = 1 + 4 + 9 + 16 +...........

PROGRESSIONS

A progression is a series that advances in a logical and predictable pattern.

Types of Progressions

1. Arithmetic Progression (AP)
2. Geometric Progression (GP)
3. Harmonic Progression (HP)

1. Arithmetic Progression (AP)

If the difference between two consecutive terms of a series is always equal, then that series is called an **Arithmetic Progression (AP)**

Here - this difference "d" is called the common difference of the sequence.

For example 3, 5, 7, 9,........

Here, common difference d = 5 - 3 = 7 - 5 = 9 - 7 = ...= 2

TYPE I

n^{th} term of an arithmetic progression

(i) If m^{th} term of an A.P. is P and n^{th} term is q then

$$c.d. = \frac{p-q}{m-n}$$

Example. If 5^{th} term and 12^{th} terms of an AP are 14 and 35 respectively. Find its common difference.

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) None

Solution :- (b) is correct.

Let 1st term = a and c.d. = d of an A.P.

$\therefore t_5 = a+(5-1)d = 14 \Rightarrow a+4d = 14$ ....(1)

$t_{12} = a+(12-1)d = 35 \quad or \quad a+11d = 35$ .....(2)

Eqn. (2) - Eqn. (1) ; we get

$$\begin{array}{r} a+11d = 35 \\ -a \pm 4d = \_14 \\ \hline 7d = 21 \end{array} \quad \therefore d = 3. \& a = 2$$

(ii) If m^th term of an A.P. is P and n^th term is q then r^th term is

t_r = P +(r - m)d.

or t_r = q + (r - n)d.

Example. If 5^th term and 12^th terms of and AP are 14 and 35 respectively, Find 25^th term.

(*a*) 74 (*b*) 75

(*c*) 73 (*d*) None

[Sum of "n" terms of an AP.]

Formula

(i) $S=\frac{n}{2}[2a+(n-1)d]$

(ii) $S=\frac{n(a+L)}{2}$

Where $L = t_n = a + (n - 1)d$.

TYPE V

For Consecutive Terms

(A) For odd number of terms c.d = d.

Ass. a - d ; a ; a + d 3 consecutive terms

a - 2d, a - d, a, a + d, a + 2d ; 5 consecutive terms

(B) For Even no. of terms c.d. = 2d

....

Ass. a - 3d ; a-d ; a + d; a+3d 4 consecutive term

a-5d ; a - 3d ; a-d ; a + d ; a+3d ; a+5d

6 consecutive term

....

(Properties of AP)

(*i*) If a constant quantity is added to or subtracted from every term of an AP then the resulting series is also in AP.

Example. $S = 2 + 5 + 8 + 11 + \dots\dots$ in AP.

Adding 10 to each term ; we get

$S_1 = 12 + 15 + 18 + 21 + \dots\dots\dots\dots$ also in AP.

(*ii*) If all terms of an A.P. are multiplied or divided by a constant quantity then the resulting series is also in A.P.

Example. $S = 2 + 4 + 6 + 8 + \dots\dots\dots\dots$ in AP. Multiplying each term by 5; we get

$S_1 = 10 + 20 + 30 + 40 + \dots\dots$ are also in A.P.

(*iii*) If corresponding terms of two A.Ps. are added or subtracted then the resulting series is also in AP.

Example. $S_1 = 2 + 4 + 6 + 8 + \dots\dots\dots\dots\dots\dots$ in AP.

$S_2 = 3 + 6 + 9 + 12 + \dots\dots\dots\dots\dots\dots$ in AP.

$S = 5 + 10 + 15 + 20 + \dots\dots\dots\dots\dots\dots$ in AP.

[Corresponding terms are added]

Example 1. If a, b, c are in AP then the value of is

$$\frac{(a^3 + 4b^3 + c^3)}{b(a^2 + c^2)} \text{ is}$$

Dividing each term by $(a+b+c)$; we get

$\frac{1}{a}; \frac{1}{b}; \frac{1}{c}$ are in AP

So, a; b; c are in HP.

$\therefore$ (c) is correct.

TYPE VII

Arithmetic Mean

I. Arithmetic Mean (A.M.) :- The number "A" is said to be A.M. between a and b

So, a, A, b ae in AP.

$\therefore$ its common difference is same

So, $\quad A - a = b - A$

or $\quad A + A = a + b$

or $\quad 2A = a + b$

$$\therefore A = \frac{a+b}{2}$$

III. Sum

$= A_1 + A$

Exampl
that the

(*a*) 2

(*b*) -2,

(*c*) 4;

(*d*) -4;

Solutio

Tricks:

Option
sum is

(c) is al
equal to

Solution :- (a) is correct

$$AM = \frac{2+18}{2} = 10.$$

II. Insertion of **"n" Arithmetic Means** between two unequal quantities.

Let $A_1 ; A_2 ; A_3 ; \ldots\ldots\ldots\ldots A_n$ are n AMs. between a and b.

$\therefore$ a, $A_1 ; A_2 ; A_3 ; \ldots\ldots\ldots ; A_n$; b are in AP.

Total no. of terms = n + 2 last term = t_{n+2} = b. Let common difference = d.

$$\therefore t_{n+2} = a + (n+2-1)d = b$$

$$or \quad (n+1)d = b - a$$

$$or \quad d = \frac{b-a}{n+1}$$

1^{st} A.M. = A_1

$$= t_2 = a + (2-1)d = a + d = a + \frac{b-a}{n+1}$$

$$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$$

....

....

$$An = a + nd = a + n\left(\frac{b-a}{n+1}\right)$$

III. Sum of n - AMs.

$$= A_1 + A_2 + A_3 + \ldots\ldots\ldots\ldots + An = n\left(\frac{a+b}{2}\right)$$

Example 2. Divide 12.50 into five parts in A.P. such that the first part and the last part are in the ratio 2 : 3.

(a) 2 ; 2.25; 2.50; 2.75; 3

n^{th} term of a Geometric Progressions

n^{th} term of a G.P $= t_n = a.r^{n-1}$

Where a = 1^{st} term

r = common ratio

n = No. of terms.

Example 1. Find 16th term of the series $S = 2 + 4 + 8 + \ldots\ldots\ldots\ldots\ldots$

(*a*) 65536 (*b*) 64536

(*c*) 66536 (*d*) None

Solution : (a) is correct.

Detail Method.

Given that a = 2 ; c.r = r = 2

$\therefore t_n = ar^{n-1} \therefore t_{16} = 2.2^{16-1} = 2^{15} = 65536.$

Calculator Trick

I Sum of n terms of a G.P.

$$S=\frac{a(r^n-1)}{r-1}; \quad when \ r>1$$

II $$S=\frac{a(1-r^n)}{1-r}; \quad when \ r<1$$

III $$S_\infty=\frac{a}{1-r}; \quad when \ r<1$$

Example 1 Th...

(a) $\frac{353}{1650}$ (b) $\frac{367}{1650}$

(c) $\frac{359}{1650}$ (d) None

Solution :- (c) is correct

Tricks:- $0.21\overline{75} = 0.21757575$........

Go by choices & the check the result For (c) 359 ÷ 1650 = 0.21757575.................

Clearly (c) is the correct ans.

TYPE III (CONSECUTIVE TERMS)

when No. of terms = odd Then c.r. = r

(i) $\frac{a}{r}; a; ar$ are 3 consecutive terms

$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$ are 5 consecutive terms

(*ii*) When No. of terms = even.

Then c.r. = r^2 (Let)

$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$ are 4 consecutive terms

$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$ are 6 consecutive terms

TYPE V (GEOMETRIC MEANS)

Definition:-

(*i*) If a, G, b are in G.P. ; Then G is called the GM of a & b

$\therefore$ a, G, b are in G.P.

$\therefore$ Common ratio $= r = \dfrac{G}{a} = \dfrac{b}{G}$ or $G^2 = ab$ $G = \sqrt{ab}$

Example. Find G.M. of 3 and 27.

Solution :- $G.M = \sqrt{3\times 27} = 9$

(*ii*) Insertion of n G.Ms. between a and b.

Let G_1 ; G_2 ; G_3 ; ....; G_n are n GMs. between a and b.

$\therefore$ a; G_1 ; G_2 ; G_3 ; ....; G_n ; b are in G.P.

First term = a

Let common ratio = r.

Total no. of terms = n + 2

$\therefore (n+2)^{th}$ term = b.

$$or\ \ a.r^{n+2-1} = 6 \quad or\ \ r^{n+1} = b/a \quad or\ \ r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$\therefore G_1 = t_2 = a.r^{2-1} = ar = a.\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = t_3 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$G_n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}.$$

Example. Four Geometric Means between 4 and 972 are

(*a*) 12, 30,100; 324 (*b*) 12, 24,108, 320

(*c*) 10, 36,108, 320 (*d*) 12, 36, 108, 324

Solution :- (d) is correct

Tricks:-

G.Ms are also in G.P.

HARMONIC PROGRESSION (HP)

Definition:- A series of numbers is said to be Harmonic progression when their reciprocals are in arithmetic progression (AP.)

Example. $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots\dots\dots$ are in HP because their reciprocals 3, 5, 7,..........are in AP.

Note:- Questions of HP are first converted into AP and then it is proceed.

HARMONIC MEAN (HM)

Definition:- The number H is said to be HM of two numbers a and b if a, H, b are in HP.

Harmonic mean of two numbers a & b is formulated as $H = \frac{2ab}{a+b}$

Relationship between AM ; GM and HM

If A, G and H are Arithmetic Mean; Geometric mean and Harmonic mean of two numbers a and b respectively then

(i) $A = \frac{a+b}{2}$

$G = \sqrt{ab}$

$H = \frac{2ab}{a+b}$

(ii) $AM \geq G.M \geq HM$.

(iii) AM; GM and HM are in GP.

$\therefore$ A, G, H are in G.P.

$\therefore$ $G^2 = AH$.

or $G = \sqrt{AH}$

Example 1. If AM and HM of two numbers are 32 and

Tricks:-

$\because$ a, x, b are in AP $\Rightarrow$ x is the AM of "a" and "b"

$\because$ a, y, b are in G.P. $\Rightarrow$ y is the GM of "a" and "b"

$\because Z = \frac{2ab}{a+b} \Rightarrow Z$ is the HM of a & b.

From properties.

AM ; GM & HM are in G.P.

So, x, y, z are in G.P.

and $AM \geq GM \geq HM$ So $x \geq y \geq z$

$\therefore$ (c) is correct

SUMMATION OF SERIES

I Notation Σ (sigma) means Summation.

II $\sum_{r=1}^{n} t_r = t_1 + t_2 + t_3 + ... + t_n$

= Summation of 1st n terms of the series.

III $\sum 1 = 1+1+1+............$ to n times = n.

$\sum 2 = 2+2+2+............$ to n times = 2n.

IV Sum of 1st n natural numbers

$$= 1+2+3+........+n = \sum n = \frac{n(n+1)}{2}$$

Proof:-

S = 1 + 2 + 3 + ...+ n are in AP.

1st term = a = 1 ; c.d. = 1 last term l = n.

$$\therefore S = \frac{n}{2}(a+l) = \frac{n}{2}(1+n) = \frac{n(n+1)}{2}$$

Note:- Proof not required.

V. Sum of squares of 1st n natural numbers

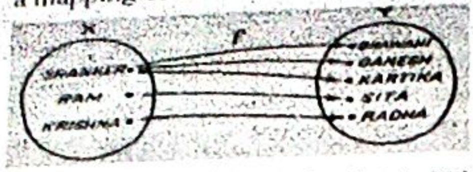
$$\sum n^2 = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

VI. Sum of cubes of 1st n natural numbers =

$$1^3 + 2^3 + 3^3 + \dots\dots\dots\dots + n^3 = \sum n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

The correspondence shown above also describes a mapping of the set X into the set Y.

(iii) The following correspondence does not describe a mapping of the set X into the set Y.

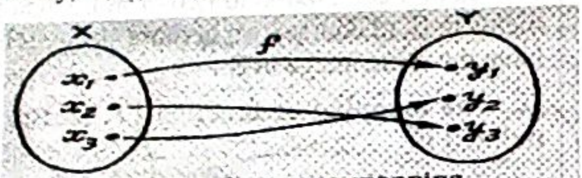


This is so because an element Shanker in X is associated with three different elements Bhawani, Ganesh and Kartika in Y.

TYPES OF MAPPINGS

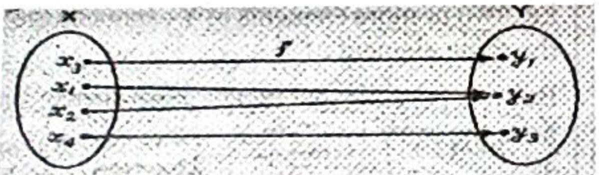
Let $f : X \rightarrow Y$.

(i) The mapping f is said to be one-one if different elements in X have different f-images in Y, i.e., if $x_1 \neq x_2; x_1, x_2 \in X \Rightarrow f(x_1) \neq f(x_2)$ or equivalently, $f(x_1) = f(x_2); x_1, x_2 \in X \Rightarrow x_1 = x_2$.



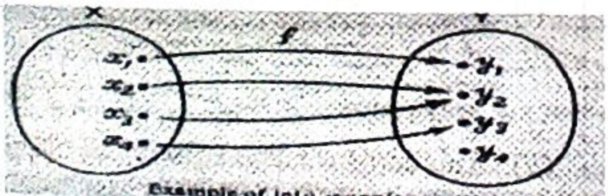
Example of one-one mapping

(ii) The mapping f is said to many-one if two or more different elements in X have the same f-image in Y, i.e., if $(x_1) = f(x_2); x_1, x_2 \in X \Rightarrow x_1 \neq x_2$.



Example of many-one mapping

(iii) The mapping f is said to be into if there is at least one element in Y which is not the f-image of any element in X In such mapping f-image of X is a proper subset of Y, i.e., $\{f(x)\} \subset Y, x \in X$.



Example of into mapping

(iv) The mapping is said to be onto if every element in Y is the f-image of at least one element in X. In such mapping f-image of X is equal to Y, i.e., $\{f(x)\} \subset Y, x \in X$.

Example of onto mapping

Hence mapping can be of the following four types

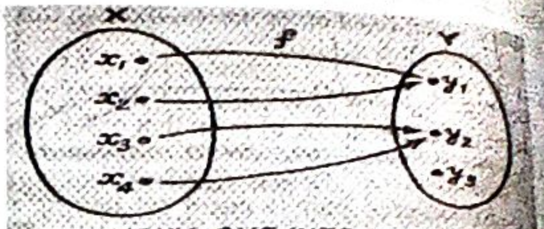
(i) Many-one into mapping,

(ii) Many-one onto mapping,

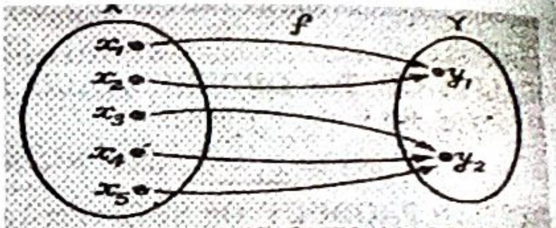
(iii) One-one into mapping, and

(iv) One-one onto mapping.

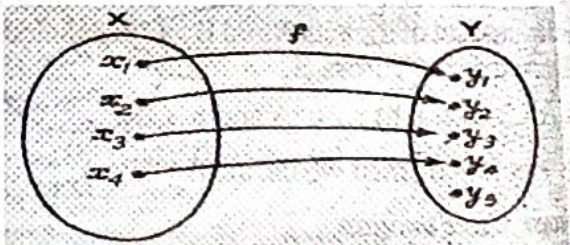
The following diagrams will make mappings clear:



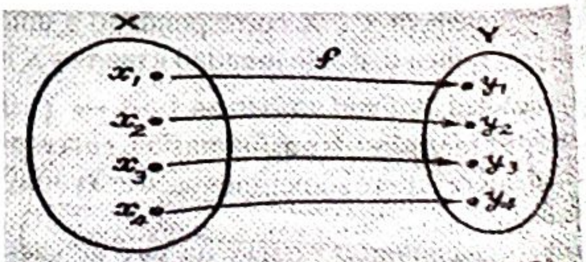
MANY-ONE INTO MAPPING



MANY-ONE ONTO MAPPING



ONE-ONE INTO MAPPING



ONE-ONE ONTO MAPPING

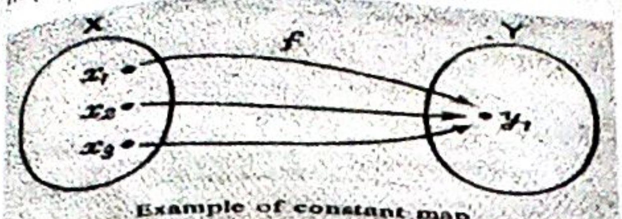
Note : - An onto mapping is also called a **Surjection**.

A one-one mapping is also known as **Injection**.

A one-one onto mapping is also known as **Bijection**.

CONSTANT MAP OR CONSTANT FUNCTION

Let X and Y be two non-empty sets. Then the map $f: X \to Y$ given by $f(x) = c \, \forall x \in X$ is called a constant map.

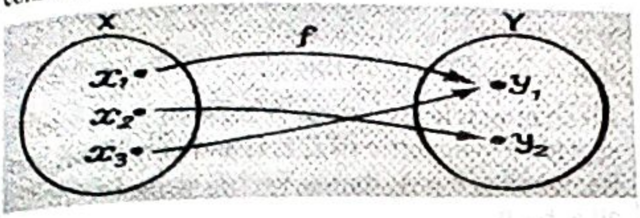


Example of constant map

Here $f(x_1) = f(x_2) = f(x_3) = y_1$.

So, f is a constant function.

The map f defined by the diagram given below is not a constant function.



EQUAL MAPPINGS OR EQUAL FUNCTIONS

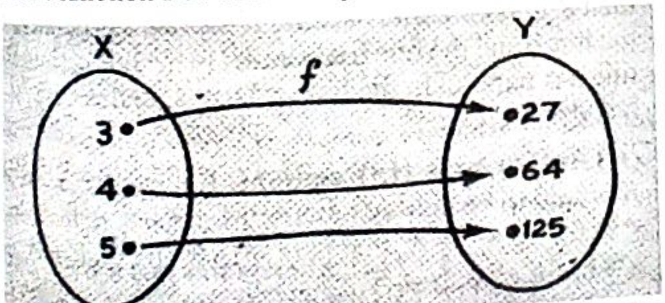
Let X, Y and Z non-empty sets, and $f: X \to Y$; $g: X \to Z$ be two maps.

Then the mappings f and g are said to be equal if and only if $f(x) = g(x), \forall x \in X$.

In case of equal mappings, the domains of mappings must be the same.

Example

Let a function f be defined by the following diagram.



Let g be a function defined by the formula $g(x) = x^3$ with {3, 4, 5} as its domain. Then f = g.

IDENTITY MAPPING

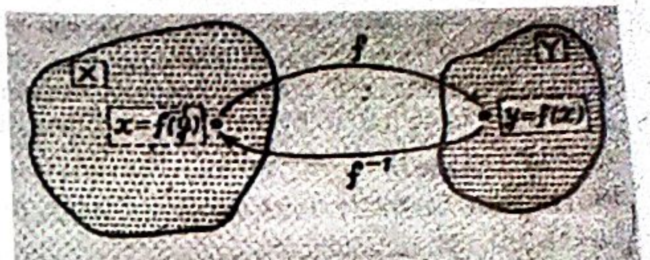
If each element of a set X is mapped onto itself, then such a mapping is called identity mapping and is denoted by I_x.

Symbolically, $I_x : X \to X$ is the identity map given by $I_x(x) = x, \forall x \in X$.

INVERSE FUNCTION OR INVERSE MAPPING

If $f : X \to Y$ is both onto and one-one, then we can define its inverse mapping $f^{-1} : X \to Y$ as follows : for each y in Y, we find that unique element x in X such that f(x) = y (x exists and is unique since f is onto and one-one), we then define x to be $f^{-1}(y)$.

The figure given below illustrates the concept of the inverse of a mapping.



The inverse of a mapping

Inverse function definition

An inverse function is a function that undoes the action of the another function. A function g is the inverse of a function f if whenever y = f(x) then x = g(y). In other words, applying f and then g is the same thing as doing nothing. We can write this in terms of the composition of f and g as g(f(x)) = x.

A function f has an inverse function only if for every y in its range there is only one value of x in its domain for which f(x)=y. This inverse function is unique and is frequently denoted by f^{-1} and called "f inverse."

Example. If $f(x) = \dfrac{2+x}{2-x}$, then $f^{-1}(x)$:

(a) $\dfrac{2(x-1)}{x+1}$ (b) $\dfrac{2(x+1)}{x-1}$

(c) $\dfrac{x+1}{x-1}$ (d) $\dfrac{x-1}{x+1}$

[June 2008]

Solution (a) $\because f(x) = \dfrac{2+x}{2-x} = y(let)$

$2 + x = 2y - xy$

or, $x + xy = 2y - 2$

or, $x(1 + y) = 2(y - 1)$

or, $x = \dfrac{2(y-1)}{1+y}$

Some Important Limits

1. $\lim_{x\to 0}\frac{(e^x-1)}{x}=1$

2. $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a(a>0)$

3. $\lim_{x\to 0}\frac{\log(1+x)}{x}=1$

4. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$

5. $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$

6. $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x=e^a$

7. $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

8. $\lim_{x\to 0}\frac{(1+x)^n-1}{x}=n$

9. $e^x\to\infty, as\ x\to\infty$

10. $e^{-x}\to 0, as\ x\to\infty$

11. $\lim_{x\to 0}\frac{1}{x}$ does not exist.

exist.

Some Results on Limits

The calculation of limits is based on the following results:

1. $\lim_{x \to c}(f(x) \pm g(x)) = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x).$

2. $\lim_{x \to c}[kf(x)] = k \lim_{x \to c} f(x)$ where k is a scalar.

3. $\lim_{x \to c}(f(x).g(x)) = \lim_{x \to c} f(x).\lim_{x \to c} g(x).$

4. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c}} (provided \lim_{x \to c} g(x) \neq 0).$

Concept of Differentiation

The rate of change of function of one variable with respect to another on which it depends is called the **derivative of the function.** The process of finding the derivative in terms of a limit involving the increments of the independent 'Δx' and the dependent variables 'Δy' is called **differentiation.**

Let $y = f(x)$ be a function of x.

$\therefore$ The rate of change is given by

$$\frac{\Delta y}{\Delta x} = \frac{\text{increment in the value of } y \text{ (dependent variable)}}{\text{increment in the value of } x \text{ (independent variable)}}$$
$$= \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

If this ratio tends to a definite finite limit as Δx tends to zero from either side, then this limit is called the differential coefficient (or derivative) of $f(x)$ with respect to x. Symbolically, the differential coefficient of y with respect to x is denoted by

$$\frac{dy}{dx} \text{ or } f'(x) = y_1 \text{ or } \frac{d}{dx}[f(x)] \text{ or } Df(x).$$

Thus, $$\frac{dy}{dx} = f'(x) = y_1 = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The process of finding the differential coefficient is known as **differentiation.**

Example Consider the function, $y = f(x) = x^2$.

By definition,

$$\frac{dy}{dx} = \frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x + 0 = 2x.$$

SOME STANDARD RESULTS (FORMULAE)

1. $\frac{dy}{dx}(x^n) = nx^{n-1}$

2. $\frac{d}{dx}(e^x) = e^x$

3. $\frac{d}{dx}(a^x) = a^x \log_e a$

4. $\frac{d}{dx}$ (constant) = 0

5. $\frac{d}{dx}(e^{ax}) = ae^{ax}$

6. $\frac{d}{dx}(\log x) = \frac{1}{x}$

7. $\frac{d}{dx}(\log_a x) = \frac{1}{x} \log_a e$

Type I (Scalar Multiple Rule)

Formula. $\frac{d}{dx}\{kf(x)\} = k\frac{df(x)}{dx}$

Example 1. Differentiate the following with respect to x.

(*i*) $5x4$ (*ii*) $3 \log x$

(*iii*) $7e^x$ (*iv*) $3\sqrt{x}$

(*v*) 4.2^x

Solution :-

(*i*) $\frac{d(5x^4)}{dx} = 5.\frac{dx^4}{dx} = 5 \times 4x^3 = 20x^3$

(*ii*) $\frac{d}{dx}\frac{(3\log x)}{} = 3.\frac{d}{dx}\log x = 3 \times \frac{1}{x} = \frac{3}{x}$

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$ Where u and v are functions of x.

Type III (Product Rule)

$$\frac{duv}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$ Where u and v are functions of x.

Example 1. Differentiate the following functions

Type IV Quotient/Division Rule

Formula

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

Where u and v are functions of x.

(III) Formula

(*i*) $$\frac{da^x}{dx} = a^x.\log_e a.$$

(*ii*) $$\frac{da^{f(x)}}{dx} = a^{f(x)}.\log a.\frac{df(x)}{dx}$$

Example 1. Differentiate the following f...

Applications of Differential Calculus in Business

Demand Function. : It is a relationship between demand and price of a commodity. If the price of a commodity is 'x' and its demand is 'y', then this fact can be expressed mathematically as

$y = f(x)\ ;\ x, y > 0$

It can also be expressed as $x = g(y)$

Cost Function. The amount spent on the production of a commodity is called its cost. Cost of production of x units 'C' of a commodity can be expressed as $C = f(x)$

Total cost = TC = Fixed Cost + Variable Cost

Variable Cost = VC = (variable cost per unit). (No. of units produced)

$$\text{Average Cost} = AC = \frac{TC}{x}$$

$$\text{Average Variable Cost} = AVC = \frac{VC}{x}$$

$$\text{Marginal Cost} = MC = \frac{dC}{dx}$$

$$\text{Marginal Variable Cost} = MVC = \frac{dVC}{dx}$$

$$\text{Marginal Average Cost} = MAC = \frac{dAC}{dx}$$

Revenue Function. It is the revenue obtained by selling x units produced.

Let No. of units sold = x & selling price per unit = p

R(x) = p.x ; where p ; x > 0

$$\text{Average Revenue} = AR = \frac{R(x)}{x} = p$$

$$\text{Marginal Revenue} = MR = \frac{dR(x)}{dx}$$

$$\text{Marginal Average Revenue} = \frac{dAR}{dx}$$

Profit Function. The excess of total revenue over the total cost of production is called the profit and is denoted by P.

P(x) = R(x) - C(x), x > 0.

where, R(x)= revenue function and

C(x) = cost function.

Average Profit = AP = AR - AC

$$\text{Marginal Profit} = MP = MR - MC = \frac{dP}{dx}$$

CONSUMPTION FUNCTION, MPC AND MPS

Consumption Function. The consumption function expresses a relationship between the total income (I) and the total national consumption (C). It is denoted by C=f(I).

Marginal Propensity to consume. It is the rate of change of the consumption with respect to income.

$$\therefore \text{Marginal Propensity to Consume} = MPC = \frac{dC}{dI}.$$

Marginal propensity to Save.

Let S denote the saving, then Saving S = (Total income - Total consumption) = I - C

$$\therefore \textbf{Marginal propensity to Save} = \textbf{MPS} = \frac{dS}{dI}.$$

It indicates how fast saving changes with respect to income.

Q.1. The total cost function of a commodity is given by $C(x) = 0.5x^2 + 2x + 20$. Where C denotes the total cost and x denotes the quantity produced. Find the average cost and the marginal cost.

Ans. $\textbf{AC} = \textbf{Average Cost} = \dfrac{0.5x^2 + 2x + 20}{x}$

$= 0.5x + \dfrac{20}{x} + 2$

50 units are produced. Also interpret the result.

Ans. Total Cost = Average Cost × Quantity Produced

AVERAGE REVENUE ; MARGINAL REVENUE & MARGINAL REVENUE PRODUCT

Q.4. Let p be the price per unit of a certain product, when there is a sale of q units. The relation between p and q is given by $p = \dfrac{100}{3q+1} - 4$

(*i*) Find the marginal revenue function.

(*ii*) When q = 10, find the relative change of R, i.e., (Rate of change of R with respect to q)/R and also the percentage rate of change of R at q = 10.

ated is approximately 20. ... revenue gener...

CONSUMPTION FUNCTION, MPC AND MPS

Q.12. If the consumption function if given by $C=9+9I^{3/2}$, determine the marginal consumption function.

Ans. ∴ Marginal consumption function =

$$\frac{dC}{dI}=\frac{27}{2}I^{1/2}$$

Q.13. The consumption function C=f(I) gives relationship between the total income (I) and the total consumption (C). What are marginal propensity to consume' and 'marginal propensity to save.' If $C=5\sqrt[3]{I}$, determine the marginal propensity to save when $I=\sqrt{27}$

Ans. $C=5I^{1/3}$ and $S=I-C=I-5I^{1/3}$ When $I=\sqrt{27}$,

$$\text{MPS}=\frac{dS}{dI}=1-\frac{5}{3(\sqrt{27})^{2/3}}=1-\frac{5}{3\times 3}=\frac{4}{9}$$

Q.14. If the consumption is given by $C=7I+15\sqrt{I}$, where I is the income. When I = 25

(*a*) determine the marginal propensity to consume;

(*b*) marginal propensity to save.

Ans. (*a*) When I=25, then the marginal propensity to

$$\text{consume}=\left.\frac{dC}{dI}\right]_{I=25}\left.7+\frac{8}{\sqrt{7}}\right]_{at\ I=25}=7+\frac{8}{5}=\frac{43}{5}$$

(*b*) Also marginal propensity to save =

$$\text{When I= 25, then }\left.\frac{dS}{dI}\right]_{at I=25}=-6-\frac{8}{\sqrt{25}}$$

$$=-6-\frac{8}{5}=-\frac{38}{5}.$$

of f(x) or simply integral of $f(x)$. ... written as $\int f(x)dx = F(x)$.

The process of finding the integral is called Integration and f(x) is called integral and x is called variable of integration.

Constant of Integration

If $\frac{d}{dx}F(x) = f(x)$; then we also have

$\frac{d}{dx}[F(x)+c] = f(x)$; where c is an arbitrary constant.

Thus we may write $\int f(x)dx = F(x)+c$

Example: (*i*) $\frac{d}{dx}\frac{(x^2+c)}{} = 2x \quad \therefore \int 2xdx = x^2+c$

where c = Arbitrary or Integration constant.

(*ii*) $\frac{d}{dx}\log x = \frac{1}{x}; \quad \therefore \int \frac{1}{x}dx = \log x + c$

(*iii*) $\frac{de^x}{dx} = e^x; \therefore \int e^x dx = e^x + c.$

(*iv*) $\frac{da^x}{dx} = a^x \log_e a; \therefore \int a^x dx = \frac{a^x}{\log_e a} + c$

Some Important Results /Formula

1. (*i*) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ where $n \neq -1$.

(*ii*) $\int (ax+d)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c$ where $n \neq -1$

2. (*i*) $\int e^x dx = e^x + c$

(ii) $\int e^{ax+b}dx = \frac{e^{ax+b}}{a} + c$

3. (i) $\int \frac{1}{x}dx = \int x^{-1}dx = \log_e x + c$

(ii) $\int \frac{1}{ax+b}dx = \int (ax+b)^{-1} = \frac{\log_e(ax+b)}{a} + c$

4. (i) $\int a^x dx = \frac{a^x}{\log_e a} + c$

(ii) $\int a^{mx+n}dx = \frac{a^{mx+n}}{m\log_e a} + c$

Where c = Arbitrary / Integration constant.

where c = I . c.

TYPE V (RULES FOR INTEGRATION)

I. Scalar multiple rule.

$\int kf(x)dx = k\int f(x)dx$; where k = constant.

II. Addition Rule.

$$\int (u \pm v)dx = \int udx \pm \int vdx$$

where u and v are functions of x.

Example : Integrate the following functions

(*i*) 8^{x7} (*ii*) $4x^3 - \frac{1}{x} + 5$

Solution:

(*i*) $\int 8x^7 dx = 8\int x^7 dx = 8.\frac{x^8}{8} + c = x^8 + c$

where c = I.c

(*ii*) $\int\left(4x^3 - \frac{1}{x} + 5\right)dx$

$= 4\int x^3 dx - \int \frac{1}{x}dx + \int 5dx$

$= 4.\frac{x^4}{4} - \log x + 5x + c$

$= x^4 - \log x + 5x + c$

where $c =$ integration constant.

Methods of Integration

(*i*) Transformation Method
(*ii*) Substitution Method
(*iii*) Integration by parts
(*iv*) Integration by partial fraction.

Formula :- $\int uv\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$

Where u and v are two functions of x

SOME SPECIAL INTEGRALS

1. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right)(x > a)$

2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left(\frac{x+a}{x-a}\right)(x < a)$

3. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left[x + \sqrt{x^2 - a^2}\right]$

4. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left[x + \sqrt{x^2 + a^2}\right]$

5. $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left[x + \sqrt{x^2 + a^2}\right]$

6. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left[x + \sqrt{x^2 - a^2}\right]$

7. $\int e^x [f(x) + f'(x)] dx = e^x f(x). + c$

Applications of Integral calculus

1. $C = \int MC dx + K$

Where K is an arbitrary constant of integration and it can be evaluated if the fixed cost (i.e., the cost when x = 0 or the total cost at any arbitrary value is given.)

2. $AC = \int MAC dx.$

3. $VC = \int MVC dx.$

4. $R = \int MR dx. + K$

where k is an arbitrary constant of integration. But when x =0, then R = 0 (i.e., when output is zero, then the revenue is also zero)

5. $AR = \int MAR dx.$

the difference between them. The two types of information for each year are placed in such a way that a comparison may be made between them. So a grouped bar diagram is drawn in order to represent the given data.

III. PIE DIAGRAM OR CIRCLE DIAGRAM OR ANGULAR DIAGRAM

A circular graph which represents the total value with its components is a Pie Diagram. Area of a circle shows the total value and its each sectors of the circle shows

and between ... *diagram, the data can also be* ... *form.* A pie diagram is also known as A...

In determining the circumference of a ... quantity 'pie' (written as π) that's why ... Diagram.

Method of construction. The data rep... Pie diagram may be presented through ... 360 degrees, parts or sections of a c... formula to determine Central Angle of

sector as

$$\text{Central Angle} = \frac{360^0}{\text{Total value}} \times \text{Value of desired section}$$

Class-mark or Mid-point or Mid-value / Point :- *The central value of the class interval is called the mid-point or mid-value or class mark.* It is the arithmetic mean of the lower class and upper class of the same class.

$$\text{Mid-value of Class} = \frac{\text{Lower class limit + Upper class limit}}{2}$$

$$\text{Class mark} = \frac{\text{True upper class limit + True lower class limit}}{2}$$

or $$= \frac{\text{LCB + UCB}}{2}$$

The class mark of the class 11 - 20 = $\frac{11+20}{2} = 15.5.$

Frequency Density : - Frequency density of a class interval = $\frac{\textit{frequency} \text{ of the Class Interval}}{\text{Length or size or width of class interval}}$

Relative Frequency :- A relative frequency distribution is a distribution in which relative frequencies are recorded against each class interval.

Relative Frequency = $\frac{\textit{frequency} \text{ of the Class Interval}}{\sum f}$

Percentage Frequency :- When Relative frequency is converted in percentage form then it is called Percentage frequency.

Percentage frequency = $\frac{\textit{frequency} \text{ of the Class Interval}}{\sum f} \times 100$

Note 1 :- Sum of relative frequency of all class interval = 1

Note 2 :- Sum of percentage frequency of all class interval = 100

Example :- If frequency of the class interval 10-20 is 4 and total frequency is 25. Then

Frequency Density = $\frac{4}{10} = 0.4$

Relative Frequency = $\frac{4}{25} = 0.16$

Percentage Frequency = $\frac{4}{25} \times 100 = 16$

CUMULATIVE FREQUENCY DISTRIBUTION

In statistics, absolute frequency refers to the number of times a particular value appears in a data set. Cumulative frequency is different: it is the sum (or running total) of all the frequencies up to the current point in the data set. *The frequency of a particular class is obtained by adding to the frequency of that class to all the frequencies of its previous class.* Thus, the cumulative frequency table is obtained from the ordinary frequency table by successively adding the several frequencies.

Cumulative frequency series are of two types:

(*i*) Less than Series Cumulative frequency

(*ii*) More than Series Cumulative frequency

Suppose, we are given the following discrete series of marks obtained by 100 students. With the help of this series, we shall form the 'Less than' and 'More than' series.

Marks obtained:	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
No. of Students:	8	12	20	25	18	17

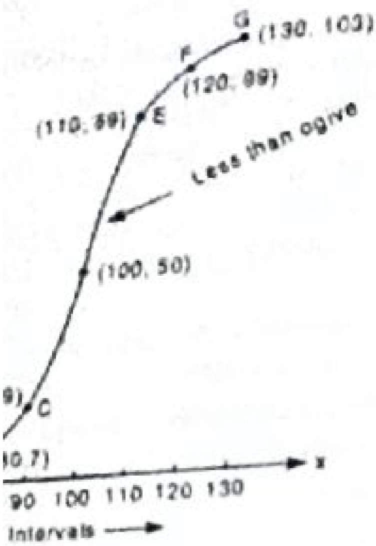
Less than Cumulative Series

No. of Students

More than Cumulative Series

Marks obtained No. of Students

...umulative Frequency	Plots to be plotted
0	(60,0)
2	(70,2)
2+5=7	(80,7)
7+12=19	(90, 19)
19+31=50	(100, 50)
50+39=89	(110, 89)
89+10=99	(120,99)
99+4=103	(130, 103)

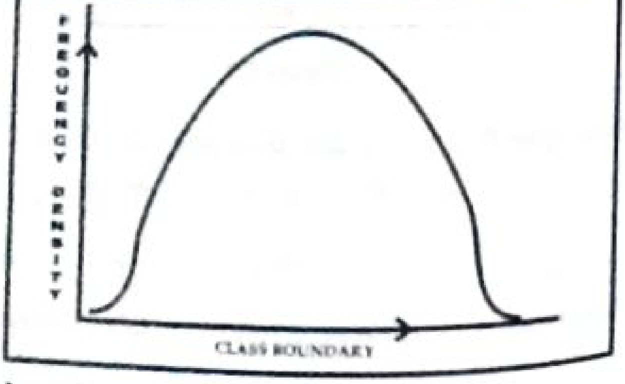


...ure 1.10

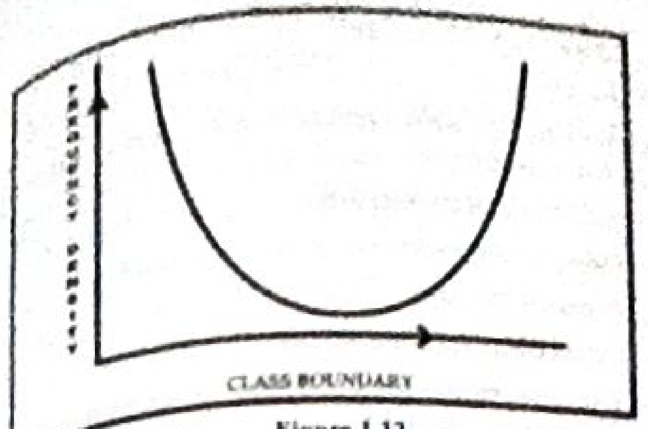
(*iii*) J-Shaped curve: Curve starts with a minimum frequency and then gradually reaches its maximum frequency at the other extremity. As graph of commuters coming to Delhi from the early morning hour to peak morning hour.

(*iv*) S-Shaped curve : Less than ogive curve may be S-Shaped curve.

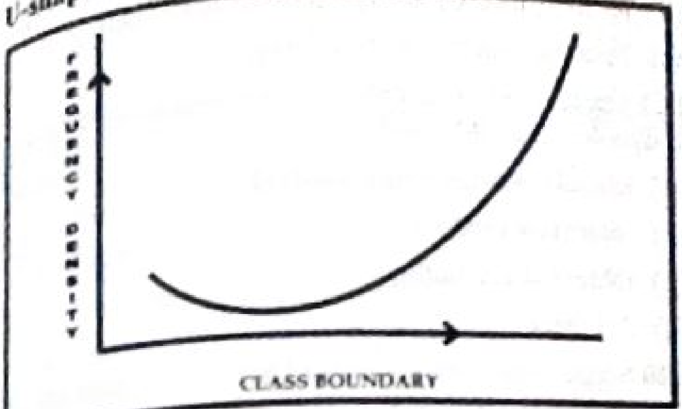
(*v*) Mixed curve : Combination of varieties of different shapes is called Mixed shaped.



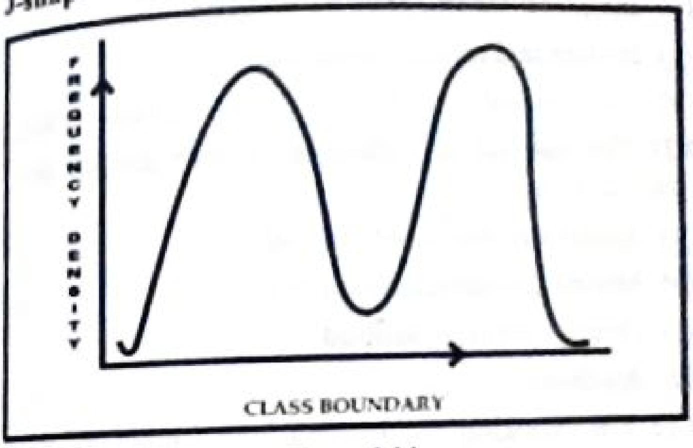
Bell-shaped curve **Figure 1.11**



U-shaped curve **Figure 1.12**



J-shaped curve **Figure 1.13**



Mixed curve **Figure 1.14**

NOTE : -

Remember:-

1. **Anything which represents only length is : - ONE DIMENSIONAL**
2. **Anything which represents Area is : - TWO DIMENSIONAL**
3. **Anything which represents Volume is : - THREE DIMENSIONAL**
4. **Anything which has no length, no breadth & no height is : - ZERO DIMENSIONAL**

Central Tendency

Formula to Find Arithmetic Mean.

<table>
<tr><th rowspan="2">Sr. No.</th><th rowspan="2">Raw Data or Data</th><th colspan="2">Frequency Distributions</th></tr>
<tr><th>Discrete Data</th><th>Continuous Data Or Grouped Data</th></tr>
<tr><td></td><td>Ex- 5, 7, 8, 12, 13, 15...</td><td>Ex-<table><tr><td>x</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>f</td><td>2</td><td>3</td><td>8</td><td>5</td><td>3</td></tr></table></td><td>Ex-<table><tr><td>C.I.</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td></tr><tr><td>f</td><td>3</td><td>5</td><td>8</td><td>4</td></tr></table>Where C.I. = Class-Interval</td></tr>
</table>

			Frequency Distributions
I. **Direct Method**	$\overline{X}=\frac{\sum X}{N}$ When x are smaller in Size.	$\overline{X}=\frac{\sum fX}{\Sigma f}$ When x are smaller in Size.	$\overline{X}=\frac{\sum fX}{\Sigma f}$ When x are smaller in Size. and x = Mid-Point of the C.I.
II. **Short-cut Method**	$\overline{X}=A+\frac{\sum x}{N}$ Where A = Assumed Mean id = X - A; When x are larger in Size.	$\overline{X}=A+\frac{\sum fx}{\Sigma f}$ Where A = Assumed Mean; id = X - A; When x are larger in Size.	$\overline{X}=A+\frac{\sum fx}{\Sigma f}$ Where A = Assumed Mean d = X -A. ,X = Mid-point of the class-interval When length of each class-interval are not equal.
III. **Step-Deviation Method**	$\overline{X}=A+\frac{\sum d}{N}\times i$ Where $d=\frac{X-A}{i}$ i = difference between two consecutive obs. values	$\overline{X}=A+\frac{\sum fd}{\Sigma f}\times i$ Where $d=\frac{X-A}{i}$ deviation of variate X from A. i = difference between two consecutive obs. values	$\overline{X}=A+\frac{\sum fd}{\Sigma f}\times i$ Where $d=\frac{X-A}{i}$, Dev. of variate x from A;i = length of each class-interval. [when length of each C.I. is equal]

Combined/Pooled/Grouped Mean

I. If Arithmetic means of two group containing N_1 and N_2 observed values are $\overline{X_1}$ and $\overline{X_2}$ respectively. Then the combined mean $\overline{X_{12}} = \dfrac{N_1\overline{X_1} + N_2\overline{X_2}}{N_1 + N_2}$

II. If there are a three groups containing N_1 ; N_2 and N_3 observations with means $\overline{X_1}$; $\overline{X_2}$ and $\overline{X_3}$ respectively Then the combined mean

$$\overline{X_{123}} = \frac{N_1\overline{X_1} + N_2\overline{X_2} + N_3\overline{X_3}}{N_1 + N_2 + N_3}$$

Ungrouped/Raw Data

Example 7; 8; 10; 12; 15;....

1. Arrange all obs. Values in ascending order.

2. Use formula.

$$M = \left(\frac{N+1}{2}\right) \text{th obs. value.}$$

Note:-

(i) If N = odd number

$$M = \left(\frac{N+1}{2}\right)^{th} \text{obs. value}$$

(ii) If N = even No.

$$M = \frac{1}{2}\left[\frac{N}{2} \text{th obs. value} + \left(\frac{N}{2}+1\right)^{th} \text{obs. value}\right]$$

Discrete Data

x	5	8	10	12	15
f	3	5	8	4	2

1. Arrange all obs. value in ascending order and write their corresponding frequencies

2. Find less than cumulative frequencies of all obs. values.

3. Use formula

$$M = \left(\frac{N+1}{2}\right)^{th} \text{obs. value} =$$

The obs. value having c.f. just equal to $\frac{N+1}{2}$ or just greater than $\frac{N+1}{2}$.

Note:-

(i) If N = odd Number

$$M = \left(\frac{N+1}{2}\right)^{th} \text{obs. value}$$

(ii) If N = even Number

$$M = \frac{1}{2}\left[\frac{N}{2} \text{th obs. value} + \left(\frac{N}{2}+1\right)^{th} \text{obs. value}\right]$$

Continuous Data

C.I.	0-10	10-20	20-30	30-40
f	3	5	8	4

1. Arrange all class-Intervals in ascending order and write their corresponding frequencies.

2. Find less than c.f. of all class intervals.

3. All C. Is must be in overlapping form.

4. Find median- class = The class-interval having c.f. just equal to or just greater than $\frac{N}{2}$.

5. Use Formula.

$$M = L + \left(\frac{\frac{N}{2} - c}{f}\right)i \quad \text{Where}$$

L = Lower limit of M - class.
f = freq. of M - class.
i = length of M - class.
c = c.f. of just pre-M-class
N = Σf
M - Class = Median class

Partition Values or Quartiles or Fractiles

[Remember just like Median]

Data	Median	Quartiles	Deciles	Percentiles
	Median ÷ in 2 equal Parts M	**Quartiles** ÷ in 4 equal Parts Q_1 Q_2 Q_3	**Deciles** ÷ in 10 equal Parts D_1 D_2------D_9	**Percentiles** ÷ in 100 equal Parts P_1 P_2-------- P_{99}
ungrouped & Discrete Data	$M = \left(\frac{N+1}{2}\right)^{th}$ obs. Value	$Q_K = K\left(\frac{N+1}{4}\right)^{th}$ obs. value where K = 1; 2; 3	$D_K = K\left(\frac{N+1}{10}\right)^{th}$ obs. value where K = 1, 2, ...;9	$P_K = K\left(\frac{N+1}{100}\right)^{th}$ obs. value where K = 1, 2, ...;99.
Continuous Data	$M = L + \left(\frac{\frac{N}{2} - c}{f}\right) \times i$	$Q_K = L + \left(\frac{\frac{KN}{4} - c}{f}\right) \times i$	$D_K = L + \left(\frac{\frac{KN}{10} - c}{f}\right) \times i$	$P_K = L + \left(\frac{\frac{KN}{100} - c}{f}\right) \times i$

Type II (Continuous Frequency Distribution)

Working rule

(i) Arrange all class-intervals in ascending order, and write their corresponding frequencies.

(ii) All class-intervals must be in overlapping form.

(iii) Length of each class-intervals must be equal

(iv) Find modal-class = The class -interval having frequency maximum is called modal-class.

(v) Use Formula.

$$M_0 = L + \left(\frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_{+1}} \right) \times i$$

or

$$M_0 = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$$

Where $\Delta_1 = f_0 - f_{-1}$

$\Delta_2 = f_0 - f_{+1}$

L = Lower limit of modal-class

f_0 = frequency of modal - class.

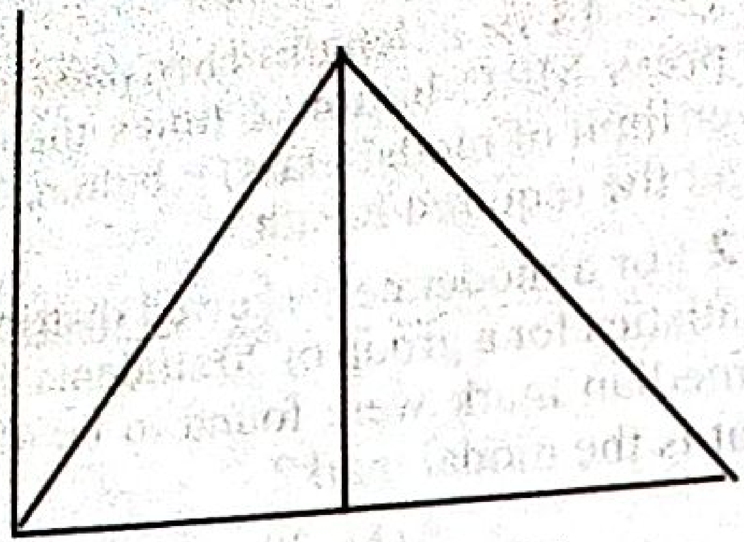
f_{-1} = frequency of just pre-modal-class.

f_{+1} = frequency of just post-modal class

i = length of modal - class.

MODE

Definition :- The observed value in a series occurring most frequently is called **Mode.** In a frequency distribution; the observed value having maximum frequency is called **Mode or NORM.** It is denoted by **M_0 or Z.**



$AM = M_e = M_0$

II Moderately skewed/Asymmetrical :-

The data having mean ; median and mode not equal is called Asymmetrical data.

Mean ≠ Median ≠ Mode.

Karl Pearson's relations [EMPIRICAL FORMULA]

$M_0 = 3M - 2\overline{X}$

Mean – Mode = 3(Mean – Median).

percentages to calculat...
portfolio of securities. It is...
cial and stock market index...
Value Line Geometric inde...

Example G.M. of 9 , 27 ...

$= (3.9.27)^{\frac{1}{3}} = (3 \times 3^2 \times 3^3 ...$

Logarithmic Formula f...

$G = \text{Antilog}\left(\frac{\Sigma \log x}{N}\right)$

For Grouped Frequency ...
variates $x_1 ; x_2 ; x_3 ; \ldots\ldots$
$f_3 ; \ldots\ldots ; f_n$ resp...

$G = \left(x_1^{f_1} x_2^{f_2} \ldots\ldots x_n^{f_n} ...\right.$

Where $N = \Sigma f_i = f_1 + ...$

Geometric Mean

Definition:- If $x_1 ; x_2 ; x_3 ; ....; x_n$ are n non-zero values of a variate x ; The Geometric Mean "G" is defined as

$$G = (x_1 . x_2 . x_3 ....x_n)^{\frac{1}{n}} = \sqrt[n]{x_1 . x_2 . x_3 ....x_n}$$

Harmonic Mean

Definition:- Harmonic Mean (H) of non-zero observed values x_1 ; x_2 ; x_3 ; .... ; x_N is defined as

$$= H = \frac{N.}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\ldots\ldots+\frac{1}{x_N}} = \frac{N}{\Sigma \frac{1}{x}}$$

Example 1 HM of 4, 6 and 10 is

(a) 5.81

(b) 3.23

(c) 4.71

(d) None

Properties of HM

(*i*) If all observations of the data are constant k [Let] (*i.e.* are equal), Then the HM=K.

Example HM of 8 ; 8 ; 8 ; 8 ; ....; 8 is 8.

(*ii*) If H_1 and H_2 are HMs. of two groups containing N_1 and N_2 observed values respectively then their

$$\text{combined } HM = \frac{N_1 + N_2}{\frac{N_1}{H_1} + \frac{N_2}{H_2}}$$

button we get the required result *i.e.* 21.74.

Relationship between Arithmetic Mean ; Geometric Mean and Harmonic Mean

1. If A ; G and H be Arithmetic mean ; Geometric Mean and Harmonic mean of same set of observations (*i.e.* of same data) Then the relationship between then are

   (*i*) A > G > H (When all obs. values are unequal)

   (*ii*) A = G = H (when all obs. values are equal)

   (*iii*) A ≥ G ≥ H (No indications about obs. values)

2. If A ; G and H are Arithmetic mean ; Geometric Mean and Harmonic Mean of two observed values a and b respectively Then

$$A = \frac{a+b}{2}$$

$$G = \sqrt{ab}$$

$$H = \frac{2ab}{a+b}$$

Relationship between A ; G & H are

(i) $G^2 = AH \Rightarrow G = \sqrt{AH}$

(ii) A ; G and H are in G.P.

These relations holds only for two observations

10 ... 0.4

...satisfies the given conditions.

∴ (c) is correct

CHOICE OF A SUITABLE AVERAGE

1. Purpose-The choices of central values are made according to their purposes . Suggested guidelines in this regard are :-

Purpose	*Suitable Central values*
(i) To focus equal importance to all observations of the data	(*a*) Arithmetic Mean
(ii) To locate the position of an item in relation to others	(*b*) Median and other Partition values
(iii) To find out the most common or most fashionable item than bigger one	(*c*) Mode
(iv) To focus more importance to smaller items	(*d*) Geometric mean
(v) To give greatest importance to small items	(*e*) Harmonic Mean

2. Nature and form of Data-The choices of central values are made according to their nature and form of data. Suggested Guidelines in this regard are :

(i) For open-end data/distributions. Prefer When the distribution are J-shaped or reverse J-shaped. For example, price distribution and Income Distribution.	(a) Median
(ii) To describe qualitative nature . For example, to study the consumer preferences for different products	(*b*) Mode
(iii) To compute average rates of increase / decrease, average ratios, average percentages.	(*c*) Geometric Mean
(iv) When the value of a variable is compared with another variable which is constant. Examples:-Varying quantities bought/sold per unit , Varying speed with constant distance, Ratio average.	(d) Harmonic Mean
(v) In all remaining cases	(*e*) Arithmetic Mean

Note: Arithmetic mean should not be used in the following cases :

(i) If the distribution is spread unevenly *i.e.* concentration being small or large at irregular points.

(ii) If the distribution is highly skewed.

(*iii*) When there are extreme values *i.e.* very large and very small.

(*iv*) If average ratios and rates of change are to be computed.

Range

Def.:- The difference between largest observed value (L) and smallest observed value (S) of the data is called RANGE of the data.

∴ Range = R = L - S. (Absolute Measure of Dispersion)

Coefficient of Range = $\frac{L-S}{L+S}\times100$ [Relative Measure of Dispersion]

It is in Percentage (%) form.

<table>
<tr><th></th><th colspan="3">DATA</th></tr>
<tr><th>Measures of Dispersion</th><th>Ungrouped data</th><th>Discrete Data</th><th>Continuous Data</th></tr>
<tr>
<td>I. Absolute measures of Range</td>
<td>Ex - 5, 8, 20, 15, 25
R = L - S; Where
R = Range
L = Largest value
S = Smallest Value
∴ R = 25 - 5 = 20</td>
<td>Ex-<table><tr><td>x</td><td>3</td><td>4</td><td>7</td><td>9</td></tr><tr><td>f</td><td>2</td><td>8</td><td>4</td><td>3</td></tr></table>R = L - S
Do
R = 9 - 3 = 6</td>
<td>Ex-<table><tr><td>C.I.</td><td>0-10</td><td>10-20</td><td>20-30</td></tr><tr><td>f</td><td>3</td><td>8</td><td>4</td></tr></table>(i) $R = U_L - L_S = 30 - 0 = 30$
Where
U_L = Upper - limit of Largest class.
L_S = Lower limit of Smallest Class.
or
(ii) $R = M_L - M_S$
Where M_L = Mid-point of largest values class
M_S = Mid-point of smallest value class</td>
</tr>
<tr>
<td>II. Relative Measure of Range</td>
<td>Co-efficient of Range = $\frac{L-S}{L+S}\times100$
$=\frac{25-5}{25+5}\times100=66.67\%$</td>
<td>Co-efficient of Range= $\frac{L-S}{L+S}\times100$
$=\frac{9-3}{9+3}\times100=50\%$</td>
<td>Co-efficient of
(i) Range = $\frac{U_L-L_S}{U_L+L_S}\times100$
or
(ii) = $\frac{M_L-M_S}{M_L+M_S}\times100$</td>
</tr>
</table>

Example 1 If Range and co-efficient of Range of a | **Trick:-** Go by ch

Quartile Deviation

Formulae.

Range	Absolute Measures of Deviation	Relative Measures of Deviation
1. Quartile	(i) Inter-Quartile Range = $Q_3 - Q_1$ = Upper Quartile - Lower Quartile	(i) Co-efficient of Quartile Deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$
	(ii) Quartile-Deviation/Semi-Inter-Quartile Range $= \frac{Q_3 - Q_1}{2}$	
2. Decile	(i) Decile-Range = $D_9 - D_1$	(i) Co-efficient of Decile Deviation $= \frac{D_9 - D_1}{D_9 + D_1} \times 100$
	(ii) Decide Deviation $= \frac{D_9 - D_1}{2}$	
3. Percentile	(i) Percentile Range = $P_{90} - P_{10}$	Coefficient of Percentile $= \frac{P_{90} - P_{10}}{P_{90} + P_{10}} \times 100$
	(ii) Percentile Deviation $= \frac{P_{90} - P_{10}}{2}$	

Note:- For Symmetrical distribution (*i.e.* Mean = Median = Mode) , Median $= \frac{Q_3 + Q_1}{\ }$

mean, median or sometimes even ...

Measures of Mean-Deviation

<table>
<tr><th>Measures of Distribution</th><th>Individual or Ungrouped data</th><th>Discrete series/ Data</th><th>Continuous series/data</th></tr>
<tr><td>Example</td><td>5, 6, 8, 12, 3</td><td><table><tr><td>x</td><td>3</td><td>5</td><td>8</td><td>10</td></tr><tr><td>f</td><td>1</td><td>3</td><td>5</td><td>2</td></tr></table></td><td><table><tr><td>C.I.</td><td>0-10</td><td>10-20</td><td>20-30</td></tr><tr><td>f</td><td>3</td><td>5</td><td>4</td></tr></table></td></tr>
<tr><td>I Absolute Measures of Mean-Deviation</td><td>(i) M.D about Mean = $\frac{\Sigma|X-\bar{X}|}{N}=\frac{\Sigma|D|}{N}$

(ii) MD about Median = $\frac{\Sigma|X-M|}{N}=\frac{\Sigma|D|}{N}$
Where D = X - M</td><td>(i) MD about Mean = $\frac{\Sigma f|X-\bar{X}|}{\Sigma f}=\frac{\Sigma f|D|}{\Sigma f}$

(ii) MD about Median = $\frac{\Sigma f|X-M|}{\Sigma f}=\frac{\Sigma f|D|}{\Sigma f}$
Where D = X - M</td><td>(i) MD about Mean = $\frac{\Sigma f|D|}{\Sigma f}$
where $D = X - \bar{X}$

(ii) MD about Median = $\frac{\Sigma f|D|}{\Sigma f}$
Where D = X - M.</td></tr>
</table>

II Relative Measures of Mean-Deviation

(i) Coefficient of MD about mean $=\frac{\text{MD about Mean}}{\text{Mean}}\times 100$

(ii) Co-efficient of Mean - Deviation about Median $=\frac{\text{MD about Median}}{\text{Median}}\times 100$

(iii) Co-efficient of Mean - Deviation about Mode $=\frac{\text{MD about Mode}}{\text{Mode}}\times 100$

...properties under one box so that we can memorize

Standard Deviation

Definition:- The arithmetic mean of the squares of deviations of all observed values from arithmetic mean [not from Median or Mode] is called standard Deviation or Root Mean Square Deviation. It is denoted by "σ" (sigma). or "s"

$$\sigma = \sqrt{\frac{\sum (X-\overline{X})^2}{N}}$$; Where X = observations

$\overline{X}$ = Their mean

N = No. of observations.

Measures of Standard Deviation

Measure of standard Deviation	*Ungrouped Data*	*Discrete Data*	*Continuous Data*
I Absolute Measure of sd.			
(i) Actual Mean Method	$\sigma = \sqrt{\frac{\Sigma(X-\overline{X})^2}{N}}$ $= \sqrt{\frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2}$	$\sigma = \sqrt{\frac{\Sigma f(X-\overline{X})^2}{\Sigma f}}$ Where X = obs. values	$\sigma = \sqrt{\frac{\Sigma f(X-\overline{X})^2}{\Sigma f}}$ Where X = Mid-point of the class-interval.
(ii) Assumed Mean Method	$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$ Where d = X - A A = Assumed Mean	$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$ Where d = X - A A = Assumed Mean	$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$ Where d = X - A A = Assumed Mean When length of each class-interval is not equal.
(iii) Step-Deviation Method		$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \times c$ When $d = \frac{X-A}{c}$ A = Assumed mean and c = Common factor of each obs. values	$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \times i$ Where $d = \frac{X-A}{i}$ A = Assumed Mean i = length of each class -interval when length of each class-interval is equal.

II Relative Measure of Standard Deviation Coefficient of variation $= C.V. = \frac{\sigma}{\overline{X}} \times 100$ to compare the variations of two or more Series

III S.D. of any two values $\sigma = \frac{1}{2}|L - S|$ or $\sigma = \frac{1}{2} \times$ Range

IV Variance $= \sigma^2$

s.d. 2

VI. The relationship between Quartile Deviation ; Mean - Deviation and standard Deviation is

(*i*) $6QD = 5\ MD = 4\ SD$.

(*ii*) $QD < MD < SD$

(*iii*) $QD = MD = SD$ [If all obs. values are equal]

VII Standard Deviation of *1st n* natural numbers :-

Standard Deviation of 1^{st} n natural number $= \sqrt{\dfrac{n^2 - 1}{12}}$

Example 1 ...

∴ option (b) is correct

VIII Combined/Pooled/Grouped SD.

(*i*) If $\overline{X_1}$; $\overline{X_2}$ are Arithmetic means and σ_1 ; σ_2 the standard deviations of two data having N_1 and N_2 no. of observed values respectively then combined s.d is

$$\sigma_{12} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$$

Where $d_1 = \overline{X_1} - \overline{X_{12}}$ and $d_2 = \overline{X_2} - \overline{X_{12}}$

and $\overline{X_{12}} = \dfrac{N_1\overline{X_1} + N_2\overline{X_2}}{N_1 + N_2}$

(*ii*) If there are three groups containing N_1 ; N_2 and N_3 observations, $\overline{X_1}$; $\overline{X_2}$ and $\overline{X_3}$ as their respective AM's, σ_1 ; σ_2 and σ_3 as their respective SD's then combined SD is given by

$$\sigma_{123} = \sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2) + N_3(\sigma_3^2 + d_3^2)}{N_1 + N_2 + N_3}}$$

Where $d_1 = \overline{X_1} - \overline{X_{123}}$; $d_2 = \overline{X_2} - \overline{X_{123}}$ and

$d_3 = \overline{X_3} - \overline{X_{123}}$

where $\overline{X_{123}} = \dfrac{N_1\overline{X_1} + N_2\overline{X_2} + N_3\overline{X_3}}{N_1 + N_2 + N_3}$

(.., 1), (4, 2);..............................;(4, 6);
(5, 1), (5, 2);..............................;(5, 6);
(6, 1), (6, 2);..............................;(6, 6)}

$\therefore\ n(s) = n(s_1).\ n(s_2) = 6\times6 = 36.$

Tricks:- For Dice

$n(S) = 6^{\text{No. of dice Thrown together.}}$

Combination Form SAMPLE SPACE

Formula $^nc_r = \dfrac{n!}{r!.(n-r)!}$, where $0 \le r \le n$.

$\text{Sample Space} = n(S) = {}^{\text{Total no. of units}}C_{\text{No. of units taken at a time.}}$

Example 1. A bag contains 5 Red and 3 black balls. Two balls are drawn at a time ; the sample

Space $= n(s) = {}^{(5+3)}C_2 = {}^8C_2 = \dfrac{8!}{2!8-2!} = \dfrac{8.7.6!}{2.1.6!} = 28$

Example 2. A card is drawn at random from a pack of 52 playing cards. The sample Space

$n(s) = {}^{52}C_1 = \dfrac{52!}{1!5!} = \dfrac{52\times51!}{1.51!} = 52.$

About Playing Cards

Total No. of playing cards = 52

Total No. of suits = 4

No. of cards in a suit = 13

No. of Ace cards = 4

No. of king cards = 4

No. of Queen Cards = 4

No. of Jack/Knive Cards = 4

No. of Cards of any Number = 4

No. of Face Cards = 12

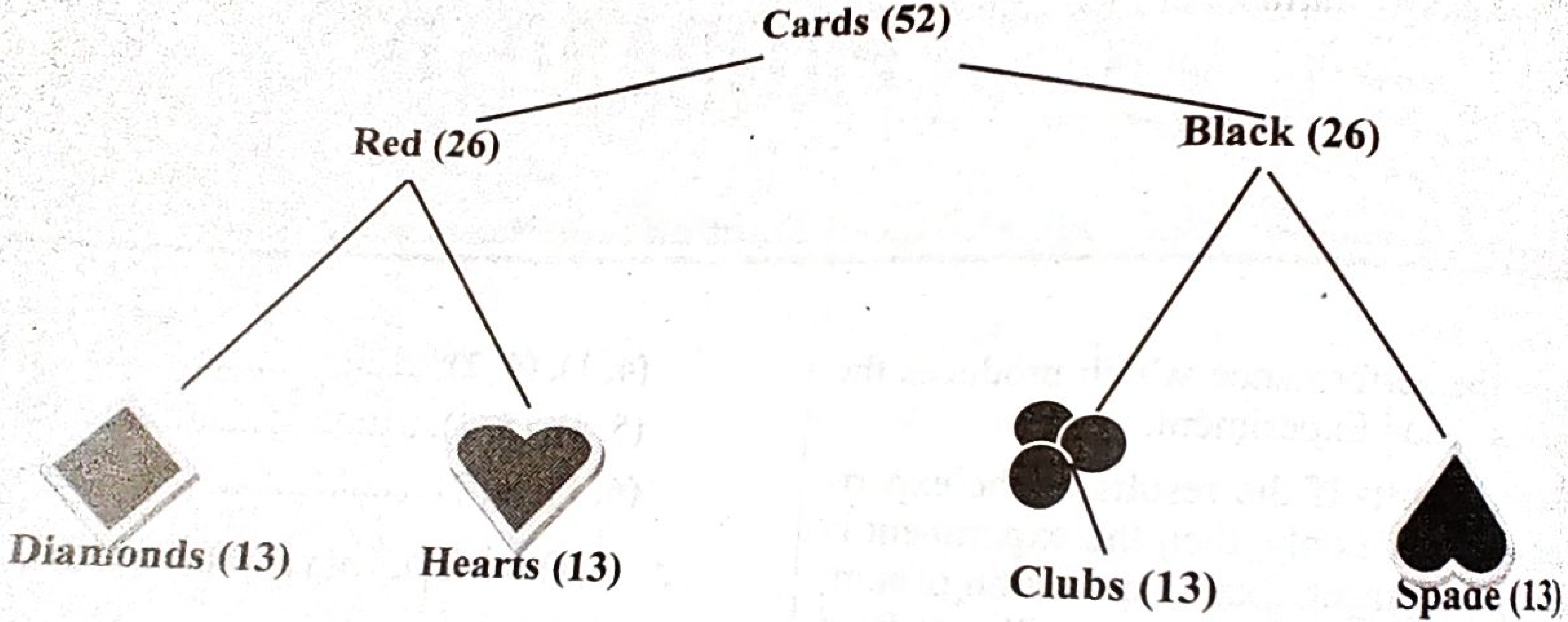
No. of face card including Aces = 16

No. of owner/Honour cards = 16

No. of poker-hands = 5 (cards)

No. of colours/colors = 2

Types of cards:-



Diamonds (13) Hearts (13)

Event :- The set or sub-collection of a number of sample points of a Sample Space under a definite rule or law is called an **Event.** It is generally denoted by E or any English capital alphabet.

Example 1: A coin is tossed at random the sample space S = {H, T}

Let E = Event of getting head = {H}, n (E) = 1

Example 2: A dice is thrown at random. The sample space S = {1, 2, 3, 4, 5, 6}.

Let E = Event of getting an odd number

$\therefore$ E = {1, 3, 5}, n(E) = 3

(i) **Null Event:-** The event having no sample point is called null event. It is denoted by φ.

Let A be a Null Event, then n(A) = 0

(ii) **Simple or Elementary Event :-** The event which cannot be decomposed into further events is called simple or Elementary event.

Example - A coin is tossed once then there may be two simple events.

A = Event of getting head.

B = Event of getting tail.

(iii) **Composite or Compound Event : -** The event which can be decomposed into two or more events is called composite or compound event.

Example-A Coin is tossed twice. It is an example of composite or compound event because it can be split into the event HT and TH . Here {TH} and {HT} are simple events.

(iv) **Mutually Exclusive Events or Incompatible Events : -** Events A and B are called mutually exclusive events if they have no common element.

A = Event of getting head = {H}and B = Event of getting tail = {T}, when a coin is tossed at random.

Clubs (13) Spade (13)

$\therefore$ A and B are mutually exclusive events.

$n(A \cap B) = 0$; $P(A \cap B) = 0$

(v) Equally Likely Events or Mutually Symmetric Events or Equi-Probable Events:

When all necessary evidences are taken into account, no event is expected to occur more frequently as compared to the other events i.e. if probability of each event is equal, then Events are called Equally Likely.

Let A = Event of getting Head & B = Event of getting Tail when a coin is tossed at random.

$n(A) = n(B)$; So, $P(A) = P(B)$. Hence A & B are Equally Likely events.

(vi) Exhaustive Events :- Let A_1 ; A_2 ; A_3; ....; A_n are set of Events such that no event can occur except these *i.e.* any one of these events must occur, then these Events are called Exhaustive Events. If those Events have no common elements, then those Events are called **Mutually Exclusive & Exhaustive Events.**

In this case $P(A_1) + P(A_2) + P(A_3) + ... + P(A_n) = 1$

Probability : - Probability of event A is defined

as $P(A) = \frac{n(A)}{n(S)}$

$P(A \cap B) = 0$

Note:- I P(A) always lies between 0 and 1 i.e. $0 \leq P(A) \leq 1$

II If $P(A) = 0$, Then event- A is called Impossible / Null event.

III If $P(A) = 1 \Rightarrow n(A) = n(S)$ Then Event - A is called Sure Event /Super Event.

Complementary Event :- The set of those elements of sample space which are not the elements of given event is called complementary Event.

Let A be an event of a sample space "S". Its complementary event is denoted as A' or $\overline{A}$ or A^c and is defined as A' or $\overline{A}$ or $A^c = S - A$

Example-A dice is thrown at random. S = {1, 2, 3, 4, 5, 6}

Let A = Event of odd numbers = {1, 3, 5}

Then its complementary event

$A^c = S - A = \{1, 2, 3, 4, 5, 6\} - \{1, 3, 5\} = \{2, 4, 6\}.$

$\therefore n(A^c) = n(s) - n(A) = 6 - 3 = 3.$

$$\because \frac{n(A^c)}{n(s)} = \frac{n(s)}{n(s)} - \frac{n(A)}{n(s)} \quad \text{or} \quad p(A^c) = 1 - p(A)$$

Exam

(a)

(c)

Solu

DEM

(i)

DEMORGAN'S Formula

(i) $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B)$

(ii) $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B)$

Odds in Favour OR odds against the Event:-

(i) **Odds in Favour of Event:-** Let A be an event of sample space. Then odds in favour of Event -A is defined as Odds in favour of Event-A

$$= \frac{n(A)}{n(A^c)} = \frac{n(A)}{n(s) - n(A)} = \frac{P(A)}{P(A^c)}$$

$$= \frac{\text{Favourable outcomes}}{\text{Not favourable i.e. against outcomes}}$$

(ii) **Odds Against Event:-** odds Against of an Event - A is defined as Odds against Event-A =

$$\frac{n(A^c)}{n(A)} = \frac{n(s) - n(A)}{n(A)} = \frac{P(A^c)}{P(A)}$$

$$= \frac{\text{Against outcomes}}{\text{Favourable outcomes}}$$

(iii) To Find Probability if Odds in favour or Odds against Event is given

If Odds in favour of Event-A $= \dfrac{n(A)}{n(A^c)} = \dfrac{p(A)}{p(A^c)} = \dfrac{x}{y}$

Then $P(A) = \dfrac{x}{x+y}$ and $P(A^c) = \dfrac{y}{x+y}$

If Odds against Event - $A = \dfrac{P(A^c)}{P(A)} = \dfrac{a}{b}$

Then $P(A) = \dfrac{b}{a+b}$ and $P(A^c) = \dfrac{a}{a+b}$.

Example 1. A card is drawn at random from a pack of 52 playing cards. Find Odds in favour of getting a king card.

(*a*) 1 : 12
(*b*) 12 : 1
(*c*) 2 : 3
(*d*) None

Solution :- (a) is correct.

$n(S) = {}^{52}c_1 = 52.$

Let A = Event of getting a king card.

$$\therefore n(A) = {}^{4}c_1 = 4$$

$$P(A) = \frac{n(A)}{n(s)} = \frac{4}{52} = \frac{1}{13}$$

$$P(A^c) = 1 - P(A) = 1 - \frac{1}{13} = \frac{12}{13}$$

Odds in favour of Event -

$$A = \frac{P(A)}{P(A^c)} = \frac{n(A)}{n(A^c)} = \frac{n(A)}{n(s) - n(A)}$$

$$= \frac{4}{52-4} = \frac{4}{48} = \frac{1}{12} = 1:12$$

odds against Event -

$$A = \frac{P(A^c)}{P(A)} = \frac{n(A^c)}{n(A)} = \frac{\frac{12}{13}}{\frac{1}{13}} = \frac{12}{1} = 12:1.$$

Theorems on Probability

I. **Addition Theorem or Theorem on Total Probability.**

II **Multiplication Theorem or Theorem on Compound Probability**

I Addition Theorem or Theorem on Total Probability :-

(*i*) If A and B are two events of a sample space "S". Then probability of either A or B or atleast one of the events A and B is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore P(A \cup B) = \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$$

Remarks

(*a*) For events A & B.

In case of either or ; or ; one of them ; at least one or options / choices ; we find $P(A \cup B)$.

(*b*) If in Question word "and" is mentioned for event A and B or "Both" or A & B both or Sense of being common ; we use $p(A \cap B)$

• Card is drawn at random from a ...

$P(A \cap B) =$

(*i*) $\therefore$ p(A

$\therefore P(A$

(*ii*) If A, B
clusive

$p(A \cup$

$-P(B$

(*iii*) If A, B
of sam
P(A) +

$\therefore P(A$

(*iv*) If A_1;
sive ev

$P(A_1 \cup$

$= P(A$

Example 1
pack

Mutually Exclusive Events:- If A and B are two mutually exclusive events $n(A \cap B) = 0$; $P(A \cap B) = 0$

(*i*) $\therefore p(A \cup B) = p(A) + p(B)$;

$$\therefore P(A \cup B) = \frac{n(A) + n(B)}{n(S)}$$

(*ii*) If A, B and C are three events (Not mutually exclusive events) of a Sample space "S". Then $p(A \cup B \cup C) = p(A) + p(B) + p(C) - p(A \cap B)$

$$-P(B \cap C) - p(C \cap A) + p(A \cap B \cap C)$$

(*iii*) If A, B and C are three mutually exclusive events of sample space "S". Then $P(A \cup B \cup C) =$ $P(A) + P(B) + P(C)$

$$\therefore P(A \cup B) = \frac{n(A) + n(B) + n(A \cap B)}{n(S)}$$

(*iv*) If $A_1; A_2; A_3;$ ....; A_n are n mutually exclusive events Then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots\ldots\ldots\ldots \cup A_n)$$

$$= P(A_1) + P(A_2) + P(A_3) + \ldots\ldots\ldots + P(A_n).$$

Remarks

I. If A ; B and C are exhaustive events. Then $P(A \cup B \cup C) = 1$.

II. If A ; B and C are equally - likely events ; Then $P(A) = p(B) = P(C)$.

Example 3. A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 5 or 9.

(*a*) 0.29 (*b*) 0.30

(*c*) 0.21 (*d*) None

Solution :- (a) is correct

$n(S) = {}^{1000}c_1 = 1000.$

Let A = Event of getting a natural number divisible by 5.

= {5, 10, 15,....................;1000} are in AP.

and B = Event of getting a natural number divisible by 9.

= {9, 18, 27,.................; 999} are also in AP.

$A \cap B$ = Event of getting a natural number divisible by 5 and 9 both i.e. LCM of 5 & 9 = 45

= {45, 90, 135;............; 990 are also in AP. }

<u>Formula for AP.</u>

$n = \frac{l-a}{d} + 1$; Where a = 1st term.; l = last term;

d = common difference

$$n(A) = \frac{1000-5}{5} + 1 = 200$$

$$n(B) = \frac{999-9}{9} + 1 = 111$$

$$n(A \cap B) = \frac{990-45}{45} + 1 = 22$$

$$\therefore P(A \cup B) = \frac{n(A) + n(B) - n(A \cap B)}{n(s)}$$

$$= \frac{200 + 111 - 22}{1000} = \frac{289}{1000} = 0.289 = 0.29$$

5)1000(200

9)1000(111

9×5 = 45)1000(22

(i) $$P(A/B) = \frac{P(A \cap B)}{P(B)} =$$

denoted as P(B/A) and read as prob. of Event-B when Event A has already occurred.

Multiplication Theorem on Probability

Total Probability = Product of probabilities of each experiments.

$\therefore$ If A & B are two simultaneous events

Then $P(A \cap B) = P(AB) = P(A).P(B/A)$. where P(A) > 0.

Similarly, If A, B and C are three simultaneous events Then

$P(A \cap B \cap C) = P(A).P(B/A)P(C/A \cap B)$, Where

P(A) > 0, $p(A \cap B) > 0$

Binomial Theorem

Bernoulli's conditions for the Applicability of Binomial Distribution.

(*i*) There should be a finite number of trials.

(*ii*) Each trial must surely result in either a success or a failure.

(*iii*) The trials should be independent.

(*iv*) The probability of success or failure should be constant for all the trials.

$$P(x = r) = {}^{n}c_{r}.p^{r}:q^{n-r}$$

Where r = 0, 1, 2,.......;n

p = Prob. of success of a single trial.

q = 1-p

n = No. of trials

r = No. of successes in "n" trials.

[It is also called Probability Mass function]

Expected Value (MEAN) : - The algebraic sum of products of the different values of taken random variable and their corresponding probabilities is called its Expected value. Expected value of random variable X is denoted as E(X) and defined as $E(X) = \Sigma PX$.

Properties of Expected Values

I Expected value of a constant K is K i.e. E(K)= K

Example:- E(5) = 5.

II E(K.X) = K.E(X) ;

Example:- E(5X) = 5. E(X).

III E(X + Y) = E(X) + E(Y) & E(X - Y) = E(X) - E(Y)

Where E(X) and E(Y) are expected values of X and Y respectively.

Example -.E(2X + 3Y) = E(2X) + E(3Y) = 2E(X) + 3E(Y).

IV E(XY) = E(X).E(Y).

where X and Y are two independent random variables

X a discrete Random Variable	X a continuous Random Variable
(i) Mean = $\mu = E(X) = \Sigma PX$ $E(X^2) = \Sigma PX^2$	(i) Mean = Expected Value $= \mu = E(X) = \int_{-\infty}^{\infty} X\, f(X) dX$
(ii) Variance of $X = V(X) = \sigma^2 = E(X - \mu)^2$ $= E(X^2) - \mu^2 = \sum PX^2 - (\sum PX)^2$	(ii) Variance of $X = V(X) = \sigma^2 = E(X^2) - \mu^2$ Where $E(X^2) = \int_{-\infty}^{\infty} X^2 f(X) dx$
(iii) $\mu = \sum_X X f(X), \quad \sigma^2 = E(X^2) - \mu^2$ where, $E(X^2) = \sum X^2 . f(x)$	

Properties of Expected Values

I Expected value of a constant K is K i.e. E(K) = K Example:-E(5) = 5.

II E(K.X) = K.E(X) ;

Example:- E(5X) = 5. E(X).

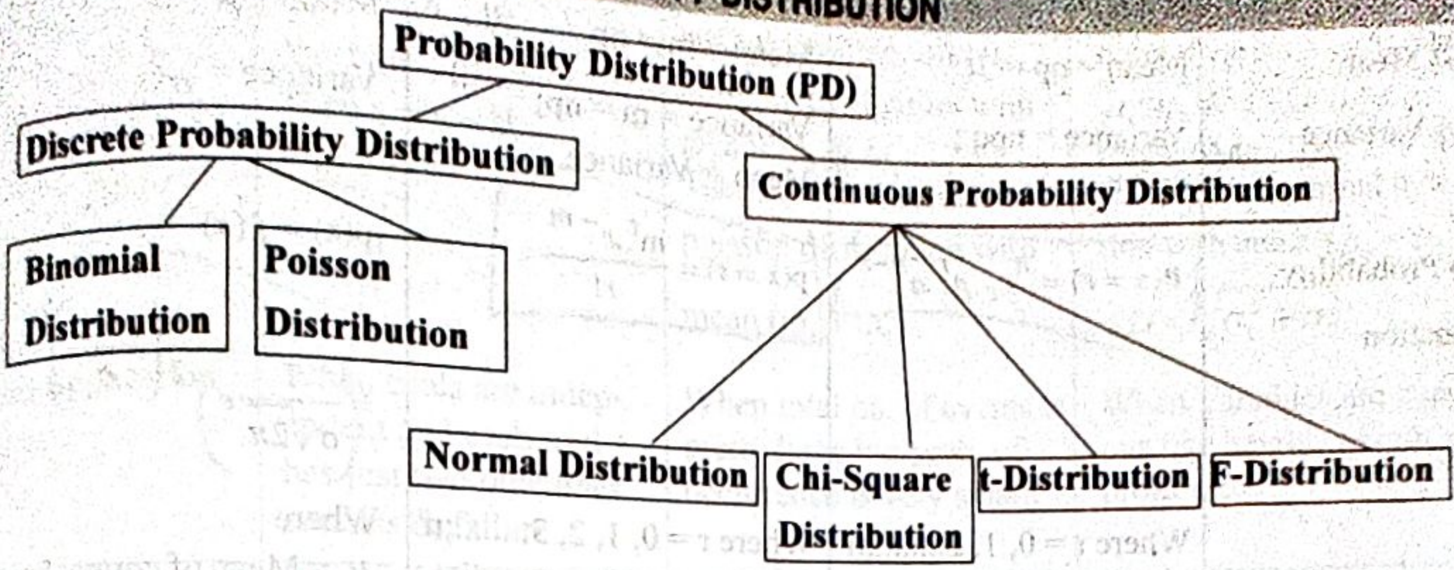
III E(X ... E(X) & E(X - Y) =

∴ Probability Distribution is

X	:	0	1	2
P(X)	:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$

X	P	PX	P.X²

PROBABILITY DISTRIBUTION



Probability/Theoretical Distribution

Characteristics	Discrete Probability Distribution		Continuous Distribution
	Binomial Distributing	**Poisson Distribution**	**Normal Distribution**
1. Conditions	**(*i*)** Same Experiments should be repeated a finite fixed number of time Let n = No. of trials	**(*i*)** It is limiting case of Binomial Distribution	**(*i*)** It is approximation of Binomial Distribution
	(*ii*) Each trial has only two events/outcomes i.e. success and failure.	**(*ii*)** n(No. of trials) is large i.e. $n \to \infty$ (Practically $n \geq 20$).	**(*ii*)** N is large.
	(*iii*) The prob. of success = P and Prob. of failure = q for each trial so that p + q = 1	**(*iii*)** P(Prob. of Success) is very small i.e. $P \to 0$. Practically $P \leq 0.1$	**(*iii*)** Neither "P" nor "q" is close to zero.
	(*iv*) The value of "P" must be equal in each trial & same for "q" ⇒ q = 1 - p	**(*iv*)** Mean = np = m (Finite)	

Characteristics	Discrete Probability Distribution: Binomial Distributing	Poisson Distribution	Continuous Distribution: Normal Distribution
	(v) Prob. of "success" for each trial must be always independent and same for "q"	(v) Trials → Always independent.	
	(vi) Variables :- Discrete	(vi) Variable:- Discrete	
II. Characteristics (i) Types of Distribution	Discrete Prob. Distribution	Discrete Prob. Distribution	Continuous Prob. Distribution
(ii) Parameters	Biparametric Distribution (Parameters are n & p)	Uniparametric Distribution (Parameter is m)	Biparametric Distribution (Parameters are μ and σ^2)
(iii) Mean	Mean = np = μ	Mean = m = np	Mean = μ
(iv) Variance	Variance = npq ; npq<np	Variance = m = np; Mean = Variance	Variance = σ^2
(v) Probability Function	$P(x=r) = {}^nc_r.p^r.q^{n-r}$ Where r = 0, 1, 2,.......;n p = Prob. of success of a single trial. q = 1-p n = No. of trials r = No. of successes in "n" trials. [It is also called Probability Mass function]	$p(x=r) = \frac{m^r.e^{-m}}{r!}$ Where r = 0, 1, 2, 3, ...;n m = Mean of Poisson-distribution = np e = 2.7183. [Probability mass function]	$p(x) = f(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ Where μ = Mean of normal Random variable x σ = SD of the given Random variable x. e = transcendental quantity = 2.71828 π = 3.14 [It is also called probability Density Function].
(vi) Expected Frequency	N.p(x = r) = N.^{n}c$_r$.p^r.q^{n-r}	$N.P(x) = N.\frac{m^r.e^{-m}}{r!}$	N. f(x) = N.P(r).
(vii) Denoted by	X ~ B(n, p)	X ~ P(x)	X ~ N (μ, σ^2)
(viii) Restriction on Parameter.	0 < p < 1.	m > 0	$-\infty < X < +\infty$
(ix) Mode	Unimodal or bi-modal (μ_0) μ_0 = Integral part of (n + 1)p ;	Unimodal or bi-modal (Depends on m) μ_0 = integral part of m if if m is non-integer,	Unimodal $\mu_0 = \mu$ = Mean

Characteristics	Discrete Probability Distribution		Continuous Distribution
	Binomial Distributing	Poisson Distribution	Normal Distribution
	if (n + 1)p is Non-integer, (n + 1)p is integer. Then $M_0 = (n+1)p - 1$	$\mu_0 = m - 1$, if m is integer	
(x) Maximum value of variance	When p = q = 0.5 and Maximum Value $= \frac{n}{4}$	-------------	-------------
(xi) Additive Property	Follows Additive Property If $X \sim B(n_1 ; p)$ $Y \sim N(n_2 ; p)$ Then $x + y \sim B(n_1 + n_2 ; p)$	Follows Additive Property If X and Y are two independent variable follows poisson distribution with means m_1 & m_2 Then X + Y also follows poisson distribution with mean $(m_1 + m_2)$.	Follows Additive Property If X and Y are Independent normal variable with mean $\mu_1 ; \mu_2$ and SD $\sigma_1 ; \sigma_2$ respectively Then Z = X + Y also follows normal distribution with mean $(\mu_1 + \mu_2)$ and $SD = \sqrt{\sigma_1^2 + \sigma_2^2}$
(xii) Application	When trials are independent and each trial has just two outcomes success & failure.	When total no. of events is pretty large but prob. of occurrence is very small.	When Variables are continuous like height, weight wage; profit etc.

For example,

Confidence Level	Level of significance	Difference between observed and expected values	Whether or not Considered significant
95 %	5%	If the difference is more than 1.96 S.E.	Significant
95%	5%	If the difference is less than 1.96 S.E.	Not significant
99%	1%	If the difference is more than 2.58 P.E.	Significant
99%	1%	If the difference is less than 2.58 P.E.	Not significant

Note: In practice, usually the hypotheses are tested at 5% level of significance. Unless otherwise stated in examination.

COMPUTATION OF STANDARD ERROR OF THE MEAN

Population Size	When "σ" is known	When "σ" is unknown i.e."s" known
(I) * N is Large ** N is unknown *** $\frac{n}{N}<0.05$ **** SRSWR (Simple Random Sampling with Replacement)	$S.E._{\bar{x}}=\frac{\sigma}{\sqrt{n}}$ Where, σ = Population S.D. n = Sample size	$S.E._{\bar{x}}=\frac{s}{\sqrt{n-1}}$ Where, S = Sample S.D. n = Sample size
(II) * $\frac{n}{N}\geq 0.05$ ** SRSWOR (Simple Random Sampling Without Replacement)	$SE(\bar{X})=\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$	$SE(\bar{X})=\frac{s}{\sqrt{n-1}}\sqrt{\frac{N-n}{N-1}}$

COMPUTATION OF STANDARD ERROR OF THE PROPORTION

Standard Error of the proportion (P) = SE(p) as follows :

Population Size	When Population proportion is known	When Population proportion is not known
(I) * N is Large ** N is unknown *** $\frac{n}{N}<0.05$ **** SRSWOR (Simple Random Sampling without Replacement)	$SE(p)=\sqrt{\frac{PQ}{n}}$ *where,* P = Population proportion Q = 1-P, n = Sample size	$SE(p)=\sqrt{\frac{pq}{n}}$ *where,* p = Sample proportion q = 1-p, n = Sample size
(II) * $\frac{n}{N}\geq 0.05$ ** SRSWOR (Simple Random Sampling Without Replacement)	$SE(p)=\sqrt{\frac{PQ}{n}}\sqrt{\frac{N-n}{N-1}}$	$SE(p)=\sqrt{\frac{pq}{n}}\sqrt{\frac{N-n}{N-1}}$

and direction of linear correlation between the variables.

COVARIANCE

Definition - How much two random variables vary together is a measured by a Mathematical tool is called Covariance. It's similar to variance, but here variance tells us how a single variable varies, Covariance tells us how two variables vary together. Let a set of N pairs of observations $(X_1, Y_1), (X_2, Y_2), ..., (X_N, Y_N)$ relating to two variables X and Y , the covariance of X and Y, usually represented by Cov. (X,Y), is defined as -

(i) $$\text{Cov.}(X, Y) = \frac{\Sigma(X-\overline{X})(Y-\overline{Y})}{N} \text{ or } = \frac{\Sigma xy}{N};$$

where, $x = (X-\overline{X}), y = (Y-\overline{Y})$

(ii) $$\text{Cov.}(X; Y) = \frac{1}{N}\Sigma XY - \overline{X}.\overline{Y}$$

$$= E(XY) - E(X) \times E(Y)$$

PROPERTIES OF COVARIANCE

Karl Pearson's Coefficient of Correlation

1. [Cov.(X ; Y) and standard deviations are given]

$$r = \frac{Cov.(X;Y)}{\sigma_X.\sigma_Y} = \frac{\Sigma(X-\overline{X})(Y-\overline{Y})}{N\sigma_x.\sigma_y}$$ Where

σ_X = s.d. of X ; σ_Y = s.d. of Y.

N = No. of observed pairs.

2. [When observed values pairs are smaller in size]

$$r = \frac{N\Sigma XY - \Sigma X.\Sigma Y.}{\sqrt{N\Sigma X^2 - (\Sigma X)^2}.\sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}$$

3. [When observed values pair are larger in size]

$$r = \frac{N\Sigma dxdy - \Sigma dx.\Sigma dy}{\sqrt{N\Sigma dx^2 - (\Sigma dx)^2}.\sqrt{N\Sigma dy^2 - (\Sigma dy)^2}}$$

Where dx = X - A ; A = Assumed Mean of X - Series.

dy = Y - B ; B = Assumed Mean of Y - Series.

4. [When $\overline{X}$ and $\overline{Y}$ are whole numbers not in fraction]

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2.\Sigma y^2}}$$ Where $x = X - \overline{X}; y = Y - \overline{Y}$.

If the covariance between variables X

Solution:- (a)

$$= r = \frac{N\sum dxdy - (\sum dx)(\sum dy)}{\sqrt{N\sum dx^2 - (\sum dx)^2}\sqrt{N\sum dy^2 - (\sum dy)^2}}$$

$$= \frac{8\times 2116 - (47)(108)}{\sqrt{18\times 1475 - (47)^2}\sqrt{8\times 3468 - (108)^2}} = 0.95$$

STANDARD ERROR AND PROBABLE ERROR OF CORRELATION COEFFICIENT

1. Standard Error - The Standard Error of Coefficient of Correlation is calculated as follows:

$$\text{Standard Error (S.E.)} = \frac{1-r^2}{\sqrt{N}}$$

where, r = Coefficient of Correlation , N = Number of pairs of observations .

Utility - Standard error of correlation coefficient is used to ascertain the probable error of correlation coefficient .

2. Probable Error

Def. - The Probable Error of Correlation Coefficient helps in determining the accuracy and reliability of the value of the coefficient that in so far depends on the random sampling. The probable error is 0.6745 times the standard error of r.

Probable Error ($P.E_r$) = .6745. Standard Error

$$= .6745 \frac{1-r^2}{\sqrt{N}}$$

where, r = Coefficient of Correlation, N = Number of pairs of observations.

COEFFICIENT OF DETERMINATION

Meaning - The coefficient of determination is define as the ratio of the explained variance to the tot variance.

$$\text{Coefficient of Determination} \quad r^2 = \frac{\text{Explained Varianc}}{\text{Total Variance}}$$

Calculation - The coefficient of determination i calculated by squaring the Coefficient of Correlatior Thus, Coefficient of Determination = r^2

Maximum Value of r^2 - The maximum value of r^2 i unity i.e. 1

Minimum value of r^2 is ZERO = 0

Example 2. If $r = 0.7$, what is the proportion o variation in the dependent variable which is explainec by the independent variable?

Solution:-If $r = 0.7$, $r^2 = (0.7)^2 = 0.49$, it means tha 49% of the variation in the dependent variable has beer explained by the independent variable. It means 49% of the data is accounted & 51% is unaccounted.

COEFFICIENT OF NON-DETERMINATION

Coefficient of Determination $r^2 = \frac{\text{Explained Variance}}{\text{Total Variance}}$

Calculation - The coefficient of determination is calculated by squaring the Coefficient of Correlation. Thus, Coefficient of Determination = r^2

Maximum Value of r^2 - The maximum value of r^2 is unity i.e. 1

Minimum value of r^2 is ZERO = 0

Example 2. If $r = 0.7$, what is the proportion of variation in the dependent variable which is explained by the independent variable?

Solution:-If $r = 0.7$, $r^2 = (0.7)^2 = 0.49$, it means that 49% of the variation in the dependent variable has been explained by the independent variable. It means 49% of the data is accounted & 51% is unaccounted.

COEFFICIENT OF NON-DETERMINATION

Meaning - The ratio of unexplained variance to the total variance is called the coefficient of non-determination. Coefficient of Non-Determination =

$$(K2) = 1 - r^2 = \frac{\text{Unexplained Variance}}{\text{Total Variance}}$$

It expresses the % of total variance which is not explained by the given independent variable.

$1 - 0.36 = 0.64$

It means that 64% of the variation in the dependent variable has not been explained by the independent variable.

COEFFICIENT OF ALIENATION

Coefficient of Alienation is the square root of coefficient of non-determination.

$$\text{Coefficient of Alienation} = \sqrt{1-r^2}$$

It is used in determining standard error.

SPEARMAN'S RANK CORRELATION

Correlation between two variables, having qualitative characteristics (as, beauty; intelligency, honesty etc.) is obtained on the basis of rankings is called **Rank correlation.** It is a **nonparametric** measure of rank correlation. It is also applied to find the level of agreement (or disagreement) between two judges to assess qualitative characteristic. Rank correlation co-efficient is denoted by r_s or R.

$$R = 1 - \frac{6\Sigma D^2}{N^3 - N}$$

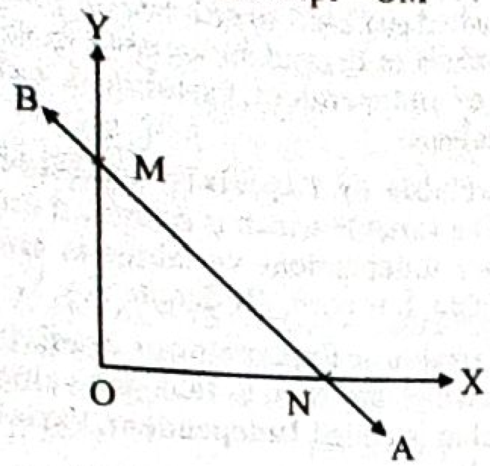
Where $D = R_1 - R_2$ = Difference of Ranks between paired items i.e. deviation in ranks

N = No. of pairs of Ranks.

TYPES OF REGRE...

Regression line of Y on X

1. It gives best estimate of Y for a given value of X
2. Regression eqn. of Y on X is
   $Y = a + bX$
   Where a = Y -intercept = OM

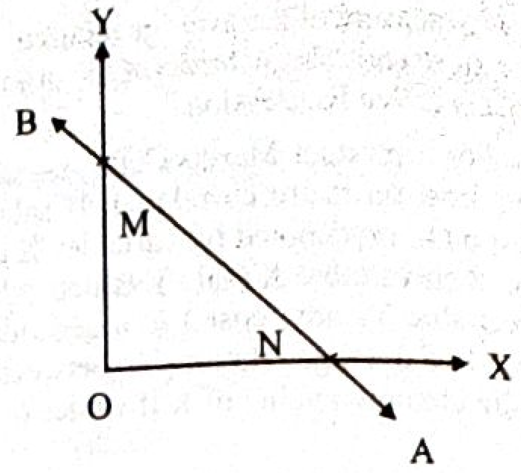


Let AB is a regression line of Y on X
where b = slope of the said line

3. **LEAST SQUARE METHOD**
   Normal Eqns. are
   $$\Sigma Y = aN + b\Sigma X$$
   $$\Sigma XY = a\Sigma X + b\Sigma X^2$$
   Solving these two normal eqns. we get the values of "a" and "b". Putting these values of a and b in the given eqns. Y = a + bX ; we get the required eqn.
4. **Second Method**
   Regression Eqn. of Y on X is
   $$Y - \overline{Y} = b_{yx}(X - \overline{X})$$
   where b_{yx} = Regression-Coefficient of Y on X.
   $\overline{X}$ = Mean of X - series
   $\overline{Y}$ = Mean of Y - series

Regression line of X on Y.

1. It gives best estimate of X for a given value of Y
2. Regression eqn. of X on Y is $X = a + bY$
   Where a = X -intercept = ON



Let AB is a regression line of X on Y
where, 1/ b = slope of the said line.

3. **LEAST SQUARE METHOD**
   Normal Eqns. are
   $$\Sigma X = aN + b\Sigma Y$$
   $$\Sigma XY = a\Sigma Y + b\Sigma Y^2$$
   Solving these two normal eqns. we get the values of "a" and "b" Putting these values of a and b in the given eqns. X = a + bY ; we get the required eqn.
4. **Second Method**
   Regression Eqn. of X on Y is
   $$X - \overline{X} = b_{xy}(Y - \overline{Y})$$
   where b_{xy} = Regression-Coefficient of X on Y.
   $\overline{X}$ = Mean of X - series
   $\overline{Y}$ = Mean of Y - series

Formula to Find Regression - Coefficient

Regression Coefficient of Y on X = b_{yx}	Regression Coefficient of X on Y = b_{xy}
1. $b_{yx} = r.\frac{\sigma_y}{\sigma_x}$ where σ_x = SD of x series σ_y = SD of y-series, r = correlation coefficient	1. $b_{xy} = r.\frac{\sigma_x}{\sigma_y}$ where σ_x = SD of x series σ_y = SD of y-series , r = correlation coefficient

Regression Coefficient of Y on X = b_{yx}	Regression Coefficient of X on Y = b_{xy}
2. $b_{yx} = \frac{cov.(X;Y)}{\sigma_x^2}$	2. $b_{xy} = \frac{cov.(X;Y)}{\sigma_y^2}$
3. $b_{yx} = \frac{N\Sigma XY - \Sigma X\Sigma Y}{N\Sigma X^2 - (\Sigma X)^2}$ when x and y are smaller in size.	3. $b_{xy} = \frac{N\Sigma XY - \Sigma X\Sigma Y}{N\Sigma Y^2 - (\Sigma Y)^2}$ when x and y are smaller in size.
4. $b_{yx} = \frac{N\Sigma dxdy - \Sigma dx.\Sigma dy}{N\Sigma dx^2 - (\Sigma dx)^2}$ where $dx = x - A_x$, $dy = y - A_y$, A_x = Assumed Mean of x. A_y = Assumed Mean of y. [when x and y are larger in size]	4. $b_{xy} = \frac{N\Sigma dxdy - \Sigma d_x.\Sigma d_y}{N\Sigma dy^2 - (\Sigma dy)^2}$ where $dx = x - A_x$, $dy = y - A_y$, A_x = Assumed Mean of x. A_y = Assumed Mean of y. [when x and y are larger in size]
5. $b_{yx} = \frac{\Sigma xy}{\Sigma x^2}$ where $x = X - \overline{X}$; $y = Y - \overline{Y}$ [when $\overline{X}$ and $\overline{Y}$ a whole No.	5. $b_{xy} = \frac{\Sigma xy}{\Sigma y^2}$ where $x = X - \overline{X}$; $y = Y - \overline{Y}$. [when $\overline{X}$ and $\overline{Y}$ a whole No.

Type I (To Find regression co-efficient)

Example 1. The regression coefficient of y on x (b_{yx}) of the following data:

{(x ; y)} = {(5, 2); (2, 3); (3 ; 1), (4, 5), (1, 4)}

(*a*) -0.20 (*b*) 0.25 (*c*) 0.32 (*d*) None

Solution:- (*a*) is correct.

X	Y	X^2	XY
5	2	25	10
2	3	4	6
3	1	9	3
4	5	16	20
1	4	1	4
$\Sigma X=15$	$\Sigma Y=15$	$\Sigma X^2=55$	$\Sigma XY=43$

$$b_{yx} = \frac{N\Sigma XY - \Sigma X\Sigma Y.}{N\Sigma X^2 - (\Sigma X)^2} = \frac{5\times 43 - 15\times 15}{5\times 55 - (15)^2} = \frac{-10}{50} = -\frac{1}{5} = -0.2$$

Example 2. The regression coefficient of x on y (b_{xy}) is if $\Sigma X = 40$; $\Sigma Y = 20$; $\Sigma XY = 800$; $\Sigma X^2 = 2500$; $\Sigma Y^2 = 1500$; N = 10.

(*a*) - 0.493 (*b*) 0.493

(*c*) -0.749 (*d*) None

Solution:- (*b*) is correct

$$b_{xy} = \frac{N\Sigma XY - \Sigma X.\Sigma Y}{N\Sigma Y^2 - (\Sigma Y)^2} = \frac{10\times 800 - 40\times 20}{10\times 1500 - (20)^2}$$

$$= \frac{7200}{14600} = 0.493$$

PROPERTIES OF REGRESSION COEFFICIENTS & LINES

1. The correlation coefficient r is the Geometric Mean of Regression Coefficients b_{xy} & b_{yx}. The product of the two regression coefficients is equal to the square of correlation coefficient. $r = \sqrt{b_{xy} . b_{yx}}$ i.e.

$$b_{yx} . b_{xy} = r^2$$

2. Signs of Regression Coefficient and Correlation Coefficient i.e. b_{yx}, b_{xy} ; & r must be same.

3. The regression lines always intersect at their means.

4. Slopes - The slopes of the regression line of Y on X and the regression line of X on Y are

respectively b_{yx} and $\frac{1}{b_{xy}}$

5. The angle between the two regression lines depends on the Correlation Coefficient (r).

(*i*) If Regression lines are perpendicular to each other, then $r = 0$.

(*ii*) If Regression lines coincide (i.e. become identical), then $r = +1$ or -1.

6. The estimated value of X or Y can be obtained through Regression equations, if $r \neq 0$.

7. The value of Regression Coefficients are always independent i.e. does not change with respect to the change of origin but changes with respect to scale i.e. dependent on scale.

8. Magnitude of Both Regression coefficients cannot be greater than one i.e. if one of the regression coefficients is greater than one (unity), the other must be less than one so that product of both regression coefficients can become less than one (unity).

9. Arithmetic mean of both regression coefficients is greater than the correlation coefficient.

i.e **A.M. > r**; $\frac{b_{xy} + b_{yx}}{2} > r$

Type III

Example 9. If $b_{xy} = 0.8$ and $b_{yx} = 0.46$ Then ...

(a) 0.26

Type-V

(To recognise regression Eqns.) Tricks

If $ax + by + c = 0$ be a regression eqn. of y on x, Then

$by = -c - ax$

$$Y = -\frac{c}{b} - \frac{a}{b}X; \therefore b_{yx} = -\frac{a}{b};$$

$$\therefore b_{yx} = \frac{-\textit{Coefficient of } X}{\textit{Coefficient of } Y}$$

Similarly:

If $ax + by + c = 0$ be a regression eqn. of X on Y.

$$\therefore aX = -c - bY;$$

$$\text{or } X = \frac{c}{a} - \frac{b}{a}Y. \therefore b_{xy} = \frac{-\textit{Coefficient of } Y}{\textit{Coefficient of } X}$$

Example 12. For variables X and Y ; the regression equations are given as $7x - 3y - 18 = 0$ and $4x - y - 11 = 0$ then the regression eqn. of y on x is

(*a*) $7x - 3y - 18 = 0$ (*b*) $4x - y - 11 = 0$

(*c*) Cannot be decided (*d*) None

Solution:-(*a*) is correct.

Let $7x - 3y - 18 = 0$ be a regr. Eqn. of y on x

$$\therefore b_{yx} = \frac{\text{Coefficient of X}}{\text{Coefficient of Y}} = -\frac{-1}{4} = \frac{1}{4} = -\frac{7}{-3} = \frac{7}{3}$$

Then $4x - y - 11 = 0$ should be the regr. eqn. of x on y.

$$\therefore b_{xy} = \frac{\textit{Coefficient of } Y}{\textit{Coefficient of } X} = -\frac{-1}{4} = \frac{1}{4}$$

$$\therefore r^2 = b_{xy}.b_{yx} = \frac{7}{3} \times \frac{1}{4} = \frac{7}{12} < 1.$$

$\therefore$ Our assumption is correct. So, $7x - 3y - 18 = 0$ is the regression Eqn. of y on x.

The Factor ... is satisfied only by Fisher's Ideal Index.	**Test is met by Simple Geometric mean of Price Relatives and the Weighted Aggregative fixed weights.**

Table showing the Methods which satisfy the test

Test	Methods which satisfy the test
1 Unit Test	All except Simple (Unweighted Aggregative) Index
2. Time Reversal Test	1. The Fisher's ideal formula, 2. Simple geometric mean of price relatives , 3. Aggregates with fixed weights, 4. The weighted geometric mean of price relatives if we use fixed weights, and 5. Marshall-Edgeworth Method. 6. Kelly's Index No.
3. Factor Reversal Test	Fisher's Index
4. Circular Test	1. Simple Geometric Mean of Price Relative. 2. Weighted Aggregative with Fixed Weights

...ases from ₹ 16,800 to ₹ 30,000. What is the gain or loss to the worker?

There is a loss of ₹ 432 *i.e.* (30,...
worker.

LIST OF FORMULAE

1. UNWEIGHTED INDEX NUMBERS

(a) Simple Aggregative

$$P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$$

(b) Simple Average of Relatives

(i) When Arithmetic Mean is used

$$p_{01} = \frac{\sum \left(\frac{p_1}{p_0} \times 100 \right)}{N}$$

(ii) **When Geometric Mean is used**

$$P_{01} = \text{Antilog}\left[\frac{\Sigma \log\left(\frac{p_1}{p_0}\times 100\right)}{N}\right]$$

WEIGHTED INDEX NUMBERS

(a) Weighted Aggregative Indices

(i) *Laspeyre's Method*

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}\times 100$$

(ii) *Paasche's Method*

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}\times 100$$

(iii) *Dosbish & Bowley's Method*

$$P_{01} = \frac{L+P}{2} = \frac{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2}\times 100$$

(iv) *Fisher's Ideal Method*

$$P_{01} = \sqrt{L\times P} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}\times\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}\times 100$$

(v) *Marshall-Edgeworth Method*

$$P_{01} = \frac{\Sigma(q_0+q_1)\times p_1}{\Sigma(q_0+q_1)\times p_0}\times 100 = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1}\times 100$$

(vi) *Kelly's Method*

$$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q}$$

(a) *If fixed quantities are given as weights*

$$P_{01} = \frac{\Sigma p_1 q}{\Sigma p_0 q}\times 100 \quad where\ q = \frac{q_0+q_1}{2}$$

(b) *If average of the quantities of two years is used as weights*

(b) Weighted Average of Relatives

$$P_{01} = \frac{\Sigma PV}{\Sigma V}$$

(i) *If Arithmetic mean is used*

where, P = Price relative = $\frac{P_1}{P_0}\times 100$, V = Value weights = $p_0 q_0$

$$P_{01} = Antilog\left[\frac{\Sigma V \log P}{\Sigma V}\right]$$

(ii) *If Geometric mean is used*

3. QUANTITY INDEX NUMBERS

(a) Weighted Aggregative Indices

(i) **Laspeyres Method** $Q_{01} = \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times 100$

(ii) ***Paasche Method*** $Q_{01} = \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1} \times 100$

(iii) ***Dosbish & Bowley's Method*** $Q_{01} = \frac{L+P}{2} = \frac{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} + \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}{2} \times 100$

(iv) ***Fisher's Ideal Method*** $Q_{01} = \sqrt{L \times P} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}} \times 100$

(v) ***Marshall-Edgeworth Method*** $Q_{01} = \frac{\Sigma(p_0 + p_1) \times q_1}{\Sigma(p_0 + p_1) \times q_0} \times 100 = \frac{\Sigma q_1 p_0 + \Sigma q_1 p_1}{\Sigma q_0 p_0 + \Sigma q_0 p_1} \times 100$

(vi) ***Kelly's Method***

(a) *If fixed price are given as weights* $Q_{01} = \frac{\Sigma q_1 p}{\Sigma q_0 p} \times 100$

(b) *If average of the price of two is used as* weights $Q_{01} = \frac{\Sigma q_1 p}{\Sigma q_0 p} \times 100$, *where* $p = \frac{p_0 + p_1}{2}$

(b) Weighted Average of Relatives

(i) **If Arithmetic mean is used** $Q_{01} = \frac{\Sigma QV}{\Sigma V}$ where, Q = Quantity relatives

$\frac{Q_1}{Q_0} \times 100$ V = value weights = $p_0 q_0$

(ii) **If Geometric mean is used** $Q_{01} = \text{Antilog}\left[\frac{\Sigma V \log Q}{\Sigma V}\right]$

4. VALUE INDEX NUMBER

$V = P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100$

5. TESTS OF ADEQUACY

(a) *Time Reversal Test* $P_{01} \times Q_{10} = 1$

(b) *Factor reversal test* $P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$

(d) *Circular test*

$$P_{01} \times P_{12} \times P_{20} = 1$$

6. Chain Base Index Number = $\frac{\text{Link relative of current year}}{100} \times \text{Chain Index of Previous Year}$

7. Conversion of Chain Base Index into Fixed Base Index

$$\text{Current year's F.B.I.} = \frac{\text{Current year's C.B.I.} \times \text{Previous year's F.B.I.}}{100}$$

8. Conversion of Fixed Base Index into Chain Base Index

$$\text{Current year's C.B.I.} = \frac{\text{Current year's F.B.I.}}{\text{Previous year's F.B.I.}} \times 100$$

9. Conversion of Link Relative to Price Relative

$$\text{Current year's Price Relative} = \frac{\text{Current year's link Relative} \times \text{Previous year's Price Relative}}{100}$$

10. Base Shifting

$$\text{New Index Number using New Base} = \frac{\text{Old Index Number using old Base}}{\text{Index Number Corresponding to New Base period}} \times 100$$

11. Splicing

(a) Index No. of old series under forward splicing

$$= \frac{100}{\text{Overlapping year's Index No. of old series}} \times \text{Given Index No. of old series}$$

(b) Index No. of new series under backward splicing

$$= \frac{\text{Overlapping Index No. of Old Series} \times \text{Given Index No. of New Series}}{100}$$

12. Deflating

(i) Real Wage = $\frac{\text{Money Wage}}{\text{Price Index}} \times 100$

(ii) Money Wage Index = $\frac{\text{Real Wage}}{\text{Money Wage of the Base Year}} \times 100$

(iii) Real Wage Index = $\frac{\text{Money Wage Index}}{\text{Price Index}} \times 100$

13. Consumer Price Index (Cost of Living Index Number)

(i) Consumer Price Index = $\frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$

(*ii*) Consumer Price Index $= \dfrac{\sum PW}{\sum W}$

(*iii*) Consumer Price Index $= \dfrac{\sum IW}{\sum W}$ Where P=I= $\dfrac{P_1}{P_0} \times 100$